EE4C5 Digital Signal Processing

Lecture 11 – Optimal FIR Design

This lecture

- Based on Chapter 5 of O&S
- All images from O&S book unless otherwise stated

Optimisation approaches

- What is the best filter of a given order M that you can get?
- Need to define “best”
- For windowing for example, might want to minimise the mean square error as:
  - $\varepsilon^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H_d\left(e^{j\omega}\right) - H_{opt}\left(e^{j\omega}\right)\right|^2 d\omega$
- Transpires that the solution to the above is our rectangular window!
  - $h_{opt}[n] = \begin{cases} h_d[n] =, \ 0 \leq n \leq M \\ 0, \ \text{otherwise} \end{cases}$
- Already know this is problematic (how exactly?)

Alternative criteria

- Minimisation of maximum error (minmax strategy)
- Frequency-weighted error criterion

Type 1 FIR linear-phase Filter

- Design the zero-phase filter with impulse response that satisfies
  - $h_e[n] = h_e[-n]$
- Then time-shift to ensure causal
  - $A_e\left(e^{j\omega}\right) = \sum_{n=-L}^{L} h_e[n]e^{-j\omega n}$
  - With $M = L/2$
- Can rewrite as:
  - $A_e\left(e^{j\omega}\right) = h_e[0] + \sum_{n=1}^{L} 2h_e[n]\cos(\omega n)$
  - $A_e\left(e^{j\omega}\right)$ is a real, even and periodic fxn of ω
- Optimally adjust parameters within (possibly fixed) tolerances

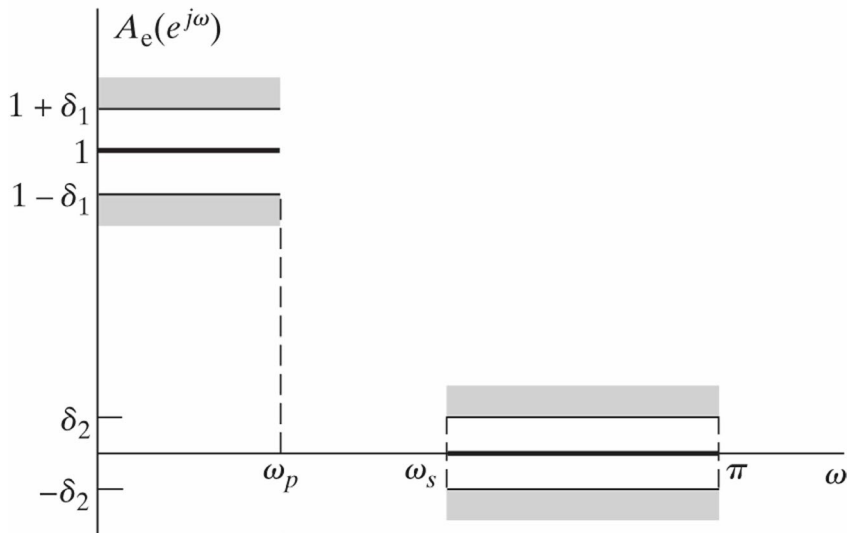


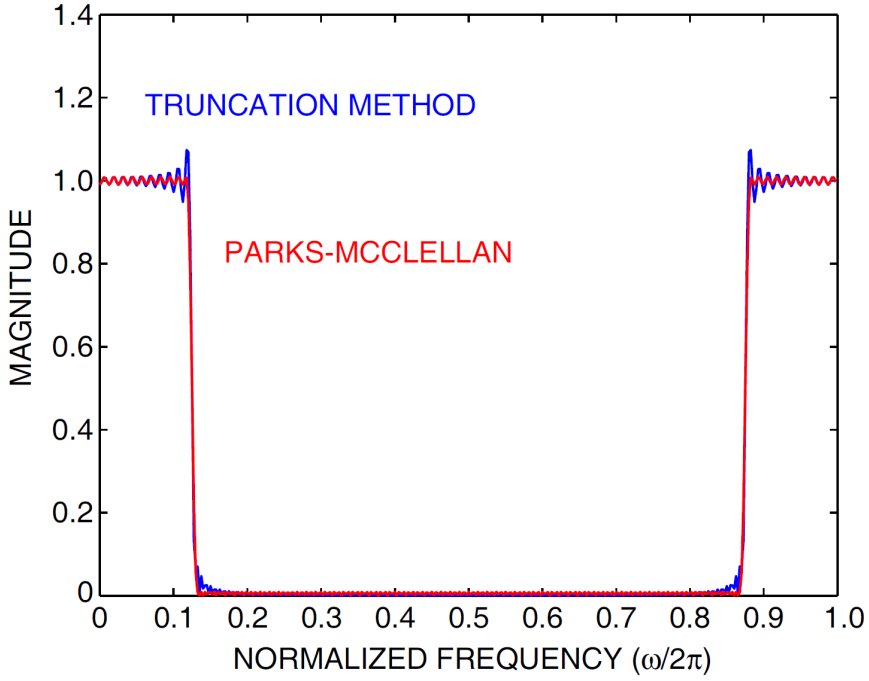
Figure 7.40 Tolerance scheme and ideal response for lowpass filter.

