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Aplicação em FTHA – Finite Time Heat Addition Otto Engine Model

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Equação de Estado (EoS): Comportamento $P-T-\frac{1}{2}$

$$Pv = RT$$

$$P\bar{v} = \bar{R}T$$
 \rightarrow





$$Pv = RT$$

$$P = \frac{RT}{V}$$

$$P\bar{v} = \bar{R}T$$
 -

$$P = rac{RT}{ar{v}}$$
 —





$$Pv = RT$$

$$P = \frac{RT}{v}$$

$$r = \frac{Pv}{R}$$

$$P\bar{v} = \bar{R}T$$

$$P=rac{ar{R}T}{ar{v}}$$
 —

$$T = \frac{P\bar{\nu}}{\bar{R}}$$





$$Pv = RT$$
$$P = \frac{RT}{}$$

$$P = \frac{\kappa T}{v}$$

$$T = \frac{Pv}{R}$$

$$v = \frac{RT}{P}$$

$$P\bar{v} = \bar{R}T$$

$$P = \frac{\bar{R}T}{\bar{v}} \qquad -$$

$$T = rac{Par{v}}{ar{R}}$$
 —

$$ar{v} = rac{ar{R}T}{P}$$





$$Pv = RT$$
 $P\bar{v} = \bar{R}T$ \neg

$$P = \frac{RT}{v}$$
 $P = \frac{\bar{R}T}{\bar{v}}$ \neg

$$T = \frac{Pv}{R}$$
 $T = \frac{P\bar{v}}{\bar{R}}$ \neg

$$v = \frac{RT}{P}$$
 $\bar{v} = \frac{\bar{R}T}{P}$ \therefore

Cada equação com forma nas bases mássica, e molar, com $R = \bar{R}/M$ — armazenar \bar{R} e M!





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1},$$

$$T_{min} \leqslant T \leqslant T_{max}$$





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1},$$

$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 -

$$T_{min} \leqslant T \leqslant T_{max}$$
 \neg





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{c}_v(T) = \bar{c}_p(T) - \bar{R} = \sum_{i=1}^4 b_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max}$$





$$ar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} - ar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} - ar{c}_v(T) = ar{c}_p(T) - ar{R} = \sum_{i=1}^4 b_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} - ar{c}_v(T) = a_1 - ar{R}.$$





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = \bar{c}_p(T) - \bar{R} = \sum_{i=1}^4 b_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = a_1 - \bar{R}, \qquad b_{i>1} = a_{i>1} \qquad \vdots$$

Armazenar a_i , T_{min} e T_{max} e saber as conversões (i) $a_i \to b_i$ e (ii) $\bar{c}_{p,v}(T) \to c_{p,v}(T)$.





$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_{\scriptscriptstyle \mathcal{V}}(T) \, dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} \, dT, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -$$





$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_v(T) dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} dT, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T_{ref}}^T, \qquad T_{min} \leqslant T \leqslant T_{max}$$





$$ar{u}(T) = \int_{T_{ref}}^{T} ar{c}_{v}(T) dT = \int_{T_{ref}}^{T} \sum_{i=1}^{4} b_{i} T^{i-1} dT, \qquad T_{min} \leqslant T \leqslant T_{max} - ar{u}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max}$$

• Armazenar T_{ref} ,





$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_v(T) dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} dT, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T=1}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max}$$

- Armazenar T_{ref} ,
- Compor eficientemente a soma de produtos, e





$$ar{u}(T) = \int_{T_{mf}}^T ar{c}_{\scriptscriptstyle V}(T) \, dT = \int_{T_{mf}}^T \sum_{i=1}^4 b_i T^{i-1} \, dT, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T=0}^T, \qquad T_{min} \leqslant T \leqslant T_{max}$$

- Armazenar T_{ref} ,
- Compor eficientemente a soma de produtos, e
- Saber as conversões $\bar{u}(T) \to u(T)$.





