Mathematical Model for Cattle Feed Mix Problem With Chance Constraint

G.Singhal[1] , M.Mann[2]

IIIT Sonepat

Abstract

This paper gives a mathematical model using GAMS STUDIO for Cattle Feed Mix Problem. It Provides a model with the help of which you can determine how much quantity of which food should be mixed or in which portion we need to divide the food, so that our subject (Cattle) meets it 's daily requirement to essential nutrients. This algorithm is designed in such a way that the money spent on the food is minimum on daily basis. Using this model many People (farmers ,diary workers ,etc.) can find that how much amount of money on which food they should mix in their food of subject(Cattle). This model is further Optimize using Chance constraint. Two problems are formulated, a deterministic model and the chance constraint version deterministic equivalent.

1. Introduction

Feed mix is a mixture of different feed ingredients used for animal feeding. A balanced feed mix is important for dairy cows as it provides them with the nutrients they need at various stages of their development. It can help them produce better yields and improve their nutrient utilization. For achieving these objectives, optimization should be done in such a manner that optimal feed mix can fulfil the nutritional requirements of the animal at different stages of production and livestock. Optimization models are in use for more than a half century at commercial level and for livestock management. A linear programming technique has been developed for defining feed formulation problem [1](Babu et al., 2014). Feed is one of the factors that play an important role in determining a successful development of an aquaculture Industry. It has been proven that feed constitutes the most expensive component of the industry, approximating 50 to 60 percent of operational costs. In this research paper we will try to find out that how much money one should spent on the food mix so that in minimum amount of money they get their desired requirement. This model is seen as a combination of two models, a linear programming model and the chance constraint version deterministic equivalent.

The goals of this work are to build the following algorithms:

1. To achieve nutrient diversity by using a low-cost feed mix that meets the nutritional requirements of the cattle.
2. Chance Constraint used to further optimise the result.

2. Linear Programming Model

[2](Waugh, 1951) was the first researcher to use mathematical programming to tackle the feed mix problem in farmed animal nutrition. It happened when he figured out that linear programming (LP) method was best suited for solving animal diet problem. [2](Waugh, 1951) stated that he was not an

animal nutritionist in his endeavour, but that he was looking for an appropriate approach to lessen the animal nutritionist's job. Since then, LP is used widely in modelling the animal feed problem [3](Alexander et al., 2006), [4](Barbieri & Cuzon, 1980), [5](Candler, 1960). The common objective in formulating the feed mix is to minimize cost while providing adequate nutrients to meet the needs of the farmed animal type being fed [6](Chappell, 1974), [7](Waugh, 1951).

Since Then ,LP is widely used in animal feeding problem and is seen in various papers [8](Alexander et al., 2006),[9](Barbieri & Cuzon, 1980),[10],[11](Chakeredza et al., 2008),[12](Chappell, 1974),[13](Swanson & Woodruff, 1964),[14](Thomson & Nolan, 2001),[15](Sirisatien et al., 2009),[16](Abd Rahman et al., 2010), etc.

However, the feed mix problem has gotten increasingly challenging due to different constraints that must be considered. As a result, LP alone will not be able to solve all of the problems. We need to consider other factors also. Issues arisen in feed mix can be:

1. feed ingredient prices are not constant, also known as price variability
2. nutrients imbalance occurs in the final solution.
3. infeasible solution occurs.
4. feed ingredient nutrients levels are unstable and fluctuate.

That is the reason we are introducing chance constraint Algorithm in our research work.

