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Network Theory
Lecture 4.09

(Tutorial #4)

EEU45C09 / EEP55C09
Self Organising Technological Networks

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Q1

The graph in Fig. Q.2, shows the interference topology for a cellular network, where an edge is present if the two base stations mutually interfere with each other.

(b) Compute the betweenness centrality for each node, considering only the geodesic paths from node 5 to node 4.

[6 marks]

(c) List all the 3-cliques.

[5 marks]

(d) Does this graph contain a k-component for $k \geq 2$?

[5 marks]

Provide an explanation for each answer.

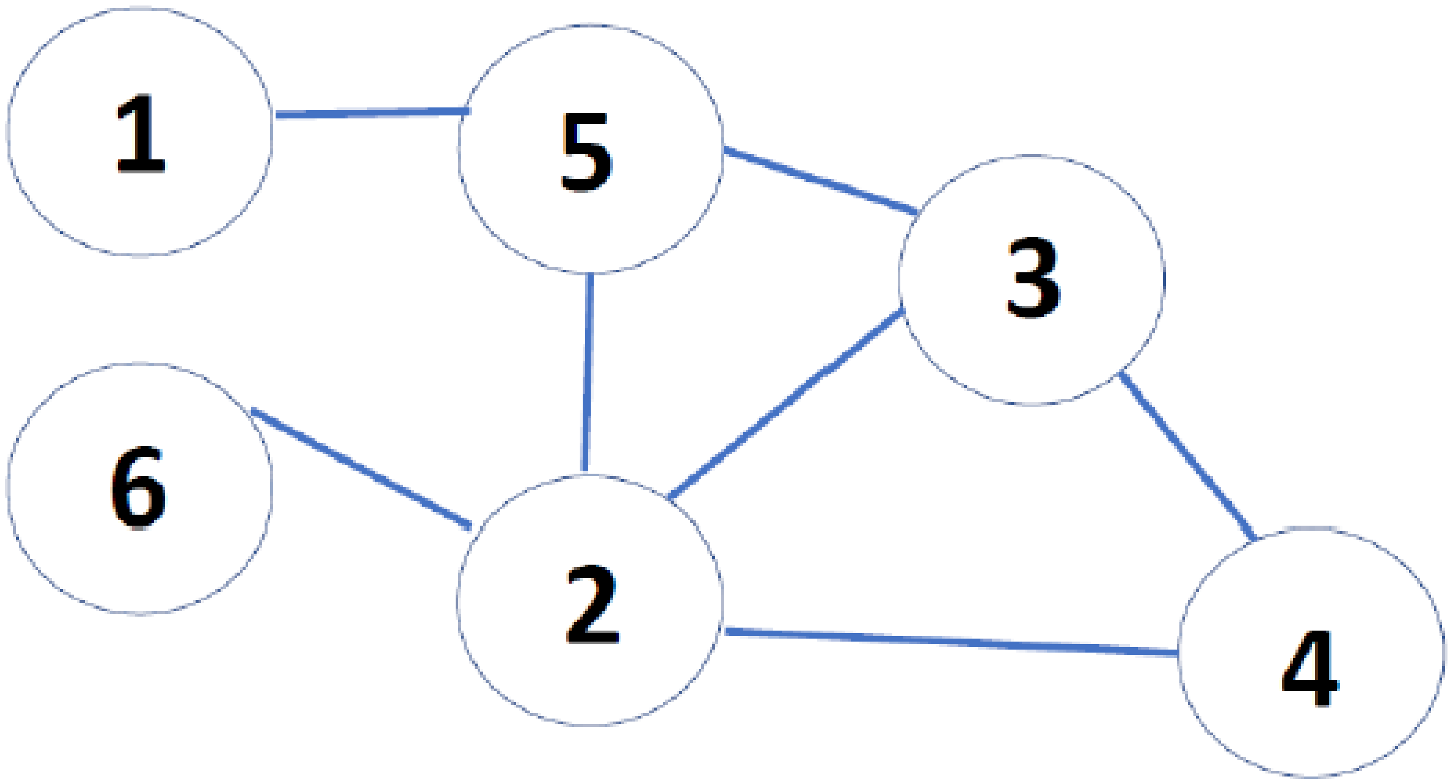


Fig. Q.2

(b)

$$x_i = \sum_{st} \frac{n_{st}^i}{g_{st}}$$

g_{54} = 2, i.e. the paths 5-3-4 and 5-2-4.

$x_1 = 0$, $x_2 = 1/2$, $x_3 = 1/2$, $x_4 = 2/2 = 1$, $x_5 = 2/2 = 1$, $x_6 = 0$

(c)

Two 3-cliques, the triangles 2-3-4 and 2-3-5.

A clique is a maximal subset of the vertices in an undirected network such that every member of the set is connected by an edge to every other.

(d)

The part of the graph encompassing nodes 2-3-4-5 is a **2-component** based on the definition here below.

A k-component is a maximal subset of vertices such that each is reachable from each of the other vertices by at least k vertex-independent paths.

