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Lecture 4: Orthogonal Signalling and Multi-tone Modulation

EE412 - Wireless Digital Communications

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Outline

- Describe the differences between single-user detection and multiuser detection in CDMA.
- Analyse the capacity of a CDMA channel when single-user detection is performed.
- Apply the method for maximum likelihood to perform multiuser detection.
- Analyse the capacity of a CDMA channel when multiuser detection is performed.

Introduction

- In CDMA systems, there are two ways to extract or ‘detect’ the information of all users using the same channel:
  - *Single-user detection* (or non-cooperative detection) extracts the user of interest through correlation with the associated PN sequence, disregarding information from other users’ signals (e.g., their spreading codes). In this method, signals from other users manifest as interference, reflecting the receiver structure discussed in the previous lecture. Typically, this type of signal detection is performed by a mobile station in the downlink, focusing on extracting the information of a specific user in the downlink channel.
  - *Multiuser detection* (or cooperative detection) simultaneously detects all K users, providing better performance than single-user detection. However, the more complex implementation poses challenges to scalability, making it difficult to support a large number of users. Base stations use multiuser detection to extract information from all users simultaneously, utilizing the same CDMA channel.

Single User Detection Channel Capacity

- In single-user CDMA detection with K users using the same channel, the signal power corresponds to the power of any single user of interest, P_k , and the noise power corresponds to the sum of the thermal noise, WN_0, and the interference power of the other $(K-1)$ users, $\sum_{n \neq k} P_n$.
- Since, at the receiver, de-spreading only affects the user of interest's information, this translates into a **processing gain** (G) with respect to the other users' signals. The processing gain can be considered to be equal to the spreading factor $(G = N)$. If the noise is considered wideband, the processing gain does not reduce the effects of the noise. This gives a Signal-to-Interference-plus-noise ratio (SINR):

$$\mathrm{SINR} = \frac{GP_k}{GWN_0 + \sum_{n \neq k} P_n} \quad \text{(wideband noise)} \tag{1}$$

Single User Detection Channel Capacity...

- In the presence of narrowband noise, this noise can be considered as another interferer which effect is reduced by the de-spreading operation. Therefore, the SINR is:
$$\mathrm{SINR} = \frac{GP_k}{WN_0 + \sum_{n \neq k} P_n} \quad \text{(narrowband noise)} \tag{2}$$
- From these two expressions we can see the near-far problem. Users with a high received power cause a large noise for other users. To overcome this problem power control must be implemented.
- From (1), given appropriate power control, the receive power due to each one of the K users will be approximately equal and the capacity per user may be written:
$$C_k = W \log_2 \left(1 + \frac{GP}{GWN_0 + (K-1)P}\right) \quad \text{bits/s} \tag{3}$$

Single User Detection Channel Capacity...

- The efficiency is specified as capacity per unit bandwidth which is:

$$\frac{C_k}{W} = \log_2\left(1 + \frac{GP}{GWN_0 + (K-1)P}\right) \quad \text{bits/s/Hz} \tag{4}$$

and it is also more interesting to look at capacity in terms of the energy-per-bit to noise density ratio, i.e., E_b/N_0 ,where:

$$E_b = \frac{P}{C} \implies P = CE_b \tag{5}$$

thus

$$\begin{aligned} \frac{C_k}{W} &= \log_2\left(1 + \frac{GC_kE_b}{GWN_0 + (K-1)C_kE_b}\right) && (6) \\ &= \log_2\left(1 + \left(\frac{C_k}{W}\right)\frac{G\left(\frac{E_b}{N_0}\right)}{G + (K-1)\left(\frac{C_k}{W}\right)\left(\frac{E_b}{N_0}\right)}\right) \quad \text{bits/s/Hz} && (7) \end{aligned}$$

Single User Detection Channel Capacity...

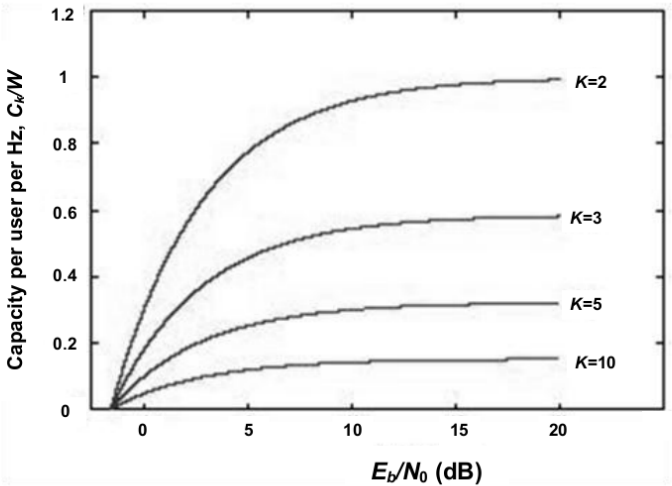

Figure: Capacity per user per Hz vs energy-per-bit in CDMA.