3.Chance Constraint Algorithm

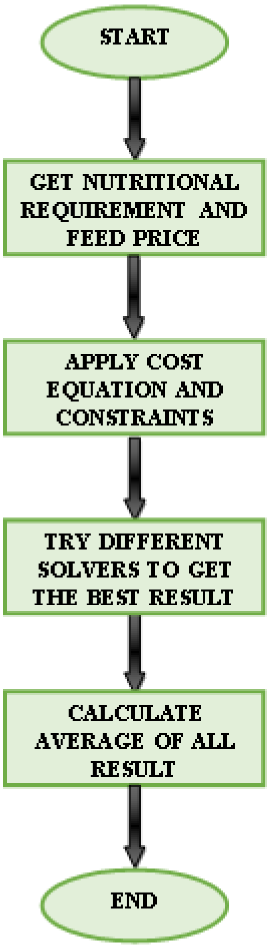
One of the most common ways to addressing optimization problems under varied uncertainties is the chance-constrained method. It is an optimization problem formulation that ensures that the probability of achieving a given constraint is greater than a particular threshold. In other words, it limits the viable region so that the solution's confidence level is high. Although the chance-constrained method is a rather robust strategy, it is frequently difficult to solve.

CCP is a nonlinear feed formulation method [17](Roush et al., 1994).CCP was developed in response to a deterministic situation in which some of the parameters were not constant.[18](Dupačová & Kopa, 2014). [19](de Panne & Popp, 1963) is one of the first researchers to employ stochastic techniques, assuming that the nutrients in the ingredients are not predictable. This approach showed that the probability of the nutritional requirements will be met an increase from 50% to 95%.By comparing LP and CCP, the CCP shows that the probability that nutritional requirements will be met increases from 60% up to 90%.So with the help of chance constraint we can optimize our result as best as we can.

4. Mathematical Model

4.1 Flow Chart

- In This model, first we need to get the nutritional requirement value of cattle as it is necessary to know the food requirement of cattle. By knowing this we get to know that how nutrient is required for cattle so that the cattle will be healthy and good.



- After knowing the nutrient requirement of cattle, we need to know that what type of food cattle will eat and what is the cost per kg of food in the market. Types of food eatable by cow is present in later section.
- After knowing both the things i.e nutrient requirement and food price per kg, now we need to work on our constraints and mathematical model. Constraints and model are described on further section.
- After Applying all the constraints, now we will try different solvers on our constraints and check which solver is giving us the best answer.
- After the previous step, we will come to know that what portion of which food should mix so that the cost of food mix is minimum and our cattle will meet its daily requirement of nutrients.

4.2 Nutritional Requirement

4.2.1 Water

Water is the most necessary ingredient for life, second only to oxygen [20](Kononoff et al., 2017). Milk is made up of 87 percent water and is essential for the formation of milk. In the dairy cow, water accounts for 56 percent to 81 percent of total body weight [21](Murphy, 1992), whereas in the preruminant calf, it accounts for 68 percent to 72 percent [22](Chapman et al., 2017). Water is required for the transit of substances throughout the body, as well as temperature regulation, insulation, and waste elimination. Free-water (drinking water), feed water, and water from endogenous processes are all sources of water. Free water intake is the primary source of water for cattle. As a result, anything that restricts free water intake will have a negative impact on animal performance. Free water consumption rises in tandem with milk production, as one might predict. When the cow is in an environment where the temperature humidity index is above 68, she drinks more water.

Water Requirement:- Mature dairy cows should have access to at least 4 inches of linear water area. Although this is a little space per animal, it is adequate because cattle do not all drink at the same time if they have constant access to water. On grassland, dairy cattle should always have access to free-choice water. If you don't provide your animals enough water, they'll produce less and eat less.

4.2.2 Protein and amino acids

Protein is commonly quantified in feedstuffs as crude protein (CP), which is calculated by multiplying the percent N in a feed by 6.25. Crude protein, on the other hand, contains not only real protein, but also amino acids, dipeptides, nucleic acids, NH3eN, and other nonprotein nitrogen (NPN) molecules. It's also important to understand early on that dairy cows, like all other animals, don't require protein; instead, they require amino acids, and true proteins are defined as chains of amino acids. Amino acids are used by cattle to make enzymes, milk proteins, immunoglobulins, muscle, and other organs and tissues in the body. Milk proteins are required for the formation of bioactive proteins found in the whey fraction of milk, which have a variety of protective roles for new borns. Casein and whey protein production offers the amino acids required for young people's growth.

