Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF ENGINEERING

Electronic and Electrical Engineering

Engineering
Junior Sophister
Annual Examinations

Semester 1, 2019

Signals and Systems (3C1)

13th December 2019 **Venue: RDS Simmonscourt** **Time: 17:00 – 19:00**

Dr. W. Dowling

Instructions to Candidates:

Answer FOUR questions. All questions carry equal marks.

Materials permitted for this examination:

Mathematical Tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination.

Please indicate the make and model of your calculator on each answer book used.

Q.1 (a) Obtain the complex Fourier series representation for the ideal impulse train, $p(t)$, given by

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

where T is a positive constant. **[7 marks]**

(b) A continuous-time signal, $x_a(t)$, has the Fourier transform, $X_a(j\omega)$, shown in Figure Q.1. Let $x_p(t)$ denote the ideal impulse-sampled signal:

$$x_p(t) = x_a(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

where T is a positive constant.

(i) Show that $X_p(j\omega)$, the Fourier transform of $x_p(t)$, is given by

$$X_p(j\omega) = \frac{1}{T} \sum_{r=-\infty}^{\infty} X_a\left(j\left(\omega - \frac{2\pi r}{T}\right)\right)$$

[7 marks]

(ii) If $T = 10^{-3}$, sketch $X_p(j\omega)$ for $-3000\pi \leq \omega \leq 3000\pi$. **[6 marks]**

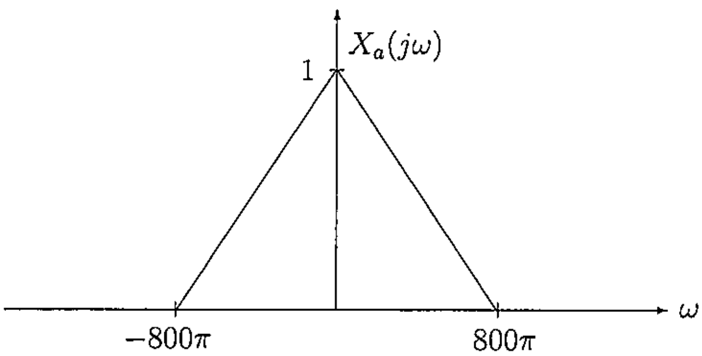

Figure Q.1

Q.2 (a) Let $X(j\omega)$ and $Y(j\omega)$ denote the Fourier transform of the signals $x(t)$ and $y(t)$ respectively.

(i) If $g(t) = x(t)\,y(t)$ show that $G(j\omega)$, the Fourier transform of $g(t)$, is given by

$$G(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)Y(j(\omega-\theta))\,d\theta$$

[6 marks]

(ii) Let $r(t) = x(t - t_0)$ where t_0 is a constant. Show that $R(j\omega)$, the Fourier transform of $r(t)$, is given by

$$R(j\omega) = e^{-j\omega t_0}X(j\omega)$$

[3 marks]

(b) Compute the Fourier transform of the signal

$$x(t) = e^{-a\,t}u(t)$$

where a is a positive constant and $u(t)$ is the unit-step function. **[3 marks]**

(c) Consider a causal, linear, time-invariant system with frequency response

$$H(j\omega) = \frac{1}{1+j\omega}$$

For a particular input, $x(t)$, this system is observed to produce the output

$$y(t) = e^{-t}u(t) - e^{-2\,t}u(t)$$

Determine $x(t)$. **[8 marks]**

Q.3 (a) Consider a continuous-time system with input, $x(t)$, and output, $y(t)$.
The input-output relationship for this system is

$$y(t) = x(t) - 3$$

(i) Is the system linear?

(ii) Is the system time-invariant?

Justify your answers. **[8 marks]**

(b) The input, $x(t)$, and output, $y(t)$, of a causal, linear, time-invariant system satisfy the following differential equation:

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt}$$

(i) Determine the transfer function of the system, $H(s)$. **[3 marks]**

(ii) Plot the pole and zero of $H(s)$ in the s-plane and indicate the region of convergence of $H(s)$. **[5 marks]**

(iii) Sketch the magnitude of the frequency response of the system. **[4 marks]**

Q.4 (a) Let $X(e^{j\Omega})$ and $H(e^{j\Omega})$ denote the discrete-time Fourier transform of the sequences $x[n]$ and $h[n]$ respectively. The sequence $y[n]$ is given by

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Show that $Y(e^{j\Omega})$, the discrete-time Fourier transform of $y[n]$, is given by

$$Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega})$$

[6 marks]

(b) Compute the discrete-time Fourier transform, $X_1(e^{j\Omega})$, of the sequence

$$x_1[n] = \delta[n+1] + \delta[n-1]$$

where $\delta[n]$ is the unit-sample sequence.
Sketch the magnitude of $X_1(e^{j\Omega})$ for $-\pi \leq \Omega \leq \pi$.

[4 marks]

(c) Determine the z-transform and the associated region of convergence for the sequence $x_2[n]$ given by

$$x_2[n] = (0.75)^n u[n] + \delta[n-1]$$

where $u[n]$ is the unit-step sequence.

[4 marks]

(d) Determine the inverse z-transform of

$$X(z) = \frac{3 - z^{-1}}{(1 - 0.25z^{-1})(1 - 0.5z^{-1})}, \quad |z| > 0.5$$

[6 marks]

Note that: $\displaystyle\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}, \quad |\alpha| < 1.$

Q.5 (a) Show that a linear, time-invariant, discrete-time system is stable in the bounded-input bounded-output sense if, and only if, the unit-sample response of the system, $h[n]$, is absolutely summable, that is,

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

[13 marks]

(b) Consider a linear, time-invariant discrete-time system with unit-sample response, $h[n]$, given by

$$h[n] = (0.5)^n u[n]$$

where $u[n]$ is the unit-step sequence.

(i) Is the system stable in the bounded-input bounded-output sense? Justify your answer.

(ii) Determine the unit-step response of the system, $s[n]$.

[7 marks]

Note that: $$\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}, \quad \alpha \neq 1.$$