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Lecture 7: Spread Spectrum Communications

EE412 - Wireless Digital Communications

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2023

Learning Outcomes

- Explain what spread spectrum transmission is.
- Represent different spreading sequences and determine if they are orthogonal to each other.
- Analyse the transmitter and receiver structure for spread spectrum communications.
- Calculate the bandwidth and transmission rate of spread spectrum signals.
- Define code division multiple access (CDMA).
- Explain the principles of RAKE receivers.

Introduction

- Spread spectrum is a modulation technique employed by digital communications in which the bandwidth of the transmitted signal is largely increased in comparison to the bandwidth that would be required to transmit the underlying information bits.
- There are three properties needed in order to modulate a signal using spread spectrum:
    - The bandwidth reserved for the transmitted signal is much larger than for the normal bandwidth for the information signal.
    - The spread spectrum modulation is done using a spreading code, which is independent of the information signal.
    - In the receiver, de-spreading is done by correlating the received signal with a copy of the spreading code employed in the transmitter.

Introduction

- Spread spectrum modulation techniques were partially applied to 2G mobile communications and they became the basis for 3G mobile communication standards (eg. UMTS also known as WCDMA).
- The benefits offered by spread spectrum modulation are:
  - Reduction in the effect of interference signals onto the information signal. This means an increase in the Signal-to-Interference Ratio (SIR).
  - It allows multiple user access techniques such as code division multiple access (CDMA), which will be seen in the next section of class notes.
- There are two types of spread spectrum modulation techniques:
  - Frequency hopping spread spectrum (FHSS).
  - Direct sequence spread spectrum (DSSS).

Frequency-Hopping Spread Spectrum

- Frequency hopping is based on the continuous change of the channel centre carrier frequency over a wide bandwidth according to the value of a spreading code.
- The period of time between hops is known as chip period (T_{chip}).
- If the chip period is smaller than the symbol period ($T_{\text{chip}} < T$), it is called fast frequency hopping; whereas if the chip period is larger than the symbol period ($T_{\text{chip}} > T$), it is called slow frequency hopping.

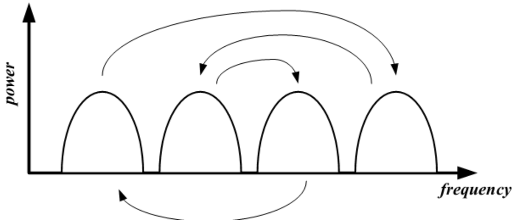

Figure: Example of frequency hopping

Frequency-Hopping Spread Spectrum...

- The total bandwidth occupied by the frequency hopping system is NW, where N is the number of different carrier frequencies, and W is the bandwidth of the information signal.
- For multiple users, FHSS offers a poorer performance than DSSS in terms of the number of users that can share the channel simultaneously.
- However, FHSS offers some benefits versus DSSS, such as robustness versus strong interferers.
- Slow FHSS has been employed in GSM to reduce the interference from adjacent cells. In addition, FHSS is the modulation scheme employed by Bluetooth.

Direct-Sequence Spread Spectrum

- We know that the bandwidth of a digital signal depends on the sampling rate and that the sampling rate is directly related to the symbol rate in many types of digital modulation, e.g., 4 QAM symbols/second $\Leftrightarrow$ $W = 4$Hz bandwidth[1]
- A system which spreads the signal over a large bandwidth (much larger than that required for transmission) is called a spread spectrum communication system, we discuss a particular system known as Direct-Sequence Spread Spectrum (DSSS) below:
    - The transmitted power per Hz will decrease since the signal is now spread over a greater bandwidth. The signal power per Hz may be small enough that the signal is hard to detect (appears to be background noise to the enemy!)
    - The signal is more robust against effects which are localised in frequency. This introduces robustness against jamming signals.

[1] assuming ideal low pass pulse shaping

Direct-Sequence Spread Spectrum...

- Transmitting a signal which requires a higher sampling rate than the symbol rate spreads the signal over frequency.
- Consider the following waveform as a modulating waveform:

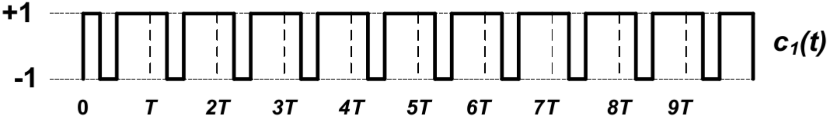

Figure: Modulating waveform

In this case the chip period (T_{chip}) is four time smaller than the symbol period (T), in other words there are 4 chips per symbol. This particular chip sequence (spreading sequence) corresponds to $[1\ -1\ 1\ 1]$. Therefore, any symbol $\{s_n\}$, at a sampling rate $f_s = 1/T$ produces a sequence, $\{g_n\}$, where $g_n = [s_n\ -s_n\ s_n\ s_n]$, at a chip rate of $f_{\text{chip}} = 1/T_{\text{chip}}$.