Amino Acids:-
In nature, there are hundreds of amino acids. Both amino and acid groups can be found in an amino acid. There are ten essential amino acids in most species, with the others considered non-essential. Some amino acids ae considered limiting for production. These limiting amino acids are determined by the cattle's diet. Corn is commonly fed in two forms: silage and grain. Corn is reported to be low in lysine, an important amino acid. Corn feedstuffs are frequently supplemented with soybean co-products as the principal protein source. The amino acid methionine is lacking in soybean feedstuffs. This means that the amount of these amino acids in the cow's diet limits protein synthesis and yield [23](Council & others, 2001). Suggestion: Lactating cows should be given a variety of protein sources rather than just one.

4.2.3 Carbohydrates

Carbohydrates are carbon, hydrogen, and oxygen-based nutrients. Carbohydrates have more oxygen molecules than lipids. They are also the most important part of a dairy cow's diet. Lactating dairy calves consume up to 70% of their diet, while developing heifers and non-lactating cows consume even more. Sugars and chains of simple sugars are the most common types of carbohydrate. Carbohydrates can be found in forages, roughages, cereals, and sweets. Cellulolytic bacteria breakdown forages such as hay, hay-crop silage, and grain-based silage (corn or small grains), resulting in the generation of acetic and butyric acid.

Recommendation:-

Most cows should be fed diets that comprise more than 50% forages, but this might vary greatly depending on the proportion of fibre wastes.

4.2.4 Fats

Saturated fats can be found in a variety of sweet and savoury meals. The majority of them originate from animal sources, such as meat and dairy, as well as some plant items, such as palm and coconut oil.

With 2.25 times the energy of carbohydrates or protein, fat is the most energy dense nutrient. Fat is not significantly fermented in the rumen, resulting in little heat of fermentation, which can help cattle maintain caloric intake, particularly when they are under stress.

Types of Fats:-

Fats are of two types: 1. Glycerol and 2. Non-glycerol [24](Ni et al., 2020).

Non-glycerol fats, such as waxes and sterols, have little to no nutritional value, whereas glycerol fats, such as triglycerides, phospholipids, and glycolipids, have nutritional benefit [25]. These lipids have a long carbon chain, a glycerol backbone, and a carbohydrate or phosphate component.

It's important to note that practically all feeds contain fat, with the exception of water and minerals. Isoprenes are the carbon backbones of many lipid-soluble vitamins.

4.2.5 Minerals

Minerals are elements found on the soil and in foods that our bodies require for appropriate development and function. Calcium, phosphorus, potassium, sodium, chloride, magnesium, iron, zinc, iodine, chromium, copper, fluoride, molybdenum, manganese, and selenium are all necessary minerals. Minerals are inorganic chemicals that are needed for a variety of body processes, including structure, nerve impulses, and osmotic balance.

Some minerals are required for enzyme function or act as catalysts for processes (e.g. glutathione peroxidase).

There are two types of minerals: 1) those that are found in the earth and 2) those that are found in the macrominerals (Calcium, Phosphorus, Magnesium, K,Cl,Na and S), both of which are required in gramme amounts.

Microminerals, often known as trace minerals, are a type of mineral needed in milligrammes (mg).

Mineral diet is critical for the lactation performance of dairy cattle. There is a considerable demand on Ca due to the enormous volume of milk that cows produce at parturition. In this situation we will put cow into a hypocalcemic state, known as milk fever or parturient paresis. In this, cow will have inability to stand, cold ears, and reduced body temperature. Normally, the parathyroid hormone will respond to low blood calcium levels by inducing the bone to release calcium, the kidney to limit Ca excretion, and the kidney to begin manufacturing 1,25 dihydroxy vitamin D to begin efficient Ca absorption from the gut.

The pH of cattle's blood is tightly controlled, hovering at 7.37 [26]. As per [26] The pH of blood is determined by three factors: (1) respiration, which eliminates the bicarbonate anion from the blood, (2) the balance of positively and negatively charged minerals (cations and anions), and (3) the content of blood proteins.

