# PARUL UNIVERSITY
## FACULTY OF ENGINEERING & TECHNOLOGY
### B.Tech. Winter 2021 - 22 Examination

Semester: 3                                             Date: 13/10/2021
Subject Code: 203191202                                 Time: 2:00pm to 4:30pm
Subject Name: Discrete Mathematics                      Total Marks: 60

**Instructions:**
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

**Q.1 Objective Type Questions** (All are compulsory) (Each of one mark)                 **(15)**

1. If the truth value of $p \vee q$ is F then the truth value of $(\neg p \wedge \neg q)$ is _____.

2. The value of $1.0 + \left(\overline{0} + \overline{1}\right)$ is _____.

3. The degree of an isolated vertex is _____ .
4. Cyclic group is an _____ group.
5. A vertex having no child is called a _____.
6. **If f(n+1) = $3^{f(n)/3}$ and f (0) =3, then f(2) is _____.**
7. A completely bipartite graph need not be a simple graph. T/F
8. Let $R_1$ = {(1, 2) (1, 6) (2, 4) (3, 4) (3, 6) (3, 8)} and

   $R_2$ = {(2, x) (4, y) (4, z) (6, z) (8, x)}. Then $R_2 o R_1$ = _____.

9. The truth value of $p \wedge \neg p$ is not a tautology.  T/F

10. If a relation $R$ satisfies reflexive, symmetric and Transitive property then the relation is called _____ relation.

11. $p \wedge (p \vee q)$ is known as
    **(a)** Distributive Law    **(b)** Idempotent Law    **(c)** Absorption Law    **(d)** Domination Law

12. If the relation $R$ on a set $A$ is called a void relation if
    **(a)** $R=AXA$            **(b)** $R=\emptyset$            **(c)** $R=\{(a,a)/a \in A\}$    **(d)** $R=R-1$

13. If a group satisfies the commutative property then it is known as
    **(a)** Abelian Group      **(b)** Symmetric group    **(c)** Semi-group        **(d)** Monoid

14. A graph, in which there is only an edge between a pair of vertices, is called a
    **(a)** Simple graph       **(b)** Pseudo-graph       **(c)** multi-graph        **(d)** weighted graph

15. The adjacency matrix of a simple graph is _____ matrix.
    **(a)** Skew-Hermitian     **(b)** Skew-symmetric     **(c)** Hermitian          **(d)** Symmetric

**Q.2 Answer the following questions. (Attempt any three)**                                 **(15)**

**A)** Let $R = \{(1,b),(2,a),(2,c)\}$ be a relation from $A = \{1,2,3\}$ to $B = \{a,b,c\}$ and let $S = \{(a,y),(b,x),(c,y),(c,z)\}$ be a relation from B to $C = \{x,y,z\}$.
Check if the composition relations $S o R$ and *RoS* can exist. Write the relation as set if they exists. If not, give the reason.

**A)** Check whether the "greater than or equal to" ($\geq$) relation is a partially ordered set or not on the set of integers Z.

**B)** Find the adjacency and incidence matrix for the following graph
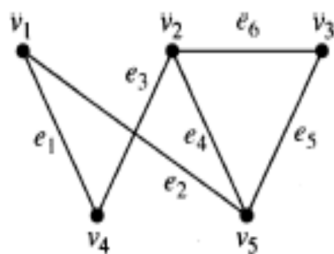
**C)** Use Kruskal's algorithm for finding minimum spanning tree for the given weighted graph.
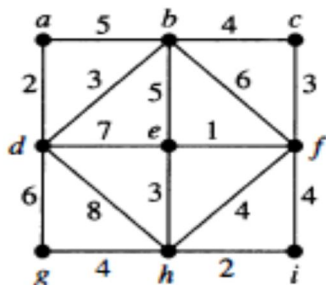


**Q.3**

**A) (1)** If $\alpha = \begin{pmatrix} 1\ 2\ 3\ 4\ 5\ 6 \\ 3\ 1\ 5\ 4\ 6\ 2 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1\ 2\ 3\ 4\ 5\ 6 \\ 2\ 5\ 1\ 3\ 6\ 4 \end{pmatrix}$ are two element of symmetric group $S_6$, **(04+03)**

find using right permutation $\alpha\beta$, $\beta\alpha$, $\beta^2$ and $\alpha^{-1}$ .

