EE4C5 Digital Signal Processing

Lecture 15 – Spectral Analysis

This lecture

- Based on Chapter 10 of O&S
- All images from O&S book unless otherwise stated

Window length for analysis

- Lecture #13 showed that long window (of the correct shape) yields better frequency resolution in DFT
- The example of two sinusoids were fixed in time
  - No time varying property, no change in frequency content
- Many real signals of interest are non-stationary
  - Radar, sonar, comms data stream, audio, video…
- Conflicts with the idea of a single long window
  - Not adequate or informative for such a signal
- Short-time Fourier Transform
  - Aka the time-dependant Fourier Transform

Short-time Fourier Transform

- STFT of signal $x[n]$ defined as:

$$X[n, \lambda) = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\lambda m}$$

- where $w[n]$ is the window signal
- 1-D sequence $x[n]$ which is a function of a single discrete variable, is now converted into a 2-D function of the time variable n , which is discrete, and the frequency variable λ , which is continuous.
- STFT is periodic in λ with period 2π => only need to consider values of λ for $0 \leq \lambda < 2\pi$
- STFT can be interpreted as the DTFT of the shifted signal $x[n+m]$ as it moves past the stationary window $w[m]$.

Chirp signal

- Consider a discrete time signal which is a linear chirp

$$x[n] = \cos(\alpha_0 n^2)$$

- Over a short interval, the signal looks sinusoidal, but the spacing between peaks becomes smaller and smaller as time progresses
    - increasing frequency with time.
- Next page shows taking different windows of the signal

