

DATA 301: Decision Trees

A Supervised Algorithm

Topics

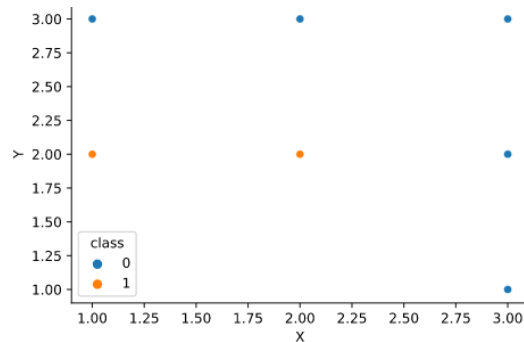
Demonstration

Impurity

Choosing which feature to split on
scikit-learn

Problem

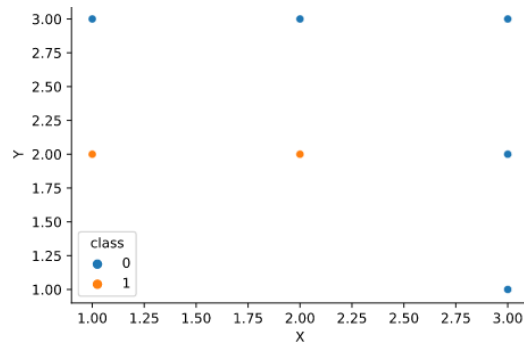
What if data is not linearly separable?



For this data you cannot draw a straight line that cleanly separates class 0 from class 1

Problem

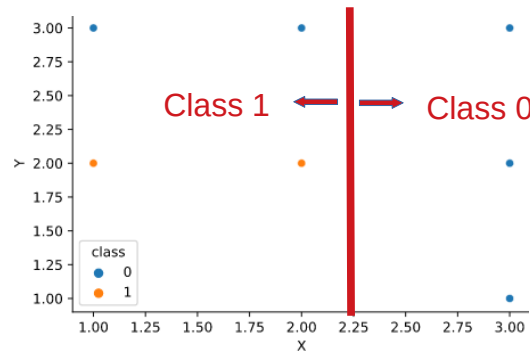
What if data is not linearly separable?



But you can use multiple line segments to do so.

Problem

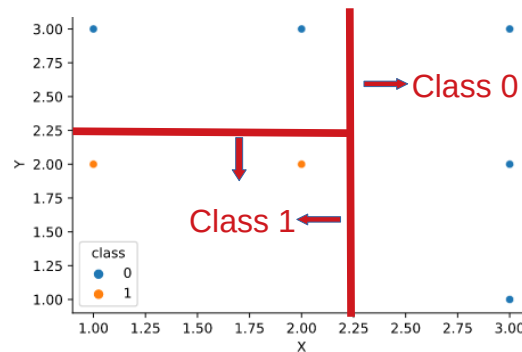
What if data is not linearly separable?



But you can use multiple line segments to do so.
The first

Problem

What if data is not linearly separable?



But you can use multiple line segments to do so.

The first

The second

Problem

Easy to do for 2 dimensions, what about 4 or 8 dimensions?

How to do this algorithmically?

Especially since data looks like this.

	X	Y	class
0	1	3	a
1	2	3	a
2	3	3	a
3	1	2	b
4	2	2	b
5	3	2	a
6	3	1	a

Solution

Using 2 common algorithms:

- entropy and information gain
- gini impurity

We will use gini impurity for this class. It's calculated with the following equation

$$\text{gini_impurity} = 1 - (\text{probability class A})^2 - (\text{probability class B})^2 - (\text{probability class C})^2 \dots (\text{probability class n})^2$$

Solution

	X	Y	class
0	1	3	a
1	2	3	a
2	3	3	a
3	1	2	b
4	2	2	b
5	3	2	a
6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Solution



