Tutorial 1: Solutions

1. What is analogue modulation and main reasons for analogue modulation.

2. Describe the steps involved in the development of the complex baseband equivalent model of a modulated signal, in both frequency and time domain, in as much detail as possible.

Solution:

Given a real-valued signal $s(t)$, with frequency content concentrated about some centre frequency f_c (i.e. a modulated signal), consider the two following steps

(a) Construct a signal which contains only the positive frequencies in $s(t)$. Explicitly, let $S(f)$ is the FT of $s(t)$.

$$S_+(f) = 2u(f)S(f) \tag{1}$$

where $u(f)$ is the unit step function, i.e.,

$$u(f) = \begin{cases} 0 & f < 0 \\ 1/2 & f = 0 \\ 1 & f > 0 \end{cases} \tag{2}$$

(b) Translate this signal to centre around the zero-frequency

$$S_l(f) = S_+(f + f_c) \tag{3}$$

In the time domain: Let $s_+(t)$ is time domain representation of $S_+(f)$, i.e.,

$$\begin{aligned} s_+(t) &= \text{invFT}(S_+(f)) = \text{invFT}\left(2u(f)S(f)\right) && (4) \\ &= \underbrace{\text{invFT}\left(2u(f)\right)}_{\delta(t)+j\frac{1}{\pi t}} * \underbrace{\text{invFT}\left(S(f)\right)}_{s(t)} && (5) \\ &= s(t) + j\left(\frac{1}{\pi t} * s(t)\right) && (6) \\ &= s(t) + j\hat{s}(t) && (7) \end{aligned}$$

Note that $\hat{s}(t)$ may be viewed as the original signal $s(t)$ passed through a filter with impulse response $\frac{1}{\pi t}$; such a filter is called a Hilbert transformer.

(c) Low pass equivalent signal

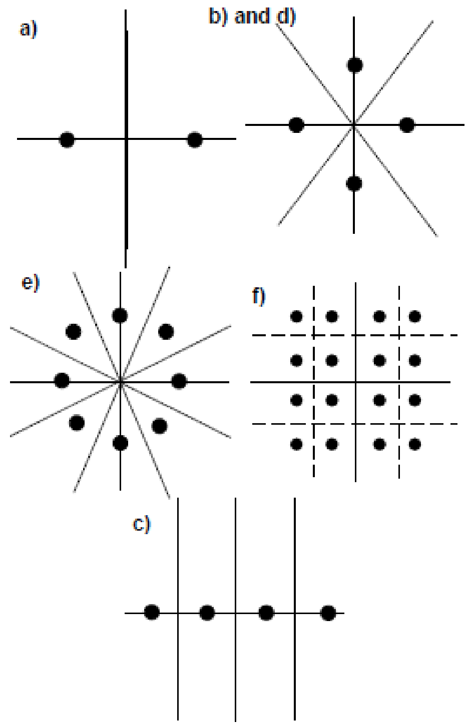
$$s_l(t) = s_+(t)e^{-j2\pi f_c t} \tag{8}$$
$$= (s(t) + j\hat{s}(t))\, e^{-j2\pi f_c t} \tag{9}$$

thus,

$$s_l(t)e^{j2\pi f_c t} = s(t) + j\hat{s}(t) \tag{10}$$

3. Draw a signal space diagram for the following constellations and their receiver decision areas: a) BPSK b) QPSK c) 4-PAM d) 4-QAM e) 8-PSK f) 16-QAM.

Solution



4. Assuming each constellation in Q3 is allocated unit average energy (corresponding to average squared distance from the origin of unity)

$$E_S = \frac{1}{M}\sum_{i=1}^{M} A_i^2 = 1 \tag{11}$$

where M is the total number of symbols in the constellation and A_i is the distance between every symbol and the constellation origin.

- For each constellation, calculate the minimum distance between any two signal points in the constellation.

- Explain which one is the most robust constellation.

Solution

(a) It is necessary first to scale each of the constellations to unit energy.

i. BPSK with signal points -1 and +1 is already unit energy. The minimum distance is the distance between the two points -1 and +1

$$d_{\text{BPSK}} = 2 \Rightarrow d^2_{\text{BPSK}} = 4 \tag{12}$$

ii. QPSK with signal points -1, $+1$, $-j$ and $+j$ is already unit energy. The minimum distance between two points is the distance between 1 and j, thus

$$d_{\text{QPSK}} = \sqrt{2} \Rightarrow d^2_{\text{QPSK}} = 2 \tag{13}$$

and thus

$$d^2_{\text{BPSK}}/d^2_{\text{QPSK}} = 2 \tag{14}$$

iii. 4-PAM with signal points -3, -1, +1 and +3 does not have unit energy. The points are scaled to unit energy by the factor

