PHYSICS WITHOUT FRONTIERS

Physics Research and Education Bootcamps
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Quantum Information Science & Technology

Quantum circuits & implementations

Quantum Computing – Lecture 3

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Centre Européen de Science Quantiques (CESQ)
Apr.2022

Université de Strasbourg

QUSTEC
Quantum Science and Technologies at the European Campus

Scope of the course

- *Five lectures of 1 hour*

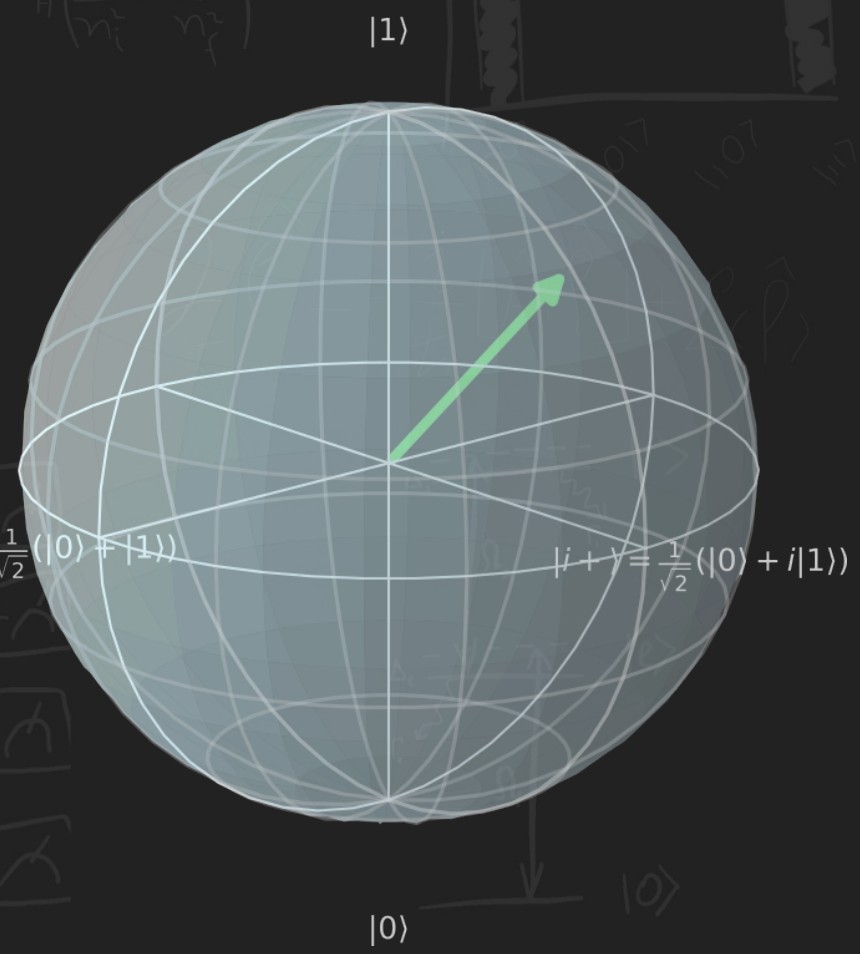
  1) Introduction to Quantum Science & Technology
  2) From classical logic to quantum gates
  3) Quantum circuits & implementations
  4) Quantum algorithms & error correction
  5) Quantum stack

Qubit

Bloch Sphere (Graphical representation)

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$\theta, \phi \in \mathbb{R}$$



Quantum dynamics

Schrödinger Equation

We can describe the evolution of the quantum system by solving the **Schrödinger equation**, under the *action* of a Hamiltonian:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{\mathcal{H}} |\psi(t)\rangle$$

Quantum systems

Bloch Sphere (Graphical representation)

We can describe the evolution of the quantum system by solving the **Schrödinger equation**, under the *action* of a Hamiltonian:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{\mathcal{H}} |\psi(t)\rangle$$

This will lead to the **evolution operator**:

$$U = e^{i\hat{\mathcal{H}}\Delta t/\hbar}$$

Quantum systems

Bloch Sphere (Graphical representation)

Thus the **quantum logic gate** it is a unitary transformation (*reversible*) that takes the initial (*input*) state and evolve (*transform*) it into a final state (output) state

$$|\psi_{\text{in}}\rangle - \boxed{U} - |\psi_{\text{out}}\rangle$$

Quantum systems

Bloch Sphere (Graphical representation)

Thus the **quantum logic gate** it is a unitary transformation (*reversible*) that takes the initial (*input*) state and evolve (*transform*) it into a final state (output) state

