#### C.01.01.Z1 – Biblioteca Simplificada de Gás Ideal

Aplicação em FTHA – Finite Time Heat Addition Otto Engine Model

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https://github.com/CNThermSci/ApplThermSci Compiled on 2020-09-10 02h09m11s UTC





$$Pv = RT$$

$$P\bar{v} = \bar{R}T$$
  $\rightarrow$ 





$$Pv = RT$$

$$P = \frac{RT}{v}$$

$$P\bar{v} = \bar{R}T$$

$$P = rac{RT}{ar{v}}$$
 —





$$Pv = RT$$

$$P = \frac{RT}{v}$$

$$T = \frac{Pv}{R}$$

$$P\bar{v} = \bar{R}T$$

$$P = rac{ar{R}T}{ar{v}}$$
 —

$$T = \frac{P\bar{\nu}}{\bar{R}}$$





$$Pv = RT$$

$$P = \frac{RT}{v}$$

$$T = \frac{Pv}{R}$$

$$RT$$

$$P\bar{v} = \bar{R}T$$
  $\rightarrow$ 
 $P = \frac{\bar{R}T}{\bar{v}}$   $\rightarrow$ 
 $T = \frac{P\bar{v}}{\bar{R}}$   $\rightarrow$ 
 $\bar{v} = \frac{\bar{R}T}{\bar{R}}$   $\therefore$ 





$$Pv = RT$$
  $P\bar{v} = \bar{R}T$   $\neg$ 

$$P = \frac{RT}{v}$$
  $P = \frac{\bar{R}T}{\bar{v}}$   $\neg$ 

$$T = \frac{Pv}{R}$$
  $T = \frac{P\bar{v}}{\bar{R}}$   $\neg$ 

$$v = \frac{RT}{P}$$
  $\bar{v} = \frac{\bar{R}T}{P}$   $\vdots$ 

Cada equação com forma nas bases mássica, e molar, com  $R = \bar{R}/M$  — armazenar  $\bar{R}$  e M!





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1},$$

$$T_{min} \leqslant T \leqslant T_{max}$$
  $\rightarrow$ 





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1},$$

$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 -

$$T_{min} \leqslant T \leqslant T_{max}$$
  $\neg$ 





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{c}_v(T) = \bar{c}_p(T) - \bar{R} = \sum_{i=1}^4 b_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max}$$





$$ar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} - ar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} - ar{c}_v(T) = ar{c}_p(T) - ar{R} = \sum_{i=1}^4 b_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} - ar{c}_v(T) = a_1 - ar{R}.$$





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = \bar{c}_p(T) - \bar{R} = \sum_{i=1}^4 b_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = a_1 - \bar{R}, \qquad b_{i>1} = a_{i>1} \qquad \vdots$$

Armazenar  $a_i$ ,  $T_{min}$  e  $T_{max}$  e saber as conversões (i)  $a_i \to b_i$  e (ii)  $\bar{c}_{p,v}(T) \to c_{p,v}(T)$ 





### Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{u}(T)$

$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_{\scriptscriptstyle \mathcal{V}}(T) \, dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} \, dT, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -$$





# Modelo de $\bar{c}_n(T)$ Polinomial: $\bar{u}(T)$

$$ar{u}(T) = \int_{T_{vorf}}^T ar{c}_{\scriptscriptstyle \mathcal{V}}(T) \, dT = \int_{T_{vorf}}^T \sum_{i=1}^4 b_i T^{i-1} \, dT,$$

$$ar{u}(T) = \left(b_1 T + rac{b_2 T^2}{2} + rac{b_3 T^3}{3} + rac{b_4 T^4}{4}
ight)_{T_{ref}}^T, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 -

$$\leqslant T \leqslant T_{max}$$





# Modelo de $\bar{c}_n(T)$ Polinomial: $\bar{u}(T)$

$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_{v}(T) dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} dT, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T=1}^T, \qquad T_{min} \leqslant T \leqslant T_{max}$$

Armazenar  $T_{ref}$ ,





# Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{u}(T)$

$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_v(T) \, dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} \, dT, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T=1}^T, \qquad T_{min} \leqslant T \leqslant T_{max}$$

- Armazenar  $T_{ref}$ ,
- Compor eficientemente a soma de produtos, e





# Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{u}(T)$

$$ar{u}(T) = \int_{T_{min}}^T ar{c}_v(T) dT = \int_{T_{min}}^T \sum_{i=1}^4 b_i T^{i-1} dT, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T=1}^T, \qquad T_{min} \leqslant T \leqslant T_{max}$$

- Armazenar  $T_{ref}$ ,
- Compor eficientemente a soma de produtos, e
- Saber as conversões  $\bar{u}(T) \rightarrow u(T)$ .





