Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF ENGINEERING

Electronic and Electrical Engineering

Engineering **Annual Examinations 2021**
Senior Sophister

Digital Signal Processing (4C5/5C5)

Date: 20th January 2021 **Venue: Online** **Time: 12.00 – 14.00**

Dr. W. Dowling

Instructions to Candidates:

Answer FOUR questions. All questions carry equal marks.

You have 30 minutes at the end of the examination to: (i) scan your written answers and the signed plagiarism declaration form; and (ii) upload your submission as a single pdf file.

If you need to contact me during the examination please send an e-mail message to wdowling@tcd.ie

Q.1 (a) The Nyquist rate is twice the highest frequency in a bandlimited signal. If the Nyquist rate for a signal $x(t)$ is w_0, find the Nyquist rate for the signal

$y(t) = x(3t)$ **[5 marks]**.

(b) Consider the sequence

$$x[n] = \cos\left(\frac{\pi n}{10}\right).$$

Find two different continuous-time signals that would produce this sequence when sampled at a frequency of $f_s = 100$ Hz. [**5 marks**]

(c) A system for sampling rate reduction by a factor of 1.25 is shown in Fig. Q1-1.

$x[n]$ ⁼ | " 4 | —— $r[n]$ ⁼ | $h[n]$ | —— $w[n]$ ⁼ | # 5 | $y[n]$

Fig. Q1-1 8 $x[n/4]$, n =

0,±4,±8,... $r[n]$ = ><>:

$$w[n] = \sum_{k=-\infty}^{\infty} r[k]h[n \quad k]$$ 0, otherwise

$y[n] = w[5n]$

The ideal discrete-time low-pass filter has a unit sample response, $h[n]$, and a frequency response, $H(e^{j⌦})$, given by

$$H(e^{j\Omega}) = \begin{cases} 4, & |\Omega| < \frac{\pi}{5} \\ 0, & \frac{\pi}{5} < |\Omega| \leq \pi \end{cases}$$

Let $R(e^{j⌦})$ and $Y(e^{j✓})$ denote the discrete-time Fourier transforms of the sequences $r[n]$ and $y[n]$ respectively. A continuous-time signal $x_a(t)$ has the Fourier transform $X_a(j!)$ shown in Fig. Q1-2. continued ...

[Q.1 continued]

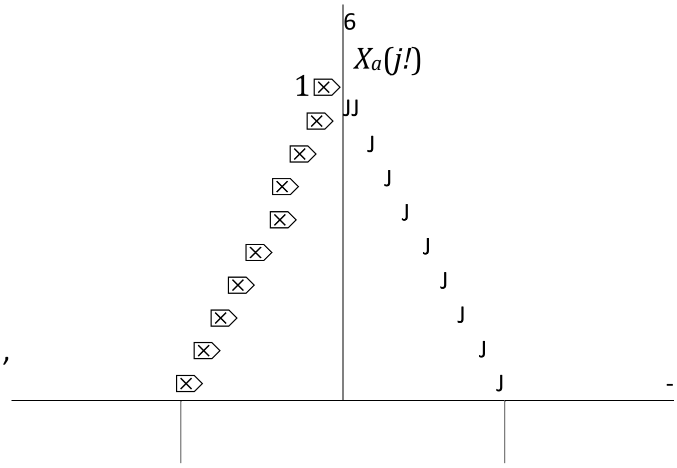

Fig. Q1-2

If $x[n] = x_a(nT)$, and the sampling period $T = 1$ millisecond,

(i) sketch $R(e^{j})$ for , and [**5 marks**] (ii) sketch $Y(e^{j})$ for . [**5 marks**]

Q.2 (a) A discrete-time filter has a unit sample response, $h[n]$, that is zero for $n < 0$ and for $n >$ N 1. If $h[n] = h[N\ 1\ n]$ and N is even, show that the filter has a frequency response with generalized linear phase. [**8 marks**]

(b) An ideal discrete-time high-pass filter has a frequency response, $H_{id}(e^{j})$, given by

$$\left|H_{id}\left(e^{j\Omega}\right)\right| = \begin{cases} 0, & |\Omega| < \frac{\pi}{3} \\ 1, & \frac{\pi}{3} < |\Omega| < \frac{2\pi}{3} \\ 0, & \frac{2\pi}{3} < |\Omega| \leq \pi \end{cases}$$

Obtain an expression for the unit-sample response of this filter. [**7 marks**]

