Trinity College Dublin
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EEU44C18 / EEP55C28
Digital Wireless Communications

Lecture 2: Wireless Communication System Modelling

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Review of Lecture 1

- In general a digital communications system consists of the **transmitter**, the **channel**, and the **receiver**
- The **transmitter**: 1) Coding for error detection/correction; 2) Pulse-shaping; 3) Digital-to-analogue conversion; 4) Analogue Modulation.
- The **channel**: distorts the signal applied at its input.
  - The channel passes certain frequencies more easily than others.
  - Can be represented as a filter.
  - Wireless channels are more challenging.
- The **receiver**: 1) Band pass filtering; 2) Demodulation; 3) Analogue-to-digital conversion; 4) Equalization; 5) Decoding for robust data recovery.

Lecture 2 - Outline

- Explain the steps involved in the development of the complex baseband equivalent model of a modulated signal.
  - Analogue modulation (single tone).
  - Complex baseband representation.
- Analyse undesirable channel effects such as inter-symbol interference and additive white Gaussian noise on the transmitted signal

Analogue Modulation – Single Tone

Single tone signal:

- Time domain

$$s(t) = 2R\cos(2\pi f_s t + \theta) \tag{1}$$

- Frequency domain

$$S(f) = R\left[\delta(f - f_s)e^{-j\theta} + \delta(f + f_s)e^{j\theta}\right] \tag{2}$$

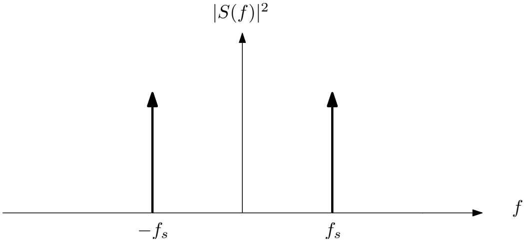


Figure: Information signal spectrum

Analogue Modulation – Single Tone...

Single tone amplitude modulation

- Time domain

$$
\begin{aligned}
s_{\mathrm{m}}(t) &= s(t)\cos(2\pi f_{\mathrm{c}} t) && (3)\\
&= R\left[\cos(2\pi(f_{\mathrm{c}} - f_{\mathrm{s}})t - \theta) + \cos(2\pi(f_{\mathrm{c}} + f_{\mathrm{s}})t + \theta)\right] && (4)
\end{aligned}
$$

- Circuitry representation

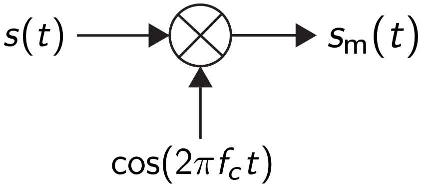


Figure: Shifting a signal in frequency using an oscillator

Analogue Modulation – Single Tone...

Single tone amplitude modulation...

- Frequency domain:

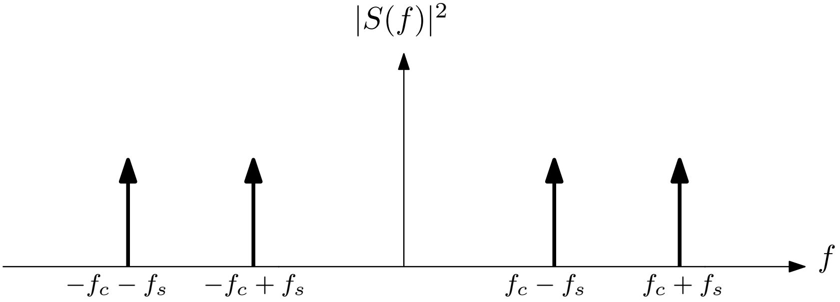


Figure: Spectrum of modulated signal

Analogue Modulation – Single Tone...

- How to recover the original signal? down shifting and low pass filtering
  - Mathematical representation

$$
\begin{aligned}
r(t) &= s_m(t)\cos(2\pi f_c t) && (5)\\
&= s(t)\cos^2(2\pi f_c t) && (6)\\
&= \frac{1}{2}\left[s(t) + s(t)\cos(4\pi f_c t)\right] && (7)
\end{aligned}
$$

  - If we apply a proper LPF to $r(t)$, the high frequency component $s(t)\cos(4\pi f_c t)$ is eliminated and then $s(t)$ is recovered.

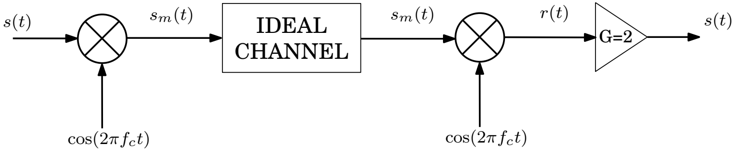


Figure: Block diagram of Single-tone Modulation and Demodulation

Analogue Modulation – Single Tone...

