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Lecture #4

Probabilistic Seismic Hazard Analysis

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PSHA

Probabilistic Seismic Hazard Analysis (PSHA) evaluates the exceedance (or occurrence) probability of a given ground motion intensity measure threshold at given site and time interval.

PSHA provides a framework in which uncertainties, typically include magnitude size, earthquake location, soil condition, and rate of occurrence of earthquakes, are quantified.

The calculation of seismic hazard is based on the Total Probability Theorem*

$$P(IM > im) = \int_S P(IM > im \,|M = m, R = r)\; f_{R|M}^{(n)}\, f_M^{(n)} dr\, dm \qquad (1)$$

<< the probability that a fixed value of ground motion im is exceeded at a given site, given the occurrence of random earthquake from the seismic source n >>

**see Lecture Notes*

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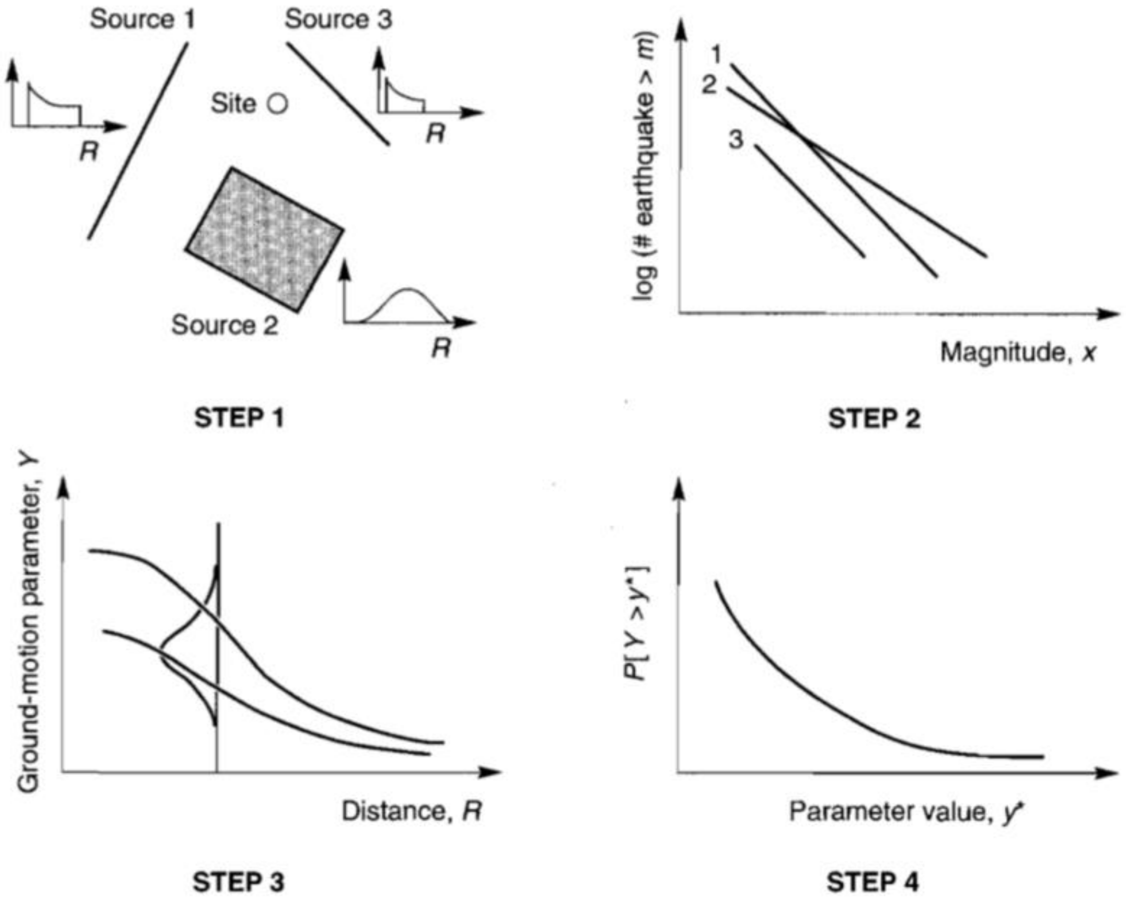
PSHA - Steps

i. **Source Characterization**: probability distribution of potential earthquake locations within the source → Definition of $f_{R|M}^{(n)}$

ii. **Magnitude Characterization**: temporal distribution of earthquake recurrence and size distribution $f_M^{(n)}$

iii. **Ground Motion Estimation**: empirical regression models named ground motion prediction equations (GMPE) → Definition of $P(IM > im\,|M = m, R = r)$

iv. **Hazard Computation**: solution of the integral *(1)* $\forall\, N_s$ → probability that the ground motion parameter will be exceeded during particular time period