	X	Y	class
0	1	3	a
1	2	3	a
2	3	3	a
3	1	2	b
4	2	2	b
5	3	2	a
6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Start with first column (X)

Solution



	X	Y	class
0	1	3	a
3	1	2	b
1	2	3	a
4	2	2	b
2	3	3	a
5	3	2	a
6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Start with first column (X)

Sort it ascending

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Start with first column (X)

Sort it ascending

Find midpoints between adjacent values

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Start with first column (X)

Sort it ascending

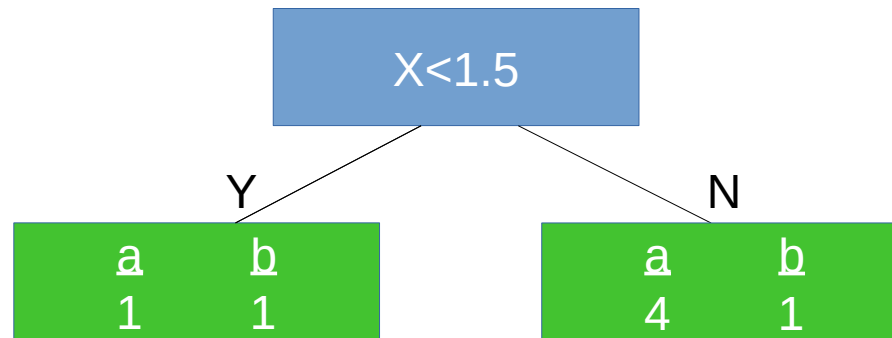
Find midpoints between adjacent values

Find gini impurity using each of these midpoints

Solution

$$\text{gini_impurity} = 1 - (\text{probability class A})^2 - (\text{probability class B})^2$$

	X	Y	class	
	0	1	3	a
	3	1	2	b
1.5	1	2	3	a
	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a



Left node

$$\begin{aligned} \text{gi} &= 1 - (1/2)^2 - (1/2)^2 \\ \text{gi} &= .5 \end{aligned}$$

Right node

$$\begin{aligned} \text{gi} &= 1 - (4/5)^2 - (1/5)^2 \\ \text{gi} &= .32 \end{aligned}$$

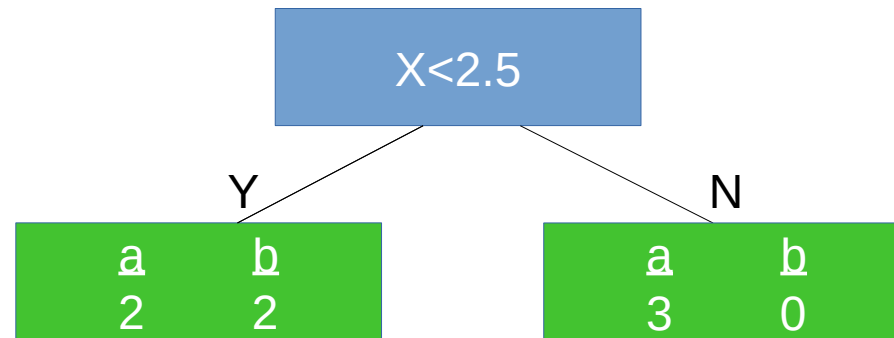
Gini impurity for column X at split 1.5 is a weighted average though
Weights = (total values in node)/(total values in both nodes)

$$\begin{aligned} \text{GI} &= 2/(2+5) \cdot .5 + 5/(2+5) \cdot .32 \\ \text{GI} &= .37 \end{aligned}$$

Solution

$$\text{gini_impurity} = 1 - (\text{probability class A})^2 - (\text{probability class B})^2$$

