Join the class

Quick Revision

In the last class you read:

- Divisible Rules
- Prime Factorization
- LCM and GCD
- Relatively Prime

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Logic and Propositions

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CSA102 Mathematics-1

Logics


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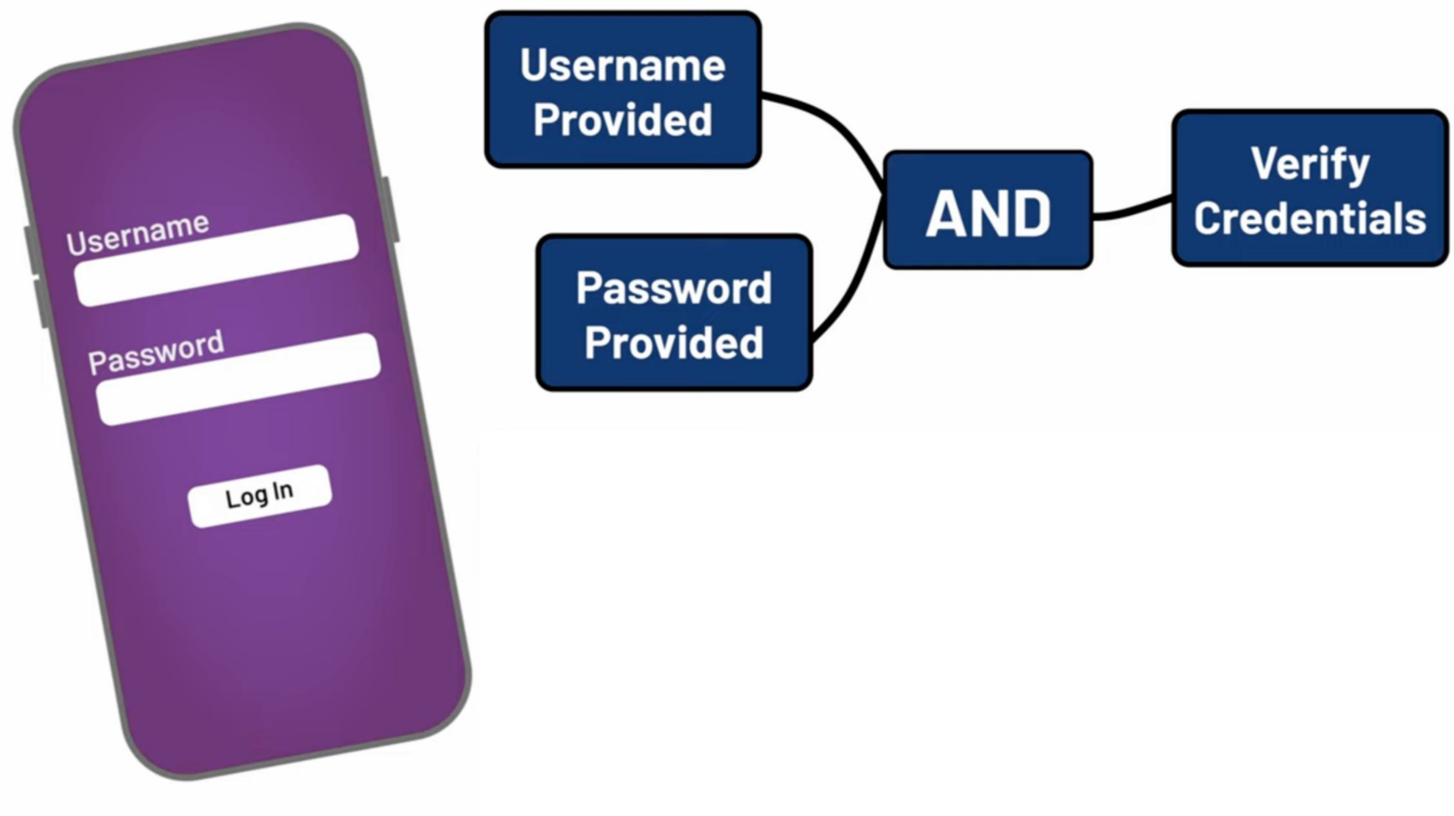
Log In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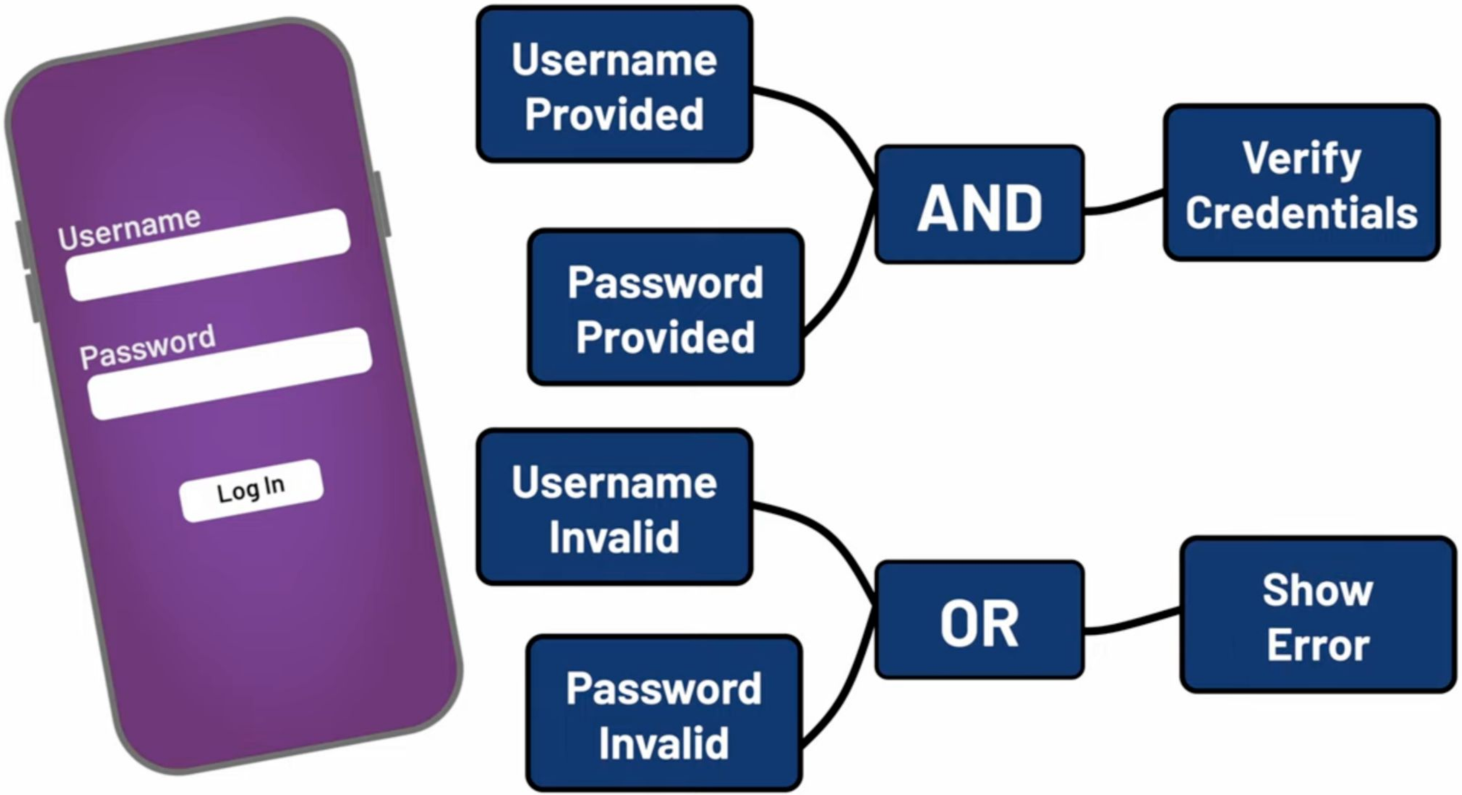
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Logics

