Chapter 4

1. $x[n] = x_c(nT)$

$$= \sin\left(2\pi\, 100\, n \frac{1}{400}\right) = \boxed{\sin\left(n\frac{\pi}{2}\right)}$$

2. We start from $\cos\left(n\frac{\pi}{4}\right) = x[n] = x_c(nT) = \cos(\Omega_0 n T)$

$\Leftrightarrow$ For all $k \in \mathbb{Z}$, $\frac{n\pi}{4} + 2k\pi = \Omega_0 n T$

$\Leftrightarrow$ $\frac{\pi}{4} + 2k'\pi = \Omega_0 T$

$k' = 0$ $\quad \Omega_0 = \frac{\pi}{4}.1000 = \boxed{250\pi}$

$k' = 1$ $\quad \Omega_0 = \left(2\pi + \frac{\pi}{4}\right)1000 = \boxed{2250\pi}$

5. (a) The Nyquist rate is two times the highest frequency.
To avoid aliasing, $f \geq 2 \times 5{,}000 \Rightarrow \boxed{T \leq \frac{1}{10{,}000} \text{ sec}}$.

(b) $\frac{1}{T} = 10$ kHz

$\omega = T\Omega$

$\frac{\pi}{8} = \frac{1}{10{,}000}\Omega_c$

$\Omega_c = 2\pi \cdot 625$ rad/sec

$\boxed{f_c = 625 \text{ Hz}}$

(c) $\frac{1}{T} = 20$ kHz

$\omega = T\Omega$

$\frac{\pi}{8} = \frac{1}{20{,}000}\Omega_c$

$\Omega_c = 2\pi \cdot 1250$ rad/sec

$\boxed{f_c = 1250 \text{ Hz}}$

19. The Nyquist sampling property must be satisfied : $\Omega_s \geq 2\Omega_0$

$\Leftrightarrow \frac{2\pi}{T} \geq 2\Omega_0 \Leftrightarrow T \leq \frac{\pi}{\Omega_0}$

So the range of values for T for which $x_r(t) = x_c(t)$ is $\boxed{\left]0, \frac{\pi}{\Omega_0}\right]}$.

20. (a) Similarly to 19., we need:

$$\Omega_s \geq 2\Omega_0$$

$$\Omega_s = \frac{2\pi}{T} \iff \frac{1}{T} = \frac{\Omega_s}{2\pi}$$

Hence, $$\frac{1}{T} \geq \frac{2\Omega_0}{2\pi} = \frac{2(2\pi)1000}{2\pi} = 2000 \text{ Hz}$$

$$\boxed{f_s \geq 2 \text{ kHz}}$$

(b) if $w_c = \frac{\pi}{2}$, we need $|w| < \frac{\pi}{2}$ so that $y_r(t) = x_c(t)$

Let's take a specific value of w and call it w_N

Then, $w_N = \Omega_N . T \leq \frac{\pi}{2}$, then $\frac{1}{T} \geq \frac{2\Omega_N}{\pi}$

