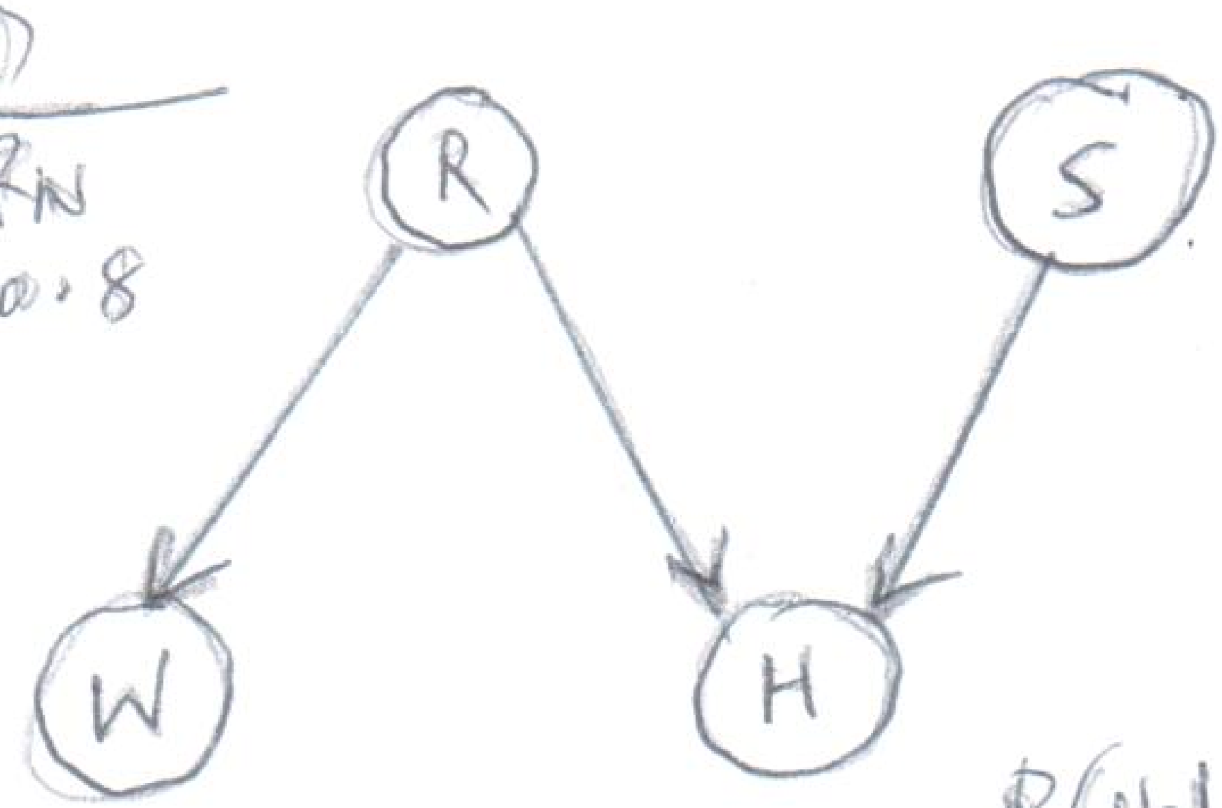
Wet Grass Example!

P(R)	
R_Y	R_N
0.2	0.8



P(S)	
S_Y	S_N
0.1	0.9

$P(W|R)$

	R_Y	R_N
W_Y	1	0.2
W_N	0	0.8

$P(H|R,S)$

	R_Y, S_Y	R_N, S_Y	R_Y, S_N	R_N, S_N
H_Y	1	0.9	1	0
H_N	0	0.1	0	1

Initial values

	b(x)	π(x)	λ(x)
R	(0.2, 0.8)	(0.2, 0.8)	(1,1)
S	(0.1, 0.9)	(0.1, 0.9)	(1,1)
W			(1,1)
H			(1,1)

$$\boxed{\begin{aligned} \lambda_W(R) &= [1, 1]\begin{bmatrix}1 & 0.2\\ 0 & 0.8\end{bmatrix} = [1, 1] \\ \lambda_H(R) &= [1, 1]\begin{bmatrix}1 & 0.09\\ 0 & 0.91\end{bmatrix} = [1, 1]\end{aligned}}$$

