Tutorial EE4C5

Q1

Design a low-pass discrete-time Butterworth filter that has a 3dB cutoff frequency of 1.5 kHz and attenuation of at least 40 dB at 3.0 kHz.

Use the impulse invariance method, noting any assumptions made in your method..

[Note that this is posed differently from the example done in class. Here you are given the 3dB frequency i.e. Ω_c. Additionally, the frequencies are given in Hz.]

Q2

(a) Let $H_c(s)$ denote the transfer function of a continuous-time filter. The transfer function of a discrete-time filter, $H(z)$, is obtained by applying the bilinear transformation to $H_c(s)$:

$$H(z) = H_c(s)\Big|_{s = (1-z^{-1})/(1+z^{-1})}$$

Show that the frequency responses of the discrete-time and continuous-time filters are related by

$$H\left(e^{j\omega}\right) = H_c(j\Omega)\Big|_{\Omega = \tan(\omega/2)}$$

(b) A discrete-time low-pass filter with frequency response, $H\left(e^{j\omega}\right)$, is to be designed to meet the following specifications:

$$\begin{aligned} 0.89 \le \left|H\left(e^{j\omega\Omega}\right)\right| \le 1, &\qquad |\omega| \le 0.2\pi\ 0.6\pi \\ \left|H\left(e^{j\omega}\right)\right| \le 0.18, &\qquad \le |\omega| \le \pi \end{aligned}$$

The filter is to be designed by applying the bilinear transformation to the transfer function of an appropriate Butterworth continuous-time filter. Verify that a second order filter is sufficient to meet the specifications. Determine the transfer function, $H(z)$, of the discrete-time filter.
Note that the transfer function of a second order Butterworth low-pass prototype filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Q3

(a) Show that the bilinear transformation, $s = (1 - z^{-1})/(1 + z^{-1})$, has the following properties:

(i) The imaginary axis in the s-plane maps to the unit circle in the z-plane.

(ii) The left half of the s-plane maps to the inside of the unit circle in the z-plane.

Q4

A high-pass discrete-time filter with frequency response, $H\left(e^{j\omega}\right)$, is to be designed to meet the following specifications:

$$\begin{array}{ll} 0.89 \le \left|H\left(e^{j\omega}\right)\right| \le 1, & 0.6\pi \le |\omega| \le \pi \\ \left|H\left(e^{j\omega}\right)\right| \le 0.18, & |\omega| \le 0.2\pi \end{array}$$

The filter is to be designed by applying the bilinear transformation to the transfer function of an appropriate Butterworth continuous-time filter.
Verify that a second order filter is sufficient to meet the specifications.
Determine the transfer function, $H(z)$, of the discrete-time filter.
Note that the transfer function of a second order Butterworth low-pass prototype filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$