Tutorial 5 - Solutions

1. A space station is communicating with a probe heading for Mars at a distance of 1,000,000 km at a rate of 12 data bits/second. The space station and probe communicate at 15 GHz, and have directional gains of 6 dBi each. The space channel may be modelled as an infinite bandwidth all-pass AWGN channel. The receive noise spectral density at the probe is $N_0 = 10^{-12}$ W/Hz. Assuming that we may achieve capacity over this channel using a code which adds 25% overhead (in terms of number of bits) and using 4-QAM modulation shaped with an ideal LPF, what is the minimum required transmit power of the space station ?

Solution:

The channel is modelled as an AWGN channel and thus the capacity equation is

$$C = W \log_2 \left(1 + \frac{P_R}{WN_0}\right) \tag{1}$$

where P_R is the average receive power at the probe. We may rearrange this to

$$P_R = (2^{C/W} - 1)WN_0 \tag{2}$$

From the above equation, it is clear that we need to determine the bandwidth used by the transmission to determine the average receive power (since we know the capacity required is 12 bits/second from the question).

The total number of transmitted bits is the 12 data bits/second plus the 25% overhead due to coding. The total number of transmitted bits is thus 15 bits/second. 4-QAM modulation transmits 2 bits per symbols and thus 15 bits/second corresponds to 7.5 symbols/second. Using an ideal LPF as pulse shaping filter, the bandwidth occupied by 7.5 complex symbols is 7.5 Hz. The bandwidth is thus 7.5 Hz. We may now determine the average receive power as

$$P_R = (2^{12/7.5} - 1) \times 7.5 \times 10^{-12} = 1.524 \times 10^{-11} \tag{3}$$

which when expressed in dBm is -78 dBm (equivalently −108 dB referenced to 1 W). The transmit power is related to the receive power by the free space path loss equation

$$\text{Path loss} = 10 \log_{10} \left(\frac{4\pi d}{\lambda}\right)^2 - 10 \log_{10} (G_T G_R) \text{ dB} \tag{4}$$

The frequency of transmission is $f = 15$ GHz, and thus the wavelength is

$$\lambda = \frac{c}{f} = 0.02 \text{ m} \tag{5}$$

Path loss is calculated as

$$\text{Path loss} = 10\log_{10}\left(\frac{4\pi 10^9}{0.02}\right)^2 - 12 \quad (6)$$

$$= 224 \text{ dB} \quad (7)$$

Thus, give a required receive power at the probe of -78 dBm, the required transmit power at the space station is

$$P_T = P_R + 224 = 146 \text{ dBm} \quad (8)$$

2. Given an OFDM system transmitting using a 64-point IDFT and transmitting at a rate of 8 complex samples/second over a multipath channel with associated scattering function

$$S(\tau,\lambda) = \begin{cases} \frac{3}{4}\lambda^2\tau e^{-\tau^2} & \tau > 0, -1 \leq \lambda \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

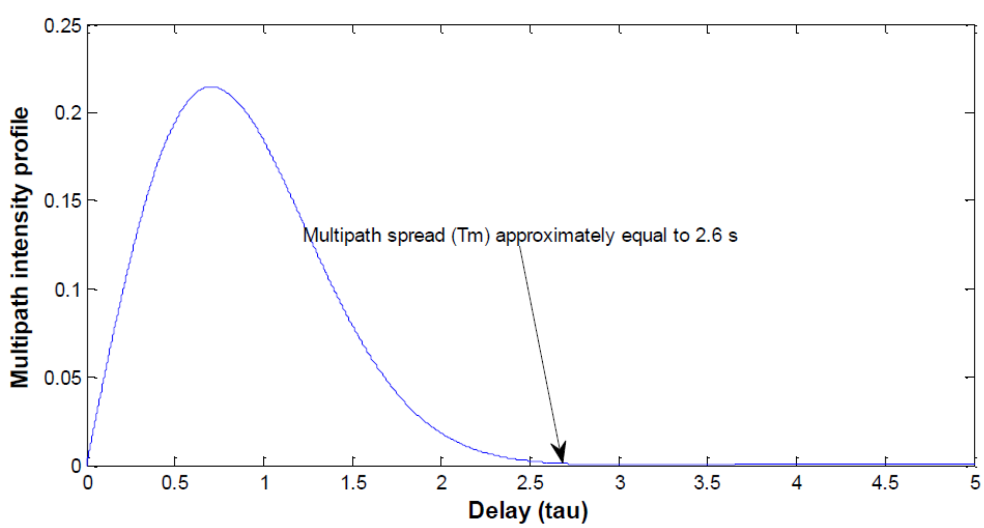
   (a) Estimate:
      i. The multipath spread
      ii. The coherence bandwidth
      iii. The Doppler spread
   (b) Suggest an appropriate cyclic prefix length for the OFDM transmission.