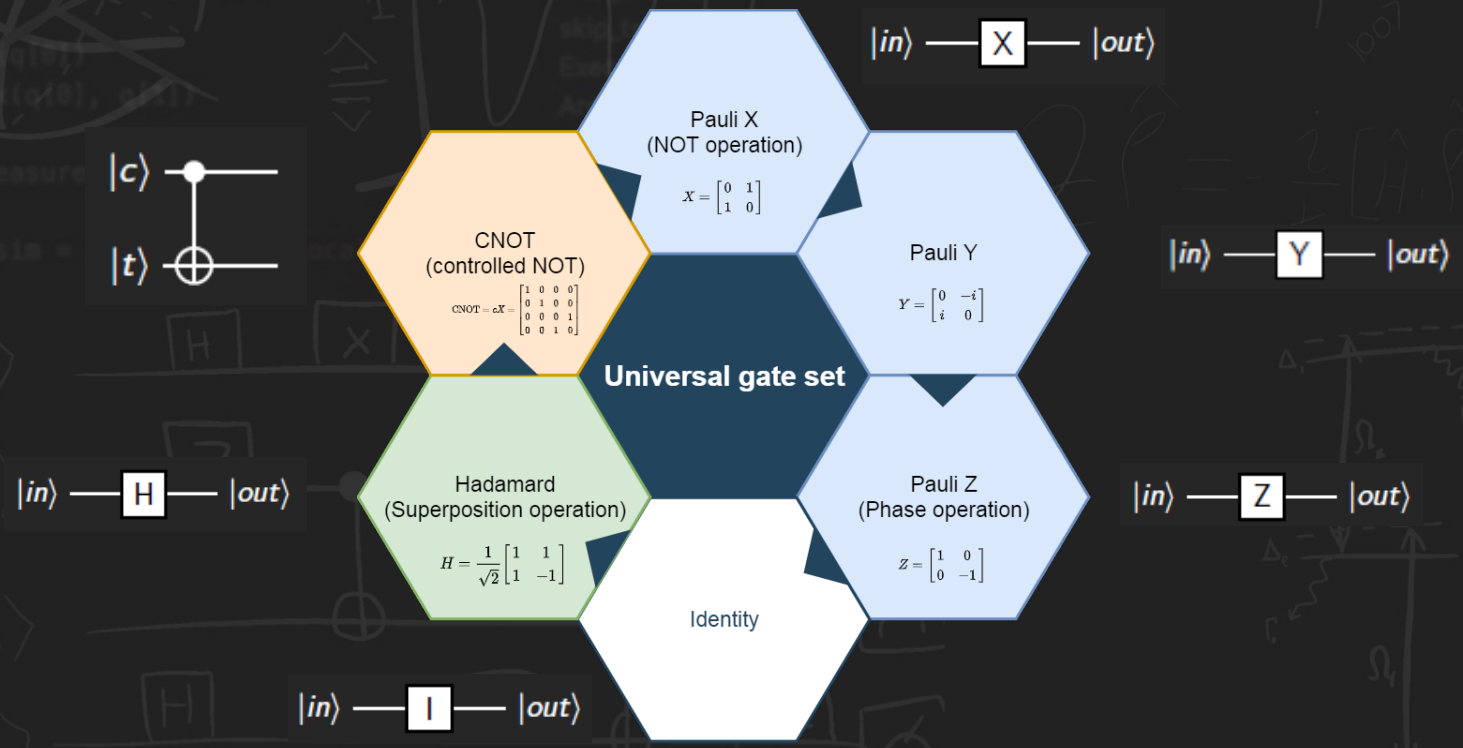
$$|\psi_{\text{in}}\rangle - \boxed{U} - |\psi_{\text{out}}\rangle$$

This unitary transformation can be obtained from **expanding the Taylor series of the exponential matrix:**

