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Lecture #4

Fragility analysis

Chiara Nardin – Ph.D., M.Sc., Eng. in Civil Engineering

https://github.com/CNardin/Lecture/tree/2021/ISPS

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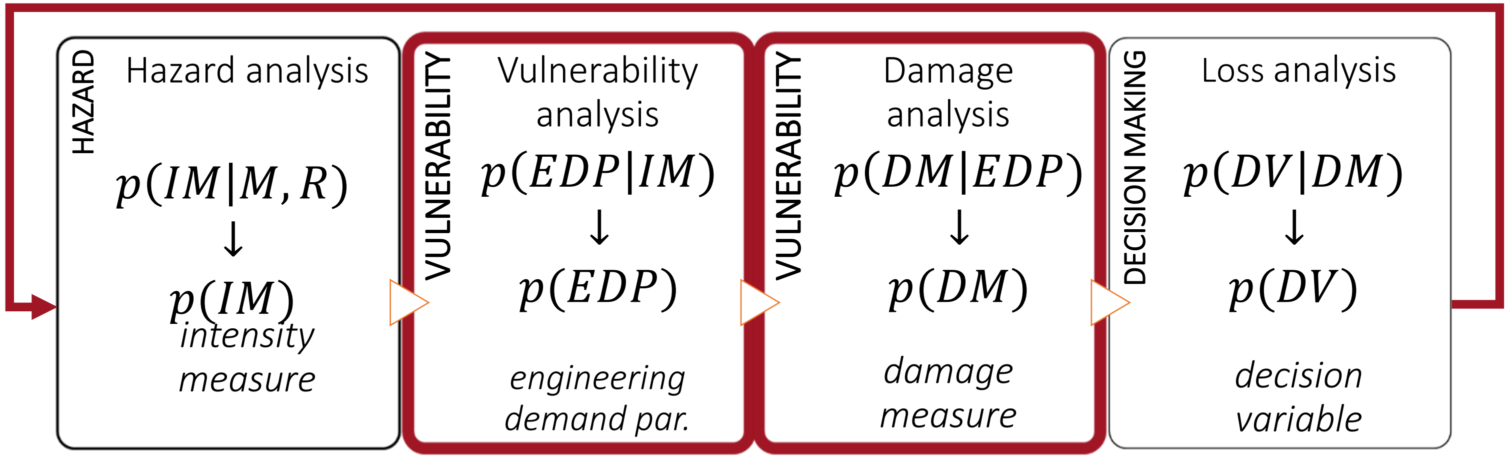
Fragility Analysis

Fragility (or Vulnerability) Analysis is the second step of the PBEE-PEER framework



Useful definitions:

seismic fragility function := the conditional probability of an event (e.g. a defined limit/damage state) given the observation of an intensity measure which describe the seismic event.

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Fragility Analysis

VULNERABILITY

Vulnerability analysis

$p(EDP|IM)$
$\downarrow$
$p(EDP)$

engineering demand par.

VULNERABILITY

Damage analysis

$p(DM|EDP)$
$\downarrow$
$p(DM)$

damage measure

The fragility curve is defined as the conditional probability of failure of a structure, or its critical components, at given values of seismic intensity measures (*IMs*).

$$\lambda(dv) = \sum_d \int_{edp} \int_{im} G(dv|d) P(d|im) |d\lambda(im)$$

In practice, a fragility curve is calculated as the conditional probability that the damage measure (*D*) exceeds a critical threshold, for a given seismic *IM*.

$$P\ (D > d_{threshold}\ |\ IM = im)$$

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Fragility Analysis: key aspects

VULNERABILITY

Vulnerability analysis

$$p(EDP|IM)$$
$$\downarrow$$
$$p(EDP)$$

engineering demand par.

VULNERABILITY

Damage analysis

$$p(DM|EDP)$$
$$\downarrow$$
$$p(DM)$$

damage measure

INTENSITY MEASURE (*IM*)

in terms of:

- ***Efficiency***, i.e. variability of an *EDP* for a given *IM*;
- ***Robustness***, i.e. efficiency between *IM*-*EDP* at different period ranges;
- ***Practicality***, i.e. correlation to known and easy identifiable engineering quantities;
- ***Sufficiency***, i.e. validity of *EDP*|*IM* as statistically independent from gm site characteristics;
- ***Effectiveness***, i.e. ability to evaluate an analytical relation.

DAMAGE STATE (*D*)

Should suit the specific structural problem → associate each damage state to a specific *EDP*

- Categorical variables, i.e. D_0 *no damages* – D_1 *minor* – D_2 *moderate* – … – $D_f = C$ *collapse* ;
- Probabilistic or deterministic relationship between *EDP* and *D*

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Class of fragilities

EMPIRICAL	ANALYTICAL	EXPERT OPINION	HYBRID
• by fitting a function to observational data from past earthquakes or lab tests ↕ collections pairs of level of excitation and categorical variables of damage or collapse	• by defining analytical structural model and analyzing its performance under different levels of the seismic hazard ↕ ***static***, i.e. hazard as response spectrum and push-over analysis *vs* ***dynamic***, i.e. collection of gms and simulations on FEM via NLA	• by polling one or more experts of the given structural asset ↕ to guess or estimate the failure probability for a given hazard level	• based on combination of the different methods

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Analytical fragility functions: steps

i) Definition of a numerical model: $y(t) = \mathcal{M}\left[\ddot{x}_g(t|IM = im); \boldsymbol{\theta}_{\mathcal{M}}(t)\right]$

ii) Selection of a suitable IM given the structure

iii) Selection of a suitable set of N gms for the location

iv) Selection of an EDP of interest

v) Definition of damage limit states D via EDP thresholds

vi) Scale each gm based on the given IM <u>eventually</u> until collapse

- *a) Full IDA*
- *b) Truncated IDA*

vii) Save each $[EDP \rightarrow D]$ threshold-im_n pair for each gm

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Computation of fragility function

Hp: - assume a *lognormal probability distribution* for the random variable IM associated with given D

FULL IDA	TRUNCATED IDA
$\forall$ damage state $\rightarrow N$ results, since $\forall\, IM_n = im_n$, $y(t)$ reached the given damage state $$P(D > d \mid IM = im) = \Phi\left(\frac{\ln(im_n) - \hat{\mu}}{\hat{\sigma}}\right)$$ where $\Phi(\cdot)$ is the CDF of the normal distribution and $$\hat{\mu} = \frac{1}{N}\sum_{n=1}^{N}\ln(im_n)$$ $$\hat{\sigma} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\ln(im_n) - \hat{\mu})^2}.$$	$\overline{IM}$ upper limit; a) data that causes collapse $n \in [1, \bar{N}]$, b) data that do not cause collapse $n \in [\bar{N}+1, N]$ a) $\rightarrow$ $$\mathcal{L}(\mu,\sigma)\alpha\prod_{n=1}^{\bar{N}}\varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right)$$ b) $\rightarrow$ $$\bar{\mathcal{L}}(\mu,\sigma)\alpha\prod_{n=\bar{N}+1}^{N}\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right] = \left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right]^{N-\bar{N}}$$