	X	Y	class	
	0	1	3	a
	3	1	2	b
	1	2	3	a
	4	2	2	b
2.5	2	3	3	a
	5	3	2	a
	6	3	1	a



Left node

$$\text{gi} = 1 - (2/4)^2 - (2/4)^2$$
$$\text{gi} = .5$$

Right node

$$\text{gi} = 1 - (3/3)^2 - (0/3)^2$$
$$\text{gi} = 0$$

Gini impurity for column X at split 2.5 is a weighted average though
Weights = (total values in node)/(total values in both nodes)

$$\text{GI} = 4/(2+5) \cdot .5 + 3/(2+5) \cdot 0$$
$$\text{GI} = .28$$

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Start with first column (X)

Sort it ascending

Find midpoints between adjacent values

Find gini impurity using each of these midpoints

1.5 \Rightarrow GI=.37

2.5 \Rightarrow GI=.28

Choose split with lowest value

Col X \rightarrow Choose split 2.5 with GI=.28

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Start with first column (X)

Sort it ascending

Find midpoints between adjacent values

Find gini impurity using each of these midpoints

1.5 => GI=.37

2.5=> GI=.28

Choose split with lowest value

Col X → Choose split 2.5 with GI=.28

Repeat for all other columns (except the target 'class')

Solution



	X	Y	class
0	1	3	a
3	1	2	b
1	2	3	a
4	2	2	b
2	3	3	a
5	3	2	a
6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Choose column (Y)

Solution



	X	Y	class
6	3	1	a
1.5	3	1	2
4	2	2	b
5	3	2	a
2.5	1	3	a
0	1	3	a
1	2	3	a
2	3	3	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Choose column (Y)

Sort it ascending

Solution



	X	Y	class	
	6	3	1	a
1.5	3	1	2	b
	4	2	2	b
5	3	2	a	
2.5	0	1	3	a
	1	2	3	a
	2	3	3	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Choose column (Y)

Sort it ascending

Find midpoints between adjacent values

Solution



	X	Y	class
6	3	1	a
1.5	3	1	b
4	2	2	b
5	3	2	a
2.5	0	1	a
1	2	3	a
2	3	3	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Choose column (Y)

Sort it ascending

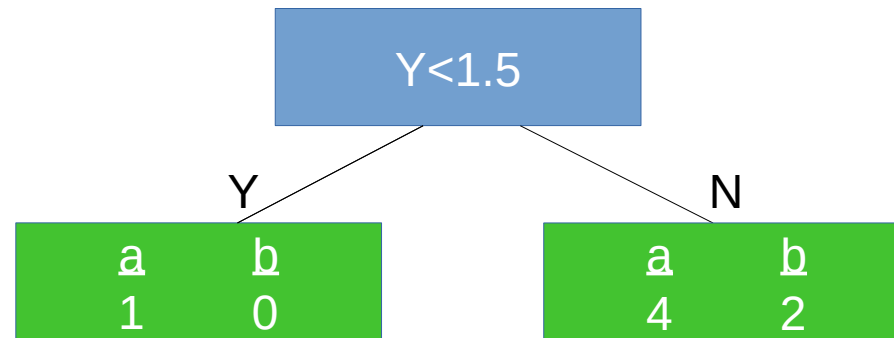
Find midpoints between adjacent values

Find gini impurity using each of these midpoints

Solution

$$\text{gini_impurity} = 1 - (\text{probability class A})^2 - (\text{probability class B})^2$$