#### ropicos de impenientaç

Modelo de  $\bar{c}_p(T)$  Polinomial:  $\bar{h}(T) = \bar{u}(T) + \bar{R}T$ 

$$ar{h}(T) = \int_{T_{ref}}^T ar{c}_p(T) dT + ar{R}T_{ref} = \int_{T_{ref}}^T \sum_{i=1}^4 a_i T^{i-1} dT + ar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max}$$





# Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{h}(T) = \bar{u}(T) + \bar{R}T$

$$ar{h}(T) = \int_{T_{ref}}^T ar{c}_p(T) dT + ar{R}T_{ref} = \int_{T_{ref}}^T \sum_{i=1}^4 a_i T^{i-1} dT + ar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max} \qquad -$$

$$\bar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_{T_{ref}}^T + \bar{R} T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \quad \therefore$$





# Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{h}(T) = \bar{u}(T) + \bar{R}T$

$$ar{h}(T) = \int_{T_{ref}}^{T} ar{c}_p(T) dT + ar{R}T_{ref} = \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-1} dT + ar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max}$$
 $ar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{2} + \frac{a_4 T^4}{2}\right)^T + ar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max}$ 

• Compor eficientemente a soma de produtos, e





# Modelo de $\bar{c}_p(T)$ Polinomial: $\bar{h}(T) = \bar{u}(T) + \bar{R}T$

$$\bar{h}(T) = \int_{T_{ref}}^{T} \bar{c}_p(T) dT + \bar{R}T_{ref} = \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-1} dT + \bar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max} \qquad \neg T_{min} \qquad \neg T_{min} \leqslant T_{max} \qquad \neg T_{min} \leqslant T_{$$

$$ar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_{T=1}^{T} + ar{R} T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \quad .$$

- Compor eficientemente a soma de produtos, e
- Saber as conversões  $\bar{h}(T) \rightarrow h(T)$ .





Modelo de 
$$\bar{c}_p(T)$$
 Polinomial:  $\bar{s}^{\circ}(T)$ 

$$ar{s}^{\circ}(T) = \int_0^T rac{ar{c}_p(T)}{T} \, dT = \int_0^{T_{ref}} rac{ar{c}_p(T)}{T} \, dT + \int_{T_{ref}}^T rac{ar{c}_p(T)}{T} \, dT, \quad T_{min} \leqslant T \leqslant T_{max}$$





$$\bar{s}^{\circ}(T) = \int_0^T \frac{\bar{c}_p(T)}{T} dT = \int_0^{T_{ref}} \frac{\bar{c}_p(T)}{T} dT + \int_{T_{ref}}^T \frac{\bar{c}_p(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad -$$

$$ar{s}^{\circ}(T) = ar{s}_{ref}^{\circ} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-2} dT$$
  $T_{min} \leqslant T \leqslant T_{max}$ 





$$ar{s}^{\circ}(T) = \int_{0}^{T} rac{ar{c}_{p}(T)}{T} dT = \int_{0}^{T_{ref}} rac{ar{c}_{p}(T)}{T} dT + \int_{T_{ref}}^{T} rac{ar{c}_{p}(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max}$$

$$- ar{s}^{\circ}(T) = ar{s}^{\circ}_{ref} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_{i} T^{i-2} dT \qquad T_{min} \leqslant T \leqslant T_{max} \qquad - ar{s}^{\circ}_{ref} + ar{s}^{\circ}_{ref} +$$

$$\int_{T_{ref}} i=1$$

$$\int_{T_{ref}} a_{1}T^{2} = a_{1}T^{3} \setminus_{T}^{T}$$

$$ar{s}^{\circ}(T) = ar{s}_{ref}^{\circ} + \left(a_1 \ln(T) + a_2 T + rac{a_3 T^2}{2} + rac{a_4 T^3}{3}
ight)_T^T$$
,  $T_{min} \leqslant T \leqslant T_{max}$ .





$$\bar{s}^{\circ}(T) = \int_{0}^{T} \frac{\bar{c}_{p}(T)}{T} dT = \int_{0}^{T_{ref}} \frac{\bar{c}_{p}(T)}{T} dT + \int_{T_{ref}}^{T} \frac{\bar{c}_{p}(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_{i} T^{i-2} dT$$

$$T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \left(a_{1} \ln(T) + a_{2} T + \frac{a_{3} T^{2}}{2} + \frac{a_{4} T^{3}}{3}\right)_{T}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\therefore$$

• Armazenar  $\bar{s}_{ref}^{\circ}$ , compor eficientemente a soma de produtos, e





$$\bar{s}^{\circ}(T) = \int_{0}^{T} \frac{\bar{c}_{p}(T)}{T} dT = \int_{0}^{T_{ref}} \frac{\bar{c}_{p}(T)}{T} dT + \int_{T_{ref}}^{T} \frac{\bar{c}_{p}(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_{i} T^{i-2} dT \qquad \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \left( a_{1} \ln(T) + a_{2} T + \frac{a_{3} T^{2}}{2} + \frac{a_{4} T^{3}}{3} \right)_{T}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$

- Armazenar  $\bar{s}_{ref}^{\circ}$ , compor eficientemente a soma de produtos, e
- Saber as conversões  $\bar{s}^{\circ}(T) \to s^{\circ}(T)$ .





