UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF ENGINEERING

Electronic & Electrical Engineering

Senior Sophister **Trinity Term, 2013**
Engineering
Annual Examinations

DIGITAL SIGNAL PROCESSING (4C5)

Date: 10th May 2013 **Venue: LUCE LOWER** **Time: 09.30 – 11.30**

Anthony Quinn

ANSWER QUESTION 1, and any TWO of the remaining four questions.

Question 1 is worth 40 marks.
Each of the two remaining questions is worth 30 marks.

Permitted Materials:
Calculator
Drawing Instruments
Graph Paper

Conventions used in this paper

DSP:	digital signal processing
LTI:	linear, time-invariant
IIR / FIR:	infinite / finite "impulse" (*i.e.* unit sample sequence) response
DTFT:	discrete-time Fourier transform
(G)LP:	(generalized) linear phase

$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$:	the unit step function
sinc$(x) = \frac{\sin(x)}{x}$:	the sinc function
$\delta[n]$:	the unit sample sequence
$h[n]$:	the unit sample sequence response of a LTI discrete-time system
$H(z)$:	the system function (z-transform of $h[n]$)
$x(t)$; $X(j\omega)$:	continuous-time signal; its Fourier transform

Q.1 [COMPULSORY]

Answer ALL the following questions.

(a) Consider a DSP-enabled analogue signal processing system. Assume that 5-times over-sampling is implemented, and that the output signal is reconstructed by way of a zero-order hold (ZOH). Provide a calibrated sketch of the amplitude response of a suitable smoothing filter for the output of the ZOH.

Note:
the amplitude spectrum of the duration-T rectangular pulse, $x_R(t) = u(t) - u(t-T)$, is

$$|X_R(j\omega)| = T \left|\text{sinc}\left(\frac{\omega T}{2}\right)\right|.$$

[8 marks]

(b) Let $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{4}\delta[n-2]$, and let $x_E[n] = x[n] * x[-n]$ be the deterministic autocorrelation of $x[n]$. Sketch the amplitude and phase spectra of $x_E[n-1]$.

[8 marks]

(c) Prove that the stable system,

$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}},$$

is an all-pass system for any *real* $a \neq \pm 1$. **[8 marks]**

(d) A stable, *lowpass*, IIR, discrete-time filter, $H(z)$, is designed via the bilinear transformation, $s = \frac{1-z^{-1}}{1+z^{-1}}$. It is found that the pre-warped gain specification is satisfied using a 2nd-order Butterworth polynomial,

$$B_2(s) = s^2 + \sqrt{2}s + 1,$$

under a lowpass-to-lowpass transformation, $s \to \frac{s}{\omega_c}$, with $\omega_c = 2$ rads/sec. Roughly sketch of the pole-zero diagram of the designed $H(z)$.

[8 marks]

(e) A noisy sinusoidal sequence,

$$X_n = 1.5\sin(\frac{\pi}{3}n) + E_n,$$

where E_n is unit variance white noise, is applied at the input of an ideal brickwall, unity gain, bandpass filter, with passband between $\frac{\pi}{4}$ and $\frac{\pi}{2}$ rads/sample. Evaluate the total average power of Y_n, the output of the filter.

[8 marks]

Q.2

(a) Consider an analogue signal, $x(t)$, and the sequence, $x[n] = x(nT)$, $n \in \mathbb{Z}$, resulting from its ideal, periodic sampling every T seconds. Define the unique baseband reconstruction, $x_r(t)$, based on $x[n]$, and prove that $x_r(t) = x(t)$ if the sampling procedure is Shannon-compliant.

[17 marks]

(b) In (a) above, let $x(t) = u(t+1.5) - u(t-1.5)$, *i.e.* the *even* rectangular pulse of amplitude 1 and duration 3 seconds, and let $T = 1$ second.

(i) By considering the DTFT of $x[n]$ in this case, or otherwise, provide a calibrated sketch of $X_r(j\omega)$, the Fourier transform of $x_r(t)$.

[8 marks]

(ii) Evaluate $x_r(\frac{1}{2})$, and explain why $x_r(\frac{1}{2}) \neq x(\frac{1}{2})$.

[5 marks]

Q.3

(a) Consider the following analogue signal, being the superposition of two cisoids:

$$x_a(t) = e^{j2\pi f_1 t} + e^{j2\pi f_2 t},$$

where $f_1 = 1$ Hz and $f_2 = 2$ Hz. Let $x[n] = x_a(nT)$, for a chosen sampling period, T (seconds), and let $x_w[n] = x[n]w_N[n]$, for a chosen length-N windowing sequence, $w_N[n]$. Design appropriate values for T and N which allow the cisoids to be resolved via the DTFT of $x_w[n]$. As part of your answer, provide sketches of the amplitude spectrum of $x[n]$, and of $x_w[n]$ for both a rectangular and a Bartlett window of chosen length, N.

[15 marks]

(b) Design a causal GLP FIR discrete-time filter with group delay 5 samples. Aim for an ideal *bandstop* brickwall gain specification, where the stopband is between $\frac{\pi}{4}$ and $\frac{\pi}{2}$ rads/sample, with gain 1 and 0 in the pass- and stopbands, respectively.

[15 marks]

Q.4

(a) A digital recording system captures audio sequences, ideally $x[n]$, but introduces a distortion, yielding sequences, $x_d[n]$. The distortion process yielding $x_d[n]$ from $x[n]$ is identified as the following LTI discrete-time system:

$$h[n] = \delta[n] + 1.21\delta[n-2].$$

(i) Design a *causal, stable* LTI discrete-time system to cancel the amplitude distortion introduced by this system.

[8 marks]

(ii) If

$$x[n] = 5\sin(\frac{\pi}{2}n),$$

then show that the resulting output of the system you designed in (i) is $y[n] = x[n-2]$ (*i.e.* a pure 2-sample delay of $x[n]$).

[5 marks]

(b) A FIR LTI discrete-time system is to be designed, with local gain minima at the Nyquist and half-Nyquist frequencies, being 0.1 and zero, respectively. Informally design *both* a GLP and a LP system with these properties. Provide a minimum-multiplication realization of *either* one of your filters.

[17 marks]

Q.5

(a) Let X_n be a *regular random process*, with autocorrelation sequence, $\phi_{XX}[m]$, and let x_n, $n = 0, 1, 2, ..., N-1$, be a length N realization of X_n.

(i) Define the phrase in italics. **[5 marks]**

(ii) Specify a procedure for estimation of $\phi_{XX}[m]$ from x_n, up to an appropriate lag, $|m|$. **[10 marks]**

(b) Let the autocorrelation sequence be

$$\phi_{XX}[m] = 3(-0.5)^{|m|}.$$

(i) Provide a calibrated sketch of the power spectral density (PSD) of X_n, and state the total average power. **[10 marks]**

(ii) Design a *whitening filter* for X_n, *i.e.* one whose output is unit variance white noise, E_n, if driven by X_n above. **[5 marks]**

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