Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

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Digital Wireless Communications
Lecture 5: Wireless Channel Modelling

Dr. Arman Farhang (arman.farhang@tcd.ie)
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Outline

- Calculate the attenuation on the transmitted signal produced by the wireless channel considering free space transmission.
- Explain how signals propagate in the wireless channel and what effects are produced.
- Define multipath intensity profile, coherence time, coherence bandwidth and Doppler spread.
- Enumerate different propagation models used in wireless communications.
- Calculate the multipath intensity profile, coherence time, coherence bandwidth and Doppler spread from the scattering function associated to a wireless channel.
- Classify a wireless channel based on fast/slow fading and frequency selective/non-selective fading.

Free Space Transmission

- In free space (i.e. no ground, no buildings), the receive power is related to the transmit power via the 'path loss equation'.
- Assuming isotropic transmission (uniform in all directions), the power, P_T, is radiated equally in all directions.

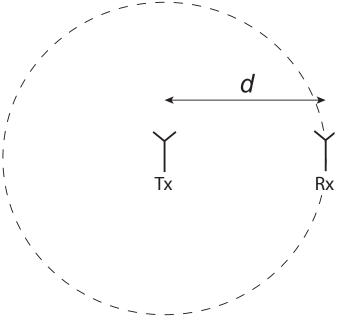


Figure: Illustration of path loss – 2-D projection.

Free Space Transmission

- At a given distance, d, from the transmit antenna we may imagine a sphere over which the entire power is equally distributed.
  - The surface area of a sphere is $4\pi d^2$.
  - The receive power per unit surface area of the sphere is:

$$\frac{P_T}{4\pi d^2} \quad (W/m2) \tag{1}$$

- The path loss in free space is proportional to the squared distance, d^2.

Free Space Transmission

- The transmit and receive antennas have a number of important properties:
  - The **effective aperture** of an antenna is the ratio of power available at the antenna terminals to the power per unit area of the incoming electromagnetic wave.
  - The power received depends on the effective aperture of the receive antenna, A_R (m^2):
$$P_R = \frac{A_R P_T}{4\pi d^2} \quad (W) \tag{2}$$
  - The directional **gain** of an antenna is the ratio of the power radiated in the direction of interest to the average radiated power. Consequently, the power received from the direction of interest is increased by a factor of G_T , the transmit antenna gain
$$P_R = \frac{A_R G_T P_T}{4\pi d^2} \quad (W) \tag{3}$$

Free Space Transmission

- An antenna's gain is related to its effective aperture as

$$A = \frac{G\lambda^2}{4\pi} \quad (m^2) \tag{4}$$

  where λ is the wavelength of interest. This allows the use of only gain or aperture terms in the receive power expression

$$P_R = \frac{G_T G_R P_T \lambda^2}{16\pi^2 d^2} \quad (W) \tag{5}$$

  - This is called the Friis free-space equation.

Path Loss in Free Space

- The **path loss** is defined as the attenuation over the radio link
- Path loss is usually expressed in dB.

$$
\begin{aligned}
\text{Free path loss} &= 10\log_{10}\left(\frac{P_T}{P_R}\right) \\
&= 10\log_{10}\left(\frac{16\pi^2 d^2}{G_T G_R \lambda^2}\right) \\
&= 10\log_{10}\left(\frac{4\pi d}{\lambda}\right)^2 - 10\log_{10}(G_T G_R) \quad (\text{dB})
\end{aligned} \tag{6}
$$

- The second term is constant only dependant on the antenna gains.
- Note that as distance increases, it may be desirable to lower the frequency (increase the wavelength) in order to keep path loss from becoming unworkably large.
- Receive power in free space is inversely proportional to d^2 .

Path Loss in Free Space

- Typical communication links do not take place over free-space but rather in harsher environments with reflections, scattering, atmospheric absorption, etc.
  - In mobile radio, the receive power is inversely proportional to d^p , where the value of p (typically $2 < p < 4$) determines the severity of the channel.
  - A large number of factors contribute to the path loss in a radio channel:
    - Difference in antenna height.
    - Frequency of transmission.
    - Presence of obstacles.
    - Atmospheric conditions, e.g. hazy, humid, raining, snowing, etc.

