PHYSICS WITHOUT FRONTIERS

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Quantum Information Science & Technology

Classical logic gates to quantum gates

Quantum Computing – Lecture 2

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Université de Strasbourg

QUSTEC
Quantum Science and Technologies at the European Campus

Scope of the course

- *Five lectures of 1 hour*

  1) Introduction to Quantum Science & Technology

  2) From classical logic to quantum gates

  3) Quantum circuits & implementations

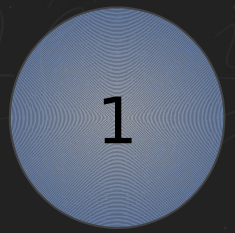
  4) Quantum algorithms & error correction

  5) Quantum stack

Classical Information

Binary unit - Bit

$< 5\mathrm{V}$



$\geq 5\mathrm{V}$

Classical Information

Logical gates

INPUT	OUTPUT
A	Q
0	1
1	0

$$f(a) = 1 - a$$

Classical Information

Logical gates

INPUT		OUTPUT
A	B	Q
0	0	0
0	1	0
1	0	0
1	1	1

$$f(a, b) = a * b$$

Classical Information

Logical gates

INPUT		OUTPUT
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	1

$$f(a,b) = a + b - a * b$$

Classical Information

Logical gates

INPUT		OUTPUT
A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0

$$f(a,b) = \overline{a * b}$$

Classical Information

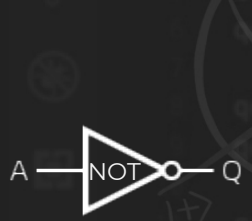
Logical gates

INPUT		OUTPUT
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

$$f(a,b) = \overline{a + b - a * b}$$

Classical Information

Logical gates

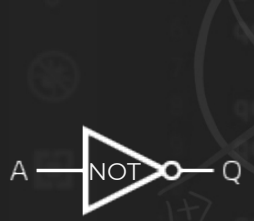


INPUT		OUTPUT
A	B	Q
0	0	0
0	1	1
1	0	1
1	1	0

$$f(a,b) = a + b - 2ab$$

Classical Information

Logical gates

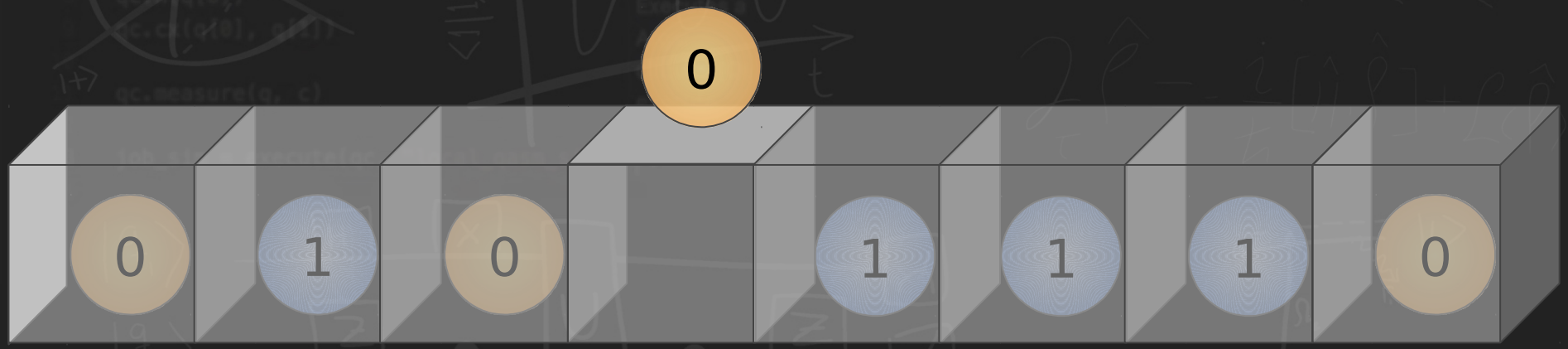


INPUT		OUTPUT
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	1

$$f(a,b) = a \cdot b + \bar{a} \cdot \bar{b}$$

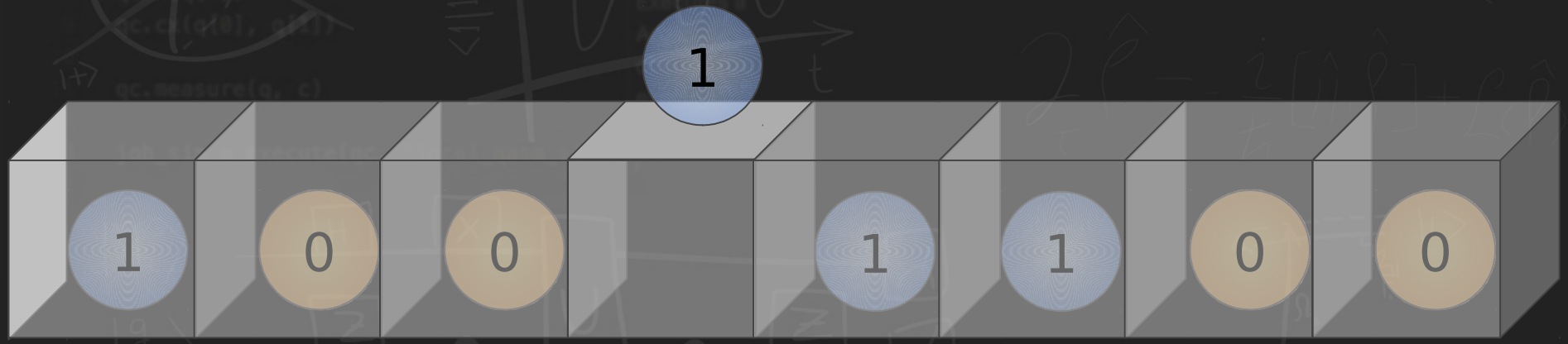
Classical Information

Classical register



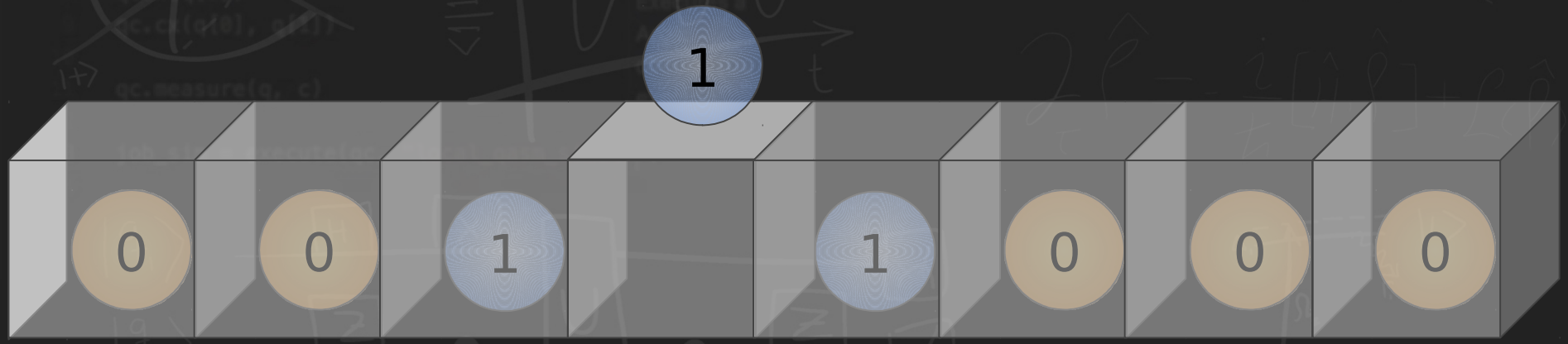
Classical Information

Classical register



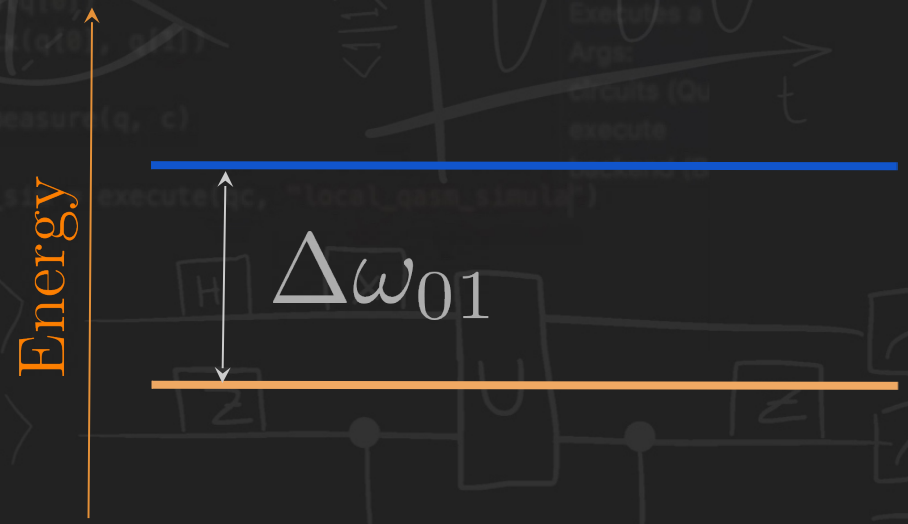
Classical Information