- Important observations:
  - when at baseband, our signal contains the **same information** on either side of the zero frequency
  - After modulation, we are essentially transmitting the *same thing* twice, once on either side of f_c $\rightarrow$ spectrum inefficient.
  - One solution is to band-pass filter after modulation, in order to remove all components either above or below f_c. However it is very difficult to perform such accurate band-pass filtering without damaging the signal, and demodulation becomes complicated.
- Can we do better?

Analogue Modulation – Single Tone...

- What happens if we demodulated $s_{\mathrm{m}}(t)$ with $\sin(2\pi f_{\mathrm{c}} t)$
$$2s_{\mathrm{m}}(t)\sin(2\pi f_{\mathrm{c}} t) = 2s(t)\cos(2\pi f_{\mathrm{c}} t)\sin(2\pi f_{\mathrm{c}} t) = s(t)\sin(4\pi f_{\mathrm{c}} t) \tag{8}$$
which will be removed by LPF.
- The same happens when we attempt to demodulate $s(t)\sin(2\pi f_{\mathrm{c}} t)$ with $\cos(2\pi f_{\mathrm{c}} t)$.
- The above observations mean that we can transmit two signals in the same bandwidth at the same frequency, i.e.,
$$s(t) = \underbrace{s_{\mathrm{I}}(t)\cos(2\pi f_{\mathrm{c}} t)}_{\text{in-phase component}} + \underbrace{s_{\mathrm{Q}}(t)\sin(2\pi f_{\mathrm{c}} t)}_{\text{quadrature commponent}} \tag{9}$$
- In fact $\cos(2\pi f_{\mathrm{c}} t)$ and $\sin(2\pi f_{\mathrm{c}} t)$ are orthogonal functions. In general we can use orthogonal functions to modulate signals.
- Two signals $x_1(t)$ and $x_2(t)$ are orthogonal if their inner product is equal to zero
$$\langle x_1(t), x_2(t)\rangle = \int_{-\infty}^{\infty} x_1(t)x_2^*(t) = 0 \tag{10}$$

Analogue Modulation – Single Tone...

Orthogonal modulation

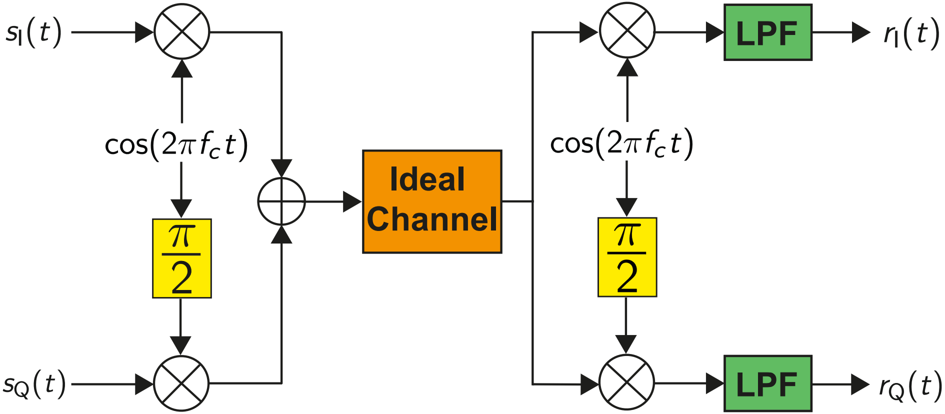


Figure: Orthogonal Modulation

Complex Baseband Representation

- The orthogonal modulation model of the figure in previous slide can be simplified using the complex baseband model.
- Why we need equivalent baseband representation
  - Almost every communication system operates by modulating an information bearing signal onto a modulating signal.
  - Different transmission types differ only by their centre frequency f_c and bandwidth.
  - In some cases, we want to simulate the system by numerical methods $\rightarrow$ too many samples will be generated.
- A method of characterizing a communication channel that is independent of f_c is required.
- This method is known as the complex baseband model and is used to analyse all of the transmission types. This model ignores the analogue modulation and demodulation part of the communication system since this only determines the carrier centre frequency.

Complex Baseband Representation

Frequency Domain

- As discussed earlier, a communication channel is limited in bandwidth to an interval of frequencies centred about the carrier.

