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Lecture 6: Diversity Reception

EE412 - Wireless Digital Communications

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Learning Outcomes

- Define the diversity reception concept.
- Explain the difference between time diversity, frequency diversity and space diversity.
- Explain the advantages and drawbacks between different diversity combination methods.
- Calculate the Signal-to-Noise Ratio (SNR) of the received signal depending on the combination method employed.

Multipath Fading Channels

- Flat fading channels may, at any instant, have very high attenuation, thus causing errors at the receiver.
  - This causes *burst errors* (i.e. a large number of errors in close succession).
- In wireless digital communications, diversity is based on the reception of multiple copies of the transmitted data such that, at any instant, the probability of all copies being strongly attenuated by a fading channel is low.
  - If the probability of one received copy fading is p_f , then the probability of N copies fading simultaneously is p_f^N (assuming the copies are independently fading).

Multipath Fading Channels...

- Diversity may be achieved in a number of ways.
  - Several copies of the data may be transmitted at different frequencies (*frequency diversity*)
    - If the copies are **spaced by greater than the coherence bandwidth**, then the correlation between the fading components is negligible (independent fading).
  - Several copies of the data may be transmitted at different times (*time diversity*)
    - If the copies are **spaced in time by greater than the coherence time**, then the correlation between fading components is negligible (independent fading).

Multipath Fading Channels...

- The previously described diversity schemes are based on repetition of the data (either across time or frequency) to obtain diversity.
  - Repetition of data for error robustness is a form of *repetition coding*.
  - Repetition coding is in general wasteful compared to non-trivial coding methods
- A further type of diversity is *spatial diversity*:
  - Diversity is obtained by the reception of the signal at physically distinct locations, i.e., multiple receive antennas.
  - Independent fading is obtained if the antennas are sufficiently spaced (because antennas which are right beside each other receive highly correlated signals).
  - A spacing of the order of a few wavelengths is required between two antennas for independent fading.

Multipath Fading Channels...

- Diversity obtained by having a signal bandwidth much greater than the coherence bandwidth will be discussed later with respect to code-division multiple access (CDMA).
- Other diversity techniques exist which obtain multiple copies by other means (e.g. receiving based on incident angle).

Selection Combining

- Given a set of copies of the signal, each with a different attenuation, we must consider how we combine these copies.
  - Since not all signals have the same characteristics, one branch will have the highest signal-to-noise ratio (SNR). *Selection combining* is the extraction of only the branch with the highest SNR.
  - It is not possible to obtain a SNR higher than the branch with the highest SNR using selection combining.
  - Theoretically, the implementation of selection combining diversity implies constant monitoring of the SNR in all branches of the reciver.
  - In practice, the continual monitoring of branches for SNR is not implemented since it requires a continuous monitoring/switching. Instead, initially the branch with highest receive signal power is selected and maintained until its SNR drops below some predefined value, at which point the branch which now has the highest receive power is selected. This is then undertaken continuously.

Maximal-Ratio Combining

- When multiple copies of a signal are received, each with different SNR, it is desirable to combine these in such a way that we obtain the highest possible overall SNR.
- When this is done by means of a linear combination, this is called *maximal-ratio combining*.
- **Problem**: Given a set of received signals, $r_k(t)$, corresponding to a single diversity input and corrupted by AWGN so that:

$$r_k(t) = K_k s(t) + \eta_k(t) \quad k = 1, \ldots, N \tag{1}$$

where K_k describes the attenuation during transmission over every copy of the transmitted signal $s(t)$. The noise $\eta_k(t)$ is AWGN (Gaussian stochastic process with zero-mean) with variance σ_k^2 . Assuming that $s(t)$ has unit energy, and that K_k are uncorrelated, find the combination:

$$\hat{r}(t) = \sum_k \alpha_k r_k(t) \tag{2}$$

which maximizes the output SNR where α_k are the coefficients applied to each $r_k(t)$.

Maximal-Ratio Combining...

- **Solution**: The SNR of each received sample (also called instantaneous SNR) can be calculated as:

$$(\mathrm{SNR})_k = \frac{E\left\{|K_k s(t)|^2\right\}}{E\left\{|\eta_k(t)|^2\right\}} = \frac{E\left\{|K_k s(t)|^2\right\}}{N_0 W} \tag{3}$$

  where $E\left\{|K_k s(t)|^2\right\}$ is the power or mean squared value of the received copy and $E\left\{|\eta_k(t)|^2\right\}$ is the power of the noise.
- Considering that the power of the signal is equal to its energy (E) divided by the symbol period ($T_s = 1/2W$ for an ideal baseband brick-wall filter), the SNR can be rewritten as

$$(\mathrm{SNR})_k = \frac{K_k^2 E_s}{N_0/2} \tag{4}$$

Maximal-Ratio Combining...

