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Number Theory

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CSA 102 Mathematics

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Join the class

Quick Revision:

In the last class you read:

- Divisibility rule of 2, 3, 4, 5, 8, 10
- How to find divisibility rule of any number
- Doing prime factorization
- Finding number of divisors using prime factors

Recap Question:

What is the largest 4 digit number exactly divisible by 88?

A) 9944

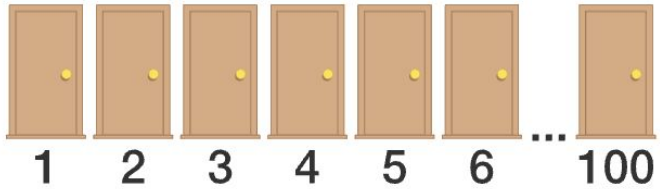
B) 9900

C) 9988

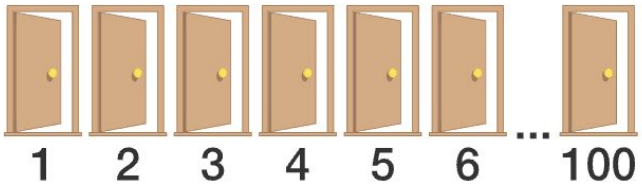
D) 9999

100 doors revisited

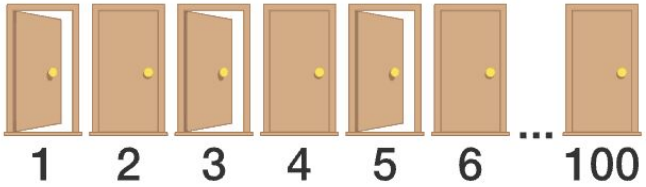
- Let's take another look at the hallway of **100 doors**.
- In the hallway of 100 doors, **100 people** numbered 1 to 100 are standing in a long hallway that has 100 closed doors also numbered 1 to 100:



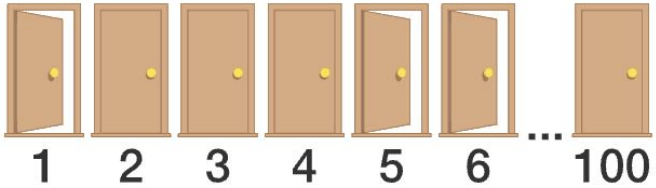
- **Person 1** walks down the hallway and opens every door:



- **Person 2** walks down the hallway and closes every door that is a multiple of 2:



- **Person 3** walks down the hallway and changes every door that is a multiple of 3. That is, if the door is open, they close it, and if it is closed, they open it:



- **Person 4** changes every door that is a multiple of 4, **Person 5** every door that is a multiple of 5, etc. This continues until all **100 people** have walked down the hallway and changed their doors.

  Q. What is the number of the first door changed by both Person 6 and Person 8?

  Ans: 14 / 24 / 48

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Today's Agenda:

- LCM
- Real life based problems on LCM
- GCD
- Real life based problems on GCD

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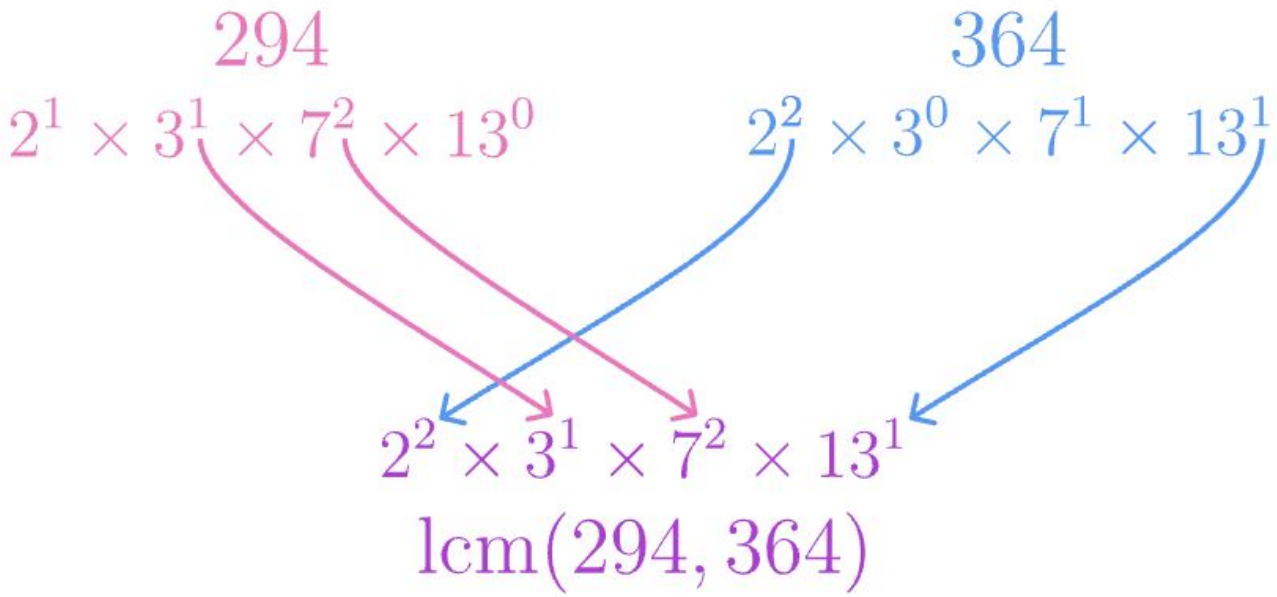
Lowest Common Multiple