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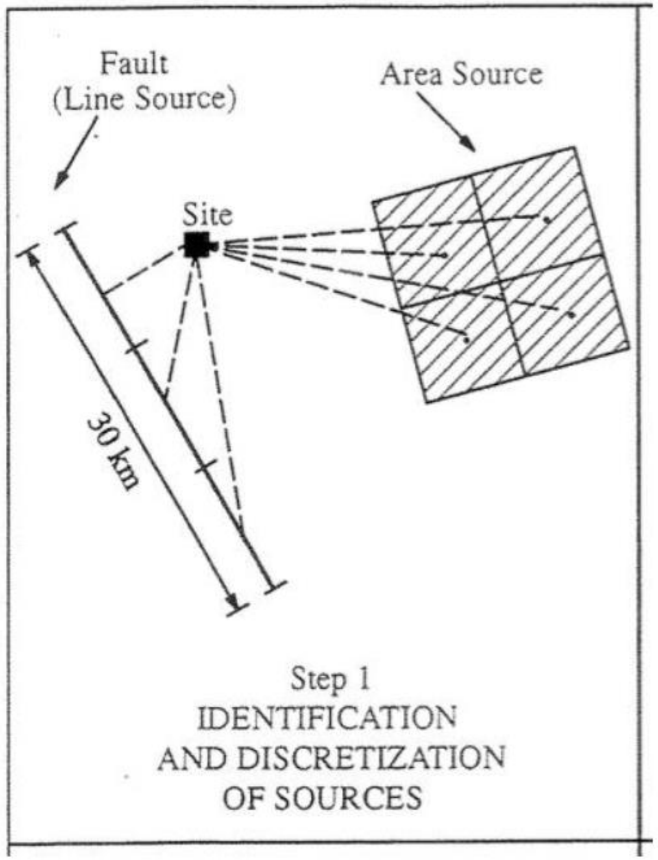
SOURCES

PSHA – Step 1: Earthquake source characterization

Goal: **to identify**

- *Fault sources:* individual or multiple **identified** faults
- *Area sources:* defined by polygons in which seismicity is **assumed** uniform

(Identification based upon the interpretation of geological, geophysical and seismological – historical data)



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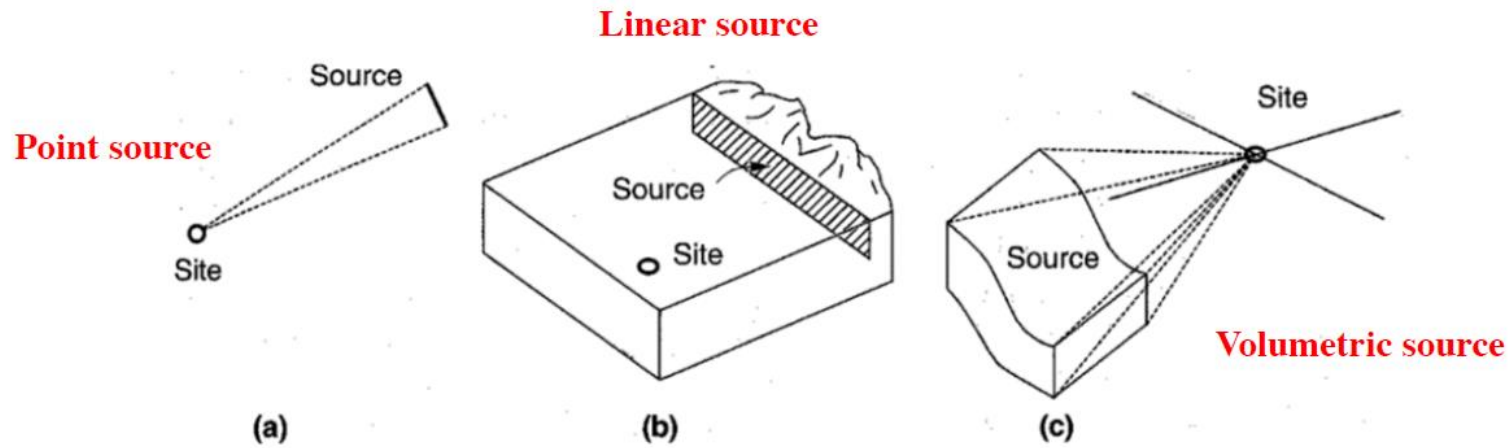
DISTANCES