Signal moving past the window…

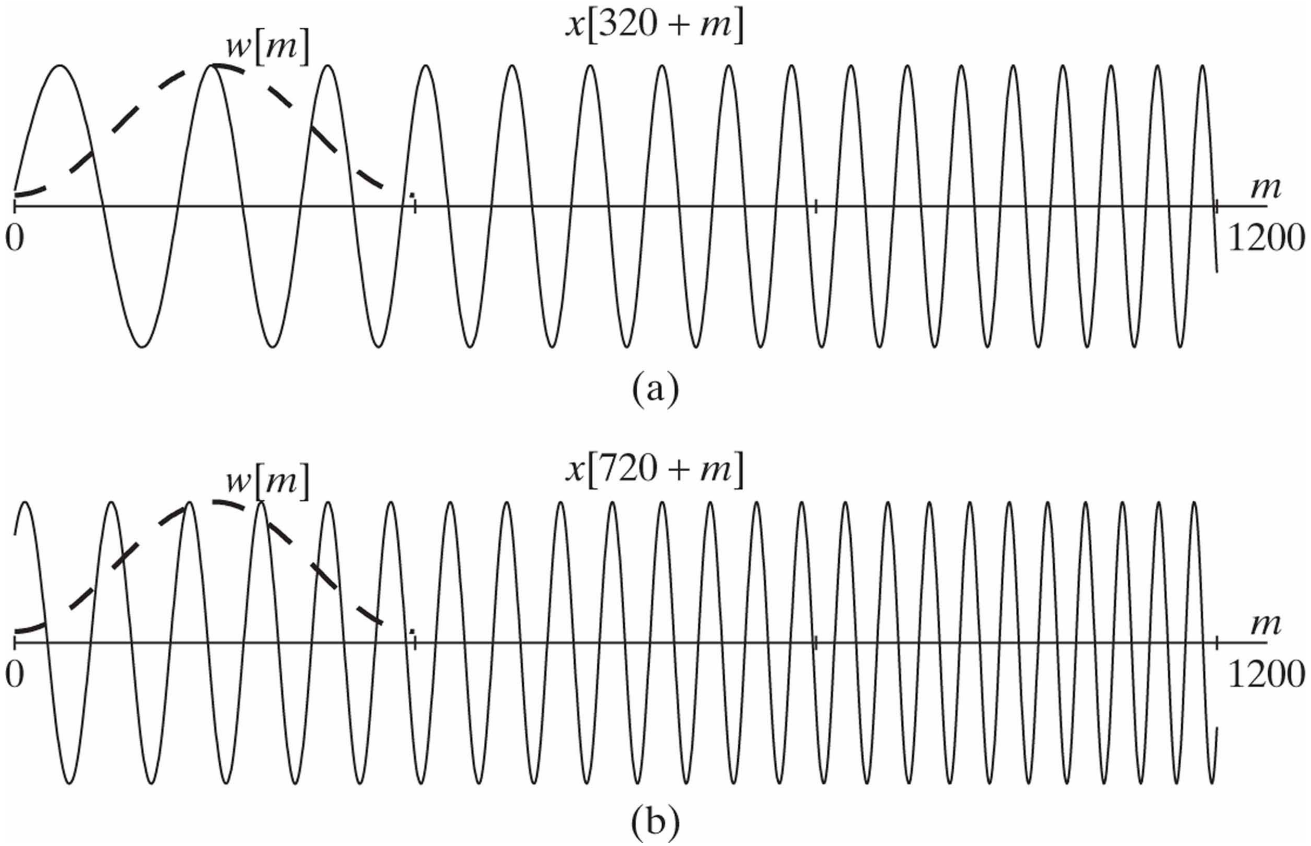


Figure 10.11 Two segments of the linear chirp signal $x[n] = \cos(\alpha_0 n^2)$ for $\alpha_0 = 15\pi \times 10^{-6}$ with a 400-sample Hamming window superimposed. (a) $X[n, \lambda)$ at $n = 320$ would be the DTFT of the top trace multiplied by the window. (b) $X[720, \lambda)$ would be the DTFT of the bottom trace multiplied by the window.