Classical register



Quantum systems

2-level system



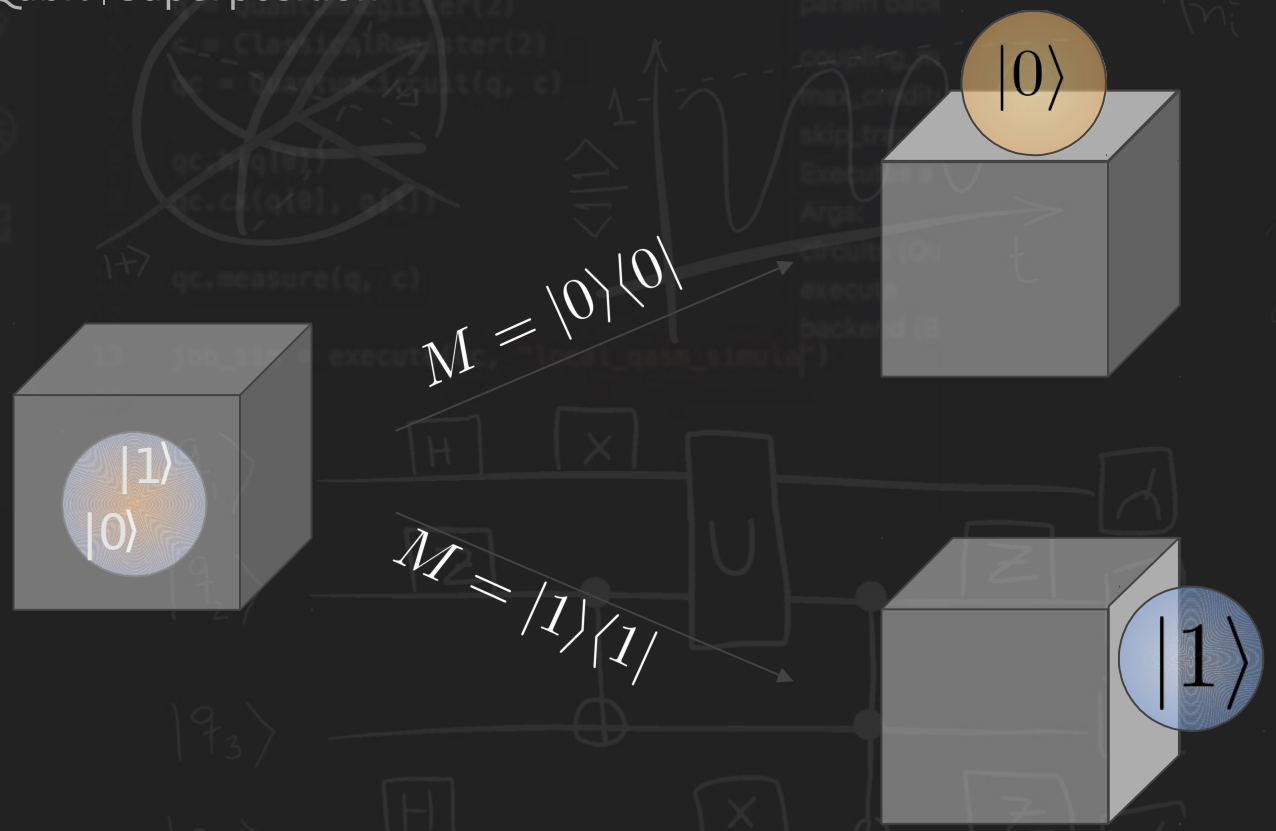
Quantum systems

Qubit

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Quantum systems

Qubit | Superposition

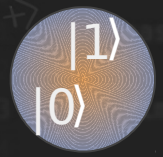


$$\langle\psi|0\rangle\langle 0|\psi\rangle = |\alpha|^2$$

$$\langle\psi|1\rangle\langle 1|\psi\rangle = |\beta|^2$$

Quantum systems

Qubit | Superposition



Quantum systems

Qubit | Superposition

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

Quantum systems

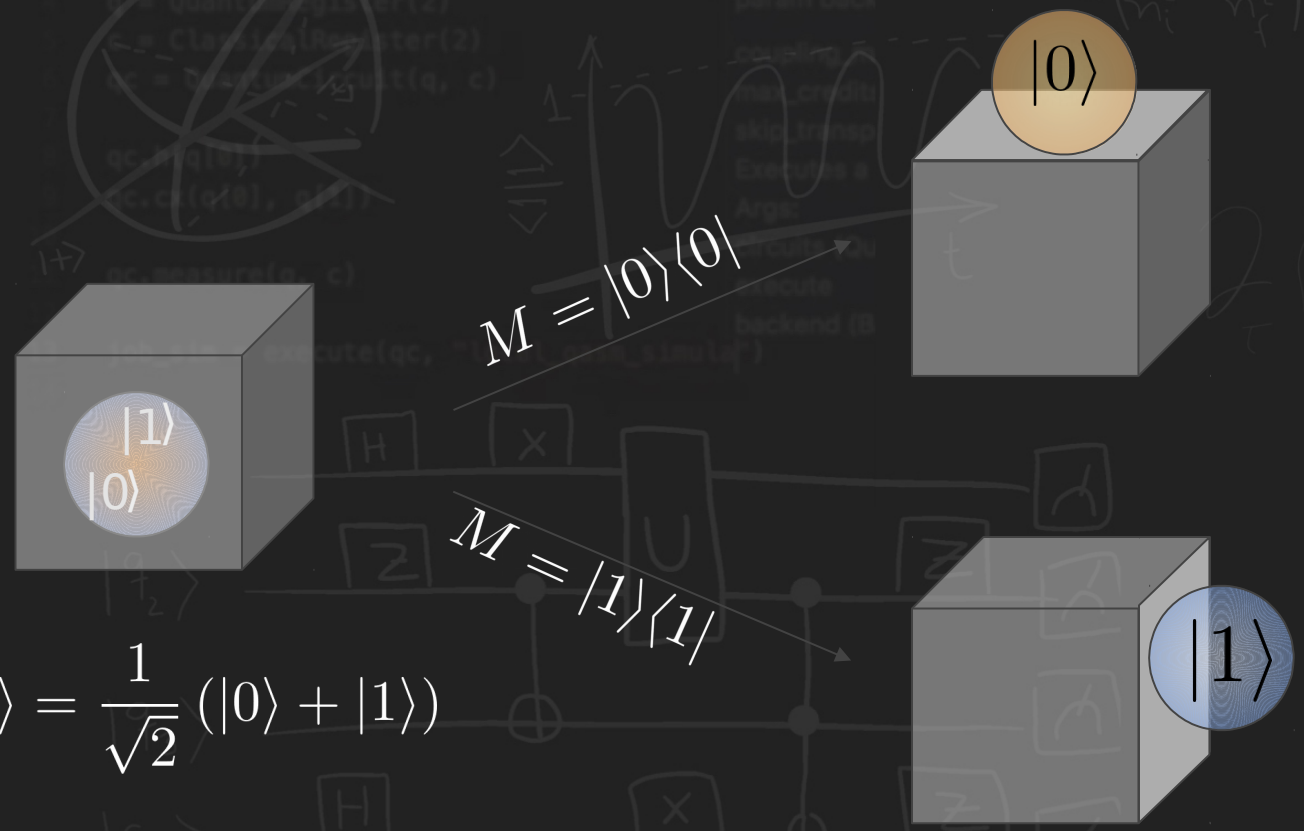
Qubit | Superposition

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

Quantum systems

Qubit



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|\psi\rangle \rightarrow |0\rangle$$

$$|\alpha|^2 = 0.5$$

$$|\psi\rangle \rightarrow |q\rangle$$

$$|\beta|^2 = 0.5$$

Quantum systems

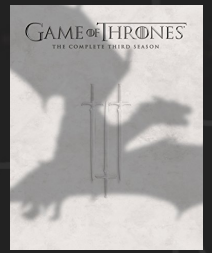
Qubit | Entanglement

Let's consider your favorite classical TV series:

Season 1

Season 2

Season 3

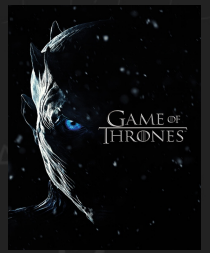


Season 4

Season 5

Season 6

Season 7



Season 8

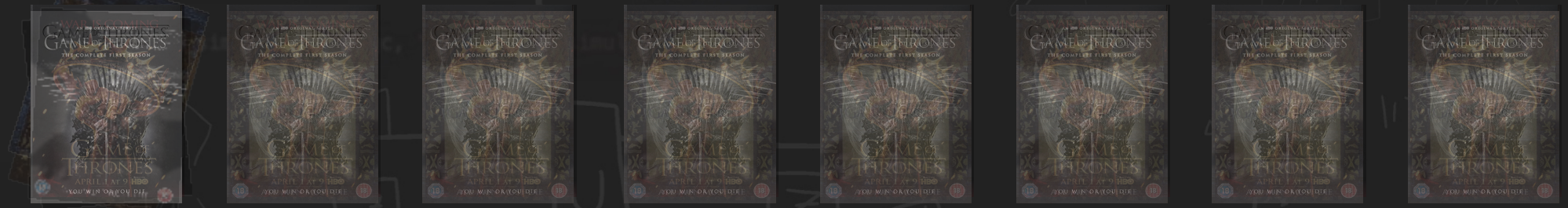
With a classical TV we could get what the series is about if we see one episode at the time. But let's now consider the quantum version, where the information is encoded in the correlations among the episodes.

Quantum systems

Qubit | Entanglement

In contrast to the classical version, in the quantum version, **you get a random piece of information every time you look for an episode**

Season ? Season ? Season ? Season ? Season ? Season ? Season ? Season ?



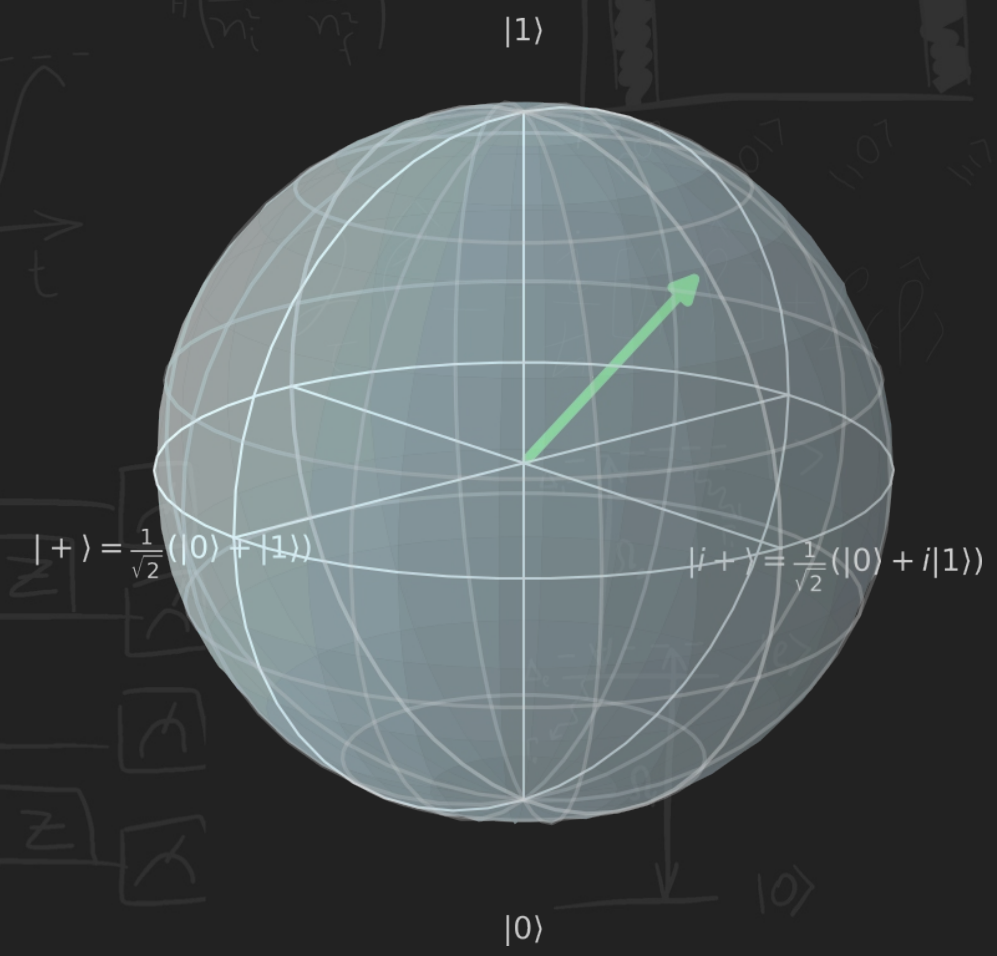
The information of the system can only be accessed via a collective observation on many episodes at the time!!!