- Logics in mathematics is the formal study of reasoning and making decisions.
- Here we will read rules and regulations that allow mathematics to make valid deduction. In today's class we will read
    - Propositions
    - Logical Connectives:
        - Negation
        - Conjunction
        - Disjunction
        - Exclusive OR
        - Implication
    - Negation of operators

Logic

If

Then

Not

And

Or

Propositions

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Statement (Proposition)

A *Statement* is a sentence that is either **True** or **False**

Examples:

$2 + 2 = 4$ True

$3 \times 3 = 8$ False

787009911 is a prime

Non-examples:

$x+y>0$

$x^2+y^2=z^2$

They are true for some values of x and y
but are false for some other values of x and y.

Example

Decide which of the following is a proposition or not.

• The number 4 is even and less than 12.	Proposition (T)
• New Delhi is the capital of India.	Proposition (T)
• Covid19 is a Bacteria.	Proposition (F)
• 5 is an even number.	Proposition (F)
• I am lying.	Not a Proposition. It’s a Paradox.

Proposition

1) Simple - a single, complete, statement -
    a) New Delhi is the capital of India.
    b) Covid19 is a Bacteria.
    c) 5 is an even number.

2) Compound - two or more conjoined statements -
    a) The number 4 is even ***and*** less than 12.
    b) ***Either*** it is a Maths class, ***or*** it is a programming class.
    c) ***If*** Husky is a dog, ***then*** Husky is a mammal.

