Tutorial 6 EE4C5

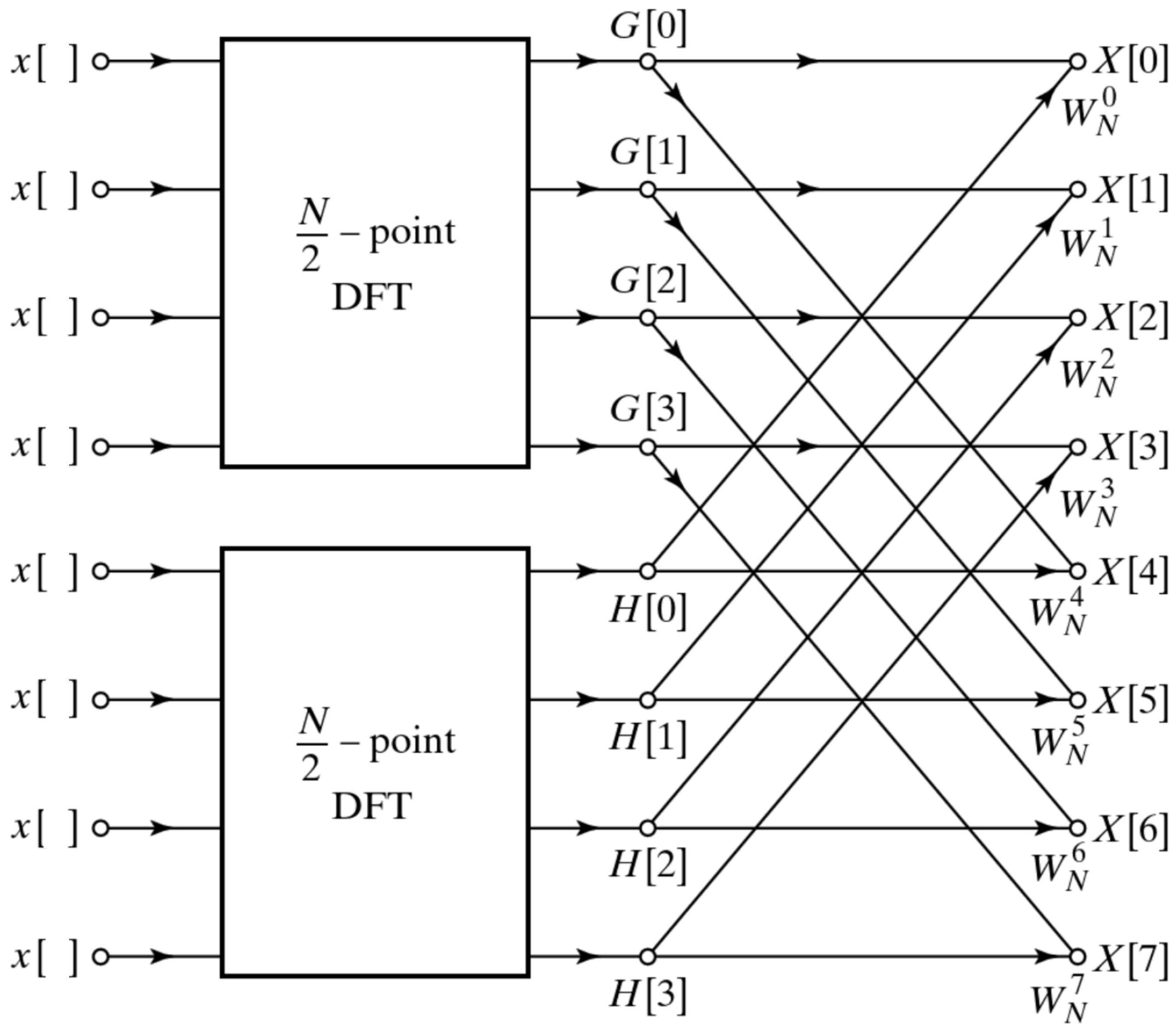
Q1

In lectures we derived the relevant expressions that allowed us to construct the general flow graph of the decimation-in-time decomposition of an N-point DFT computation into two ($N/2$)-point DFT computations.

