Lecture #4

PSHA – MatLab Tutorial

Chiara Nardin – Ph.D., M.Sc., Eng. in Civil Engineering

PSHA

Probabilistic Seismic Hazard Analysis (PSHA) evaluates the exceedance (or occurrence) probability of a given ground motion intensity measure threshold at given site and time interval.

PSHA provides a framework in which uncertainties, typically include magnitude size, earthquake location, soil condition, and rate of occurrence of earthquakes, are quantified.

The calculation of seismic hazard is based on the Total Probability Theorem*

$$P(IM > im) = \int_S P(IM > im \,|M = m, R = r)\; f_{R|M}^{(n)}\, f_M^{(n)} dr\, dm \qquad (1)$$

<< the probability that a fixed value of ground motion im is exceeded at a given site, given the occurrence of random earthquake from the seismic source n >>

**see Notes at the end of Presentation*

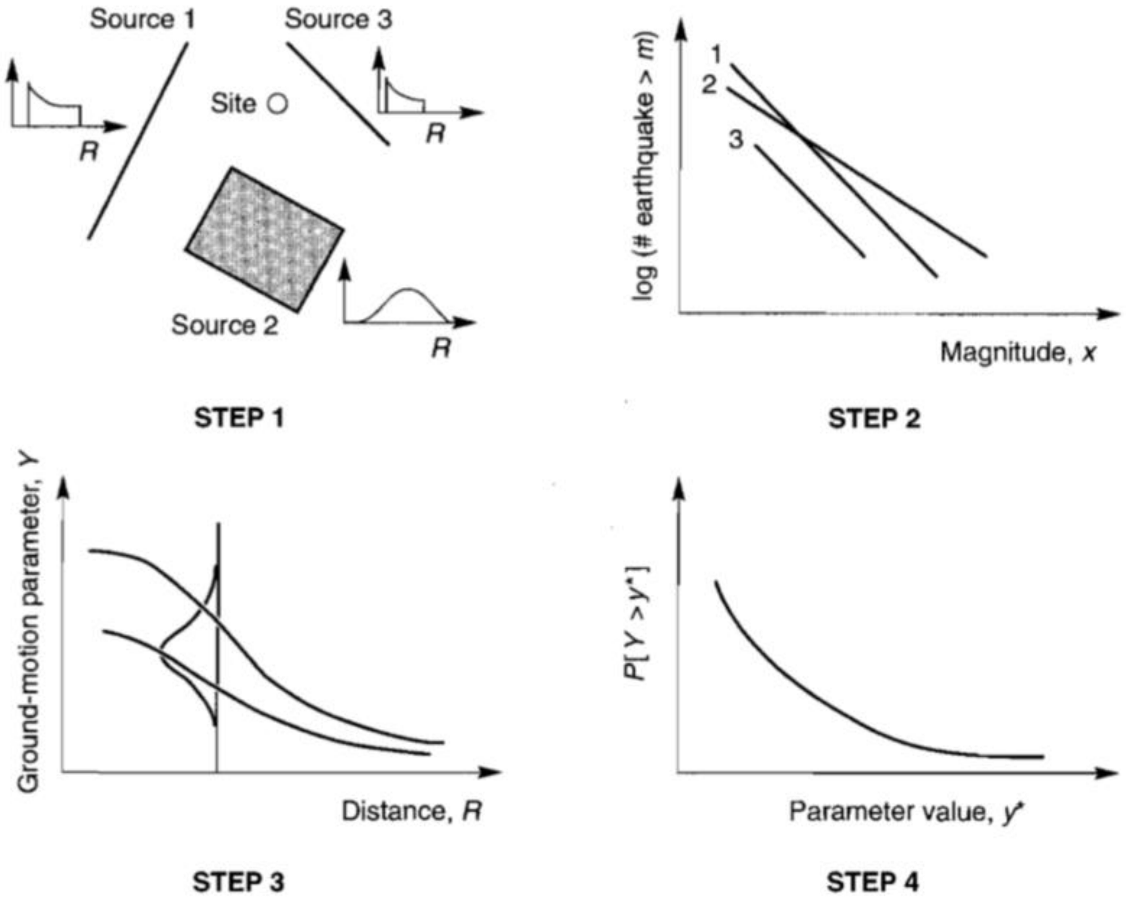
PSHA - Steps

i. **Source Characterization**: Identification and classification of the Ns source → Definition of $f_{R|M}^{(n)}$

ii. **Earthquake Size**: for each source based on magnitude recurrence relationship $f_{M}^{(n)}$

iii. **Ground Motion Estimation**: empirical regression models named ground motion prediction equations (GMPE) → Definition of $P(IM > im\,|M = m, R = r)$

iv. **Hazard Computation**: solution of the integral *(1)* $\forall\ N_s$



PSHA – Step 1: Earthquake source characterization

SOURCES

Goal: **to identify**

- *Fault sources:* individual or multiple identified faults
- *Area sources:* defined by polygons in which seismicity is assumed uniform

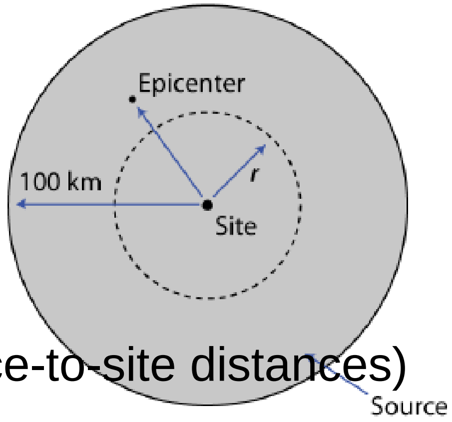
(Identification based upon the interpretation of geological, geophysical and seismological – historical data)

DISTANCES

and **to characterize seismic sources**

- *Point source*
- *Linear source*
- *Area source*

(the geometry of the source is used to identify the probability distribution of source-to-site distances)



PSHA – Step 2: Earthquake size (Recurrence law)

Goal: **to define distribution of** $f_M^{(n)}$

the chance of an earthquake of a given size occurring anywhere inside the source during a specified period of time

The Gutenberg-Richter law (G-R law) expresses the relationship between the magnitude and rate of cumulative number of earthquakes in any given region:

$$\log \lambda(m) = a - b \cdot m$$

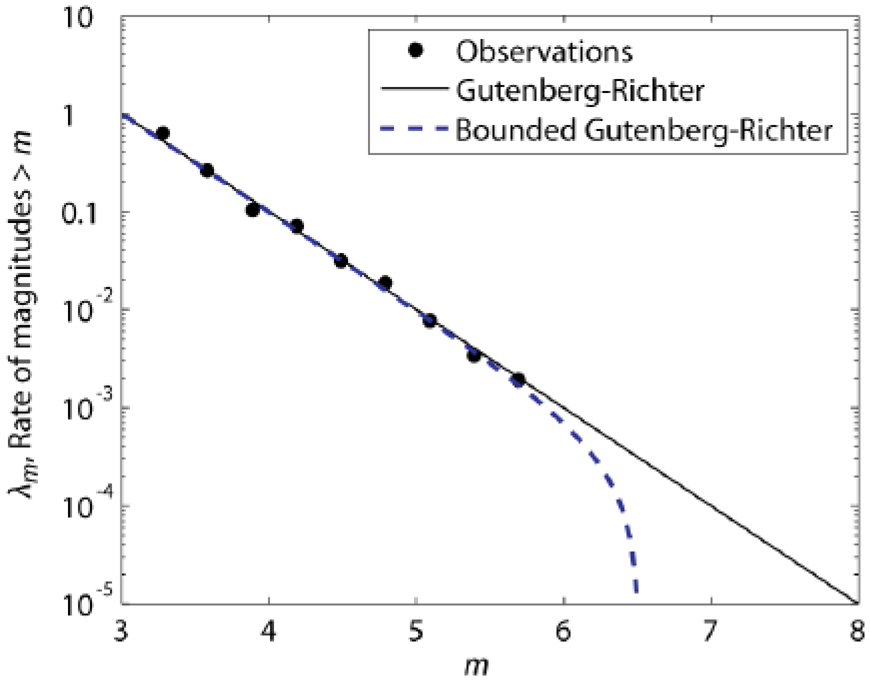
$\log \lambda(m)$ logarithm base 10 of the mean annual rate of exceedance of magnitude m, a overall rate of earthquakes of the source and b relative ratio of small vs large magnitudes

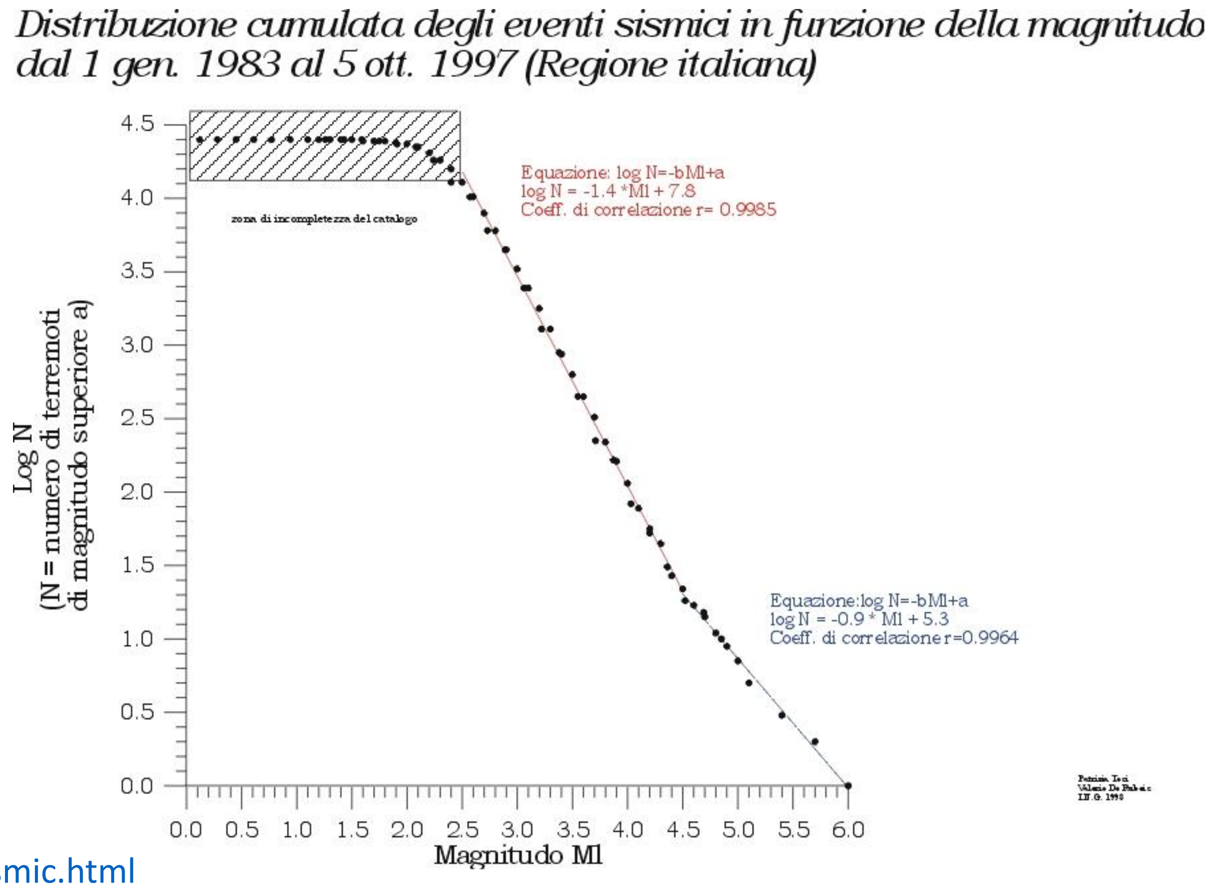
PSHA – Step 2: Earthquake size (Recurrence law)

Observations:

- $m_{min} - m_{max}$ linked to the minimum magnitude capable of causing damages and to physically capability of seismic zone to generate magnitude with these values
- G-R law *bounded*
- given a seismic source, $a - b$ parameters are estimated through statistical analysis of historical data with constraints from geological evidence
- paramount aspect regarding completeness and undistortion of the reference catalogue in terms of intensity/magnitude range and time intervals

PSHA – Step 2: Earthquake size (Recurrence law)





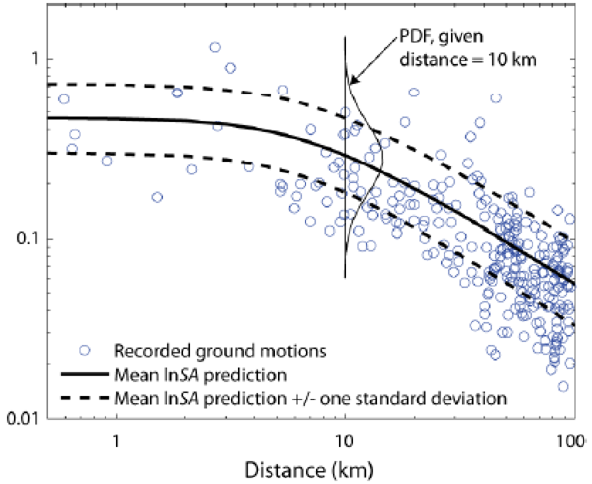
http://legacy.ingv.it/roma/attivita/pererischio/macrosismica/Seismic/seismic.html

PSHA – Step 3: Ground motion predictive equations (GMPE)

Goal: **estimate ground motion at the site**

- Identify the IM of interest for the situation and purposes;
- Estimation of the PDF of the selected IM by referring to predictor variables such as the earthquake source properties $(M, R \ \ldots)$

GMPEs are usually adopted to evaluate the probability that a particular IM exceeds a certain value, im, for a given earthquake $M = m$, occurring at a given distance, $R = r$



PSHA – Step 3: Ground motion predictive equations (GMPE)