PSHA – Step 1: Earthquake source characterization

and **to characterize seismic sources**

- *Point source*
- *Linear source*
- *Area source*

(the geometry of the source is used to identify the probability distribution of source-to-site distances)



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DISTANCES

PSHA – Step 1: Earthquake source characterization

The geometry of the source is used to identify the probability distribution of source-to-site distances

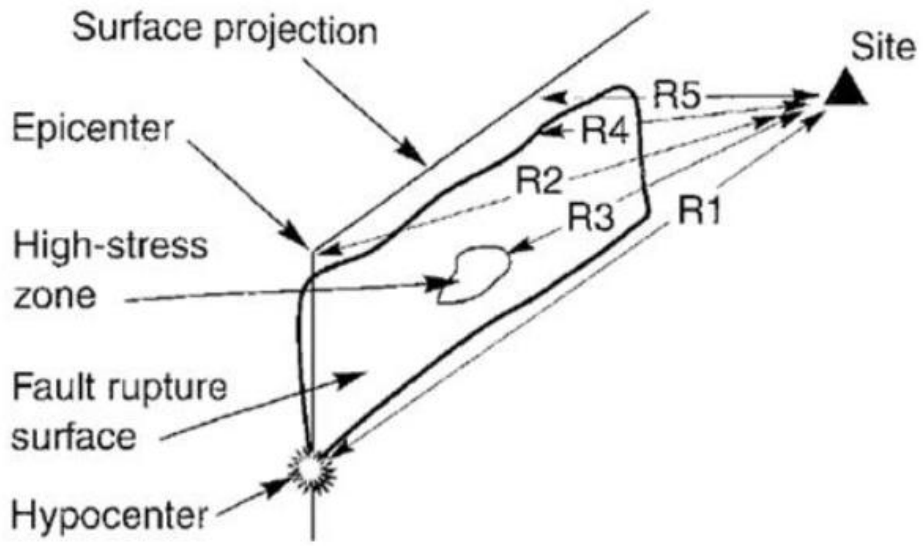
R1: hypocentral distance

R2: epicentral distance

R3: distance to the zone of highest energy release.

R4: closest distance to the zone of rupture

R5: closest distance to the surface projection of the fault rupture (also known as **Joyner-Boore** distance)



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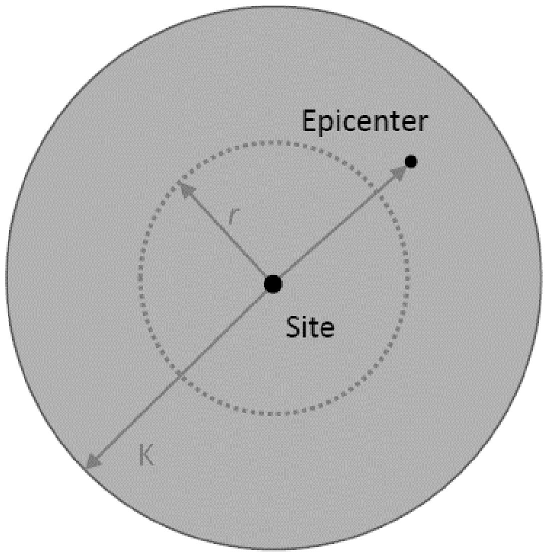
PSHA – Step 1: Earthquake source characterization

EXAMPLE

Earthquakes are equally likely to occur anywhere in the area within K km of the site.



$$F(r) = P(R \le r) = \frac{\text{area of circle with radius r}}{\text{area of circle with radius K}}$$

$$F(r) = \begin{cases} \dfrac{r^2}{K^2} & \text{if } 0 \le r < K \\ 1 & \text{if } r \ge K \end{cases}$$

$$f(r) = \frac{d}{dr}F(r) = \begin{cases} \dfrac{r}{0.5K^2} & \text{if } 0 \le r < K \\ 0 & \text{otherwise} \end{cases}$$

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PSHA – Step 2: Earthquake size (Recurrence law)