$$e^{\hat{A}} = \sum_{n=0}^{\infty} \frac{\hat{A}^n}{n!}$$

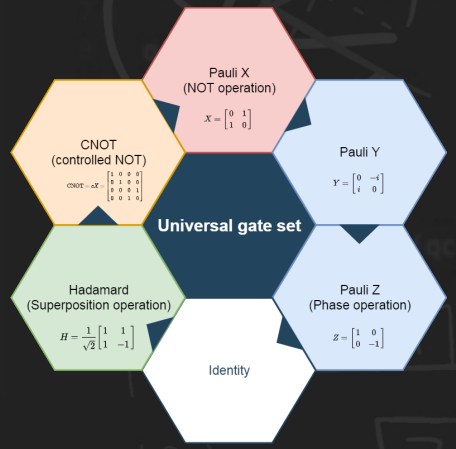
Gate-based Quantum Computing

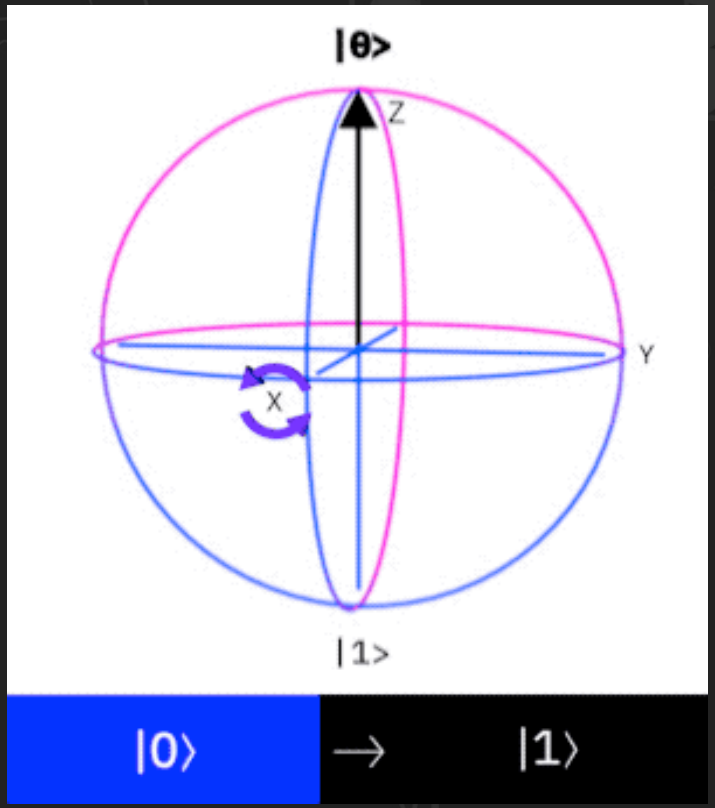
Universal gate set



Gate-based Quantum Computing

Universal gate set

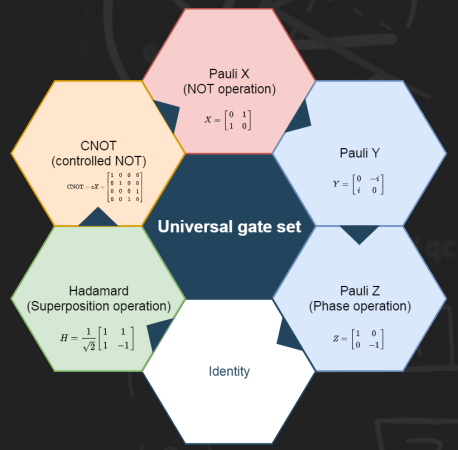




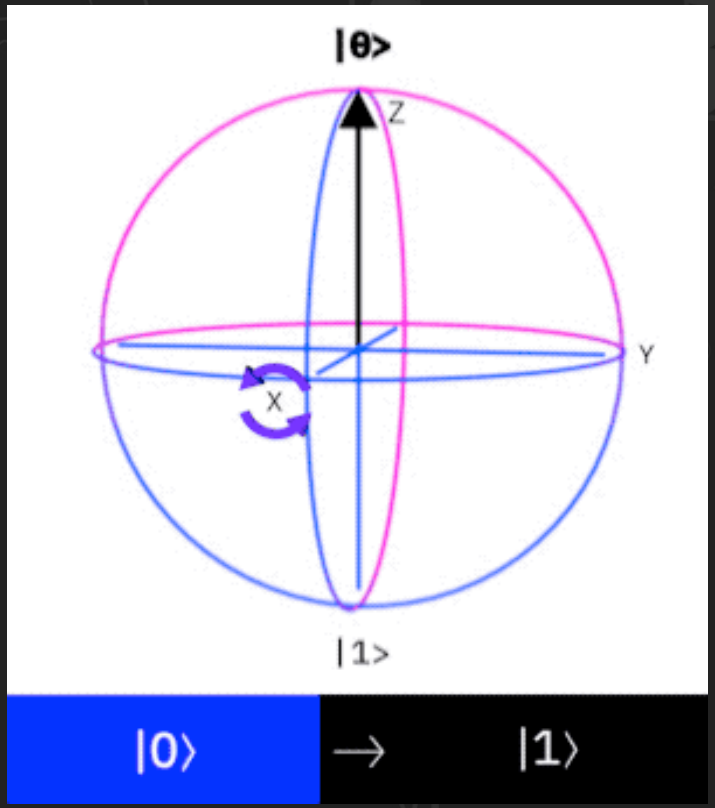
Source: https://quantum-computing.ibm.com/

Gate-based Quantum Computing

Universal gate set



$\lvert$ Input	$\lvert$ Output $\rangle = \mathbf{X} \lvert$ Input $\rangle$
$\lvert 0 \rangle$	$\lvert 1 \rangle$
$\lvert 1 \rangle$	$\lvert 0 \rangle$
$\alpha \lvert 0 \rangle + \beta \lvert 1 \rangle$	$\alpha \lvert 1 \rangle + \beta \lvert 0 \rangle$



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Gate-based Quantum Computing

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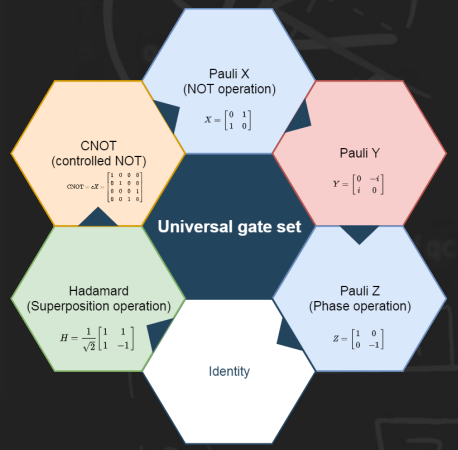