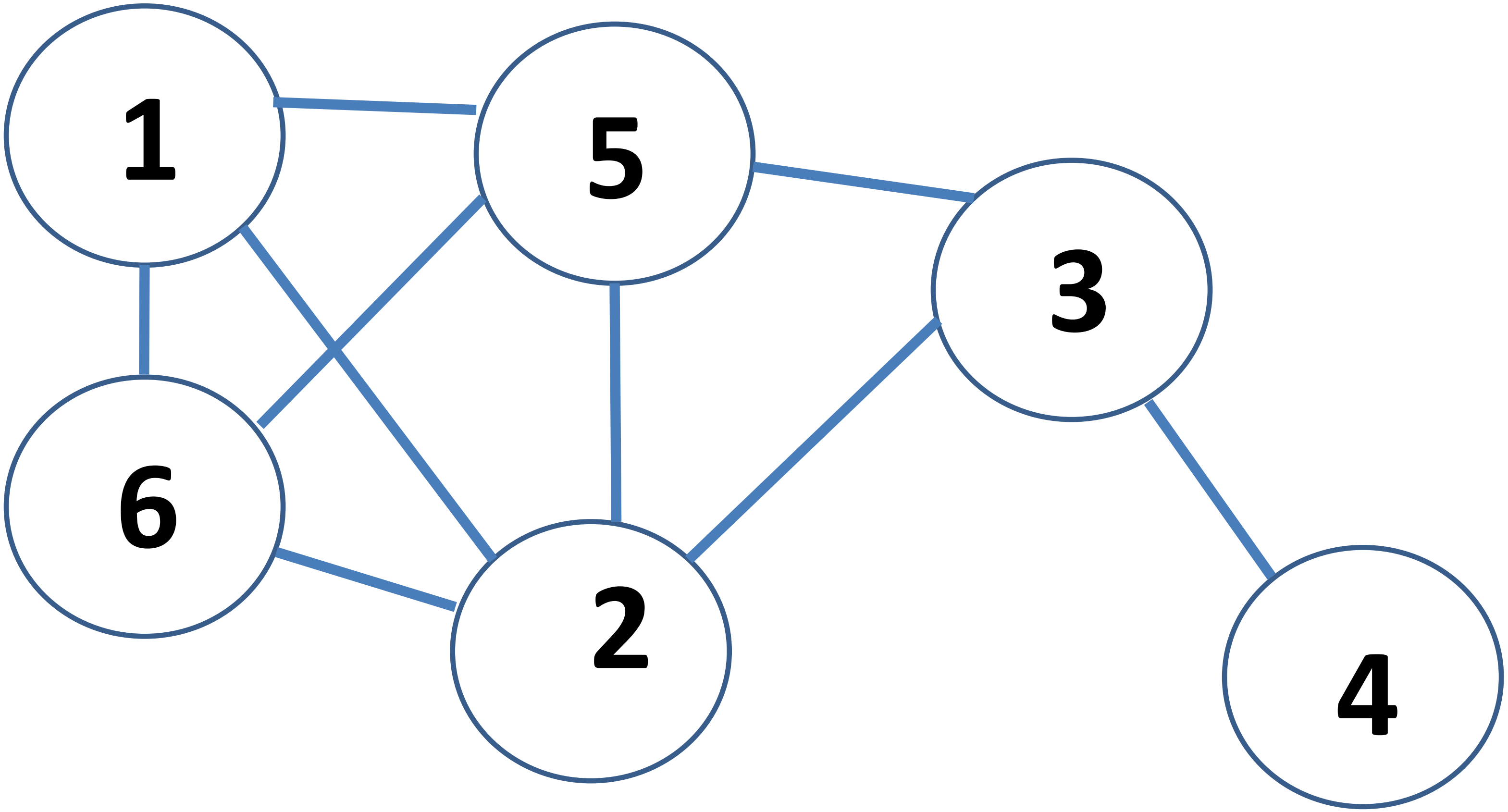
In fact removing two edges (e.g. e_{24} and e_{34}) makes the graph disconnected.

Q2

Create a graph containing at least one 3-clique and one 4-clique, clearly indicating what nodes compose the different cliques.