In probabilistic terms

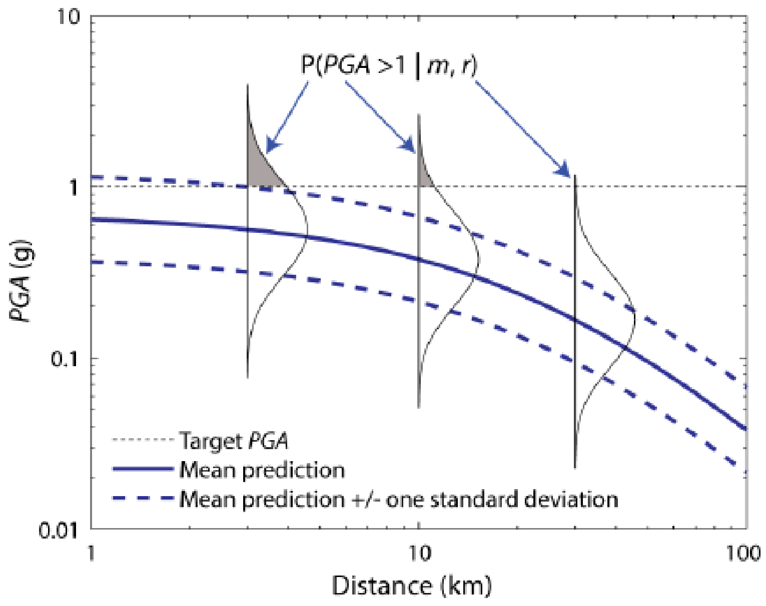
$$P[IM > im|R = r, M = m] = 1 - F_{IM|RM}(im|r,m)$$

Usually, the conditional distribution of the ground motion intensity measure, i.e. $F(im|r,m)$ is assumed log normal.

Observations:

- GMPEs are developed independently for each region;
- Faulting mechanism, effects of local site conditions are of

paramount importance



Schematic illustration of conditional probability of exceeding a particular value of a ground motion parameter for a given magnitude and distance.

PSHA – Step 4: Hazard Computation

The seismic hazard curve is a function representing the annual frequency of exceeding various levels of ground shaking (i.e. the IM) at a specific site. The curve is obtained by integration of the previously three steps over all possible magnitudes and earthquakes locations.

Seismic hazard curves are obtained for individual sources and, then, combined to express the aggregate hazard at a particular site.

Then

$$\lambda(im) = \sum_{n=1}^{N_s} \lambda_{min}^{(n)} \left[\left[\int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \,|M = m\,, R = r)\; f_R^{(n)}(r) f_M^{(n)}(m) dr dm \right] \right] \qquad (2)$$

Numerically

$$\lambda(im) \approx \sum_{n=1}^{N_s} \sum_{m=1}^{N_r} \sum_{l=1}^{N_m} \lambda_{min}^{(n)} P\left(IM > im \middle| M^{(n)} = m_l, R^{(n)} = r_m\right) f_R^{(n)}(r_m|m_l) f_M^{(n)}(m_l) \Delta r\, \Delta m \qquad (3)$$

References

- Baker J. W. (2008). *An Introduction to Probabilistic Seismic Hazard Analysis (PSHA)*, White Paper, Version 1.3, 72 pp.
- Kramer, S.L. (1996) *Geotechnical earthquake engineering*. Prentice Hall, Upper Saddle River, N.J.
- Wells, D.L. and Coppersmith, K.J. (1994) *New empirical relationships among magnitude, rupture length, rupture width, rupture area, and surface displacement.* Bull. Seism. Soc. Am., 84, 974-1002.
- Cornell, C.A. (1968). *Engineering seismic risk analysis*, Bull. Seism. Soc. Am., 58, 1583-1606.
- Broccardo, M. (2018) *Probabilistic seismic risk analysis for civil systems*, Lecture Notes

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MATLAB®

Lecture #4

PSHA – MatLab Tutorial

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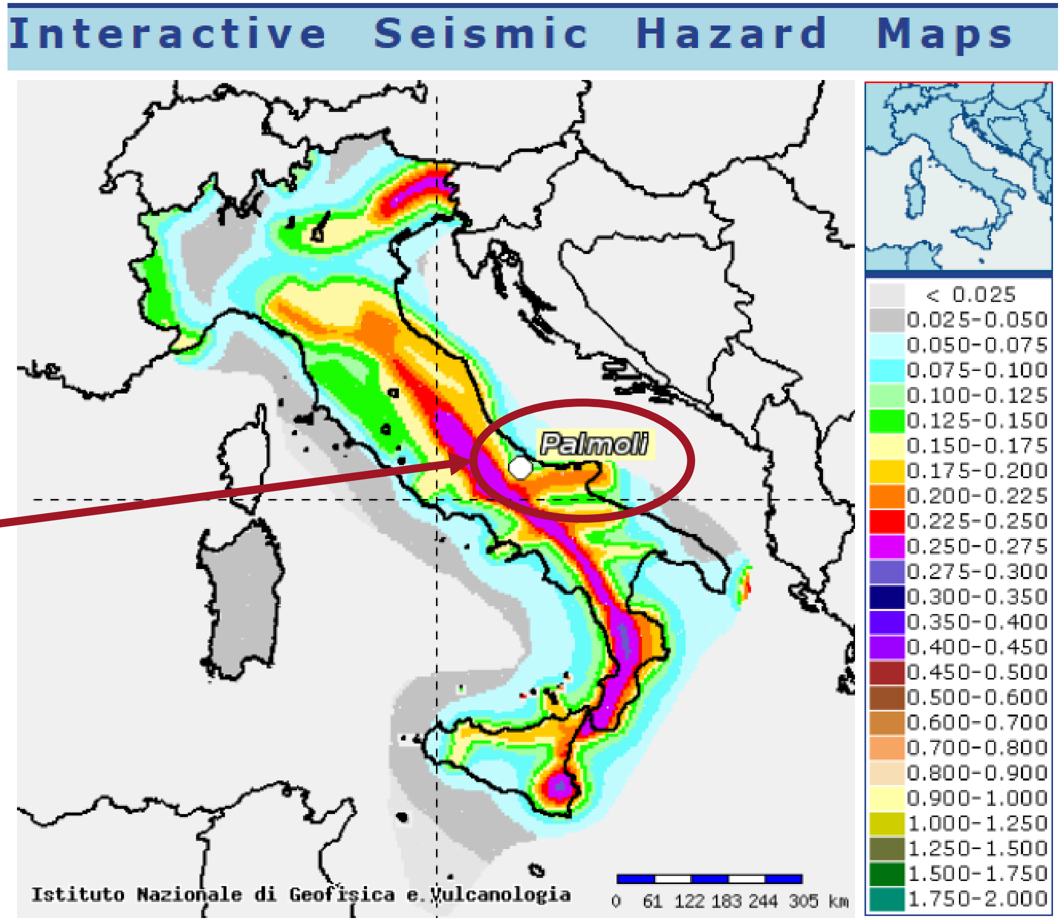
https://github.com/CNardin/ISPS.git

Goal: to perform a PSHA analysis

Through the scheme depicted in *(1)*, compute:

- the annual hazard curve for each fault;
- the 50 years hazard curve for each fault;
- the 475 years hazard curve for each fault

for the highlighted seismic site.



