#### C.01.01.Z1 – Biblioteca Simplificada de Gás Ideal

Aplicação em FTHA – Finite Time Heat Addition Otto Engine Model

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https://github.com/CNThermSci/ApplThermSci Compiled on 2020-09-11 18h33m12s UTC





### Equação de Estado (EoS): Comportamento P - T - v

$$Pv = RT$$

$$P\bar{v} = \bar{R}T$$
  $\rightarrow$ 





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$$P = \frac{RT}{v}$$

$$P\bar{v} = \bar{R}T$$
 —

$$P = rac{RT}{ar{v}}$$
 —





### Equação de Estado (EoS): Comportamento P - T - v'

$$Pv = RT$$

$$P = \frac{RT}{v}$$

$$T = \frac{Pv}{R}$$

$$P\bar{v} = \bar{R}T$$
 —

$$P = rac{ar{R}T}{ar{v}}$$
 —

$$T = rac{Par{v}}{ar{R}}$$





#### Equação de Estado (EoS): Comportamento P - T - v

$$Pv = RT$$

$$P = \frac{RT}{v}$$

$$T = \frac{Pv}{R}$$

$$v = \frac{RT}{P}$$

$$P\bar{v} = \bar{R}T$$

$$P = \frac{\bar{R}T}{\bar{v}} \qquad -$$

$$T = rac{Par{v}}{ar{R}}$$

$$ar{v} = rac{ar{R}T}{P}$$







### Equação de Estado (EoS): Comportamento P - T - v

$$Pv = RT$$
  $P\bar{v} = \bar{R}T$   $\neg$ 

$$P = \frac{RT}{v}$$
  $P = \frac{\bar{R}T}{\bar{v}}$   $\neg$ 

$$T = \frac{Pv}{R}$$
  $T = \frac{P\bar{v}}{\bar{R}}$   $\neg$ 

$$v = \frac{RT}{P}$$
  $\bar{v} = \frac{\bar{R}T}{P}$   $\therefore$ 

Cada equação com forma nas bases mássica, e molar, com  $R = \bar{R}/M$  — armazenar  $\bar{R}$  e M!





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1},$$

$$T_{min} \leqslant T \leqslant T_{max}$$





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1},$$

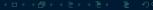
$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 -

$$T_{min} \leqslant T \leqslant T_{max}$$







$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1},$$

$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$

$$ar{c}_{v}(T) = ar{c}_{p}(T) - ar{R} = \sum_{i=1}^{4} b_{i} T^{i-1},$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 -

$$T_{min} \leqslant T \leqslant T_{max}$$
 —

$$T_{min} \leqslant T \leqslant T_{max}$$
 —





$$\bar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1},$$

$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$

$$ar{c}_{\scriptscriptstyle V}(T) = ar{c}_{\scriptscriptstyle P}(T) - ar{R} = \sum_{i=1}^4 b_i T^{i-1},$$

$$b_1 = a_1 - \bar{R},$$

$$T_{min} \leqslant T \leqslant T_{max}$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 —

$$T_{min} \leqslant T \leqslant T_{max}$$
 -

$$b_{i>1} = a_{i>1}$$
 :.





$$ar{c}_p(T) = \sum_{i=1}^4 a_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = \overline{c}_p(T) - \overline{R} = \sum_{i=1}^4 b_i T^{i-1}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_p(T) = a_1 - \overline{R}, \qquad b_{i>1} = a_{i>1} \qquad \therefore$$

Armazenar  $a_i$ ,  $T_{min}$  e  $T_{max}$  e saber as conversões (i)  $a_i \to b_i$  e (ii)  $\bar{c}_{p,v}(T) \to c_{p,v}(T)$ .