Direct-Sequence Spread Spectrum...

- In general, the chip rate is given by

$$f_{\text{chip}} = \frac{1}{T_{\text{chip}}} = \frac{N}{T} \tag{1}$$

  where N is known as the **spreading factor**.
- The required bandwidth is expanded by a factor of four in this case.
- Note that this is the digital equivalent and pulse shaping is still required on these before transmission (ideal filtering/raised cosine shaping).
- Note that the in-phase and quadrature components of the transmitted signal are usually spread separately.
- The set of chips chosen was completely arbitrary, and any set of chips at 4 times the symbol rate may be used.
- We know that given a set of orthogonal signals, $s_k(t)$, we may modulate onto these signals, $r(t) = \sum_k a_k(t)s_k(t)$, and uniquely retrieve the modulating data, a_k at the receiver.

Direct-Sequence Spread Spectrum...

- It is possible to derive a set of spreading sequences which are orthogonal to each other and thus we may transmit a symbol corresponding to each unique spreading sequence, i.e., the spreading sequence$[1\ -1\ 1\ 1]$ is orthogonal to the sequence $[1\ 1\ -1\ 1]$.

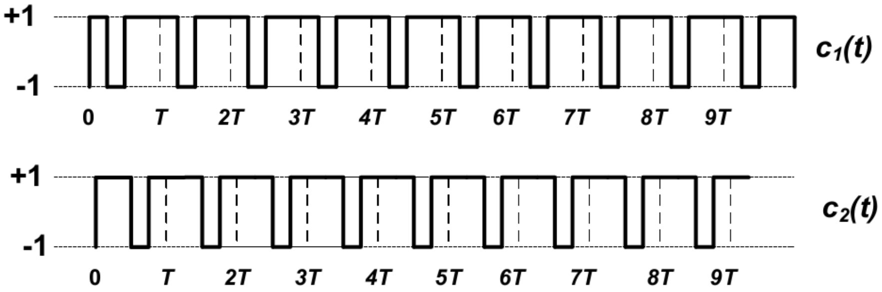

Figure: Orthogonal modulating waveforms

Direct-Sequence Spread Spectrum...

- Considering two spreading sequences (c_1 and c_2) expressed as bipolar sequences with values$\{-1, 1\}$, and both with lengths equal to n, the two spreading sequences are orthogonal to each other if their cross-correlation is equal to zero as:

$$\phi_{12} = \sum_{i=1}^{n} c_{1,i} c_{2,i} = 0 \tag{2}$$

- If the spreading sequences are expressed as unipolar sequences with values $\{0, 1\}$, their cross-correlation can be calculated as

$$\phi_{12} = \sum_{i=1}^{n} (2c_{1,i} - 1)(2c_{2,i} - 1) \tag{3}$$

Direct-Sequence Spread Spectrum...

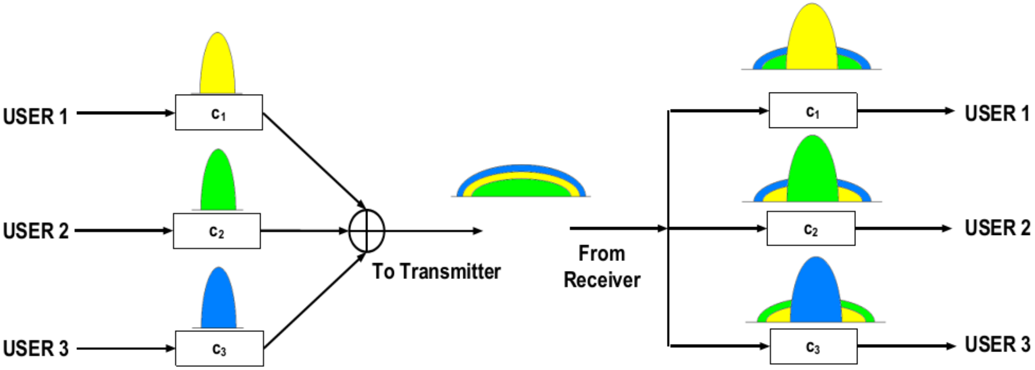

Figure: Block diagram of multiuser system

Direct-Sequence Spread Spectrum...