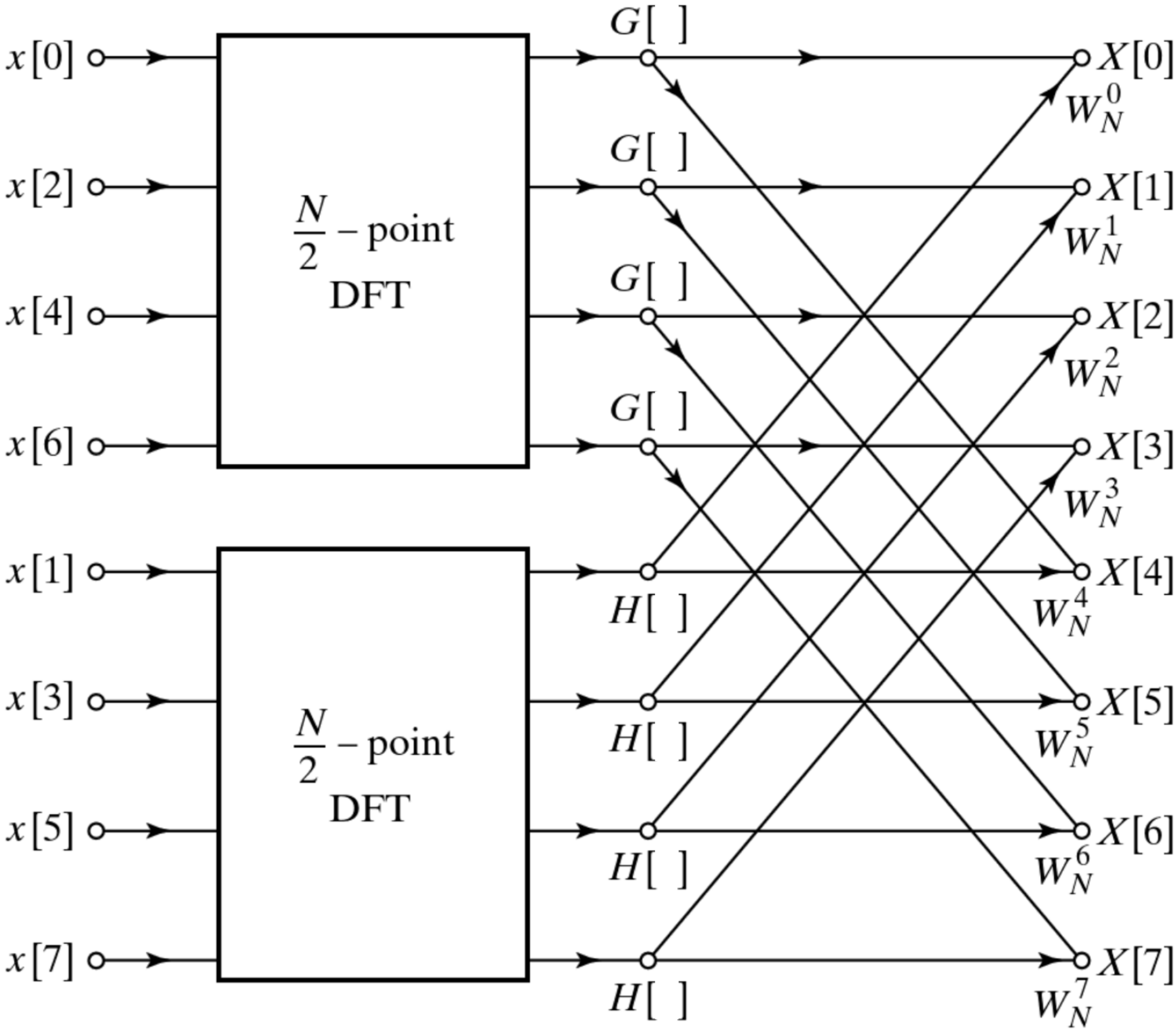
To be useful, it's best if you study that material and then attempt these questions which are based on the flow graph of the decimation-in-time decomposition of an *8*-point DFT computation into two 4-point DFT computations. Then attempt these questions WITHOUT the notes in front of you! (Otherwise, it's a bit pointless)

In each case, you need to complete the missing elements in the diagram.

a.

