Linear Algebra Practice Questions

1 Types of Solutions and their Conditions

Question 1.1

Consider the system of linear equations:

$$x + 2y - z = 4 \quad (1)$$
$$2x + 4y - 2z = k \quad (2)$$
$$3x + y + z = 5 \quad (3)$$

(a) For what value(s) of k does this system have:

- No solution
- Infinitely many solutions
- A unique solution

(b) Explain your reasoning for each case.

Question 1.2

Consider the following three systems of equations:

System A:

$$x + y = 3 \quad (4)$$
$$2x + 2y = 6 \quad (5)$$
$$3x + 3y = 9 \quad (6)$$

System B:

$$x + y = 3 \quad (7)$$
$$2x + 2y = 6 \quad (8)$$
$$3x + 3y = 10 \quad (9)$$

System C:

$$x + y = 3 \quad (10)$$
$$x - y = 1 \quad (11)$$

For each system, determine:

- Does it have no solution, a unique solution, or infinitely many solutions?
- If a system does not have solution, what changes would you make to make the system consistent?
- If the system has solutions, find them.

2 Homogeneous Systems and their Solutions

Question 2.1

Consider the homogeneous system of linear equations:

$$x + 2y - z = 0 \tag{12}$$
$$2x + 4y - 2z = 0 \tag{13}$$
$$x + y + z = 0 \tag{14}$$

(a) Without actually finding the solution determine if the solution have trivial solution or non trivial solution. If the solution is trivial change the system in such a way that the new system has non trivial solution or vice versa.

(b) Solve the system completely. Does it have only the trivial solution or are there non-trivial solutions?

(c) If a homogeneous system has more unknowns than equations, what can you say about its solutions? Explain with an example.

3 Gaussian Elimination and LU Decomposition

Question 3.1

Solve the following system of linear equations using **both methods**:
System:

$$2x + y - z = 8 \tag{15}$$
$$-3x - y + 2z = -11 \tag{16}$$
$$-2x + y + 2z = -3 \tag{17}$$

Part A: Gaussian Elimination Method

- Write the augmented matrix
- Apply row operations step-by-step to convert to upper triangular form
- Clearly show each row operation you perform (e.g., $R_2 \to R_2 + \frac{3}{2}R_1$)
- Use back substitution to find x, y, and z

- Show all your work

Part B: LU Decomposition Method

- Write the coefficient matrix A
- Find the L (lower triangular) and U (upper triangular) matrices such that $A = LU$
- Solve $Ly = b$ by forward substitution (where b is the right-hand side vector)
- Solve $Ux = y$ by backward substitution
- Show all steps clearly