Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

FACULTY OF SCIENCE, TECHNOLOGY, ENGINEERING AND MATHEMATICS

SCHOOL OF ENGINEERING

Electronic and Electrical Engineering

Engineering **Semester 1, 2023**
4th year, MAI, MSc

Digital Signal Processing

XX/YY/2023 **ANY VENUE** **XX:00–YY:00**

Prof. Naomi Harte

Instructions to Candidates:

Answer question one (1) and any other three (3) questions. All questions carry equal marks.

Materials permitted for this examination:

Mathematical Tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination.

Please indicate the make and model of your calculator on each answer book used.

Additional information:

Useful formulae are included at the end of the exam paper.

THIS QUESTION IS COMPULSORY. ALL STUDENTS MUST ANSWER THIS QUESTION

Q.1

(a) You need to make a recording of birdsong. The particular species sings at frequencies up to 10 kHz. Is a sampling rate of 16 kHz suitable for the task? Explain why.

[5 marks]

(b) What is a random process? Give one example of a real-world signal that is modelled as a random process.

[5 marks]

(c) Give an example of a signal processing application where analog filtering remains the dominant approach. Briefly explain why digital filtering is not used in the application you have given.

[5 marks]

(d) Give three differences between IIR and FIR filters which explain why FIR filters can be preferred in many applications.

[5 marks]

(e) Outline how much more efficient the FFT is than a direct implementation of the DFT, with reference to the number of operations.

[5 marks]

Q.2

(a) Explain what is meant by distortionless transmission in a system and how this relates to phase.

[6 marks]

(b) If an FIR filter has linear phase, how will this manifest in its impulse response?

[6 marks]

(c) Choose one type of window function (other than a rectangular window) that you have studied or used during this module. Explain how the properties of this window make it a better option than a rectangular window when using the windowing method of FIR filter design. Support your answer with the aid of suitably labelled time domain and frequency domain sketches.

[13 marks]

Q.3

(a) Let $H_c(s)$ denote the transfer function of a continuous-time filter. The transfer function of a discrete-time filter, $H(z)$ is obtained by applying the bilinear transformation to $H_c(s)$ as:

$$H(z) = H_c(s)\Big|_{s = (1 - z^{-1})/(1 + z^{-1})}$$

Show that the frequency response of the discrete-time and continuous-time filters are related by:

$$H\left(e^{j\omega}\right) = H_c(j\Omega)\Big|_{\Omega = \tan\left(\omega/2\right)}$$

[6 marks]

(b) As an intern in a bioacoustics lab, you are tasked with designing a discrete-time filter to analyse noise in audio recordings of bat echolocation signals. The filter should attenuate frequencies above 20 kHz in order preserve the low-frequency ambient noise of interest, while removing the critical frequency range of 20 kHz to 200 kHz, where bat calls are prominent. The sampling rate used is 500 kHz.

You decide to design a lowpass Butterworth filter to preserve the frequency range of interest. Give a set of filter specifications that you deem suitable for this task, with important frequencies given in radians. Make a labelled sketch of the expected magnitude frequency response, highlighting important characteristics.

[6 marks]

(c) You need to design the filter in (b) by applying the bilinear Transformation to the transfer function of a suitably designed continuous-time filter. Determine the required filter order to meet your specifications.

[8 marks]

(d) How would you expect a reduction in the transition band to change the filter order?

[5 marks]

Q.4

(a) Find the circular convolution of the following sequences using the concentric circle method. Explain your method clearly.

x[n]={2,1,3,-2} and y[n]={1,2,-3,4}

[8 marks]

(b) The sequence $x[n]$ is zero for $n < 0$ and for $n > N - 1$. Assume that $N = 2^M$, where M is a positive integer. Let $g[n] = x[2n]$ and $h[n] = x[2n + 1]$.

Prove that the N-point discrete Fourier transform (DFT) of the sequence $x[n]$ can be obtained by appropriately combining the $N/2$-point DFTs of the sequences $g[n]$ and $h[n]$.