Dietary cations, particularly Na^+ and K^+, are practically completely absorbed.

Some essentials Minerals:-

- Calcium
- Phosphorus
- Potassium
- Magnesium
- Sodium
- Sulfur
- Chloride
- Copper
- Iodine
- Iron
- Manganese
- Molybdenum
- Zinc
- Selenium

4.2.6 Vitamins

Vitamins are chemical compounds that can be classed as fat-soluble or water-soluble. Vitamins that dissolve in fat (vitamin A, vitamin D, vitamin E, and vitamin K) tend to accumulate in the body.

Vitamins are required for metabolic function. They are organic compounds that are classified as either water-soluble or lipid-soluble. Vitamins that dissolve in water are referred to as water-soluble vitamins. They are usually generated in sufficient quantities in the rumen. Vitamins that are lipid-soluble are those that are lipid-based. Vitamins A, D, E, and K are the ones in question. Various quantities of vitamin A, D, and E supplementation are required based on the diet consumed.

Some essential Vitamins :-

- Vitamin A
- Vitamin D
- Vitamin E

4.3 Common Food Product

In this section, we are going to describe some common eatable food products by cattle. These are some kinds of things which are easily available in market and are good for cattle health.

List of common food products:-

Food
Grass
Wheat-Straw
Mustard-Cake
Wheat Bran
Chick peel
Binola khal
Porridge (Daliya)

4.4 Objective function

The objective of our work is to find the least cost diet mix for cattle so that the nutritional requirement for cattle will be fulfilled.

Objective function (in linear programming) the function that it is desired to maximize or minimize. In our Case, it is the cost equation which is need to minimize. Cost Equation is defined as the sum of product of price per kg of feed and amount taken over all feed.

Cost = sum(f, price(f)*x(f));

Where :
price(f) – price of feed f per kg.
x(f) – Feed mix.
Cost – total cost per kg.

In mathematical equation:-

$$C = \sum_{f=1}^{f=n} p(f) \times x(f)$$

4.5 Constraint

In the form of linear inequalities, the variables are also susceptible to conditions. These are referred to as limitations. The variables must also meet the non-negativity requirement, which means they can't be negative. The feasible set is the set of points, or variable values, that meet the constraints and the non-negativity criterion.

In our there are majorly three constraints which are derived on this problem. We are going to discuss each and every one by one. Constraints are something which need to be properly implemented and taken care off. Whole accuracy depends upon these constraints and one can increase more accuracy by introducing more constraints.

Main constraints are:-

- **Mix Constraint**

  Mix Constraint states that the mixing portion of our feed mix should be equal to 1 or 100%. This means that the sum of amount of feed mix over all the food products should be 1.

  Sum(f,x(f)) = 1;

  Where x(f) – feed mix of feed f.

  In mathematical Terms:-

  $$\sum_{f:1}^{n} x\,(f) = 1$$

- **Nutrient Balance**
  Nutrient Balance state that the summation of product of feed characteristics and feed mix over all food products should be greater than requirement of cattle.
  In other terms:-
  Sum(f,char("mean",n,f)*x(f)) > = req(n);
  Where :-
  x(f) - feed mix of feed f.
  char("mean", n, f) – average value of nutrient n in food f.
  req(n) – requirement of nutrient n for cattle.

  In Mathematical Terms:-

$$\sum_{f=1}^{f=n} char(\text{mean}, n, f) \times x(f) \geq req(n)$$

- **Chance Constraint**
  We work with differentiable nonlinear random functions and discrete distributions to solve chance constrained problems (CCP).

  In our case, Chance Constraint equation is given as:-

$$\sum_{f=1}^{f=n} char(\text{mean}, n, f) * x(f) - 1.645 * (\sqrt{\Sigma char(variance, n, f) * x(f) * x(f)} \geq req(n)$$

  This constraint is used to increase the efficiency of the model.

  It insures that the probability of that the previous constraints are following is greater than 95%.