- The spreading process at the transmitter side can be carried out in two equivalent different ways:
  - A modulo-2 addition operation is carried out between the information bits and the direct spreading sequence followed by the digital modulation mapping (i.e. QPSK).
  - The symbols obtained at the output of the digital modulator (i.e. QPSK) are multiplied by the spreading sequence in the form of unit amplitude rectangular pulses.

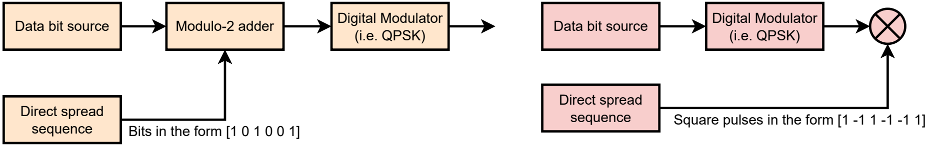

Figure: Equivalent implementations of DS spread spectrum transmitter

Direct-Sequence Spread Spectrum...

- There are many different spreading sequences and each may be extracted uniquely.
    - We must know the chip of the signal we wish to extract.
    - In a multi-user system each user is allocated a spreading sequence which allow access to that user's data but which does not allow access to other users' data.
    - It is possible to add users to the system without change to the transmitter and receiver architectures or overall bandwidth, just a small increase in transmit power per Hz.
- By the use of different spreading sequences for different users, it is possible for many different users to simultaneously make use of the available bandwidth, separable by their specific spreading sequences.

Direct-Sequence Spread Spectrum...

- When DS spread spectrum is used to support multiple users simultaneously, it is known as a code division multiple access (CDMA) system.
- It may seem that when the spreading sequences are mutually orthogonal, there is no interference between users, however this is only the case when each user's receive signal is properly synchronized (i.e. the start points match):
  - Proper synchronization is not possible in the mobile environment, since the channels and path delays are random.
  - There is no way to ensure that spreading sequences are orthogonal for all possible delays.
  - The presence of other users causes interference.
  - As the number of users increases in a CDMA system, the total interference increases, limiting the performance of each individual user.

Direct-Sequence Spread Spectrum...

- The lack of orthogonality between users' spreading codes causes interference between users.
- In a mobile system, this is a particular problem when one user has a lower power at a receiver than other users (the near -far problem), since the interference from the other users may be substantial in this case:
  - Ideally, all users' signals should be at the same power at the receiver.
  - In a CDMA system each user is required to continually adjust their power to avoid the near-far problem at the receiver.

Direct-Sequence Spread Spectrum...

- Given a DS spread spectrum system subject to narrowband interference, e.g.,
  - Fading of the channel over a small range of frequencies (small compared to the bandwidth of the spread spectrum signal).
  - A leaky microwave.
  - Someone attempting to jam communications.
- The de-spreading is performed in the same way as the spreading, thus, although the previously spread signal is de-spread, the narrowband interference is spread.
- Consequently, the power in the narrowband interference which occupies the frequency band of interest is reduced by the spreading factor, N. This makes DS spread spectrum a suitable choice for frequency selective fading channels.

Example

- A system uses the spreading sequence $c = [10101010]$, which spreads a binary input sequence b_k by modulo-2 addition, such that the output sequence, s_n , is given by

$$s_{8k+n} = c_n \bigoplus b_k, \quad \text{for } n = 0, 1, 2, ..., 7 \quad \text{and } k = 0, 1, 2, ... \tag{4}$$

  where $\bigoplus$ denotes modulo-2 addition.
  - Given an input binary sequence, b_k , at a rate of 15 bits/second and using BPSK modulation, what is the minimum bandwidth of the output sequence?
  - Given unit input power per unit bandwidth, what is the transmit power per Hz after spreading?
  - Draw a suitable transmitter structure.
  - What would the bandwidth be if the sequence $c = [11001100]$ was used instead?
  - Are the two spreading sequences given above orthogonal to each other?

Example...

- Solution:
  - BPSK modulates 1 bit/symbol, and thus 15 bits/second corresponds to 15 symbols/second. The minimum bandwidth which this may occupy before spreading is thus 15 Hz. The spreading sequence spreads this over 8 times the original bandwidth to 120 Hz.
  - Given unit power, the power per unit bandwidth is 1/120 W/Hz.
  - The transmitter structure is given earlier.
  - The sequence $c_n = [11001100]$ only has a total of a transition every second element and thus is equivalent to spreading by a code at half the rate. The bandwidth using this code would thus be 60 Hz.
  - The two sequences are orthogonal to each other.

