Tutorial 3: Solutions

1. Given

$$u(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $T = 1/f_c$.

(a) Which of the following form sets are orthogonal functions?

i. $u(t)\cos(2\pi f_c t)$ and $u(t)\cos(2\pi f_c t)$.
ii. $u(t)\sin(2\pi f_c t)$ and $u(t)\cos(2\pi f_c t)$.
iii. $u(t)\cos(4\pi f_c t)$ and $u(t)\sin(2\pi f_c t)$.
iv. $u(t)\cos(4\pi f_c t)$ and $u(t-T)\sin(2\pi f_c t)$.
v. $u(t)\cos(\pi f_c t)$ and $u(t-T)$.

(b) Is it possible to transmit
$r(t) = a_1 u(t)\cos(2\pi f_c t) + a_2 u(t)\sin(2\pi f_c t) + a_3 u(t)\cos(4\pi f_c t) + a_4 u(t)\cos(64\pi f_c t)$ and recover a_1, a_2, a_3 and a_4 uniquely back from $r(t)$? Explain your reasoning.

(c) If so, sketch the recovery circuitry.

(d) If not, is there any combination of the above functions that may be transmitted which allow the recovery of a_k.

Solution:

(a) We test orthogonality by examining the inner product of each pair of continuous functions as

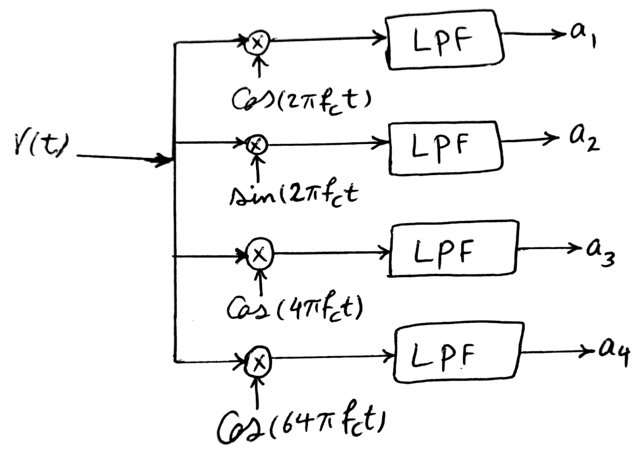
$$< f_1(t), f_2(t) >= \int_0^T f_1(t) f^*(t) dt \tag{2}$$

If the inner product of the two functions is zero then they are orthogonal.

i. $u(t)\cos(2\pi f_c t)$ and $u(t)\cos(2\pi f_c t)$ are NOT orthogonal.
ii. $u(t)\sin(2\pi f_c t)$ and $u(t)\cos(2\pi f_c t)$ are orthogonal.
iii. $u(t)\cos(4\pi f_c t)$ and $u(t)\sin(2\pi f_c t)$ are orthogonal
iv. $u(t)\cos(4\pi f_c t)$ and $u(t-T)\sin(2\pi f_c t)$ are orthogonal.
v. $u(t)\cos(\pi f_c t)$ and $u(t-T)$ are orthogonal.

(b) It is possible to uniquely recover a_1, a_2, a_3 and a_4 from $r(t)$ since each of the functions employed to modulate them are orthogonal to each other.

(c) The general recovery circuitry is for each signal is the following



2. Given $s(t) = a(t)cos(2\pi f_c t + \phi(t))$, where $a(t)$ and $\phi(t)$ are rectangular waveforms of the form given in Figures 1 and 2 below:

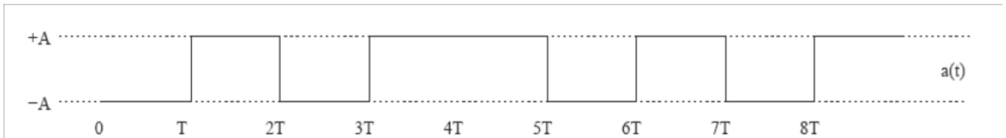


Figure 1. Waveform for *a(t)*

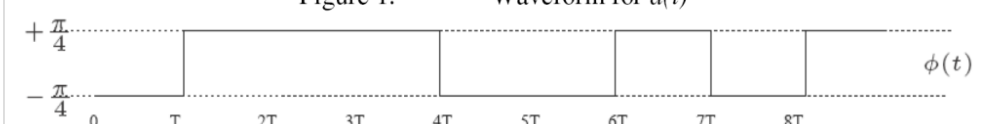


Figure 2. Waveform for *ϕ(t)*

(a) Draw the signal space diagram.

(b) What kind of constellation does this correspond to?

