Q3 specifications are:

$$0.95 < H(e^{j\omega}) < 1.05 \qquad 0 \leq |\omega| \leq 0.25\pi$$

$$-0.1 < H(e^{j\omega}) < 0.1 \qquad 0.35\pi \leq |\omega| \leq \pi$$

so this filter requires a maximal passband error of $\delta_p = 0.05$ and a maximal stopband error of $\delta_s = 0.1$

Convert these values to dB: $\delta_p(\text{dB}) = 20\log_{10}(0.05) = -26\,\text{dB}$

$\delta_s(\text{dB}) = 20\log_{10}(0.1) = -20\,\text{dB}$

This requires a window with a peak approximation error less than -26 dB.
From the table, the Hann, Hamming and Blackman windows meet this criterion.

The minimum length L required for each of these filters can be found using the "approximate width of mainlobe" column since the mainlobe width is about equal to the transition width.
Note that the actual length of the filter is $L = M+1$

Hanning / Hann: $0.35\pi - 0.25\pi = \frac{8\pi}{M} \Leftrightarrow M = 80$

Hamming: $0.1\pi = \frac{8\pi}{M} \Leftrightarrow M = 80$

Blackman: $0.1\pi = \frac{12\pi}{M} \Leftrightarrow M = 120$

Q4.

$$|H_d(e^{j\omega})| = \begin{cases} 1 & |\omega| \leq 0.2\pi \\ 0 & 0.2\pi \leq \omega \leq \pi \end{cases}$$

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$ (from (7.54) in O&S book)

$$= \frac{1}{2\pi} \int_{-0.2\pi}^{0.2\pi} e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi j n} \left(e^{j0.2\pi n} - e^{-j0.2\pi n} \right)$$

$$= \frac{1}{2\pi j n} 2j \sin(0.2\pi n)$$

$$= \frac{\sin(0.2\pi n)}{\pi n}$$

As $N = 24$, the delay of $h(n)$ is $\frac{N}{2} = 12$ and the ideal unit sample response that is to be windowed is

$$h_d(n) = \frac{\sin(0.2\pi(n-12))}{\pi(n-12)}.$$

All that is left is to select a window. With the length of the window fixed, there is a trade-off between the width of the transition band and the amplitude of the passband and stopband ripple. With a rectangular window, which provides the smallest transition band: $\Delta\omega = 2\pi \Delta f = 2\pi \frac{0.9}{N} = 2\pi \frac{0.9}{24} = 0.075\pi$

the filter is $h(n) = \begin{cases} \frac{\sin(0.2\pi(n-12))}{\pi(n-12)} & 0 \leq n \leq 24 \\ 0 & \text{otherwise} \end{cases}$

However the stopband attenuation is only 21 dB (from the table).

With a Hamming window, $h(n) = \left[0.54 - 0.46\cos\left(\frac{2\pi n}{24}\right)\right] \frac{\sin(0.2\pi(n-12))}{\pi(n-12)}$ $0 \leq n \leq 24$

$\Delta\omega = 2\pi \frac{3.3}{24} = 0.275\pi$ and $\delta_s = -53\text{dB}$

Q5. (a)

$$H_{id}(e^{j\omega}) = \begin{cases} 0 & |\omega| < \frac{\pi}{2} \\ 1 & \frac{\pi}{2} < |\omega| < \pi \end{cases}$$

$$h_{id}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{id}(e^{j\omega}) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\frac{\pi}{2}} e^{j\omega n} \, d\omega + \frac{1}{2\pi} \int_{\frac{\pi}{2}}^{\pi} e^{j\omega n} \, d\omega$$

$$= \frac{(-1)}{2\pi j n} \left(e^{-j\pi n} - e^{-j\frac{\pi}{2} n} \right) + \frac{1}{2\pi j n} \left(e^{j\pi n} - e^{j\frac{\pi}{2} n} \right)$$

$$= \frac{1}{2\pi j n} \left(e^{j\pi n} - e^{-j\pi n} \right) + \frac{1}{2\pi j n} \left(e^{-j\frac{\pi}{2} n} - e^{j\frac{\pi}{2} n} \right)$$

$$= \frac{\sin(\pi n)}{\pi n} - \frac{\sin\left(\frac{\pi n}{2}\right)}{\pi n}$$

(b) the impulse response filter is:

$$h[n] = h_{id}[n] \cdot w_H[n]$$

the problem here is that the resulting sequence $h[n]$ will be non-zero from $n = -5$ up to $n = 5$, so the filter is non-causal.

Then, we must shift 5 samples to the right $(0 \leq n \leq 10)$

The unit-sample response $h_c[n]$ of the causal 11-point FIR filter is given by:

$$h_c[n] = h[n-5]$$

$$= h_{id}[n-5] \, w_H[n-5]$$

$$h_c[n] = \begin{cases} \dfrac{\sin(\pi(n-5)) - \sin\left(\frac{\pi}{2}(n-5)\right)}{\pi(n-5)} \left(0.54 + 0.46 \cos\left(\frac{\pi}{5}(n-5)\right)\right), & 0 \leq n \leq 10 \\ 0 & , \text{otherwise} \end{cases}$$

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