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# Number Theory

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**CSA 102 Mathematics**

# Join the class

# Quick Revision:

In the last class you read:

- Percentages
- Ratio and Proportion
- Mixtures and Allegations

# Example:

A dealer has 1000 kg sugar and he sells a part of it at 8% profit and the rest of it at 18% profit. The overall profit he earns is 14%. What is the quantity which is sold at 18% profit.

# NUMBER THEORY

# Example 2

Q. Given a cyclical pattern with three colors.
What color is 8 × 17?

Ans. Blue / Green / Orange.


| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

# Example 2

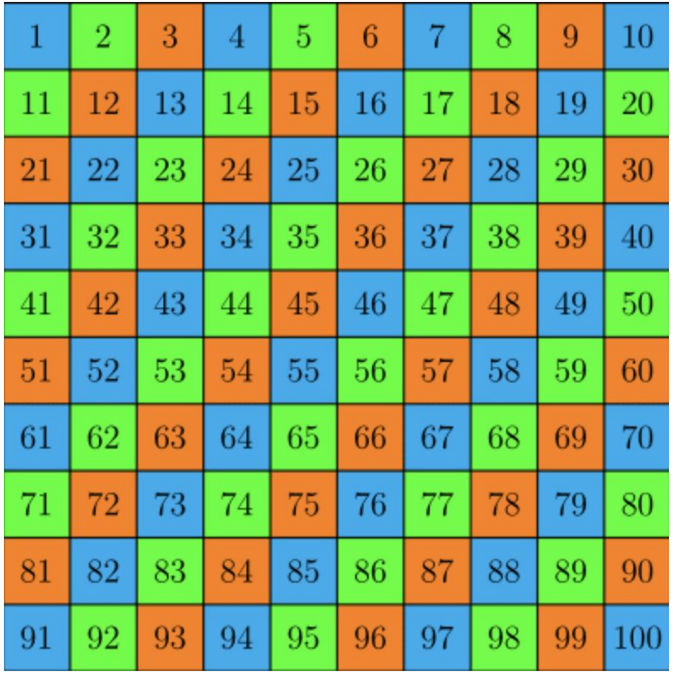
Q. Given a cyclical pattern with three colors. What color is $8 \times 17$?

Ans. Blue

$8 \times 17 = 136$

$136 \% 3 = 1$

Since numbers that leave a remainder of 1 when divided by 3 are blue, $8 \times 17$ is blue.

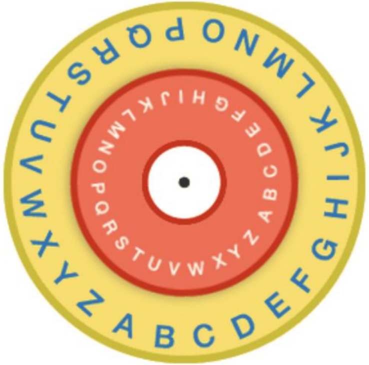


# What is Number Theory ?

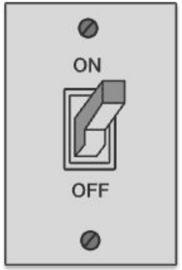
- Number Theory is about numbers themselves and how they relate to each other
- **Short-cut seeking** is a major skill you will be learning here

# Number Theory





| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |




A cycle of 2 values

A cycle of 3 values


A cycle of 7 values


A cycle of 12 values

# Agenda

- Divisibility rule of 2, 3, 4, 5, 9
- How to find divisibility rule of any number
- Doing prime factorization
- Finding number of divisors using prime factors

# Divisibility Rules

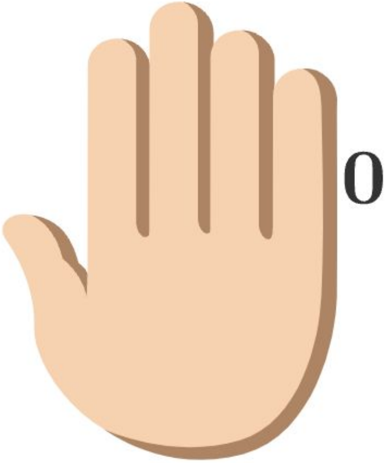
- **Divisible :**
  - We say a number n is divisible by m if the remainder is 0.
  - Example:
    - 12 is divisible by 4

- **Not-divisible:**
  - We say a number n is not divisible by m if the remainder is not 0 .
  - Example:
    - 12 is not-divisible by 5

# Example

Q. Your friend writes down a five-digit number, and then covers all digits except the last digit, which is a 0, with his hand.