DTFT

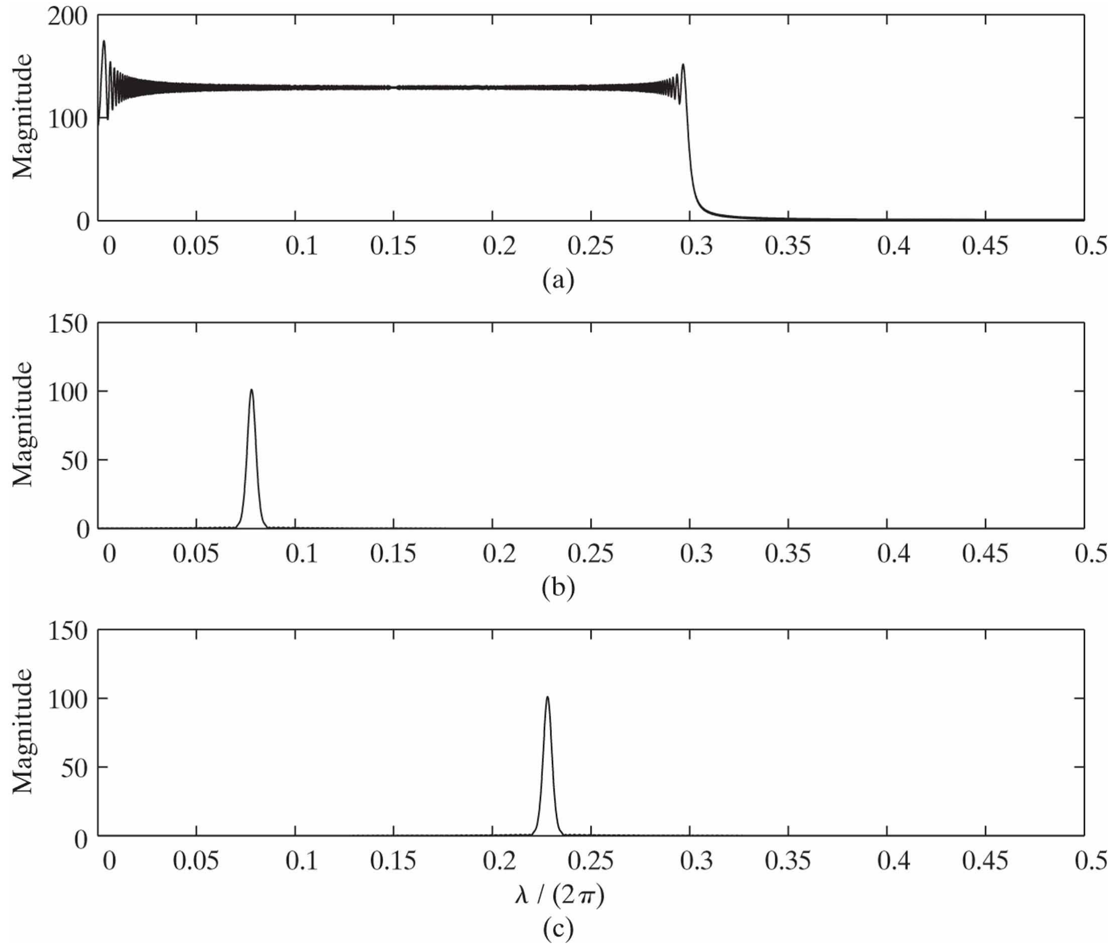


Figure 10.12 DTFTs of segments of a linear chirp signal:

(a) DTFT of 20,000 samples of the signal $x[n] = \cos(\alpha_0 n^2)$.

(b) DTFT of $x[5000 + m]w[m]$ where $w[m]$ is a Hamming window of length $L = 401$; i.e., $X[5000, \lambda)$.

(c) DTFT of $x[15{,}000 + m]w[m]$ where $w[m]$ is a Hamming window of length $L = 401$; i.e., $X[15{,}000, \lambda)$.

STFT is more informative

- To illustrate what we will see in a STFT, let's use a new signal:

$$y[n] = \begin{cases} 0 & n < 0 \\ \cos(\alpha_0 n^2) & 0 \leq n \leq 20{,}000 \\ \cos(0.2\pi n) & 20{,}000 < n \leq 25{,}000 \\ \cos(0.2\pi n) + \cos(0.23\pi n) & 25{,}000 < n. \end{cases}$$

- Signal $y[n]$ is equal to $x[n]$ of slide #5 for $0 \leq n \leq 20{,}000$
- Then it abruptly changes to cosine components with fixed frequencies for $n > 20{,}000$