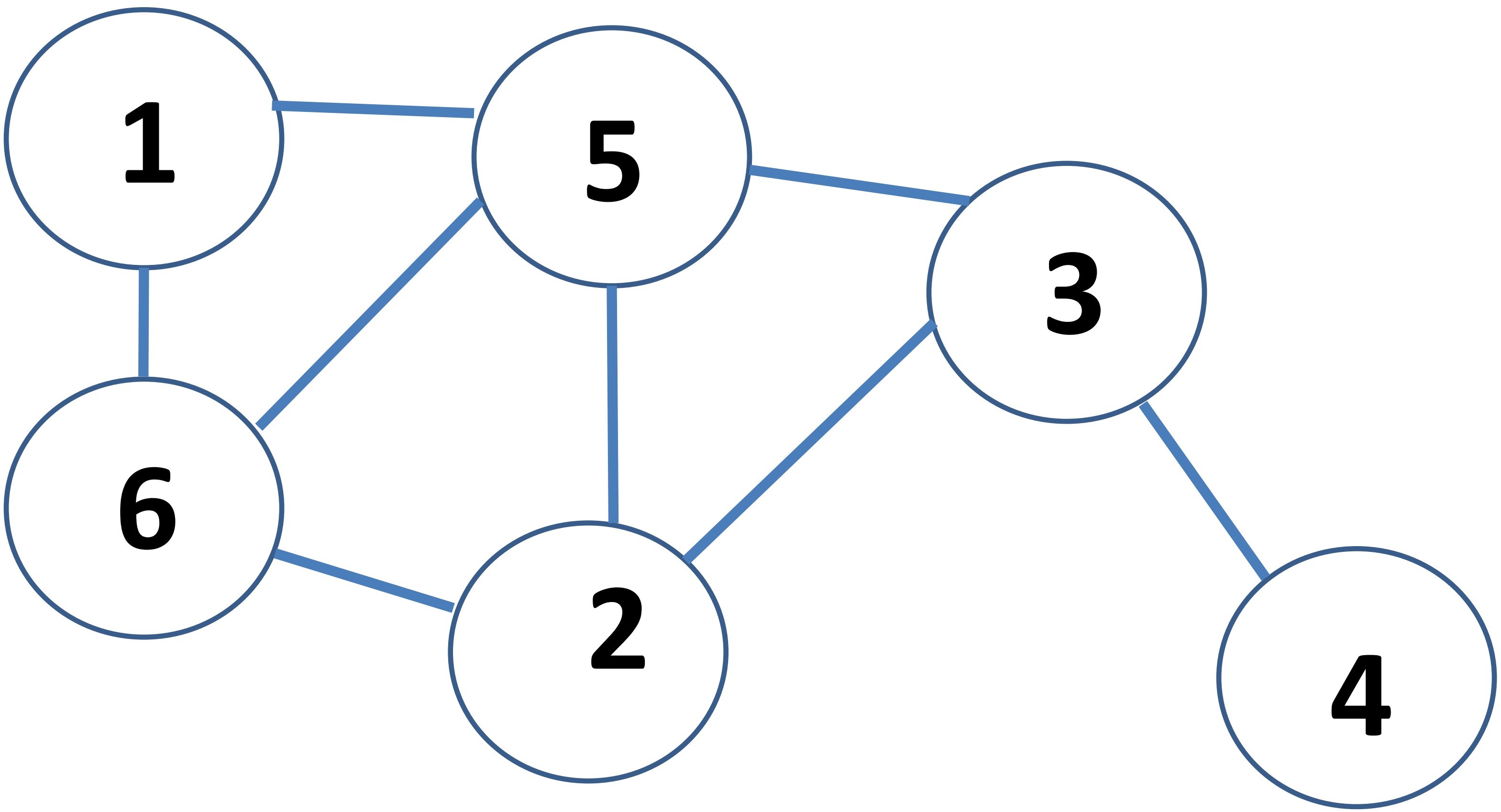
A clique is a maximal subset of the vertices in an undirected network such that every member of the set is connected by an edge to every other.