(c) A 17-point Hamming window, $w_H[n]$, is given by

$$w_H[n] = \begin{cases} 0.54 + 0.46\cos\left(\frac{\pi n}{8}\right), & 8 \leq n \leq 8 \\ 0, & \text{otherwise.} \end{cases}$$

Using the Hamming window, design a causal, 17-point, generalised linear phase filter that approximates the magnitude response of the ideal band-pass filter in part (b). [**5 marks**]

Q.3 (a) A continuous-time filter has an impulse response, $h_a(t)$, given by

a 8><>:e t, t 0,

$h(t) =$

$0, \qquad t < 0.$

A discrete-time filter has the unit-sample response, $h[n]$, given by

$$h[n] = Th_a(nT),$$

where T is a positive constant.

Let $H(z)$ denote the transfer function of the discrete-time filter. Show that

$$H(z) = \frac{T}{1 - e^{-T}z^{-1}}, \qquad |z| > e^{-T}.$$

[4 marks]

Note that: $$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}, \qquad |\alpha| < 1.$$

(b) A discrete-time high-pass filter with frequency response, $H\left(e^{j\Omega}\right)$, is to be designed to meet the following specifications:

$$0.9 \leq \left|H\left(e^{j\Omega}\right)\right| \leq 1, \qquad 0.7\pi \leq |\Omega| \leq \pi$$
$$\left|H\left(e^{j\Omega}\right)\right| \leq 0.2, \qquad |\Omega| \leq 0.3\pi$$

The filter is to be designed by applying the bilinear transformation

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

to the transfer function of an appropriate Butterworth continuous-time filter.

Verify that a second order filter is sufficient to meet the specifications.

Determine the transfer function, $H(z)$, of the discrete-time filter.

Note that the transfer function of a second order Butterworth low-pass prototype filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

[16 marks]

Q.4 (a) Let $\tilde{x}[n]$ denote a periodic sequence with period N. This sequence is also periodic with period $2N$. Let $\tilde{X}_1[k]$ denote the Discrete Fourier Series (DFS) coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period N and $\tilde{X}_2[k]$ denote the DFS coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period $2N$. Determine $\tilde{X}_2[k]$ in terms of $\tilde{X}_1[k]$.

[**10 marks**]

(b) Consider two finite length sequences, $x[n]$ and $h[n]$, where both are zero for $n < 0$ and where

$x[n] = 0,$ n

$32\ h[n] = 0,$ n

5.

The 32-point DFTs of the two sequences are multiplied and the inverse DFT computed. Let $r[n]$ denote this inverse DFT.

The sequence $y[n]$ is obtained by linearly convolving $x[n]$ and $h[n]$.

Specify the values of n for which $r[n]$ is guaranteed to be equal to $y[n]$.

[**6 marks**]

(c) A 50,000 point sequence is to be linearly convolved with a sequence that is 128 points long. The convolution is to be implemented using DFTs and inverse DFTs of length 1024. If the overlap-add method is used, what is the minimum number of 1024-point DFTs and the minimum number of 1024-point inverse DFTs needed to implement the convolution for the entire 50,000 point sequence?

[**4 marks**]

Q.5 (a) Let $\{X[n], n \; 2\mathbb{Z}\}$ be a discrete-time random process, defined by

$$X[n] = \cos\left(\frac{\pi n}{4} + \Theta\right)$$

where ⇥ is a random variable that is uniformly distributed on the interval (⍰,⍰).

(i) Find the mean of the random process, $m_X[n]$. [**2 marks**]

(ii) Find the autocorrelation $_{XX}[n_1,n_2]$ of $X[n]$. [**4 marks**]

(iii) Is $X[n]$ a wide-sense stationary process? [**2 marks**]

(b) Let $X[n]$ be a zero mean, white-noise process with autocorrelation sequence

$_{XX}[m] = {}_X^2\,[m]$, where $[m]$ is the unit-sample sequence. $X[n]$ is applied to the input the of a linear, shift-invariant filter with unit-sample response, $h[n]$, given by

$$h[n] = [n] + [n \quad 1].$$

(i) Obtain an expression for the output autocorrelation sequence, $_{YY}[m]$.

[6 marks]

(ii) Determine the power spectral density, S_{YY} (⌦), of the output process.

[3 **marks**]

(iii) Determine the mean, m_Y, and the variance, $_Y{}^2$, of the output process.

[3 **marks**]