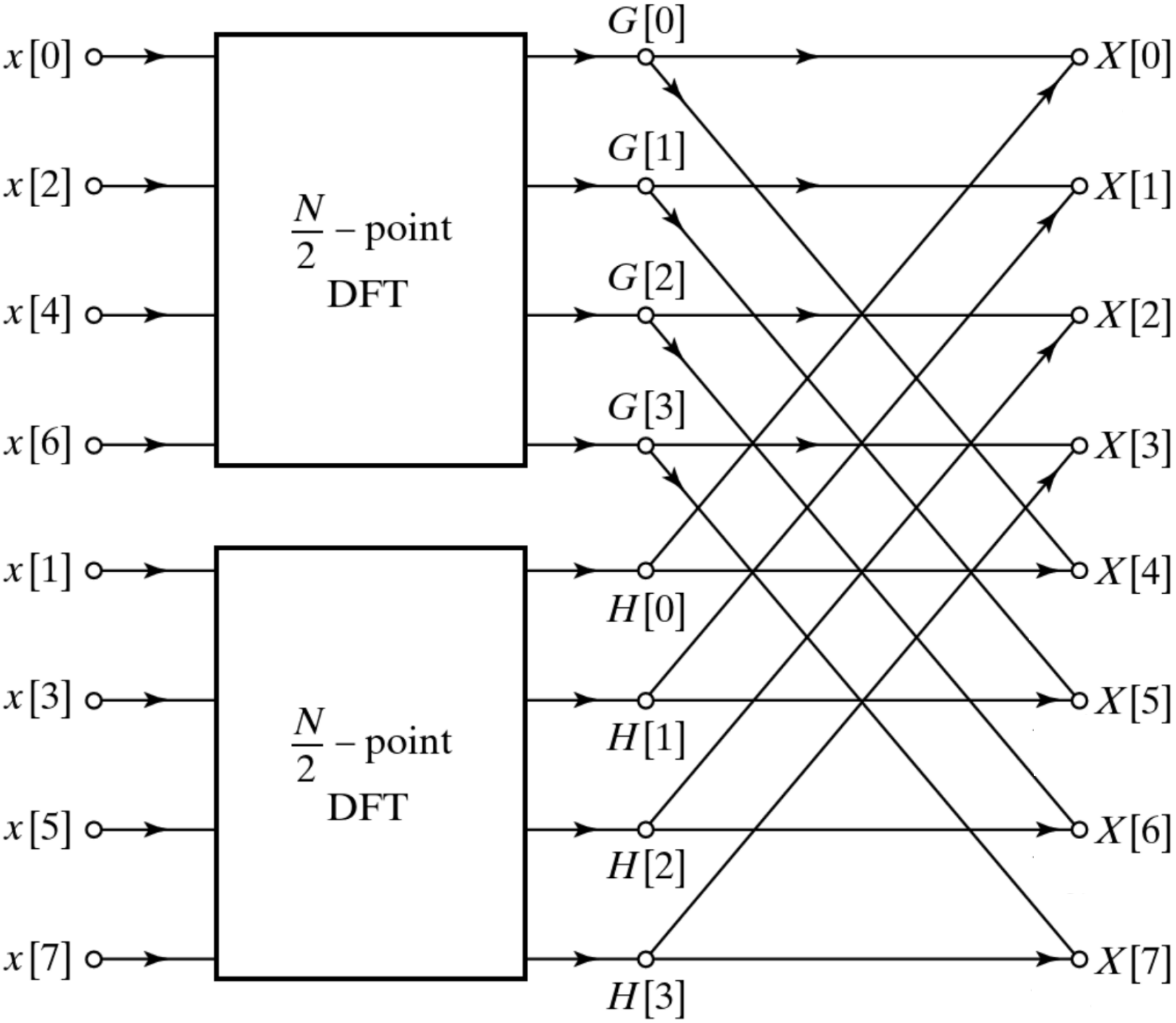


b.