Goal: **to define distribution of** $f_M^{(n)}$

the chance of an earthquake of a given size occurring anywhere inside the source during a specified period of time

The Gutenberg-Richter law (G-R law) expresses the relationship between the magnitude and rate of cumulative number of earthquakes in any given region:

$$\log \lambda(m) = a - b \cdot m$$

$\log \lambda(m)$ logarithm base 10 of the mean annual rate of exceedance of magnitude m, a overall rate of earthquakes of the source and b relative ratio of small vs large magnitudes

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PSHA – Step 2: Earthquake size (Recurrence law)

Bounded G-R law

Truncated exponential distribution:

- M_{min}: not all the earthquakes can produce damages to the structures and a threshold on the magnitude is thus fixed. It is usually set at values from about 4.0 to 5.0.

- M_{max} : magnitude scale saturates and seismic zones in question cannot generate magnitudes above this value. Estimation based on geologic evidence, geophysical data, analogies to similar tectonic regimes, or other methods.

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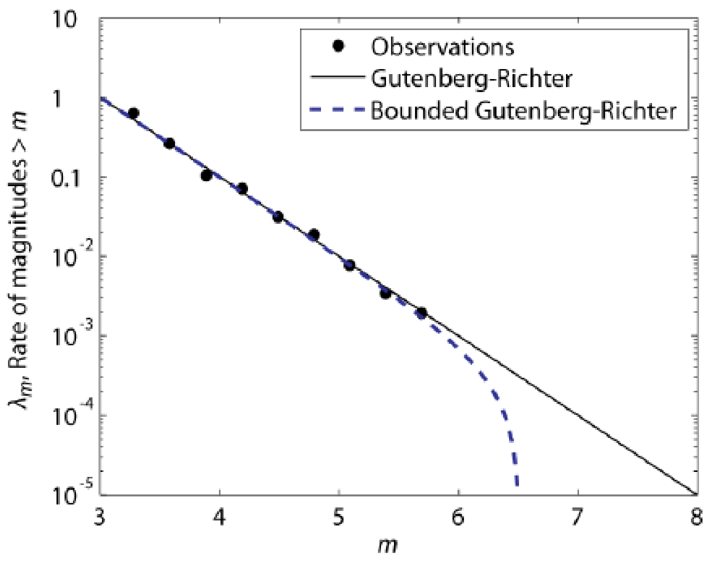
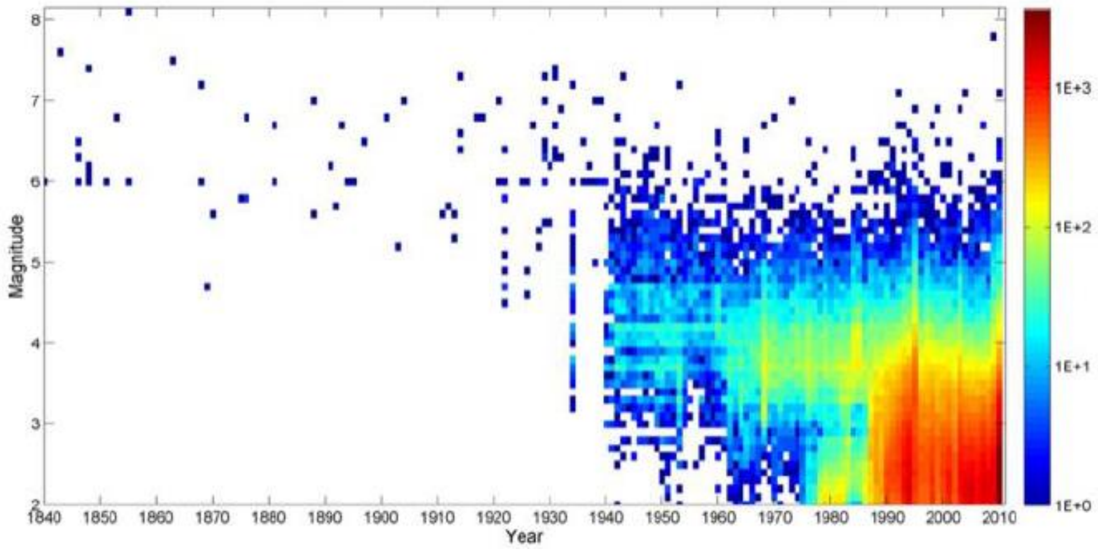
PSHA – Step 2: Earthquake size (Recurrence law)

