EE4C5 Digital Signal Processing

Lecture 4 – Sampling and reconstruction

Labs

- Available to entire class 1 week before first associated clinic.
- Must submit “initial” or “rough work” BEFORE the scheduled clinic.
    - Deadline on Blackboard
    - Just code in .m file or cut&paste of command window operations or initial figures
- Can only attend assigned clinic (CadLab will be full). Check Lab Group assigned.
- Attendance at clinic is optional but will be recorded.
- Submission for each clinic NOT a full engineering report.
    - Pdf with figures and observations from each task.
    - Number them by the tasks in handout.
    - All code in .m files.
    - Submitted according to deadline in Blackboard.
    - Due approx. 1 week later (but before next clinic)
    - Gather material as you work
- Any late submissions must be due to exceptional circumstances
    - https://www.tcd.ie/teaching-learning/academic-affairs/ug-regulations/assets/appeals/Exceptional%20Circumstances%2026-02-16.pdf

CA Mark

Lab	Completion Mark (0/1)	Individual Grade (0-5)	
1	c1	m1	
2	c2	m2	
3	c3	m3	
4	c4	m4	

CA mark $=\left(\frac{c1+c2+c3+c4}{4}\right)(m2+m3+m4)$

Completion => Reasonable attempt made and all required submissions (before and after clinic) made on time

Late submission policy: Up to one week late, completion mark halved but individual grade given. More than one week late, completion mark 0, individual grade 0 for that lab.

This lecture

- Based on (part of) Chapter 4 of O&S
- All images from O&S book unless otherwise stated

Importance of sampling

- Discrete-time signals often represent sampled continuous-time signal
- Ambiguity introduced in the signal
- Potential for aliasing
- How to not lose information?
  - Or at least not the important part
  - Trade-offs

Periodic Sampling

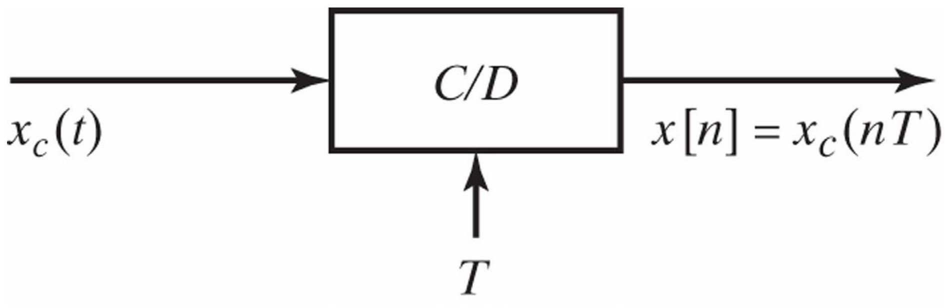
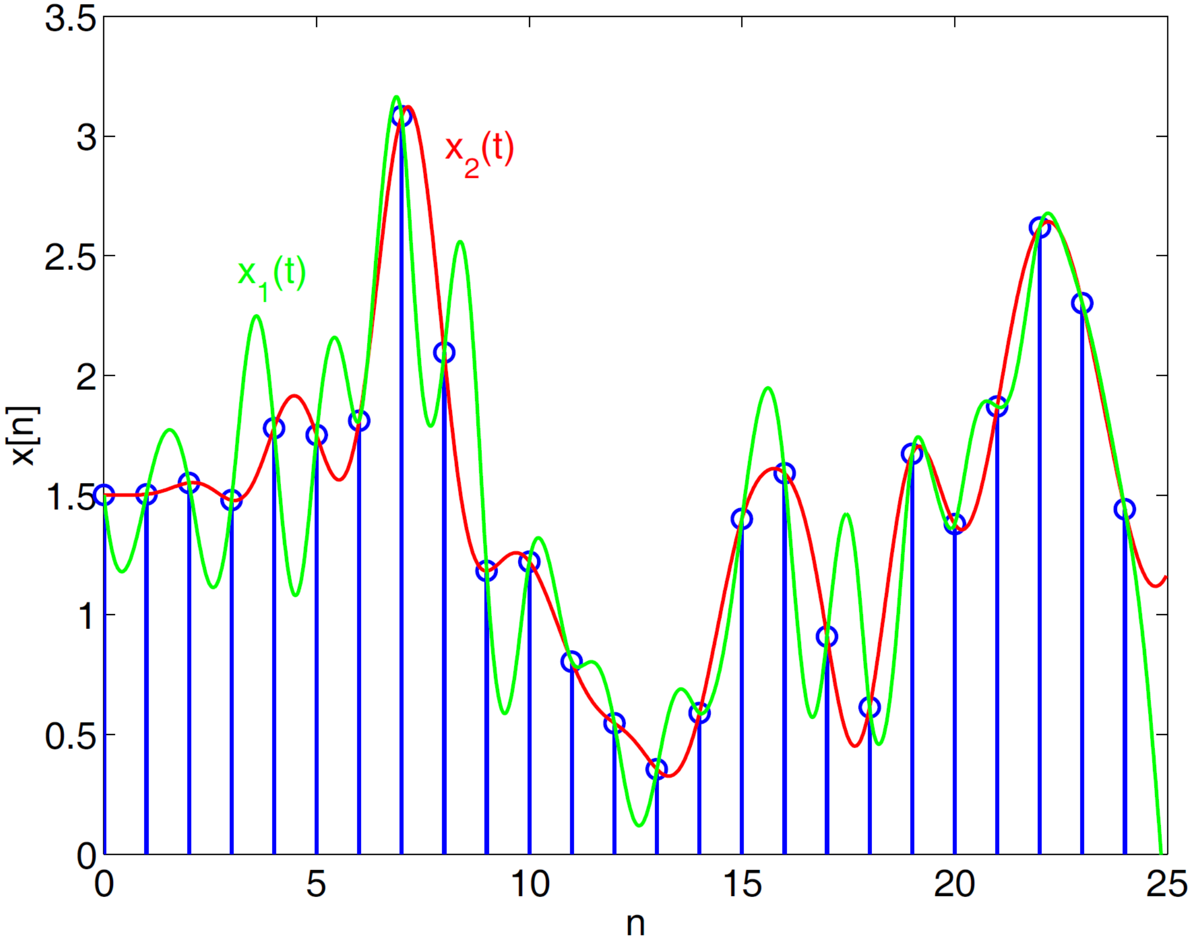