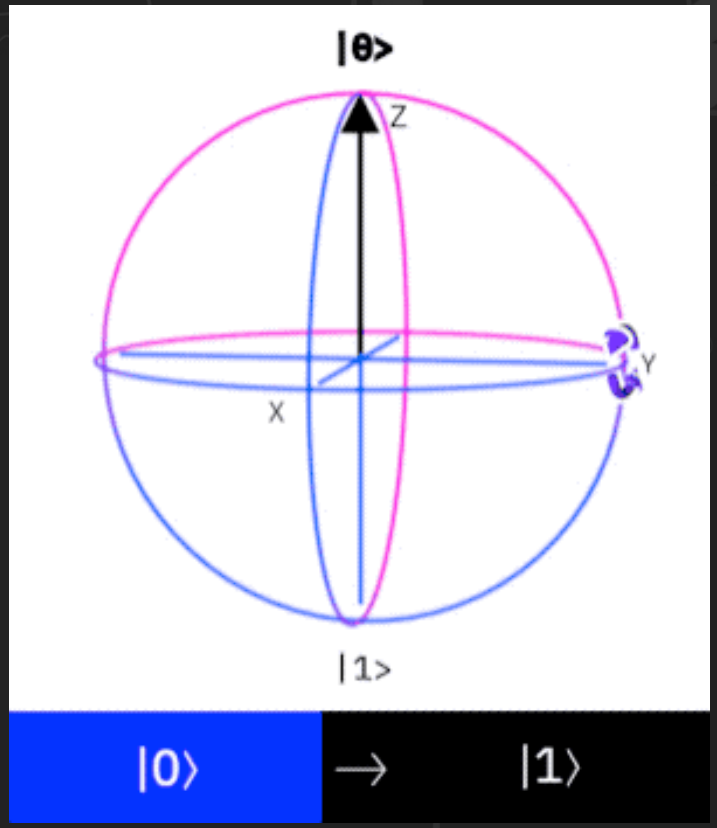

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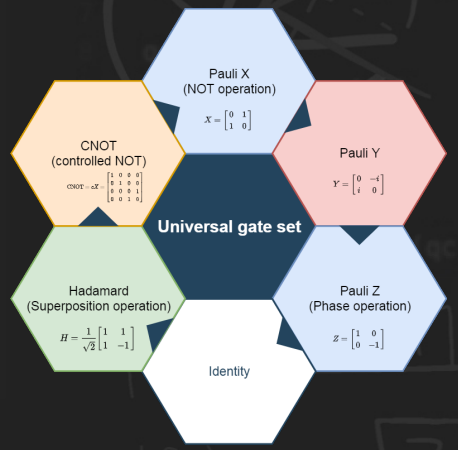
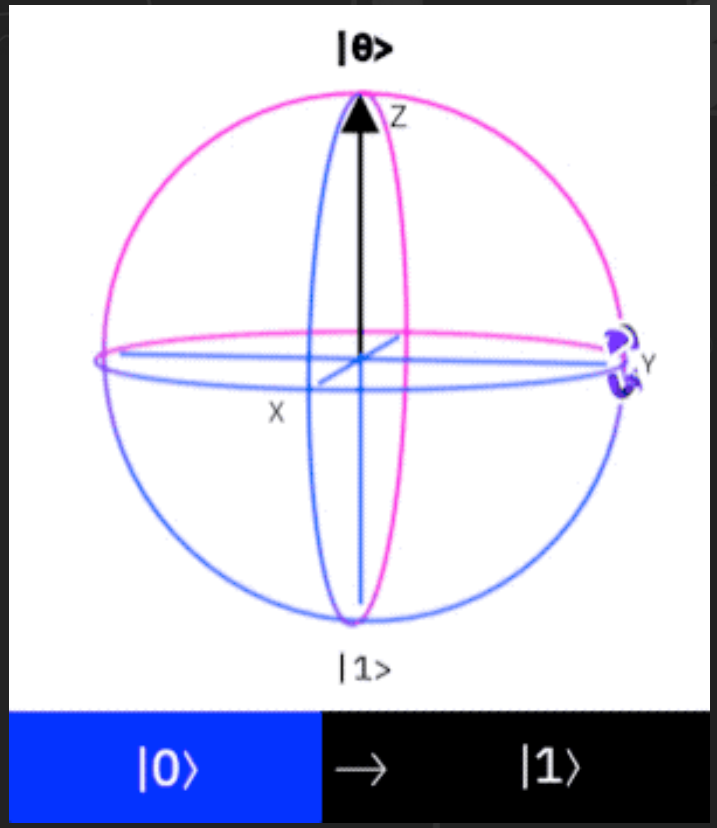
Gate-based Quantum Computing

Universal gate set



$\lvert\text{Input}\rangle$	$\lvert\text{ Output }\rangle = \text{Y }\lvert\text{Input}\rangle$
$\lvert 0\rangle$	$i\lvert 1\rangle$
$\lvert 1\rangle$	$-i\lvert 0\rangle$
$\alpha\lvert 0\rangle + \beta\lvert 1\rangle$	$\alpha\lvert 1\rangle - \beta\lvert 0\rangle$



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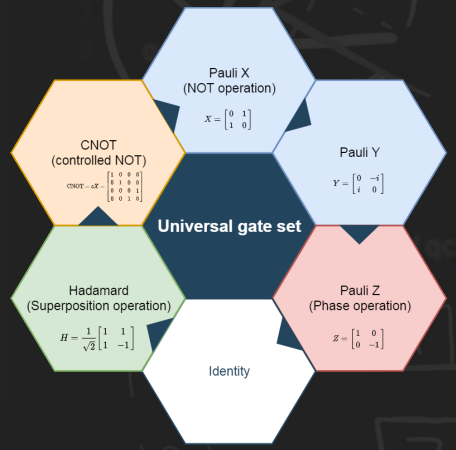
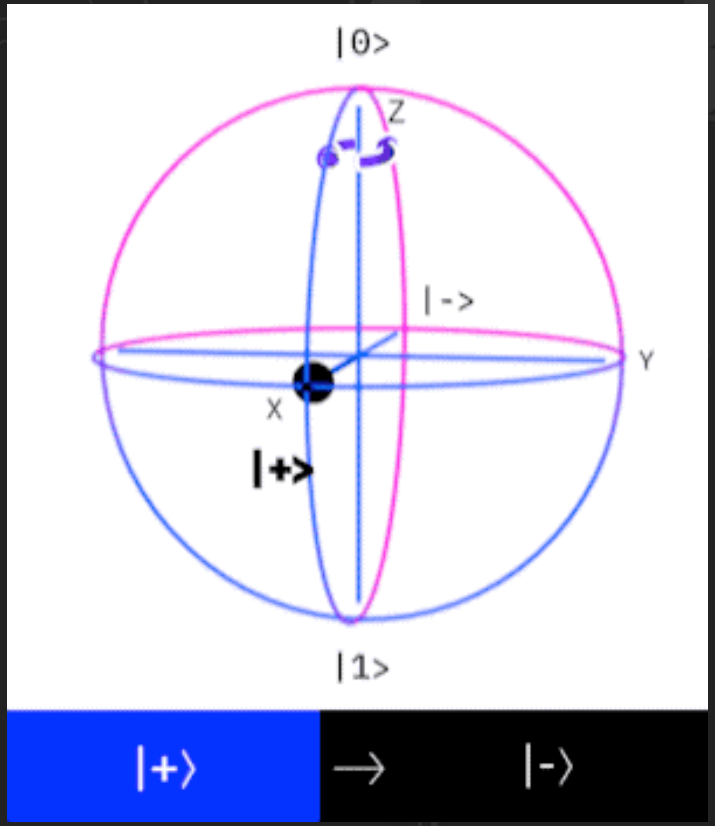
Gate-based Quantum Computing

Universal gate set





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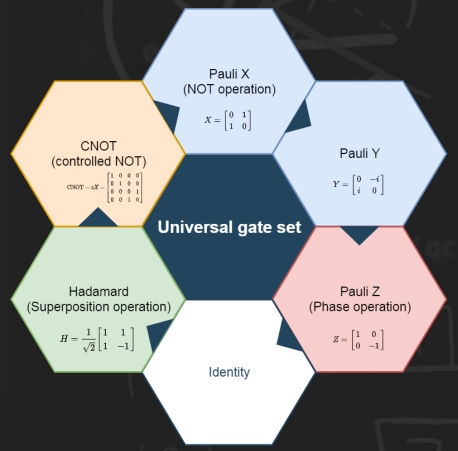
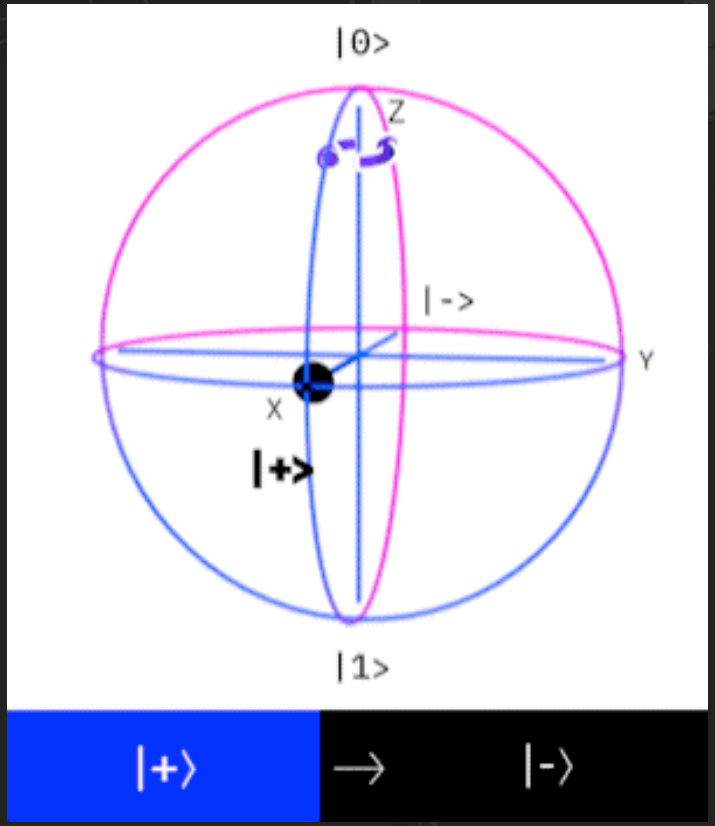
Gate-based Quantum Computing