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Computation of fragility function

TRUNCATED IDA

The likelihood for the entire set of data

$$\mathcal{L}(\mu,\sigma) = \underline{\mathcal{L}}(\mu,\sigma)\overline{\mathcal{L}}(\mu,\sigma) = \prod_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right)\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right]^{N-\overline{N}} \quad (1)$$

And the log likelihood

$$\ln\mathcal{L}(\mu,\sigma) = \sum_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right) + (N-\overline{N})\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right] \quad (2)$$

Estimation of parameters by optimization

$$[\hat{\mu},\hat{\sigma}] = \underset{\mu,\sigma}{\operatorname{argmin}}[-\ln\mathcal{L}(\mu,\sigma)].$$

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WUHAN UNIVERSITY OF TECHNOLOGY

MATLAB®

Lecture #4

Fragility Analysis – MatLab Tutorial

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https://github.com/CNardin/ISPS.git

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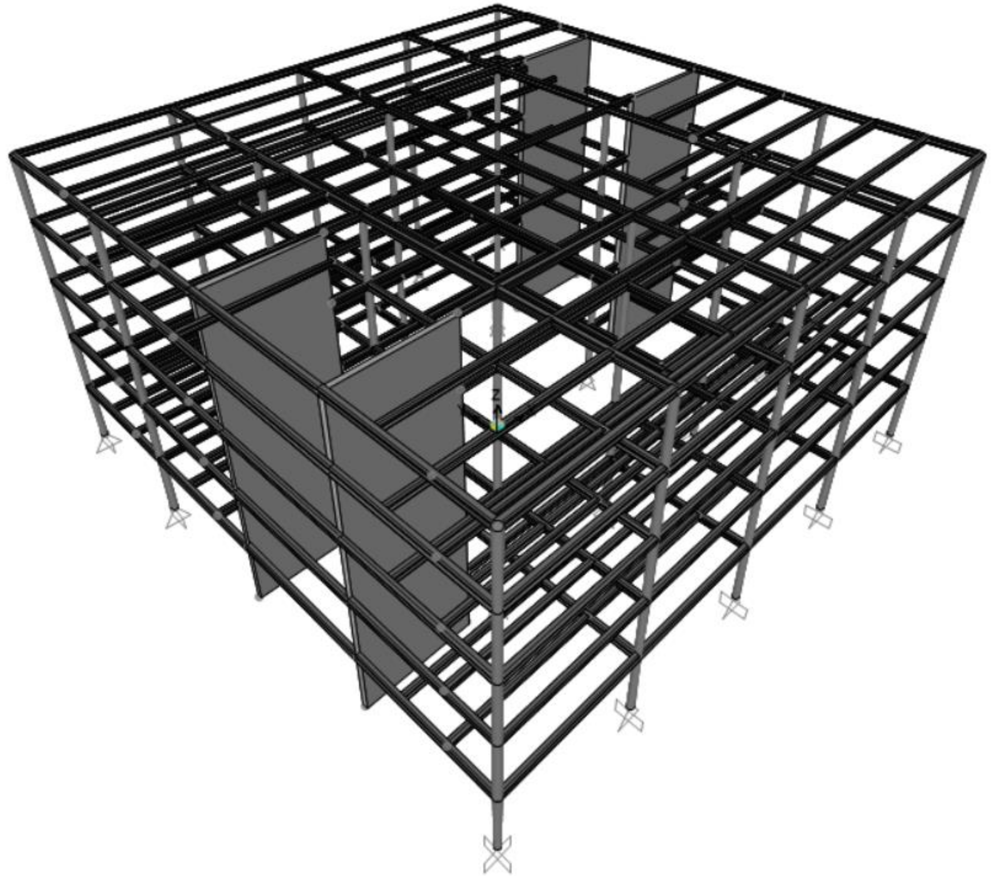
Goal: to perform fragility analysis

Given the provided set of ground motions, perform a classical and truncated incremental dynamic analysis (*IDA*) and determine fragility curves for:

- ATTEL – moment resistant frame (*MRF*);
- ATTEL – braced frame (*BF*)

by considering both

- Linear elastic behaviour
- Bouc – Wen model for hysteresis



3D model of the case study ATTEL – SERA project.

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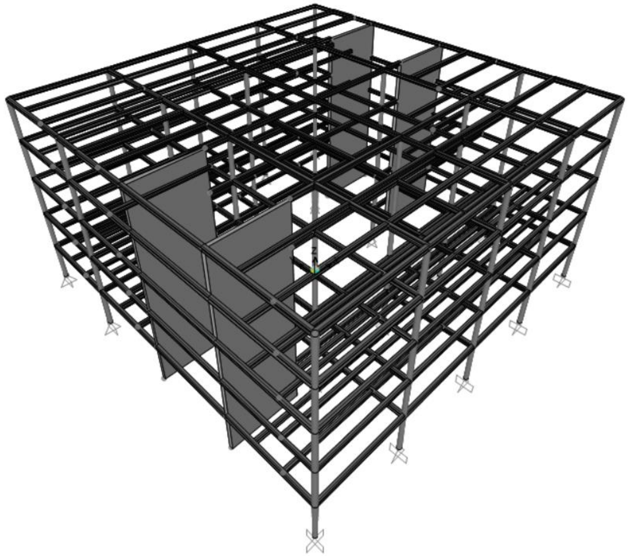
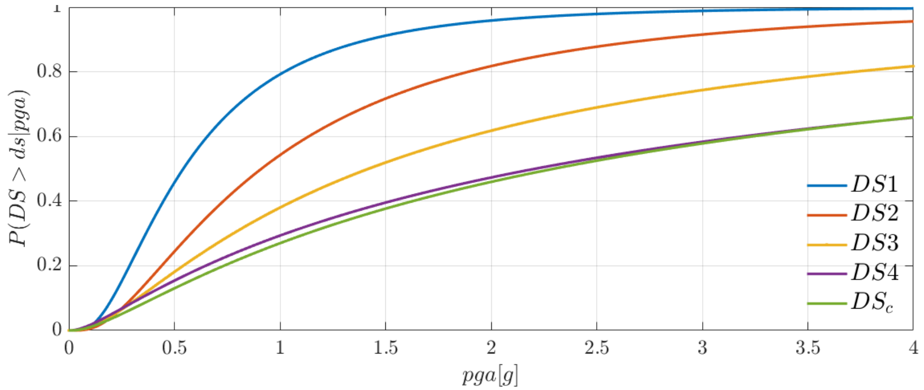
Main steps:

1) Definition of the numerical model
2) Input and *IM* selection
3) Definition of *damage limit states* and reference *EDP*
4) Performing non-linear time histories analysis (IDA, truncated IDA, cloud, MSA …)
5) Collecting results pairs and computing fragility





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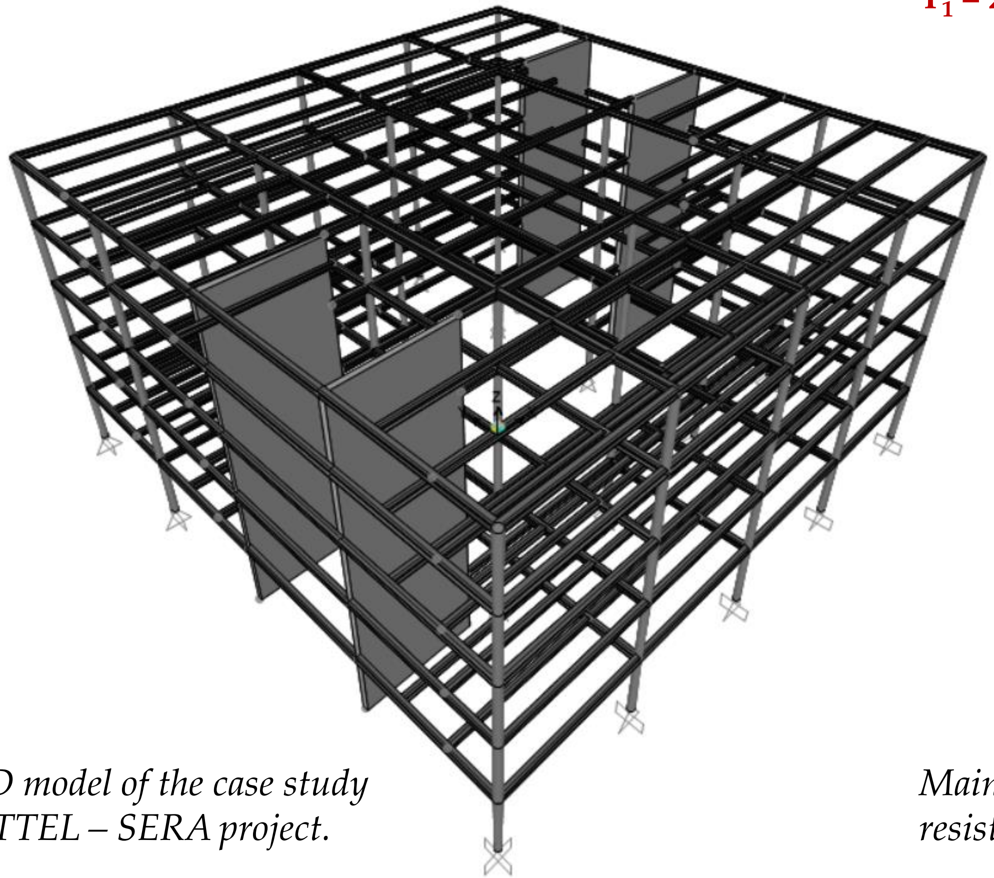
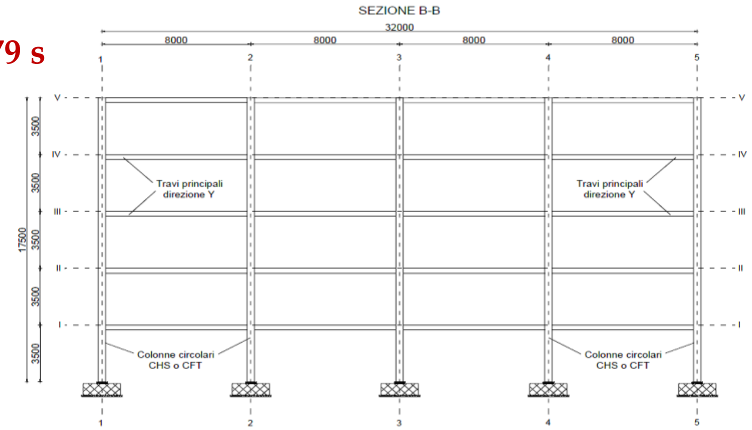
Step 1: the case study ATTEL(1)

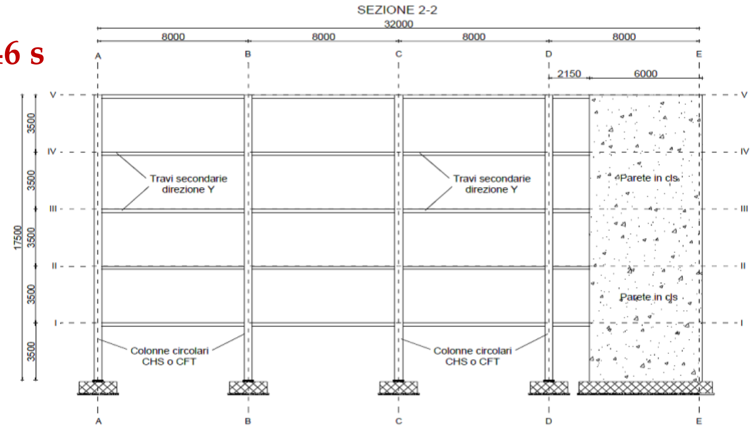


3D model of the case study ATTEL – SERA project.

T_1 = 2.79 s



T_1 = 0.46 s



Main sections of the moment resistent frame and the braced one.

[1]BURSI, PUCINOTTI, TONDINI, ZANON, Tests and model calibration of high strength steel tubular beam-to-column and column-base composite joints for moment-resisting structures, Earthquake Engineering and Structural Dynamics, (2015).