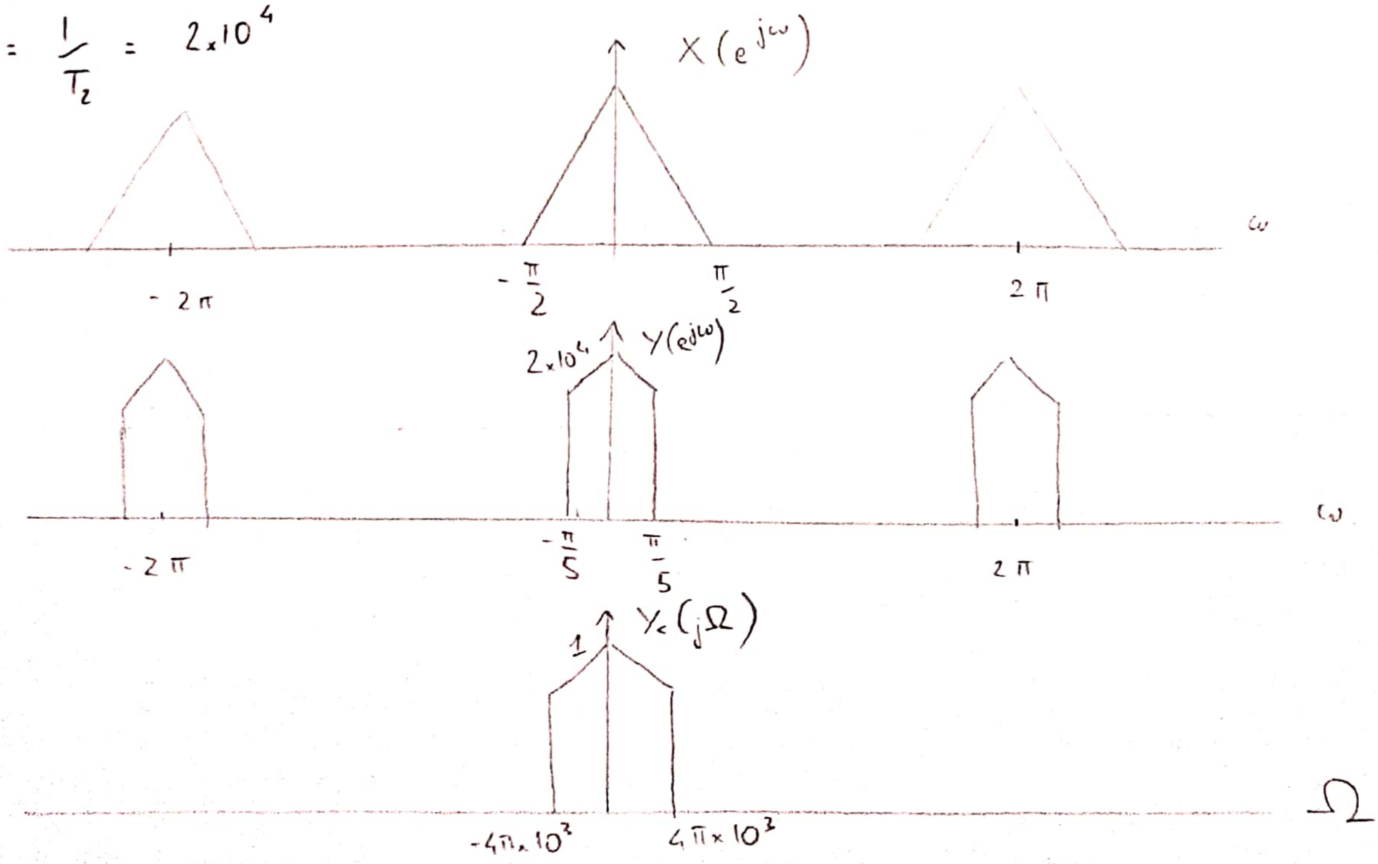
as the maximum value of Ω_N is Ω_0, we must verify

$$\frac{1}{T} \geq \frac{2\Omega_0}{\pi} = \frac{2(2\pi)1000}{\pi} = 4000 \text{ Hz}$$