Universal gate set



$\mid \text{Input} \rangle$	$\mid \text{Output} \rangle = \mathrm{Z} \mid \text{Input} \rangle$
$\lvert 0 \rangle$	$\lvert 0 \rangle$
$\lvert 1 \rangle$	$-\lvert 1 \rangle$
$\alpha\lvert 0 \rangle + \beta\lvert 1 \rangle$	$\alpha\lvert 0 \rangle - \beta\lvert 1 \rangle$



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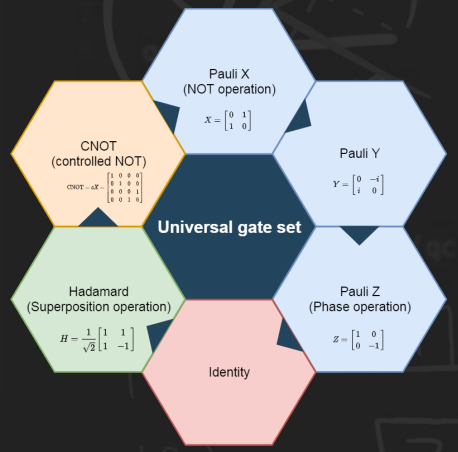
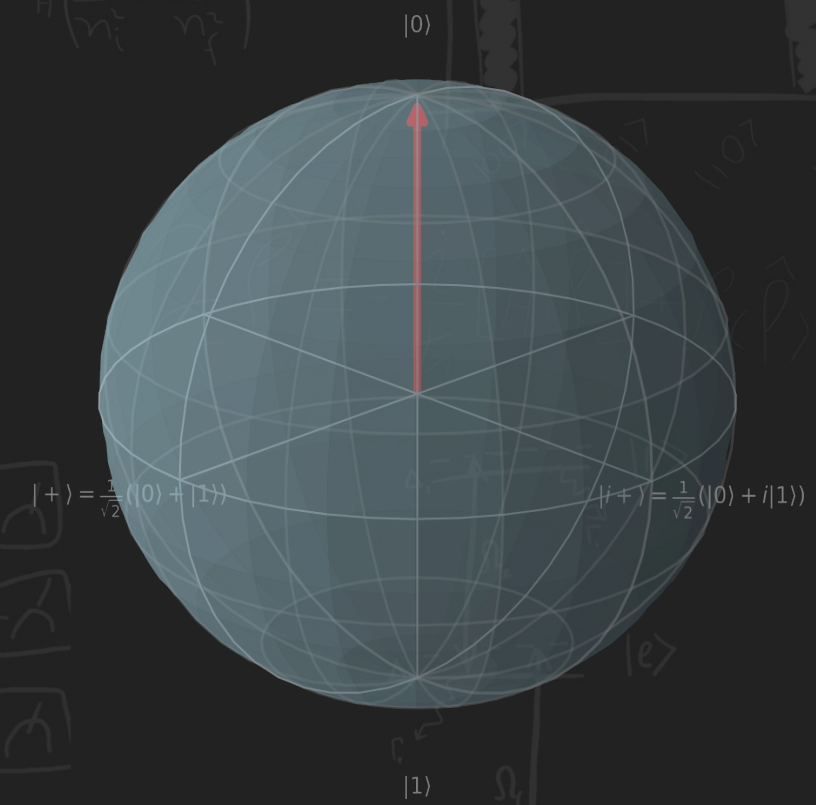
Gate-based Quantum Computing

