Tutorial 8: Solutions

1. The base station of a CDMA communication system uses maximal-length sequences to perform direct sequence spread spectrum (DSSS) on data transmitted to mobile stations. The base station transmits the information to six mobile stations simultaneously using the same carrier frequency and six different orthogonal maximal-length sequences. The sequences are generated using feedback shift registers of length $m = 5$. The spreading factor (N) is equal to the length of the spreading sequences. The base station transmits with a power of 30 dBm per user. The channel is considered an AWGN channel with power noise density $\Phi = N_0/2 = 10^{-12}$ W/Hz. The receiver of each mobile station is a CDMA single-user detector where the Signal-to-Interfere-and-Noise Ratio (SINR) is given by

$$SINR = \frac{GP_n}{GN_0W + \sum_{k \neq n} P_k} \tag{1}$$

where P_n is the power of the received signal of interest, P_k is the received power for the kth user, and G is the processing gain ($G = N$).

   (a) What is the value of the spreading factor N?

   (b) What is the bandwidth of the signal before and after the spreading action, if each channel has a transmit rate of 750 kbit/s using 8-PSK modulation and a raised cosine filter with 50% excess bandwidth as pulse shape?

   (c) Assuming an attenuation of 25 dB on the received signal for each user, what is the maximum transmit data rate that can be achieved for the down-link communication for each user?

   For the same spreading factor calculated in 1a:

   (d) Calculate the maximum number of CDMA users in the down-link to ensure a transmit data rate of 750 kbit/s.

   (e) Repeat the calculation in 1d if 64-QAM is used instead.

Solution:

   (a) We are told that the spreading factor is equal to the length of the spreading sequences. We are also told that the PN spreading sequences are generated using shift registers, therefore we can assume that they are maximum-length sequences, which length is equal to $n = 2^m - 1 = 31$. So the spreading factor is $N = 31$.

(b) A 8-PSK modulation scheme maps 3 bits into each symbol at the output of the digital modulator. This means that the symbol rate is 250 ksym/s. If a raised cosine pulse shape was used with an excess bandwidth of 50%, the bandwidth of this channel before the spreading operation is 375 kHz. Since the spreading factor is 31, the bandwidth of the channel after the spreading operation is 31×375 kHz = 11.625 MHz.

(c) Each channel suffers the same attenuation, so in each mobile station, the received signal for each user is $30 - 25 = 5$ dBm which is equal to 3.2 mW. The capacity for each use can be calculated using the expression of the SINR as

$$\begin{aligned} C &= W \log_2\left(1 + \frac{GP_n}{GN_0W + \sum_{k\neq n} P_k}\right) = W \log_2\left(1 + \frac{GP}{GN_0W + (K-1)P}\right) \\ &= 375 \times 10^3 \log_2\left(1 + \frac{31 \times 3.2 \times 10^{-3}}{31 \times 2 \times 10^{-12} \times 375 \times 10^3 + (6-1) \times 3.2 \times 10^{-3}}\right) \\ &= 1.067 \text{ Mbit/s} \end{aligned} \quad (2)$$

(d) From the previous question we have

$$\begin{aligned} K &= \frac{\frac{GP}{2^{C/W}-1} - GN_0W}{P} + 1 = \frac{\frac{31\times 3.2\times 10^{-3}}{2^{750\times 10^3/375\times 10^3}-1} - 31 \times 2 \times 10^{-12} \times 375 \times 10^3}{3.2 \times 10^{-3}} + 1 \\ &= 11.32 \approx 11 \text{ users} \end{aligned} \quad (3)$$