Single User Detection Channel Capacity (cont)

- Figure 1 illustrates the representation of C_k/W versus E_b/N_0 with K as a parameter. Unlike TDMA and FDMA, in CDMA systems, even though E_b/N_0 increases, C_k/W tends to a limit.
- As for TDMA and FDMA, letting $C_n = KC_k/W$ being the total capacity of the channel, it is obtained that:

$$C_n \leq \frac{1}{\ln 2} - \frac{1}{E_b N_0} \leq \frac{1}{\ln 2} \tag{8}$$

- Unlike TDMA and FDMA, in CDMA when the number of users K increases the total capacity C_n does not increase indefinitely but tends to a limit even though the total power increases with the number of users.

Optimum CDMA Multiuser Detection

- In optimum multiuser detection (MUD)::
  - The multiuser receiver knows the spreading codes of the K different users.
  - In the demodulation of the information of the user of interest, the receiver takes into account the information of the other $K-1$ users to determine the transmitted information.
  - The maximum likelihood algorithm is an optimum MUD method.
  - In this type of detection, the number of operations that are required increases exponentially with the number of users K.
  - Sub-optimum detectors also exist and are constantly being developed:
    - The multiuser receiver knows the spreading sequences of the K different users.
    - In the demodulation process of the information of the user of interest, the signals from the other $K-1$ users are considered as additive noise.
    - Single-user detection, minimum mean square error detection (MMSE) and successive interference cancellation (SIC) are different sub-optimal CDMA multiuser receivers.
    - The number of operations that are required linearly increases with the number of users K.

The optimum detector: Maximum Likelihood...

- The optimum detector selects the set of transmit bits which correspond to the receive bits with the highest probability:
  - This is known as the Method of Maximum Likelihood (ML).
  - The Method of Maximum Likelihood determines the transmission bit with the highest probability.
- For simplicity, we make the following two assumptions:
  - We assume cooperative MUD, where:
    - All signals are synchronized at the receiver.
    - The multiuser receiver knows the spreading sequences of all the users.
  - Only a single symbol from each user may interfere with any symbol from the desired user.

The optimum detector: Maximum Likelihood...

- Considering that the users employ antipodal signals to transmit their information, the receive signal over a single symbol interval is of the form:

$$r(t) = \sum_{k=1}^{K} s_k g_k(t) + n(t) \tag{9}$$

$s_k(t)$ is the kth user transmit symbol, $g_k(t)$ is the spreading sequence plus pulse shaping of the kth user, and $n(t)$ is the noise term.

- ML detection finds the set of transmissions which minimizes the distance between the actual received signal and the signal corresponding to each possible input:
  - At any time instant, the Euclidean distance between the receive signal, $r(t)$, and the candidate signal is:

$$\left| r(t) - \sum_{k=1}^{K} s_k^{(\text{candidate})} g_k(t) \right|^2 \tag{10}$$

  - Given K users each transmitting B bits per symbol, the total number of candidate sets is 2^{KB} .

The optimum detector: Maximum Likelihood...

- The function to be minimized associated with this detector is squared Euclidean distance over the entire signal interval. This is called the log-likelihood function:

$$\Lambda = \int_0^T \left[r(t) - \sum_{k=1}^{K} s_k^{(\text{candidate})} g_k(t) \right]^2 dt \tag{11}$$

- The log-likelihood function expands as (where we drop the candidate superscript):

$$\begin{aligned}
\Lambda &= \int_0^T \left[r(t) - \sum_{k=1}^{K} s_k^{(\text{candidate})} g_k(t) \right]^2 dt \qquad (12)\\
&= \int_0^T r^2(t)dt + \sum_{i=1}^{K}\sum_{j=1}^{K} s_i s_j \int_0^T g_i(t) g_j(t) dt \qquad (13)\\
&\quad -2\sum_{k=1}^{K} s_k \int_0^T r(t) g_k(t) dt \qquad (14)
\end{aligned}$$

The optimum detector: Maximum Likelihood...