Universal gate set

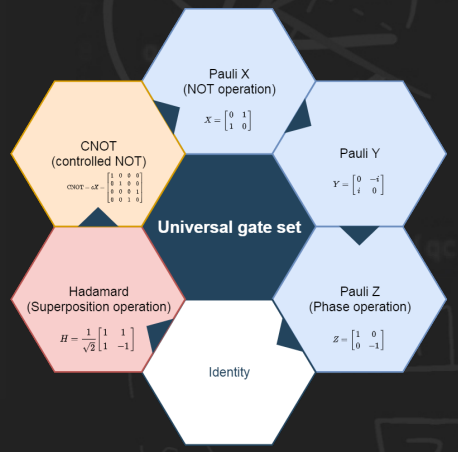


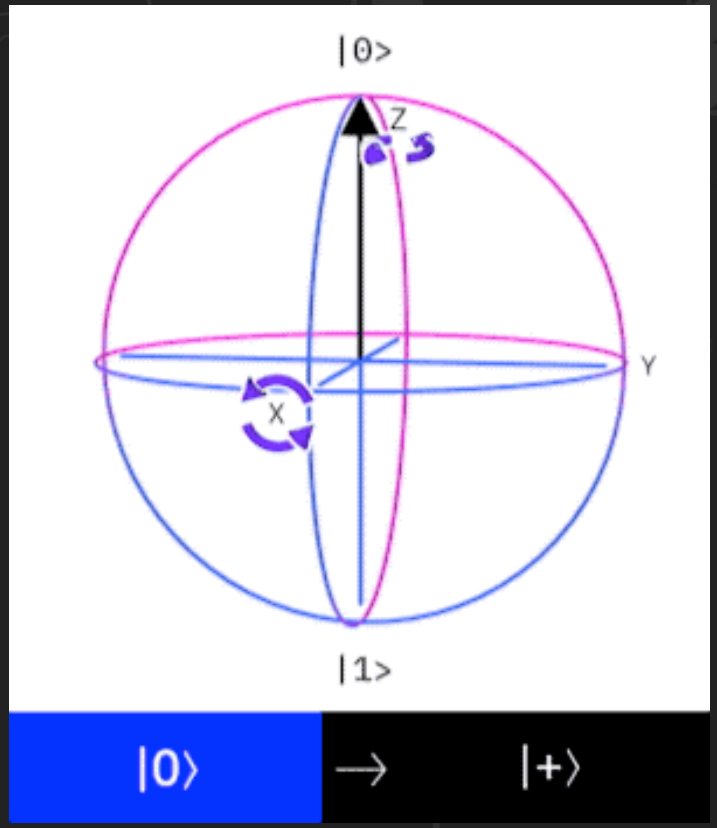
$\lvert\text{Input}\rangle$	$\lvert\text{ Output }\rangle = \text{I}\ \lvert\text{Input}\rangle$
$\lvert 0\rangle$	$\lvert 0\rangle$
$\lvert 1\rangle$	$\lvert 1\rangle$
$\alpha\lvert 0\rangle + \beta\lvert 1\rangle$	$\alpha\lvert 0\rangle + \beta\lvert 1\rangle$



Gate-based Quantum Computing

Universal gate set





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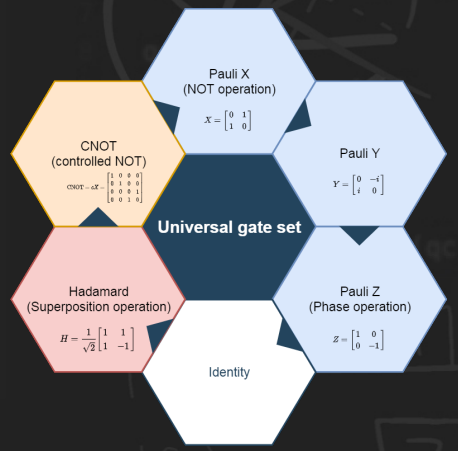
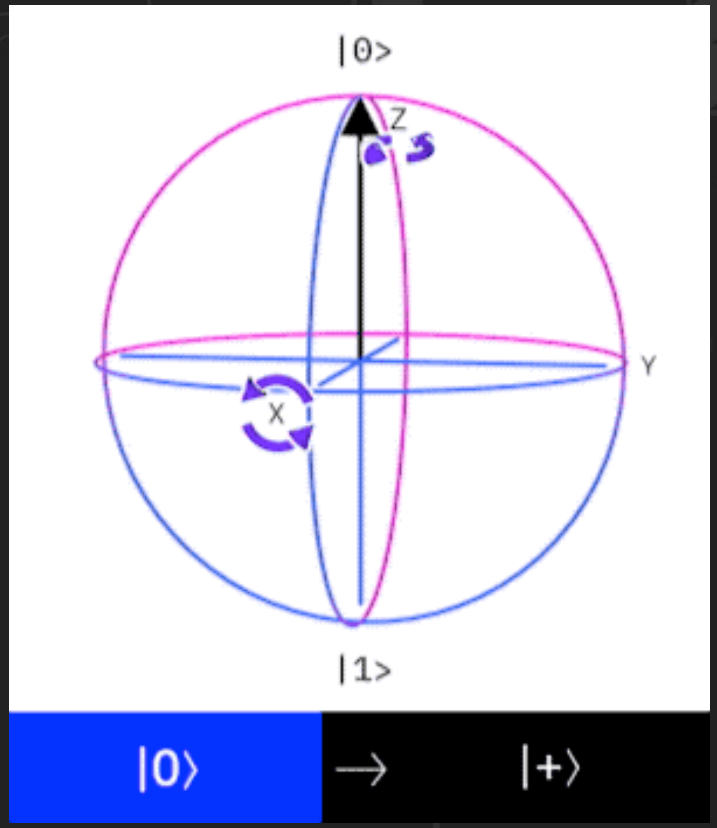
Gate-based Quantum Computing

Universal gate set



$\mid \text{Input} \rangle$	$\mid \text{Output} \rangle = \text{H} \mid \text{Input} \rangle$
$\lvert 0 \rangle$	$(\lvert 0 \rangle + \lvert 1 \rangle)/\sqrt{2}$
$\lvert 1 \rangle$	$(\lvert 0 \rangle - \lvert 1 \rangle)\sqrt{2}$
$\alpha\lvert 0 \rangle + \beta\lvert 1 \rangle$	$\alpha\lvert 1 \rangle + \beta\lvert 0 \rangle$



