Tutorial 2

Q1 From equation 135 of the O&S book chapter 4, we have:

$$SNR_Q = 10 \log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right)$$

$$= 10 \log_{10}\left(\frac{12 \cdot 2^{2B}\ \sigma_x^2}{X_m^2}\right)$$

$$= 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right)$$

We know that $X_m = 3\sigma_x$, hence:

$$SNR_Q = 6.02B + 10.8 - 20\log_{10}\left(\frac{3\cancel{\sigma_x}}{\cancel{\sigma_x}}\right)$$

$$= 6.02B + 1.27$$

If we want a signal-to-quantization noise ratio of 90 dB, we require

$$B = \frac{90 - 1.27}{6.02} = 14.74$$

that is $\boxed{\lceil B + 1 \rceil = 16 \text{ bits}}$

Q2 For a bipolar signal with amplitudes that fall within the range $[-X_m, X_m]$, the signal-to-quantization noise ratio is

$$SQNR = 6.02B + 10.8 - 20\log_{10}\frac{X_m}{\sigma_x}$$

For a nonnegative signal confined to the interval $[0, 1]$, the SQNR is equivalent to the bipolar case if we set $X_m = 0.5$.

We assume that the intensities X are uniformly distributed over $[0, 1]$.

That is $\sigma_x^2 = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

$$= \left(\frac{1}{3} - 0\right) - \left(\frac{1}{2} - 0\right)^2$$

$$= \frac{1}{12}$$

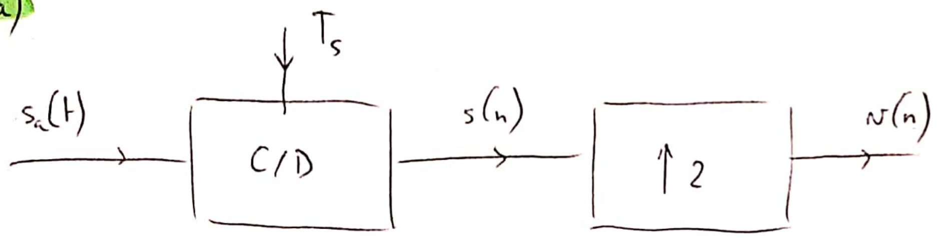
Therefore, $SQNR = 6.02B + 10.8 - 20\log_{10}\left(\frac{0.5}{1/\sqrt{12}}\right) = 6.02B + 6.03$

and For a signal-to-quantization noise ratio of 80dB, we require

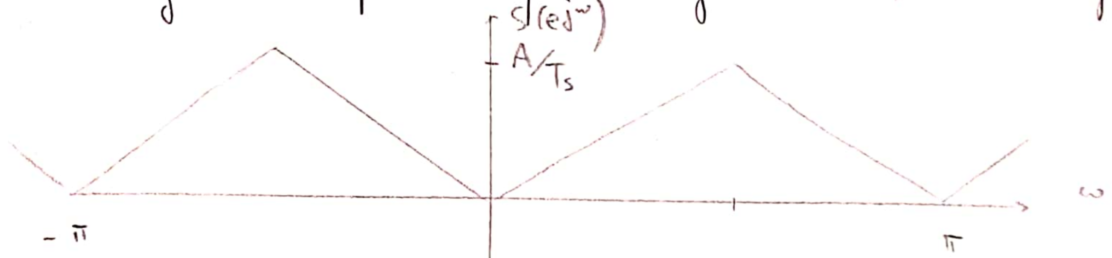
$$B = \frac{80 - 6.03}{6.02} = 12.29$$

that is $\boxed{\lceil B + 1 \rceil = 14 \text{ bits}}$

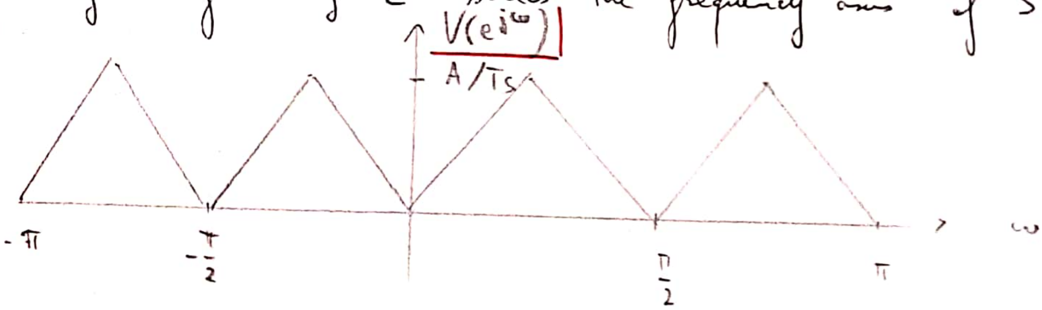
Q3. (a)