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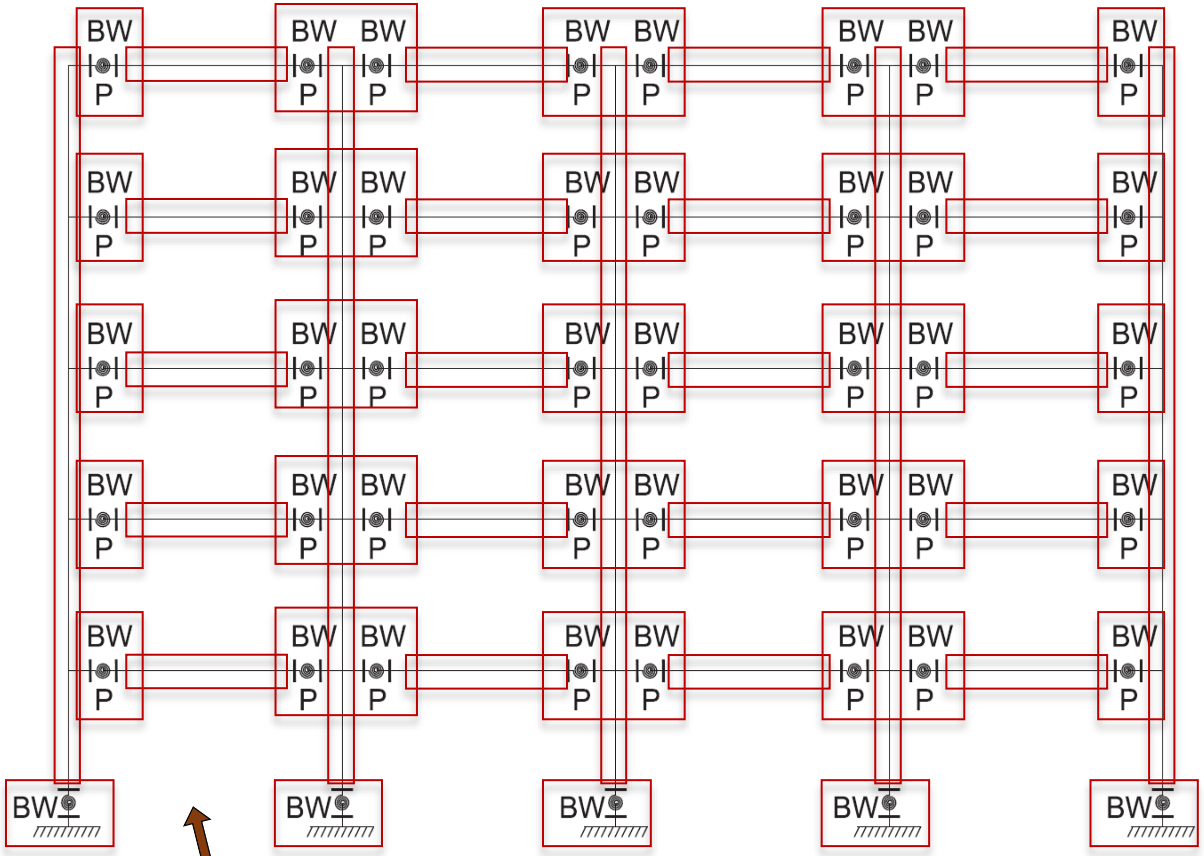


Model of the structure in OpenSees.

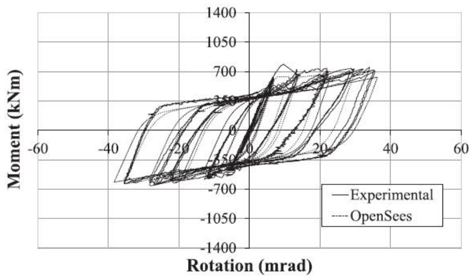
Designed according to EC8 and modelled in OS:

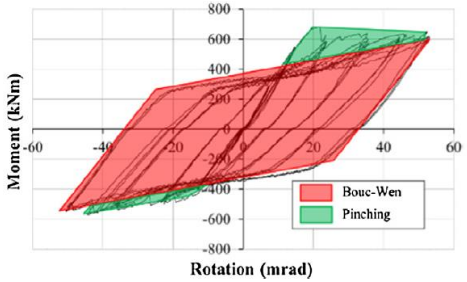
- beam and column elements with linear elastic behavior

```
elasticBeamColumn
```

- mechanical nonlinearities

```
uniaxialMaterial BoucWen
uniaxialMaterial Pinching4
uniaxialMaterial Parallel
```






```
e uniaxialMaterial Parallel $matBoucWen $matPinching  $Iz $transfTag
```

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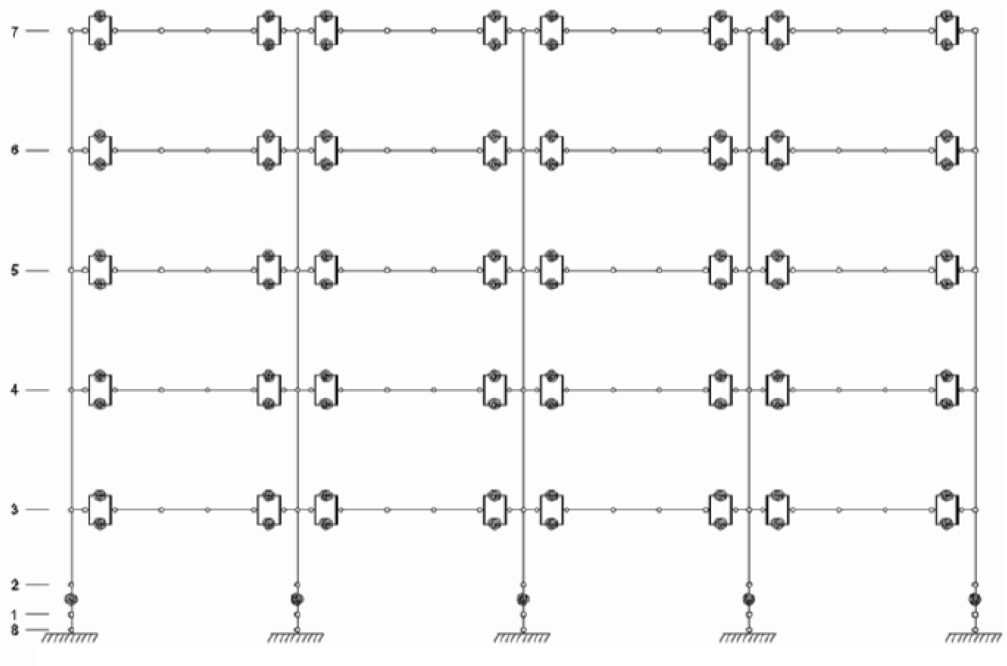
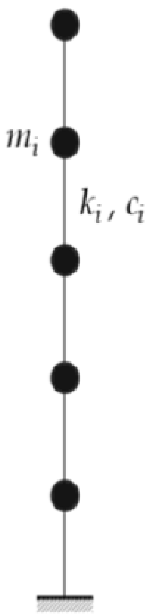
High number of analysis for seismic simulations →

- to reduce computational burden
- to reduce required simulation times





Calibration oriented to correspondence of:

- main periods
- modes of vibrating
- dissipative behavior

high fidelity model in *OpenSees - OS* → simplified model MDOF in *MATLAB® - ML*

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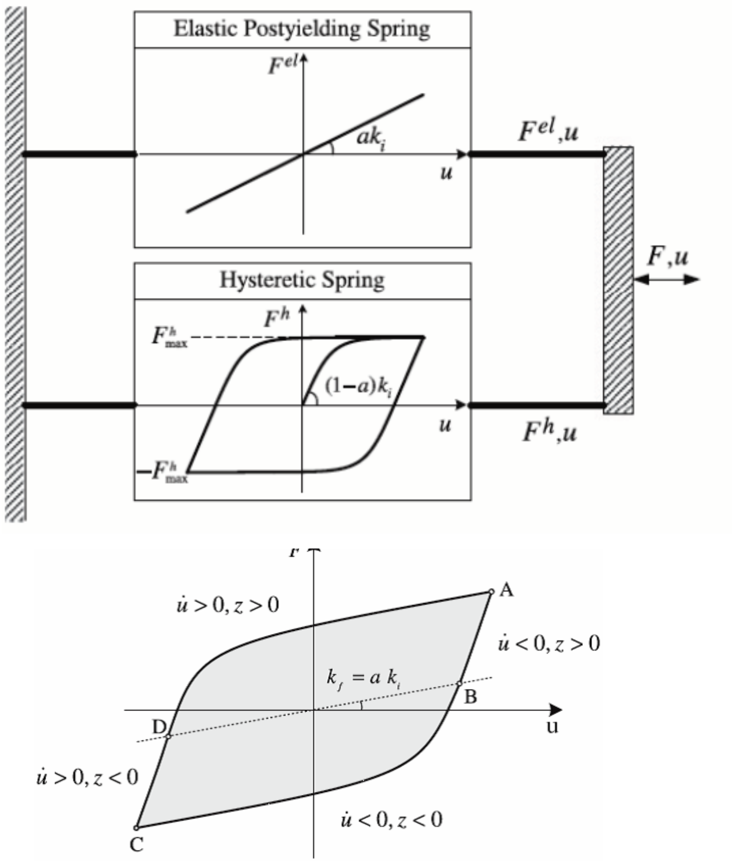


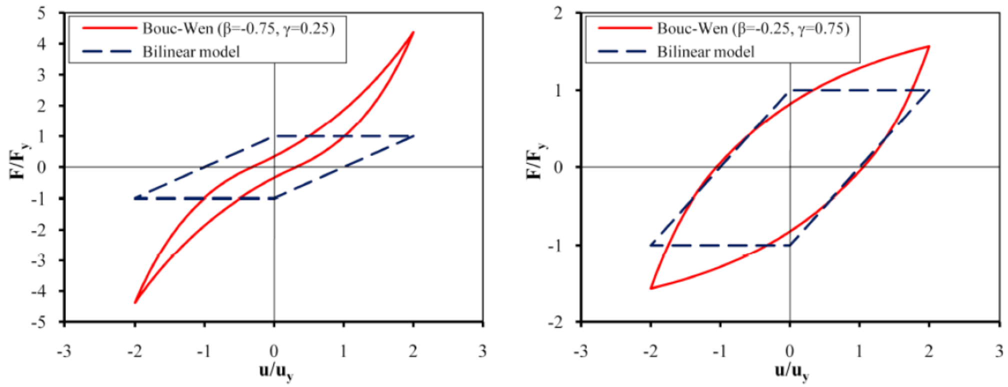
Hysteretic model of Bouc Wen.

Hysteretic model of Bouc Wen

$$m\ddot{u}(t) + c\dot{u}(t) + F_s(t) = F(t)$$

$$F_s(u(t), \dot{u}(t), z(t)) = F_{el}(t) + F_h(t) = \alpha k_i u(t) + (1-\alpha) k_i z(t)$$

$$\dot{z} = \frac{A\dot{u} - \{\beta \,|\, \dot{x} \,|\, z \,|\, z \,|^{n-1} + \gamma \dot{u} \,|\, z \,|^{n}\} \nu}{\eta}$$



Formulation of the problem and examples of hysteretic cycles.

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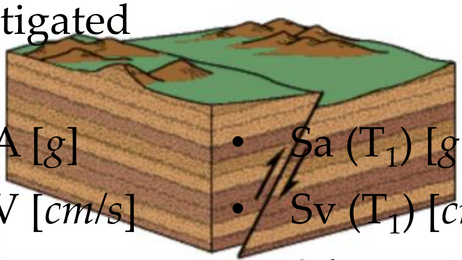
Step 2: input and *IM* selection

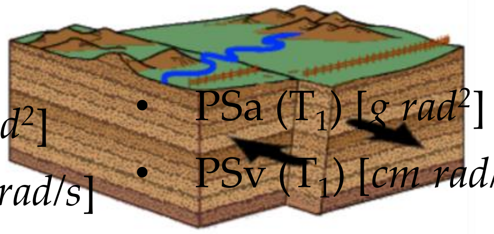
Dataset NGA-WEST 2 → *206 ground motions*

main features
- crustal earthquakes
- $M_w > 6$
- $R_{rup} > 10$ km
- $V_{s30} > 600$ m/s

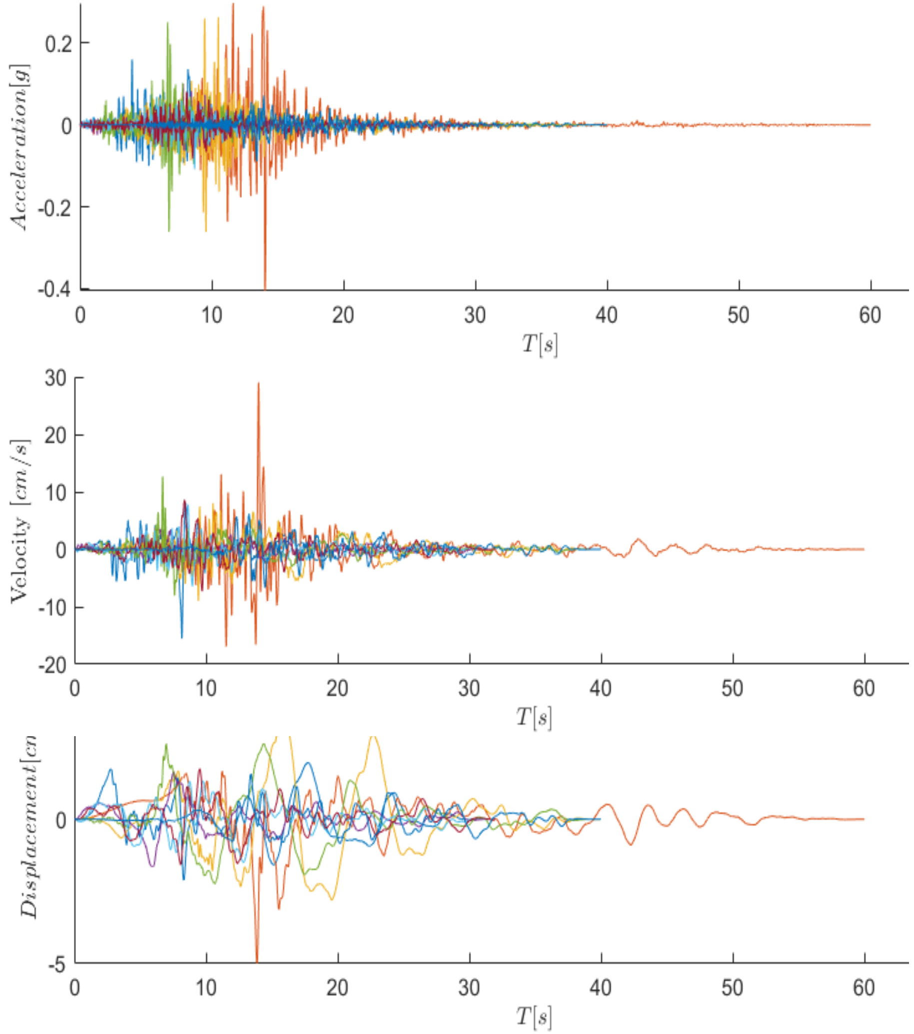
fault mechanism
- *reverse REV*
- *strike slip SS*