- 3-clique: {2,3,5}
- 4-clique: {1,2,5,6}



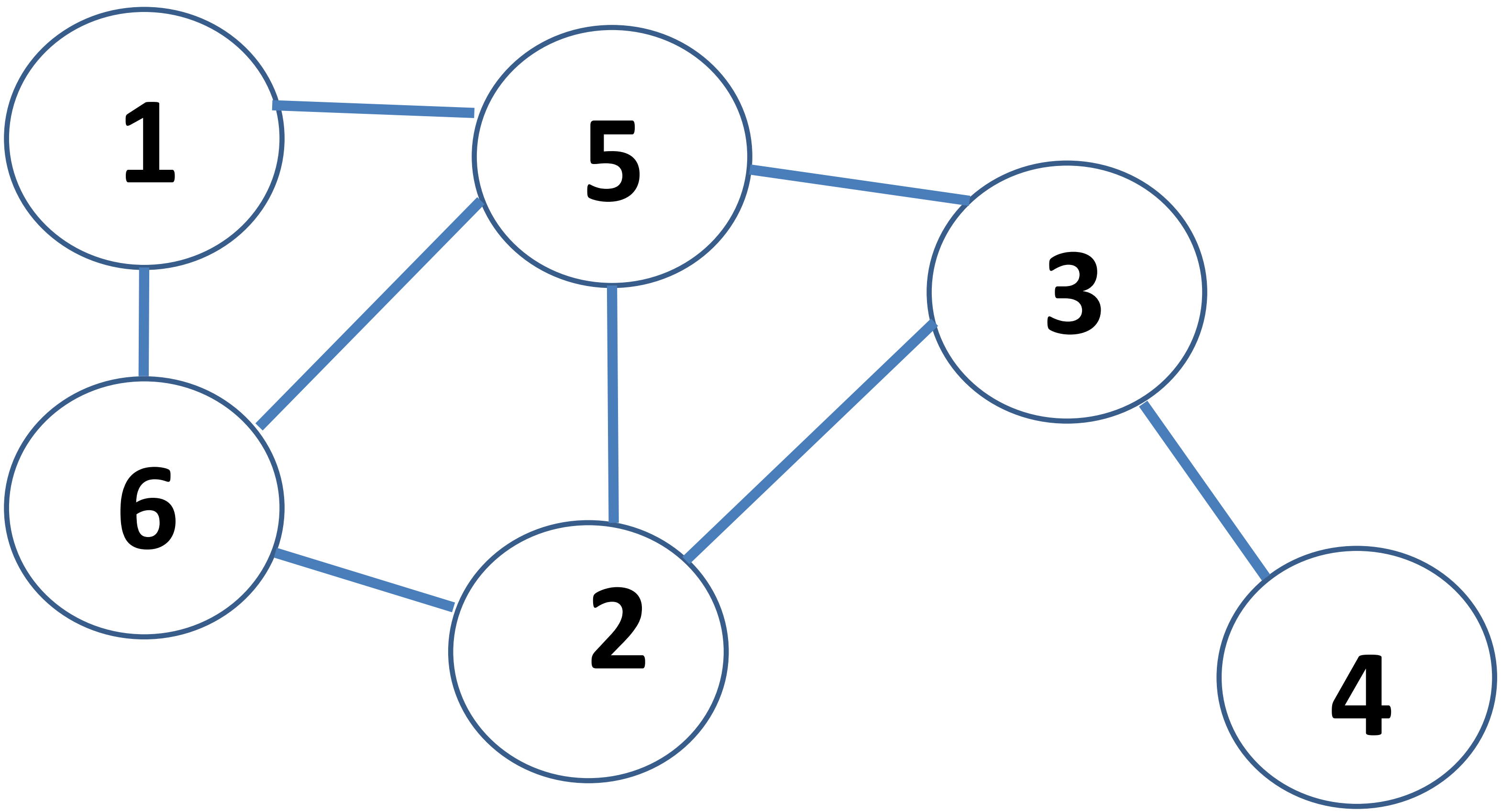
Q3

Is the graph below a k-plex of size 6 for k ≥ 1 ?



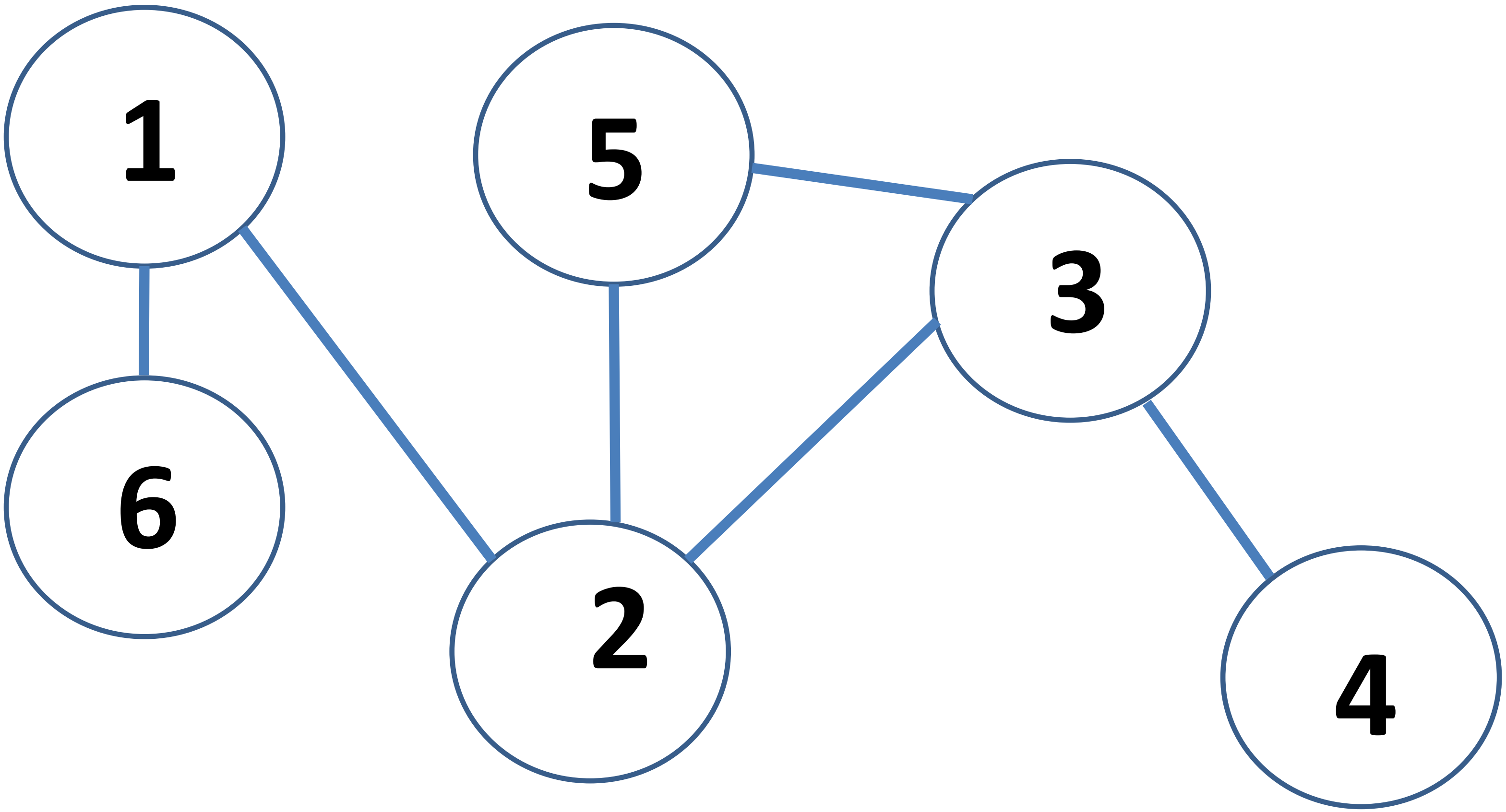
We can relax the stringent clique condition to define a k-plex: a $k-$plex of size n is a maximal subset of n vertices such that each vertex is connected to at least $n-k$ of the other vertices.