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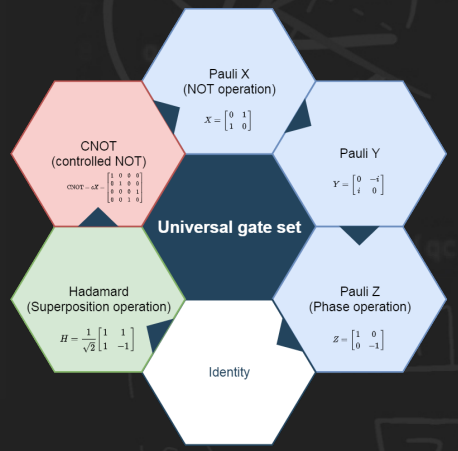
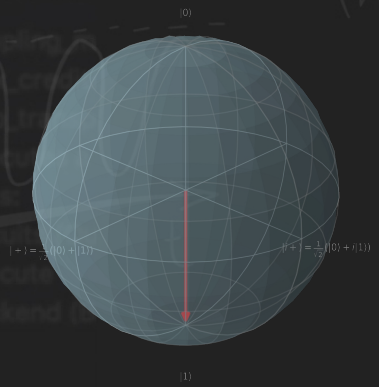
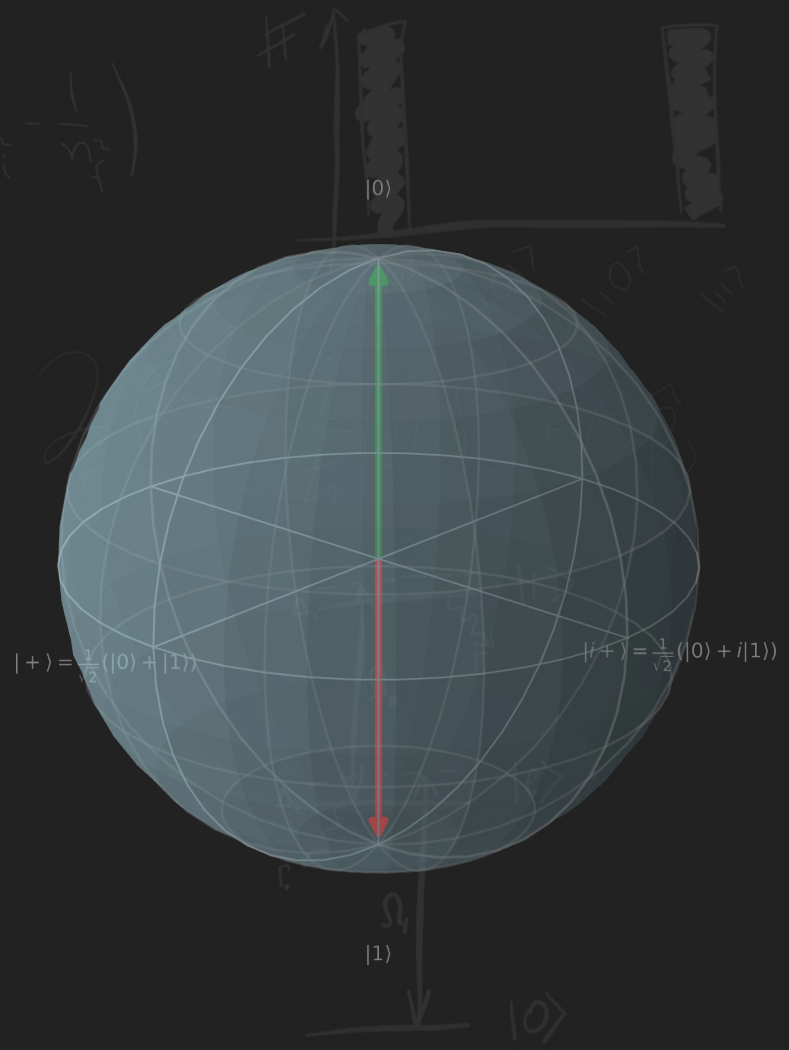
Gate-based Quantum Computing

Universal gate set







$\lvert c\rangle$	$\lvert t\rangle$	$\lvert\ \text{out}\ \rangle = \lvert 0\rangle\langle 0\rvert \otimes I + \lvert 1\rangle\langle 1\rvert \otimes X(\lvert c\rangle \otimes \lvert t\rangle)$
$\lvert 0\rangle$	$\lvert 0\rangle$	$\lvert 0\rangle \otimes \lvert 0\rangle$
$\lvert 0\rangle$	$\lvert 1\rangle$	$\lvert 0\rangle \otimes \lvert 1\rangle$
$1\rangle$	$0\rangle$	$\lvert 1\rangle \otimes \lvert 1\rangle$
$\lvert 1\rangle$	$\lvert 1\rangle$	$\lvert 1\rangle \otimes \lvert 0\rangle$

Qubit

Fidelity

The Fidelity is a figure of merit that maps the qubit states into real numbers, quantifying the "overlapping" between two quantum states. For two mixed states, it is defined as:

$$\mathcal{F}(\rho, \sigma) = \mathrm{Tr}\left(\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2$$

Gate-based Quantum Computing

Native gates | Single-qubit gate: U_{xy}

Using the **atom-light Hamiltonian**:

$$\hat{\mathcal{H}}_j^{ab} = \left(\frac{\Omega_j(t)}{2} e^{i\varphi_j(t)} |a_j\rangle\langle b| + \text{h.c.} \right) - \Delta_j(t) |b_j\rangle\langle b|$$

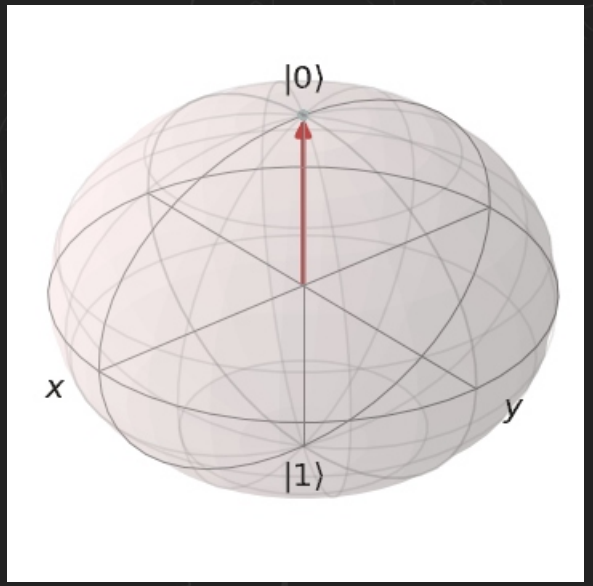
One can follow the **sequence**:

$$\begin{aligned} U = & \exp\left(-i\hat{\mathcal{H}}_j^{01} \tau_g\right) \\ = & \exp\left(\frac{i\Delta\tau_g}{2}\right) \times \exp\left(-\frac{i\tilde{\Omega}\tau_g}{2} \vec{v} \cdot \vec{\sigma}\right) \end{aligned}$$

And doing the exponential matrix to the evolution operator one can obtain the **unitary transformation**:

$$\mathrm{U}_{xy}(\theta, \varphi) = \begin{pmatrix} cos(\theta/2) & -isin(\theta/2)e^{i\varphi} \\ -isin(\theta/2)e^{-i\varphi} & cos(\theta/2) \end{pmatrix}$$

The **symbol for the quantum circuit** formalism, we defined as:



Representation of the qubit state in the Bloch sphere for a sequence of values of the unitary gate parameters.

