Seat No: _____ **Enrollment No: _____**

PARUL UNIVERSITY

FACULTY OF ENGINEERING & TECHNOLOGY

B.Tech. Winter 2021 - 22 Examination

Semester: 3 **Date: 13/10/2021**
Subject Code: 203191202 **Time: 2:00pm to 4:30pm**
Subject Name: Discrete Mathematics **Total Marks: 60**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

Q.1 Objective Type Questions (All are compulsory) (Each of one mark) **(15)**

1. If the truth value of $p \vee q$ is F then the truth value of $(\neg p \wedge \neg q)$ is _____.

2. The value of $1.0 + (\overline{0} + \overline{1})$ is _____.

3. The degree of an isolated vertex is _____ .

4. Cyclic group is an _____ group.

5. A vertex having no child is called a _____.

6. **If f(n+1) = $3^{f(n)/3}$ and f (0) =3, then f(2) is _____.**

7. A completely bipartite graph need not be a simple graph. T/F

8. **Let R_1 = {(1, 2) (1, 6) (2, 4) (3, 4) (3, 6) (3, 8)} and**

R_2 = {(2, x) (4, y) (4, z) (6, z) (8, x)}. Then $R_2 o R_1$ = _____.

9. The truth value of $p \wedge \neg p$ is not a tautology. T/F

10. If a relation *R* satisfies reflexive, symmetric and Transitive property then the relation is called _____ relation.

11. $p \wedge (p \vee q)$ is known as

(a) Distributive Law **(b)** Idempotent Law **(c)** Absorption Law **(d)** Domination Law

12. If the relation *R*on a set *A* is called a void relation if

(a) $R=AXA$ **(b)** $R=\emptyset$ **(c)** $R=\{(a,a)/a \in A\}$ **(d)** $R=R-1$

13. If a group satisfies the commutative property then it is known as

(a) Abelian Group **(b)** Symmetric group **(c)** Semi-group **(d)** Monoid

14. A graph, in which there is only an edge between a pair of vertices, is called a

(a) Simple graph **(b)** Pseudo-graph **(c)** multi-graph **(d)** weighted graph

15. The adjacency matrix of a simple graph is _____matrix.

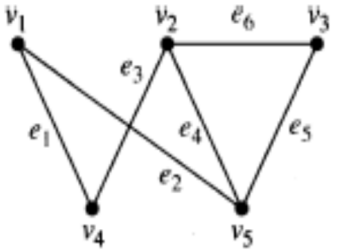
(a) Skew-Hermitian **(b)** Skew-symmetric **(c)** Hermitian **(d)** Symmetric

Q.2 Answer the following questions. (Attempt any three) **(15)**

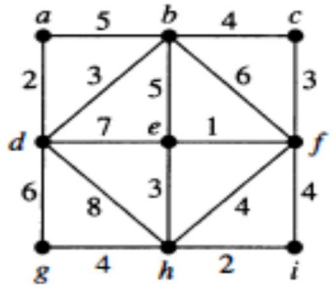
A) **Let $R = \{(1,b),(2,a),(2,c)\}$ be a relation from $A = \{1,2,3\}$ to $B = \{a,b,c\}$ and let $S = \{(a,y),(b,x),(c,y),(c,z)\}$ be a relation from B to C = $\{x,y,z\}$.**
Check if the composition relations SoR **and RoS can exist. Write the relation as set if they exists. If not, give the reason.**

A) Check whether the "greater than or equal to" ($\geq$) relation is a partially ordered set or not on the set of integers Z.

B) Find the adjacency and incidence matrix for the following graph



C) Use Kruskal's algorithm for finding minimum spanning tree for the given weighted graph.



Q.3 **(04+03)**

A) (1) If $\alpha = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 3\,1\,5\,4\,6\,2 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 2\,5\,1\,3\,6\,4 \end{pmatrix}$ are two element of symmetric group S_6, find using right permutation $\alpha\beta, \beta\alpha, \beta^2$ and α^{-1} .

(2) Let G be a group under usual multiplication. Show that $G = \{1, -1, i, -i\}$ is a cyclic group under usual multiplication.

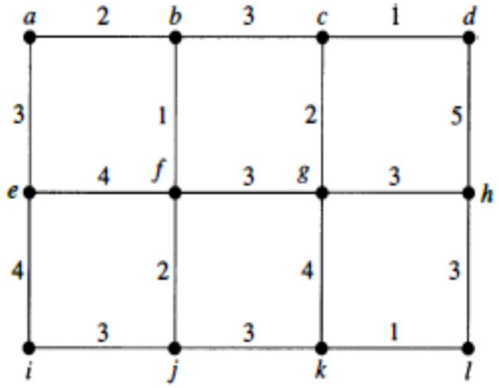
B) (1) Using mathematical induction prove the following **(05+03)**

$$1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

(2) With the help of truth table prove that $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$.

OR

B) Define Spanning Tree. Use Prim's algorithm for finding the minimum spanning tree for the following weighted graph. **(08)**



Q.4 **A) (1)** Find the order of every element of the multiplication group $G = \{a^2, a^3, a^4, a^5, a^6 = e\}$. **(05+02)**

(2) Check whether the usual multiplication on the set of natural numbers given by $a * b = \frac{a+b}{ab}$ is a binary operation.

OR

A) (1) Use the method of contradiction to prove that $\sqrt{2}$ is not a rational number. **(04+03)**

(2) State the converse, contrapositive and Inverse of the proposition, "If it snow tonight, then I will stay at home."

B) (1) Prove that $(z_4, +_4)$ is an Abelian Group. **(05+03)**

(2) Use a direct proof to show that, "If x is an even integer then x^2 is an even integer".