Practice Questions - Set A

September 2, 2025

Question 1: Given the matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{pmatrix}$$

Find the number of Pivot columns and Free Variables. Also find the Rank and Nullity.

Question 2: For the matrix:

$$B = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 1 & 2 & 5 \end{pmatrix}$$

perform Gaussian elimination and find the number of pivot columns and the rank of the matrix.

1. A: Rank = 3, Pivot columns = 3.
2. B: Rank = 2, Pivot columns = 2.
3. C: Rank = 1, Pivot columns = 1.
4. D: Rank = 2, Pivot columns = 1.

Question 3: If a matrix has n columns and r pivot columns, what is the dimension of the null space of the matrix?

1. A: r
2. B: n
3. C: n r
4. D: r 1

Question 4: Given the matrix:

$$C = \begin{pmatrix} 3 & 4 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 4 & 6 & 8 & 3 \end{pmatrix}$$

find nullity and the number of free variables after Gaussian elimination.

1. A: 1
2. B: 2
3. C: 3
4. D: None

Question 5: Solve the system of Linear equations using LU decomposition $\begin{aligned} 2x + 3y + z &= 9, \\ 4x + 7y + 5z &= 23, \\ 6x + 18y + 19z &= 72. \end{aligned}$

Question 6: For the matrix:

$$F = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

find the null space of F by solving Fx = 0.

1. A: Null space is spanned by $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$.
2. B: Null space is spanned by $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.
3. C: Null space is spanned by $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.
4. D: Null space is spanned by $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

Question 7: Given the system of equations:

$$Ax = 0, A = \begin{pmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 2 & 6 & 8 \end{pmatrix}$$

find the dimension of the null space.

1. A: 1
2. B: 2
3. C: 3
4. D: 0

Question 8: For the matrix:

$$G = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{pmatrix}$$

determine the basis for the null space of G.

1. A: Null space is spanned by $\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.
2. B: Null space is spanned by $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.
3. C: Null space is spanned by $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.
4. D: Null space is spanned by $\begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.

Question 9: If the null space of a matrix A has dimension 3 and the rank of A is 2, how many variables does A have?

1. A: 3
2. B: 4
3. C: 5
4. D: 6

Question 10: Let $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ be two vectors in $\mathbb{R}^3$. Which of the following statements about the set of all affine combinations of v_1 and v_2 is true?

1. A: The set forms a vector space.
2. B: The set forms an affine space, but not a vector space.
3. C: The set is a subspace of $\mathbb{R}^3$.
4. D: The set forms a hyperplane.

Question 11: Which of the following is true about an affine space in $\mathbb{R}^3$?

1. A: It contains the zero vector.

2. B: It is closed under addition.
3. C: It is not closed under addition.
4. D: It is always a subspace.

Question 12: Let $S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x + y + z = 1 \right\}$ be a set in $\mathbb{R}^3$. Which of the following statements is true?

1. A: S forms a vector space.
2. B: S forms an affine space.
3. C: S forms a subspace.
4. D: S is not closed under scalar multiplication.

Question 13: Consider the set:

$$S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x + y = 0 \right\}$$

Which of the following is true?

1. A: S is a subspace of $\mathbb{R}^3$.
2. B: S is not a subspace of $\mathbb{R}^3$ because it does not contain the zero vector.
3. C: S forms an affine space.
4. D: S is closed under scalar multiplication.

Question 14: Consider the system:

$$\begin{cases} x + 2y - z + w = 3 \\ 2x + 4y - 2z + 2w = 6 \\ x + 2y + z - w = 1 \end{cases}$$

The solution set forms:

1. A: A shifted line (1-dimensional affine space)
2. B: A shifted plane (2-dimensional affine space)
3. C: A single point
4. D: The empty set (inconsistent system)

Question 15: The general solution to the system $Ax = b$ where:

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix}$$

The solution set is:

1. A: A shifted line through the origin
2. B: A shifted plane not passing through the origin
3. C: A shifted line not passing through the origin
4. D: A single point

Question 16: Consider the non-homogeneous system:

$$\begin{cases} x_1 + 2x_2 + x_3 = 2 \\ 2x_1 + 4x_2 + 3x_3 = 5 \\ x_1 + 2x_2 + 2x_3 = 3 \end{cases}$$

Which geometric object best describes the solution set?

1. A: A line parallel to the vector $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$
2. B: A plane parallel to the x_2-axis
3. C: A line parallel to the vector $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$
4. D: A point in 3-dimensional space

Question 17: The system of equations:

$$\begin{cases} 2x - y + 3z = 1 \\ 4x - 2y + 6z = 2 \\ x + y - z = 0 \end{cases}$$

has a solution set that is:

1. A: A line in $\mathbb{R}^3$
2. B: A plane in $\mathbb{R}^3$
3. C: The empty set

4. D: A single point

Question 18: For the matrix:

$$H = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{pmatrix}$$

find the column space of H.

1. A: The column space is spanned by $\begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix}$.
2. B: The column space is spanned by $\begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 7 \\ 10 \end{pmatrix}$.
3. C: The column space is spanned by $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$.
4. D: The column space is spanned by $\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$.

Question 19: Consider the matrix:

$$I = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

determine the rank and column space of I.