**(2)** Let $G$ be a group under usual multiplication. Show that $G = \{1, -1, i, -i\}$ is a cyclic group under usual multiplication.
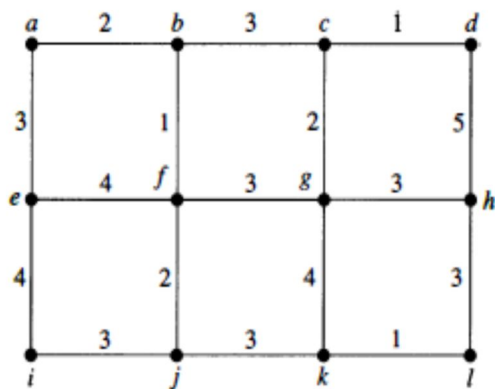
**B) (1)** Using mathematical induction prove the following **(05+03)**

$$1^3 + 2^3 + 3^3 + \cdots\cdots\cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

**(2)** With the help of truth table prove that $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$ .

**OR**

**B)** Define Spanning Tree. Use Prim's algorithm for finding the minimum spanning tree for the following weighted graph. **(08)**



**Q.4 A) (1)** Find the order of every element of the multiplication group $G = \{a^2, a^3, a^4, a^5, a^6 = e\}$. **(05+02)**

**(2)** Check whether the usual multiplication on the set of natural numbers given by

$a * b = \dfrac{a+b}{ab}$ is a binary operation.

**OR**

**A) (1)** Use the method of contradiction to prove that $\sqrt{2}$ is not a rational number. **(04+03)**

**(2)** State the converse, contrapositive and Inverse of the proposition, "If it snow tonight, then I will stay at home."

**B) (1) Prove that (z₄, +₄) is an Abelian Group.** (05+03)

(2) Use a direct proof to show that, "If $x$ is an even integer then $x^2$ is an even integer".

# PARUL UNIVERSITY
### FACULTY OF ENGINEERING & TECHNOLOGY
**B. Tech. Summer 2021 - 22 Examination**

**Semester : 3**                                                    **Date: 25/04/2022**
**Subject Code: 203191202/ 03191202**                 **Time: 2:00pm to 4:30pm**
**Subject Name: Discrete Mathematics**                   **Total Marks: 60**

**Instructions:**
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Make suitable assumptions wherever necessary.
4. Start new question on new page.

**Q.1** **Objective Type Questions** - (All are compulsory)  (Each of one mark)                **(15)**
1. In directed graph if there are never two edges in opposite directions between distinct vertices then relation is
   A) Symmetric                          B) Anti-Symmetric
   C) Reflexive                          D) Transitive
2. What is the remainder when - 17 is divided by 5?
   A) 3                                  B) -3
   C) 2                                  D) -2
3. If U= {1, 2, 3, 4, 5, 6} A= {1, 2, 3, 4} B={3, 4, 5, 6} using bit string find the intersection of the sets A and B.
   A) 11 11 00                           B) 00 11 11
   C) 11 11 11                           D) 00 11 00
4. What is the identity element in Z under the operation given as a∗b=a+b−7,for any a , b∈Z.
   A) -7                                 B) 14
   C) 7                                  D) None of these
5. Consider $Z_{10}$ under addition modulo 10. If $2 \in Z_{10}$ then the order of 2 is
   A) 2                                  B) 5
   C) 1                                  D) 3
6. . $P(n,n)$ ... ... ....
7. The degree of isolated vertex is -------------
8. A compound proposition that always takes the final truth value as true is called ………..
9. (1 4 5 6 )(3 7 2 9 8 ) is _____ permutation (Even/Odd)
10. What is the Cardinality of the Power set of the set {0, 1, 2}?
11. Spanning tree of the simple graph is unique(True/False)
12. How many edges are there in a graph with 10 vertices of degree 8 each ?
13. If the left coset and right coset are same then the group is called cyclic (True/False)
14. How many different bit strings of length eight are there?
15. Product of two irrational is _____.