STFT of $y[n]$

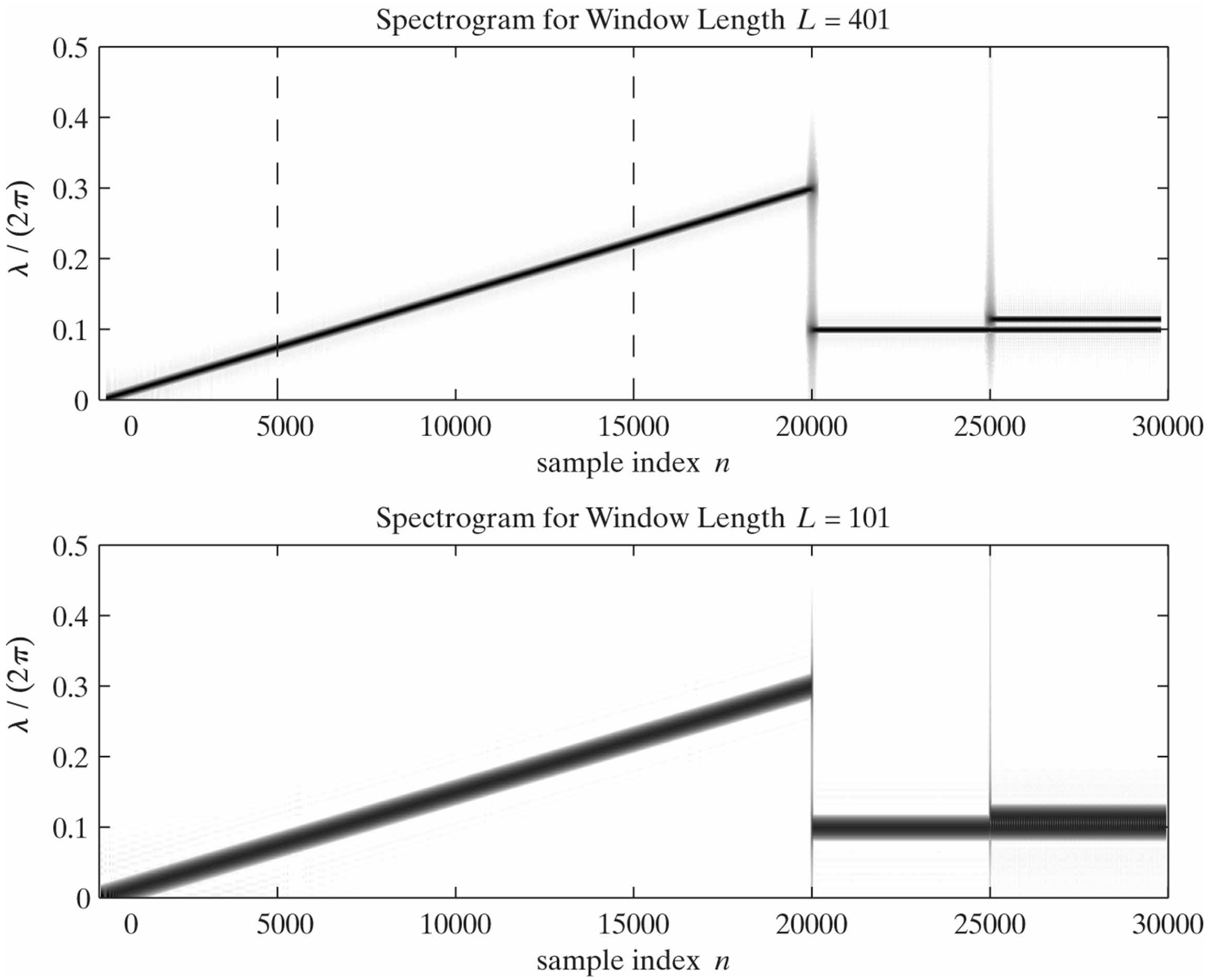
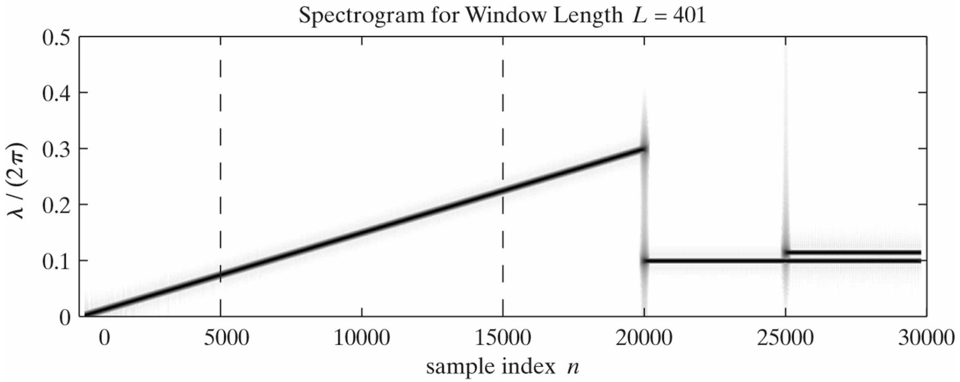


Figure 10.13 The magnitude of the time-dependent Fourier transform of $y[n]$ from slide #8

(a) Using a Hamming window of length L = 401.

(b) Using a Hamming window of length L = 101.