- **1**-plex → Each node is connected to at least 6-**1** = 5 nodes → NO
- **2**-plex → Each node is connected to at least 6-**2**=4 nodes → NO
- …
- **5**-plex → Each node is connected to at least 6-**5**=1 node → YES
  - it’s like saying it’s not disconnected



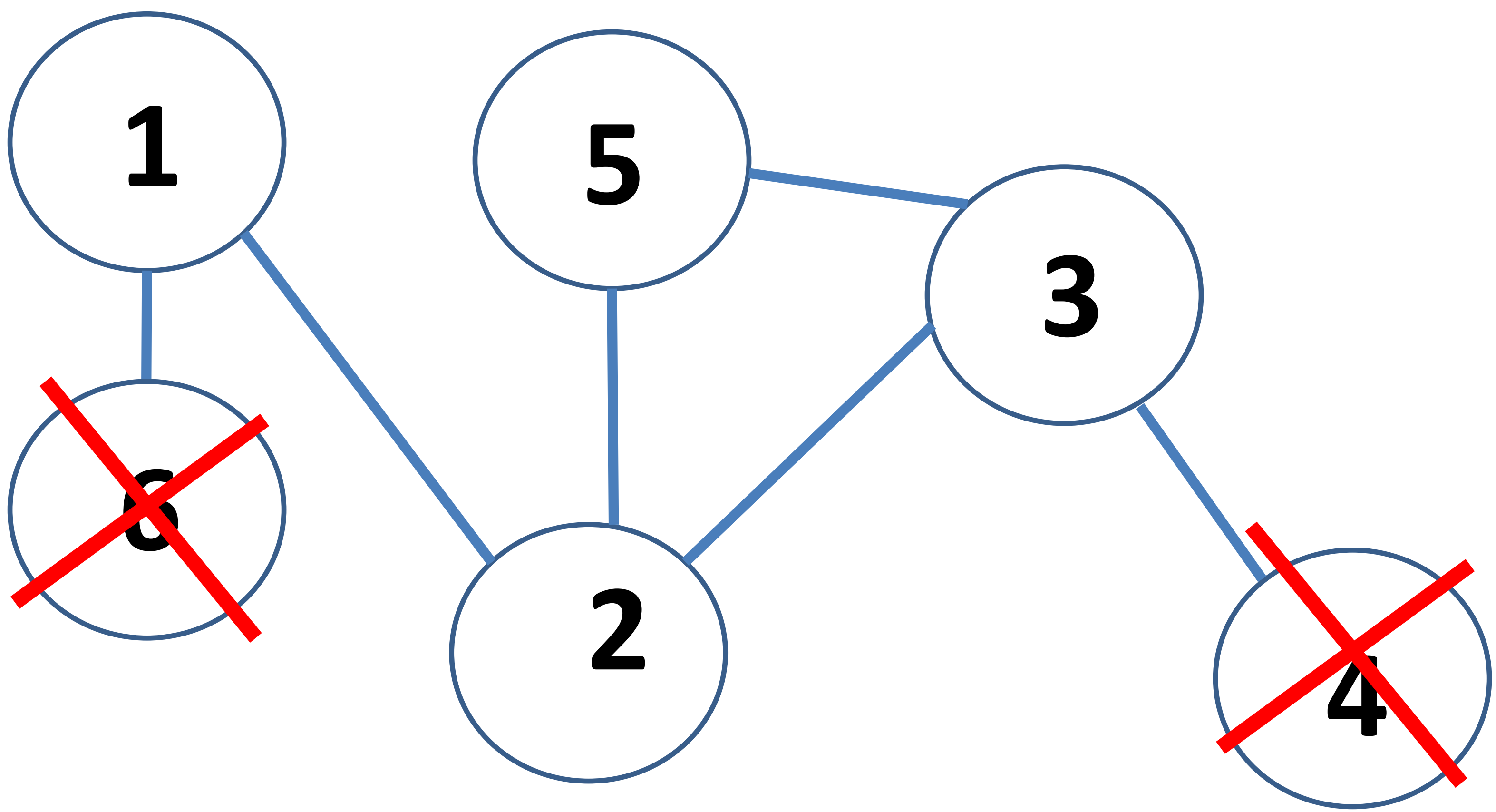
Q4

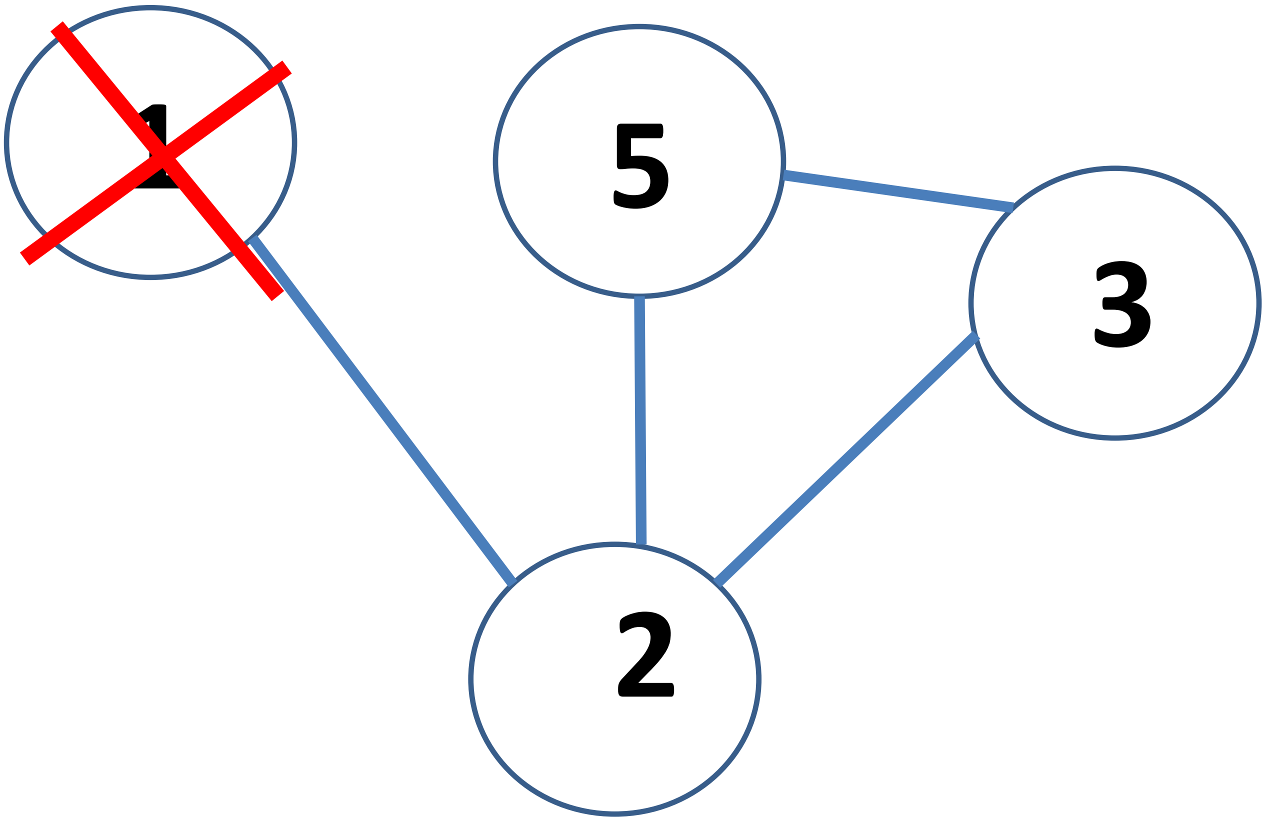