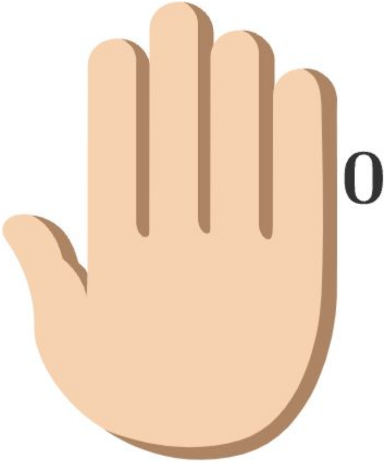
- Is this number divisible by 2?
- Is this number divisible by 4?
- Is this number divisible by 5?



# Example

Q. Your friend writes down a five-digit number, and then covers all digits except the last digit, which is a 0, with his hand.

- Is this number divisible by 2? Yes.
- Is this number divisible by 4? Not possible to be certain.
- Is this number divisible by 5? Yes.



# Divisibility by 2

- A number is divisible by 2 if its last digit is divisible by 2.

# Divisibility by 5

- A number is divisible by 5 if its last digit is 0 or 5.

# Divisibility by 10

- A number is divisible by 10 if its last digit is 0.

# Divisibility of 4

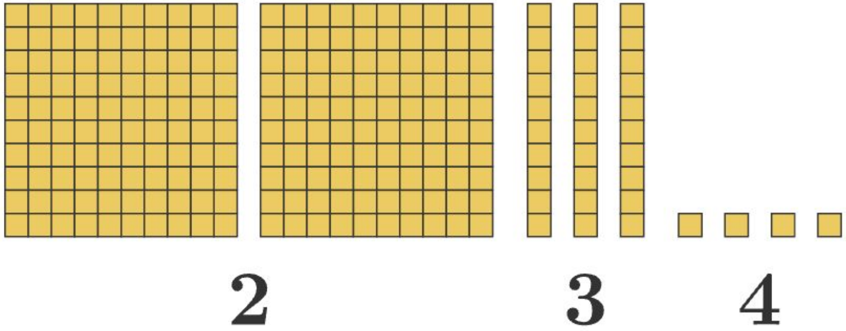
- What is divisibility rule of 4 ?

A number is divisible by 4 if last 2 digits of the number are divisible by 4.

**But why ?**

# Let’s Decode the magic technique

- Any number can be written as sum of hundreds, tens, and units.
  - Example: Check 234 is divisible by 4 or not



# Let’s Decode the magic technique

# Example:

Q. Determining whether 234 is divisible by 4.

Ans: There are 2 sets of 100 squares each, 3 sets of 10 squares each and 4 set of 1 square each.

- Dividing 100 by 4 leaves a remainder of 0.
- Dividing 10 by 4 leaves a remainder of 2
- Dividing 1 by 4 leaves a remainder of 1

The sum of all these remainders is $(2\times0) + (3\times2) + (4\times1) = 10$.
Dividing 10 by 4 leaves a remainder of 2, which corresponds to the remainder when 234 is divided by 4.

**Note:** Since every power of 10 greater than 10 itself is divisible by 4, the remainder when a number is divided by 4 is the same as the remainder when the last two digits of the number are divided by 4.

# Revisiting Divisibility Rule of 2,5,10

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. . . 10000s 1000s 100s | 10s 1s

These are all divisible by 4.

So the entire number is divisible by 4 if this part is divisible by 4.

. . . 10000s 1000s 100s 10s | 1s

These are all divisible by 2.

So the entire number is divisible by 2 if this part is divisible by 2.

. . . 10000s 1000s 100s 10s | 1s

These are all divisible by 5.

So the entire number is divisible by 5 if this part is divisible by 5.

. . . 10000s 1000s 100s 10s | 1s

These are all divisible by 10.

So the entire number is divisible by 10 if this part is divisible by 10.