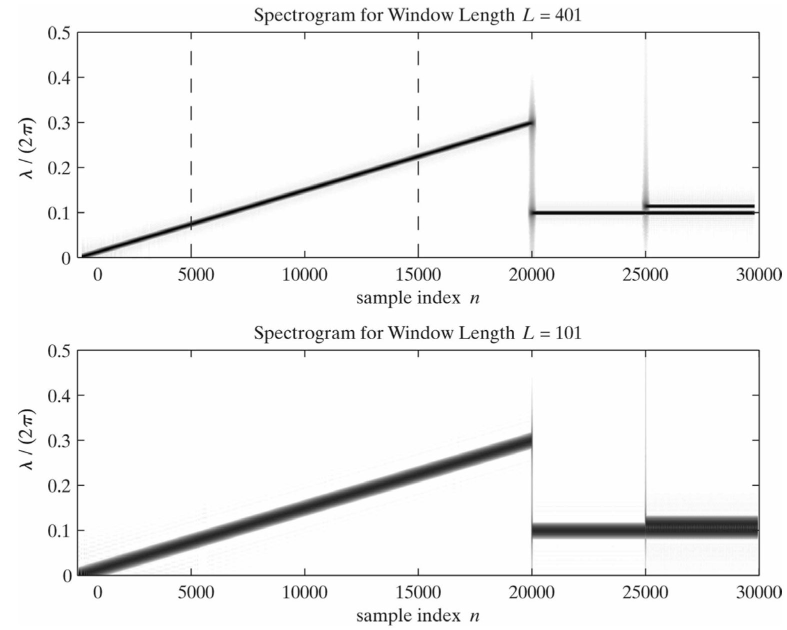
Spectrogram



- This time-frequency plot is a spectrogram
  - a graphical display of the magnitude of the time-varying discrete STFT
- Have plotted $20\log_{10}|Y[n,\lambda)|$ as a function of $\lambda/2\pi$ in the vertical dimension
  - Will also just see it plotted in Hz
  - Typically linear or log spacing (other perceptual scales for speech)
  - The "frequency" axis
- Plot the time index n in the horizontal dimension
  - The time axis
- Value $20\log_{10}|Y[n,\lambda)|$ over a restricted range of $50dB$ is represented by the darkness of the marking at $[n,\lambda)$