Path Loss in Urban Areas – Hata Model

- One path loss equation for a large city is the Hata model, where the path loss is:

$$\begin{aligned} \text{Path loss} &= 69.55 + 26.16 \log_{10} f - 13.82 \log_{10} h_t \\ &\quad - a(h_r) + (44.9 - 6.55 \log_{10} h_t) \log_{10} d \end{aligned} \qquad \text{(dB)} \tag{7}$$

where h_t, h_r are the respective heights of the receive and transmit antennas (in metres, $1 < h_r < 10$ and $30 < h_t < 200$), f is the frequency of transmission (in MHz, $150 < f < 1500$), d is the distance from transmit to receive antenna (in km, $1 < d < 20$), and $a(h_r)$ is:

$$a(h_r) = \begin{cases} 8.29 \left(\log_{10}(1.54 h_r)\right)^2 - 1.1 & 200 \geq f \geq 150 \text{ MHz} \\ 3.2 \left(\log_{10}(11.75 h_r)\right)^2 - 4.97 & f \geq 200 \text{ MHz} \end{cases} \tag{8}$$

Path Loss in Urban Areas – Hata Model

- This equation was obtained by empirical methods evolving the previous published Okumura model, this is why it is also known as Okumura-Hata model in plenty of literature. There are also Okumura-Hata models for suburban areas and open areas.
  - **Exercise:** Using the Hata model, calculate the path loss between two antennas in a city, given a transmission frequency of 1 GHz, from a transmitter at 100 m to a receiver at 2 m from the ground and separated by 5 km.

The Wireless Channel

- In the wireless environment, the signal at the receiver is the result of multiple received signals, each corresponding to separate paths for the transmitted signal.
- The multiple paths travel different distances and interact in different ways with the environment:
  - Different distances and interactions imply different propagation delays for each ray between transmitter and receiver, also different attenuation.
  - Examples of different interactions are
    - Reflections off windows vs concrete.
    - Diffracted rays vs reflected rays.
    - Line-of-sight (LOS) rays vs indirect rays.
    - Rays scattered by the atmosphere.

The Wireless Channel

- The received signal is the sum of delayed, attenuated versions of the transmitted signal.

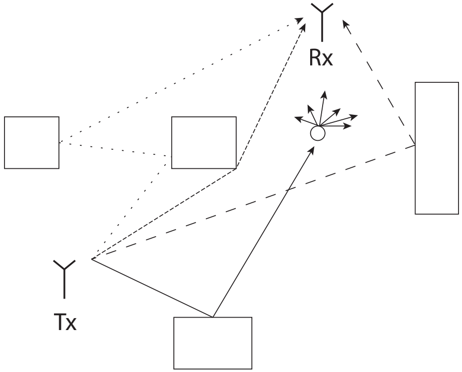
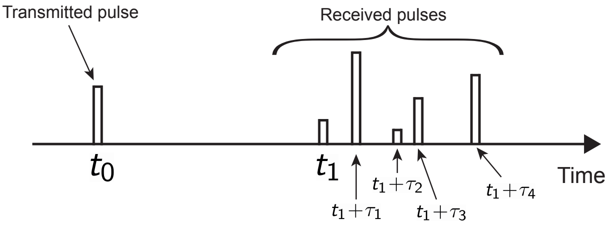


Figure: Signal propagation in wireless environment.

The Wireless Channel

- The wireless channel is in general time-varying:
  - The transmitter and/or receiver may be free to move.
  - The environment may change (e.g. someone opens a window, walks across the room, it starts to rain).
- Transmitting a narrow pulse at different time instants gives a different train of receive pulses.
  - The time-delays, amplitudes and number of received pulses vary with time.