IMs investigated





- PGA [*g*]
- PGV [*cm/s*]
- PGD [*cm*]
- Sa (T_1) [*g rad*^{2}]
- Sv (T_1) [*cm rad/s*]
- Sd (T_1) [*cm*]
- PSa (T_1) [*g rad*^{2}]
- PSv (T_1) [*cm rad/s*]



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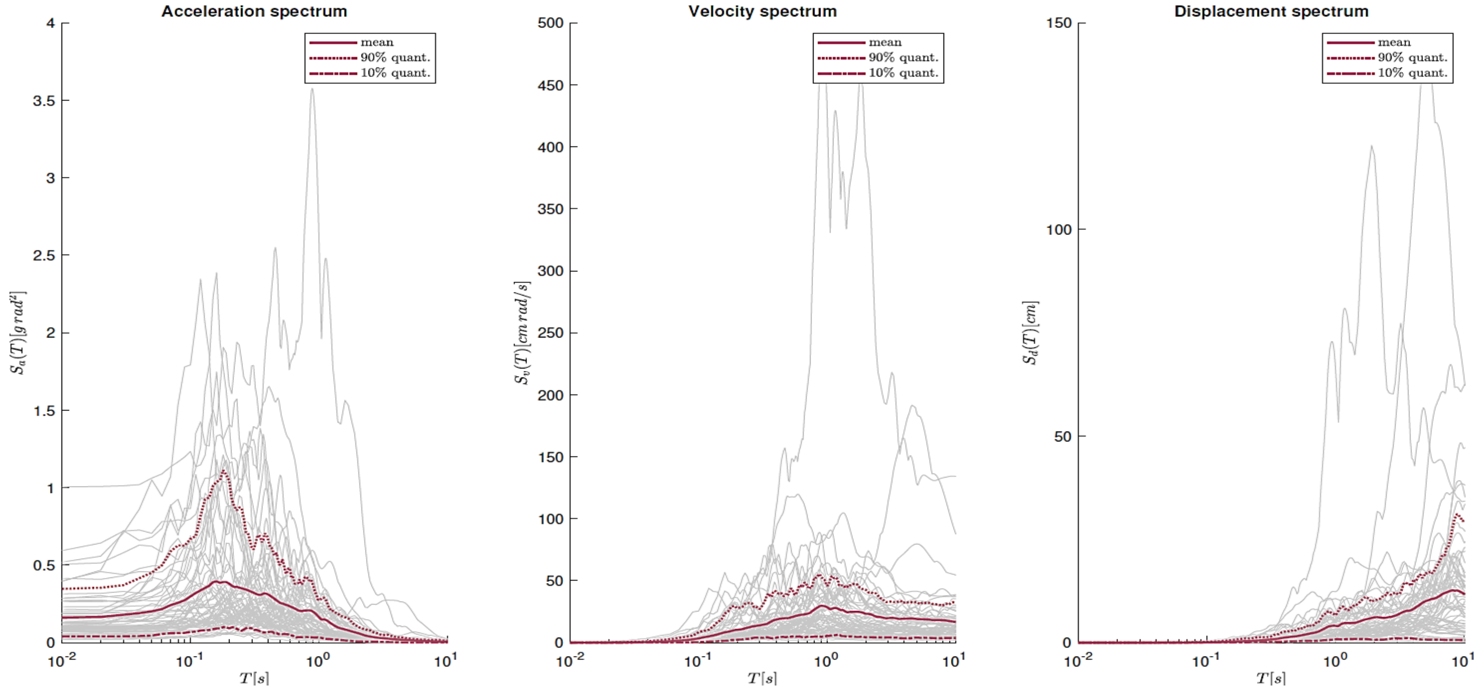
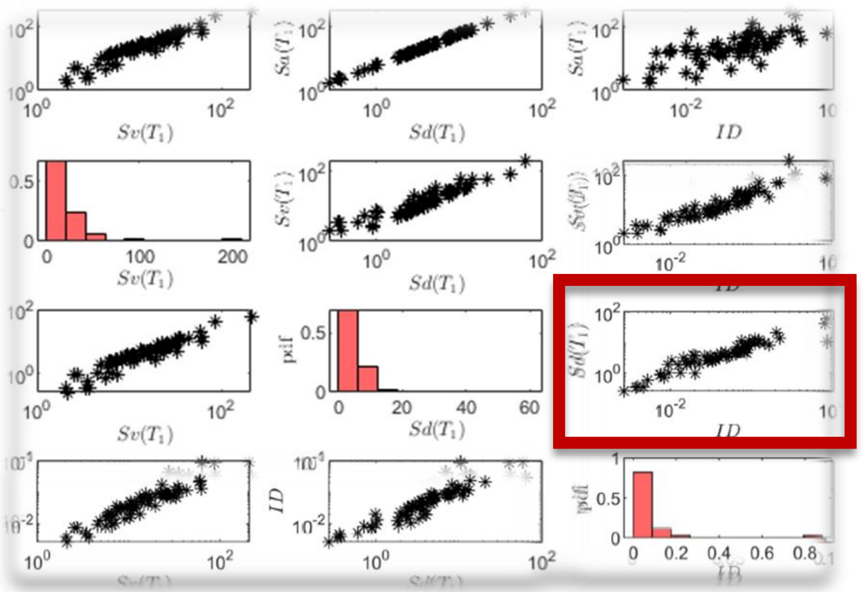
Step 2: input and *IM* selection

1) Data exploration of recorded gms
2) Scatter plot and statistic tools to evaluate proper *IM*



Acceleration, velocity and displacement response spectra with mean value, 10-th and 90-th quantile.



Scatter plot for correlation.