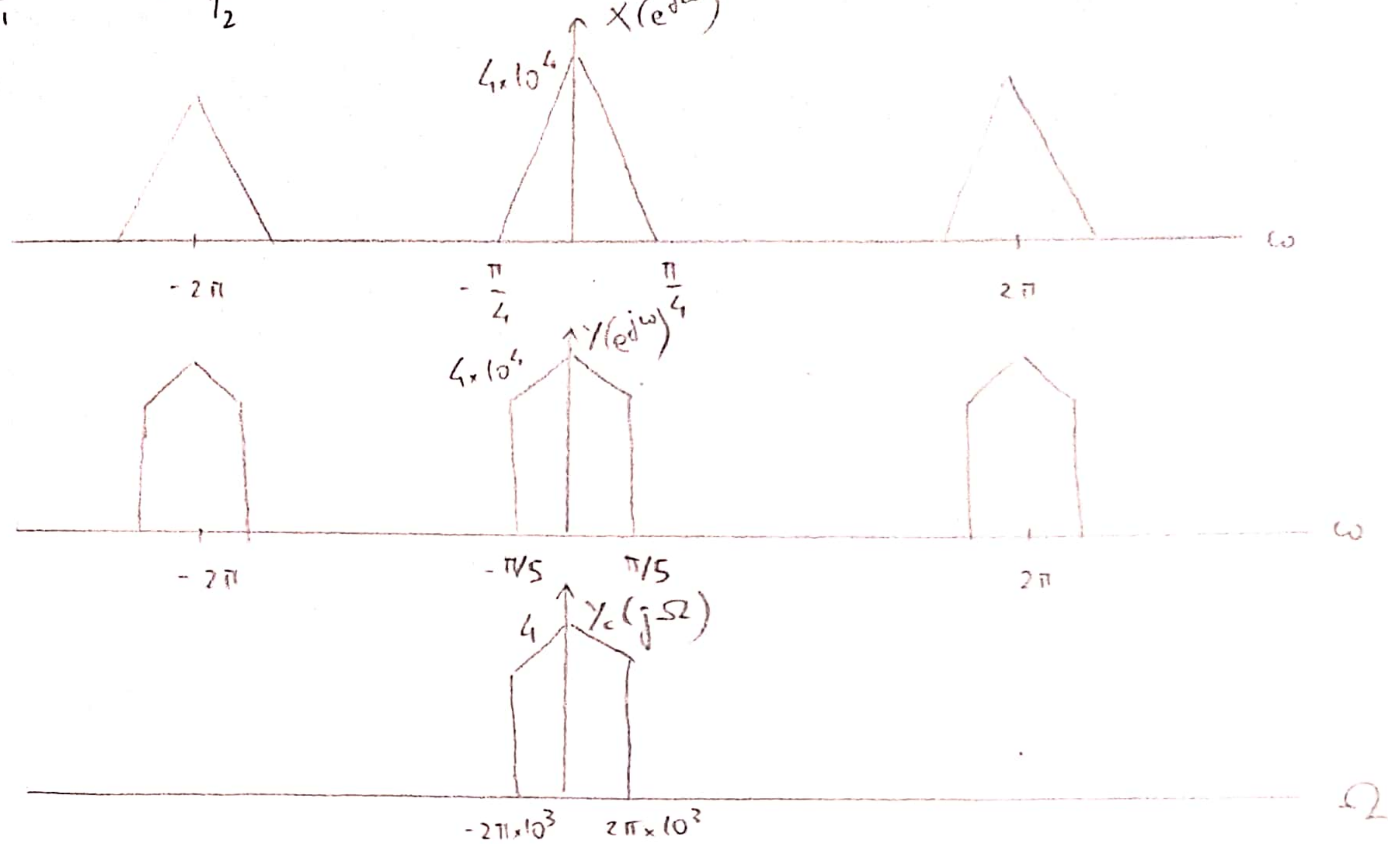
$$\boxed{f_s \geq 4 \text{ kHz}}$$

23. (a)

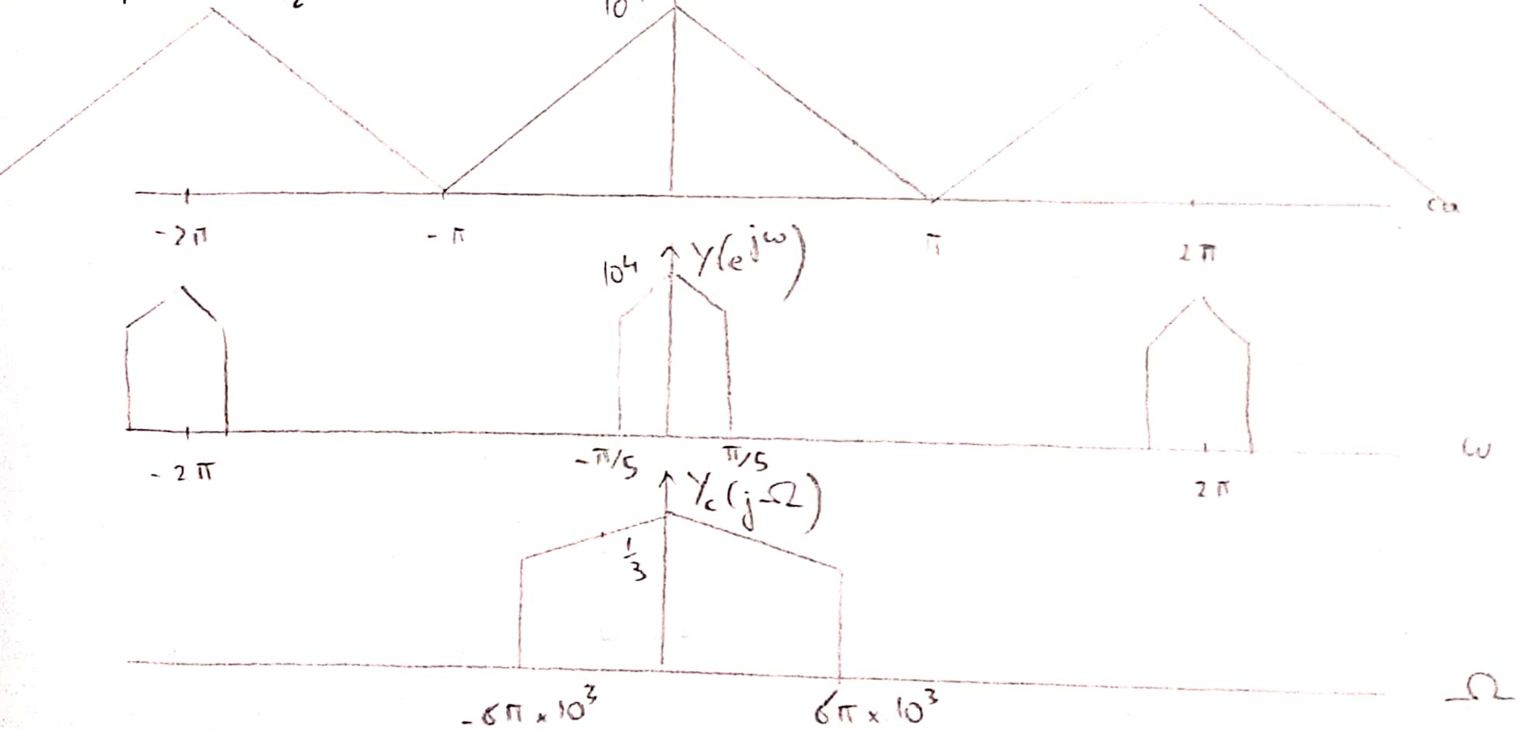
(i) $\frac{1}{T_1} = \frac{1}{T_2} = 2 \times 10^4$