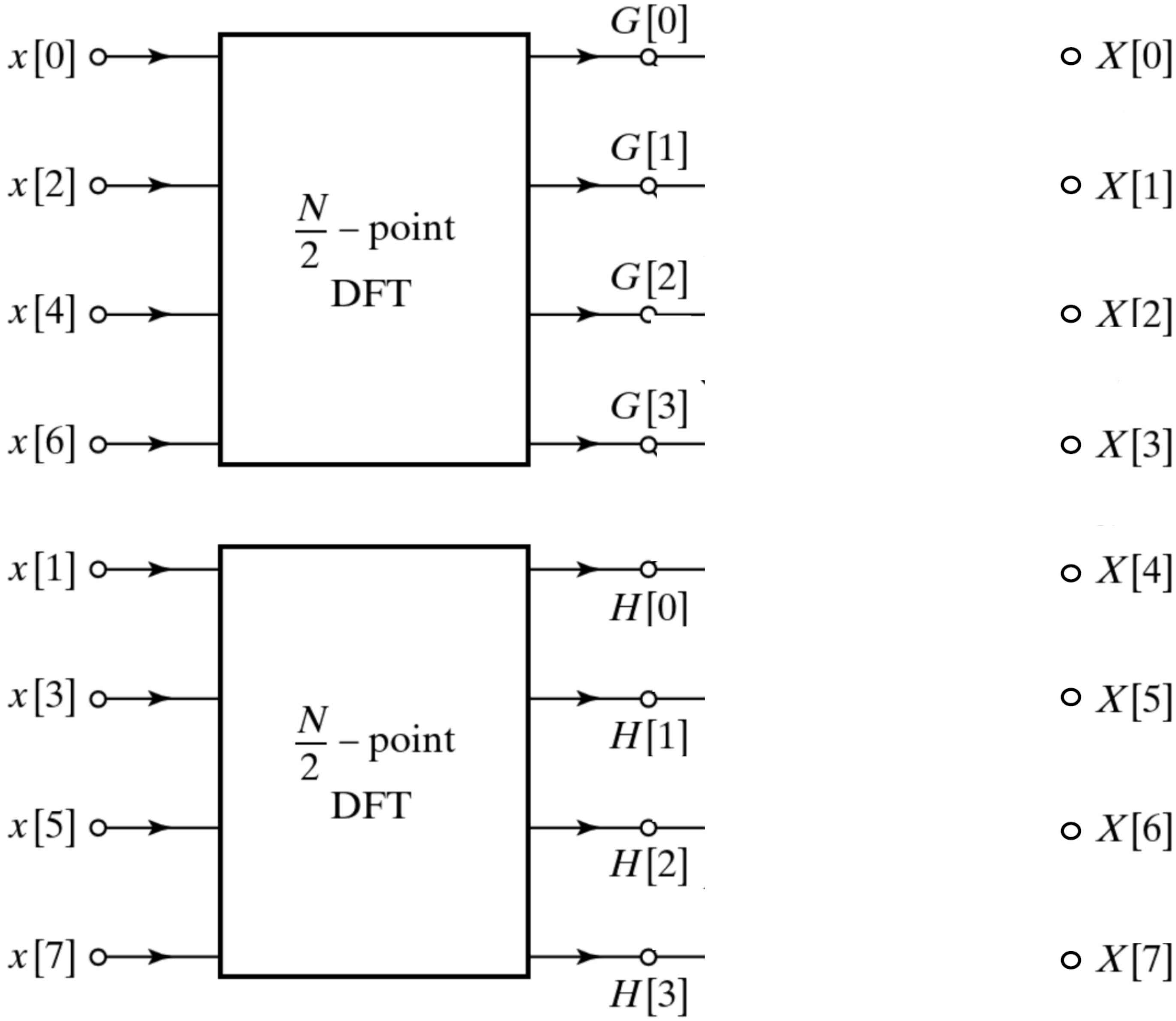
$x[0]$
$x[2]$
$x[4]$
$x[6]$
$x[1]$
$x[3]$
$x[5]$
$x[7]$
$\frac{N}{2}$ – point DFT
$\frac{N}{2}$ – point DFT
$G[\ ]$
$G[\ ]$
$G[\ ]$
$G[\ ]$
$H[\ ]$
$H[\ ]$
$H[\ ]$
$H[\ ]$
$X[0]$
W_N^0
$X[1]$
W_N^1
$X[2]$
W_N^2
$X[3]$
W_N^3
$X[4]$
W_N^4
$X[5]$
W_N^5
$X[6]$
W_N^6
$X[7]$
W_N^7

