EE4C5 Digital Signal Processing

Lecture 7 – Filters, an introduction

This lecture

- Partly based on Chapter 7 of O&S
- Chapter 14 of The Scientist and Engineer's Guide to Digital Signal Processing
- Chapter 8 Porat Book
- Some images from O&S book
- Others from Ian Bruce lectures in McMaster University

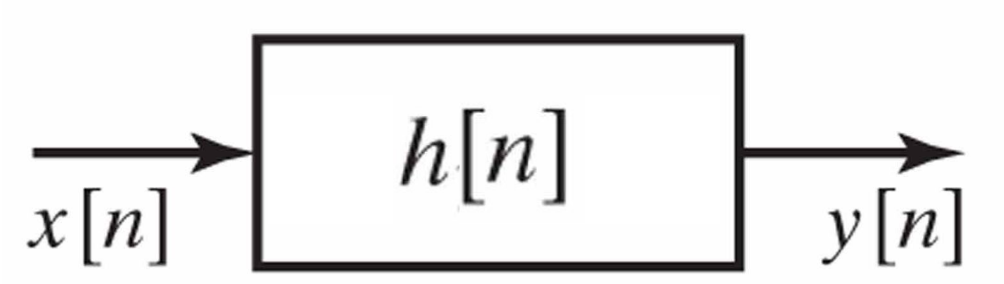
What is a digital filter?

- A digital system that modifies certain frequencies in its input relative to others
- Applications (Porat book)
  - Noise suppression (e.g. EEG, audio)
  - Signal enhancement (e.g. image enhancements, sound effects)
  - Bandwidth limiting (e.g. avoid aliasing)
  - Remove or attenuate specific frequencies (e.g. DC component)
  - Specific operations (e.g. differentiation, integration, Hilbert transform)

Note on analog filtering

- In continuous-time domain
- With operational amplifiers, resistors, and capacitors
- Noise sensitivity, non-linearities, size
- Applications where analog still dominates e.g.
  - High frequencies
  - Audio processing
  - Physical sensing

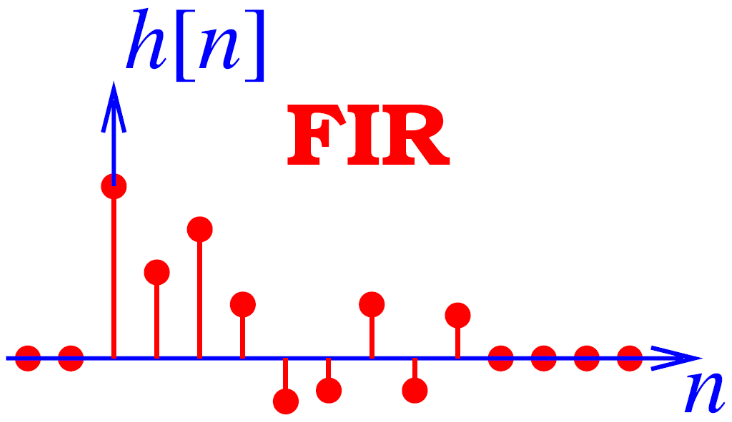
LTI system

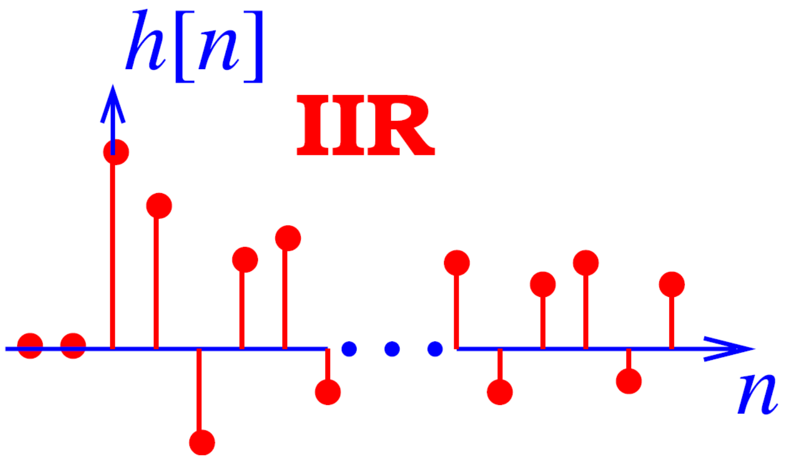


- Causal and LTI
- $y[n] = x[n] * h[n]$
- $y[n] = \sum_{k=0}^{\infty} h[k]\, \mathrm{x}[n-k]$
- $\mathrm{Y}\left(e^{j\omega}\right) = X\left(e^{j\omega}\right) H\left(e^{j\omega}\right)$
- Set of coefficients $\{h[0], h[1], h[2], \ldots\}$ are the filter coefficients

FIR and IIR filters

- If $h[n]$ is an infinite duration sequence, the corresponding filter is called an infinite impulse response(IIR) filter.
- If $h[n]$ is a finite duration sequence, the corresponding filter is called a finite impulse response(FIR) filter.