торісов ис ініріспісніка

Modelo de
$$\bar{c}_p(T)$$
 Polinomial: $\bar{h}(T) = \bar{u}(T) + \bar{R}T$

$$\bar{h}(T) = \int_{T_{ref}}^{T} \bar{c}_p(T) dT + \bar{R}T_{ref} = \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-1} dT + \bar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$





Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{h}(T) = \bar{u}(T) + \bar{R}T$

$$ar{h}(T) = \int_{T_{ref}}^T ar{c}_p(T) dT + ar{R} T_{ref} = \int_{T_{ref}}^T \sum_{i=1}^4 a_i T^{i-1} dT + ar{R} T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_{T=1}^T + \bar{R} T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max}$$





Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{h}(T) = \bar{u}(T) + \bar{R}T$

$$ar{h}(T) = \int_{T_{ref}}^{T} ar{c}_p(T) dT + ar{R}T_{ref} = \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-1} dT + ar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max}$$
 $ar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_{T}^{T} + ar{R}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$

Compor eficientemente a soma de produtos, e





Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{h}(T) = \bar{u}(T) + \bar{R}T$

$$\bar{h}(T) = \int_{T_{ref}}^{T} \bar{c}_p(T) dT + \bar{R}T_{ref} = \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-1} dT + \bar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max} \qquad \neg T_{min} \leqslant T_{min} \leqslant T_{max} \qquad \neg T_{min} \leqslant T_{min} \leqslant T_{min} \qquad \neg T_{min} \leqslant T_{min}$$

$$\bar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_T^T + \bar{R} T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max}$$

- Compor eficientemente a soma de produtos, e
- Saber as conversões $\bar{h}(T) \rightarrow h(T)$.





$$ar{s}^{\circ}(T) = \int_0^T rac{ar{c}_p(T)}{T} \, dT = \int_0^{T_{ref}} rac{ar{c}_p(T)}{T} \, dT + \int_{T_{ref}}^T rac{ar{c}_p(T)}{T} \, dT, \quad T_{min} \leqslant T \leqslant T_{max}$$





$$ar{s}^{\circ}(T) = \int_0^T rac{ar{c}_p(T)}{T} dT = \int_0^{T_{ref}} rac{ar{c}_p(T)}{T} dT + \int_{T_{ref}}^T rac{ar{c}_p(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max}$$

$$\overline{s}^{\circ}(T) = \overline{s}^{\circ}_{ref} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-2} dT$$

$$T_{min} \leqslant T \leqslant T_{max}$$





$$\bar{s}^{\circ}(T) = \int_{0}^{T} \frac{\bar{c}_{p}(T)}{T} dT = \int_{0}^{T_{ref}} \frac{\bar{c}_{p}(T)}{T} dT + \int_{T_{ref}}^{T} \frac{\bar{c}_{p}(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

$$\bar{s}^{\circ}(T) = \bar{s}_{ref}^{\circ} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_{i} T^{i-2} dT \qquad \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

 $ar{s}^{\circ}(T) = ar{s}_{ref}^{\circ} + \left(a_1 \ln(T) + a_2 T + \frac{a_3 T^2}{2} + \frac{a_4 T^3}{3}\right)_T^T$, $T_{min} \leqslant T \leqslant T_{max}$





$$\bar{s}^{\circ}(T) = \int_{0}^{T} \frac{\bar{c}_{p}(T)}{T} dT = \int_{0}^{T_{ref}} \frac{\bar{c}_{p}(T)}{T} dT + \int_{T_{ref}}^{T} \frac{\bar{c}_{p}(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_{i} T^{i-2} dT \qquad \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \left(a_{1} \ln(T) + a_{2} T + \frac{a_{3} T^{2}}{2} + \frac{a_{4} T^{3}}{3}\right)_{T}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$

• Armazenar \bar{s}_{ref}° , compor eficientemente a soma de produtos, e





$$\bar{s}^{\circ}(T) = \int_{0}^{T} \frac{\bar{c}_{p}(T)}{T} dT = \int_{0}^{T_{ref}} \frac{\bar{c}_{p}(T)}{T} dT + \int_{T_{ref}}^{T} \frac{\bar{c}_{p}(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_{i} T^{i-2} dT \qquad \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \left(a_{1} \ln(T) + a_{2} T + \frac{a_{3} T^{2}}{2} + \frac{a_{4} T^{3}}{3}\right)_{T}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$

- Armazenar \bar{s}_{ref}° , compor eficientemente a soma de produtos, e
- Saber as conversões $\bar{s}^{\circ}(T) \to s^{\circ}(T)$.