(e) If 64-QAM is used instead (6 bits/symbol) the symbol rate after the digital modulator would be equal to 125 ksymbol/s and the bandwidth after raised cosine filter 187.5 kHz. The bandwidth after the spreading operation would be 5.8125 MHz. The number of users is

$$\begin{aligned} K &= \frac{\frac{GP}{2^{C/W}-1} - GN_0W}{P} + 1 = \frac{\frac{31\times 3.2\times 10^{-3}}{2^{750\times 10^3/187.5\times 10^3}-1} - 31 \times 2 \times 10^{-12} \times 187.5 \times 10^3}{3.2 \times 10^{-3}} + 1 \\ &= 3.06 \approx 3 \text{ users} \end{aligned} \quad (4)$$

2. Consider a CDMA system with 2 users transmitting 4-PAM (i.e. levels -3, -1, 1, 3) over an AWGN channel. The users are assigned spreading sequences of unit energy corresponding to the following binary sequences shaped as *antipodal* rectangular pulses:

$$\text{User}_1 = [1\ 0\ 1\ 0\ 1\ 0\ 1\ 0] \quad (5)$$

and

$$\text{User}_2 = [0\ 1\ 0\ 1\ 0\ 0\ 1\ 1] \quad (6)$$

(a) Sketch the two shaping waveforms.

(b) Are the two waveforms orthogonal to each other?

(c) Given a sampled received sequence

$$r_n = [0.63915\ -0.94800\ 0.32026\ -1.48780\ 0.46229\ -1.60167\ 1.56136\ -0.23358]. \tag{7}$$

Use the method of maximum-likelihood to decide the symbols transmitted by each user (assuming synchronous reception).

Solution:

(a) Shaping the sequences as antipodal rectangular pulses of unit energy involves the mapping

$$\begin{aligned} 0 &\rightarrow -\frac{1}{\sqrt{8}} \\ 1 &\rightarrow \frac{1}{\sqrt{8}} \end{aligned}$$

Recall that energy is related to the sum of the squares of the amplitudes.

(b) The cross-correlation of the two waveforms is $-1/2$. So, they are not orthogonal to each other.

(c) Consider the log-likelihood function

$$\Lambda = \sum_{i-1}^{2}\sum_{j-1}^{2} s_i s_j \phi_{ij} - 2\sum_{k=1}^{2} s_k \phi_k \tag{8}$$

which may be equivalently written:

$$\Lambda = s_1^2\phi_{11} + s_2^2\phi_{22} + 2s_1s_2\phi_{12} - 2s_1\phi_1 - 2s_2\phi_2 \tag{9}$$

where $\phi_{11} = \phi_{22} = 1$ since the waveforms are unit energy, and $\phi_{12} = \phi_{21} = -1/2$ as we determined in (b). We need to calculate ϕ_1 and ϕ_2, the <u>cross-correlation of the received signal with each of the received spreading waveforms</u>

$$\begin{aligned} \phi_1 &= \frac{0.63915 + 0.94800 + 0.32026 + 1.48780 + 0.46229 + 1.60167 + 1.56136 + 0.23358}{\sqrt{8}} \quad (10) \\ &= 2.5647 \quad (11) \end{aligned}$$

$$\begin{aligned} \phi_2 &= \frac{-0.63915 - 0.94800 - 0.32026 - 1.48780 - 0.46229 + 1.60167 + 1.56136 - 0.23358}{\sqrt{8}} \quad (12) \\ &= -0.32812 \quad (13) \end{aligned}$$

There are a total of 16 possibilities for $\{s_1, s_2\}$. We tabulate the log-likelihood function in the following table.

s_1	s_2	Λ
-3	-3	44.357
-3	-1	29.045
-3	+1	21.732
-3	+3	22.42
-1	-3	20.098
-1	-1	8.7857
-1	+1	5.4732
-1	+3	10.161
+1	-3	3.8393
+1	-1	-3.4732
+1	+1	-2.7857
+1	+3	5.9019
+3	-3	-4.4196
+3	**-1**	**-7.732**
+3	+1	-3.0445
+3	+3	9.643

From the calculated table, the transmitted data is determined to be: User 1 = +3 User 2 = -1.