- We note the following facts about the log-likelihood function:
- $\int_0^T r^2(t)dt$ is independent of the sequence and thus may not be relevant for determining the transmitted sequence.
  - $\int_0^T g_i(t)g_j(t)dt$ is the cross-correlation between two spreading waveforms and is known for each pair (i, j) because the individual spreading waveforms are assumed known, we denote this expression as ϕ_{ij} .
  - $\int_0^T r(t)g_k(t)dt$ is the cross-correlation of the received signal with each of the received spreading waveforms, we denote this as ϕ_k.
- We can now express the log-likelihood function in terms of its correlation metrics as

$$\Lambda = \sum_{i=1}^{K} \sum_{j=1}^{K} s_i s_j \phi_{ij} - 2 \sum_{k=1}^{K} s_k \phi_k \tag{15}$$

- The sequence of symbols $b_1, \ldots, b_k$ that minimizes the value of the expression above is selected as the transmitted sequence.

Multiuser detection channel capacity

- In multiuser detection, all the users signals are demodulated and received simultaneously:
  - Here we assume that the receiver has access to all the spreading waveforms and all the users are transmitting cooperatively (i.e. synchronously in time - best case scenario).
- Assuming that each user has adjusted such that each is received with equal power, P, and letting R_k be the transmission rate of the kth user, then the following are true:

$$R_k < W \log_2\left(1 + \frac{P}{WN_0}\right) \quad \forall k \leq K \tag{16}$$

$$R_{k_1} + R_{k_2} < W \log_2\left(1 + \frac{2P}{WN_0}\right) \quad \forall k_1, k_2 \leq K, \ k_1 \neq k_2 \tag{17}$$

$$\vdots \tag{18}$$

$$\sum_{j=1}^{K} R_{k_j} < W \log_2\left(1 + \frac{KP}{WN_0}\right) \tag{19}$$

Multiuser detection channel capacity...

- If all the rates are identical (equal to R) then the final inequality is dominant and defines achievable performance given K users, i.e.,

$$KR < W \log_2\left(1 + \frac{KP}{WN_0}\right) \Rightarrow R < \frac{W}{K}\log_2\left(1 + \frac{KP}{WN_0}\right) \tag{20}$$

  note that this is exactly the same constraint on total transmission rate as with FDMA/TDMA.
- Note that this is only the constraint when all the rates are identical.

Multiuser detection channel capacity...

- **Example**: Given a CDMA system with two users, using a bandwidth W, having equal receive power from each user, P, the following are the constraints on transmission rate:

$$
\begin{aligned}
R_1 &< W \log_2 \left(1 + \frac{P}{WN_0}\right) && (21) \\
R_2 &< W \log_2 \left(1 + \frac{P}{WN_0}\right) && (22) \\
R_1 + R_2 &< W \log_2 \left(1 + \frac{2P}{WN_0}\right) && (23)
\end{aligned}
$$

Multiuser detection channel capacity...

- Assume that user 1 is transmitting at capacity,

$$R_1 = C_1 = W \log_2 \left(1 + \frac{P}{WN_0} \right) \tag{24}$$

then from the third inequality:

$$\begin{aligned} R_2 &< W \log_2 \left(1 + \frac{2P}{WN_0} \right) - R_1 && (25) \\ &= W \log_2 \left(1 + \frac{2P}{WN_0} \right) - C_1 && (26) \\ &= W \log_2 \left(1 + \frac{2P}{WN_0} \right) - W \log_2 \left(1 + \frac{P}{WN_0} \right) && (27) \\ &= W \log_2 \left[1 + \frac{P}{WN_0 + P} \right] && (28) \end{aligned}$$

Multiuser detection channel capacity...

- Similarly, assuming that user 2 is transmitting at capacity, then:

$$R_1 = W \log_2 \left[1 + \frac{P}{WN_0 + P} \right] \tag{29}$$

- These two inequalities define bounds on possible rates.
- Note that when one user is transmitting at capacity, the other user's capacity is reduced as if the first user is additive noise. This corresponds to single-user detection (i.e. the interfering user's power goes to reducing the rate to the other user).

Multiuser detection channel capacity...