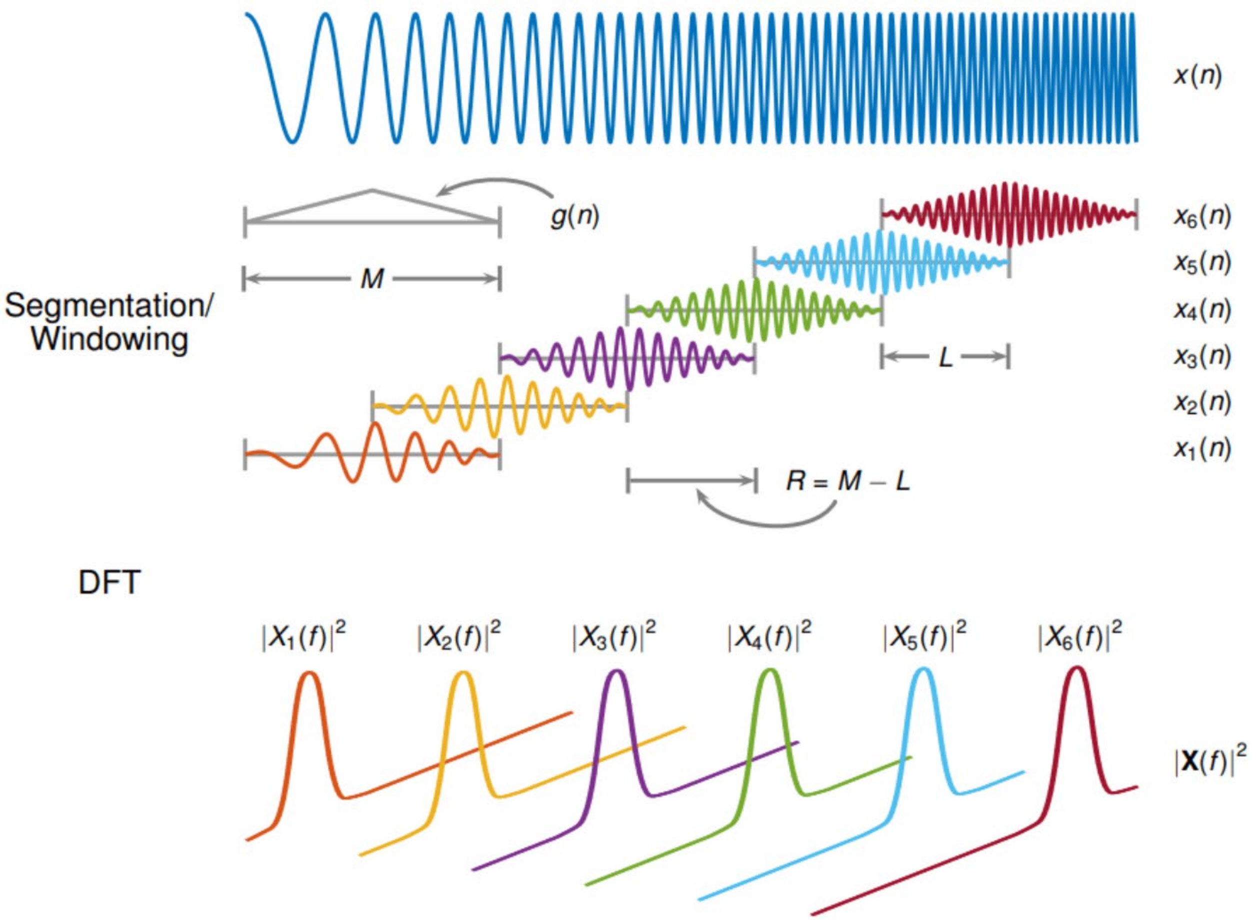
Shorter window?

- 401-sample window provides good frequency resolution at almost all points in time
- at $n = 20{,}000$ and $25{,}000$ the signal properties change abruptly,
  - the window contains samples from both sides of the change.
- can improve the ability to resolve events in the time dimension by shortening the window to 101-samples
  - But now can't resolve our two frequencies
- Time-frequency resolution trade-off
  - Different windows will emphasise different features



Another conceptual diagram

- Taken from Matlab documentation at https://uk.mathworks.com/help/signal/ref/spectrogram.html
- Though I haven't matched notation to ours so watch out!



Examples in class

- In lectures, we'll have a look at many different spectrograms and see what features we can pick out for different signals…

Note C5 Piano versus Violin (in class)

- Mechanical differences in sound production result in different spectrogram patterns between different instruments producing the same isolated note
- Piano
  - Observe a well-defined attack with a sharp onset
    - when a key is pressed, a hammer strikes the strings abruptly, creating a sudden and well-defined start to the sound
  - Rich set of harmonics that decay gradually over time.
  - The sustain in a piano note contributes to a consistent presence of harmonics throughout its duration
- Violin
  - The attack gentler due to the nature of bowing
  - Harmonics exhibit a different distribution, influenced by the violin's unique tonal qualities
  - Resonances, sympathetic vibrations, and expressive nature of bowing add complexity to the spectrogram, creating peaks and troughs

Required Reading & other material

- Oppenheim & Schafer, Chapter 10
- Entertaining animated spectrograms (but little to do with 4C5)
  - https://www.youtube.com/watch?v=Hxx6Gqf1Q4w
- Audacity is free software for recording and editing audio
  - Great spectrogram capabilities
  - https://www.audacityteam.org/
- We’ll be using MATLAB in Lab 4, some intro material at:
  - https://uk.mathworks.com/help/signal/ug/spectrogram-computation-with-signal-processing-toolbox.html