The Wireless Channel

- The wireless channel is modelled statistically.
  - We make some definitions:
    - Assume that there are multiple propagation paths, and that each path has an associated delay and attenuation.
    - Assume that these delays and attenuation are time-varying.
    - Let the k^{th} delay be $\tau_k(t)$ and the k^{th} attenuation be $\alpha_k(t)$.
  - Given a transmitted signal $s(t) = \mathrm{Re}\left[s_l(t)e^{j2\pi f_c t}\right]$ passed through this channel, the receive signal is:

$$r(t) = \sum_n \alpha_n(t)s(t - \tau_n(t)) \tag{9}$$

The Wireless Channel

by substitution, we may express this in terms of the low-pass equivalent model:

$$\begin{aligned} r(t) &= \mathrm{Re}\left[\left(\sum_n \alpha_n(t) s_l(t-\tau_n(t)) e^{-j2\pi f_c \tau_n(t)}\right) e^{j2\pi f_c t}\right] \\ &= \mathrm{Re}\left[r_l(t) e^{j2\pi f_c t}\right] \end{aligned} \tag{10}$$

thus

$$r_l(t) = \sum_n \alpha_n(t) s_l(t-\tau_n(t)) e^{-j2\pi f_c \tau_n(t)} \tag{11}$$

The Wireless Channel

- This is the low-pass equivalent of the received signal. We can thus write the low-pass equivalent channel as:

$$c(\tau; t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \delta(t - \tau_n(t)) \tag{12}$$

- To model the channel components, we need to examine the effect of the varying attenuation and phase components on the received signal.
- Imagine that $s_l(t) = 1$, i.e. transmission of $\cos(2\pi f_c t)$, in this case the received signal is:

$$r_l(t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \tag{13}$$

The Wireless Channel

- **Received phase:**
  - The phase of the received components is rotated by $2\pi f_c \tau_n(t)$.
  - The phase rotates a complete 2π for each change of $1/f_c$ in the delay $\tau_n(t)$.
  - The carrier frequency is generally very high.
  - At $f_c = 2.4$ GHz, $1/fc = 0.416$ ns A
  - At the speed of light ($c \sim 3 \times 10^8$ m/s), this is a change in distance from transmitter to receiver of $\Delta = 125$ mm.
  - A very small change in the delay/length of a multipath causes a large change in the phase of a multipath component at the receiver.

The Wireless Channel

- **Attenuation:**
  - A large change in the medium is required to cause a significant change in $\alpha_n(t)$.
  - Small changes in distance between the transmitter and receiver do not significantly change the attenuation.
- Changes in the amplitude of the received signal are primarily the result of destructive interference between the phases and are known as fading.
- The received signal is a random process.

The Wireless Channel

- If there are a large number of received paths then the central limit theorem suggests that $r_l(t)$ may be modelled as a complex-valued Gaussian random process:
  - Therefore channel, $c(\tau; t)$, is modelled as a complex-valued Gaussian random process. Recall that $c(\tau; t)$ is the response of the channel at time t due to an impulse at time $t - \tau$.
  - The envelope of the channel response, $|c(\tau; t)|$, is **Rayleigh distributed** at any t. The Rahleigh model is generally employed in urban areas where there are not fixed scatters.
  - The envelope of the channel response is better modelled by a **Ricean distribution in the case of fixed scatters**. This is generally the model used in rural environments.

The Wireless Channel

- Other possible models for the envelope of the channel exist, e.g. **Nakagami-m**, used for urban radio. The Rayleigh model can be considered as a particular case of the Nakagami-m model.
- Since wireless channels are modelled statistically, it is useful/instructive to define certain useful correlations and power spectral density functions.