[8 marks]

(c) Draw the complete signal flow graph for an 8-point decimation-in-time fast Fourier transform (FFT) algorithm.

[9 marks]

Q.5

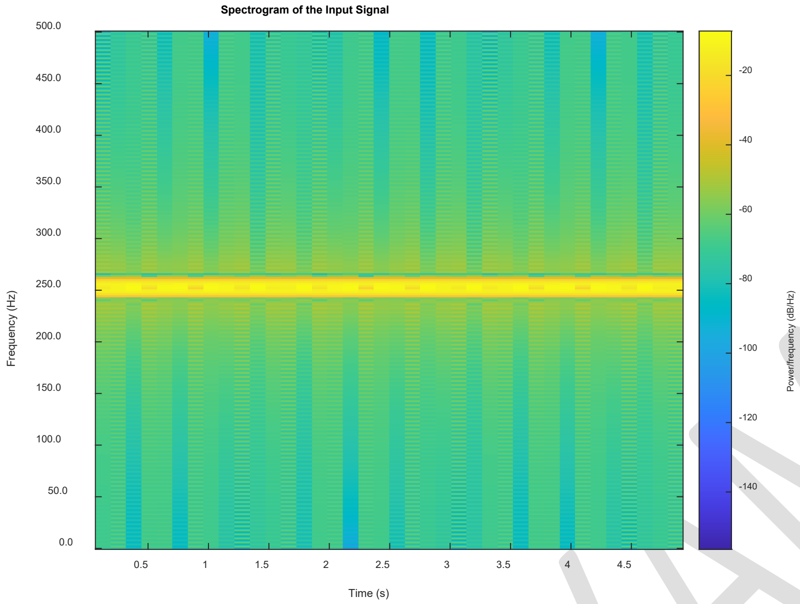

Figure Q5

You are given a.wav file and are told that it contains a signal which is a mixture of two sinusoids, both of fixed frequency. The sampling rate is 1000 Hz. You are given MATLAB code to plot a spectrogram of the signal, and the figure generated by the code is shown above in Figure Q5. In the call to the spectrogram, the code uses the settings shown below in terms of analysis window, window size and the size of the FFT.

```
window_size = 256;  % Size of the analysis window
overlap = 128;      % Overlap between consecutive windows
nfft = 512;         % Number of FFT points

% Plot the spectrogram
spectrogram(signal, window_size, overlap, nfft, Fs, 'yaxis');
```

Code snippet Q5

(a) Explain the difference between a Short-time Fourier transform and a regular Discrete Fourier Transform.

[5 marks]

(b) With reference to the spectrogram shown in Figure Q5, outline what you can say about the properties of the signal from this spectrogram.

[8 marks]

(c) What parameter(s) above would you change to improve the informativeness of this spectrogram, such that you could resolve the two frequencies. Explain your reasoning. Sketch what you would expect to see in your new spectrogram.

[12 marks]

Useful Formulae

$H(s) = \frac{1}{Q(s)}$	
Filter Order	**Polynomial Q(s)**
1	$s + 1$
2	$s^2 + \sqrt{2}s + 1$
3	$s^3 + 2s^2 + 2s + 1$
4	$s^4 + 2.6131s^3 + 3.4142s^2 + 2.6131s + 1$

Table 1 Summary of Butterworth Filter Transfer Functions

Squared magnitude response of Butterworth filter	$\lvert H_c(j\Omega)\rvert^2 = \frac{1}{1 + \left(\Omega/\Omega_c\right)^{2N}}$
DTFT $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{j\omega n}\,d\omega,$	$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
DFT $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j2\pi\frac{kn}{N}}$	$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j2\pi\frac{kn}{N}}$
Euler's Formula	$\cos x = \mathrm{Re}\left(e^{ix}\right) = \frac{e^{ix} + e^{-ix}}{2},$ $\sin x = \mathrm{Im}\left(e^{ix}\right) = \frac{e^{ix} - e^{-ix}}{2i}.$