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Step 2: input and *IM* selection

Codes:

```
%%  Ground motions
Ground_motions = load('accelrot_cellarray.mat');
%
NN = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 19, 21, 23, 24,...
    25, 26, 28, 29, 30, 31, 32, 33, 34, 36, 37]; % number ID of the SS ground motions
DT = [0.01 , 0.01 , 0.005, 0.005, 0.005, 0.005, 0.02 , 0.02, 0.02, 0.01,...
    0.01 , 0.01 , 0.02 , 0.02 , 0.02 , 0.02 , 0.02 , 0.02, 0.01, 0.01,...
    0.005, 0.005, 0.005, 0.005, 0.02 , 0.005, 0.005, 0.01, 0.01, 0.01,...
    0.01 , 0.005, 0.005, 0.02 , 0.005, 0.01 , 0.01]; % integration time step
```

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Step 3: Definition of *damage limit states* and reference *EDP*

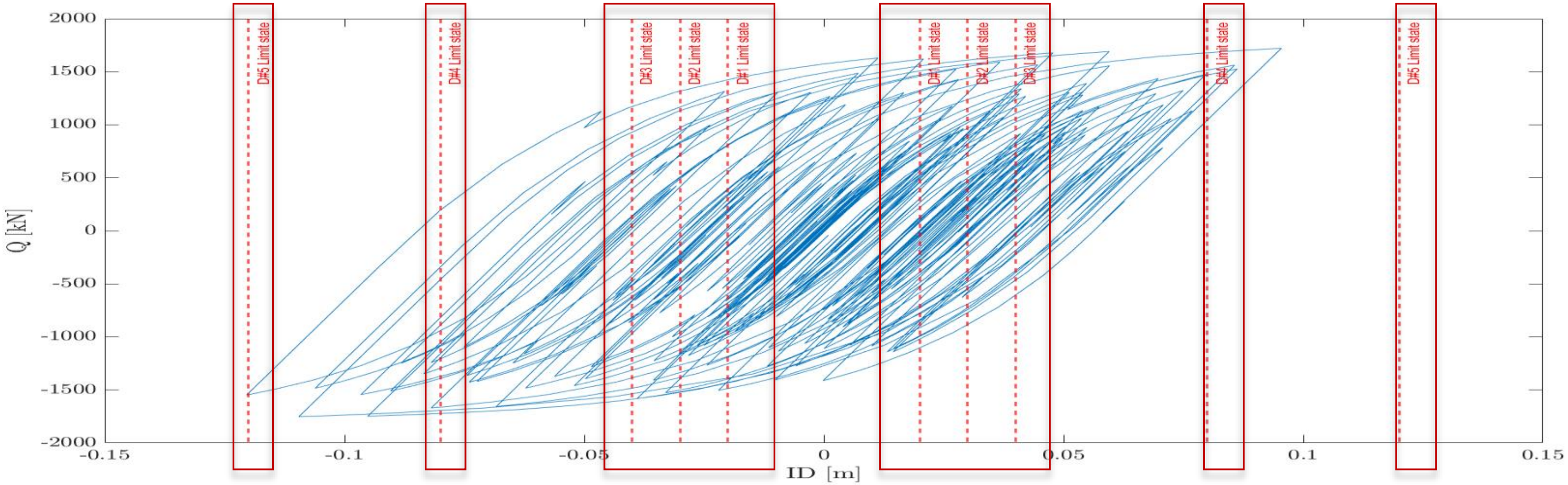
MRF

		Structural Performance Levels			
		Collapse Prevention	**Life Safety**	**Service**	**Immediate Occupancy**
Drift	[%]	5%	2,50%	1%	0,70%
	[m]	0,175	0,088	0,035	0,025

BF

		Structural Performance Levels			
		Collapse Prevention	**Life Safety**	**Service**	**Immediate Occupancy**
Drift	[%]	2,00%	1,00%	0,50%	0,30%
	[m]	0,07	0,035	0,0175	0,0105

Document *FEMA 356 - Prestandard and Commentary for the Seismic Rehabilitation of Buildings; Table C1-3 - Structural Performance Levels and Damage.*



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Step 3: Definition of *damage limit states* and reference *EDP*

<u>Codes:</u>

```
%% MDOF Properties
% Choose between the structural system
MDOF_properties_BW_MRF
MDOF_properties_BW_BF
% and  between linear or hysteretic behaviour
%% Structural behaviour
System_type = 'le'; % 'bw'
% bw = bouc-wen
% le = linear elastic
```

```
…
%% Limit States
LS = [0.50 0.75 1 2 3]*4/100;
for ls_i = 1:numel(LS)
ls_val = LS(ls_i)
Main_IDA_o_t
ls_i = ls_i + 1;
end
```

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Step 4: Performing non-linear time histories analysis

Codes:

```
 %% Initial condition
Mat.dFe=zeros(Mat.NDOF,numel(a_g_norm));    % Preallocation for the load for the time series
a_g = a_g_norm*scale;                       % Scaled ground motion
Mat.Fe=Mat.M*Mat.r'*a_g'*g;
%% Computation response
[HistVarBw]=ResponceMDF_Bw(Mat);
edp = max(abs(HistVarBw.eps(1,:)));
EDP(i) = edp; %store the EDP for each time history analysis
SCALE(i) = scale; %store the scale factor for each time history analysis…
```

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Step 5: Computing fragilities

$$\mathcal{L}(\mu,\sigma) = \underline{\mathcal{L}}(\mu,\sigma)\overline{\mathcal{L}}(\mu,\sigma) = \prod_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right)\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right]^{N-\bar{N}} \quad (1)$$

$$\ln \mathcal{L}(\mu,\sigma) = \sum_{n=1}^{\bar{N}} \varphi\left(\frac{\ln(im_n)-\mu}{\sigma}\right) + (N-\bar{N})\left[1-\Phi\left(\frac{\ln(\overline{IM})-\mu}{\sigma}\right)\right] \quad (2)$$

<u>Codes:</u>

```
 %% Untruncated IDA
[parmhat,parmci] = lognfit(IM_t_c ,0.01);
mu_IDA = parmhat(1);
sigma_IDA = parmhat(2);
%% Truncated IDA
IM_max = 2.2;
IM_trunc = IM_t_c(IM_t_c < IM_max); % take only the results with IM < IM_max
eq_over = sum(IM_t_c >= IM_max);    % number of analyses reached IM_max without collapsing
% Maximum likelihood fit, using equation (1) and (2) of previously slides
[mu_IDA_t, sigma_IDA_t ] = truncated_ida(IM_trunc, IM_max, eq_over);
```

