Tutorial 6: Solutions

1. Digitized voice data is to be transmitted. The transmitted data stream is subject to an AWGN channel with noise density $N_0 = 10^{-7}$ W/Hz. The voice stream is at a rate of 3000 bits/s and the total available transmit power is 1 mW. A capacity achieving code is available at an overhead of 10% to the number of bits transmitted.

   Choose a digital modulation scheme suitable for this system. Motivate your choice, including (but not necessarily limited to) the following

   (a) A sketch of the transmitter associated with the chosen modulation scheme.

   (b) The bandwidth occupied, assuming an ideal brick-wall LPF is used as the pulse shaping filter.

   (c) The capacity of the channel given the bandwidth used

   (d) The number of data bits transmitted per symbol.

   ## Solution:

   There is no unique solution to this question. A sample solution is as follows:

   - The receive signal power is equal to the transmit signal power in this case since the channel is an AWGN channel, i.e. Pr = 1 mW.
   - The data rate required for transmission is 3000 bits/s but since an overhead is required for the transmission of data at capacity, the modulation method must be chosen accordingly.
   - The total number of bits which need to be modulated is thus 3300 bits/second.
   - The bandwidth used is related to the number of bits per symbol used, i.e. 3300 bits/second requires $3300/B$ symbols/second where B is the number of bits/symbol. Thus, using QAM/QPSK the bandwidth used is $W = 3300/B$ Hz (assuming ideal LPF). A large transmitter constellation leads to a low bandwidth and a small constellation leads to a high bandwidth. We choose the largest constellation for which capacity may be reached. Given the equation

$$C = W \log_2 \left(1 + \frac{P}{W \times N_0}\right) \tag{1}$$

     the requirement is that

$$\frac{3300}{B} \log_2 \left(1 + \frac{B \times 10^{-3}}{3300 \times 10^{-7}}\right) \geq 3000 \tag{2}$$

by trial and error, the largest value of B for which this holds is $B = 4$. Choosing $B = 4$ allows the use of a 16-QAM constellation which uses a bandwidth of 825 Hz. The capacity at this bandwidth is 3064 bits/second. The number of data bits transmitted per symbol is

$$\text{data bits per symbol} = \frac{\text{data bits per second}}{\text{symbols per second}} = \frac{3000}{825} = 3.6364 \text{ databits/symbol} \tag{3}$$

2. ("Wireless Communications", Andrea Goldsmith, Cambridge University Press, 14.1) Consider an FDMA system for multimedia data users. The modulation format requires 10 MHz of spectrum for each user, and guard bands of 1 MHz are required on each side of the allocated spectrum in order to minimize the out-of-band interference. What total bandwidth is required to support 100 simultaneous users in this system?

   Solution:

   The problem does not specify that any guard band should be allocated between the channels of the users, only at the beginning and the end of the frequency band to avoid out-of-band interference. Therefore, the total bandwidth required by the channels of 100 simultaneous users is 100 x 10 MHz = 1,000 MHz or 1 GHz. In addition, 1 MHz is required for each guard band, so in total the required bandwidth for the system is 1,002 MHz or 1.002 GHz.

3. ("Wireless Communications", Andrea Goldsmith, Cambridge University Press, 14.2) GSM systems have 25 MHz of bandwidth allocated to their uplink and downlink, divided into 125 TDMA channels with 8 user timeslots per channel. A GSM frame consists of the 8 timeslots, preceded by a set of preamble bits and followed by a set of trail bits. Each timeslot consists of 3 start bits at the beginning, followed by a burst of 58 data bits, then 26 equalizer training bits, another burst of 58 data bits, 3 stop bits, and a guard time corresponding to 8.25 bits. The transmission rate is 270.833 kbps.

   (a) Calculate the bandwidth of a GSM channel.

   (b) Sketch the structure of a GSM frame and a timeslot within the frame.

   (c) Find the fraction of data bits within a timeslot as well as the information data rate for each user. neglecting the duration of the preamble and trail bits.

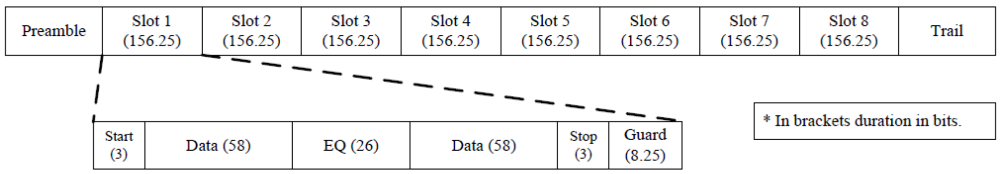
   (d) Find the duration of a frame and the latency between timeslots assigned to a given user in a frame, neglecting the duration of the preamble and trail bits.

   (e) What is the maximum delay spread in the channel such that the guard band and stop bits prevent overlap between time slots?

   Solution:

(a) The problem specifies that there are 125 GSM channels into a 25 MHz bandwidth, dividing the second by the first it is calculated that the bandwidth of a GSM channel is 200 kHz.

(b) The structure of a GSM frame and a timeslot within the frame:



(c) Within a time slot, out of 156.25 bits, 116 of them are data bits, this represents 74.24% of the timeslot bits. Considering a transmission rate of 270.833 kbps and ignoring the preamble and trail bits, effectively the information data rate would be the 74.24% of 270.833 kbps, which is 201.066 kbps. This is divided within 8 users, so the data rate per user would be 25.133 kbps.

(d) The transmission rate is 270.833 kbps, its inverse is the bit period which is equal to 3692.3 ns. The total duration of a GSM frame, ignoring the preamble and trail bits, is $156.25 \times 8 = 1250$ bits, which in time represents 461.54 μs. The latency between the timeslots assigned to a same users is the duration of the other 7 timeslots assigned to other users, this is $7 \times 156.25 \times 3.6923$ μs $= 4.0385$ ms.

(e) The maximum delay spread will be equal to the duration of the stop bits plus the guard band bits, this is $(3 + 8.25) \times 369.23$ ns $= 4.154$ μs.