Solution:

(a) i. The multipath spread is estimated from the multipath intensity profile which is related to the scattering function as

$$\phi(\tau) = \int_{-\infty}^{\infty} S(\tau,\lambda)d\lambda = \int_{-1}^{1}\frac{3}{4}\lambda^2\tau e^{-\tau^2}d\lambda = \frac{1}{2}\tau e^{-\tau^2} \quad (10)$$

The following figure plots $\phi(\tau)$ with τ



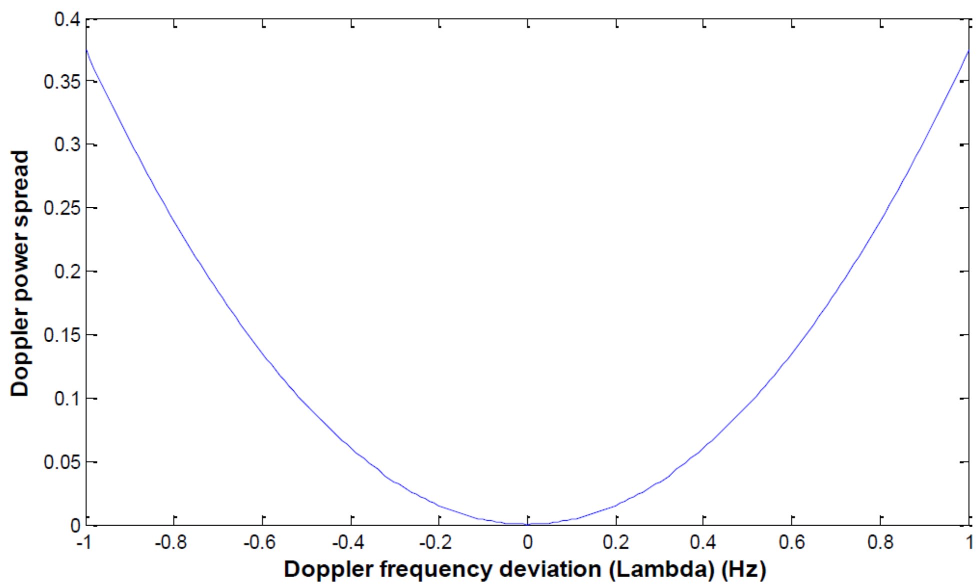
From the plot, it can be seen how the multipath spread (T_m) is approximately equal to 2.6 seconds

ii. The coherence bandwidth is inversely related to the multipath spread

$$(\Delta f)_c = \frac{1}{2.6} = 0.385 \text{ Hz} \tag{11}$$

iii. The Doppler bandwidth is estimated from the Doppler power spread function, which can be calculated from the Scattering function as

$$S(\lambda) = \int_{-\infty}^{\infty} S(\tau, \lambda) d\tau = \frac{3}{8}\lambda^2 \tag{12}$$



The Doppler spread is estimated as the range over which this function is approximately nonzero. In this case, this is the entire range of allowable values, i.e. $-11 \leq \lambda \leq 1$. Thus the Doppler spread bandwidth (B_d) is estimated as 2 Hz.

(b) The system is an OFDM system transmitting 8 complex samples/second, thus the sample (or symbol) period is equal to $T = 1/8 = 0.125$ s. Thus, the 2.6 s of the multipath spread correspond to 20.8 complex samples. An appropriate cyclic prefix length should be longer than this, i.e. 21 complex samples.

3. One base station is transmitting information to a mobile phone situated 100 m from the base station in an urban area. The base station antenna is situated at 200 m height whereas the mobile phone antenna is situated at 2 m height. An overhead of 25% is introduced to the digital information bits at the transmitter for forward error correction purposes at the receiver. The digital modulation scheme employed is QPSK, with a low-pass brick wall filter for pulse shaping and a carrier frequency of 500 MHz. The channel may be considered to have a power density $N_0 = 10^{-12}$ W/Hz. The path-loss is calculated using the Okumura-Hata model

$$\begin{aligned} \text{Path loss} \quad &= 69.55 + 26.16 \log_{10} f - 13.82 \log_{10} h_t \\ &\quad -a(h_r) + (44.9 - 6.55 \log_{10} h_t) \log_{10} d \end{aligned} \quad \text{(dB)} \tag{13}$$

where

$$a(h_r) = \begin{cases} 8.29\,(\log_{10}(1.54 h_r))^2 - 1.1 & 200 \geq f \geq 150 \text{ MHz} \\ 3.2\,(\log_{10}(11.75 h_r))^2 - 4.97 & f \geq 200 \text{ MHz} \end{cases} \quad (14)$$

h_t and h_r are the respective heights of the receive and transmit antennas (in metres), f is the frequency of transmission (in MHz), and d is the distance from transmit to receive antenna (in km).

(a) Is it possible to achieve an information capacity of 200 kbits/second between the base station and mobile station considering that the base station can transmit with a maximum power of 60 W?

(b) What would be the benefit of using a BPSK modulation instead of QPSK?

(c) If the signal bandwidth was limited to 200 kHz, could BPSK still be used? Explain your answer.

Solution:

(a) Substituting all the values in the Okumura-Hata equation with the correct units it is calculated that the path-loss attenuation is equal to 77.48 dB. At the transmitter, the digital source produces a bit rate at 200 kbits/second, and extra 25% is added for FEC, obtaining 250 kbits/second. The QPSK modulation employs 2 bits/symbol, therefore the symbol rate is equal to 125 ksymbols/second. Since a low-pass brick-wall filter is used for pulse shaping, the bandwidth of the transmitted signal is equal to 125 kHz. To calculate the necessary received power at the mobile phone to allow the required data rate the channel capacity equation for non-dispersive channels (i.e., AWGN channels) is used

$$C = C = W \log_2\left(1 + \frac{P_R}{WN_0}\right) \Rightarrow P_R = (2^{C/W} - 1)WN_0 = -35.95 \text{ dBm} \quad (15)$$

Therefore, the transmitting power should be at least

$$P_T = P_R + 77.48 = 41.52 \text{ dBm} \quad (16)$$

(b) If BPSK is employed, only 1 bit per symbol is employed, increasing the bandwidth to 250 kHz. Doing the same calculations, the necessary received power must be now at least $P_R = -37.32$ dBm. Therefore, the advantage is that less transmitting power is required to achieve the same capacity.

(c) No, because the BPSK signal would require 250 kHz.