**Q.2** Answer the following questions. (Attempt any three)                                      **(15)**
A) Determine whether each of the compound propositions is satisfiable.
$$(p \lor \neg q) \land (q \lor \neg r) \land (r \lor \neg p)$$

B) Show that (G ,+₅ ) is a cyclic group, where G={ 0, 1, 2, 3, 4 }.
C) Prove using Principle of mathematical induction,

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n (n + 1) (2n + 1)}{6}$$

E)  Check if the set of all non-negative integers is an abelian group under usual addition of integers.

**Q.3 A)**

1. Construct a truth table for each of these compound propositions (03)
   $(p \rightarrow q) \rightarrow (q \rightarrow p)$

2. Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent ,without using (04)
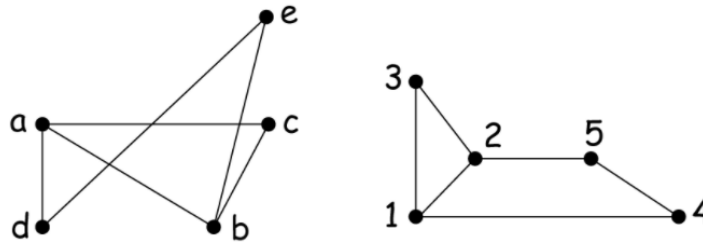   truth table.

**B)**

1. Find the zero–one matrix of the transitive closure of the relation R where (04)

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

2. Show that if $n$ is an integer and $n^3 + 5$ is odd, then $n$ is even using a proof by (04)
   contraposition and Contradiction.

**OR**

**B)**

1. Determine whether the given pair of graphs is isomorphic? (04)



(04)

2. What are the negations of the statements $\forall x(x^2 > x)$ and $\exists x(x^2 = 2)$?

**Q.4 A)**

1. Find the bitwise OR, bitwise And & bitwise XOR of each of the following (04)
   pairs of bit strings:
   (a) 1111 0000, 1010 1010
   (b) 00 0111 0001, 10 0100 1000

2. Solve the equation for $x$ under $S_6$: (03)

$$(1 \quad 4 \quad 2 \quad 5)^3 \, x = (2 \quad 3 \quad 5 \quad 4)^{-1}$$
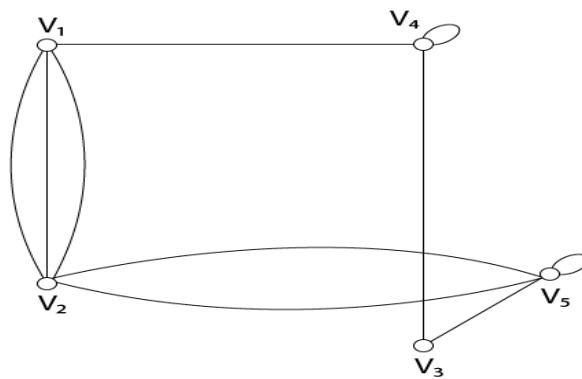
**OR**

A) Use Prim's and Kruskal's algorithm for finding the minimum spanning tree from (07)
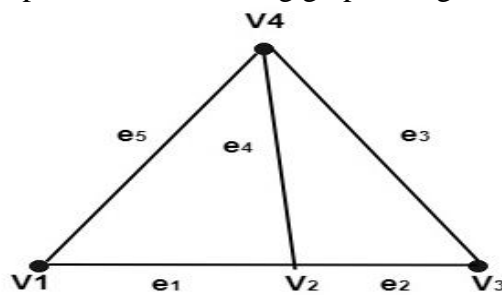   the following weighted graphs ?

B)

1) A) Represent the following graph using adjacency matrix. **(04)**



B) Represent the following graph using incidence matrix.



2) A) Define Complete Bipartite Graphs also draw $K_{2,3}$, $K_{3,3}$ and $K_{3,5}$ **(04)**

B) Give the statement of THE HANDSHAKING THEOREM and use it ,to find the number of vertices If simple graph G has 24 edges and degree of each vertex is 4.