Multipath Intensity Profile and Coh. Bandwidth

- We define the autocorrelation of the channel at two time instants t and $t + \Delta t$ due two impulses sent τ_1 and τ_2 seconds earlier as

$$\phi(\tau_1, \tau_2; \Delta t) = \frac{1}{2} E\left\{c^\star(\tau_1; t) c(\tau_2; t + \Delta t)\right\} \tag{14}$$

  where $E\{\cdot\}$ represents the mean or expected value.
- It is reasonable in radio transmission to assume uncorrelated channel response (attenuation and phase shift) at two different delays. Thus, we consider

$$\phi(\tau; \Delta t) = \frac{1}{2} E\left\{c^\star(\tau; t) c(\tau; t + \Delta t)\right\} \tag{15}$$

  which is the autocorrelation of the channel at two time instants, Δt apart.
- By letting $\Delta t = 0$ we get the **multipath intensity profile**, $\phi(\tau) = \phi(\tau; 0)$
  - This corresponds to the average power at the receiver due to a signal transmitted τ seconds earlier.

Multipath Intensity Profile and Coh. Bandwidth

- In practice, $\phi(\tau)$ is measured by transmitting extremely short pulses over the channel and correlating.
- The multipath intensity profile becomes approximately zero after T_m seconds.
- The range over which the multipath intensity profile is non-zero is called the **multipath spread of the channel**.

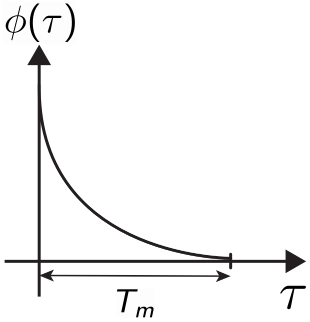


Figure: Typical Multipath Intensity Profile with T_m multipath spread

Multipath Intensity Profile and Coh. Bandwidth

- The frequency response of a channel is related to the impulse response via the Fourier transform

$$C(f;t) = \int c(\tau;t)e^{-j2\pi f\tau}d\tau \tag{16}$$

  - As before, since this is defined statistically, we examine the autocorrelation.

$$\phi(f_1, f_2; \Delta t) = \frac{1}{2}E\left\{C^\star(f_1;t)C(f_2;t+\Delta t)\right\} \tag{17}$$

    which is the autocorrelation between the response of different frequencies of the channel at two time instants Δt apart.

Multipath Intensity Profile and Coh. Bandwidth

- This is related to $\phi(\tau; \triangle t)$ via the Fourier transform

$$F\left[\phi(\tau; \triangle t)\right] = \phi(\triangle f; \triangle t) \tag{18}$$

  where $\triangle f = f_2 - f_1$

- $\phi(\triangle f; \triangle t)$ is a measure of the relation between the frequency responses at two frequencies spaced by $\triangle f$.
- $\phi(\triangle f; \triangle t)$ is called the **spaced-frequency**, **spaced-time** correlation function.
- $\phi(\triangle f; \triangle t)$ is measured by transmitting sinusoids separated by $\triangle f$ and correlating the two of them at the receiver with delay $\triangle t$.

Multipath Intensity Profile and Coh. Bandwidth

- The multipath intensity profile is related to the spaced-frequency, spaced-time correlation function of the channel by setting $\triangle t = 0$

$$\phi(\triangle f) = F\left[\phi(\tau)\right] \tag{19}$$

  - The function $\phi(\triangle f)$ is called the **spaced-frequency correlation function** and provides an idea of the coherence frequency of the channel.
  - As a result of this relationship between $\phi(\tau)$ and $\phi(\triangle f)$, the **coherence bandwidth** of the channel may be defined as:

$$(\triangle f)_c = \frac{1}{T_m} \tag{20}$$

Multipath Intensity Profile and Coh. Bandwidth

- If $(\triangle f)_c$ is large compared to the bandwidth of the signal, the channel is called **frequency non-selective**, and it is considered to affect similarly all the frequency components of the transmitted signal.
- If the channel bandwidth was larger than the coherence bandwidth, two frequency components spaced more than $(\triangle f)_c$ apart are affected differently by the channel (different attenuation and phase shift), producing a received signal severely affected by the channel.

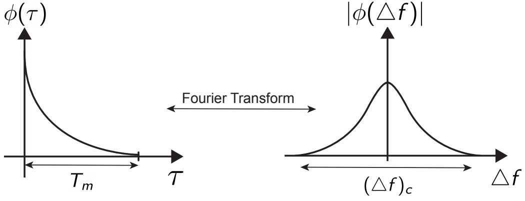


Figure: Fourier transform pair between coherence bandwidth (left) and the multipath intensity profile (right).