# The magic technique

- To find out if a number n is divisible by another number m,
  - Do the division by m on each power of 10 in n separately, and
  - If there are remainders, add them together and determine whether this sum is divisible by m

# Example:

Q. A number is divisible by 8 if and only if the number formed by its last ________ digits are divisible by 8.

2 / 3 / 4 / 5

# Example:

Q. A number is divisible by 8 if and only if the number formed by its last ___3___ digits are divisible by 8.

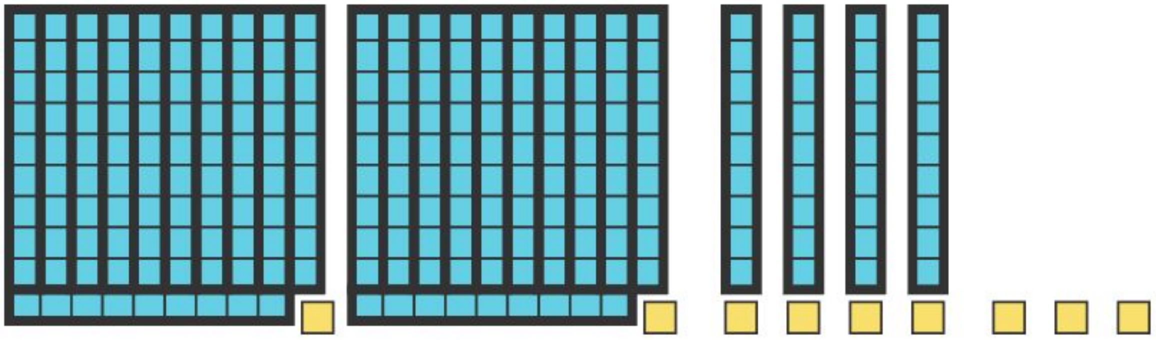
# Divisibility Rule of 3

# Divisibility Rule of 3

**Example :** Check Divisibility of 243 with 3?

$243 = (2\times100) + (4\times10) + (3\times1)$

Sum of remainders = 2 + 4 + 3 = Sum of digits



# Divisibility Rule of 3

A number is divisible by 3 if and only if the sum of its digits is divisible by 3.

# More Divisibility Rules: 9,11,125 etc

Will cover in Labs

# Key Takeaways

Till now we have learnt :

- Divisibility rule of 2, 3, 4, 5, 10
- How to find divisibility rule of any number

# Quiz Time!

# Prime Factorization

# Prime Factorization

- What are **Prime numbers** ?
  - Any number which is only divisible by 1 and the number itself.
- What are **factors** ?
  - Numbers that divide another number without leaving a remainder.
- What is **prime factorization** ?
  - Prime factorization is the process of expressing a number as a product of its prime factors.

# Example:

| | |
|---|---|
| 2 | 40 |
| 2 | 20 |
| 2 | 10 |
| 5 | 5 |
| | 1 |

Prime factorization of 40 $= 2 \times 2 \times 2 \times 5$

$= 2^3 \times 5$

# Fundamental theorem of Arithmetic

- By the fundamental theorem of arithmetic, any number n has a unique prime factorization.

# Example:

**Scenario:**

You have a pizza that is cut into 300 slices. You need to determine the number of all the possible group sizes that can share the pizza evenly, meaning no slices are left over.

# Finding number of divisors from Prime Factorization

| $n$ | Prime Factorization | # of Divisors |
|---|---|---|
| 20 | $2^2 \times 5^1$ | $3 \times 2 = 6$ |
| 30 | $2^1 \times 3^1 \times 5^1$ | $2 \times 2 \times 2 = 8$ |
| 50 | $2^1 \times 5^2$ | $2 \times 3 = 6$ |
| 60 | $2^2 \times 3^1 \times 5^1$ | $3 \times 2 \times 2 = 12$ |
| 90 | $2^1 \times 3^2 \times 5^1$ | $2 \times 3 \times 2 = 12$ |
| 150 | $2^1 \times 3^1 \times 5^2$ | $2 \times 2 \times 3 = 12$ |
| 360 | $2^3 \times 3^2 \times 5^1$ | ? |

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# Finding number of divisors from Prime Factorization

In general, if $n$ has the prime factorization

$$n = p_1^{k_1} \times p_2^{k_2} \times \cdots \times p_m^{k_m},$$

then $n$ has

$$(k_1 + 1) \times (k_2 + 1) \times \cdots \times (k_m + 1)$$

divisors.

# Example: Dividing the pizza

Ans.