Compound Proposition

Consider a statement - It is hot ***and*** sunny.

p = “it is hot”   q = “it is sunny”

Examples: It is hot ***and*** sunny

It is ***not*** hot ***but*** sunny

It is ***neither*** hot ***nor*** sunny

Logical connectives / operators

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Negation

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Negation : Definition and Notation

- When **P** is a proposition, the negation of **P** is denoted by :
  - **¬P or ~P** and is read **"not P"**.
  - The negation of **P** has the **opposite truth value** from P.

Truth Table for Negation

P	¬P
True	False
False	True

Example

P : Pakistan is independent then ¬P is

-

Example

P : Pakistan is independent then ¬P is

- Pakistan is not independent

Conjunction (AND)

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Conjunction (AND)

If **P** and **Q** are two statements, the conjunction of **P** and **Q** is denoted by:

- **P∧Q** and is read, **"P and Q**".
- The conjunction of **P** and **Q** is true only when each of **P** and **Q** is true.

Truth table:

P	Q	$P \wedge Q$
True	True	True
True	False	False
False	True	False
False	True	False

Example

Is the following proposition true: "All rectangles are squares, and there is no life outside Earth."?

Solution:

Lazy Evaluation

```python
if 2+2==5 and 5%0==0:
    print("True")
else:
    print("False")
```

Disjunction (OR)

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Disjunction (OR)

- The Logical **OR** (denoted by $\vee$) of two statements is true if and only if ***at least one*** of them is true.
- **Ex:** All squares are rectangles OR all rectangles are squares.

Truth table:

P	Q	P∨Q
True	True	True
True	False	True
False	True	True
False	False	False

Example:

$P(n)$:n is a multiple of 4 $\lor$ n is a multiple of 6

Determine whether the disjunction $P(n)$ is true or false for the following values of n:

1. $n=8$
2. $n=9$
3. $n=12$
4. $n=5$

If-Then/ Implication

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Passing the Exam

Statement- “If I teach Maths, then all students will pass.”

Let:

- T: “I teach Maths.”
- P: “All students will pass.”

Under what condition(s) will this proposition be considered false?

If-then

Formally, the statement ***P*⟹*Q*** (which we read "***P* implies *Q***" or "**If *P* then *Q***"}) is true whenever *P* is true or *Q* is false. In other words, P ⟹ Q is false if and only if *P* is true but *Q* is false.

P	Q	If P then Q
T	T	T
T	F	F
F	T	T
F	F	T

Example:

Is **P⟹Q** true or false?

- P: "n is divisible by 6 and n is divisible by 8"
- Q: "n is divisible by 12"

Exclusive OR

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Light Switches

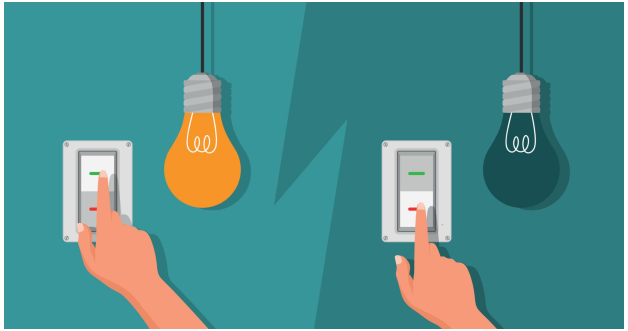
Imagine a room with two light switches controlling the same light. What is the condition for the light to be ON?

The light is on if exactly one of the switches is on.

Switch 1: ON (1)

Switch 2: OFF (0)

Light: ON



Truth table:

P	Q	P⊕Q
True	True	False
True	False	True
False	True	True
False	False	False

Example:

Given two sets A={2,3,5,7,11} and B={3,5,8,13}:

- Compute A⊕B.

Negations of Operations

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Example

What is the negation of the statement "Anna speaks French and German"?

Note: Negation of an AND - statement is an Or of negations.

De Morgan's Law

- $\neg(S \wedge T) = \neg S \vee \neg T$
  - The negation of an AND-statement is an OR of negations.

- $\neg(S \vee T) = \neg S \wedge \neg T$
  - The negation of an Or -statement is an AND of negations.

Example:

What is the negation of the statement "Either he will do it on time, or he won't get paid"?

We negate each part of this statement, and then connect them with an AND: "He will not do it on time and will still get paid".Since the only way to falsify the statement $T \Longrightarrow U$ is to have T true and U false, the negation of that statement is

$$\neg(\mathbf{T} \Rightarrow \mathbf{U}) = \neg\mathbf{T} \wedge \neg\mathbf{U}$$

Problem

Use De Morgan’s law to simplify the following code:

```
if      ((x <= 5) and (y >= 7)):
```

By De Morgan's law, the negation of an AND is an OR of negations:

```
if ((x > 5) or (y < 7)):
```

Quiz Time!

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Key takeaways:

Today we learnt:

- Propositions
- Logical Connectives:
  - Negation
  - Conjunction
  - Disjunction
  - Exclusive OR
  - Implication
- Negation of operators

Provide feedback