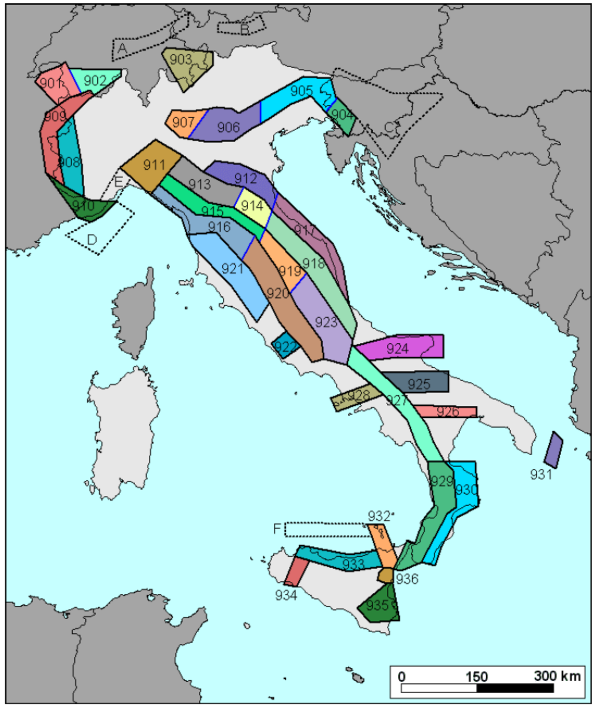
http://esse1-gis.mi.ingv.it/

Seismic hazard map: shaking parameter PGA, site Palmoli (CH)
Latitude 41,527 – Longitude 14,483.

Step 1: Source characterization

I. Localize seismogenetic zones: Italian ZS9* model for application of the attenuation law

Italian database and catalogue of the overall seismicity
http://zonesismiche.mi.ingv.it/



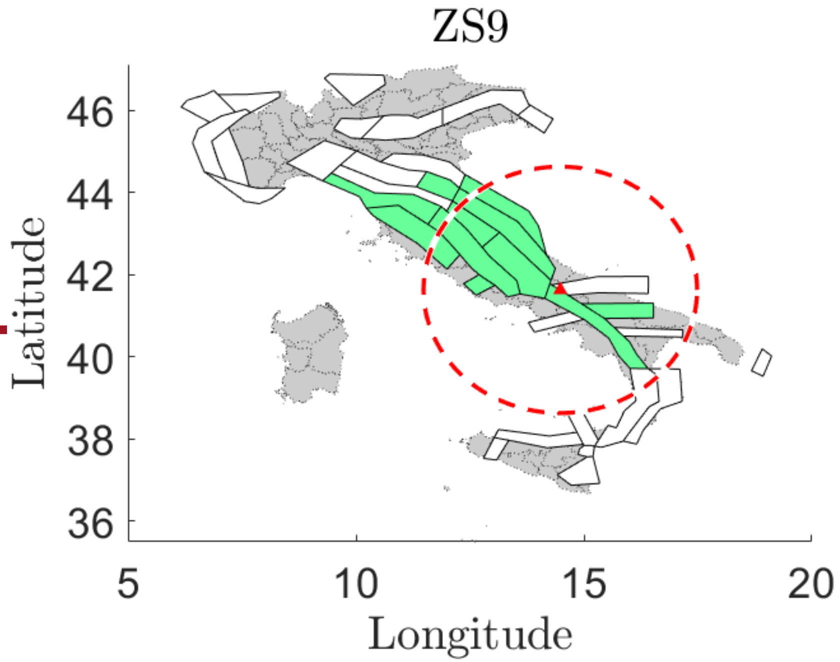
Different characteristics of zonation by:

- Seismogenetic faults and mechanisms (direct, inverse, strike-slip …)
- Hypocenter depth (shallow, intermediate, deep)
- ect.

Step 1: Source characterization

ZS Name	ZS9	MwMax AR	Tassi Mwmax Co-04.2 AR	Tassi Mwmax Co-04.4 AR	b Co-04.2	b Co-04.4	MwMax GR	Tassi Mwmax (Co.04.2) GR
Savoia	901	5,91	0,21	0,21	-1,18	-1,26	6,14	0,11
Vallese	902	6,14			-1,26	-1,05	6,14	0,14
Grigioni - Valtellina	903	5,91	0,21	0,21	-1,26	-1,05	6,14	0,14
Trieste - Monte Nevoso	904	5,68			-1,12	-1,32	6,14	0,14
Friuli - Veneto Orientale	905	6,60			-1,06	-1,12	6,60	0,37
Garda - Veronese	906	6,60		0,14	-1,14	-1,70	6,60	0,11
Bergamasco	907	5,91	0,14	0,14	-1,71	-1,48	6,14	0,04
Piemonte	908	5,68			-1,91	-1,67	6,14	0,04
Alpi Occidentali	909	5,68	0,21	0,33	-1,27	-1,38	6,14	0,10
Nizza - Sanremo	910	6,37			-1,12	-1,06	6,37	0,14
Tortona - Bobbio	911	5,68			-1,47	-1,33	6,14	0,05
Dorsale Ferrarese	912	6,14	0,12	0,12	-1,35	-1,32	6,14	0,12
Appennino Emiliano-Romagnolo	913	5,91		0,21	-1,80	-1,53	6,14	0,07
Forlivese	914	5,91			-1,33	-1,23	6,14	0,14
Garfagnana - Mugello	915	6,60			-1,34	-1,36	6,60	0,11
Versilia-Chianti	916	5,68	0,21	0,33	-1,96	-1,58	6,14	0,04
Rimini - Ancona	917	6,14	0,12	0,12	[illegible]	[illegible]	6,14	0,12
Medio-Marchigiana/Abruzzese	918	6,37	0,14	0,21	-1,10	-1,11	6,37	0,14
[illegible]	919	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]
Val di Chiana - Ciociaria	920	5,68	0,28	0,33	-1,96	-1,58	6,14	0,06
Etruria	921	5,91		0,08	-2,00	-2,01	6,14	0,05
Colli Albani	922	5,45			-2,00	-2,01	5,45	0,37
Appennino Abruzzese	923	7,06			-1,05	-1,09	7,06	0,14
Molise-Gargano	924	6,83			-1,04	-1,06	6,83	0,13
Ofanto	925	6,83			-0,67	-0,75	6,83	0,17
Basento	926	5,91			-1,28	-1,38	6,14	0,10
Sannio - Irpinia - Basilicata	927	7,06			-0,74	-0,72	7,06	0,43
Ischia - Vesuvio	928	5,91	0,21	0,21	-1,04	-0,66	5,91	0,21
Calabria tirrenica	929	7,29			-0,82	-0,79	7,29	0,17
Calabria ionica	930	6,60			-0,98	-0,89	6,60	0,17
Canale d'Otranto	931	6,83			-0,63	-0,63	6,83	0,21
Eolie - Patti	932	6,14			-1,21	-1,08	6,14	0,21
Sicilia settentrionale	933	6,14	0,21	0,33	-1,39	-1,24	6,14	0,20
Belice	934	6,14			-0,96	-0,93	6,14	0,20
Iblei	935	7,29			-0,72	-0,69	7,29	0,12
Etna	936	5,45	0,33	0,33	-1,63	-1,22	5,45	0,33

