Tutorial 4 - Solutions

A BPSK signal of unit energy is transmitted simultaneously at a rate of 1 bit/second at the frequencies $f_1 = 1$ kHz, $f_2 = 1.5$ kHz and $f_3 = 2$ kHz, over a channel with response

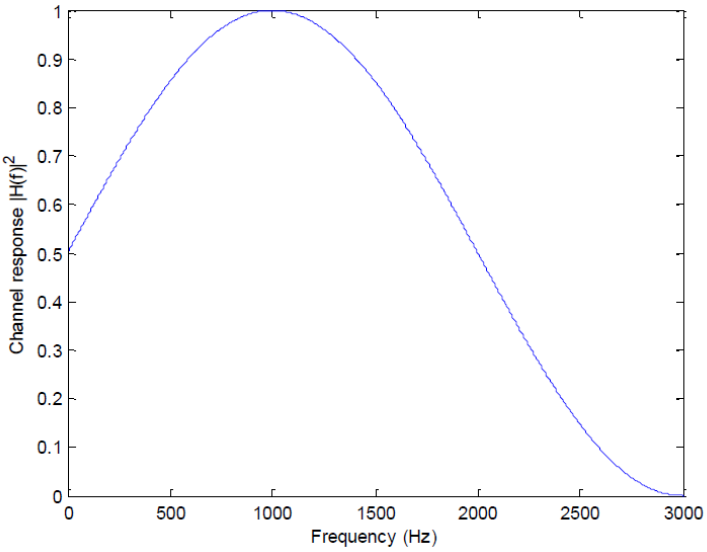
$$|H(f)|^2 = \cos^2\left(\frac{(f-1000)\pi}{4000}\right) \tag{1}$$

Given a noise variance $\sigma^2 = 10^{-3}$ across the whole band.

1. Sketch the channel response and the maximal-ratio combiner for this system
2. What is the output SNR (in dB) associated with the maximal-ratio combiner?
3. What is the output SNR (in dB) associated with the selection combiner?
4. What is the output SNR (in dB) associated with the equal-ratio combiner?

Solution:

1. The channel response is plot in the following figure



2. We note that since the bandwidth required to transmit 1 bit/s using BPSK is small relative to the total frequency band, we can consider that the channel is AWGN and the attenuation produced by the channel on each of the copies of the signal is constant (i.e.$|H(f)|^2$).

   The SNR for each copy of the received signal is given by:

$$SNR_k = \frac{E\{|K_k s(t)|^2\}}{\sigma^2} = \frac{K_k^2}{\sigma^2} = \frac{|H(f)|^2}{\sigma^2} \tag{2}$$

   The individual SNR are

$$SNR_1 = \frac{K_1^2}{\sigma^2} = \frac{|H(1000)|^2}{\sigma^2} = 1000 \tag{3}$$

$$SNR_2 = \frac{K_2^2}{\sigma^2} = \frac{|H(1500)|^2}{\sigma^2} = 853.55 \tag{4}$$

$$SNR_3 = \frac{K_3^2}{\sigma^2} = \frac{|H(2000)|^2}{\sigma^2} = 500 \tag{5}$$

   The weighting coefficients of the MRC are given by

$$\alpha_1 = \frac{K_1}{\sigma^2} = 1000 \tag{6}$$

$$\alpha_2 = \frac{K_2}{\sigma^2} = \frac{|H(1500)|}{\sigma^2} = \frac{\sqrt{\cos^2(\pi/8)}}{\sigma^2} = 923.9 \tag{7}$$

$$\alpha_1 = \frac{K_1}{\sigma^2} = \frac{|H(2000)|}{\sigma^2} = \frac{\sqrt{\cos^2(\pi/4)}}{\sigma^2} = 707.1 \tag{8}$$

   For maximal-ratio combining, the SNR of the combination of the three signals is:

$$\begin{aligned} SNR &= SNR_1 + SNR_2 + SNR_3 \\ &= 1000 + 853.55 + 500 \\ &= 2353.6 \rightarrow 33.7172 \text{ dB} \end{aligned} \tag{9}$$

3. The selection combiner chooses the single received signal with the highest SNR. i.e., $SNR = \max(SNR_1, SNR_2, SNR_3) = SNR_1 = 1000 \rightarrow 30$ dB.

4. Equal-combining is equivalent to maximal-ratio combining where the same weight is applied to all copies of the signal (i.e. $\alpha_1 = \alpha_2 = \alpha_3 = 1$), therefore:

$$SNR = \frac{\left(\sum_{i=1}^{3} \alpha_i K_i\right)^2}{\sum_{i=1}^{3} \alpha_i^2 \sigma^2} = \frac{\left(\sum_{i=1}^{3} K_i\right)^2}{\sum_{i=1}^{3} \sigma^2} \tag{10}$$

$$= \frac{\left(\sqrt{1} + \sqrt{\cos^2(\pi/8)} + \sqrt{\cos^2(\pi/4)}\right)^2}{3 \times 10^{-3}} \tag{11}$$

$$= 2307.4 \rightarrow 33.6312 \text{ dB} \tag{12}$$

   As expected, the diversity combination method with the highest SNR is maximal-ratio combining, followed by equal-gain combining and finally selection combining.