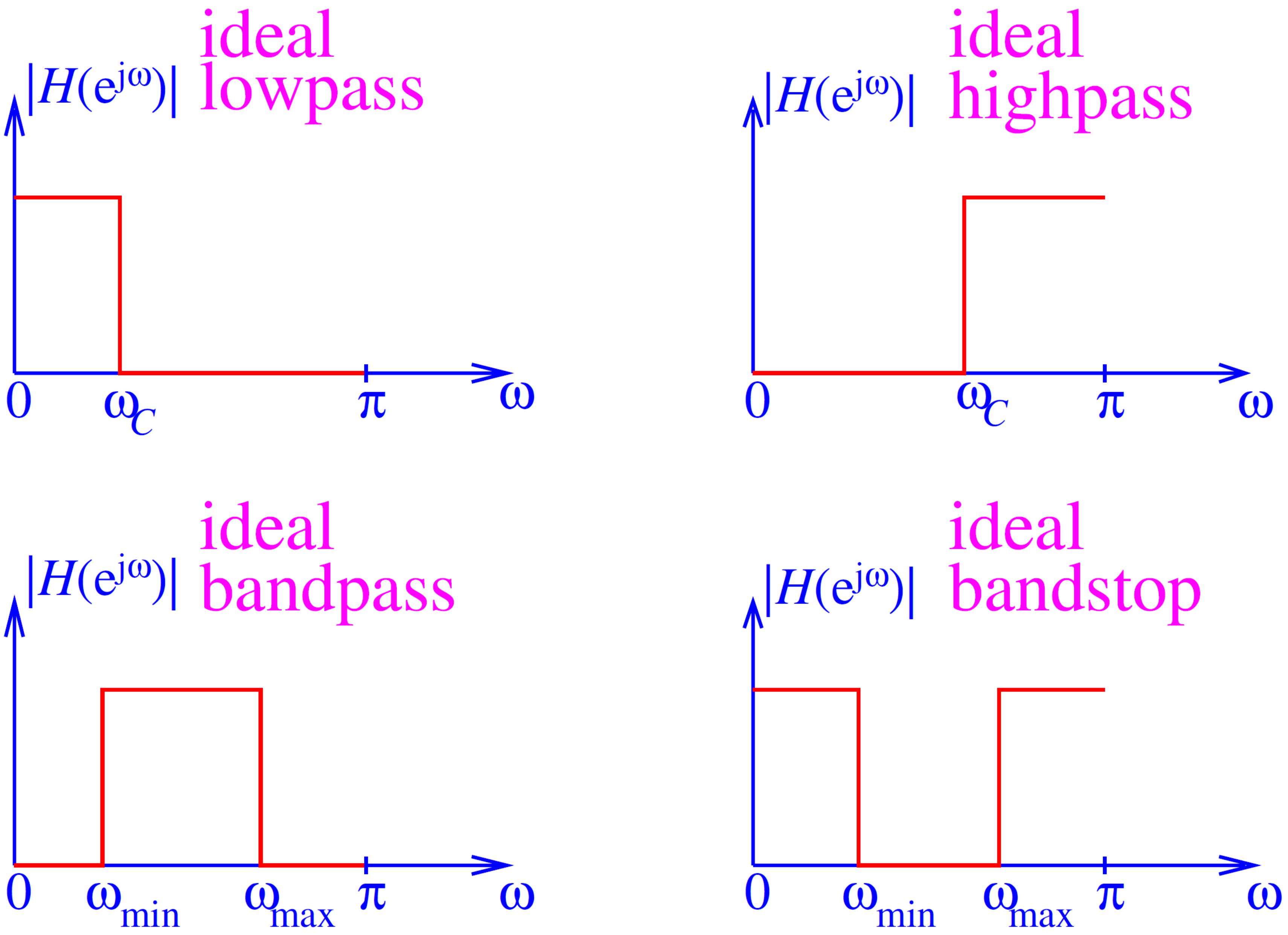
Transfer Function

- Difference equation
  - $\sum_{k=0}^{N} a[k]y[n-k] = \sum_{k=0}^{M} b[k]x[n-k]$
- Gives:
  - $H(z) = \frac{X(z)}{Y(z)} = \frac{\sum_{k=0}^{M} b[k]z^{-k}}{\sum_{k=0}^{N} a[k]z^{-k}}$
- $N = 0$ FIR
- $N > 0$ IIR