QUESTION 1/1

100 people, numbered 1 to 100, are standing in a long hallway that has 100 closed doors also numbered 1 to 100:

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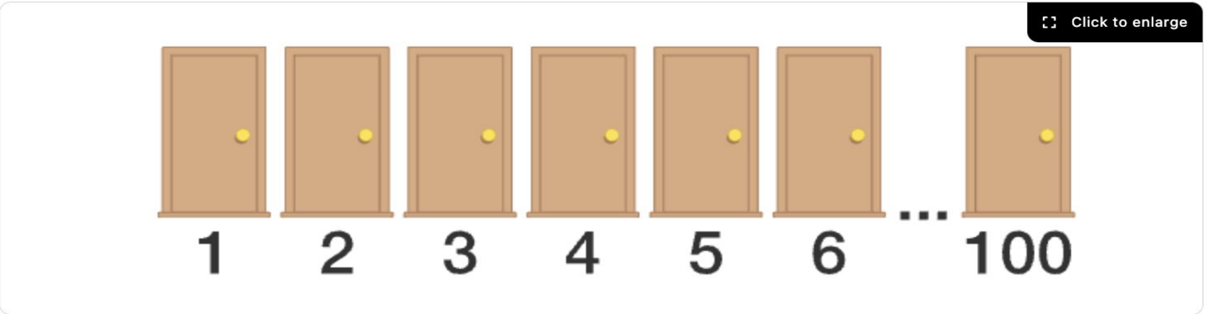
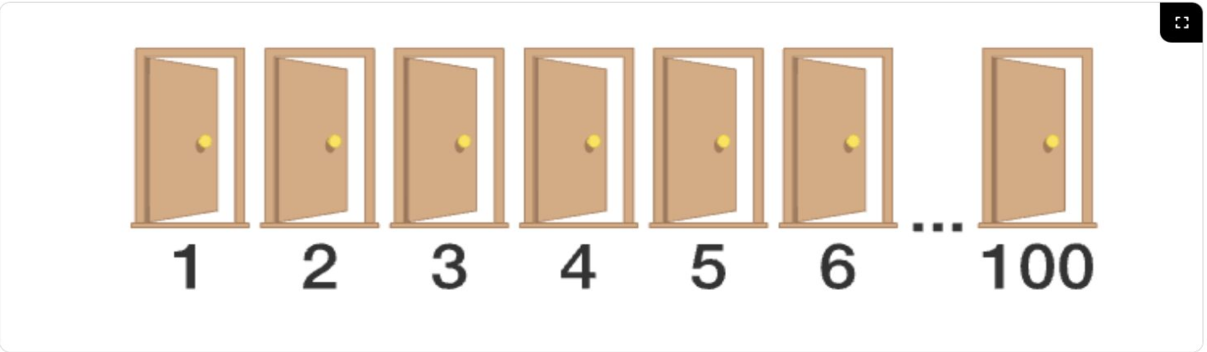
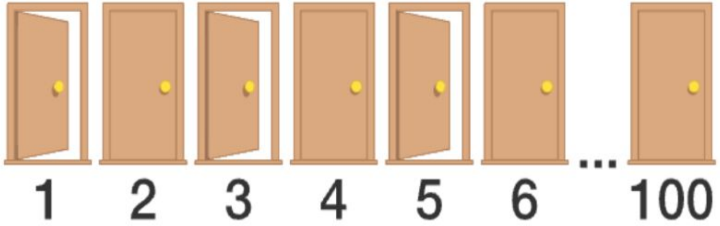


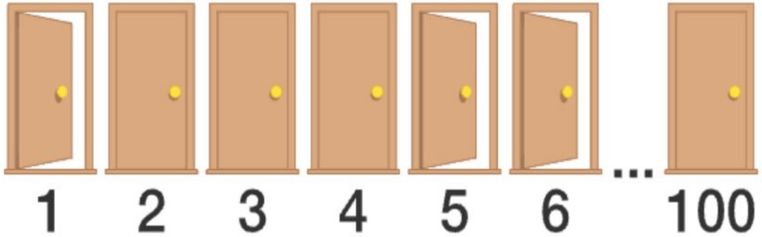

Person 1 walks down the hallway and **opens** every door:




Person 2 walks down the hallway and **closes** every door that is a multiple of 2:




Person 3 walks down the hallway and **changes** every door that is a multiple of 3. That is, if the door is open, they close it, and if it is closed, they open it:




Person 4 **changes** every door that is a multiple of 4, Person 5 every door that is a multiple of 5, etc. This continues until all 100 people have walked down the hallway and changed their doors.

# Example:

For how many times door no 88 is changed.

# Quiz Time!

# Key Takeaways

Today we learnt :

- Divisibility rule of 2, 3, 4, 5, 9
- How to find divisibility rule of any number
- Doing prime factorization
- Finding number of divisors using prime factors

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# Provide feedback