(c) Sketch $x(t)$ and $y(t)$ where $s(t) = x(t)\cos(2\pi f_c t) + y(t)\sin(2\pi f_c t)$

Solution:

(a) In the expression $s(t) = a(t)\cos(22\pi f_c t + \phi(t))$

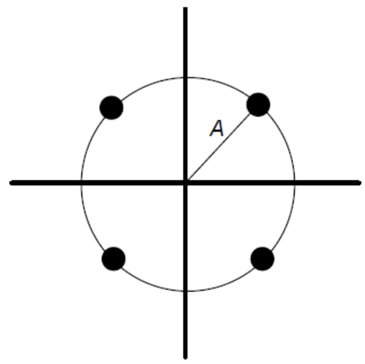
i. $a(t)$is modulating the amplitude and from the given waveform may take the values A or $+A$.

ii. $\phi(t)$ is modulating the phase and from the given waveform may take on the values $-\pi/4$ and $\pi/4$.

Therefore there are four possible combinations or symbols:

$$\begin{aligned}
s_1(t) &= A\cos(2\pi f_c t + \pi/4) \\
s_2(t) &= A\cos(2\pi f_c t - \pi/4) \\
s_3(t) &= -A\cos(2\pi f_c t + \pi/4) = A\cos(2\pi f_c t - 3\pi/4) \\
s_4(t) &= -A\cos(2\pi f_c t - \pi/4) = A\cos(2\pi f_c t + 3\pi/4)
\end{aligned}$$

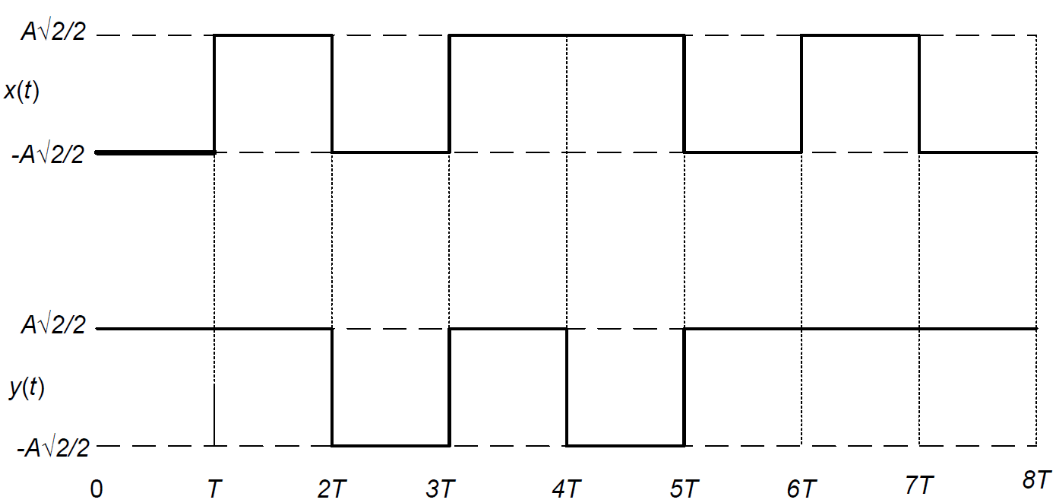
That is, all the four symbols are situated on a circle with radius A with four possible phases.



(b) This constellation corresponds to QPSK or 4-QAM.

(c) The projections of those points on the orthogonal axes correspond to

$$\begin{aligned}
x(t) &= \pm A\frac{\sqrt{2}}{2} \\
y(t) &= \pm A\frac{\sqrt{2}}{2}
\end{aligned}$$