Quantum systems

Bloch Sphere (Graphical representation)

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$\theta, \phi \in \mathbb{R}$$



Quantum systems

Bloch Sphere (Graphical representation)

We can describe the evolution of the quantum system by solving the **Schrödinger equation**, under the *action* of a Hamiltonian:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{\mathcal{H}} |\psi(t)\rangle$$

Quantum systems

Bloch Sphere (Graphical representation)

We can describe the evolution of the quantum system by solving the **Schrödinger equation**, under the *action* of a Hamiltonian:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{\mathcal{H}} |\psi(t)\rangle$$

This will lead to the **evolution operator**:

$$U = e^{i\hat{\mathcal{H}}\Delta t/\hbar}$$

Quantum systems

Bloch Sphere (Graphical representation)

Thus the **quantum logic gate** it is a unitary transformation (*reversible*) that takes the initial (*input*) state and evolve (*transform*) it into a final state (output) state

$$|\psi_{\text{in}}\rangle - \boxed{U} - |\psi_{\text{out}}\rangle$$

Quantum systems

Bloch Sphere (Graphical representation)

Thus the **quantum logic gate** it is a unitary transformation (*reversible*) that takes the initial (*input*) state and evolve (*transform*) it into a final state (output) state

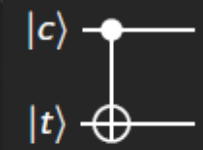
$$|\psi_{\text{in}}\rangle - \boxed{U} - |\psi_{\text{out}}\rangle$$

This unitary transformation can be obtained from **expanding the Taylor series of the exponential matrix:**

$$e^{\hat{A}} = \sum_{n=0}^{\infty} \frac{\hat{A}^n}{n!}$$

Gate-based Quantum Computing

Universal gate set



Universal gate set

CNOT (controlled NOT)

$$\mathrm{CNOT} = cX = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Pauli X (NOT operation)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$|in\rangle$ — X — $|out\rangle$

Pauli Y

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$|in\rangle$ — Y — $|out\rangle$

Hadamard (Superposition operation)

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$|in\rangle$ — H — $|out\rangle$

Pauli Z (Phase operation)

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

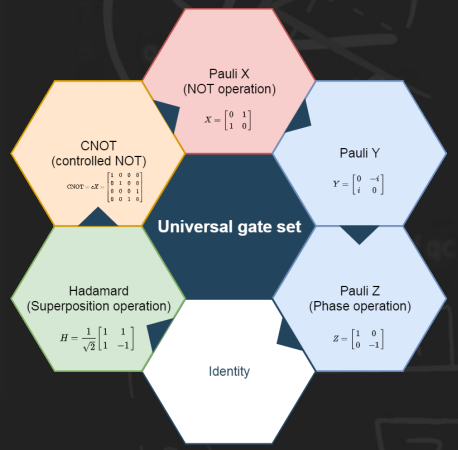
$|in\rangle$ — Z — $|out\rangle$

Identity

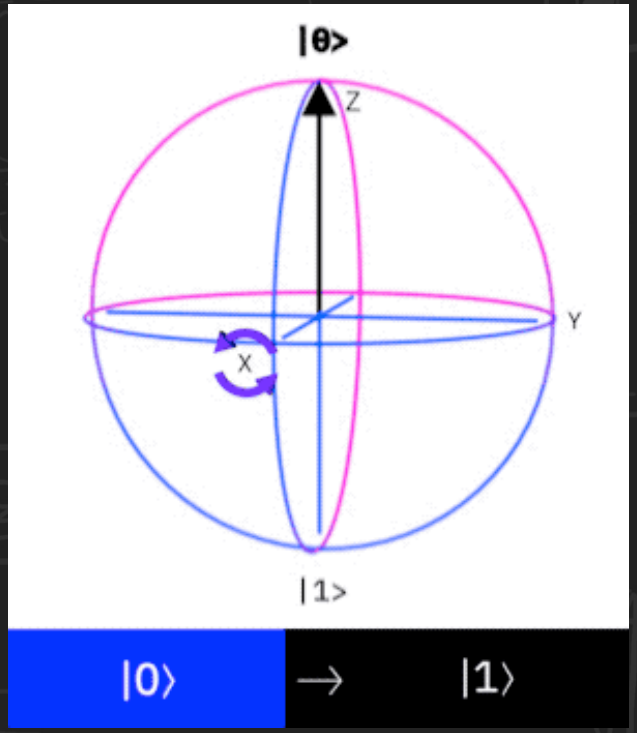
$|in\rangle$ — I — $|out\rangle$

Gate-based Quantum Computing

Universal gate set



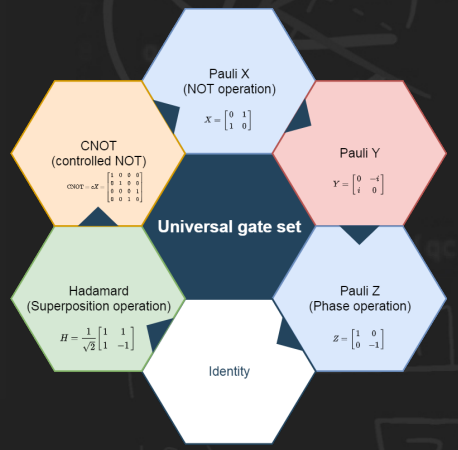
$\lvert$ Input	$\lvert$ Output $\rangle = \mathbf{X} \lvert$ Input $\rangle$
$\lvert 0 \rangle$	$\lvert 1 \rangle$
$\lvert 1 \rangle$	$\lvert 0 \rangle$
$\alpha \lvert 0 \rangle + \beta \lvert 1 \rangle$	$\alpha \lvert 1 \rangle + \beta \lvert 0 \rangle$

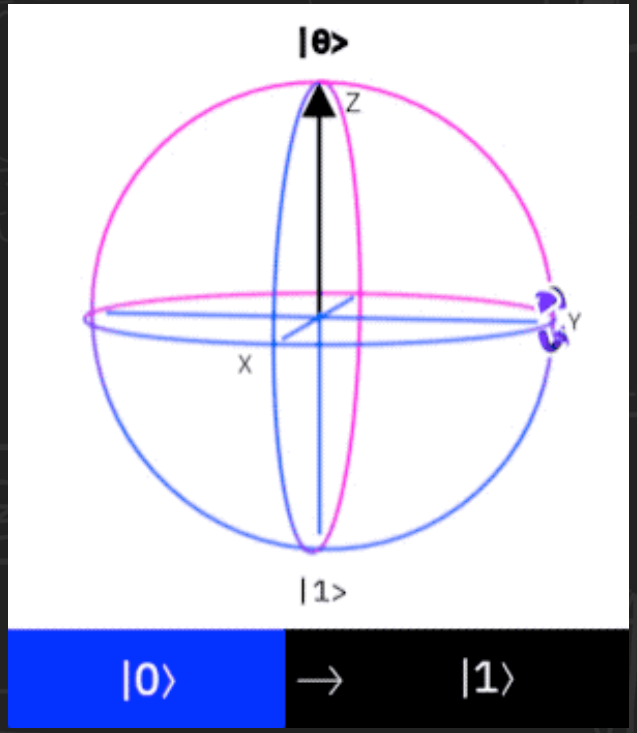


Source: https://quantum-computing.ibm.com/

Gate-based Quantum Computing

Universal gate set





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Gate-based Quantum Computing

