Tutorial 7: Solutions

1. Two users have binary data spread by the spreading codes

$$c_1 = [1\ -1\ 1\ -1] \tag{1}$$

and

$$c_2 = [-1\ 1\ 1\ -1] \tag{2}$$

transmitted as BPSK.

   (a) Show that the above spreading codes are orthogonal.

   (b) Draw a suitable receiver structure.

   (c) If user 1 transmits binary data [−1 1 1 −1] and user 2 transmits [1 −1 1 −1], write down the transmitted sequences.

   (d) Show the effect of de-spreading user 2's data sequence with user 1's spreading code.

   (e) In the above, we have assumed that each user's data is received synchronously. Show the effect of de-spreading user 2's data sequence with user 1's spreading code when there is a timing offset of one chip.

Solution:

   (a) $\phi_{12} = \sum_{i=1}^{4} c_{1i}c_{2i} = -1-1+1+1 = 0$. So the spreading codes are orthogonal.

   (b) The receiver structure is given in the notes.

   (c) Transmitted sequence for user 1 is $[-1\ 1\ -1\ 1\ 1 - 1\ 1\ 1\ 1\ -1\ 1\ -1\ -1\ 1\ -1\ 1]$ and that for user 2 is $[-1\ 1\ 1\ -1\ 1\ -1\ -1\ 1\ -1\ 1\ 1\ -1\ 1\ -1\ -1\ 1]$.

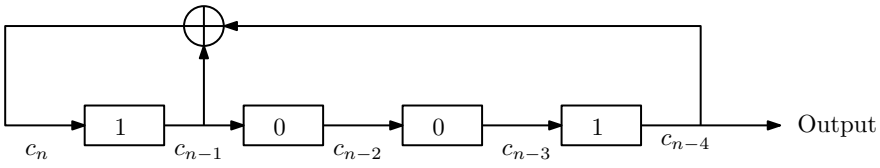
2. A direct sequence spread spectrum (DSSS) mobile communications transmitter employs a pseudo noise (PN) maximal-length spreading sequence to spread the transmitted signal. The PN sequence is obtained using a shift-register structure given by $c_n = c_{n-1} \bigoplus c_{n-4}$. The PN sequence has a period of 15 and is used to spread binary data b_k by a factor of 4. Given the first four initialisation bits of the registers as $[1\ 0\ 0\ 1]$, write down one period of the sequence and use it to encode the following binary data for BPSK transmission

$$b_k = [1\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 1] \tag{3}$$

Draw a suitable receiver structure.

Solution:

The shift register structure used is shown in the following figure.



The obtained spreading sequence has a length $2^4 - 1 = 15$ given by the following table:

	c_n	c_{n-1}	c_{n-2}	c_{n-3}	c_{n-4}	Output
Shift 1	0	1	0	0	1	1
Shift 2	0	0	1	0	0	0
Shift 3	0	0	0	1	0	0
Shift 4	1	0	0	0	1	1
Shift 5	1	1	0	0	0	0
Shift 6	1	1	1	0	0	0
Shift 7	1	1	1	1	0	0
Shift 8	0	1	1	1	1	1
Shift 9	1	0	1	1	1	1
Shift 10	0	1	0	1	1	1
Shift 11	1	0	1	0	1	1
Shift 12	1	1	0	1	0	0
Shift 13	0	1	1	0	1	1
Shift 14	0	0	1	1	0	0
Shift 15	1	0	0	1	1	1
Shift 16	0	1	0	0	1	1

One period of the PN sequence is

$$c = [100100011110101] \tag{4}$$

Encoding for BPSK is equivalent to the mapping $0 \to -1$ and $1 \to +1$. Thus

$$c \to [1\ -1\ -1\ 1\ -1\ -1\ -1\ 1\ 1\ 1\ 1\ -1\ 1\ -1\ 1] \tag{5}$$

and

$$b_k = [1\ -1\ 1\ 1\ 1\ 1\ -1\ -1\ 1] \tag{6}$$

Since the data is being spread by a factor of 4, the sequence is taken as sets of 4 from the sequence c. The transmit sequence is

$$s = [1\ -1\ -1\ 1\ 1\ 1\ 1\ -1 \cdots] \tag{7}$$