Spread Spectrum Comms. – Jamming

- In order to jam a digital communication system, the 'enemy' places a signal over the same bandwidth as the signal to be received.
- Ideally, the jammer places its energy so that it causes maximum damage, thus all the jamming energy should be in the signal space to cause maximum displacement of the received samples.
- Given a known system which is not DS spread spectrum, i.e.,

$$s(t) = x(t)\cos(2\pi f_c t) + y(t)\sin(2\pi f_c t) \tag{5}$$

the jammer places a noisy signal in the same band

$$j(t) = a(t)\cos(2\pi f_c t) + b(t)\sin(2\pi f_c t) \tag{6}$$

Spread Spectrum Comms. – Jamming

- Assuming an ideal channel, the received signal is then

$$\begin{aligned} r(t) &= s(t) + j(t) + \eta(t) \\ &= [(x(t) + a(t))\cos(2\pi f_c t) + (y(t) + b(t))\sin(2\pi f_c t)] + \eta(t) \end{aligned} \tag{7}$$

  where $\eta(t)$ is the noise in the received signal. Since the signal space corresponds to the components modulating cos and sin, it is clear that all the energy the jammer added goes toward the destruction of the signal reliability.
- Given a DS spread spectrum system with a symbol period, T, and a chip period of T/N only energy which corresponds to the particular spreading sequence has an effect on reception.

Spread Spectrum Comms. – Jamming...

- The jammer places a noisy signal over the entire band of the spread spectrum signal, because without knowledge of the spreading sequence the jamming energy cannot be targeted specifically at the signal space of the transmitted signal.
- The received signal is:

$$r(t) = s(t) + j(t) + \eta(t) \tag{8}$$

  but only the component modulating the spreading sequence is decoded as desired receive signal. Since the spreading sequence is N-dimensional and the jammer was forced to equally space the energy across the band, **the corrupting signal contributes only $1/N$ times the total jamming energy towards signal corruption**.
- A spread spectrum system is robust against this form of jamming since the jammer must use a very large energy jamming signal to obtain a useful jamming effect.

Spreading Codes

- In a wireless system, each receiver receives a signal with a random delay and phase shift.
- It is impossible in this case to ensure that the spreading codes are orthogonal at the receiver.
- A desirable property for spreading sequences is that they have an auto-correlation of the form:

$$\phi(0) \gg \phi(j), \quad j \neq 0, \quad j_{min} \leq j \leq j_{max} \tag{9}$$

  where $\phi(0)$ is the auto-correlation of the spreading sequence with zero-samples delay between the two copies of the spreading sequence, and $\phi(j)$ is the auto-correlation with j-samples delay between the two copies of the sequence.
- Sequences with this property are called pseudo-noise (PN) sequences and may be used as spreading codes.

Spreading Codes...

- PN sequences are often implemented by means of a feedback shift register with *m* memory elements.
- A feedback shift register is linear when the logic consists entirely of modulo-2 additions.
- PN sequences generated in this way are periodic.

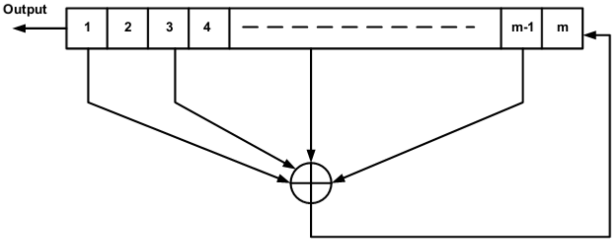

Figure: Example maximum-length shift register

Spreading Codes...

- When the period is $n = 2^m - 1$, the PN sequence is called a maximal-length sequence. Each period contains 2^{m-1} ones and $2^{m-1} - 1$ zeros.
  - Maximal-length sequences have a large number of transitions which make them good for spreading.
- Considering that the PN sequence is expressed as a bipolar sequence with values $\{-1, 1\}$, and has period equal to n, the auto-correlation function is also periodic and it is defined according to

$$\phi(j) = \sum_{i=1}^{n} c_i c_{i+j}, \quad 0 \leq j \leq n-1 \tag{10}$$

- Since $\phi(j)$ is periodic, $\phi(j + rn) = \phi(j)$ for any integer value r.
- For maximal-length PN sequences with n-elements, the auto-correlation function is

$$\phi(j) = \begin{cases} n & j = 0 \\ -1 & 1 \leq j \leq n-1 \end{cases} \tag{11}$$