	X	Y	class
6	3	1	a
3	1	2	b
4	2	2	b
5	3	2	a
0	1	3	a
1	2	3	a
2	3	3	a



Left node

$$\text{gi} = 1 - (1/1)^2 - (0/1)^2$$
$$\text{gi} = 0$$

Right node

$$\text{gi} = 1 - (4/6)^2 - (2/6)^2$$
$$\text{gi} = .44$$

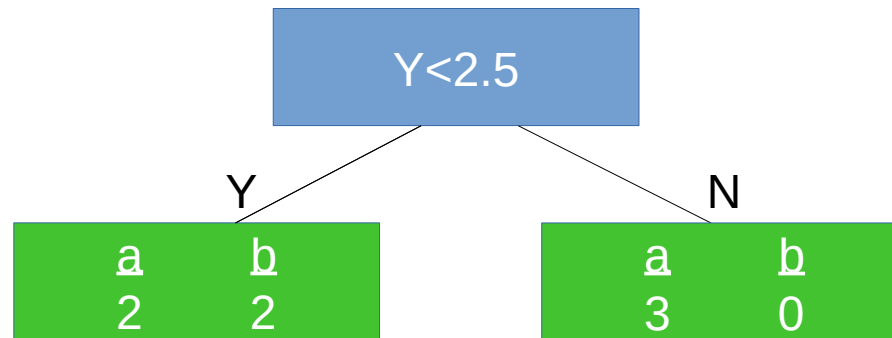
Gini impurity for column Y at split 2.5 is a weighted average though
Weights = (total values in node)/(total values in both nodes)

$$\text{GI} = 1/(1+6)*0 + 6/(1+6)*.44$$
$$\text{GI} = .37$$

Solution

$$\text{gini_impurity} = 1 - (\text{probability class A})^2 - (\text{probability class B})^2$$

	X	Y	class
6	3	1	a
3	1	2	b
4	2	2	b
5	3	2	a
2.5	1	3	a
1	2	3	a
2	3	3	a



Left node

$$\text{gi} = 1 - (2/4)^2 - (2/4)^2$$
$$\text{gi} = .5$$

Right node

$$\text{gi} = 1 - (3/3)^2 - (0/3)^2$$
$$\text{gi} = 0$$

Gini impurity for column X at split 2.5 is a weighted average though
Weights = (total values in node)/(total values in both nodes)

$$\text{GI} = 4/(2+5) \cdot .5 + 3/(2+5) \cdot 0$$
$$\text{GI} = .28$$

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Start with first column (X)

Sort it ascending

Find midpoints between adjacent values

Find gini impurity using each of these midpoints

1.5 => GI=.37

2.5=> GI=.28

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
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Find gini impurity using each of these midpoints

1.5 => GI=.37

2.5=> GI=.28

Choose split with lowest value

Col X → Choose split 2.5 with GI=.28

Col Y → Choose split 2.5 with GI=.28

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
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Choose split with lowest value

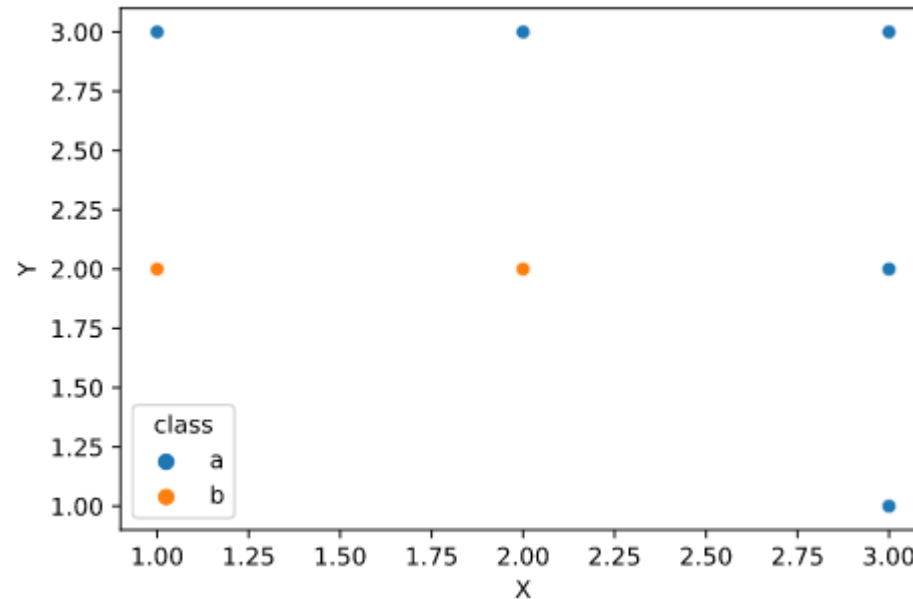
Col X → Choose split 2.5 with GI=.28

Col Y → Choose split 2.5 with GI=.28

Why are they both the same?