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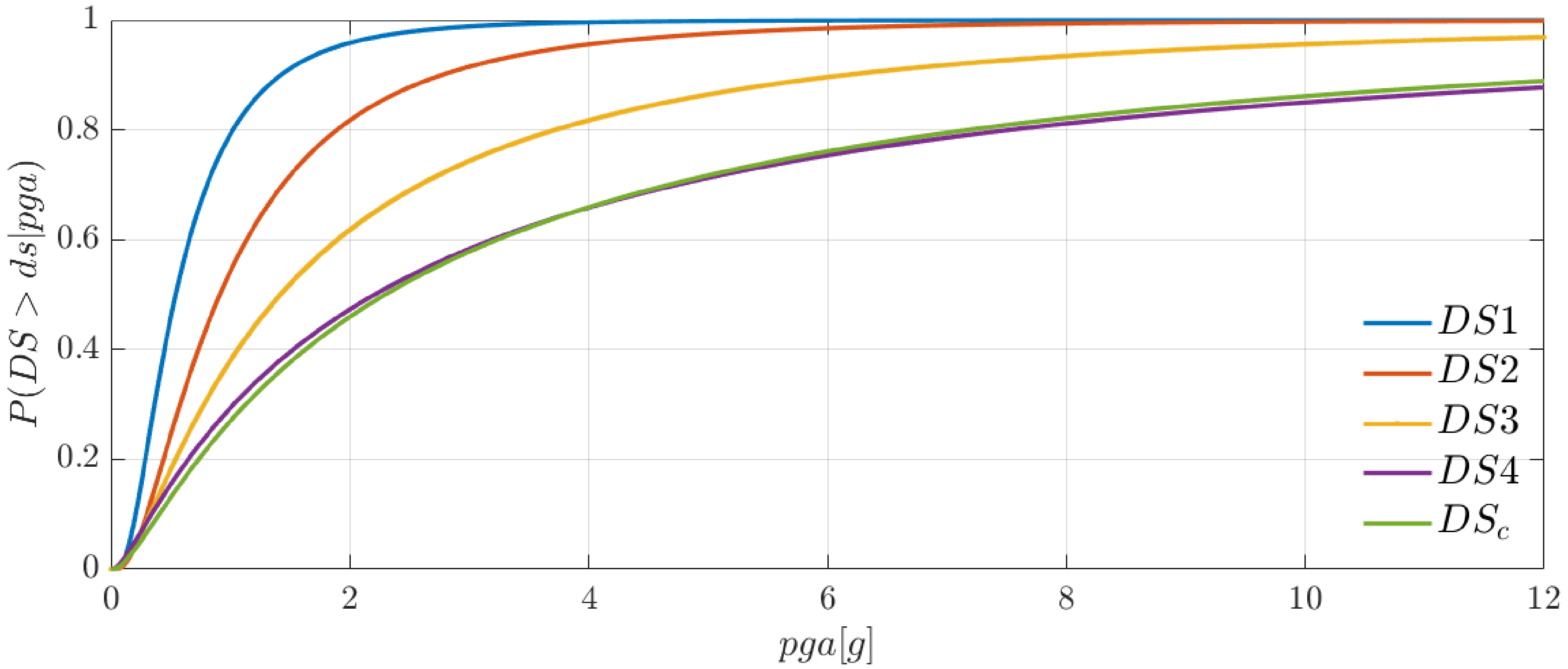
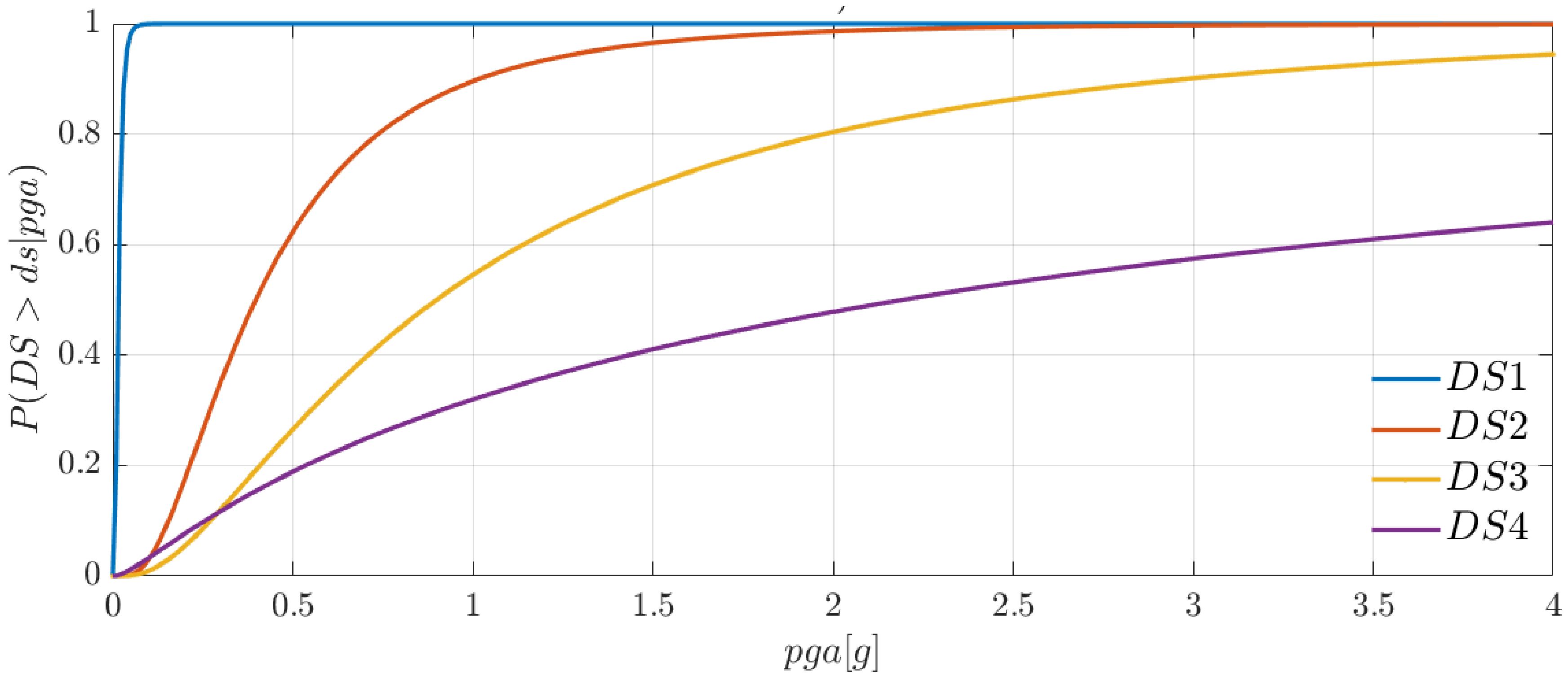
Step 5: Computing fragilities - Results



Step 5: Computing fragilities - Results



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- http://zonesismiche.mi.ingv.it/ → Italian database

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Thanks for the attention!

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