- From the example, we may draw a diagram of achievable rates

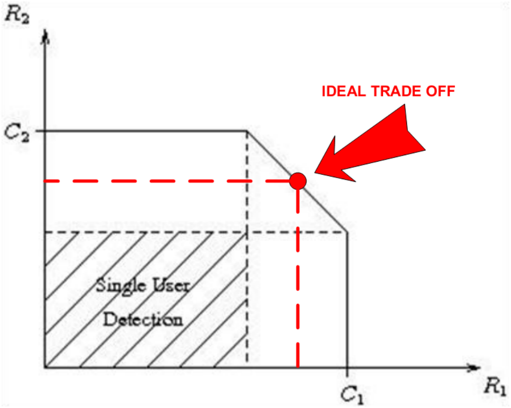

Figure: Achievable rate region for two-user CDMA

Optimum Detection Example Exercise

- Consider a CDMA system with 2 users transmitting BPSK (i.e., levels -1, 1) over an AWGN channel. The users are assigned spreading sequences of unit energy corresponding to the binary sequences:

$$\text{User 1} = [110101001] \tag{30}$$
$$\text{User 2} = [101010101] \tag{31}$$

  and shaped as antipodal rectangular pulses.

  1. Sketch the two shaping waveforms.
  2. Are the two waveforms orthogonal to each other?
  3. Given a sampled receive sequence, $r_n = [0.012\ 0.05\ -0.96\ 0.71\ -0.11\ 0.60\ 0.05\ -0.11\ -0.11]$, use the method of maximum-likelihood to decide the symbols transmitted by each user (assuming synchronous reception).

Optimum Detection Example Exercise...

- **Solution**
- Shaping the sequences as antipodal rectangular pulses of unit energy involves the mapping (recall that energy is related to the sum of the squares of the amplitudes):

$$1 \rightarrow \frac{1}{\sqrt{9}} \quad 0 \rightarrow \frac{-1}{\sqrt{9}} \tag{32}$$

We determine whether the waveforms are orthogonal by examining the effect of despreading user 1 with the waveform of user 2, i.e., multiply the elements at each index and add (corresponds to the receiver given in the notes):

$$\begin{array}{l|ccccccccc}
\text{User 1} & \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\
\text{User 2} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\
\hline
 & \frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
\hline
\end{array} = -\frac{1}{3}$$

since this is not equal to zero, the waveforms are not orthogonal to each other.

Optimum Detection Example Exercise...

We choose as the transmitted bits the bits which minimize the log-likelihood function:

$$\Lambda = \sum_{i=1}^{2}\sum_{j=1}^{2} s_i s_j \phi_{ij} - 2\sum_{k=1}^{2} s_k \phi_k \tag{33}$$

which may be equivalently written:

$$\Lambda = s_1^2\phi_{11} + s_2^2\phi_{22} + 2s_1 s_2 \phi_{12} - 2s_1\phi_1 - 2s_2\phi_2 \tag{34}$$

where $\phi_{11} = \phi_{22} = 1$ since the waveforms are unit energy, and $\phi_{12} = \phi_{21} = -1/3$ as we determined in the first part of the question.
We calculate ϕ_1 and ϕ_2 as :

$$\phi_1 = \frac{0.012 + 0.05 + 0.96 + 0.71 + 0.11 + 0.6 - 0.05 + 0.11 - 0.11}{3} = 0.797 \tag{35}$$

and

$$\phi_2 = \frac{0.012 - 0.05 - 0.96 - 0.71 - 0.11 - 0.6 + 0.05 + 0.11 - 0.11}{3} = -0.789 \tag{36}$$

Optimum Detection Example Exercise...

The log-likelihood function is thus:

$$\Lambda = s_1^2 + s_2^2 - \frac{2}{3} s_1 s_2 - 1.594 s_1 + 1.579 s_2 \tag{37}$$

There are a total of 4 possibilities for $\{b_1, b_2\}$. We tabulate the log-likelihood function in the following table:

s_1	s_2	Λ
-1	-1	1.348
-1	+1	5.84
+1	-1	-0.506
+1	+1	1.318

The most likely set of inputs is determined by the values of Λ, the most likely input will result in the smallest value of Λ. Note that Λ is a log number therefore a negative value corresponds to a fractional value between 0 and 1.

Optimum Detection Example Exercise...

Thus, in the example above the input of maximum-likelihood is:

$$\text{User } 1 = +1 \quad \text{User } 2 = -1 \tag{38}$$

The ML procedure given above is an idealised version of the true problem because:

- The above treatment assumed synchronous reception of the spreading waveforms.
- Given asynchronous reception, it is highly probable that more than one symbol from an interfering user is responsible for the interference.
- The log-likelihood function in this case depends not only on a candidate set of symbols (a larger set since we need to consider two symbols from each interferer), but also the delays between the received signals (since this will affect the cross-correlation terms).
- The total complexity of the more realistic case of asynchronous reception is significantly higher than for the synchronous case and is thus limited in usefulness to the case of very few users.