Doppler Spread and Coherence Time

- The multipath intensity profile function and the spaced-frequency correlation function provide information about the distortion suffered by the different frequency components of the transmitted signal due to the Intersymbol Interference produced by the multipath spread.
- However, the channel varies not only at different frequencies and at different delays, but also as time progresses. The time variations in the channel are evidenced as a Doppler broadening and Doppler shift.
- From the spaced-frequency spaced-time correlation function $\phi(\triangle f; \triangle t)$, the **spaced-time correlation function** $\phi(\triangle t)$ can be obtained by considering the correlation of the same frequency component $(\triangle f = 0)$ transmitted at two time instants $\triangle t$ apart. Taking the Fourier transform of $\phi(\triangle t)$ with respect to the variable $\triangle t$:

$$S(\lambda) = F\left[\phi(0; \triangle t)\right] = F\left[\phi(\triangle t)\right] \tag{21}$$

Doppler Spread and Coherence Time

- $S(\lambda)$ is called the **Doppler power spectrum**.
  - $S(\lambda)$ shows the Doppler broadening associated with the time variations.
  - Fast variations of the channel result in a greater broadening.
  - The range of values over which $S(\lambda)$ is approximately non-zero is called the **Doppler spread** of the channel, B_d .

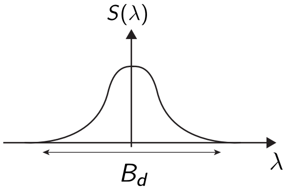


Figure: Doppler power spectrum

  - If the channel is time invariant then there is no spectral broadening and $S(\lambda) = \delta(\lambda)$.

Doppler Spread and Coherence Time

- The **coherence time** is the length of time over which the channel is "approximately" constant.
- The coherence time, $(\triangle t)_c$ is related to the Doppler spread.

$$(\triangle t)_c = \frac{1}{B_d} \tag{22}$$

- A slowly changing channel has a large coherence time and a small Doppler spread.
- Finally, we examine the channel variations at path delay τ.
- The power spectrum associated with the time variations at delay τ is called the **scattering function**, it is the Fourier transform of $\phi(\tau; \triangle t)$ in the variable $\triangle t$:

$$S(\tau; \lambda) = F\left[\phi(\tau; \triangle t)\right] \tag{23}$$

Doppler Spread and Coherence Time

- The scattering function is a measure of the average power output of the channel as a function of time delay τ and Doppler frequency λ.
  - The scattering function is related to the spaced-frequency spaced-time correlation function as:

$$
\begin{aligned}
S(\tau;\lambda) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(\triangle f;\triangle t)e^{-j2\pi\lambda\triangle t}e^{j2\pi\tau\triangle f}\,d\triangle t\,d\triangle f \\
\int_{-\infty}^{\infty} S(\tau;\lambda)\,d\lambda &= \phi(\tau) \quad \text{Multipath intensity profile} \\
\int_{-\infty}^{\infty} S(\tau;\lambda)\,d\tau &= S(\lambda) \quad \text{Doppler power spectrum}
\end{aligned}
\tag{24}
$$

Wireless Channel Modelling – Summary

- Mobile/wireless channels fit into four categories:
  - Frequency non-selective
    - Bandwidth is much smaller than the coherence bandwidth $\leftrightarrow$ the signalling interval is greater than T_m , the multipath spread. There is no ISI introduced by the channel. The channel is modelled as $C(t) = \alpha(t)e^{j\phi}$. The multiple paths cannot be individually resolved.
  - Frequency selective
    - The bandwidth is greater than the coherence bandwidth $\leftrightarrow$ the signalling interval is lower than T_m . The channel introduces ISI. The multiple paths may be individually resolved.
  - Slow fading
    - The signalling interval is smaller than the coherence time of the channel, $T < (\triangle t)_c$. The channel is essentially fixed for at least one signalling period.
  - Fast fading
    - The signalling interval is greater than the coherence time of the channel, $T > (\triangle t)_c$.