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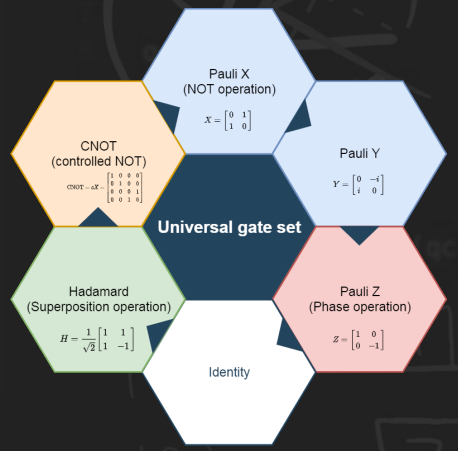

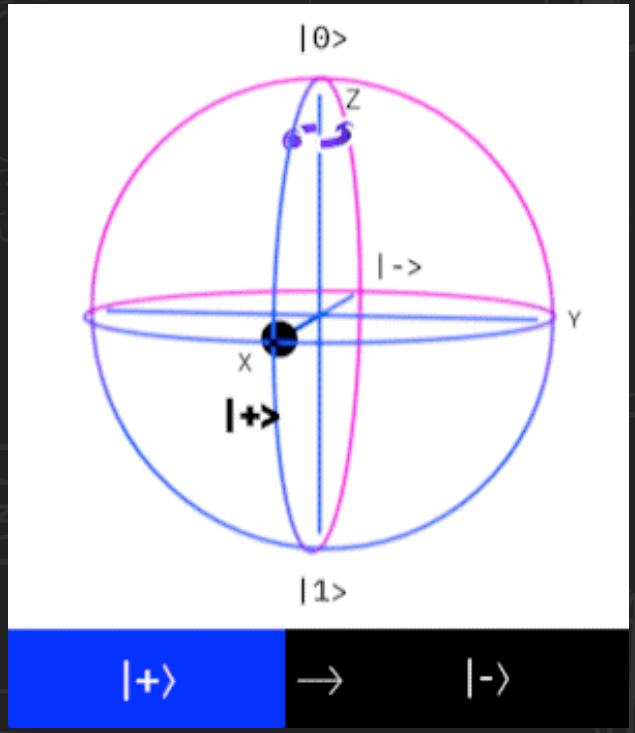

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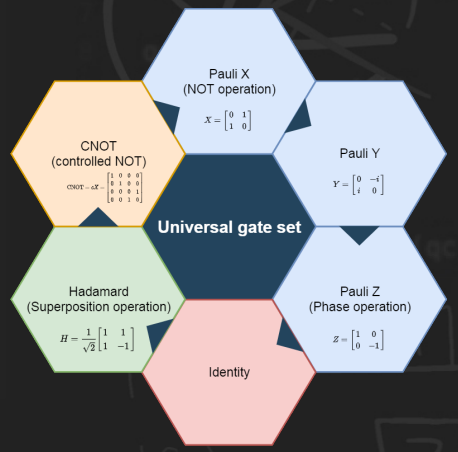
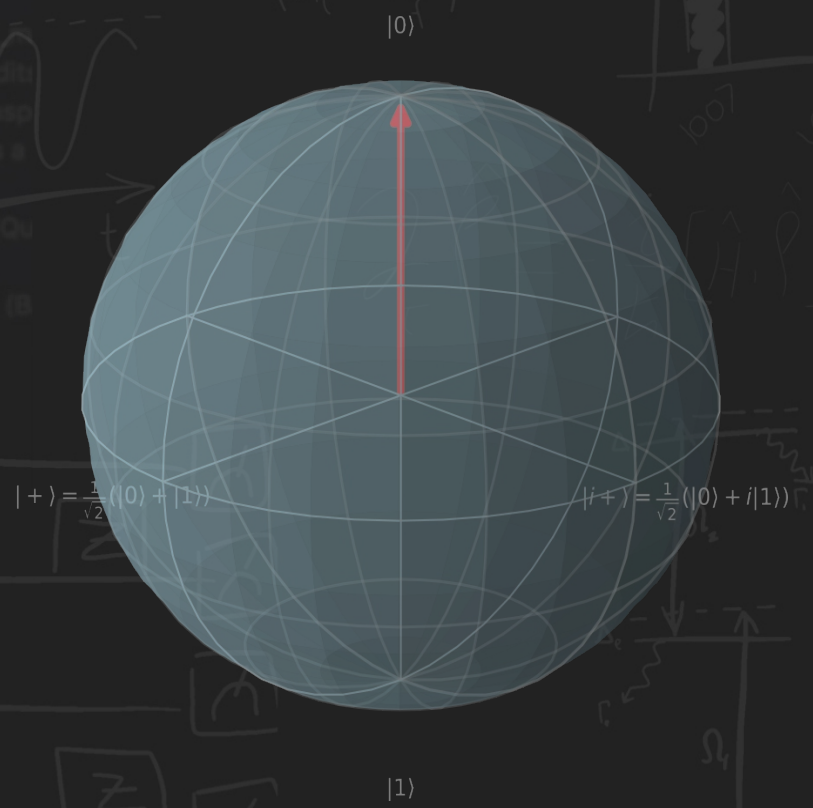
Gate-based Quantum Computing