Types of filters

- lowpass (LP) filter
  - pass low frequencies from zero to a certain cut-off frequency ωC and to block higher frequencies
- –highpass(HP) filter
  - pass high frequencies from a certain cut-off frequency ωC to π and to block lower frequencies
- –bandpass(BP) filters
  - pass a certain frequency range $[\omega_{min}, \omega_{max}]$ which does not include zero, and to block other frequencies
- –bandstop(BS) filters
  - block a certain frequency range $[\omega_{min}, \omega_{max}]$ which does not include zero, and to pass other frequencies

Filters



ideal lowpass
|H(e^jω)|
0
ω_C
π
ω
ideal highpass
|H(e^jω)|
0
ω_C
π
ω
ideal bandpass
|H(e^jω)|
0
ω_min
ω_max
π
ω
ideal bandstop
|H(e^jω)|
0
ω_min
ω_max
π
ω

Designing Filters

- Specification of desired properties of the system
- The approximation of the specifications using a causal discrete-time system
- Verification of performance
- Implementation/realisation of the system

Real implementations

- Lowpass filter example
- Examine frequency response
- Passband not unity
- Note transition region
- Stopband not absolute

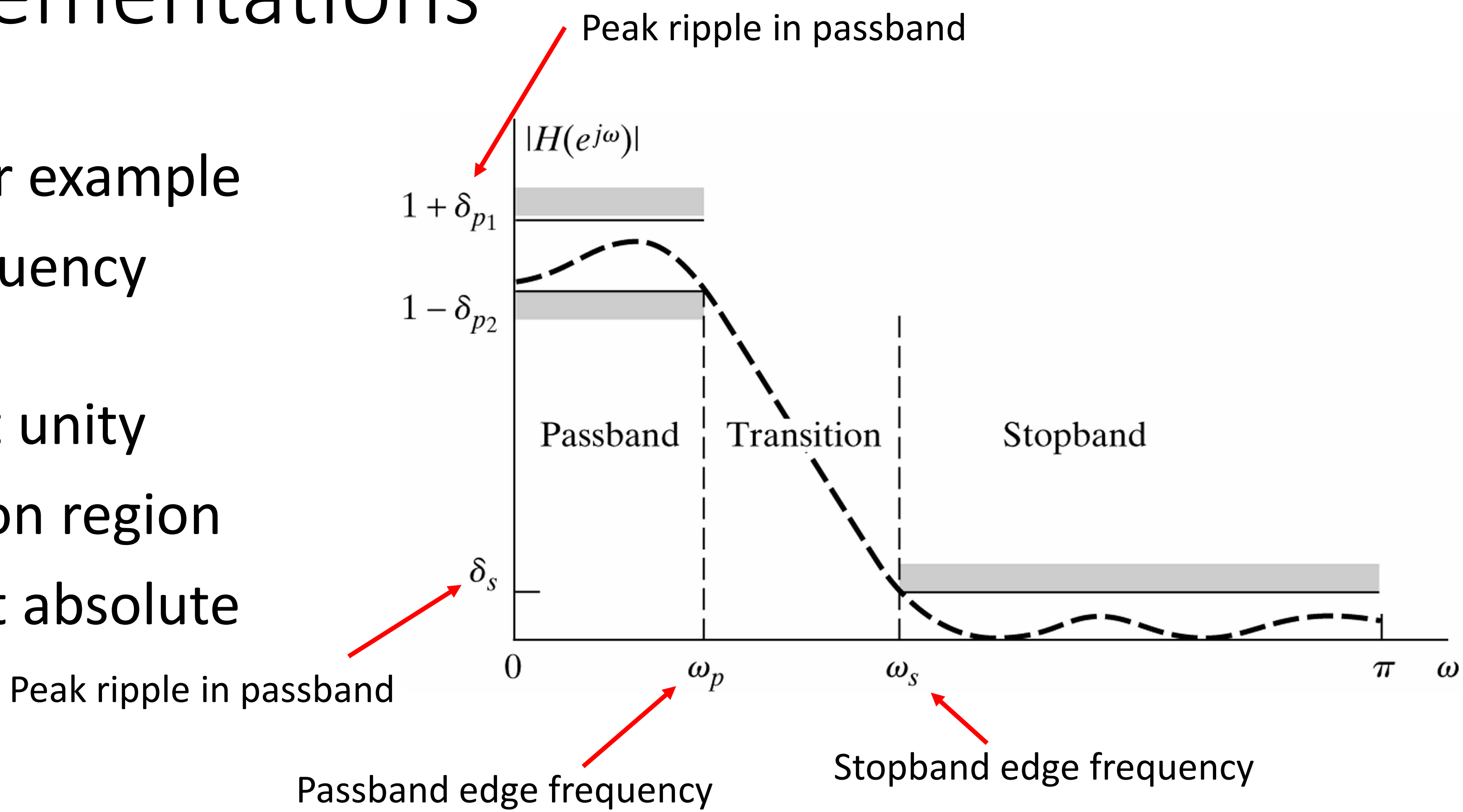


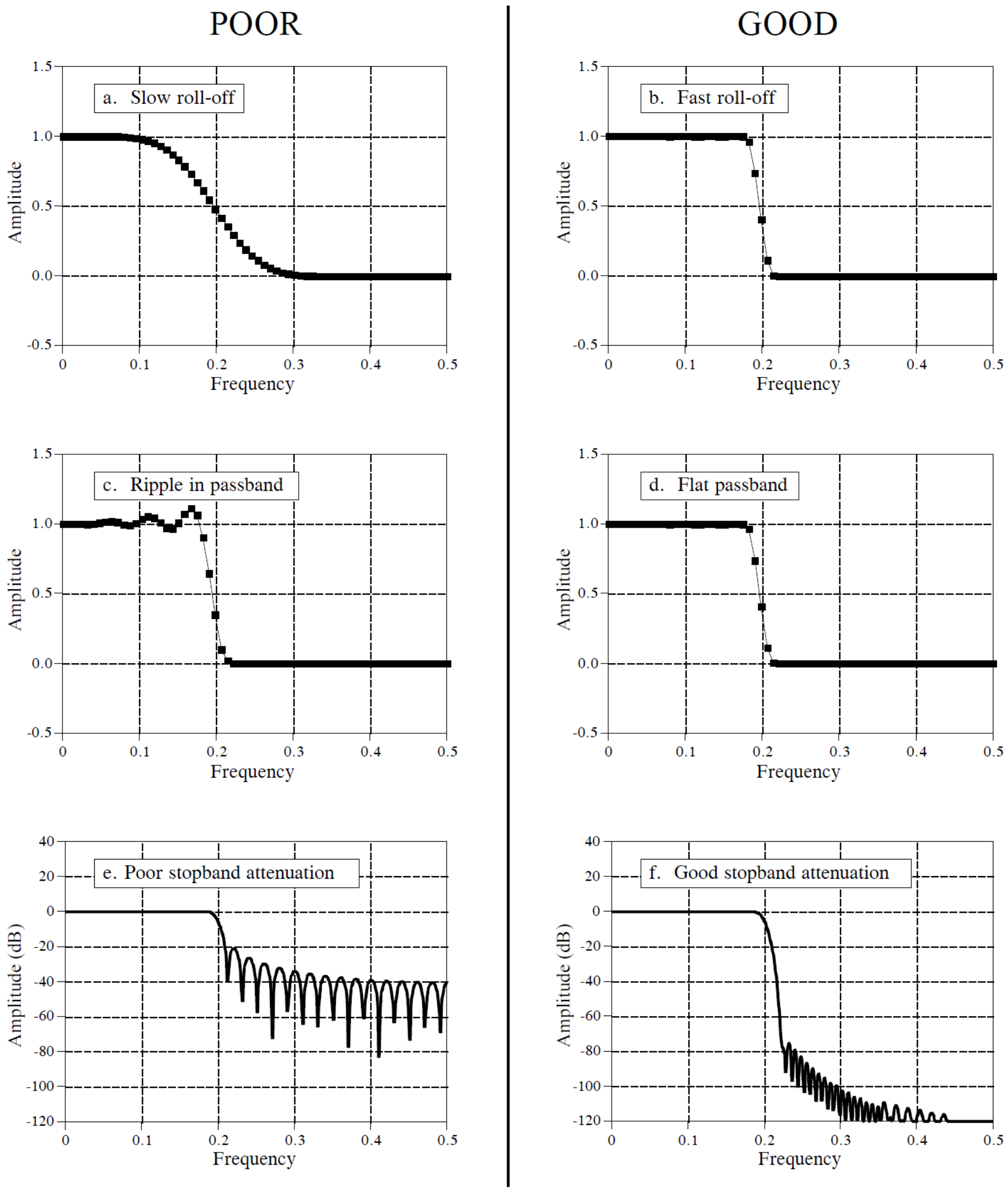
Figure 7.1 Lowpass filter tolerance scheme.

Digital Filter Specifications

- Specifications are often given in terms of loss function in dB $\mathrm{H}(\omega) = -20\log_{10}\left|H(e^{j\omega})\right|$
- Peak passband ripple
- $\alpha_p = -20\log_{10}(1 - \delta_p)$
- Minimum stopband attenuation
- $\alpha_s = -20\log_{10}(\delta_s)$

Evaluating Design

- Frequency domain performance
- Roll-off sharpness (a and b)
- Passband ripple (c and d)
- Stopband attenuation (e and f)
- Common to plot amplitude on dB scale



Required Reading & other material

- Oppenheim & Schafer, Chapter 7, Intro and section 1.
- Chapter 14, The Scientist and Engineer's Guide to Digital Signal Processing
- Chapter 8, Porat, A course in digital signal processing, initial sections only