UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF ENGINEERING

Electronic & Electrical Engineering

Senior Sophister
Engineering
Annual Examinations

Hilary Term, 2015

Digital Signal Processing (4C5)

Date: Tuesday 13th January **Venue: Luce Upper** **Time: 09.30 – 11.30**

Dr. W. Dowling

Answer THREE questions
All questions carry equal marks

Permitted Materials:

Calculator
Drawing Instruments
Mathematical Tables
Graph Paper

Q.1 **(a)** Show that the bilinear transformation, $s=(1-z^{-1})/(1+z^{-1})$, has the following properties:

(i) The imaginary axis in the s-plane maps to the unit circle in the z-plane.

[4 marks]

(ii) The left half of the s-plane maps to the inside of the unit circle in the z-plane.

[4 marks]

(b) A discrete-time bandpass filter with frequency response $H(e^{j\Omega})$ is to be designed to meet the following specifications:

$$\frac{1}{\sqrt{2}} \le \left|H(e^{j\Omega})\right| \le 1, \quad 0.4\pi \le |\Omega| \le 0.6\pi,$$

$$\left|H(e^{j\Omega})\right| \le 0.2, \quad 0.9\pi \le |\Omega| \le \pi,$$

and $\left|H(e^{j\Omega})\right| \le 0.2, \quad |\Omega| \le 0.1\pi.$

The filter is to be designed by applying the bilinear transformation to the transfer function of an appropriate Butterworth continuous-time filter.

Verify that a second order bandpass filter is sufficient to meet the specifications. Determine the transfer function, $H(z)$, of the discrete-time filter.

Note that the transfer function of a first order Butterworth lowpass prototype filter is

$$H(s)=\frac{1}{s+1}$$

and the lowpass to bandpass transformation for a continuous time filter is

$$s \to \frac{s^2+\omega_1\omega_2}{s(\omega_2-\omega_1)}$$

where ω_1 and ω_2 are the lower and upper cut-off frequencies respectively.

[12 marks]

Q.2 (a) A continuous-time filter has an impulse response, $h_a(t)$. The unit-sample response of a discrete-time filter, $h[n]$, is given by

$$h[n] = T\,h_a(nT)$$

where T is a constant. Let $H_a(j\omega)$ and $H(e^{j\Omega})$ denote the frequency response of the continuous-time filter and the frequency response of the discrete-time filter respectively. Starting from first principles, show that

$$H(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} X_a\left(j\left(\frac{\Omega}{T} - \frac{2\pi k}{T}\right)\right).$$

[12 marks]

(b) A continuous-time filter has an impulse response, $h_a(t)$, given by

$$h_a(t) = e^{-t}u(t)$$

where $u(t)$ is the unit-step function.

Let $h[n]$ and $H(z)$ denote the unit sample response and transfer function of a discrete-time filter.

(i) If $h[n] = T\,h_a(nT)$, where $T = 0.5$ seconds, obtain an expression for $H(z)$.

[4 marks]

(ii) Determine the response of the filter, $y[n]$, to the input signal, $x[n]$, given by

$$x[n] = \begin{cases} (0.75)^n, & n \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

[4 marks]

Q.3 **(a)** A discrete-time filter has a unit sample response, $h[n]$, that is zero for $n<0$ and for $n>N-1$. If $h[n]=h[N-1-n]$ and N is odd, show that the filter has a frequency response with generalized linear phase.

[8 marks]

(b) An ideal discrete-time highpass filter has a frequency response, $H_{id}(e^{j\Omega})$, given by

$$H_{id}(e^{j\Omega})=\begin{cases}0, & |\Omega|<\dfrac{\pi}{3}\\ 1, & \dfrac{\pi}{3}<|\Omega|\leq\pi\end{cases}$$

Obtain an expression for the unit-sample response of this filter.

[7 marks]

(c) Using a rectangular window sequence, design a causal 11-point finite impulse response (FIR) filter which approximates the magnitude response of the ideal highpass filter in part (b).

[5 marks]

Q.4 **(a)** The sequence $x[n]$ is zero for $n<0$ and for $n>N-1$. Assume that $N=2^M$, where M is a positive integer. Let $g[n]=x[2n]$, and $h[n]=x[2n+1]$.
Show that the N-point discrete Fourier transform (DFT) of the sequence $x[n]$ can be obtained by appropriately combining the $N/2$-point DFTs of the sequences $g[n]$ and $h[n]$.

[8 marks]

(b) Draw a complete signal flow graph for an 8-point decimation-in-time fast Fourier transform (FFT) algorithm.

[12 marks]

Q.5 (a) Let $x[n]$ denote an infinite length sequence with discrete-time Fourier transform $X(e^{j\Omega})$ and let $X_1[k]$ denote the N-point discrete Fourier transform (DFT) of the N-point sequence $x_1[n]$ Determine the relation between $x_1[n]$ and $x[n]$ if $X_1[k]$ and $X(e^{j\Omega})$ are related by

$$X_1[k] = X(e^{j2\pi k/N}), \qquad k = 0, 1, \ldots, N-1.$$

[8 marks]

(b) The finite length sequences, $x[n]$ and $y[n]$, are zero for $n < 0$ and for $n > N-1$. Let $X[k]$ and $Y[k]$ denote the N-point DFT of $x[n]$ and $y[n]$ respectively.

(i) $g[n]$ is the N-point circular convolution of the sequences $x[n]$ and $y[n]$. Let $G[k]$ denote the N-point DFT of $g[n]$. Show that $G[k] = X[k]Y[k]$.

[6 marks]

(ii) The finite-duration sequence $r[n]$ is given by $r[n] = x[n]y[n]$. Let $R[k]$ denote the N-point DFT of $r[n]$. Show that

$$R[k] = \frac{1}{N}\sum_{l=0}^{N-1} X[l]Y[((k-l))_N]$$

where $((k-l))_N$ denotes $(k-l)$ modulo N.

[6 marks]