Universal gate set





Gate-based Quantum Computing

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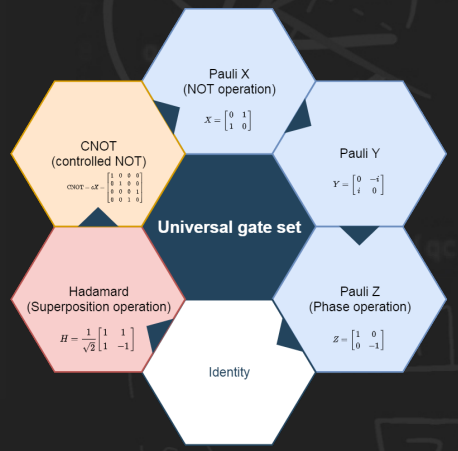

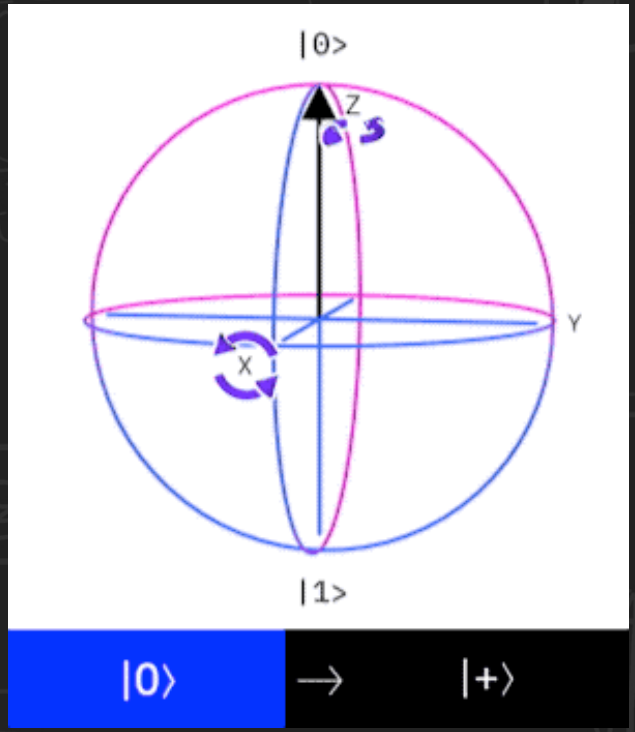

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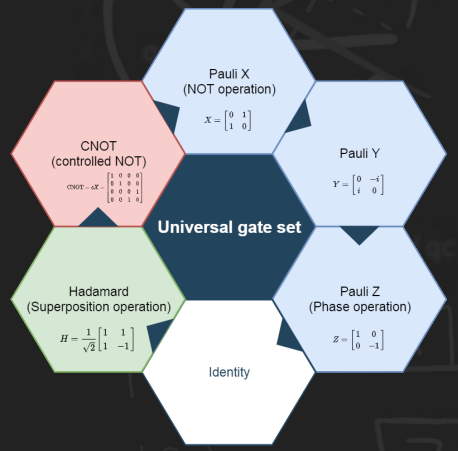
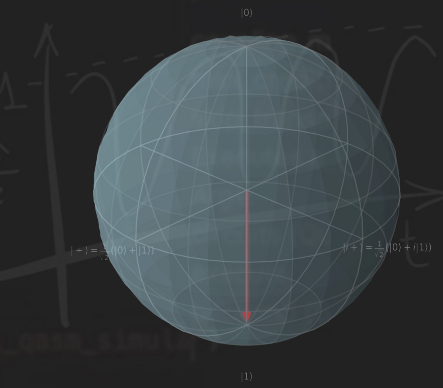
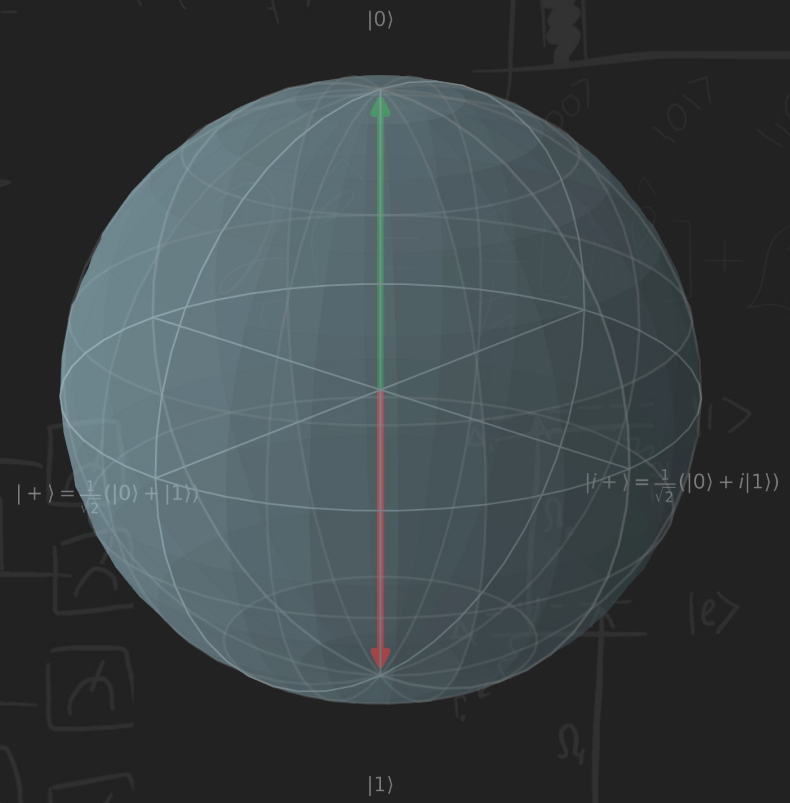
Gate-based Quantum Computing