I. Localize seismogenetic zones: Italian ZS9* model for application of the attenuation law



http://zonesismiche.mi.ingv.it/

Step 1: Source characterization

Extrapolate information regarding the source and its bound limits: convert coordinates from *deg2utm* for each sources

	Coordinates				
	WGS 84		UTM		UTM-Z
Location	long (x)	lat (y)			
Palmoli	14,48	41,63	465.293	4643124	33 T

Source	b	Mw Max	Mw Min	Rate
S1	-1,11	6,37	4,76	0,21
S2	-1,09	7,06	4,76	0,14
…	…	…	…	…

ZS9-918	S1					
WGS 84		UTM		UTM-Z	UTM-INPORT	
12,11651	43,79397	268015,9	4853032	33 T	-197.277	209.907
12,27012	44,10645	281523,5	4887320	33 T	-183.769	244.196
13,54116	43,2943	381660,3	4794530	33 T	-83.633	151.405
14,181	42,47483	432678,9	4702824	33 T	-32.614	59.699
14,37249	42,14993	448153,3	4666614	33 T	-17.140	23.489
14,27923	41,85719	440174,4	4634171	33 T	-25.118	-8.953
13,1518	43,02822	349423	4765606	33 T	-115.870	122.481
12,11651	43,79397	268015,9	4853032	33 T	-197.277	209.907

Step 1: Source characterization - Location

Define the distribution of $f_R(r)$

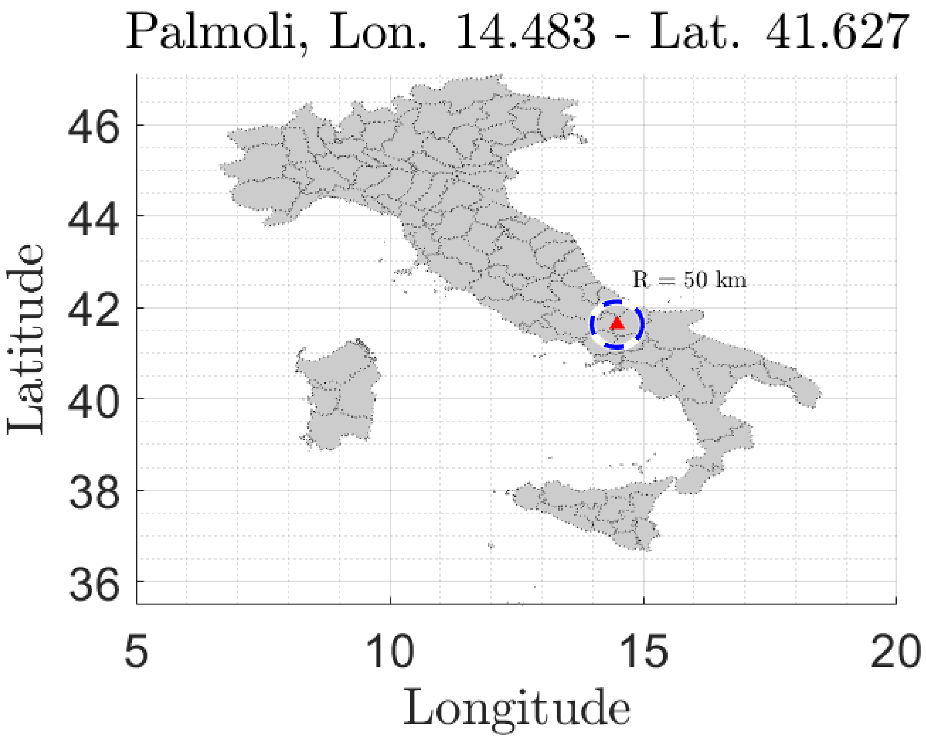
To simplify, consider 1 source with a regular zonation of $R_{fix} = 50\ km$.

So, the distribution of an epicenter being located at a distance of less than *r* is:

CDF: $$F(r) = P(R \leq r) = \frac{area\ of\ circle\ with\ radius\ r}{area\ of\ circle\ with\ radius\ R_{fix}}$$

$$= \frac{r^2}{R_{fix}^2}, 0 \leq r \leq R_{fix} \quad \wedge \quad 1, r \geq R_{fix}$$

PDF: $$f(r) = \frac{r}{2\ R_{fix}^2}, 0 \leq r \leq R_{fix} \quad \wedge \quad 0, r \geq R_{fix}$$

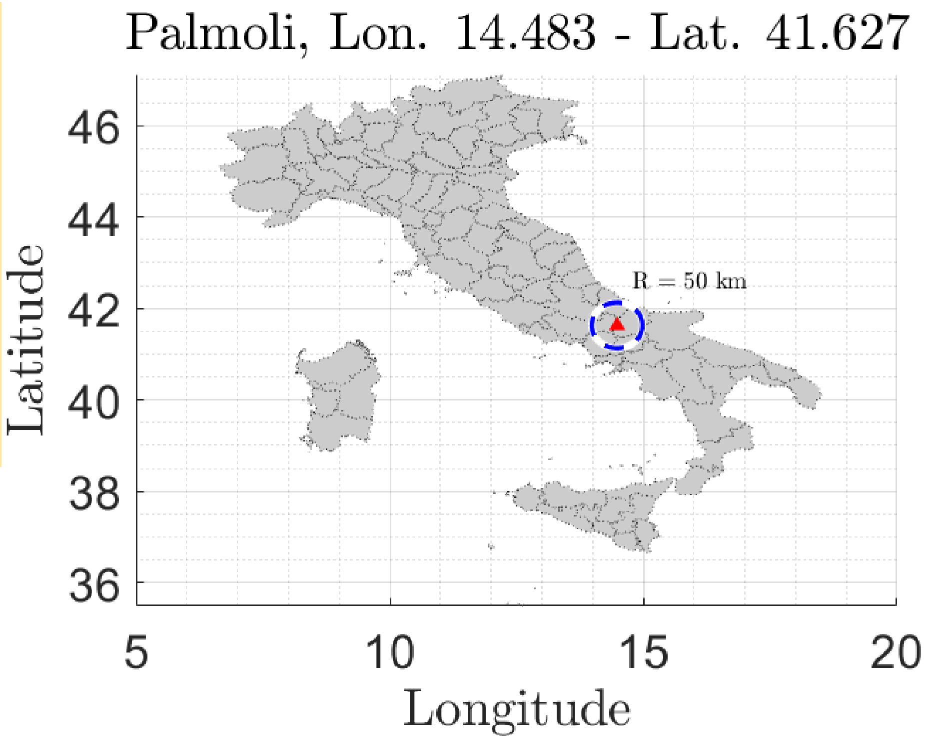


Step 1: Source characterization

Codes:

i. Read *source.xls* file

ii. Run section *Earthquake source characterization* in the attached Matlab code or load variable *seismic_source.mat* already prepared with the previously seen data