4.6 Different Solvers

GAMS [36] [37](Bussieck & Meeraus, 2004) includes all of the tools needed to create, debug, deploy, and maintain optimization models. GAMS can be used to formulate a wide range of mathematical model types (linear, mixed-integer, nonlinear, mixed-integer nonlinear, mixed complementary, and so on).
GAMS generates optimization issues from your models and data and extracts the results for further analysis and processing, but it does not solve the problem.
Table of Different Solvers is given as below:-

Different Solvers used in Various Programming Methodology.
Source: https://www.gams.com/latest/docs/S_MAIN.html

In this work, we are going to use some solvers of linear and non – linear paradime, which will be helpful for us and are given as:

- CPLEX [38](Fourer, 2011)
- CONOPT 4 [39](Shepero et al., 2020)
- GUROBI[40](Bixby, 2007)
- LINDO [41](Lin & Schrage, 2009)
- MINOS[42](Smith & Lasdon, 1992)
- XPRESS[43](Anand et al., 2017)

We will run our code with all these solvers and then try to get optimal answer.

5. Result

We have run our code with different solvers and tried to obtain our result. Running code multiple times on different solvers will take huge time and showing them all one by one will not be healthy. We will show the average value of objection function obtain by applying all multiple solvers in our code and represent them in one table, so that it will not become complex to read and understand.

The final result obtained by applying all the solvers on our algorithm and is shown below:

Optimal solution found:

Objective: **22.542857**

Feed	Lower	Level	Upper
Grass	.	.	+INF
Wheat-Straw	.	.	+INF
Mustard-Cake	.	.	+INF
Wheat-Bran	.	0.5143	+INF
Binola-Khal	.	0.4857	+INF
Chickpea-peel	.	.	+INF
Porridge	.	.	+INF

Fig2: Table obtained by applying all solvers on our algorithm.

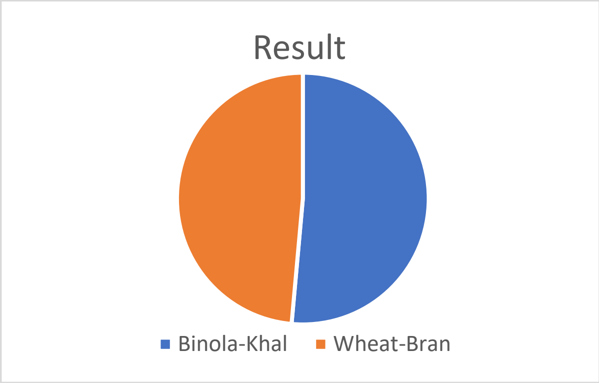
Conclusion:

Min Cost : 22.5

Ratio of wheat-bran: 51.43%

Ratio of Binola-Khal: 48.57%

Output shows that the result obtained using our algorithm is good enough so that animal can meet it’s daily requirement of nutrients with money spent as less as possible.



Output in pie-chart

We can further optimize this result by including more cheap eatable items with rich quantities of nutrients.

6. Conclusion

Our problem statement was to find out the least cost diet for dairy cow such that the cow will meet its daily nutritional requirement. Using Linear programming methodology, we are successfully able to built an algorithm such that it can find least cost mix ratio efficiently. Further with the help of Chance Constraint we are able to increase it’s effectiveness.

It can also be improvised by considering more nutrient and more feed items but as for now this is good enough and can be used in further studies.

7. Acknowledgement

Garv Singhal thanks Indian Institute of Information Technology, Sonepat, India and Ministry of Higher Education India.

8. References

[1] S. Babu and P. Sanyal, Food Security, Poverty and Nutrition Policy

Analysis: Statistical Methods and Applications. Washington, DC, USA: Academic Press. 2009. pp. 304.

[2] F. V. Waugh, "The minimum-cost dairy feed," Journal of Farm Economics, vol. 33, pp. 299-310, 1951.

[3] D. L. J. Alexander, P. C. H. Morel, and G. R. Wood, "Feeding strategies for maximising gross margin in pig production," in Global optimization: Scientific and Engineering Case Studies (pp. 33-43). United State: Springer US. 2006.