Observations:

- given a seismic source, $a - b$ parameters are estimated through statistical analysis of historical data with constraints from geological evidence
- paramount aspect regarding completeness and undistortion of the reference catalogue in terms of intensity/magnitude range and time intervals





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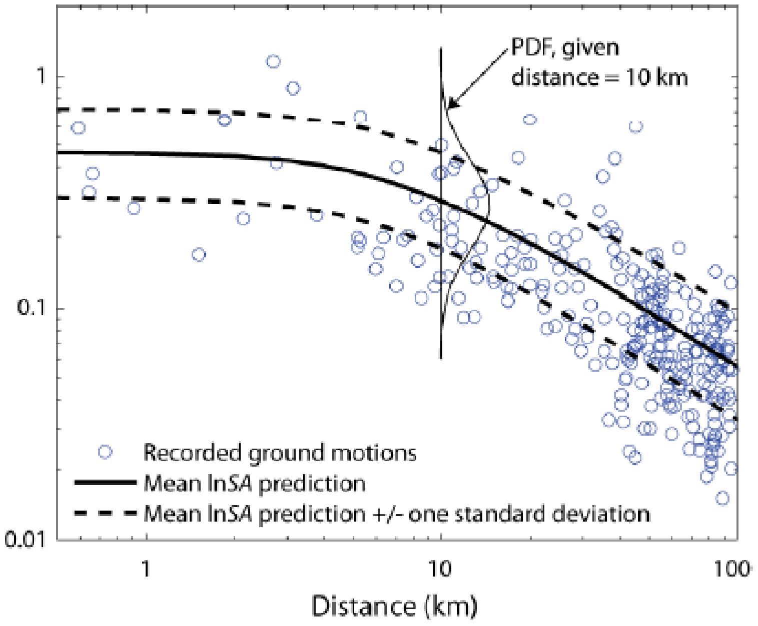
PSHA – Step 3: Ground motion predictive equations (GMPE)

The most used explanatory variables are the magnitude (M), the source-to-site distance (R) and coefficients to take into account for style of faulting (F), wave propagation path, and/or local site conditions (S).

$$\ln(IM) = \overline{\ln IM(M,R,\theta)} + \sigma(M,R,\theta) \cdot \varepsilon$$



where

- $\ln(IM)$ is the natural log of the ground motion *intensity measure* of interest;
- $\overline{\ln IM(M,R,\theta)}$ and $\sigma(M,R,\theta)$ are the predicted mean and standard deviation of $\ln(IM)$;

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PSHA – Step 3: Ground motion predictive equations (GMPE)

Goal: Identify the *IM* of interest for the situation and purposes

Ground motion parameters (also called Intensity Measures, IMs) describe the most important characteristics of strong ground motion in compact and quantitative form:

- Amplitude – how large is the shaking?
- Frequency content – what frequencies are particularly prevalent in the ground motion?
- Duration – how long does the strong shaking last?

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PSHA – Step 3: Ground motion predictive equations (GMPE)