Gold Codes

- For multiple access systems, it is desirable for the PN sequence to minimize the interference between users. This means reducing the value of the cross-correlation operations between different PN sequences (generally used for spreading information from different users).
- PN sequences in general do not have this property, so they do not represent the best option for CDMA systems.
- The maximum (peak) cross-correlation value between two Gold sequences is:

$$\phi_{max} = \begin{cases} 2^{\frac{m+1}{2}} + 1 & m \text{ odd} \\ 2^{\frac{m+2}{2}} + 1 & m \text{ even} \end{cases} \tag{12}$$

- Considering this value, the cross-correlation function between two Gold codes can only take three possible values $\{-1, -\phi_{max}, \phi_{max} - 2\}$.

Gold Codes...

- The Gold sequences autocorrelation values are the same as for the maximum-length sequences, as shown in equation 11.
- Gold codes are generated in two steps:
  - First, two maximum-length sequences which are compliant with the cross-correlation values $\{-1, -\phi_{max}, \phi_{max} - 2\}$ are chosen. These are called **preferred sequences**.
  - From two preferred sequences, a set of $n + 2$ Gold sequences can be generated by taking the modulo-2 addition of the first one with a shifted version of the second one, and vice versa.

Gold Codes...

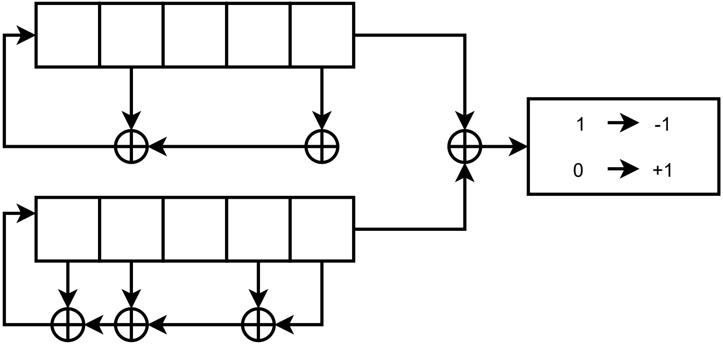

Figure: Example Gold sequence generation

Gold Codes...

- **Example**: Given two Gold sequences generated by linear feedback shift registers of length 7, the maximum value of the autocorrelation function is $\phi(0) = 2^7 - 1 = 127$, and the maximum value of the cross-correlation function is $\phi_{max} = 2^{\frac{7+1}{2}} + 1 = 17$. The ratio of these is $17/127 = 0.1339$, i.e., the maximum interference from an undesired spreading sequence is 13% of the desired sequence, assuming equal receive power.

Other spreading codes

- Kasami codes are generated in a very similar way to Gold codes but they offer better cross-correlation properties than Gold codes.
- The maximum cross-correlation value for two Kasami codes is:

$$\phi_{max} = 2^{m/2} + 1 \tag{13}$$

- Walsh-Hadamard codes are employed when time synchronisation between transmitter and receiver is possible. Under these circumstances, they provide zero interference between users.
- The requirement of time synchronisation makes Walsh-Hadamard more common in DSSS down-link channels than in DSSS up-link channels.

Reception of DSSS signals

- Consider a DSSS system where an information signal $s(t)$ is transmitted using a Gold code over an ideal channel.
- The Gold code is also known to the receiver, therefore the receiver recovers the original signal by correlating the received signal $r(t)$ with the same Gold code.
- If the receiver spreading sequence is assumed to be perfectly synchronised with the transmitter spreading sequence, it can be considered that the information signal $\hat{s}(t)$ extracted is identical to the transmitted $s(t)$.
- Consider now a K-user system where each kth user transmits using an orthogonal Gold code $(c_1(t), ..., c_K(t))$ over the same bandwidth. The received signal $r(t)$ will be the addition of the K received users,$r_1(t), \ldots, r_K(t)$.

$$r(t) = \sum_{k=1}^{K} s_k(t) c_k(t) \tag{14}$$

Reception of DSSS signals...

- The receiver associated to each user signal will correlate the signal $r(t)$ with the specific Gold code of the particular user of interest. As an example, the output of the correlator which extracts the kth user signal is the correlation of $r(t)$ with $c_k(t)$:

$$\hat{s}_k(t) = \int_a^{a+T} r(t)c_k(t-\tau)\, d\tau = s_k\phi_{kk}(\tau) + \sum_{i=2}^{K} s_i\phi_{ik}(\tau) \tag{15}$$

  with a zero-delay between transmitter and receiver codes, i.e., $\tau = 0$, the auto-correlation ϕ_{kk} has a peak[2] and the interfering users are attenuated by their respective cross-correlations.