[4] M. A., Barbieri, and G. Cuzon, "Improved nutrient specification for linear programming of penaeid rations." Aquaculture, vol. 19, pp. 313- 323, 1980.

[5] W. Candler, "A "short-cut" method for the complete solution of game theory and feed-mix problems". Econometrica, vol. 28, no. 3, pp. 618- 634, 1960.

[6] A. E. Chappell, "Linear programming cuts costs in production of animal feeds'" Operational Research Quarterly, vol 25, no. 1, pp. 19-26, 1974.

[7] F. V. Waugh, "The minimum-cost dairy feed," Journal of Farm Economics, vol. 33, pp. 299-310, 1951.

[8] D. L. J. Alexander, P. C. H. Morel, and G. R. Wood, "Feeding strategies for maximising gross margin in pig production," in Global optimization: Scientific and Engineering Case Studies (pp. 33-43). United State: Springer US. 2006.

[9] M. A., Barbieri, and G. Cuzon, "Improved nutrient specification for linear programming of penaeid rations." Aquaculture, vol. 19, pp. 313- 323, 1980.

[10] W. Candler, "A "short-cut" method for the complete solution of game theory and feed-mix problems". Econometrica, vol. 28, no. 3, pp. 618-634, 1960.

[11] S. Chakeredza, F. K. Akinnifesi, O. C. Ajayi, G. Sileshi, S. Mngomba,

and F. M. T. Gondwe, "A simple method of formulating least-cost diets for smallholder dairy production in sub-Saharan Africa'" African Journal of Biotechnology, vol. 7, no. 16, pp. 2925-2933, 2008.

[12] A. E. Chappell, "Linear programming cuts costs in production of animal feeds'" Operational Research Quarterly, vol 25, no. 1, pp. 19-26, 1974

[13] L. W. Swanson, and J. G. Woodruff, "A Sequential Approach to the Feed-Mix Problem," Operations Research, vol. 12, no. 1, pp. 89-109, 1964.

[14] E. Thomson, & J. Nolan, "UNEForm: A powerful feed formulation spreadsheet suitable for teaching or on-farm formulation," Animal Feed Science and Technology, vol. 91, pp. 233-240, 2001.

[15] D. Sirisatien, G. R. Wood, M. Dong, and P. C. H. Morel, "Two aspects of optimal diet determination for pig production: efficiency of solution and incorporation of cost variation," Journal of Global Optimum, vol. 43, pp. 249-261, 2009.

[16] C. Zioganas, The determination of viable, parity and optimum sizes of family-type sheep farms in the Epirus Region of Greece. Unpublished PhD Thesis, Wye College-University of London, 1981.

[17] W. B. Roush, R. H. Stock, T. L. Cravener, & T. H. D'Alfonso, "Using chance-constrained programming for animal feed formulation at Agway,"

Interfaces, The Institute of Management Sciences, vol. 24, pp. 53-58, 1994.

[18] A. Prékopa, "Probabilistic programming," in Handbooks in OR and MS, vol. 10, Elsevier Science, 2003, pp. 267-351.

[19] C. V. D. Panne, and W. Popp, "Minimum-cost cattle feed under probabilistic protein constraints," Management Science, vol. 9, no. 3, pp. 405-430, 1963.

[20]. Kononoff PJ, Snow DD, Christiansen. Drinking Water for Dairy Cattle. Large Dairy Herd Management. 3rd ed. Champaign, IL: FASS.; 2017.

[21]. Murphy MR. Water metabolism of dairy cattle. J Dairy Sci. 1992;75:326e333.

[22]. Chapman CE, Wilkinson-Stone P, Murphy MR, et al. Technical note: evaluating nuclear magnetic resonance spectroscopy for determining body composition in Holstein dairy calves using deuterium oxide dilution methods. J Dairy Sci. 2017;100:2807e2811.

[23]. NRC. Nutrient Requirements of Dairy Cattle. 7th Rev. Ed. National Acad. Press; 2001.