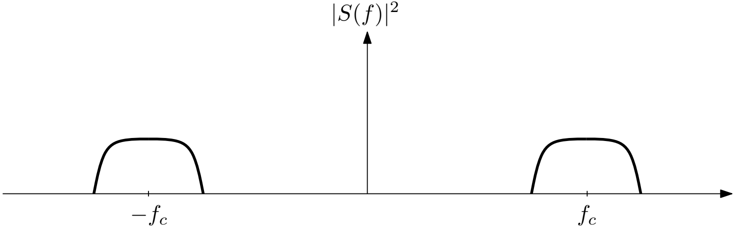


Figure: Band-pass channel representation

- For mathematical convenience, it is desirable to reduce all band-pass channels to equivalent low-pass channels $\rightarrow$ the analysis becomes independent of carrier frequency and channel bandwidth

Complex Baseband Representation

Frequency Domain

Given a real-valued signal $s(t)$, with frequency content concentrated about some centre frequency f_c (i.e. a modulated signal), consider the two following steps

1. Construct a signal which contains only the positive frequencies in $s(t)$. Explicitly, let $S(f)$ is the FT of $s(t)$.

$$S_+(f) = 2u(f)S(f) \tag{11}$$

   where $u(f)$ is the unit step function, i.e.,

$$u(f) = \begin{cases} 0 & f < 0 \\ 1/2 & f = 0 \\ 1 & f > 0 \end{cases} \tag{12}$$

2. Translate this signal to centre around the zero-frequency

$$S_l(f) = S_+(f + f_c) \tag{13}$$

Complex Baseband Representation
Time Domain

- Let us analyse steps 1 and 2 in the previous slide in time domain.
- Let $s_+(t)$ is time domain representation of $S_+(f)$, i.e.,

$$
\begin{aligned}
s_+(t) &= \mathrm{invFT}(S_+(f)) = \mathrm{invFT}\left(2u(f)S(f)\right) && (14)\\
&= \underbrace{\mathrm{invFT}\left(2u(f)\right)}_{\delta(t)+j\frac{1}{\pi t}} * \underbrace{\mathrm{invFT}\left(S(f)\right)}_{s(t)} && (15)\\
&= s(t) + j\left(\frac{1}{\pi t} * s(t)\right) && (16)\\
&= s(t) + j\hat{s}(t) && (17)
\end{aligned}
$$

- Note that $\hat{s}(t)$ may be viewed as the original signal $s(t)$ passed through a filter with impulse response $\frac{1}{\pi t}$; such a filter is called a Hilbert transformer.

Complex Baseband Representation

Time Domain

- Low pass equivalent signal

$$s_l(t) = s_+(t)e^{-j2\pi f_c t} \tag{18}$$
$$= \left(s(t) + j\hat{s}(t)\right) e^{-j2\pi f_c t} \tag{19}$$

thus,

$$s_l(t)e^{j2\pi f_c t} = s(t) + j\hat{s}(t) \tag{20}$$

Complex Baseband Representation
Time Domain

- In general the low-pass equivalent signal, $s_l(t)$, is a complex-valued function, called the complex envelope of $s(t)$:

$$s_{\mathrm{l}}(t) = \underbrace{x(t)}_{\text{in-phase}} + j \underbrace{y(t)}_{\text{quadrature}} \tag{21}$$

- The original signal may be recovered as

$$\begin{aligned} s(t) &= \Re\left(s_{\mathrm{l}}(t)e^{j2\pi f_{\mathrm{c}} t}\right) && (22) \\ &= x(t)\cos(2\pi f_{\mathrm{c}} t) - y(t)\sin(2\pi f_{\mathrm{c}} t) && (23) \\ &= a(t)\cos(2\pi f_{\mathrm{c}} t + \phi(t)) && (24) \end{aligned}$$

Modulated Signal Based on Baseband Components

- A real signal $s(t)$ occupying bandwidth about a centre frequency f_c may be expressed as
  - Two time signals individually modulated onto orthogonal carriers

$$s(t) = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \tag{25}$$

    - This corresponds to amplitude-modulation onto quadrature components, and is referred to as analogue quadrature-amplitude modulation.
  - A single carrier whose amplitude and phase are individually modulated

$$s(t) = a(t)\cos(2\pi f_c t + \phi(t)) \tag{26}$$

    - This corresponds to both amplitude and phase modulation.

Complex Baseband Representation of Channel

- We have introduced a way to represent a bandpass signal as a complex baseband signal.
- In fact the modulated signals are transmitted over a bandpass channel. Thus to model a complex system in baseband, we need to develop an equivalent baseband representation for channels.
- By extension, the filter or channel impulse response can also be expressed in terms of its complex lowpass equivalent model as

$$h(t) \quad = \quad \Re\left(h_l(t)e^{j2\pi f_c t}\right) \tag{27}$$

- The low-pass equivalent output is the result of filtering the low-pass equivalent signal with the low-pass equivalent filter $\rightarrow$ we can work exclusively with low-pass equivalent models without loss of generality.