c.



x[0]
x[2]
x[4]
x[6]
x[1]
x[3]
x[5]
x[7]
N/2 – point DFT
N/2 – point DFT
G[0]
G[1]
G[2]
G[3]
H[0]
H[1]
H[2]
H[3]
X[0]
X[1]
X[2]
X[3]
X[4]
X[5]
X[6]
X[7]

d.



$x[0]$
$x[2]$
$x[4]$
$x[6]$
$\frac{N}{2}$ – point DFT
$G[0]$
$G[1]$
$G[2]$
$G[3]$
$x[1]$
$x[3]$
$x[5]$
$x[7]$
$\frac{N}{2}$ – point DFT
$H[0]$
$H[1]$
$H[2]$
$H[3]$
$X[0]$
$X[1]$
$X[2]$
$X[3]$
$X[4]$
$X[5]$
$X[6]$
$X[7]$

Q2

Note this Q is Problem 6 in Chapter 9 of the O&S text

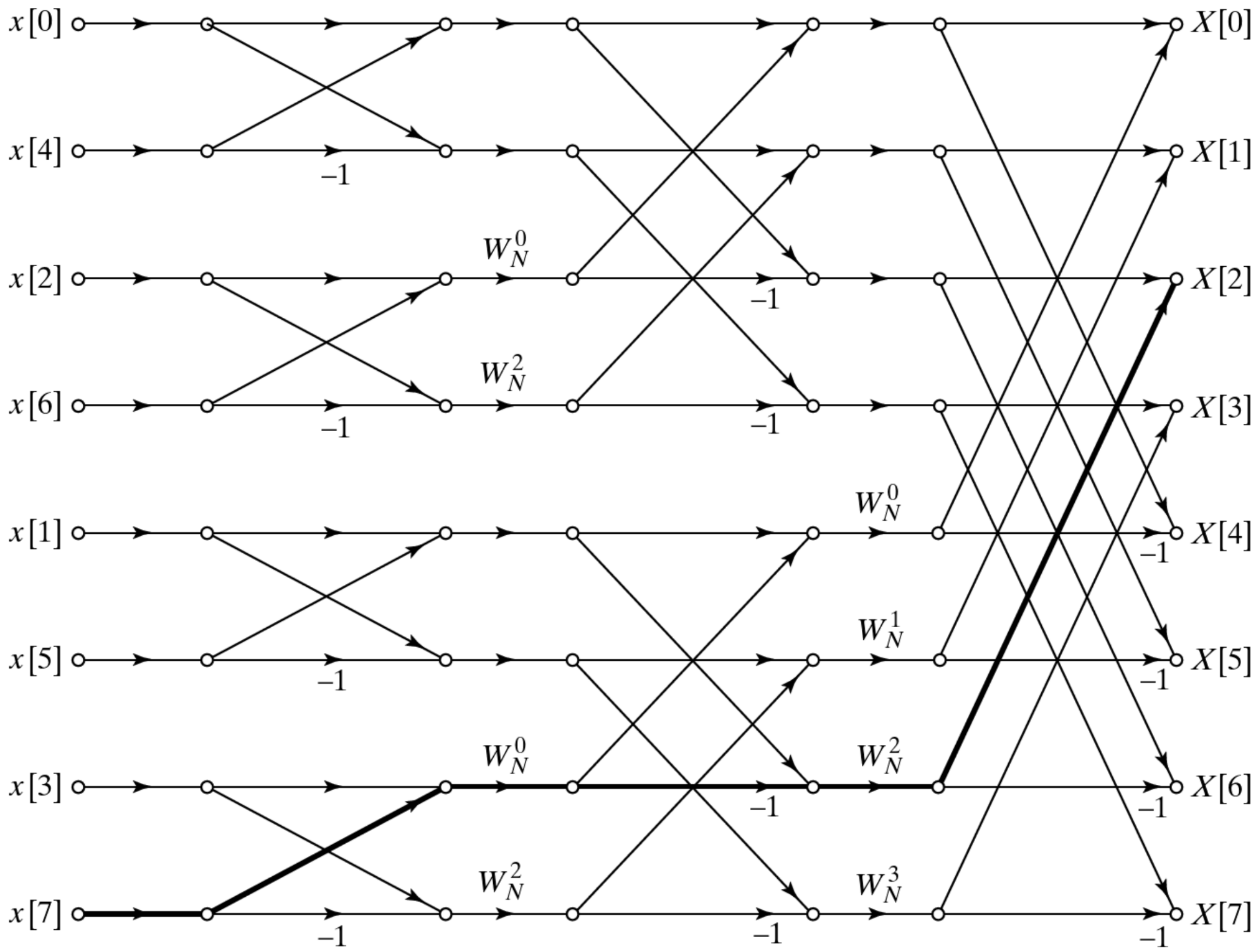


Figure P6 shows the graph representation of a decimation-in-time FFT algorithm for $N = 8$. The heavy line shows a path from sample $x[7]$ to DFT sample $X[2]$.

(a) What is the "gain" along the path that is emphasized in Figure P6?

(b) How many other paths in the flow graph begin at $x[7]$ and end at $X[2]$? Is this true in general? That is, how many paths are there between each input sample and each output sample?

(c) Now consider the DFT sample $X[2]$. By tracing paths in the flow graph of Figure P6, show that each input sample contributes the proper amount to the output DFT sample; i.e., verify that

$$X[2] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)2n}.$$

Q3

Note this Q is Problem 16 from Chapter 9 of the O&S text

