

# C.01.01.Z1 – Biblioteca Simplificada de Gás Ideal

## Aplicação em FTHA – Finite Time Heat Addition Otto Engine Model

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<https://github.com/CNThermSci/ApplThermSci>

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Cada equação com forma nas bases **mássica**, e **molar**, com  $R = \bar{R}/M$  — armazenar  $\bar{R}$  e  $M$ !

## Modelo de $\bar{c}_p(T)$ Polinomial:

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Armazenar  $a_i$ ,  $T_{min}$  e  $T_{max}$  e saber as conversões (i)  $a_i \rightarrow b_i$  e (ii)  $\bar{c}_{p,v}(T) \rightarrow c_{p,v}(T)$ .

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$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}} \qquad \left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad \longrightarrow$$

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- Sem conversões de base!

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Photo by Josh Sorenson from Pexels