1. A: Rank = 3, Column space = $\mathbb{R}^3$.
2. B: Rank = 2, Column space = $\mathbb{R}^2$.
3. C: Rank = 2, Column space = $\mathbb{R}^1$.
4. D: Rank = 1, Column space = $\mathbb{R}^2$.

Question 20: For the matrix:

$$J = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 6 & 7 \end{pmatrix}$$

determine if the vector $v = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ lies in the column space of J.

1. A: Yes, it lies in the column space.
2. B: No, it does not lie in the column space.
3. C: Yes, it can be expressed as a linear combination of the columns.
4. D: No, it cannot be expressed as a linear combination of the columns.

Question 21: Consider the set $S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} : x + y + z = 0 \right\}$ in $\mathbb{R}^3$. Which of the following statements is true?

1. A: S forms a vector space.
2. B: S is not closed under addition.
3. C: S forms a vector space but is not closed under scalar multiplication.
4. D: S does not contain the zero vector.

Question 22: Which of the following is a valid example of a vector space?

1. A: A set of Polynomials with degree at most 5; over field of scalars $\mathbb{R}$
2. B: The set of all non-negative real numbers over field of scalars $\mathbb{R}$
3. C: The set of all integers over field of scalars $\mathbb{R}$
4. D: The set of all vectors in $\mathbb{R}^2$ where the first component is always positive.

Question 23: Consider the set $S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \right\}$ in $\mathbb{R}^3$. Which of the following is true regarding the span of S?

1. A: The span of S is $\mathbb{R}^3$.
2. B: The span of S is a one-dimensional subspace of $\mathbb{R}^3$.
3. C: The set S forms a basis for $\mathbb{R}^3$.
4. D: The span of S is the line through the origin in $\mathbb{R}^3$.

Question 24: Consider the set $T = \{p(x) \in P_3 : p'(0) = 0 \text{ and } p''(1) = 0\}$ where P_3 is the space of polynomials of degree at most 3. Is T a subspace of P_3?

1. A: Yes, T is a subspace with dimension 2
2. B: Yes, T is a subspace with dimension 3

3. C: No, T is not closed under addition
4. D: No, T does not contain the zero polynomial

Question 25: Let $M_{3\times 2}$ be the space of 3×2 matrices. Consider the subset:

$$U = \left\{ \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} : a + d = 0,\ b - c = 0 \right\}$$

Which statement is true?

1. A: U is a subspace of dimension 4
2. B: U is a subspace of dimension 3
3. C: U is not a subspace because it's not closed under scalar multiplication
4. D: U is an affine space but not a vector space

Question 26: Consider the set $V = \{(x, y, z, w) \in \mathbb{R}^4 : x + 2y - z = 0, 2x + 4y - 2z = 0\}$. What is the dimension of V?

1. A: 1
2. B: 2
3. C: 3
4. D: 4

Question 27: Consider the vector space P_4 of polynomials of degree at most 4. Let W be the subspace of polynomials $p(x)$ such that $p(1) = p(-1) = 0$. What is $\dim(W)$?

1. A: 2
2. B: 3
3. C: 4
4. D: 5

Question 28: Let P_2 be the set of all polynomials of degree at most 2. Which of the following sets is a subspace of P_2?

1. A: The set of all polynomials $p(x) = a + bx$ where $a, b \in \mathbb{R}$.
2. B: The set of all polynomials $p(x) = a + bx^2$ where $a, b \in \mathbb{R}$.
3. C: The set of all polynomials of the form $p(x) = a + bx + cx^2$ where $a, b, c \in \mathbb{R}$.

4. D: The set of all polynomials of the form $p(x) = ax + bx^2$ where $a, b \in \mathbb{R}$.

Question 29: Consider the set of polynomials $S = \{p(x) \in P_2 : p(0) = 0\}$. Which of the following is true?

1. A: S is a subspace of P_2.
2. B: S is not closed under scalar multiplication.
3. C: S is not closed under addition.
4. D: S is a subset of P_1.

Question 30: Let P_2 be the set of all polynomials of degree at most 2. Which of the following sets is not a subspace of P_2?

1. A: The set of all polynomials $p(x) = a + bx + cx^2$ where $a, b, c \in \mathbb{R}$.
2. B: The set of all polynomials $p(x) = a + bx$ where $a, b \in \mathbb{R}$.
3. C: The set of all polynomials $p(x) = 0$ where $a \in \mathbb{R}$.
4. D: The set of all polynomials $p(x) = ax^2$ where $a \in \mathbb{R}$.
5. E: None

Question 31: Given the vector space P_2 and the set $S = \{1 + x, 1 + x + x^2\}$, determine if S forms a basis for P_2.

1. A: Yes, S is linearly independent and spans P_2.
2. B: No, S is linearly dependent.
3. C: Yes, but S does not span P_2.
4. D: No, S does not span P_2.

Question 32: Let $v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, and $v_3 = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$ in $\mathbb{R}^3$. Which of the following sets forms a basis for the subspace spanned by these vectors?

1. A: $\{v_1, v_2\}$
2. B: $\{v_2, v_3\}$
3. C: $\{v_1, v_3\}$
4. D: $\{v_1, v_2, v_3\}$