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a



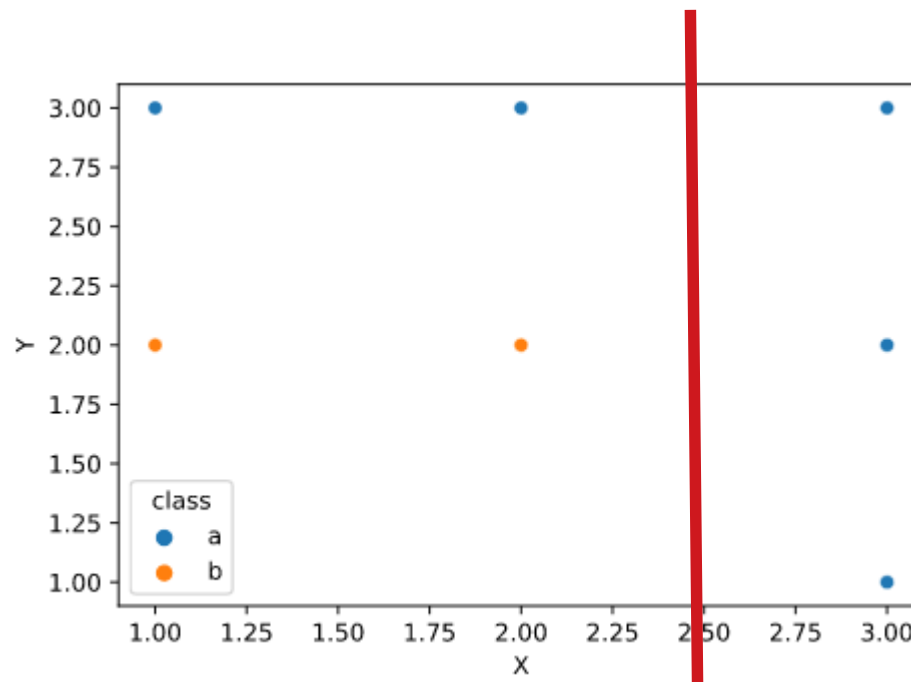
Why are they both the same?

Because it doesn't matter whether we split on $X=2.5$ or $Y=2.5$,

we still have 3 a's on 1 side and 2 a's and 2 b's on the other

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a



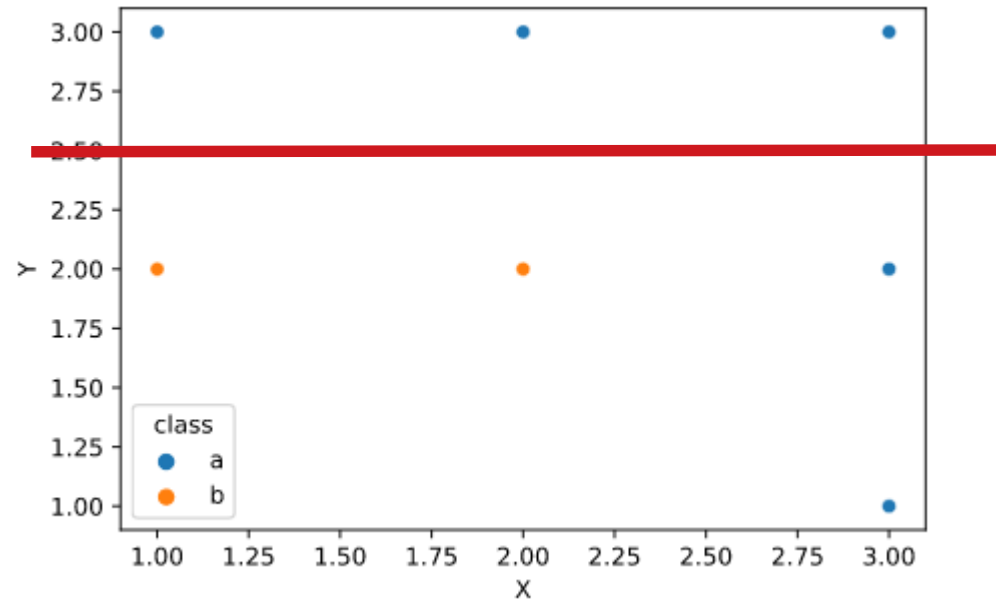
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Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a