[24] Harvatine KJ. Lipid and Fat Nutrition. Large Dairy Herd Management. 3rd ed. Champaign, IL: FASS; 2017.

[25] Goff JP. Mineral Nutrition. Large Dairy Herd Management. 3rd ed. Champaign, IL: FASS; 2017.

[26] Nutrition and feeding of dairy cattle Peter S. Ericksona , Kenneth F. Kalscheurb :: General Base Paper for Nutritional Requirement.

[27] https://www.gams.com/ :: General Algebraic Modeling system.

[28] J. Prisˇenk J, K. Pazˇek, C. Rozman, J. Turk, M. Janzˇekovicˇ and A. Borec. "Application of weighted goal programming in the optimization of rations for sport horses", Journal of Animal and Feed Sciences, vol. 22, pp. 335–41, 2013.

[29] P. Saxena and N. Khanna. "Formulation and Computation of Animal Feed Mix: Optimization by Combination of Mathematical Programming", Advances in Intelligent Systems and Computing, vol. 337, pp. 621-629, 2015.

[30] National Research Council. Nutrient requirements of Dairy Cattle. 7th Rev. ed. Natl. Acad. Sci., Washington, DC, 2001.

[31] P. Saxena and N. Khanna. "Computation of cattle feed mix by using priority function: Weighted Goal Programming", published in Computing for Sustainable Global Development(9th INDIAcom-2015), pp. 1595-1600, 2nd International Conference On Computing for sustainable global development INDIAcom-2015, BVICAM, New Delhi, March 11-13, 15.

[32] P. Saxena and N. Khanna. "Optimization of Dairy Cattle Feed by Nonlinear Programming", published in Computing for Sustainable Global Development(9th INDIAcom-2015), pp. 1579-1584, 2nd International Conference On Computing for sustainable global development INDIAcom-2015, BVICAM, New Delhi, March 11-13, 15.

[33] P.R. Tozer. "Least cost ration formulations for Holstein dairy Heifers by using linear and stochastic programming", Journal of Dairy Science, vol. 83, pp. 443-451, 2000.

[34] I.U. Udo, C.B. Ndome and P.E. Asuquo. "Use of Stochastic programming in least-cost feed formulation for African catfish (Clarias gariepinus) in semiintensive culture system in Nigeria",

Journal of Fisheries and Aquatic Science, vol. 6, pp. 447–55, 2011.

[35] V.O. Oladokun and A. Johnson. “Feed formulation problem in Nigerian poultry farms: a mathematical programming approach”, American Journal of Scientific and Industrial Research, vol. 3(1), pp. 14–20, 2012.

[36] GENERAL ALGEBRAIC MODELING SYSTEM (GAMS) Michael R. Bussieck & Alex Meeraus GAMS Development Corporation Washington D.C., U.S.A. {MBussieck,AMeeraus}@gams.com.

[37] Rimmi Anand, Divya Aggarwal & Vijay Kumar (2017) A comparative analysis of optimization solvers, Journal of Statistics and Management Systems, 20:4, 623-635.

[38] R. Fourer, Using AMPL with Guroi to apply optimization models efficiently and reliably, In: Gurobi Optimizer Solution Seminar, pp. 1-52, 2011, Tokyo.

[39] J. Renewable Sustainable Energy 11, 046301 (2019).

[40] Zonghao Gu, Ed Rothberg, Bob Bixby,(July,2008) , Gurobi Optimizer.

[41] Youdong Lin & Linus Schrage (2009) The global solver in the LINDO API, Optimization Methods and Software, 24:4-5, 657-668.

[42] Stuart Smith, Leon Lasdon, (1992) Solving Large Sparse Nonlinear Programs Using GRG. ORSA Journal on Computing 4(1):2-15.

[43] T. Berthold, J. Farmer, S. Heinz, and M. Perregaard, Paralleization of the FICO Xpress-Optimizer, In: International Congress on Mathematical Software, pp. 251-258, 2016, Cham.

[44] Github link: https://github.com/CODERGARV/Research_work_sem6 .