$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}} \qquad \left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad -$$





$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}}$$

$$V_1$$
  $V_3$   $V_{r1}$   $V_1$   $V_2$   $V_{r1}$ 

$$P_r(T) \equiv e^{s_r(T)/K}$$

$$P_r(T) = e^{s^{\circ}(T)/R}$$





$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad -$$

$$P_r(T) \equiv e^{ar{s}^\circ(T)/ar{R}}$$

$$P_r(T) = e^{s^{\circ}(T)/I}$$

$$v_r(T) \equiv \frac{T}{P_r(T)}.$$





$$\left(rac{P_2}{P_1}
ight)_s = rac{P_{r2}}{P_{r1}} \qquad \qquad \left(rac{v_2}{v_1}
ight)_s = rac{v_{r2}}{v_{r1}} \qquad \qquad \qquad P_r(T) \equiv e^{ar{s}^\circ(T)/ar{R}} \qquad \qquad P_r(T) = e^{ar{s}^\circ(T)/R} \qquad \qquad \qquad P_r(T) \equiv rac{T}{P_r(T)}.$$

Sem requisitos adicionais de armazenamento!





$$egin{align} \left(rac{P_2}{P_1}
ight)_s &= rac{P_{r2}}{P_{r1}} & \left(rac{v_2}{v_1}
ight)_s &= rac{v_{r2}}{v_{r1}} & 
ho \ & P_r(T) \equiv e^{ar{s}^\circ(T)/ar{R}} & P_r(T) = e^{s^\circ(T)/R} \ & v_r(T) \equiv rac{T}{P_r(T)}. & 
ho \ & \end{array}$$

- Sem requisitos adicionais de armazenamento!
- Sem conversões de base!





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 —





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$
  
 $\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3,$ 

$$T_{min} \leqslant T \leqslant T_{max}$$
  $T_{min} \leqslant T \leqslant T_{max}$ 





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T_{ord}}^T, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T_{ref}}^T, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_{T_{ref}}^T + \bar{R} T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\$$





$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{h}(T) = \left(a_{1}T + \frac{a_{2}T^{2}}{2} + \frac{a_{3}T^{3}}{3} + \frac{a_{4}T^{4}}{4}\right)_{T_{ref}}^{T} + \bar{R}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{s}^{\circ}(T) = \left(a_{1}\ln(T) + a_{2}T + \frac{a_{3}T^{2}}{2} + \frac{a_{4}T^{3}}{3}\right)_{T}^{T} + \bar{s}_{ref}^{\circ}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$

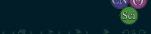




$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{v}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{v}(T) = \left(a_{1}T + \frac{a_{2}T^{2}}{2} + \frac{a_{3}T^{3}}{3} + \frac{a_{4}T^{4}}{4}\right)_{T_{ref}}^{T} + \overline{k}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{v}(T) = \left(a_{1}\ln(T) + a_{2}T + \frac{a_{3}T^{2}}{2} + \frac{a_{4}T^{3}}{3}\right)_{T_{v}}^{T} + \overline{s}_{ref}^{\circ}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$

Verificação de limites;

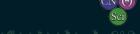




$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{min} \leqslant T \leqslant T_{min} \leqslant T_$$

- Verificação de limites;
- Coef./func. próprios; e





$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{min} \leqslant T \leqslant T_{min} \leqslant T_$$

- Verificação de limites:
- Coef./func. próprios; e
- Produtos matriciais.







```
1 # Universal gas constant
 2 \bar{R}() = 8.314472 \# \pm 0.000015 \# kJ/kmol·K
 3
 4 # Standard Tref
 5 \text{ Tref()} = 298.15 \# K
 6
 7 # IG (Ideal Gas) structure: values for each gas instance
 8 struct IG
 9
       MW
                             # Molecular "Weight", kg/kmol
10
       CP::Ntuple{4}
                             # Exactly 4 cp(T) coefficients
11
       Tmin
12
       Tmax
13
       sref
                             # soref, kJ/kmol·K
14 end
```