Apply pruning to find the 2-cores in the following graph.

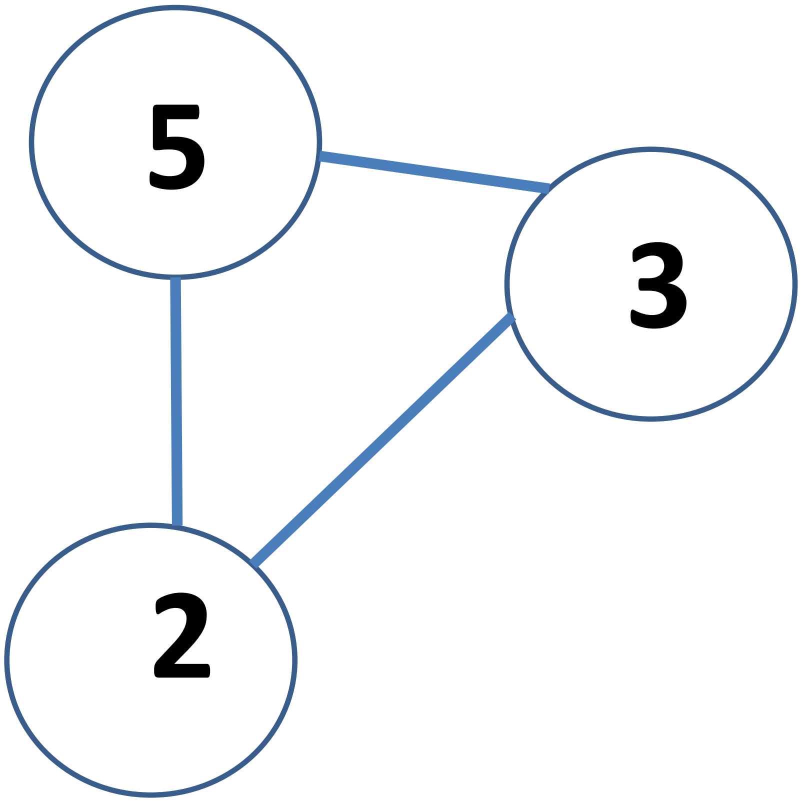


Start with the whole network and removing vertices with degree less than k, since by definition these can't be part of a k-core. This may reduce the degree of remaining vertices, so we must repeat the pruning of all vertices with degree less than k.