From $S_a(j\Omega)$, we know that $s_a(t)$ is sampled at the Nyquist rate. The DTFT of the sampled speech signal $s(n)$ is as follows:



Upsampling by a factor of 2 scales the frequency axis of $S(e^{j\omega})$ by a factor of 2 :



(b) The unit sample response of the discrete time filter is

$$h(n) = \frac{1}{2}\delta(n+1) + \delta(n) + \frac{1}{2}\delta(n-1)$$

The frequency response is

$$H(e^{j\omega}) = \frac{1}{2}e^{j\omega} + 1 + \frac{1}{2}e^{-j\omega}$$

$$= 1 + \frac{e^{j\omega} + e^{-j\omega}}{2} = \boxed{1 + \cos\omega}$$

↑ Euler Formula

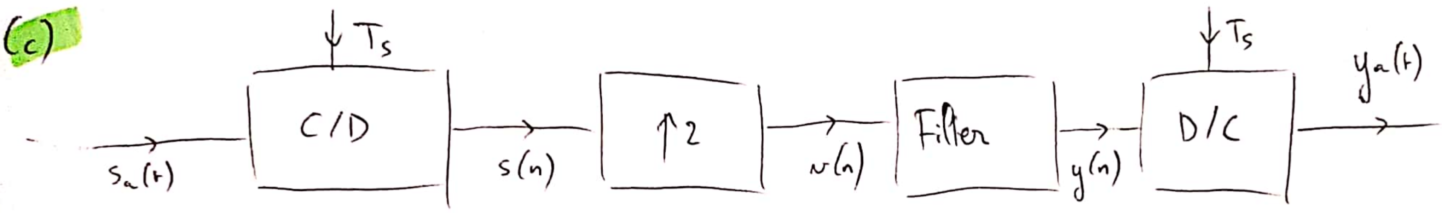
To see the effect of this filter on $v(n)$, note that due to the up-sampling $v(n) = 0$ for n odd.

Therefore,
$$y(n) = \begin{cases} v(n) & n \text{ even} \\ \frac{1}{2} v(n-1) + \frac{1}{2} v(n+1) & n \text{ odd} \end{cases}$$

Thus, the even-index values of $v(n)$ are unchanged, and the odd-index values are the average of the two neighboring values.

As a result, $h(n)$ performs a linear interpolation between the values of $v(n)$.

(c)



Let's express $Y_a(j\Omega)$ in terms of $X_a(j\Omega)$

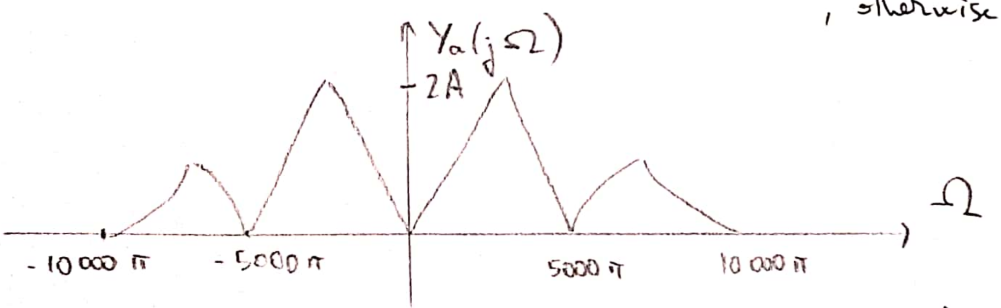
The output of the DC converter, $y_a(t)$, has a Fourier transform:

$$Y_a(j\Omega) = \begin{cases} T_s\, Y(e^{j\Omega T_s}), & |\Omega| < \pi/T_s \\ 0 & , \text{otherwise} \end{cases}$$

since $Y(e^{j\omega}) = H(e^{j\omega})\, V(e^{j\omega}) = (1 + \cos\omega)\, V(e^{j\omega})$

and $V(e^{j\omega}) = S(e^{j2\omega})$,

then
$$Y_a(j\Omega) = \begin{cases} T_s(1 + \cos\Omega T_s)\, S(e^{j2\Omega T_s}) & , |\Omega| < 10{,}000\pi \\ 0 & , \text{otherwise} \end{cases}$$



$y_a(t)$ does not correspond to slowed-down speech due to the images of $s_a(t)$ that occur in the frequency range $5000\pi < |\Omega| < 10\,000\pi$ and the non ideal linear interpolator. Note that a better approximation would be to use a DC converter with a sampling rate of $2T_s$ to eliminate the range $[5000\pi, 10000\pi]$.

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