Figure 4.1 Block diagram representation of an ideal continuous-to-discrete-time (C/D) converter.

- Discrete-time representation of a continuous-time signal obtained via periodic sampling
- $x[n] = x_c(nT) \quad -\infty < n < \infty$
- T is sampling period, sampling frequency $f_s = \frac{1}{T}$
- Can also express sampling frequency as $\Omega = \frac{2\pi}{T}$ in radians per second

Many to one…

- Ambiguity in discrete-time representation
- Sampled signal $x[n]$could come from $x_1(t)$ or $x_2(t)$
- Not invertible



Source: Ian Bruce lectures, McMaster University

Impulse train sampling

- Consider as two-stage process
  - Impulse train modulator
  - Conversion to a sequence
- Periodic impulse train $s(t)$
  - $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$
- With $\delta(t)$ the unit-impulse function
- $x_s(t) = x_c(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$
- $\quad = \sum_{n=-\infty}^{\infty} x_c(t)\delta(t - nT)$
- $\quad = \sum_{n=-\infty}^{\infty} x_c(nT)\delta(t - nT)$
- Note that $x_s(t)$ is continuous!

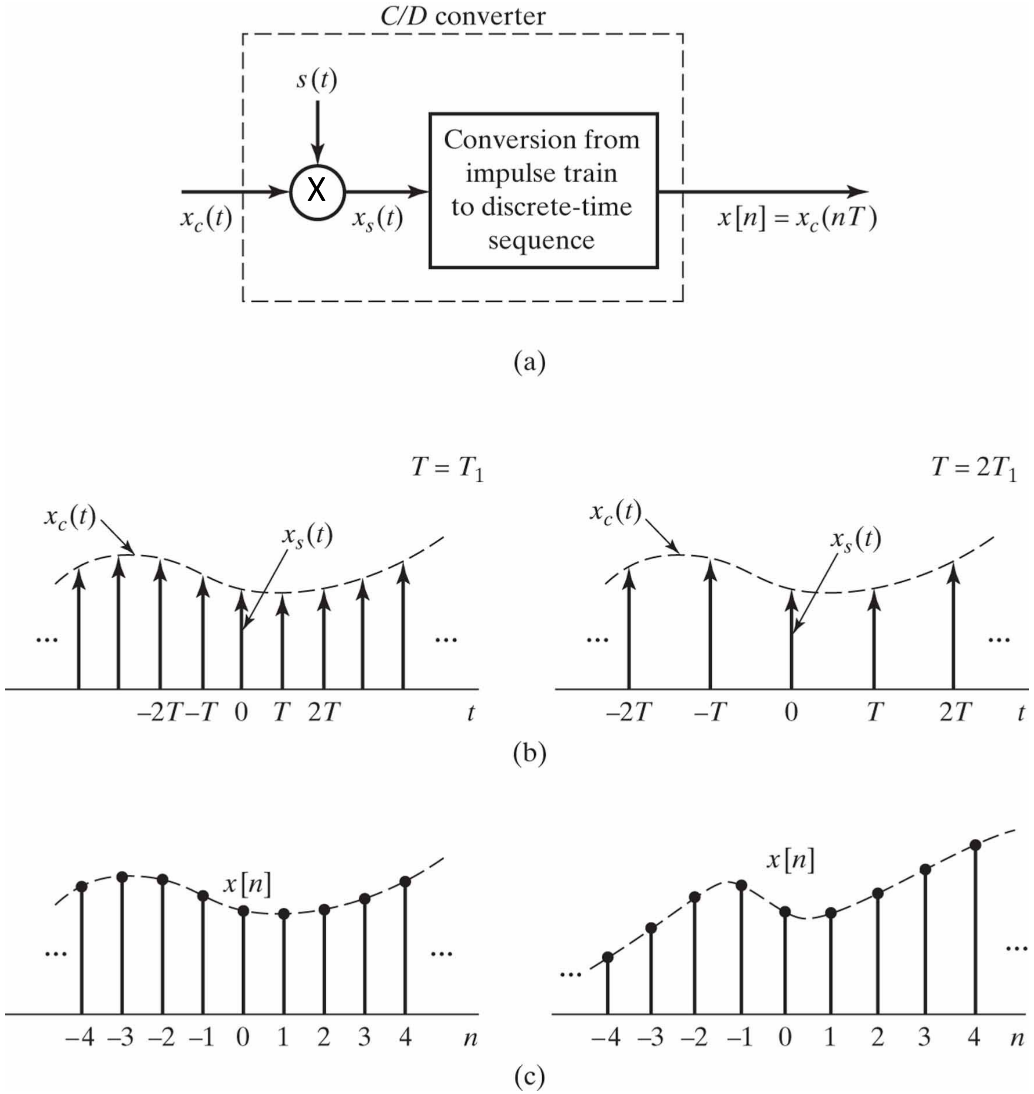


Figure 4.2 Sampling with a periodic impulse train, followed by conversion to a discrete-time sequence. (a) Overall system. (b) $x_s(t)$ for two sampling rates. (c) The output sequence for the two different sampling rates.

Frequency Domain

- Fourier transform of the periodic impulse train $s(t)$
- $S(j\Omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s)$
- with $\Omega_s = \frac{2\pi}{T}$ the sampling frequency in radians/s
- Since
  - $X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$
- Then with the continuous convolution above becomes
  - $X_s(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\big(j(\Omega - k\Omega_s)\big)$

What does this mean??

- $X_s(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\big(j(\Omega - k\Omega_s)\big)$
- Original $X_c(j\Omega)$ bandlimited such that $X_c(j\Omega) = 0$ for $|\Omega| > \Omega_N$
- $X_s(j\Omega)$ is periodically repeated copies shifted by the sampling frequency
- Need $\Omega_s - \Omega_N \geq \Omega_N$