$$\sqrt{\frac{(-3)^2 + (-1)^2 + 1^2 + 3^2}{4}} = \sqrt{5} \tag{15}$$

giving signal points $\frac{-3}{\sqrt{5}}$, $\frac{-1}{\sqrt{5}}$, $\frac{1}{\sqrt{5}}$ and $\frac{3}{\sqrt{5}}$. The minimum distance is the distance between $\frac{-1}{\sqrt{5}}$ and $\frac{1}{\sqrt{5}}$which is

$$d_{\text{4-PAM}} = \frac{2}{\sqrt{5}} \Rightarrow d^2_{\text{4-PAM}} = \frac{4}{5} \tag{16}$$

and thus

$$d^2_{\text{BPSK}}/d^2_{\text{4-PAM}} = 5 \tag{17}$$

iv. The 4-QAM constellation is equivalent to the QPSK constellation, therefore

$$d_{\text{4-QAM}} = \sqrt{2} \Rightarrow d^2_{\text{4-QAM}} = 2 \tag{18}$$

and thus

$$d^2_{\text{BPSK}}/d^2_{\text{4-QAM}} = 2 \tag{19}$$

v. 8-PSK has unit energy if spaced as points on the unit circle spaced by $\pi/4$. The distance between two adjacent points is

$$d_{\text{8PSK}} = \sqrt{\left(\frac{1}{\sqrt{2}} - 1\right)^2 + \left(\frac{1}{\sqrt{2}} - 0\right)^2} \Rightarrow d^2_{\text{8PSK}} = 2 - \sqrt{2} \tag{20}$$

and thus

$$d^2_{\text{BPSK}}/d^2_{\text{8PSK}} = \frac{4}{2 - \sqrt{2}} = 6.8284 \tag{21}$$

vi. 16-QAM with signal points -3-3j, -1-3j, +1-3j, +3-3j, -3-j, -1 -j,+1- j, +3-j, -3 + j, -1 + j, +1 + j, +3 + j, -3 + 3j, -1 + 3j, +1 + 3j and +3+3j does not have unit energy. The points are scaled to unit energy by the factor

$$\sqrt{\frac{4\times[3^2+3^2]+4\times 1^2+1^2]+8\times[1^2+3^2]}{16}}=\sqrt{10} \tag{22}$$

The distance between adjacent points is

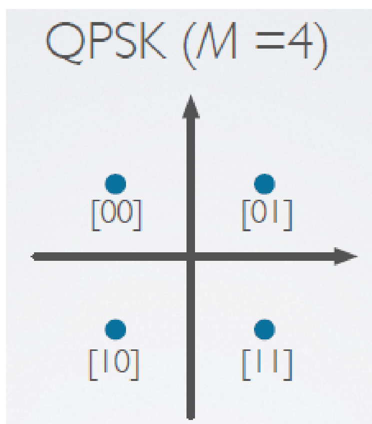
$$d_{\text{16-QAM}}=\frac{2}{\sqrt{10}}\Rightarrow d^2_{\text{16-QAM}}=2/5 \tag{23}$$

and thus

$$d^2_{\text{BPSK}}/d^2_{\text{16-QAM}}=10 \tag{24}$$

(b) The most robust constellation apart from BPSK is QPSK (and 4QAM by analogy) since its points are the farthest apart for the same amount of energy. Consequently, this digital modulation would be more robust against noise shifting the received symbols from their ideal position in the signal space to other decision areas.

5. The data source of a digital wireless communication system produces a data throughput of 2048 kbits/s. The digital modulation scheme employed is Quadrature Phase Shift Keying (QPSK), with the following symbol mapping

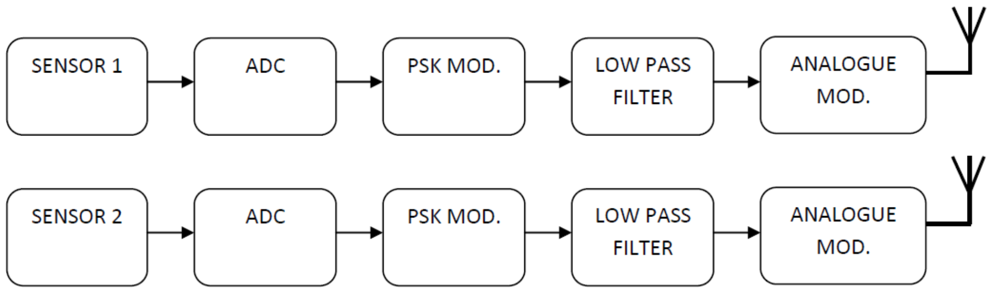


Considering that the carrier is a cosine signal with frequency 2048 kHz, draw the transmitted signal for the bit sequence: 01100011.