- This structure may also be used for synchronization by looking for the output power increase that corresponds to correct timing.

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[2] $\phi_{kk}(0) \gg \phi(\tau)$, $\tau \neq 0$

Reception of DSSS signals...

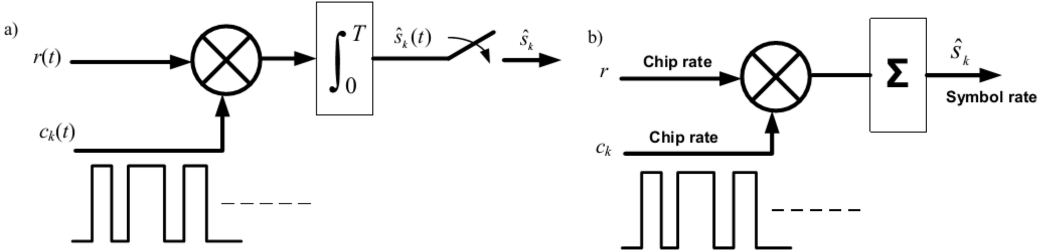


Figure: Receiver structure for DSSS a) For analogue received signals b) For digital received signals

Diversity Reception – RAKE receiver

- Let the sampling interval be equal to the symbol interval
  - In a frequency non-selective channel, the sampling interval, T, is greater than the multipath spread, T_m.
  - Each receive sample corresponds to the effect of a single transmitted symbol, subject to an unknown gain and phase rotation.
  - In a frequency selective channel, the sampling interval, T, is less than the multipath spread, T_m.
  - Each receive sample corresponds to the effect of multiple transmitted samples, each having a separate gain and rotation due to a different set of multipath components.
  - Each receive sample also corresponds to the effect of multiple symbols (ISI).

Diversity Reception – RAKE receiver...

- Let the symbol period, T, be greater than T_m, but the chip period, T_{chip}, less than T_m:
  - We receive $N = T/T_{chip}$ samples corresponding to a single symbol without ISI, each sample due to a separate set of multipath components and affected by an independent noise component.
- DS spread spectrum system uses a chip period, T_{chip}, which is smaller than the symbol period, T, to spread the data across the spectrum:
  - Given a spreading factor, $N = T/T_{chip}$, we receive N independently weighted versions of the transmitted symbol each affected by an independent noise component without ISI.
  - Maximal-ratio combining may be used to improve the output SNR at the receiver.

Diversity Reception – RAKE receiver...

- The RAKE receiver collects the signal power on each tap or finger and is so-called because it is analogous to a garden rake which collects all leaves on its fingers.
- The correlator on each finger extracts the desired signal plus noise. The coefficients, α_1 , ..., α_N , may then be chosen corresponding to the maximal-ratio combining principle (based on estimates of the respective gains and phase shift corresponding to each finger).
- In this case, the multipath nature of the channel allows an increase in the receive signal-to-noise ratio.

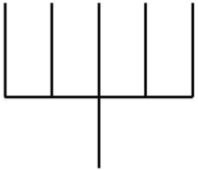
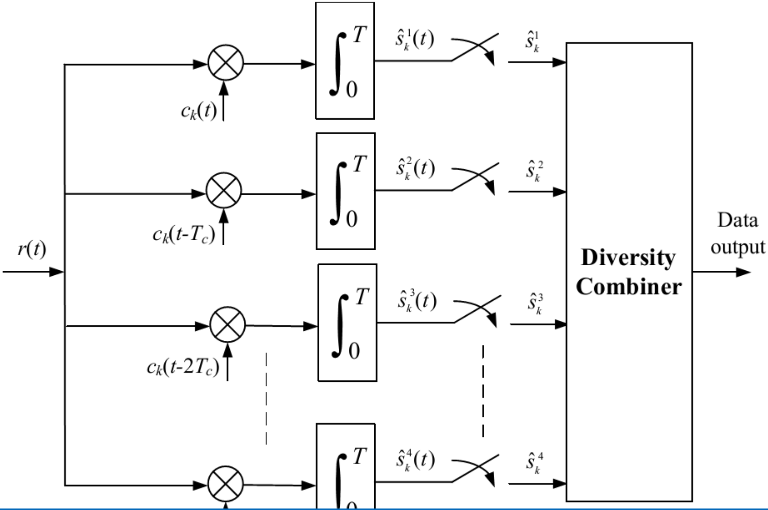


Figure: RAKE receiver fingers

Diversity Reception – RAKE receiver...