(ii) $\frac{1}{T_1} = 4 \times 10^4$; $\frac{1}{T_2} = 10^4$



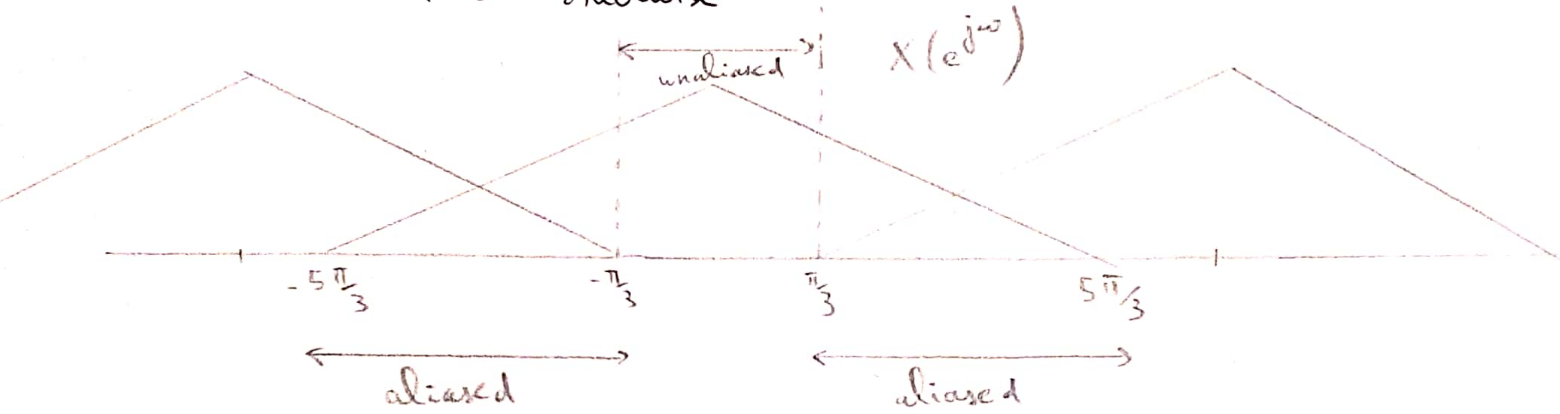
(iii) $\frac{1}{T_1} = 10^4$, $\frac{1}{T_2} = 3 \times 10^4$



23. (b)

From the figure below, it can be seen that the only portion of the spectrum which remains unaffected by the aliasing is $|\omega| < \frac{\pi}{3}$. So if we choose $\omega_c < \frac{\pi}{3}$, the overall system is LTI with a frequency response of

$$H_c(j\Omega) = \begin{cases} 1 & \text{for } |\Omega| < \omega \times 6 \times 10^3 \\ 0 & \text{otherwise} \end{cases}$$



26. For no aliasing to be present in the output (LTI means $y_c(t) = x_c(t)$) we require that

$$2\pi - \Omega_N T \geq \omega_c \quad \left(\frac{\pi}{2}\right)$$

$$2\pi - \frac{\pi}{2} \geq \Omega_N T$$

$$\frac{3\pi}{2} \geq \Omega_N T$$

$$\frac{3\pi}{2} \times \frac{1}{T} \geq \Omega_N$$

$$\frac{\cancel{2}}{2} \frac{3\cancel{\pi}}{2} \cdot f_s \geq \cancel{2\pi} f_a$$

$$\boxed{\frac{3}{4} f_s \geq f_a}$$

IF the sampling rate is $f_s = 16\,kHz$, the cutoff frequency of the C/D converter should be no more than $12\,kHz$.