Solution:

The QPSK modulator maps sets of two bits into one symbol, therefore, the symbol rate will be half of the information bit rate, or 1024 ksymbol/s. The carrier signal has a frequency of 2048 kHz, twice the symbol rate, this means that each symbol will be embedded in two periods of the carrier.
Grouping the bits from left to right, the transmitted phases for the given bit sequence are: 45º (01), 225º (10), 135º (00), 315º (11).

6. It is desired to transmit the signals from two analogue sensors installed in a weather monitoring station. To transmit both signals, the wireless communication system showed in the figure is used.



The frequency band reserved for both channels goes from 100 kHz and 114 kHz. A guard band equal to 2 kHz is necessary between both channels. The analogue signals produced by the sensors a have a peak voltage of ± 1V, which is equal to the dynamic range of the ADCs. In addition, the output signal from the sensors has a maximum frequency component equal to 375 Hz. Each of the sensor outputs is processed by an ADC which carries out the signal sampling at the Nyquist frequency. The quantification is uniform with a maximum error of 7812.5 /2 μV and the codification is binary symmetric (the first bit is used to indicate if the value is positive or negative). The output of each modulator is fed into a PSK modulator to map the information bits into symbols. The digital signal at the output of the PSK modulator is filtered by an ideal brick-wall low pass filter. Finally both digital baseband signals are modulated onto a cosine carrier signal in the analogue modulation operation. It is requested to:

(a) Calculate the number of bits used for each sample taken in each of the ADCs.

(b) Calculate the bit rate at the input of each of the PSK modulators.

(c) Explain what is the minimum number of phases needed by the PSK modulators.

(d) Calculate the carrier frequencies for each of the two analogue modulators.

(e) What is the value assigned by one of the ADCs to a sample equal to 0.6 V?

(f) If now the frequency band reserved for the two channels has a bandwidth of 8 kHz (between 100 and 108 kHz) and values for the ADC, signals and guard bands are kept from the previous sections; calculate the new minimum number of phases in the modulator and carrier centre frequencies in order to meet the new requirements.

Solution:

(a) The voltage of the analogue signal produced by the sensors has a maximum of ± 1V. Therefore, we need to find out how many bits we need to represent a value between 0 and 1 V taking into account the ADC information given. To the amount of bits necessary, we will need to add an extra one which will be allocated at the beginning of the bit word just to express if the value is negative or positive. The ADC has a dynamic range equal to the maximum peak to peak value of the input analogue signal. We are told that the maximum quantification error is 7812.5/2 μV, which means that the size of the steps between the ADC quantification levels is 7812.5 μV. The number of levels can be calculated as the division of the dynamic range between the size of the step. Since one bit is left for the sign, only the negative or the positive dynamic range needs to be considered. Then, $\frac{1}{7812.5 \times 10^{-6}} = 128$ steps or different values. The number of bits necessary to represent 128 different values is 7. In addition, we need an extra bit to represent the sign, so the length of the bit word is 8 bits.

(b) The maximum frequency component of the analogue signal from the sensors is 375 Hz. The sampling frequency used by the ADCs is the Nyquist frequency, this means twice the highest frequency component, therefore fs = 750 Hz. Each sample is represented using 8 bits, consequently the output bit rate from each ADC is 8x750 = 6000 bit/s or 6 kbit/s.

(c) The frequency band available for allocating the two channels has a bandwidth of 14 kHz. Out of them, 2 kHz need to be reserved for the guard band between channels, that means that each channel can occupy up to 6 kHz. We are also told that the pulse shape filter used in this case is an ideal brick-wall low pass filter, which means that the bandwidth of each channel at the carrier frequency will be equal to the symbol rate. This means that the symbol rate can have a maximum value of 6 ksymbol/s, which is equal to the bit rate. This means that a digital modulation which employs 1 bit-per-symbol (or 2 possible phases) is the minimum required.

(d) 103 kHz and 111 kHz this ensures the guard band of 2 kHz in the middle.

(e) First we need to find an integer k such that

$$(k-1)\Delta \leq 0.6 < k\Delta \tag{25}$$

Given that $\Delta = 7812.5 \times 10^{-6}$ we have

$$k - 1 \leq \frac{0.6}{7812.5 \times 10^{-6}} = 76.8 < k \tag{26}$$

and thus $k = 77$. The binary reprentation of $k = 77$ is 1001101. Considering that the first bit is 1 for positive values and 0 negative values, 0.6 V is represented as 11001101.

(f) Now the total bandwidth available has reduced to 8 kHz with the same requirements for the guard band. This means that only 3 kHz are available now for each channel. Using a modulation scheme that allocates 2 bits-per-symbol the symbol rate, and the bandwidth, will be equal to 3 kHz. For this, a QPSK or 4-QAM modulation scheme with 4 possible phases can be employed.