After the pruning is complete, the graph will now be a k-core or set of k-cores.


1
5
3
6
2
4


1
5
3
2


5
3
2

Q5

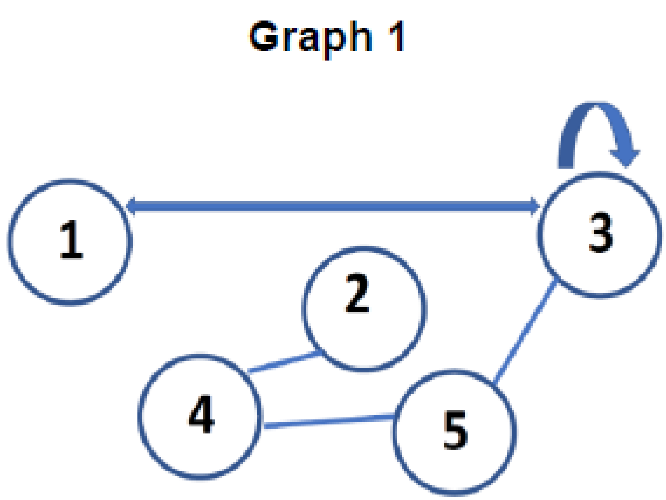
Q.1

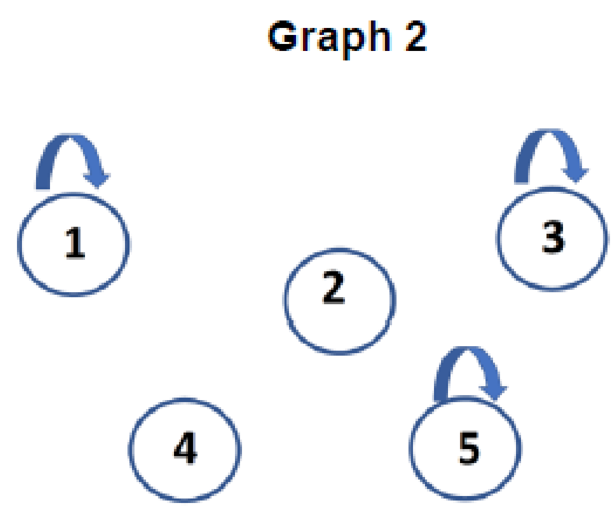
(a) Write the degree matrices of the undirected graphs, and the in-degree and out-degree matrices of the directed graphs in Fig. Q1.

[12.5 marks]

(b) Write the adjacency matrices of the graphs in Fig. Q1.

[12.5 marks]