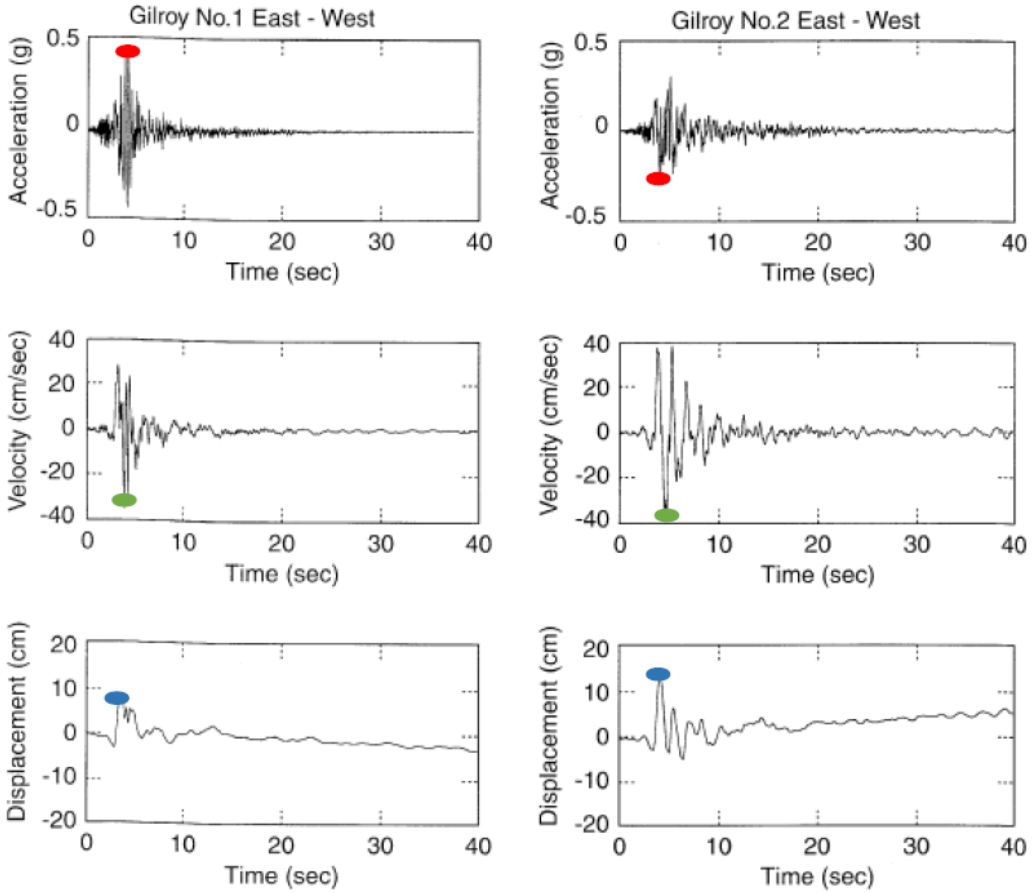
Ground motion parameters

Amplitude parameters:

• Peak ground acceleration (**PGA**) which is the maximum absolute value of acceleration at the ground level;

• Peak ground velocity (**PGV**), the maximum absolute value of velocity at the ground;

• Peak ground displacement (**PGD**) is the maximum absolute value of displacement at the ground



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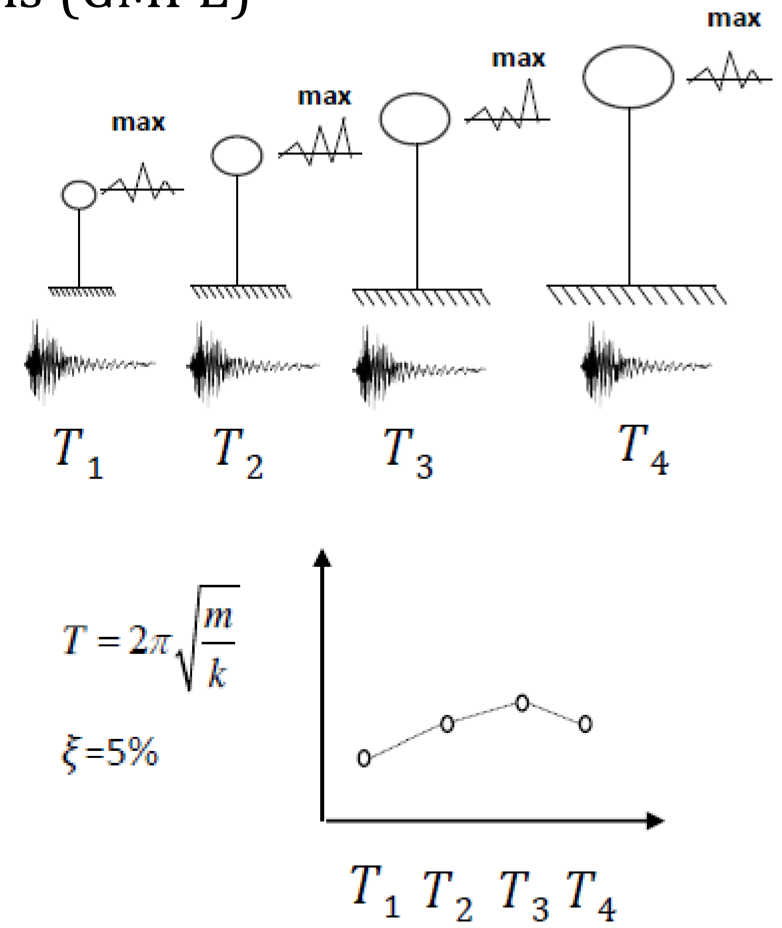
PSHA – Step 3: Ground motion predictive equations (GMPE)

Ground motion parameters

Frequency content parameters

- Response spectrum: maximum response (in terms of displacement, velocity or acceleration, S(T, ξ)) of an elastic single degree- of-freedom (SDoF) system to a particular input motion as a function of the natural period and damping ratio.