Analogue Modulation – Complex Baseband Model

- Analogue modulation may be removed from digital system model and be replaced by the use of the complex baseband (low-pass equivalent) model of data to be transmitted.

$$r_l(t) = s_l(t) * h_l(t) \tag{28}$$

- The complex baseband model

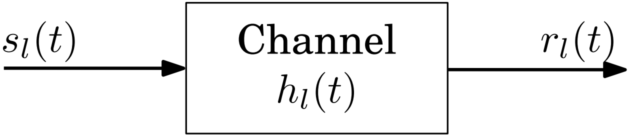


Figure: Low pass equivalent model

where $s_l(t) = s_I(t) + js_Q(t) \equiv x(t) + jy(t)$

- The system isn't quite digital yet, is continuous.

Analogue modulation of digital signals

- So far, the modulation has been replaced with complex baseband notation.
- But the modulation described until now is analogue modulation, thus any digital signals, need to be converted to analogue at the transmitter using a DAC, and back to digital at the receiver.

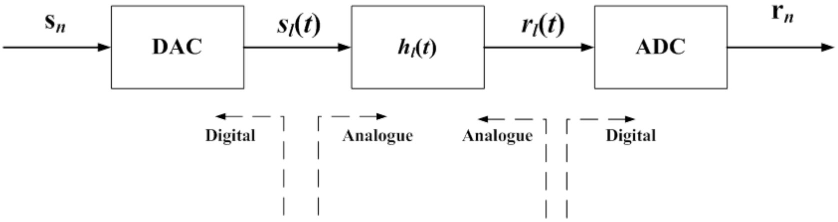


Figure: The analogue-digital division

- Both $s_\mathrm{l}(t)$ and $h_\mathrm{l}(t)$ are bandlimited and thus are perfectly represented as a sampled signal.

Analogue modulation of digital signals...

- Despite that we now assume that all signals are in low-pass equivalent form, there still is the digital/analogue division
- We remove both the DAC and ADC from our digital model by replacing $s(t)$ and $r(t)$ with sampled versions:

$$s[n] = s_l(nT) = \sum_{-\infty}^{\infty} s_l(t)\delta(t - nT) \tag{29}$$

$$r[n] = r_l(nT) = \sum_{-\infty}^{\infty} r_l(t)\delta(t - nT) \tag{30}$$

- The continuous time channel $h_l(t)$ can also be replaced by a sampled low pass model $h[n]$.

Analogue modulation of digital signals...

- Therefore, the entire digital communication system can be represented with discrete baseband signals omitting both the ADC/DAC operations and analogue modulation/demodulation actions

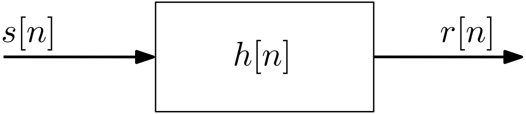


Figure: Discrete system model

- An entire digital communications system may now be represented mathematically.

Channel Modelling

The wireless channel is characterized by its hostility. When a signal is transmitted, it is distorted by several non-desired effects which change its properties. Some of these effects are:

- **Inter-symbol interference (ISI)**: The wireless channel does NOT offer us an infinite bandwidth. When a symbol sequence is transmitted by that channel, every symbol is separate from the next one by a time period. Due to the **finite bandwidth**, all these symbols suffer a spread over effect that makes them overlap each other. The higher the symbol rate is the bigger the ISI will be (a higher symbol rate means a higher signal bandwidth), due to the fact that time separation between symbols is lower. Consequently, the wireless receiver could have problems when it has to decide which symbol has been sent.

Channel Modelling...

- **Noise**: In the terms of noise, we include all kinds of undesirable effects that are added to the signal, like non-perfect components, or noise that is introduced by the transmitter or the receiver devices, and the wireless channel.
- **Co-channel interference**:
  - This kind of phenomena occurs when several users utilize the same frequency band to transmit. If the interference level is very high, the original signal can be seriously damaged and information gets lost.
  - Wireless communication systems use several techniques to reduce co-channel interference. These techniques are often regarded as user scheduling.