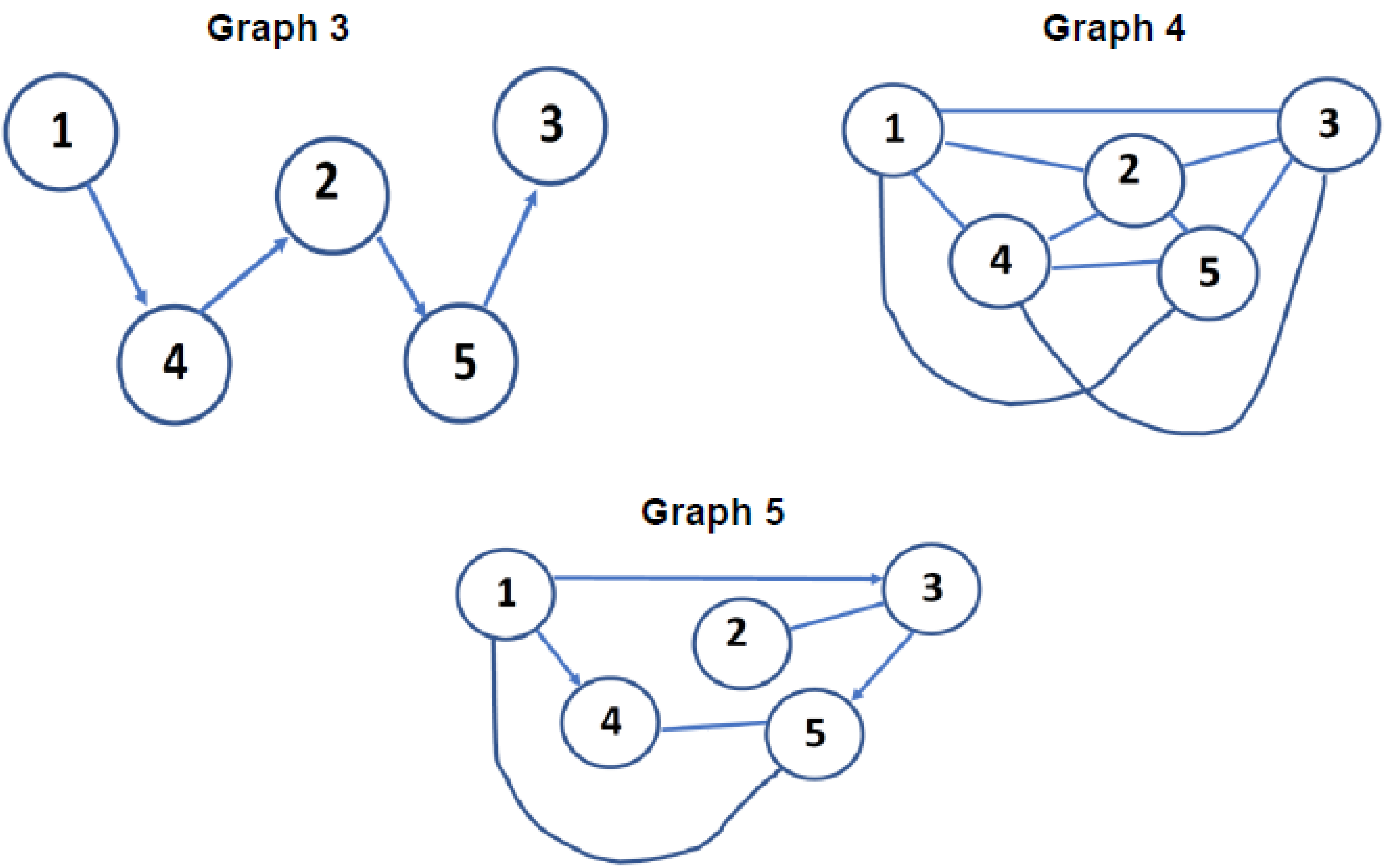


Fig. Q1

(a)

$$D_{1,in} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 3 & & \\ & & & 2 & \\ & & & & 2 \end{bmatrix}; \; D_{1,out} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 3 & & \\ & & & 2 & \\ & & & & 2 \end{bmatrix}$$

$$D_{2,in} = \begin{bmatrix} 1 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & 0 & \\ & & & & 1 \end{bmatrix}; \; D_{2,out} = \begin{bmatrix} 1 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & 0 & \\ & & & & 1 \end{bmatrix}$$

$$D_{3,in} = \begin{bmatrix} 0 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}; \; D_{3,out} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 0 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$$

$$D_4 = \begin{bmatrix} 4 & & & & \\ & 4 & & & \\ & & 4 & & \\ & & & 4 & \\ & & & & 4 \end{bmatrix}$$

$$D_{5,in} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 2 & & \\ & & & 2 & \\ & & & & 3 \end{bmatrix}; \quad D_{5,out} = \begin{bmatrix} 3 & & & & \\ & 1 & & & \\ & & 2 & & \\ & & & 1 & \\ & & & & 2 \end{bmatrix}$$

(b)

$$A_1 = \begin{bmatrix} & & 1 & & \\ & & & 1 & \\ 1 & & 1 & & 1 \\ & 1 & & & 1 \\ & & 1 & 1 & \end{bmatrix};\ A_2 = \begin{bmatrix} 1 & & & & \\ & 0 & & & \\ & & 1 & & \\ & & & 0 & \\ & & & & 1 \end{bmatrix};\ A_3 = \begin{bmatrix} & & & 1 & \\ & & & & 1 \\ & & & & \\ & 1 & & & \\ & & 1 & & \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix};\ A_5 = \begin{bmatrix} & & 1 & 1 & 1 \\ & & 1 & & \\ & 1 & & & 1 \\ & & & & 1 \\ 1 & & & 1 & \end{bmatrix}$$

Q6

Q.7

Which of the following statements about the graph Laplacian L, is **false**?

(a) The smallest eigenvalue of L is zero
(b) L is a symmetric matrix
(c) The second smallest eigenvalue of L is non-zero if and only if the network is disconnected
(d) In a network with 3 components there are always at least 3 eigenvectors of L with eigenvalue zero

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Q7

Q.8

What is one of the main issues with closeness centrality?

- **(a)** Its dynamic range is very small
- **(b)** It gives a low value for important nodes
- **(c)** It gives a low value for vertices in small components
- **(d)** None of the above is the correct answer

Q.8

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Q8

Which one among the following statements is incorrect for a directed graph?

a) The sum of 1’s in row i is the out-degree of node i
b) The sum of 1’s in column j is the in-degree of node j
c) A fully disconnected graph’s adjacency matrix will always be the identity matrix
d) A self-edge in node i corresponds to the element (i,i) of the adjacency matrix being 1

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Q9

Which one among the following statements is incorrect?

(a) A k-clique is also a k-plex
(b) A k-plex is also a k-clique
(c) A k-clique cannot contain (k-1)-cliques
(d) A k-core might exist or not, depending on the value of k

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