- Since $s(t)$ has unit energy ($E = 1$) and for an AWGN channel its variance and its power spectral density are the same value ($\sigma^2 = \Phi = N_0/2$), the SNR in each branch is equal to:

$$(\mathrm{SNR})_k = \frac{K_k^2}{N_0/2} = \frac{K_k^2}{\Phi} = \frac{K_k^2}{\sigma_k^2} \tag{5}$$

- The signal power of the combination, $\hat{r}(t)$, since $E = 1$, is:

$$E\left\{\left(\sum_k \alpha_k K_k s(t)\right)^2\right\} = \left(\sum_k \alpha_k K_k\right)^2 \tag{6}$$

- And the noise power of the combination, $\hat{r}(t)$, is:

$$E\left\{\left(\sum_k \alpha_k \eta_k(t)\right)^2\right\} = \sum_k \alpha_k^2 \sigma_k^2 \tag{7}$$

Maximal-Ratio Combining...

Therefore, the SNR of the combination is:

$$\mathrm{SNR} = \frac{\left(\sum_{k=1}^{N} \alpha_k K_k\right)^2}{\sum_{k=1}^{N} \alpha_k^2 \sigma_k^2} \tag{8}$$

- Consequently, *the goal is to find the values of* α_k *which maximize this SNR*
- Letting $u_k = \alpha_k \sigma_k$ and $v_k = \dfrac{K_k}{\sigma_k}$, we rewrite the SNR as

$$\mathrm{SNR} = \frac{\left(\sum_{k=1}^{N} u_k v_k\right)^2}{\sum_{k=1}^{N} u_k^2} \tag{9}$$

Maximal-Ratio Combining...

- Apply the Cauchy-Schwarz inequality as

$$\mathrm{SNR} \leq \frac{\sum_{k=1}^{N} u_k^2 \sum_{k=1}^{N} v_k^2}{\sum_{k=1}^{N} u_k^2} = \sum_{k=1}^{N} v_k^2 \tag{10}$$

- Therefore, there is equality in the Cauchy-Schwarz only when $u_k = v_k$, i.e., $\alpha_k \sigma_k = \frac{K_k}{\sigma_k}$.
- Thus, the optimum combining ratio coefficients for maximum output SNR are:

$$\alpha_k = \frac{K_k}{\sigma_k^2}. \tag{11}$$

Maximal-Ratio Combining...

- If the noise power is constant across all received signals, then the maximal-ratio combining principle may be stated as:
  - Maximal-Ratio Combining Principle: The SNR of a weighted sum is maximized when the amplitude weighting is performed in proportion to the signal strength (assuming equal noise power).

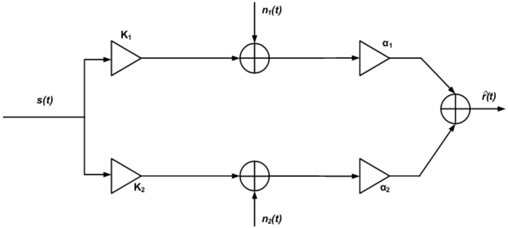

Figure: Maximal-Ratio Combining on two diversity paths

Maximal-Ratio Combining – Performance

- When using maximal-ratio combining, the optimal weights were found to be $\alpha_k = \dfrac{K_k}{\sigma_k^2}$, giving an output SNR of:

$$\mathrm{SNR} = \frac{\left(\sum_{k=1}^{N} \alpha_k K_k\right)^2}{\sum_{k=1}^{N} \alpha_k^2 \sigma_k^2} = \frac{\left(\sum_{k=1}^{N} \frac{K_k^2}{\sigma_k^2}\right)^2}{\sum_{k=1}^{N} \frac{K_k^2}{\sigma_k^2}} = \sum_{k=1}^{N} \frac{K_k^2}{\sigma_k^2} \tag{12}$$

- We can see from the above equation that maximal ratio combining gives an output whose SNR is the sum of the SNRs of each component.
  - This is **always higher than the SNR of any single branch**, thus maximal-ratio combining can far outperform selection combining.
  - Maximal-ratio combining requires a significantly greater implementation complexity than the selection combiner.

Equal-gain Combining

- A lower complexity combination in comparison with the two previous diversity combining techniques is equal-gain combining.
- By combining each incoming diversity path equally, we may help obviate the fading effect by averaging over all incoming branches which have a low probability of fading simultaneously. This is a much simpler to implement scheme.
- This is equivalent to maximal ration combining with all coefficients α_k equal to 1. Therefore, assuming a unit energy transmitted signal, by using the equal-gain combining method the obtained SNR is:

$$\mathrm{SNR} = \frac{\left(\sum_{k=1}^{N} K_k\right)^2}{N\sigma_k^2} \tag{13}$$

- The equal-gain combiner offers an increased received SNR over the selection combiner but a lower receive SNR over the maximal-ratio combiner.