Channel Modelling - ISI

- The distortion introduced in transmission through a channel is due to multiple symbols being transmitted within the time it takes for one signal to completely die away after being passed through the channel.
  - A single symbol passed through the channel in isolation is spread out in time.
  - This spreading effect in time is called the impulse response, will be different for each channel, and may be time-varying (especially in mobile/wireless channels).
  - This type of channel is known as "dispersive" channel.
- The number of symbols which interfere with each other depends on the sampling rate, and thus on the transmission rate.
  - Higher symbol rate $\rightarrow$ Greater bandwidth $\rightarrow$ More ISI.

Channel Modelling - ISI...

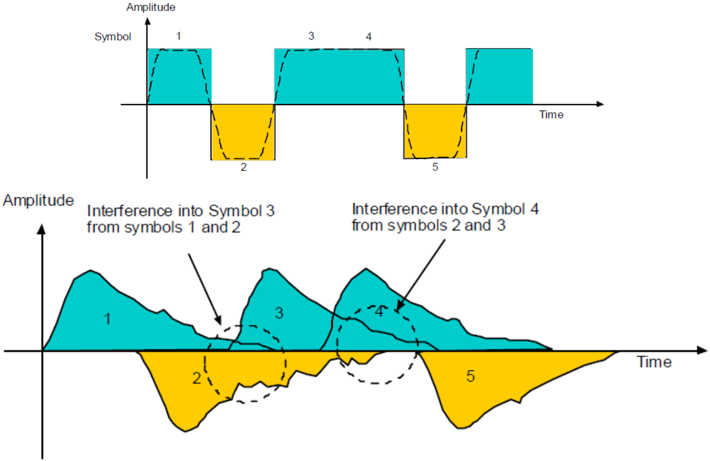


Figure: Illustration of ISI

Channel Modelling - ISI...

- At the receiver, the effects of ISI can be mitigated by using a component known as an equalizer.
- There are several equalization techniques. e.g.,
  - Zero forcing.
  - MMSE.
  - Decision feedback.
- When we have a distortionless channel (no ISI), the output signal $r[n]$ is just a weighted version of the input signal $s[n]$.
- A non-dispersive or distortionless channel is called a “flat” channel.

Channel Modelling - Noise

Noise is assumed to be the sum of a large number of independent random processes (e.g. thermal noise).

- By the central limit theorem this process approaches to a zero-mean Gaussian random process as the number independent processes becomes very large.
- Consequently, the noise in a wireless communication system is often modelled as a random process with a Gaussian distribution, independent of the output data, which is present at the channel output.
- Noise which is modelled in this way is called Additive White Gaussian Noise (AWGN).
- AWGN has constant power spectral density (PSD) over the entire frequency range, measured in Watts per Hertz of Bandwidth (W/Hz).

Channel Modelling - Noise

- A purely AWGN channel is considered the same as a flat channel. It is characterised by the single addition of AWGN to the transmitted signal with no distortion due to channel or receiver filter effects.

$$r(t) = s(t) + \eta(t) \tag{31}$$

- Despite of having constant PSD in all frequencies, since the noise is subject to the same filtering as the signal, we will assume that it occupies the same bandwidth only.
  - During reception, signals outside of the bandwidth of the signal are attenuated, and so is the noise power in those frequencies.

Channel Modelling - Noise...

Since noise has all the same characteristics which we assumed of our signal (i.e. bandlimited at the same carrier frequency), noise may be represented in an identical fashion to the signal using the complex baseband equivalent model.

- Noise is specified in terms of real and complex components:

$$\eta(t) = \eta_{\mathrm{I}}(t) + j\eta_{\mathrm{Q}}(t) \tag{32}$$

- The model of interest is an all discrete model, so noise is modelled as the samples of

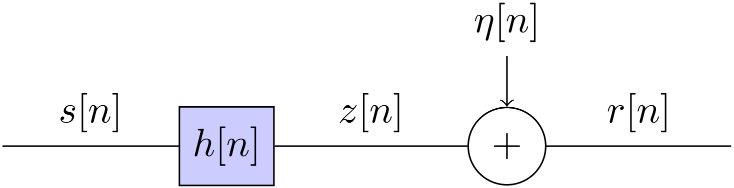
$$\eta[n] \triangleq \eta(nT) = \eta_{\mathrm{I}}(nT) + j\eta_{\mathrm{Q}}(nT) \tag{33}$$

- Consequently

$$r[n] = s[n] + \eta[n] \tag{34}$$

Channel Modelling - Noise...

- A more complete and realistic system model consists of a dispersive channel corrupted by AWGN, as represented in the following figure.



- The received signal with the discrete channel distortion and the AWGN can be expressed as

$$
\begin{aligned}
r[n] &= z[n] + \eta[n] &(35)\\
&= s[n] * h[n] + \eta[n] &(36)
\end{aligned}
$$