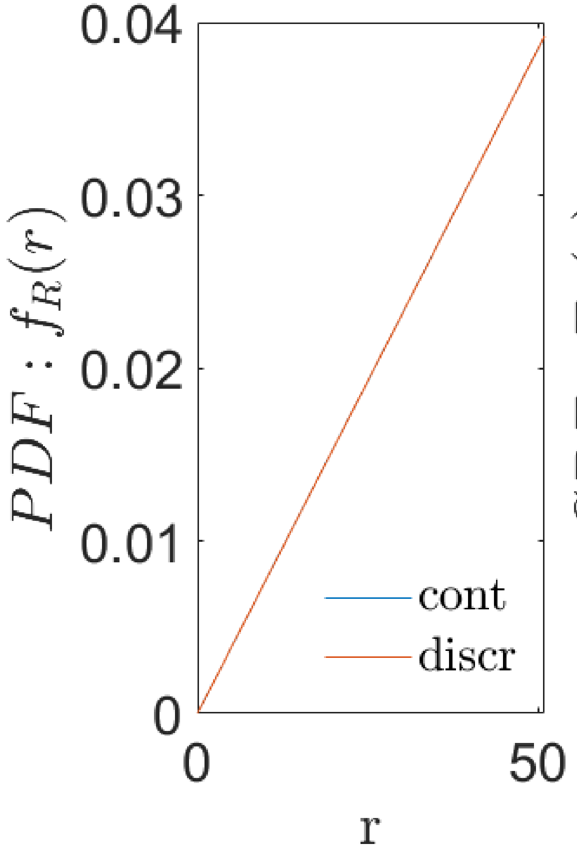
Step 1: Source characterization - Location

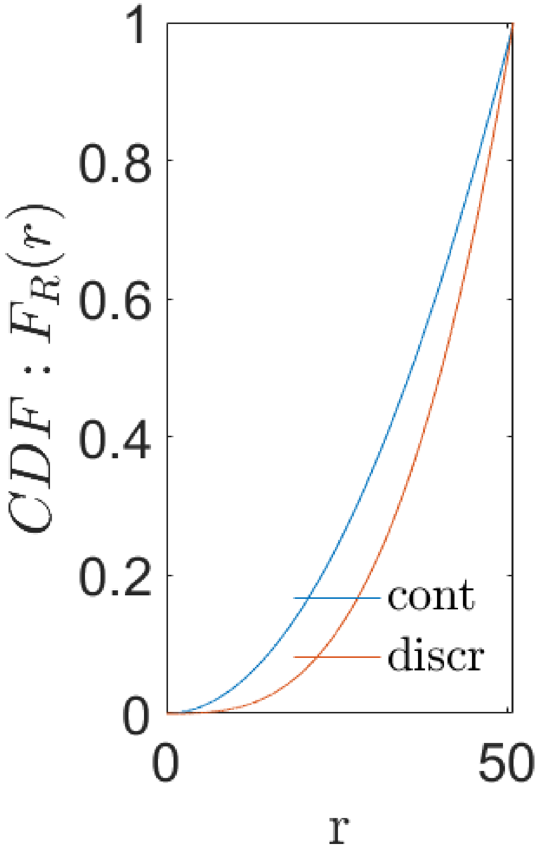
Codes:

```
% PDF of R
  rmax = Rmax*100;    % 50km
  rmin = Rmin.*100;   % site

  r_step = R_step;
  r = rmin:r_step:(rmax-r_step);
  Fr = r.^2./rmax.^2;

  fr = 2*r./rmax.^2;
  DFr = r_step;
  frdiscr = zeros(1,numel(Fr));
  Frdiscr = zeros(1,numel(Fr));
  for idr = 2:numel(Fr)
  frdiscr(idr) = -(Fr(idr-1)-Fr(idr))./(DFr);
  Frdiscr(idr) = Frdiscr(idr-1)+Fr(idr);
  end
```





Step 2: Earthquake size

Define the distribution of $f_M(m)$

$$\lambda(m) = \exp(a - b \cdot m)$$

Gutenberg-Richter bounded defines the relationship between the magnitude and rate of cumulative number of earthquakes

CDF:

$$F_M(m) = P(M \leq m | m_{max} \geq M \geq m_{min})$$

$$= \frac{\lambda_{min} - \lambda(m)}{\lambda_{min} - \lambda_{max}} = \ldots = \frac{1 - 10^{-b(m - m_{min})}}{1 - 10^{-b(m_{max} - m_{min})}} = \frac{1 - \exp[-\beta(m - m_{min})]}{1 - \exp[-\beta(m_{max} - m_{min})]}$$

PDF:

$$f(m) = \frac{d}{dm} F(m) = \frac{\beta \exp[-\beta(m - m_{min})]}{1 - \exp[-\beta(m_{max} - m_{min})]}$$

$$\beta = \ln(10)\, b$$

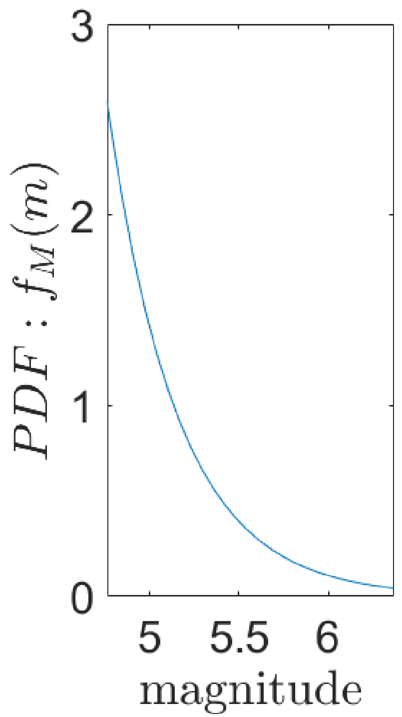
Step 2: Earthquake size

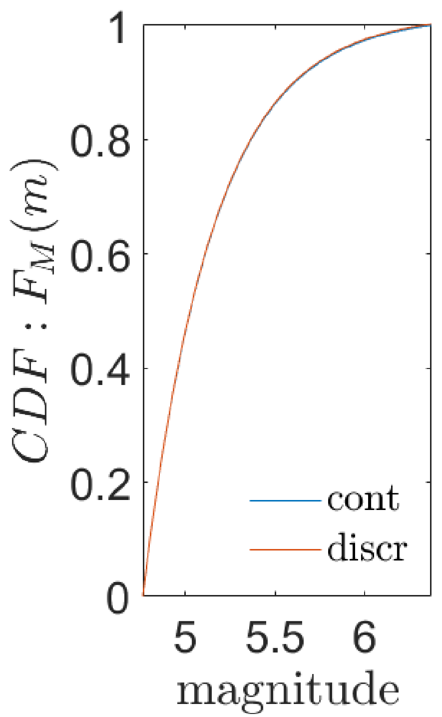
Define the distribution of $f_M(m)$: Discretize

Converting the continuous distribution of magnitudes into a discrete set of magnitudes holds

$$P(M = m_j) = F_M(m_{j+1}) - F_M(m_j)$$

where m_j are the discrete set of magnitudes ordered so that $m_{j+1} < m_j$





Codes:

```
% PDF of M
Nm = 1000; % No. of discretized points between m0 and mu for numerical integration
M = linspace(m0,mu,Nm);   %mo = m_min
fM = beta*exp(-beta*(M-m0))/(1-exp(-beta*(mu-m0)));
dM=M(2)-M(1);
Mdiscr=fM*dM;
```

Step 3: Ground motion estimation

Define effects $P(IM > x|m,r)$: Attenuation models or prediction models

These models predict the probability distribution of ground motion intensity, as a function of many predictor variables such as the earthquake's magnitude, distance, faulting mechanism, the near-surface site conditions, etc.