$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_{\scriptscriptstyle \mathcal{V}}(T) \, dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} \, dT, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -$$







$$ar{u}(T) = \int_{T_{vof}}^T ar{c}_v(T) \, dT = \int_{T_{vof}}^T \sum_{i=1}^4 b_i T^{i-1} \, dT, \qquad T$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T_{ref}}^{T},$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 —

$$T_{min} \leqslant T \leqslant T_{max}$$







$$\bar{u}(T) = \int_{T_{ref}}^{T} \bar{c}_{\nu}(T) dT = \int_{T_{ref}}^{T} \sum_{i=1}^{4} b_{i} T^{i-1} dT, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T_{ref}}^{I}, \qquad T_{min} \leqslant T \leqslant T_{max}$$

• Armazenar  $T_{ref}$ ,







$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_v(T) dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} dT, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T_{\text{mod}}}^T, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$

- Armazenar  $T_{ref}$ ,
- Compor eficientemente a soma de produtos, e





$$ar{u}(T) = \int_{T_{ref}}^T ar{c}_{\scriptscriptstyle V}(T) \, dT = \int_{T_{ref}}^T \sum_{i=1}^4 b_i T^{i-1} \, dT, \qquad T_{min} \leqslant T \leqslant T_{max}$$

$$\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T=0}^T, \qquad T_{min} \leqslant T \leqslant T_{max}$$

- Armazenar  $T_{ref}$ ,
- Compor eficientemente a soma de produtos, e
- Saber as conversões  $\bar{u}(T) \rightarrow u(T)$ .





$$ar{h}(T) = \int_{T_{ref}}^T ar{c}_p(T) dT + ar{R} T_{ref} = \int_{T_{ref}}^T \sum_{i=1}^4 a_i T^{i-1} dT + ar{R} T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max}$$







$$ar{h}(T) = \int_{T_{ref}}^T ar{c}_p(T) dT + ar{R} T_{ref} = \int_{T_{ref}}^T \sum_{i=1}^4 a_i T^{i-1} dT + ar{R} T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max}$$

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• Compor eficientemente a soma de produtos, e







$$\bar{h}(T) = \int_{T_{ref}}^{T} \bar{c}_p(T) dT + \bar{R}T_{ref} = \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-1} dT + \bar{R}T_{ref}, \quad T_{min} \leqslant T \leqslant T_{max} \qquad \rightarrow$$

$$\bar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_{T}^T + \bar{R} T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max}$$

- Compor eficientemente a soma de produtos, e
- Saber as conversões  $\bar{h}(T) \rightarrow h(T)$ .







$$ar{s}^{\circ}(T) = \int_{0}^{T} rac{ar{c}_{p}(T)}{T} \, dT = \int_{0}^{T_{ref}} rac{ar{c}_{p}(T)}{T} \, dT + \int_{T_{ref}}^{T} rac{ar{c}_{p}(T)}{T} \, dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad ext{and}$$







$$ar{s}^{\circ}(T) = \int_{0}^{T} rac{ar{c}_{p}(T)}{T} \, dT = \int_{0}^{T_{ref}} rac{ar{c}_{p}(T)}{T} \, dT + \int_{T_{ref}}^{T} rac{ar{c}_{p}(T)}{T} \, dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad -$$

$$\bar{s}^{\circ}(T) = \bar{s}^{\circ}_{ref} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-2} dT$$
  $T_{min} \leqslant T \leqslant T_{max}$ 







$$ar{s}^{\circ}(T) = \int_{0}^{T} rac{ar{c}_{p}(T)}{T} dT = \int_{0}^{T_{ref}} rac{ar{c}_{p}(T)}{T} dT + \int_{T-\epsilon}^{T} rac{ar{c}_{p}(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad -$$

$$ar{s}^{\circ}(T) = ar{s}_{ref}^{\circ} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-2} dT$$
  $T_{min} \leqslant T \leqslant T_{max}$ 