Lowest Common Multiple

- **100 Doors solved:**
  - **Person 6** changes the doors that are multiples of 6:
    6, 12, 18, 24, 30, 36.....

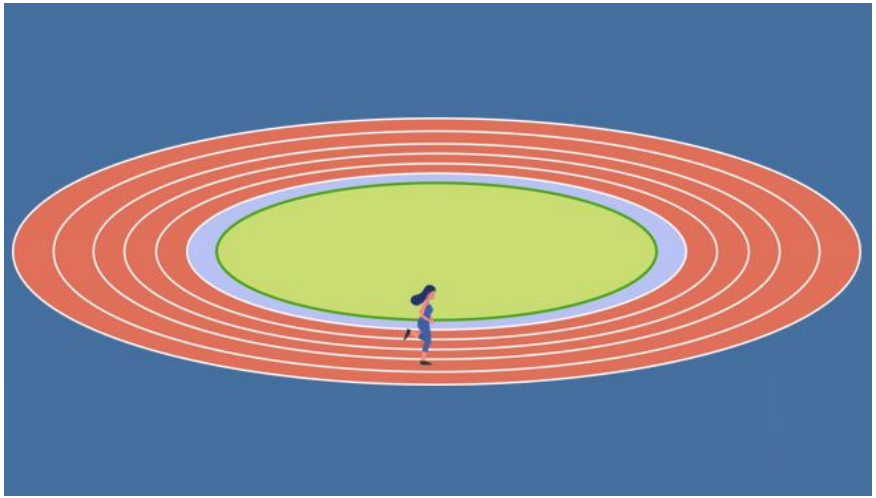
  - **Person 8** changes the doors that are multiples of 8:
    8, 16, 24, 32, 40.....

Finding lcm using prime factorization

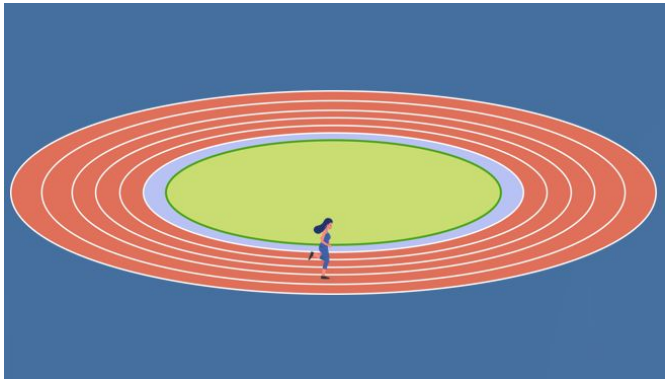


Example:

Q. Three runners Manshu, Pintu, and Dally who are running around a circular track can complete one lap in 250, 400, and 600 seconds, respectively. After how long will they meet at the starting point next if they start together?

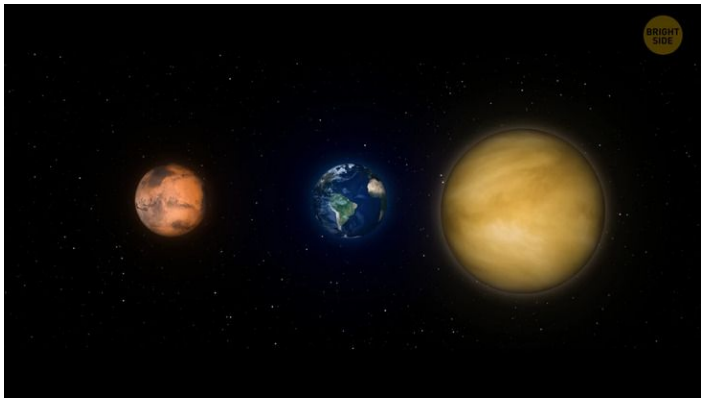


Example:



Example:

Earth rotates around the sun in 365 days and Venus rotates the sun in 730 days. Whenever Earth and Venus cross each other, a unique phenomenon occurs due to which the population of the olive ridley turtle reduces to half. Considering the initial population of Olive Ridley turtle is 10000 what will their population after 10 years.



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Quiz Time!