Example: Diversity Reception – RAKE receiver

- **Sample Exam Question**: Signals corresponding to 5 distinct users, simultaneously transmitting over a multipath channel from different physical locations, are received at a fixed base station. The multiple access method used is CDMA, and the modulation scheme is BPSK DSB-SC, pulse-shaped with the ideal LPF, with a symbol interval of $T = 1.4$ms. The spreading codes assigned to each user are unique and are Gold codes generated by a feedback shift register structure of length 3, spreading the data by a factor of 7. Each user's transmitter has power adjustment such that the received power from each user is approximately equal corrupted by noise with a spectral density $= N_0/2 = 3 \times 10^{-4}$W/Hz. The scattering function of the multipath wireless channel is given by:

$$S(\tau;\lambda) = \begin{cases} \frac{1}{2} - \frac{\pi\tau}{4T}\cos\left(\frac{\pi\lambda\tau}{2T}\right), & -\lambda_m \leq \lambda \leq \lambda_m, \quad 0 \leq \tau \leq T \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

with $T = 1$ ms and and $\lambda_m = 1$Hz.

Example: Diversity Reception – RAKE receiver...

Assuming the base station employs a RAKE receiver for single-user detection on each user, calculate (based on unit average received energy):

1. The average received power due to the user of interest on each finger of a 7 finger RAKE receiver.
2. The worst-case interference power due to the other users of the channel.
3. The signal-to-interference plus noise ratio (SINR) on each finger of the RAKE receiver, assuming maximum interference between users.
4. The SINR of the output of the RAKE receiver determined above, assuming maximum interference between users and the maximal-ratio combining diversity method.
5. The optimum coefficients of a RAKE receiver with 7 fingers based on average receive powers.
6. Calculate the SINR of the output when using equal gain combining instead of maximal-ratio combining.
7. State and explain three forms of diversity in a wireless system.
8. With reference to diversity reception, state and explain two techniques that are used to combine multiple copies of the transmitted signal at the receiver.
9. Give a reason why the transmitters might adjust their power so that each is received with equal power.

Example: Diversity Reception – RAKE receiver...

- **Solution**:
  1. *Relative power of resolved rays*: Given the multipath intensity profile, we know that the multipath spread, T_m is 1 ms. The symbol rate, T, is given in the question as 1.4 ms, thus there is no intersymbol interference. Given a spreading factor of 7, the chip period is $T_{chip} = \frac{T_S}{7} = 0.2$ ms. The RAKE receiver can resolve multipath components corresponding to channel delays, $\tau = 0, 0.2, 0.4, 0.6, 0.8$ ms. These rays have an average receive power given by the multipath intensity profile function $\phi(\tau)$.

$$\phi(\tau) = \int_{-\infty}^{\infty} S(\tau; \lambda) d\lambda = 1 - \sin\left(\frac{\pi\tau}{2}\right) \tag{17}$$

     Thus, the RAKE receiver resolves 7 multipath components with relative powers, P_k , with respect to the first component. However, there is zero receive power in two of these: (see next slide)

Example: Diversity Reception – RAKE receiver...

k	τ	$P_k = \phi(\tau)$
1	0	1
2	0.2	0.691
3	0.4	0.412
4	0.6	0.191
5	0.8	0.049
6	1	0
7	1.2	0

Example: Diversity Reception – RAKE receiver...

2. *Interference power*: Given Gold codes of shift register length 3, the maximum value of cross-correlation (corresponding to the gain of the interfering users) is:

$$\phi_{max} = 2^{\frac{3+1}{2}} + 1 = 5 \tag{18}$$

and the autocorrelation (corresponding to the gain of the desired user) is $\phi(0) = 2^3 - 1 = 7$. The signal power at each finger of the RAKE receiver is equal to the input ray power and the maximum interference power is $\left(\frac{5}{7}\right)^2$-th of the power on each finger, i.e., $\left(\frac{5}{7}\right)^2$per interfering user. Given 5 users, this corresponds to 4 interfering users and thus an interference power of $4 \times \left(\frac{5}{7}\right)^2$.

Example: Diversity Reception – RAKE receiver...