Universal gate set

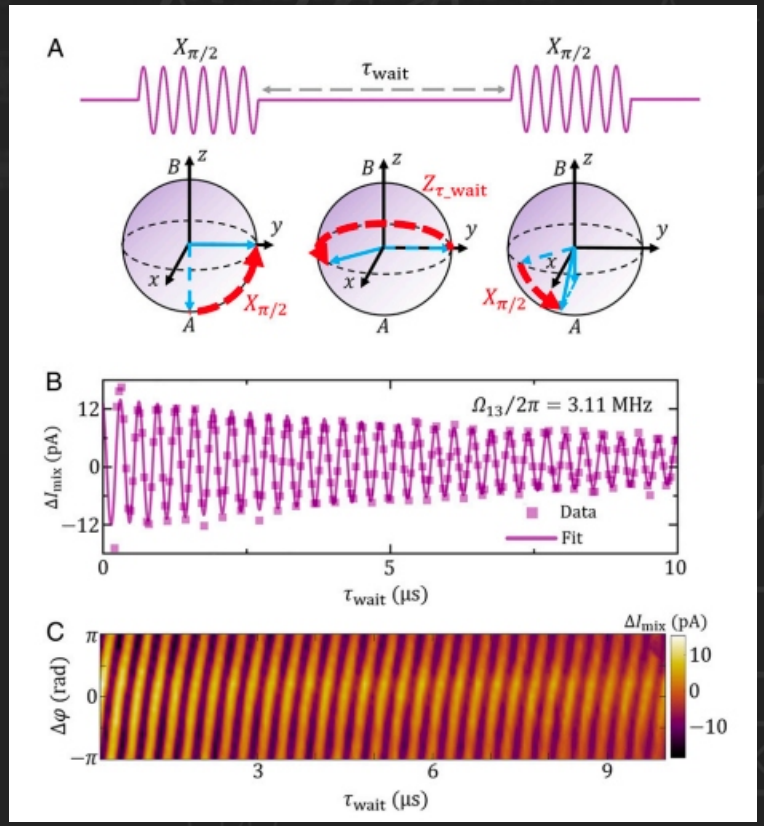


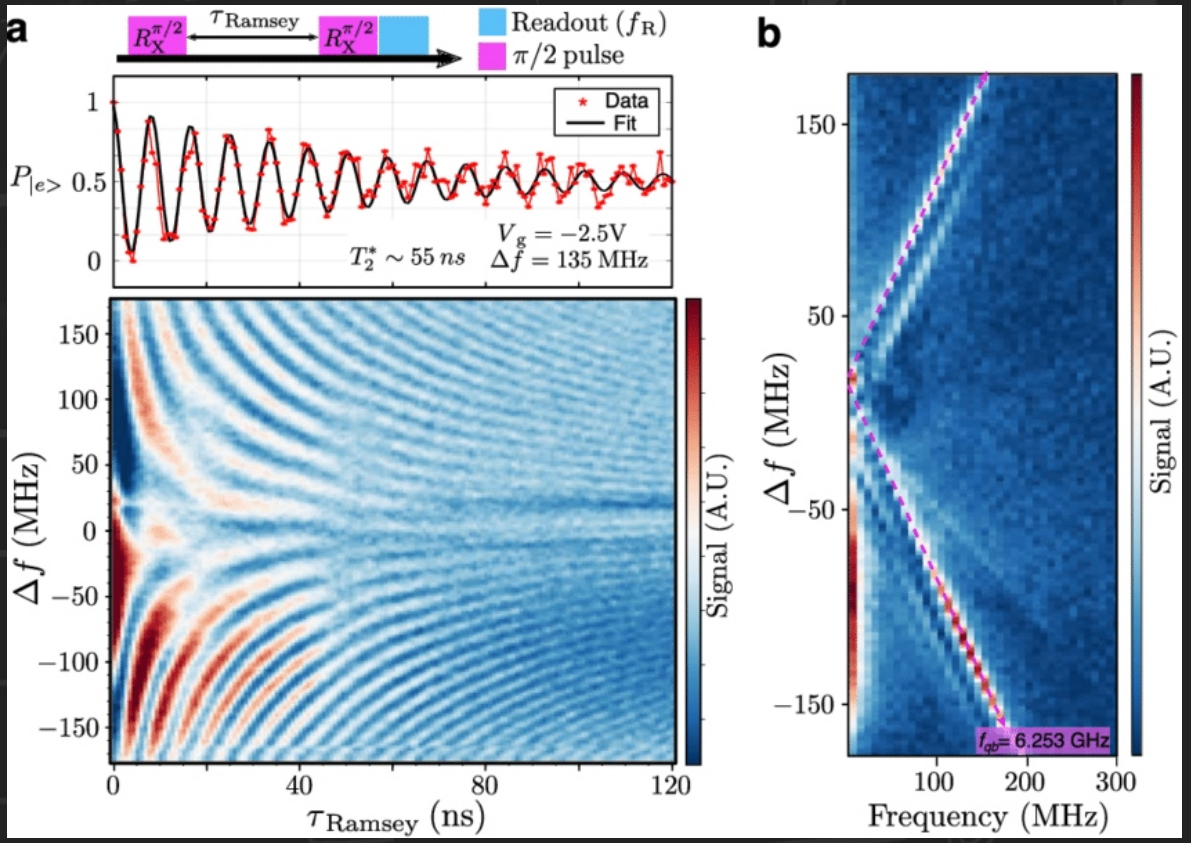




Quantum resources

Rabi oscillation and Ramsey interferometry





https://www.pnas.org/doi/pdf/10.1073/pnas.1916978117

https://bit.ly/3Mef5JI