Logic and Propositions

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CSA102 Mathematics-1

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Quick Revision

In the last class you read:

- Propositions
- Logical operators: OR, AND, NOT, XOR, If-then

Example:

Draw the truth table for following two expressions:

- $P \vee \neg P$
- $\neg P \wedge P$

Tautologies, Contradiction and Contingency

Tautology: A compound proposition that is always true irrespective of the truth values of propositions that occur in it.

Contradiction: A compound proposition that is always false.

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Negations of Operations

Example

What is the negation of the statement "Ananya speaks Hindi and Ananya speaks Telugu"?

De Morgan's Law

- $\neg(P \wedge Q) = \neg P \vee \neg Q$
- The negation of an AND-statement is an OR of negations.

Example:

What is the negation of the statement "Priya will be available in office hours or Badal will be available in office hours".

Second De Morgan's Law

- $\neg(P \lor Q) = \neg P \land \neg Q$
- The negation of an OR-statement is an AND of negations.

Example

What is an equivalent statement to ''not (A and not B)''?

- not A and B
- A or not B
- A and not B
- not A or B

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Example:

What's equivalent to ''not (A and not (B or not C))''?

- not A or not B and not C
- not A or not B or not C
- not A or B or not C
- not A or B and not C

Example:

What's equivalent to ''not (A and not (B or not C))''?

- not A or not B and not C
- not A or not B or not C
- not A or B or not C
- not A or B and not C

Logical Equivalence

Logical Equivalence :

- Compound propositions having **same truth values** in all possible cases are said to be **logically equivalent**.
- We use ≡ (read as equivalent to) sign to indicate that two expressions are equal to each other.

Example:

Is **p⟹q** and ¬**p** ⋁ **q** are logically equivalent ?

Example:

Is **p⟹q** and **¬p ∨ q** are logically equivalent ?

p	q	¬p	¬p ∨ q	p⟹q
T	T	F		
T	F	F		
F	T	T		
F	F	T		

Example:

Check if p $\oplus$ q is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$

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Check if p $\oplus$ q is equivalent to $(p \wedge \neg q) \vee (\neg p \wedge q)$

Example:

Quiz Time!

Quantifiers

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Example

Aryan says that all white lions weigh more than 100kg. What statement could Ritik say to contradict him?

Example

Consider the following statements about a number x:

- x is a multiple of 2;
- x is a multiple of 3;
- x is a multiple of 6.

Is it possible that for some integer x

A. None of them are true?
B. Exactly one of them is true?
C. Exactly two of them are true?
D. All three statements are true?

Quantifiers:

- Quantifiers: which quantifies over a range of values
- Two types of quantifiers:
    - Universal quantifier (All/every)
        - Ex: All the apples in this basket are red
    - Existential quantifier (At least one)
        - Ex: There is at least one orange in this box.

Example:

Suhana says that in every region there is a town where all inhabitants are happy. Bob wants to say that Suhana is wrong. Which of the following sentences should Bob say?

- There is a region where there is a town where all inhabitants are happy.
- In every region in all towns all inhabitants are happy.
- In every region there is a town where at least one inhabitant is unhappy.
- There is a region where in all towns at least one inhabitant is unhappy.

Example:

Alice says that all elephants are tall and heavy. What statement could Bob say to contradict her?

Example:

Mohit claims that every student in a group knows Hindi, English, or both. Which of the following sentences asserts that Mohit's statement is wrong (no more, no less)?

- All students know at most one of these two languages
- There is a student who does not know Hindi and does not know English
- There is a student who does not know Hindi or does not know English (or both)
- Every student who knows French does not know German

Quiz Time!

Key takeaways:

Today we learnt:

- Negation of operators
- Logical Equivalences
- Quantifiers

Provide feedback

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