3. *SINR*: The SNR in each finger of the RAKE receiver is calculated according to the coefficient of the signal power and the noise power.

$$(\mathrm{SINR})_k = \frac{\text{Signal power}}{\text{Noise power}} \tag{19}$$

However, to calculate the SINR the power from the interferers has to be also considered.

$$(\mathrm{SINR})_k = \frac{\text{Signal power}}{\text{Noise power} + \text{Interferers power}} = \frac{P_S}{P_N + P_I} \tag{20}$$

The SINR needs to be calculated according to the average received signal power. This means that the power of the received signal for one user in each finger of the RAKE receiver is equal to the multipath intensity profile average power ($P_S = P_k$).

Example: Diversity Reception – RAKE receiver...

Applying the same principle to the interferers' power,

$$P_I = 4 \times \left(\frac{5}{7}\right)^2 P_k \tag{21}$$

Given a symbol period of $T = 1.4$ms, the bandwidth of this signal before spreading is $W = 714.286$Hz and the noise power which will appear on each finger of a RAKE receiver is thus:

$$P_N = WN_0 = 714.286 \times 2 \times 3 \times 10^{-4} = 0.429 \text{ W} \tag{22}$$

The noise power on each finger is constant and equal to 0.429 W (since each finger corresponds to a bandwidth of 714.286 Hz, although the total occupied bandwidth of the signal is 5 kHz).

Therefore the SINR in each branch of the RAKE receiver is:

$$(\text{SINR})_k = \frac{P_k}{4 \times \left(\frac{5}{7}\right)^2 P_k + 0.429} \tag{23}$$

Example: Diversity Reception – RAKE receiver...

The SINR for each branch of the RAKE receiver is then:

k	$(\mathrm{SINR})_k$
1	0.4049
2	0.3757
3	0.3245
4	0.2333
5	0.0926
6	0
7	0

Example: Diversity Reception – RAKE receiver...

4. Maximal ratio combining SINR: Using maximal-ratio combining, the output SINR is the sum of the SINRs on each finger, i.e., the output SINR is:

$$(\mathrm{SINR})_k = \sum_{k=1}^{7} \left(\frac{P_k}{4 \times \left(\frac{5}{7}\right)^2 P_k + 0.429} \right) = 1.4309 \quad (24)$$

5. Maximal ratio combining weights:
   The optimum weights which give maximal ratio combining are:

$$\alpha_k = \frac{K_k}{4 \times \left(\frac{5}{7}\right)^2 K_k^2 + 0.429} = \frac{\sqrt{P_k}}{4 \times \left(\frac{5}{7}\right)^2 P_k + 0.429} \quad (25)$$

k	α_k
1	0.4049
2	0.4520
3	0.5055
4	0.5338
5	0.4184
6	0
7	0

Example: Diversity Reception – RAKE receiver...

6. Equal gain combining SINR:

Using equal gain combining, the signal powers are added with equal weights, giving an SINR of:

$$\mathrm{SINR} = \frac{\left(\sum_{k=1}^{7} K_k\right)^2}{\sum_{k=1}^{7}\left[4 \times \left(\frac{5}{7}\right)^2 K_k^2 + 0.429\right]} = \frac{\left(\sum_{k=1}^{7} \sqrt{P_k}\right)^2}{\sum_{k=1}^{7}\left[4 \times \left(\frac{5}{7}\right)^2 P_k + 0.429\right]} = 1.2597 \tag{26}$$

This is lower than using maximal-gain combining as expected.

Example: Diversity Reception – RAKE receiver...

7. Diversity may be achieved in a number of ways:
   1. **Frequency diversity** – Several copies of the data may be transmitted at different frequencies. If the copies are spaced by greater than the coherence bandwidth, then the correlation between the fading components is negligible (independent fading).
   2. **Time diversity** – Several copies of the data may be transmitted at different times. If the copies are spaced in time by greater than the coherence time, then the correlation between fading components is negligible (independent fading)
   3. **Spatial Diversity** – is obtained by the reception of the signal at physically distinct locations, i.e. multiple receive antennas. Independent fading is obtained if the antennas are sufficiently spaced (because antennas which are right beside each other receive highly correlated signals). A spacing of the order of a few wavelengths is required between two antennas for independent fading

Example: Diversity Reception – RAKE receiver...

8. Multiple copies of the transmitted signal can be recombined at the receiver using the following methods:
   (a) **Selection Combining**: see Lecture 6.
   (b) **Maximal-Ratio Combining**: see Lecture 6.
   (c) **Equal-Gain Combining**: see Lecture 6.
9. In single user detection, users other than the user of interest appear as interference, thus to ensure that no user has an unfair advantage in terms of SINR at the receiver, each user must adjust their power so that each is received with equal power. In this way we avoid the near-far problem in the reception.