Why are they both the same?

Because it doesn't matter whether we split on $X=2.5$ or $Y=2.5$,

we still have 3 a's on 1 side and 2 a's and 2 b's on the other

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Start with first column (X)

Sort it ascending

Find midpoints between adjacent values

Find gini impurity using each of these midpoints

1.5 => GI=.37

2.5=> GI=.28

Choose split with lowest value

Col X → Choose split 2.5 with GI=.28

Col Y → Choose split 2.5 with GI=.28

Why are they both the same?

Solution

	X	Y	class	
	0	1	3	a
1.5 →	3	1	2	b
	1	2	3	a
2.5 →	4	2	2	b
	2	3	3	a
	5	3	2	a
	6	3	1	a

Back to the problem

We want to predict **class** using features **X** and **Y**

Where to start?

Keep splitting the dataset until;

1. a node has a gini impurity less than any of the column splits, in which case leave it as is
2. a node has a gini impurity of 0, in which case leave it as is since it only has 1 class
3. you reach the maximum depth of branches. (Maximum depth is a hyperparameter)

Scikitlearns Decision Tree

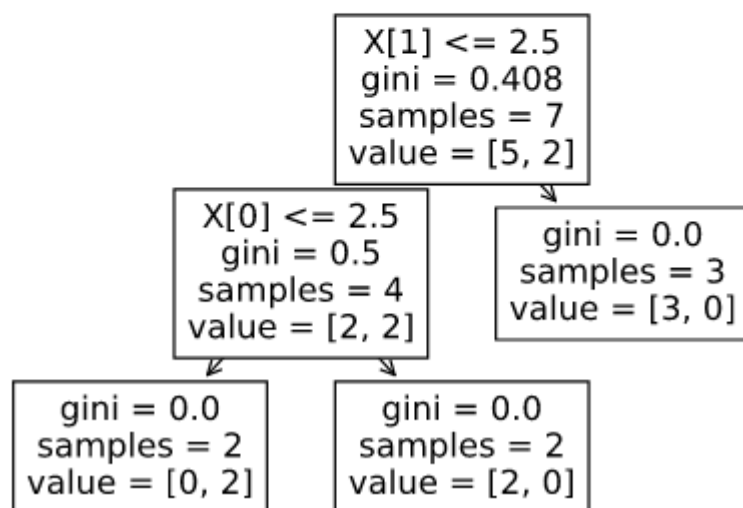
	X	Y	class
0	1	3	a
3	1	2	b
1	2	3	a
4	2	2	b
2	3	3	a
5	3	2	a
6	3	1	a

2.5

Scikitlearn Decision Tree

```
from sklearn import tree
df2=df.to_numpy();
clf = tree.DecisionTreeClassifier()
clf.fit(X=df2[:, :-1], y=df2[:, -1]);
```

```
tree.plot_tree(clf);
#for root node;  $g_i = 1 - (5/7)**2 - (2/7)**2 = 0.408$ 
```



Summary

- How to split by eye
- What gini impurity is
- How to use gini impurity to find the best features to split on and the best split points (it approximates a humans split by eye approach by splitting on midpoints of adjacent features)
- Scikit learns decision tree