$$ar{s}^{\circ}(T) = ar{s}_{ref}^{\circ} + \left(a_1 \ln(T) + a_2 T + rac{a_3 T^2}{2} + rac{a_4 T^3}{3}
ight)_T^T , \qquad T_{min} \leqslant T \leqslant T_{max} .$$







$$ar{s}^{\circ}(T) = \int_{0}^{T} rac{ar{c}_{p}(T)}{T} dT = \int_{0}^{T_{ref}} rac{ar{c}_{p}(T)}{T} dT + \int_{T_{ref}}^{T} rac{ar{c}_{p}(T)}{T} dT, \quad T_{min} \leqslant T \leqslant T_{max}$$
 $ar{s}^{\circ}(T) = ar{s}^{\circ}_{ref} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_{i} T^{i-2} dT$ 
 $T_{min} \leqslant T \leqslant T_{max}$ 
 $ar{s}^{\circ}(T) = ar{s}^{\circ}_{ref} + \left(a_{1} \ln(T) + a_{2}T + rac{a_{3}T^{2}}{2} + rac{a_{4}T^{3}}{3}
ight)_{T}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max}$ 
 $\therefore$ 

• Armazenar  $\bar{s}_{ref}^{\circ}$ , compor eficientemente a soma de produtos, e





$$ar{s}^{\circ}(T) = \int_0^T rac{ar{c}_p(T)}{T} \, dT = \int_0^{T_{ref}} rac{ar{c}_p(T)}{T} \, dT + \int_{T_{ref}}^T rac{ar{c}_p(T)}{T} \, dT, \quad T_{min} \leqslant T \leqslant T_{max} \qquad -$$

$$ar{s}^{\circ}(T) = ar{s}_{ref}^{\circ} + \int_{T_{ref}}^{T} \sum_{i=1}^{4} a_i T^{i-2} dT$$
  $T_{min} \leqslant T \leqslant T_{max}$   $T_{min} \leqslant T \leqslant T_{max}$ 

$$\bar{s}^{\circ}(T) = \bar{s}_{ref}^{\circ} + \left(a_1 \ln(T) + a_2 T + \frac{a_3 T^2}{2} + \frac{a_4 T^3}{3}\right)_{T}^T, \qquad T_{min} \leqslant T \leqslant T_{max}$$
 :

- Armazenar  $\bar{s}_{ref}^{\circ}$ , compor eficientemente a soma de produtos, e
- Saber as conversões  $\bar{s}^{\circ}(T) \to s^{\circ}(T)$ .







$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad \neg$$







$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}}$$

$$P_r(T) \equiv e^{\bar{s}^\circ(T)/\bar{R}}$$

$$\left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad \neg$$

$$P_r(T) = e^{s^\circ(T)/R}$$







$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad \neg$$

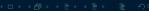
$$P_r(T) \equiv e^{\bar{s}^\circ(T)/\bar{R}}$$

$$P_r(T) = e^{s^\circ(T)/R}$$

$$v_r(T) \equiv rac{T}{P_r(T)}.$$







$$\left(rac{P_2}{P_1}
ight)_s = rac{P_{r2}}{P_{r1}} \qquad \qquad \left(rac{v_2}{v_1}
ight)_s = rac{v_{r2}}{v_{r1}} \qquad o$$
 $P_r(T) \equiv e^{ar{s}^\circ(T)/ar{R}} \qquad \qquad P_r(T) = e^{s^\circ(T)/R} \qquad \qquad v_r(T) \equiv rac{T}{P_r(T)}.$ 

• Sem requisitos adicionais de armazenamento!





$$\left(\frac{P_2}{P_1}\right)_s = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_s = \frac{v_{r2}}{v_{r1}} \qquad \neg$$

$$P_r(T) \equiv e^{ar{s}^\circ(T)/ar{R}}$$

$$P_r(T) = e^{s^{\circ}(T)/R}$$

$$v_r(T) \equiv rac{T}{P_r(T)}.$$

- Sem requisitos adicionais de armazenamento!
- Sem conversões de base!