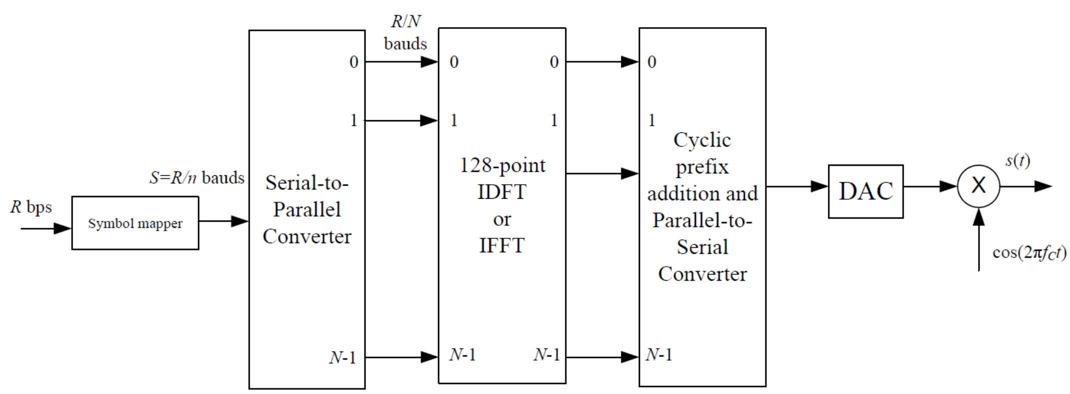


3. A given wireless communication system based on OFDM is used to transmit the data generated by a digital information source at 768 kbit/s. The OFDM system employs 128 sub-carriers where all of them are used to transmit data. All the sub-carriers are allocated around a centre carrier frequency (or radio frequency) of 10 MHz. The digital modulation scheme used in the communication system is QPSK and the pulse shape filter is considered to be an ideal brick-wall filter. The wireless transmission channel is considered dispersive with a delay spread of 8.4 ms.

   (a) Sketch the OFDM transmitter structure for this system considering that the IDFT algorithm is used for the multicarrier modulation.
   (b) Calculate the bandwidth of each sub-carrier.
   (c) Calculate approximately the total bandwidth occupied by the transmitted band-pass signal indicating the highest and lower frequency.
   (d) Calculate an appropriate size for the cyclic prefix in order to avoid ISI in the reception.

Solution:

(a) The structure of the transmitter employing the IDFT algorithm is the following.



(b) Since ideally a brick-wall filter is used for pulse shaping, the bandwidth of each sub-carrier would be equal to the symbol rate in each sub-carrier. The original bit rate is equal to 768 kbit/s. The digital modulation employs 2 bits per symbol, then the symbol rate at the exit of the digital modulator or symbol mapper is 384 ksymbol/s or kbauds. This symbol rate is divided between the 128 data sub-carriers, therefore the symbol rate in each subcarrier is 3 ksymbol/s or kbauds. Consequently, the bandwidth of each subcarrier is 3 kHz.

(c) The total bandwidth occupied by the transmitted signal is approximately $3 \times 128 = 384$ kHz. Considering that the signal is centred at 10 MHz, the maximum frequency would be 10.192 MHz and the minimum one 9.808 MHz.

(d) The period of each sub-carrier symbol is the inverse of the sub-carrier symbol rate. In this case, 333.33 µs. If we divide the delay spread of the channel by the symbol period we obtain that the delay spread is equal to 25.2 symbols. Therefore, a cyclic prefix of at least 26 symbols would be necessary to avoid ISI in the reception.

4. Given the function

$$f(t) = \begin{cases} 1 & 0 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where $T = 1/f_c$.

(a) Sketch the function $f(t)$.

(b) Which of the following is/are orthogonal to $f(t)$

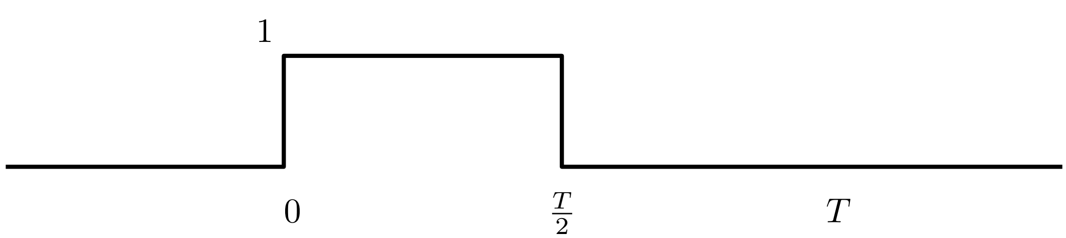
i. $g(t) = \sin(2\pi f_c t)$

ii. $h(t) = \begin{cases} 1 & T/2 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}$

iii. $l(t) = \begin{cases} 1 & T \leq t \leq \dfrac{3T}{2} \\ 0 & \text{otherwise} \end{cases}$

(c) Is it possible to transmit $r(t) = a_1 g(t) + a_2 h(t) + a_3 l(t)$ and recover a_1, a_2, and a_3 uniquely back from $r(t)$?. If not, is there any combination of the above functions that might be transmitted which will allow the recovery of a_k.

Solution:



(a)

(b) i. $g(t)$ is NOT orthogonal to $f(t)$

ii. $h(t)$ is orthogonal to $f(t)$.

iii. $l(t)$ is orthogonal to $f(t)$.

iv. It is **impossible** to recover a_1, a_2, and a_3 uniquely back from $r(t)$ since the basic functions are not orthogonal. We can use $f(t)$, $h(t)$, and $l(t)$ to transmit the 3 symbols, i.e., $r(t) = a_1 f(t) + a_2 h(t) + a_3 l(t)$.