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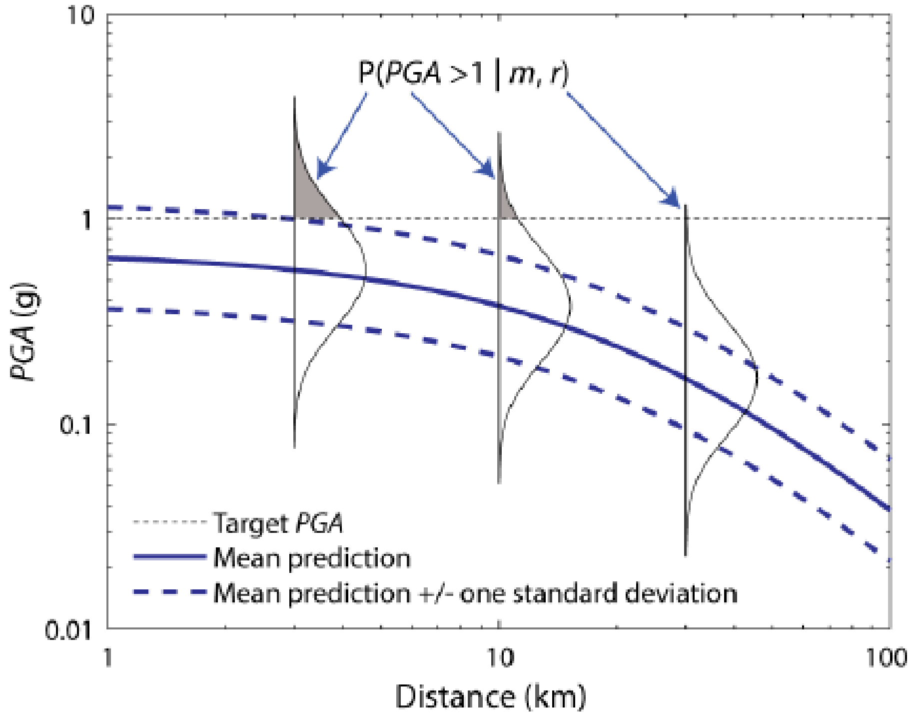
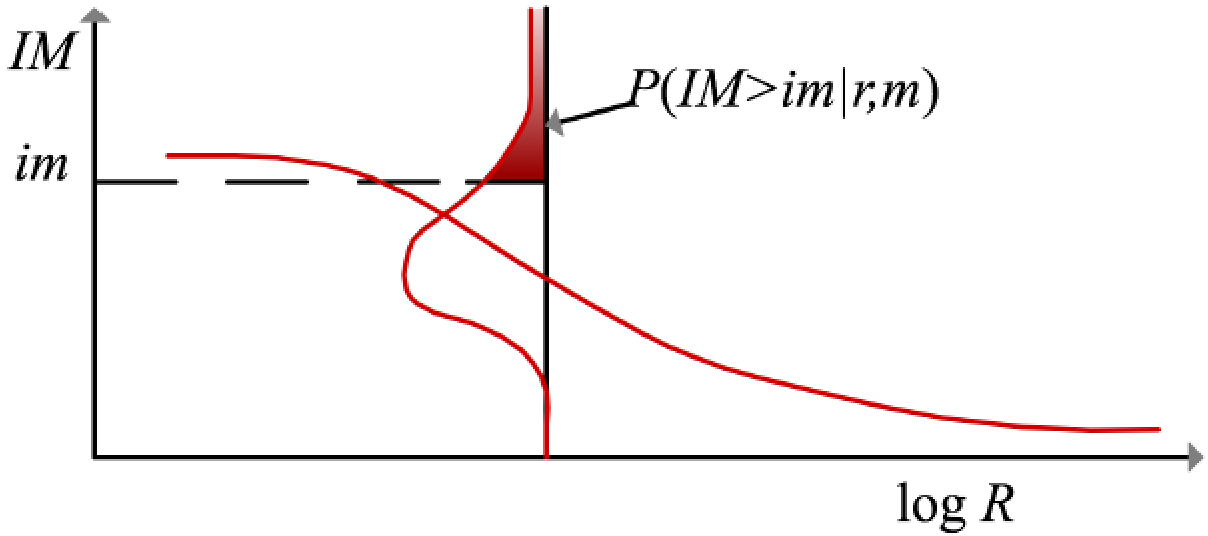
PSHA – Step 3: Ground motion predictive equations (GMPE)