Channel Modelling – Example

- We wish to transmit over a channel with scattering function:

$$S(\tau;\lambda) = \begin{cases} ae^{-k|\tau|}\cos^2\lambda & -\frac{\pi}{2} \leq \lambda \leq \frac{\pi}{2} \quad \tau \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

  such that the channel is:
  - Frequency non-selective
  - Slowly-fading
- Suggest suitable transmission rates for the parameters $a = 2$, $k = 4$. Can this channel be both frequency non-selective and slowly-fading?
  - If so, suggest a suitable transmission rate.
  - If not, suggest alternative values for the parameters a and k such that the channel may be made frequency non-selective slowly-fading, and the corresponding transmission rate.

Channel Modelling – Example

- **Solution:**
  - We may find the multipath intensity profile by integration:

$$\begin{aligned} \Phi(\tau) &= \int_{-\infty}^{\infty} S(\tau;\lambda)\, d\lambda = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2e^{-4|\tau|} \cos^2\lambda\, d\lambda \\ &= 2e^{-4|\tau|} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\lambda\, d\lambda = \pi e^{-4|\tau|} \end{aligned} \tag{26}$$

  - We may find the Doppler power spectrum by integration:

$$\begin{aligned} S(\lambda) &= \int_{-\infty}^{\infty} S(\tau;\lambda)\, d\tau = \int_{-\infty}^{\infty} 2e^{-4|\tau|} \cos^2\lambda\, d\tau \\ &= \cos^2\lambda \int_{-\infty}^{\infty} 2e^{-4|\tau|} d\tau = \cos^2\lambda \int_{0}^{\infty} 2e^{-4|\tau|} d\tau \\ &= 2\cos^2\lambda \frac{e^{-4|\tau|}}{-4} \Big|_0^{\infty} = \frac{\cos^2\lambda}{2} \end{aligned} \tag{27}$$

Channel Modelling – Example

- We can estimate the multipath spread and the Doppler spread from the graphs (below) as approximately $T_m = 1\text{s}$ and $B_d = 3\text{Hz}$.

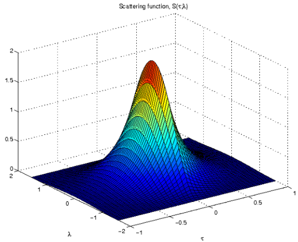


Figure: Scattering function

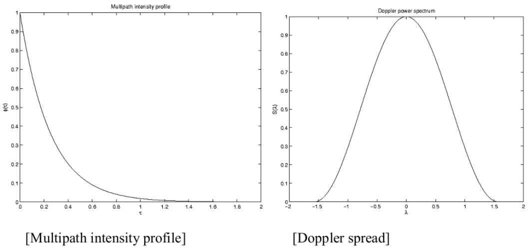


Figure: The Multipath intensity profile and the Doppler spread.

Channel Modelling – Example

- Given a multipath spread, $T_m = 1$s, and a Doppler spread $B_d = 3$Hz:
  - For a frequency non-selective channel, the signalling interval must satisfy $T > T_m$. Thus, any signalling interval of greater than one second leads to a frequency non-selective channel. We arbitrarily choose an interval of $T = 1.5$s.
  - For a channel to be slowly fading, the signalling interval must be lower than the coherence time $(\triangle t)_c$. The coherence time is related to the Doppler spread by $(\triangle t)_c = \frac{1}{B_d} = 0.333$s. Thus, any signalling interval of less than 0.333 seconds leads to a slowly fading channel. We arbitrarily choose an interval $T = 0.25$s.
  - It is not possible to satisfy the two constraints above simultaneously, thus it is not possible to choose a signalling rate for which this channel is frequency non-selective, slowly-fading.

Channel Modelling – Example

- To enable the channel to be simultaneously frequency non-selective and slowly fading, we require to either:
  - Increase the coherence time (equivalently reduce the Doppler bandwidth).
  - Decrease the multipath spread.