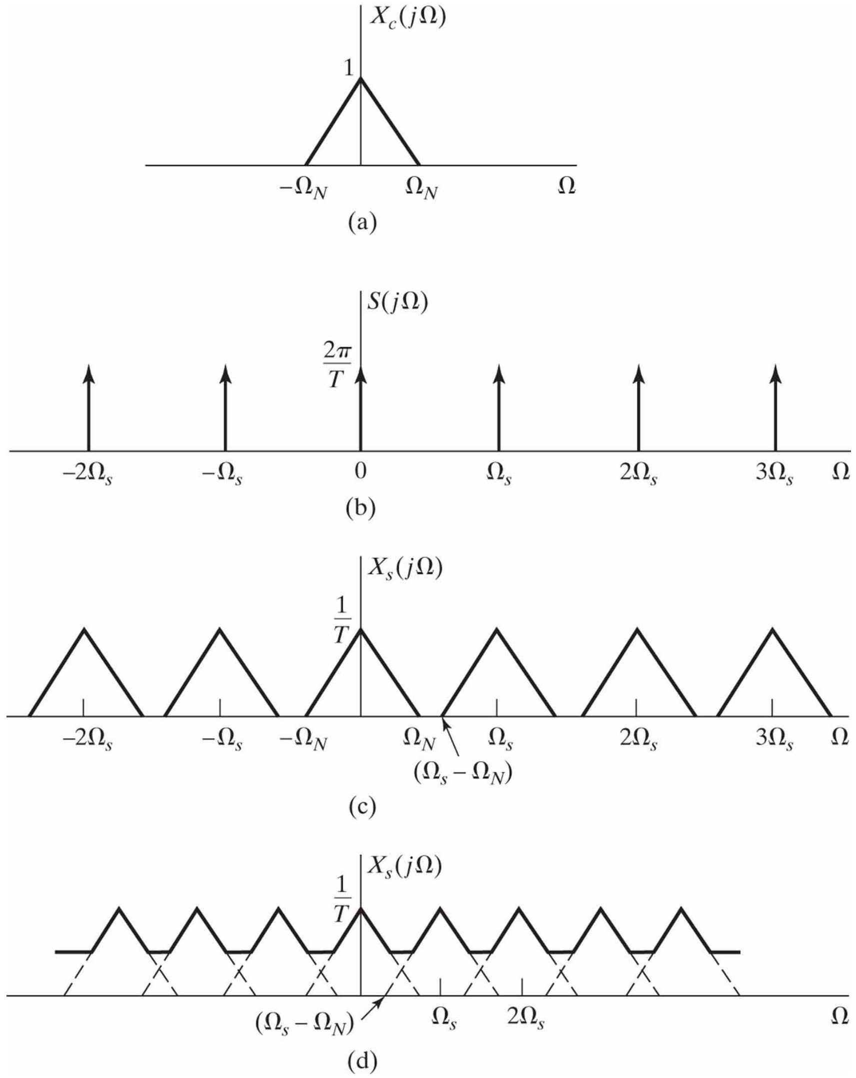


Figure 4.3 Frequency-domain representation of sampling in the time domain. (a) Spectrum of the original signal. (b) Fourier transform of the sampling function. (c) Fourier transform of the sampled signal with $\Omega_s > 2\Omega_N$. (d) Fourier transform of the sampled signal with $\Omega_s < 2\Omega_N$.

Nyquist Theorem

- Nyquist–Shannon sampling theorem
- If $x_c(t)$ is a bandlimited signal with $X_c(j\Omega) = 0$ for $|\Omega| > \Omega_N$
- Then $x_c(t)$ is uniquely determined by its samples $x[n] = \ x_c(nT), n = 0, \pm 1, \pm 2, \ldots$, if
- $\Omega_s = \frac{2\pi}{T} \geq 2\Omega_N$
- So to avoid aliasing, the sampling rate must be at least twice the bandwidth of the signal you are sampling

Terminology

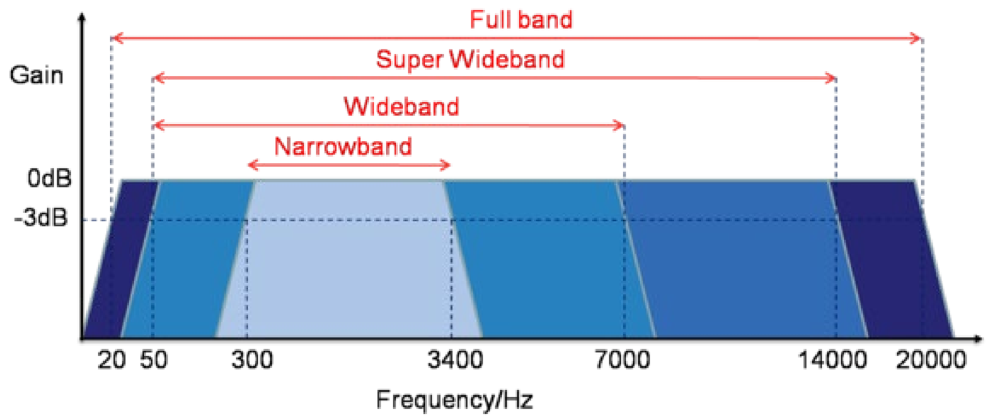
- Ω_N the Nyquist Frequency
- $2\Omega_N$ the Nyquist Rate

Effect of Sampling in the Frequency Domain

- Oversampling
  - The sampling frequency is higher than the Nyquist rate
- Undersampling
  - The sampling frequency is lower than the Nyquist rate
- Critical sampling
  - The sampling frequency is equal to the Nyquist rate
- Note: A pure sinusoid may not be recoverable from its critically sampled version

