CS 70 Fall 2024 — Discrete Mathematics and Probability Theory — Rao, Hug — DIS 1A

1 Perfect Square

Note 2

(a) Prove that if n^2 is odd, then n must also be odd.

If $2 | n \Rightarrow 4 | n^2 \Rightarrow 2 | n^2 \Rightarrow$ n^2 not odd $\Rightarrow$ contradictive

$\Rightarrow n$ odd

(b) Prove that if n^2 is odd, then n^2 can be written in the form $8k+1$ for some integer k.

n^2 odd $\Rightarrow$ n odd $\Rightarrow$ $n = 2k+1$ for any integer k

$\Rightarrow n^2 = 4k^2 + 4k + 1$

To show that $n^2 = 8k+1$, we show that $8 | 4k^2+4k$

$\frac{4k^2+4k}{8} = \frac{k^2+k}{2}$, as k^2 & k either both odd or both even

$\frac{k^2+k}{2} = c$, some integer, thus $n^2 = 8c+1$

2 Numbers of Friends

Note 2

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party. Assume that friendships are always reciprocated: that is, if Alice is friends with Bob, then Bob is also friends with Alice.

(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

For n people, the number of friends a person can have ranges from 0 . . . n-1, the only way to not have the statement be true is for the n people to respectively have number of friends 0 . . n-1 but this can't be true as 0 & n-1 can't coexist as one means befriend with everyone while the other means be friend with no one, bcs friends are mutual, can't coex -

Thus, they must have two ppl of same friend number

3 Pebbles

Note 2

Suppose you have a rectangular array of pebbles, where each pebble is either red or blue. Suppose that for every way of choosing one pebble from each column, there exists a red pebble among the chosen ones.

Prove that there must exist an all-red column.