To describe this probability distribution, prediction models take the following general form:

$$\ln(IM) = \overline{\ln IM(M,R,\theta)} + \sigma(M,R,\theta) \cdot \varepsilon$$

where

- $\ln(IM)$ is the natural log of the ground motion *intensity measure* of interest;
- $\ln(IM)$ is modeled as a random variable and is represented by a normal distribution;
- $\overline{\ln IM(M,R,\theta)}$ and $\sigma(M,R,\theta)$ are the predicted mean and standard deviation of $\ln(IM)$;
- $\overline{\ln IM(M,R,\theta)}$ and $\sigma(M,R,\theta)$ are functions of the earthquake's magnitude (M), distance (R) and other parameters (θ)

Step 3: Ground motion estimation

Define effects $P(IM > x|m, r)$: Attenuation models or prediction models

Here, for clarity, it is assumed Cornell (1979) for the mean of log peak ground acceleration (in units of *g*):

$$\overline{\ln PGA} = -0{,}152 + 0{,}859M \; - 1{,}803\ln(R + 25)$$

with

- $\sigma = 0{,}57$ of $\ln(PGA)$;
- $\ln(PGA)$ normally distributed

Step 3: Ground motion estimation

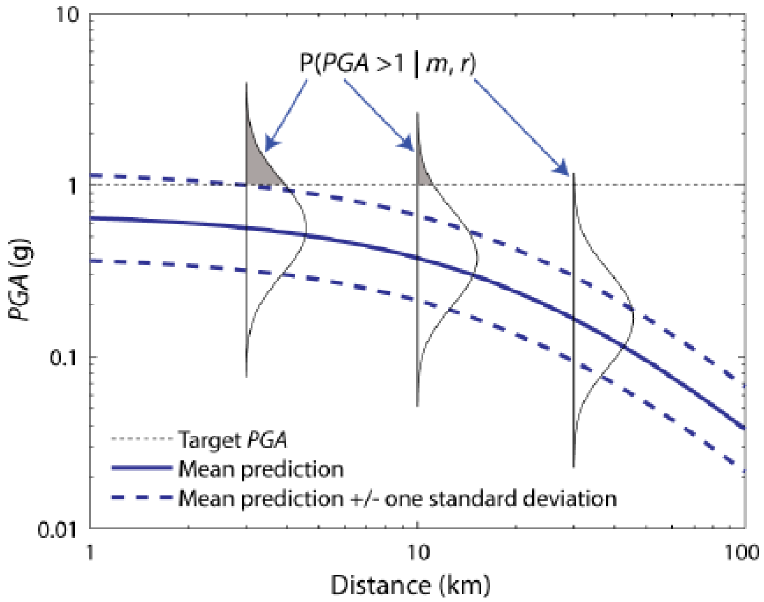
Define effects $P(IM > x|m,r)$: Attenuation models or prediction models

We can thereby compute the probability of exceeding any *PGA* level using knowledge of this mean and standard deviation:

$$P(PGA > x|m,r) = 1 - \Phi\left(\frac{\ln x - \ln \overline{PGA}}{\sigma_{\ln(PGA)}}\right)$$

where $\Phi(\,)$ is the standard normal cumulative distribution function.

N.B.: these probabilities correspond to the fraction of the corresponding PDFs that are shaded.



Step 3: Ground motion estimation

Define effects $P(IM > x|m, r)$: Attenuation models or prediction models

Codes:

```
function [mean_im, sigma_im] = GMPE(magnitude,R_distance)
%Cornell et al. (1979): Ground motion predictive equation for PGA
mean_lnPGA = -0.152+0.859*magnitude -1.803*log(R_distance+25);
mean_PGA = exp(mean_lnPGA);

mean_im = mean_PGA;
sigma_im = 0.57;
end
```

Step 4: Hazard Computation

Combine all information: compute $\int\int\int$

I. Compute the probability of exceeding an *IM* intensity level *x*, *given* occurrence of a future earthquake from a *single source*:

$$P(IM > x) = \int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \,|M = m \,, R = r) \;\; f_R^{(n)}(r) f_M^{(n)}(m) drdm$$

$from\ GMPE$ $\quad PDF\ of\ R$ $\quad PDF\ of\ M$

II. Compute the rate of *IM* > *x* for each single source:

$$\lambda\ (IM > x) = \lambda(M > m_{min}) \int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \,|M = m \,, R = r) \;\; f_R^{(n)}(r) f_M^{(n)}(m) drdm$$

Step 4: Hazard Computation

Combine all information: compute $\int\int\int$

III. Considering all sources by the sum of the rates of *IM* > *x* from each individual source, we can write:

$$\lambda(IM > x) = \sum_{i=1}^{n_{sources}} \lambda(M_i > m_{min}) \int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \,|M = m \,, R = r)\; f_{R_i}^{(n)}(r) f_{M_i}^{(n)}(m) drdm$$

where $n_{sources}$ is the number of sources considered, and $M_i \sim R_i$ denote the magnitude ~ distance distributions for source *i*

Step 4: Hazard Computation

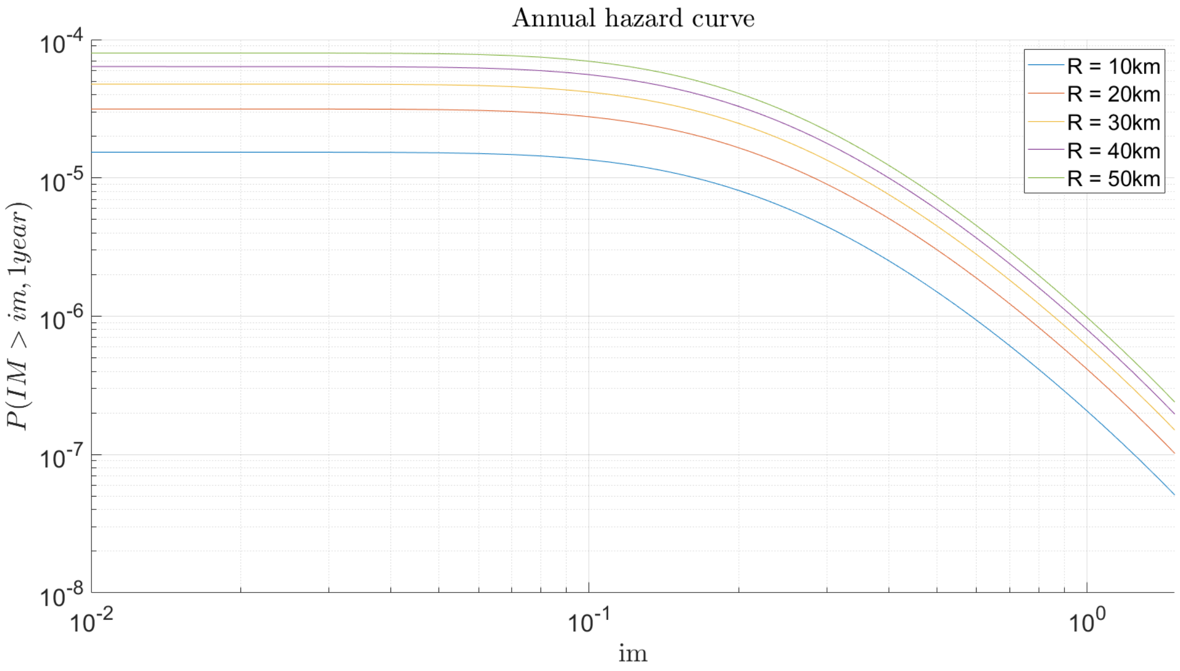
Combine all information: compute $\int\int\int$ - Discretize