Modelo de $\bar{c}_p(T)$ Polinomial: $P_r(T)$ e $v_r(T)$

$$\left(\frac{P_2}{P_1}\right)_{s} = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad -$$





Modelo de $\bar{c}_p(T)$ Polinomial: $P_r(T)$ e $v_r(T)$

$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad -$$

$$P_r(T) \equiv e^{ar{s}^\circ(T)/R}$$

$$P_r(T) = e^{s^{\circ}(T)/I}$$





Modelo de $\bar{c}_p(T)$ Polinomial: $P_r(T)$ e $v_r(T)$

$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad -$$

$$P_r(T) \equiv e^{ar{s}^\circ(T)/ar{R}}$$

$$P_r(T) = e^{s^{\circ}(T)/R}$$

$$v_r(T) \equiv \frac{T}{P_r(T)}.$$





Modelo de $\bar{c}_n(T)$ Polinomial: $P_r(T)$ e $v_r(T)$

• Sem requisitos adicionais de armazenamento!





Modelo de $\bar{c}_n(T)$ Polinomial: $P_r(T)$ e $v_r(T)$

$$egin{align} \left(rac{P_2}{P_1}
ight)_s &= rac{P_{r2}}{P_{r1}} & \left(rac{v_2}{v_1}
ight)_s &= rac{v_{r2}}{v_{r1}} &
ho \ & P_r(T) &\equiv e^{ar{s}^\circ(T)/ar{R}} & P_r(T) &= e^{s^\circ(T)/R} \ & v_r(T) &\equiv rac{T}{P_r(T)}. &
ho \ & \end{array}$$

- Sem requisitos adicionais de armazenamento!
- Sem conversões de base!





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 —





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$

 $\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3,$

$$T_{min} \leqslant T \leqslant T_{max}$$
 \neg
 $T_{min} \leqslant T \leqslant T_{max}$ \neg





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_T^T, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T_{ref}}^T, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_{T_{ref}}^T + \bar{R} T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\$$





$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{h}(T) = \left(a_{1}T + \frac{a_{2}T^{2}}{2} + \frac{a_{3}T^{3}}{3} + \frac{a_{4}T^{4}}{4}\right)_{T_{ref}}^{T} + \bar{R}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{s}^{\circ}(T) = \left(a_{1}\ln(T) + a_{2}T + \frac{a_{3}T^{2}}{2} + \frac{a_{4}T^{3}}{3}\right)_{T}^{T} + \bar{s}_{ref}^{\circ}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$





$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(a_{1}T + \frac{a_{2}T^{2}}{2} + \frac{a_{3}T^{3}}{3} + \frac{a_{4}T^{4}}{4}\right)_{T_{ref}}^{T} + \overline{k}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(a_{1}\ln(T) + a_{2}T + \frac{a_{3}T^{2}}{2} + \frac{a_{4}T^{3}}{3}\right)_{T_{v}}^{T} + \overline{s}_{ref}^{\circ}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \dots$$

Verificação de limites;





$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{h}(T) = \left(a_{1}T + \frac{a_{2}T^{2}}{2} + \frac{a_{3}T^{3}}{3} + \frac{a_{4}T^{4}}{4}\right)_{T_{ref}}^{T} + \bar{R}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{s}^{\circ}(T) = \left(a_{1}\ln(T) + a_{2}T + \frac{a_{3}T^{2}}{2} + \frac{a_{4}T^{3}}{3}\right)_{T_{mef}}^{T} + \bar{s}_{ref}^{\circ}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$

- Verificação de limites;
- Coef./func. próprios; e





$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{min} \leqslant T \leqslant T_{min} \leqslant T_$$

- Verificação de limites;
- Coef./func. próprios; e
- Produtos matriciais.





```
1 # Universal gas constant
 2 \bar{R}() = 8.314472 \# \pm 0.000015 \# kJ/kmol·K
 3
 4 # Standard Tref
 5 \text{ Tref()} = 298.15 \# K
 6
  # IG (Ideal Gas) structure: values for each gas instance
 8 struct IG
 9
       MW
                             # Molecular "Weight", kg/kmol
10
       CP::Ntuple{4}
                             # Exactly 4 cp(T) coefficients
11
       Tmin
                             # T min, K
12
       Tmax
13
       sref
                             # seref, kJ/kmol·K
14 end
```