Effect of Sampling in the Frequency Domain

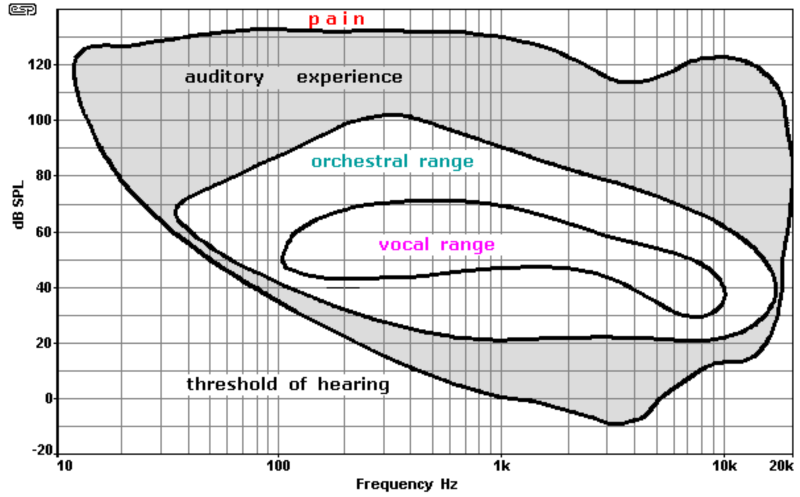
- In digital telephony ("landline"), a 3.4 kHz signal bandwidth is acceptable for telephone conversation
- Here, a sampling rate of 8 kHz, which is greater than twice the signal bandwidth, is used



https://blog.tmcnet.com/voice-of-ip/2010/03/hd_voice_-_how_much_bandwidth_do_you_need_1.html

Effect of Sampling in the Frequency Domain

- In high-quality analog music signal processing, a bandwidth of 20 kHz has been determined to preserve the fidelity
- Hence, in compact disc (CD) music systems, a sampling rate of 44.1 kHz, which is slightly higher than twice the signal bandwidth, is used



https://people.ece.cornell.edu/land/courses/ece5030/FinalProjects/s2014/kkp37_rjs483/kkp37_rjs483/AudioGram.html

Reconstruction

- Can recover original signal with an ideal lowpass filter (note scaling)
- $X_r(j\Omega) = H_r(j\Omega)\, X_s(j\Omega)$
- If $\Omega_N \leq \Omega_c \leq \Omega_s$- Ω_N
- Then
- $X_r(j\Omega) = X_c(j\Omega)$
- Works in theory!

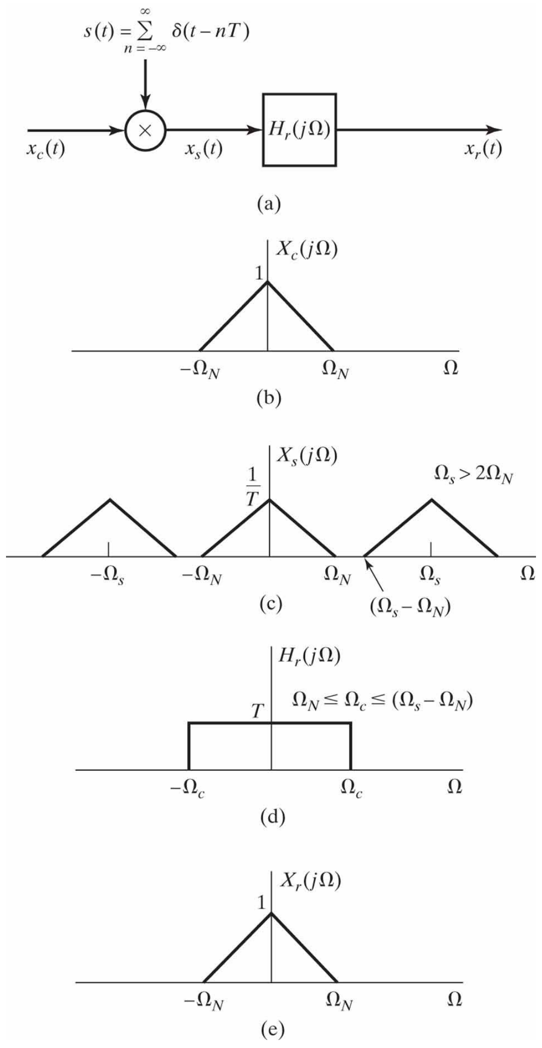


Figure 4.4 Exact recovery of a continuous-time signal from its samples using an ideal lowpass filter.