IV. By discretizing:

$$\lambda(im) \approx \sum_{n=1}^{N_s}\sum_{m=1}^{N_r}\sum_{l=1}^{N_m} \lambda_{min}^{(n)} P\left(IM > im \middle| M^{(n)} = m_l, R^{(n)} = r_m\right) P\left(R^{(n)} = r_m \middle| m_l\right) P(M^{(n)} = m_l)$$

where the range of possible M_i and R_i have been discretized into n_M and n_R intervals, respectively

Step 4: Hazard Computation



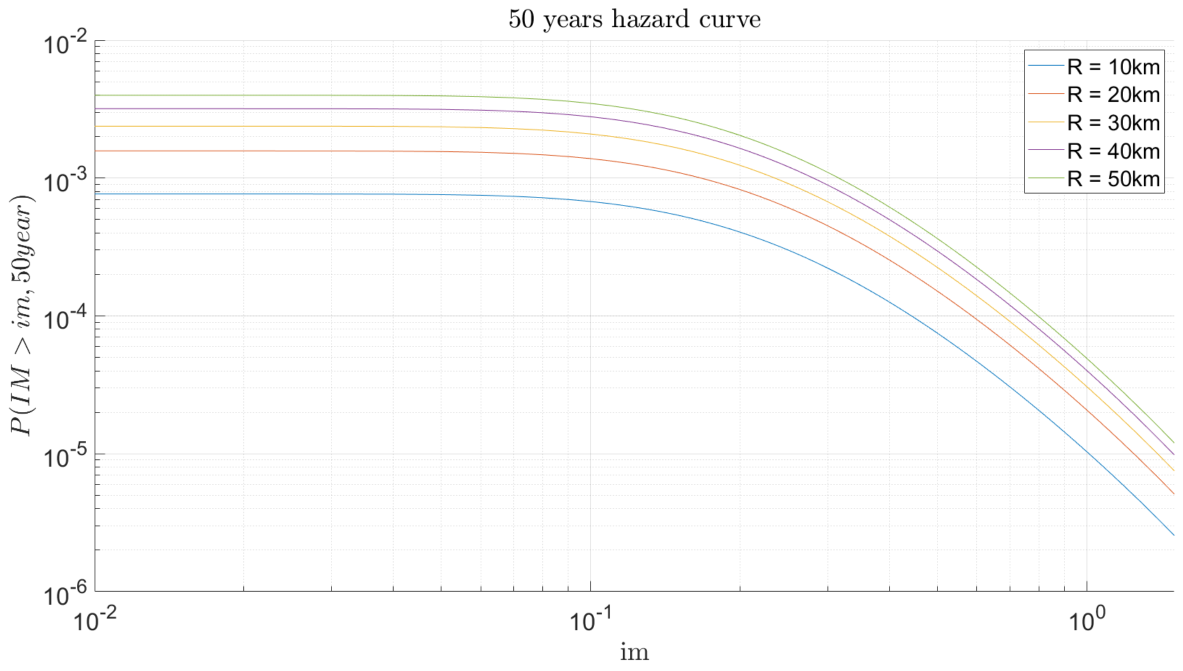
Goal and results

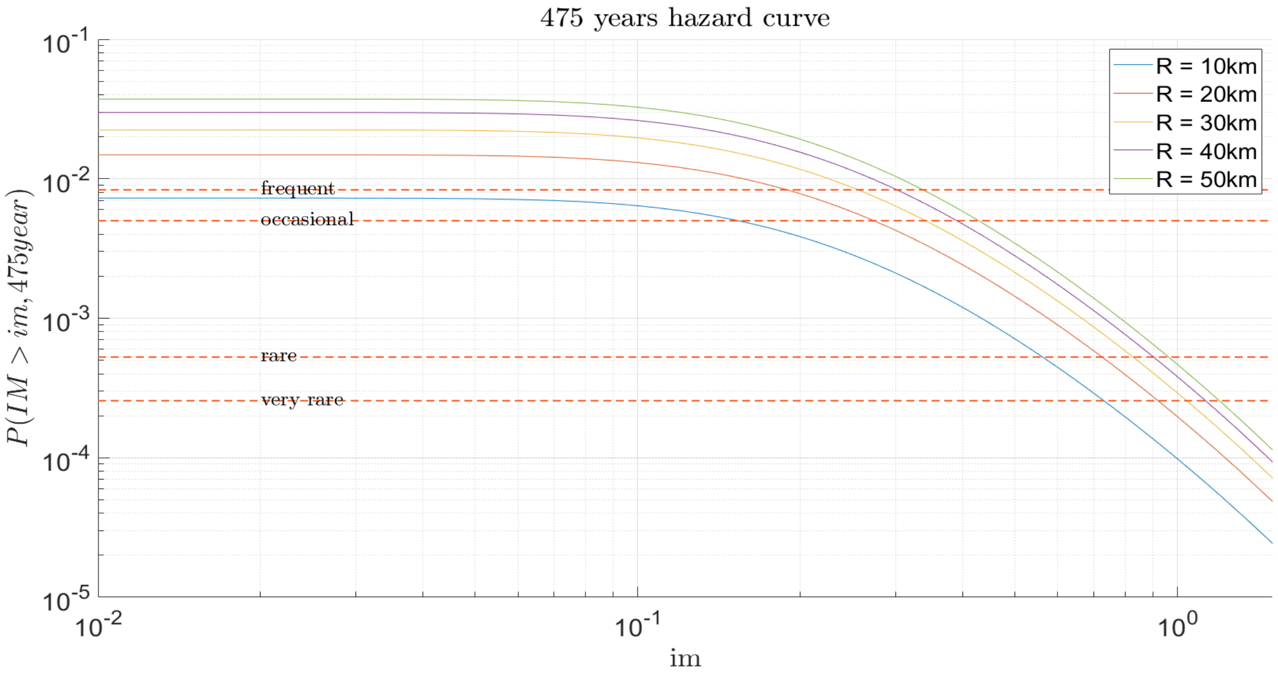
Through the scheme depicted in *(1)*, compute:

- the annual hazard curve for each fault;
- the 50 years hazard curve for each fault;
- the 475 years hazard curve for each fault

for the highlighted seismic site.

Step 4: Hazard Computation





Summary

Location	• $f_R(r)$ source site distance
Size	• $f_M(m)$ magnitude distribution
GMPE	• $P(IM > im \mid M = m , R = r)$ attenuation law
Time	• $P = 1 - e^{-\lambda t}$ Poisson model

References

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