Type 1 FIR linear-phase Filter #2

- Causal => delay by $L = M/2$ samples
- Resulting system has impulse response
  - $h[n] = h_e[n - M/2] = h[M - n]$
- And frequency response
  - $H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega M/2}$

Parks-McClellan algorithm

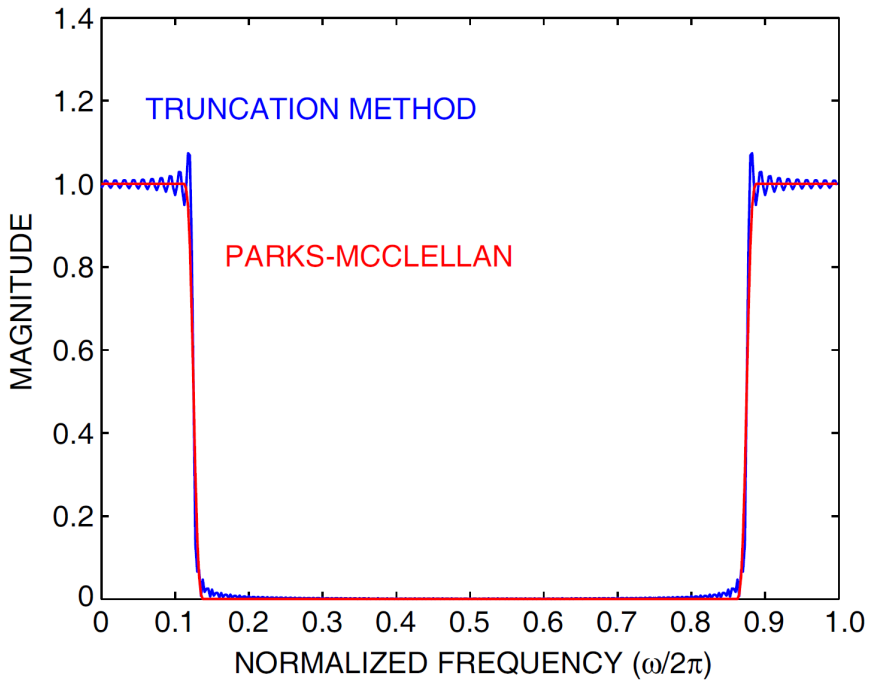
- Iteratively determines the optimal (in the minimax sense) equiripple FIR filter for a given desired frequency response.
- Ex Parks-McClellan optimal (minimax) equiripple FIR filter lowpass frequency response with M= 160
- If some nonzero passband ripple and stopband attenuation is permissible
- => a very sharp transition region matching that of the truncation method can be obtained.



Source: Ian Bruce Lectures, McMaster University

Parks-McClellan algorithm #2

- Same Ex but with just a slight relaxation of the slope of the transition region,
- => the passband ripple and stopband attenuation can be dramatically reduced.
- Overall method will yield lower order filters than windowing method



Filter Designer in Matlab

- Intro given in lecture

Required Reading & other material

- Oppenheim & Schafer, Chapter 5
  - Read closing remarks in Section 9
- Filter Designer in Matlab
- Window Designer in Matlab