Reconstruction of signal from samples

- Form an impulse train $x_s(t)$ from a sequence $x[n]$
  - $x_s(t) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n]\delta(t - nT)$

- Pass $x_s(t)$ through ideal lowpass filter with frequency response $H_r(j\Omega)$ and corresponding impulse response $h_r(t)$ (inverse FT of $H_r(j\Omega)$)
  - $x_r(t) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n]h_r(t - nT)$

- With cut-off frequency $\frac{\pi}{T}$, impulse response givenby
  - $h_r(t) = \frac{\sin \pi t/T}{\pi t/T}$

Interpolation

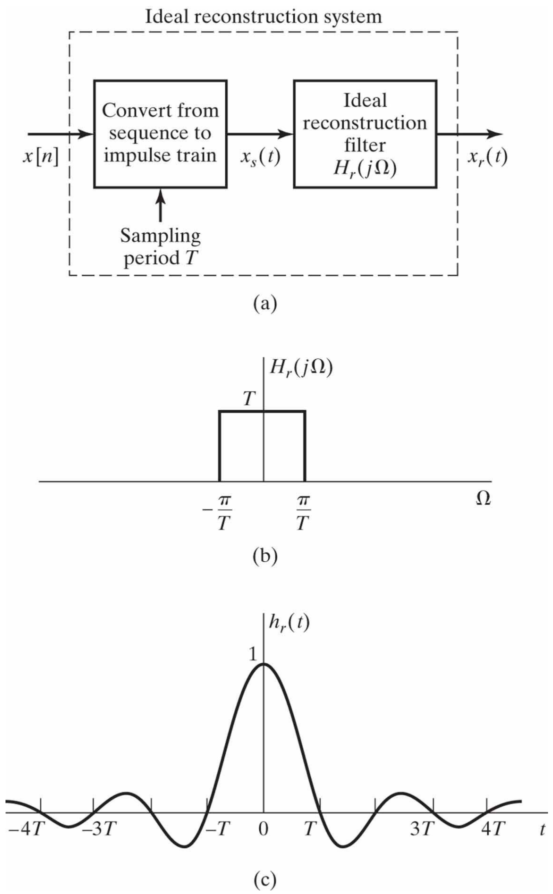


Figure 4.7 (a) Block diagram of an ideal bandlimited signal reconstruction system. (b) Frequency response of an ideal reconstruction filter. (c) Impulse response of an ideal reconstruction filter.

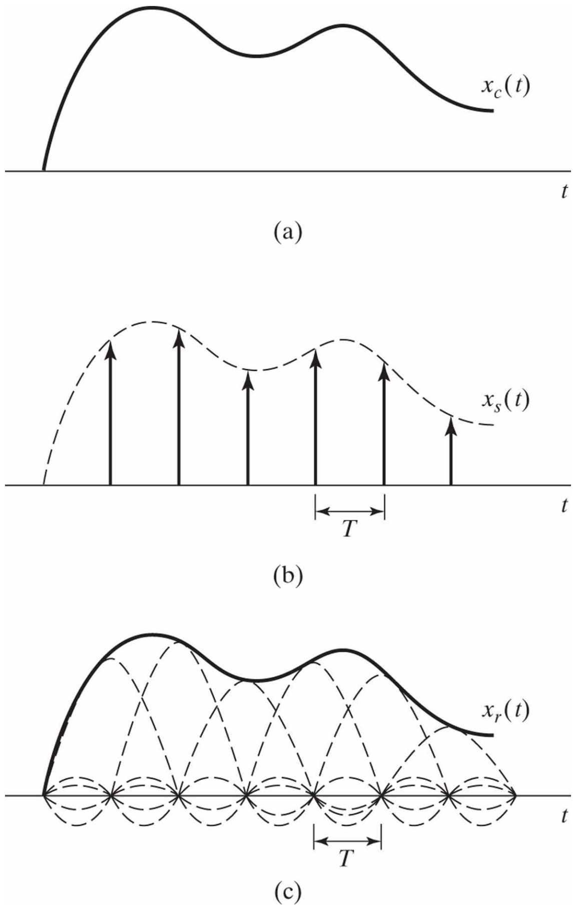


Figure 4.8 Ideal bandlimited interpolation.

Required Reading & other material

- Oppenheim & Schafer, Chapter 4, up to end Section 3