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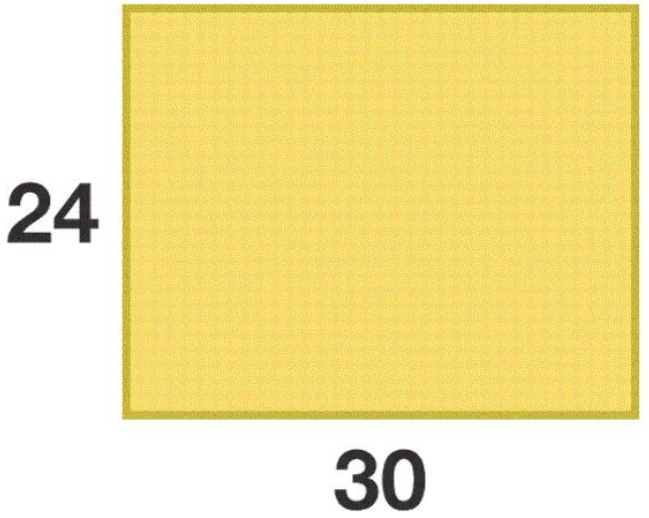
Greatest Common Divisor

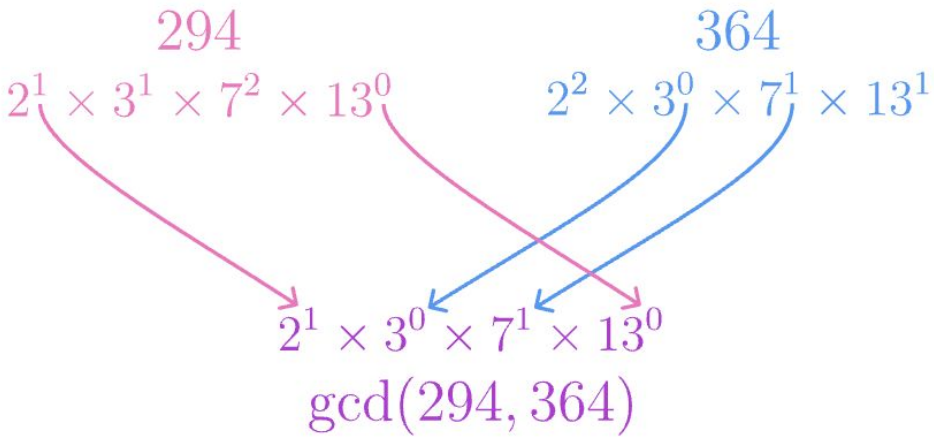
Example:

Q. If you want to tile a 24×30 rectangle with square tiles that are all the same size, what are the largest square tiles you can use?

Ans:



Finding GCD using prime factorization



Therefore, we have

$$\begin{aligned}\gcd(294, 364) &= 2^1 \times 3^0 \times 7^1 \times 13^0 \\ &= 2^1 \times 1 \times 7^1 \times 1 \\ &= 2^1 \times 7^1.\end{aligned}$$

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Example:

Q. If you want to reduce the fraction 30/24 to lowest terms, what number should you divide the numerator and denominator by?

Ans: 2 / 3 / 6 / 12

Example:

Q. Ramesh is managing 15th August parade. Three contingents, Indian Army with 54 soldiers, CRPF with 36 women soldiers and Indian air force with 48 soldiers are participating in parade. Now Ramesh has to decide what can be maximum number of rows so that all three contingents march in same number of rows. Can you help Ramesh?

Ans.

Example:

Q. Ramesh is managing 15th August parade. Three contingents, Indian Army with 54 soldiers, CRPF with 36 women soldiers and Indian air force with 48 soldiers are participating in parade. Now Ramesh has to decide what can be maximum number of rows so that all three contingents march in same number of rows. Can you help Ramesh?

Ans. 6

Relatively prime

If gcd(a,b) = 1, we say that a and b are relatively prime.

Ex: gcd(17,11) = 1

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Example:

Q. How many of the numbers 1, 2, 3 ,...., 100 are relatively prime to 101?

Ans:

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Example:

Q. How many of the numbers 1, 2, 3 ,...., 100 are relatively prime to 101?

Ans: 100

Relation between LCM and GCD

The product of the LCM and GCD of two numbers equals the product of the numbers themselves:

- LCM(12,15) × GCD(12,15) = 12×15
- 60×3 = 180, and 12×15 = 180

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Example:

You have two positive integers, A and B, with the following properties:

1. The Least Common Multiple (LCM) of A and B is 84.
2. The Greatest Common Divisor (GCD) of A and B is 6.
3. The sum of A and B is 60.

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Example:

Find sum of two positive integers x and y such that the LCM of x and y is 360 and their GCD is 15.

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Quiz Time!

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Key Takeaways:

In today's class we learnt:

- How to find LCM of 2 or 3 numbers
- How to find HCF of 2 or 3 numbers
- Relation between LCM and HCF