The butterfly in Figure P16 was taken from a decimation-in-time FFT with $N = 16$. Assume that the four stages of the signal flow graph are indexed by $m = 1, \ldots, 4$. What are the possible values of r for each of the four stages?

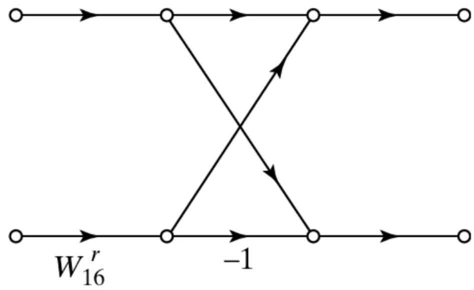


Figure P16

Q4.

In lectures, we showed that for a sequence $x[n]$ which is zero for $n < 0$ and for $n > N - 1$ and with $N = 2^{v}$, where v is a positive integer that if we et $g[n] = x[2n]$ and $\mathrm{h}[n] = x[2n + 1]$ then the N-point discrete Fourier transform (DFT) of the sequence $x[n]$ can be obtained by appropriately combining the $N/2$-point DFTs of the sequences $g[n]$ and $\mathrm{h}[n]$.

Extend this, using a similar method, to show that the sequences $g[n]$ and $\mathrm{h}[n]$ can be again decimated.

Using your findings, draw the flow graph for the case N=8, showing how the 4-point (N/2) DFTs can be performed using instead the 2-point (N/4) point DFTs.

Q5

You need to analyse the frequency content of a signal with 1024 samples. How many complex multiplications are required for a straightforward DFT calculation? Compare this with the number of complex multiplications needed for a 1024-point FFT.