In probabilistic terms

$$P[IM > im|R = r, M = m] = 1 - F_{IM|RM}(im|r,m)$$





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PSHA – Step 3: Ground motion predictive equations (GMPE)

Here, for clarity, it is assumed for the mean of log peak ground acceleration (in units of g):

1. Cornell (1979)

$\overline{\ln PGA} = -0{,}152 + 0{,}859M - 1{,}803\ln(R+25)$, with $\sigma = 0{,}57$ of $\ln(PGA)$; $\ln(PGA)$ normally distributed

2. Campbell and Bozorgnia (1994)

$$\ln(PGA) = -3.512 + 0.904M_w - 1.328\ln\sqrt{R^2 + \left[0.149\exp(0.647M_w)\right]^2} + (1.125 - 0.112\ln R - 0.0957M_w)F + (0.440 - 0.171\ln R)S_{SR} + (0.405 - 0.222\ln R)S_{HR}$$

$S_{SR} = 1, S_{HR} = 0$ *soft – rock*

$S_{HR} = 1, S_{SR} = 0$ *hard – rock*

$S_{SR} = 0, S_{HR} = 0$ *alluvium*

$$\sigma_{\ln PGA} = \begin{cases} 0.899 - 0.0691M_w & M_w \le 7.4 \\ 0.38 & M_w > 7.4 \end{cases}$$

$F = 0$ *strike – slip, normal*

$F = 1$ *reverse, oblique, thrurst*

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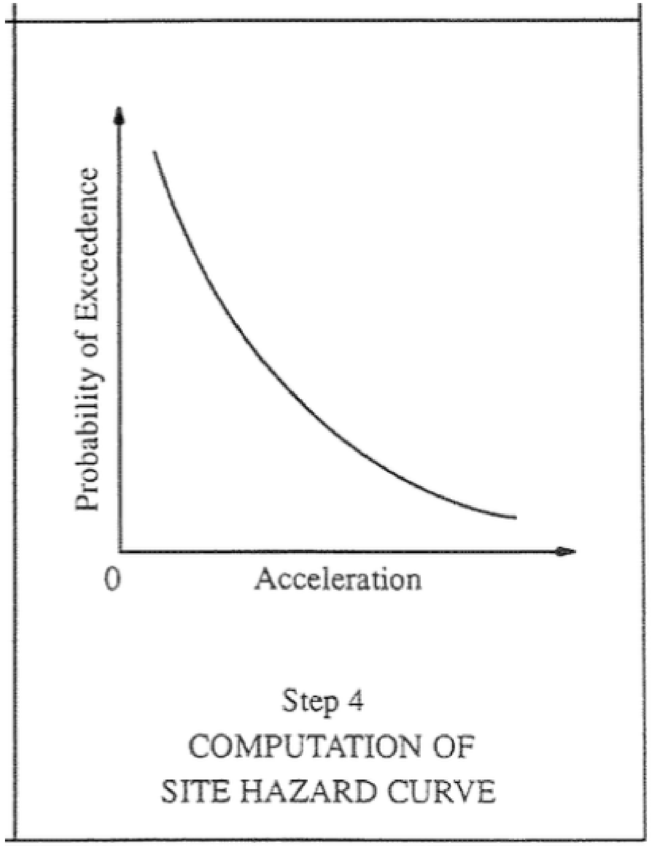
PSHA – Step 4: Hazard Computation

The seismic hazard curve is a function representing the annual frequency of exceeding various levels of ground shaking (i.e. the IM) at a specific site. The curve is obtained by integration of the previously three steps over all possible magnitudes and earthquakes locations.

Seismic hazard curves are obtained for individual sources and, then, combined to express the aggregate hazard at a particular site.

Then:

$$\lambda(im) = \sum_{n=1}^{N_s} \lambda_{min}^{(n)} \left[\int_{r_{min}}^{r_{max}} \int_{m_{min}}^{m_{max}} P(IM > im \mid M = m\,, R = r)\ f_R^{(n)}(r) f_M^{(n)}(m) dr dm \right]$$



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