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$

$$T_{min} \leqslant T \leqslant T_{max}$$
 —





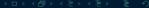


$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3,$$
  
 $\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3,$ 

$$T_{min} \leqslant T \leqslant T_{max}$$
  $\neg$ 
 $T_{min} \leqslant T \leqslant T_{max}$   $\neg$ 







$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_T^T, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg$$





$$\bar{c}_p(T) = a_1 + a_2 T + a_3 T^2 + a_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_v(T) = b_1 + b_2 T + b_3 T^2 + b_4 T^3, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_1 T + \frac{b_2 T^2}{2} + \frac{b_3 T^3}{3} + \frac{b_4 T^4}{4}\right)_{T_{ref}}^T, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{h}(T) = \left(a_1 T + \frac{a_2 T^2}{2} + \frac{a_3 T^3}{3} + \frac{a_4 T^4}{4}\right)_{T_{ref}}^T + \bar{R} T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\$$







$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{u}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{h}(T) = \left(a_{1}T + \frac{a_{2}T^{2}}{2} + \frac{a_{3}T^{3}}{3} + \frac{a_{4}T^{4}}{4}\right)_{T_{ref}}^{T} + \bar{R}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \neg \\
\bar{s}^{\circ}(T) = \left(a_{1}\ln(T) + a_{2}T + \frac{a_{3}T^{2}}{2} + \frac{a_{4}T^{3}}{3}\right)_{T}^{T} + \bar{s}_{ref}^{\circ}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \therefore$$





$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(a_{1}T + \frac{a_{2}T^{2}}{2} + \frac{a_{3}T^{3}}{3} + \frac{a_{4}T^{4}}{4}\right)_{T_{ref}}^{T} + \overline{k}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(a_{1}\ln(T) + a_{2}T + \frac{a_{3}T^{2}}{2} + \frac{a_{4}T^{3}}{3}\right)_{T_{vef}}^{T} + \overline{s}_{vef}^{\circ}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \vdots$$

Verificação de limites;





$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(b_{1}T + \frac{b_{2}T^{2}}{2} + \frac{b_{3}T^{3}}{3} + \frac{b_{4}T^{4}}{4}\right)_{T_{ref}}^{T}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(a_{1}T + \frac{a_{2}T^{2}}{2} + \frac{a_{3}T^{3}}{3} + \frac{a_{4}T^{4}}{4}\right)_{T_{ref}}^{T} + \overline{k}T_{ref}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \overline{c}_{v}(T) = \left(a_{1}\ln(T) + a_{2}T + \frac{a_{3}T^{2}}{2} + \frac{a_{4}T^{3}}{3}\right)_{T_{ref}}^{T} + \overline{s}_{ref}^{\circ}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad \vdots$$

- Verificação de limites;
- Coef./func. próprios; e



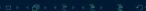


$$\bar{c}_{p}(T) = a_{1} + a_{2}T + a_{3}T^{2} + a_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{v}(T) = b_{1} + b_{2}T + b_{3}T^{2} + b_{4}T^{3}, \qquad T_{min} \leqslant T \leqslant T_{max} \qquad -\overline{c}_{min} \leqslant T \leqslant T_{min} \leqslant T_$$

- Verificação de limites;
- Coef./func. próprios; e
- Produtos matriciais.







```
1 # Universal gas constant
2 \bar{R}() = 8.314472 \# \pm 0.000015 \# kJ/kmol·K
3
4 # Standard Tref
5 \text{ Tref()} = 298.15 \# K
6
7 # IG (Ideal Gas) structure: values for each gas instance
8 struct IG
 9
       MW
                             # Molecular "Weight", kg/kmol
10
       CP::Ntuple{4}
                             # Exactly 4 cp(T) coefficients
11
       Tmin
                             # T min, K
12
       Tmax
13
       sref
                             # seref, kJ/kmol·K
14 end
```