$$P(H|R) = \sum_S P(H|R,S)\,P(S|R) = \sum_S P(H|R,S)P(S)$$

$$P(H|R_Y) = [0.1\ 0.9]\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix} = [1, 0]$$

$$P(H|R_N) = [0.1\ 0.9]\begin{bmatrix}0.9 & 0.1\\ 0 & 1\end{bmatrix} = [0.09, 0.91]$$

$$P(H|R) = \begin{bmatrix}1 & 0.09\\ 0 & 0.91\end{bmatrix}$$

$$P(H|S_Y) = [0.2\ 0.8]\begin{bmatrix}1 & 0\\ 0.9 & 0.1\end{bmatrix}$$

$$= [0.92, 0.08]$$

$$P(H|S_N) = [0.2\ 0.8]\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= [0.2, 0.8]$$

$$P(H|S) = \begin{bmatrix}0.92 & 0.2\\ 0.08 & 0.8\end{bmatrix}$$

$$\lambda(R) = [1, 1] \times [1, 1] = [1, 1]$$

Similarly

$$\lambda(S) = [1, 1] \times [1, 1] = [1, 1]$$

$$\pi(W) = [0.2\ 0.8]\begin{bmatrix}1 & 0\\ 0.2 & 0.8\end{bmatrix} = [0.36, 0.64]$$

$$\pi(H) = [1, 1] \times [0.1\ \ 0.9]\begin{bmatrix}0.92 & 0.08\\ 0.2 & 0.8\end{bmatrix} = [0.272, 0.728]$$

Prior Probabilities

	$b(x)$	$\pi(x)$	$\lambda(x)$
R	(0.2 0.8)	(0.2 0.8)	(1, 1)
S	(0.1 0.9)	(0.1 0.9)	(1, 1)
W	(0.36 0.64)	(0.36 0.64)	(1, 1)
H	(0.272 0.728)	(0.272, 0.728)	(1, 1)

$$b(R) = (0.2\ 0.8) \times (1\ 1) = (0.2\ 0.8)$$

One morning Holmes leaves his house, and finds his grass is wet.

	$b(x)$	$\pi(x)$	$\lambda(x)$
R			
S			
W			
H	(1, 0)	(0.272, 0.728)	(1, 0)

Now, $\lambda_H = [1, 0]$

$\lambda_W = [1\ 1]$

$$\lambda_H(R) = [1\ 0]\begin{bmatrix} 1 & 0.09 \\ 0 & 0.91 \end{bmatrix} = [1\ 0.09]; \quad P(R) = \alpha_1 [0.2\ 0.8] \times [1\ 0.09] = [0.7353, 0.2647]$$

$$\lambda_H(S) = [1\ 0]\begin{bmatrix} 0.92 & 0.2 \\ 0.08 & 0.8 \end{bmatrix} = [0.92\ 0.2]; \quad P(S) = \alpha_2 [0.1\ 0.9] \times [0.92\ 0.2] = [0.3382\ 0.6618]$$

$\pi_W(R)$ gets changed! $[0.7353, 0.2647]$ Increase!

$$\pi(W) = [0.7353, 0.2647]\begin{bmatrix} 1 & 0 \\ 0.2 & 0.8 \end{bmatrix} = [0.7882, 0.2118]$$

Next he notices that the grass of Dr. Watson's garden is also wet.
use previous approaches

$$\lambda_W(R) = [1\ 0]\begin{bmatrix} 1 & 0.2 \\ 0 & 0.8 \end{bmatrix} = [1, 0.2], \quad P(R) = \alpha [0.2\ 0.8] \times [1\ 0.2] = [0.55, 0.45]$$

$$\lambda_H(S) = [1\ 0]\begin{bmatrix} 0.955 & 0.55 \\ 0.045 & 0.45 \end{bmatrix} = [0.955\ 0.55], \quad P(S) = \alpha [0.1\ 0.9] \times [0.955\ 0.55] = [0.161\ 0.839]$$

$P(H|S_y)$ $\quad$ $P(H|S_n)$

$$[0.55\ 0.45]\begin{bmatrix} 1 & 0 \\ 0.9 & 0.1 \end{bmatrix} \qquad [0.55\ 0.45]\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$