Gate-based Quantum Computing

Native gates | Two-qubit gate: CZ

Using the **interaction Hamiltonian**:

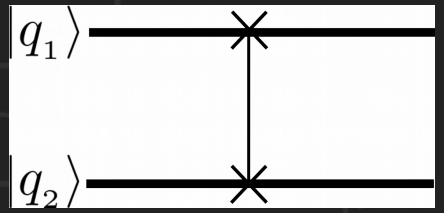
$$\hat{\mathcal{H}}_{j,k}^{abcd} = \frac{V_{j,k}}{2}|ab\rangle\langle cd| + \text{h.c.}$$

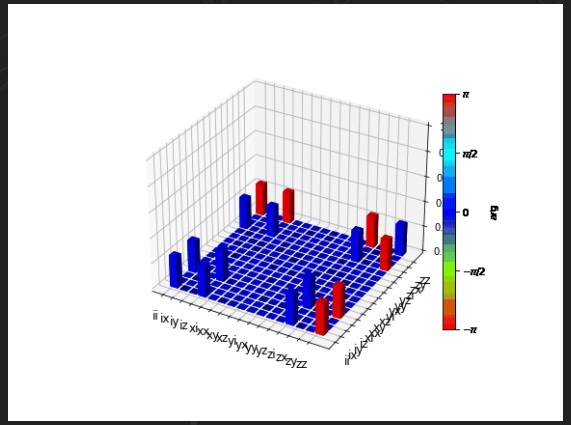
One can follow the **sequence**:

$$U = \exp\left(-i\hat{\mathcal{H}}_{c}^{r1}\tau_1\right) \times \exp\left[-i(\hat{\mathcal{H}}_{t}^{r1} + \hat{\mathcal{H}}_{c,t}^{rrrr})\tau_2\right] \times \exp\left(-i\hat{\mathcal{H}}_{c}^{r1}\tau_1\right)$$

After applying the exponential matrix to the evolution operator, we obtain the unitary transformation.

$$CZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



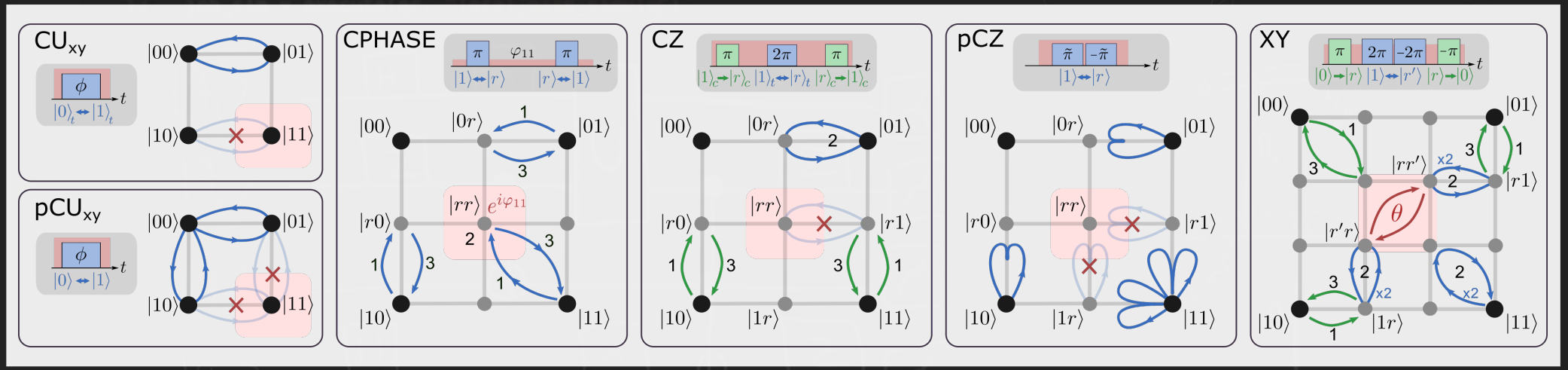


Quantum tomography for selected values of CPHASE gate.

Gate-based Quantum Computing

Quantum Circuits

An important part of the platform are the native gates that Rydberg qubits can perform, through a serie of "*physical operations*" and that can be later *transpiled* into canonical gates and viceversa:

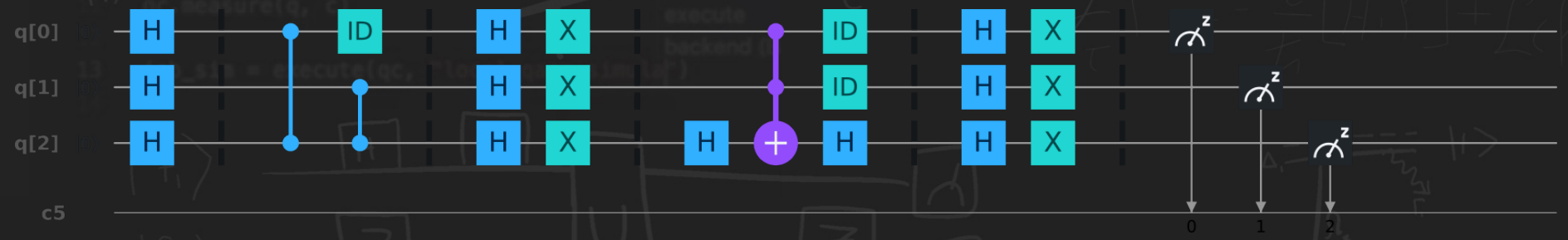


AVS Quantum Science, 3(2), p.023501

Quantum circuits

Anatomy of a quantum circuit

3-qubits Grover's Algorithm

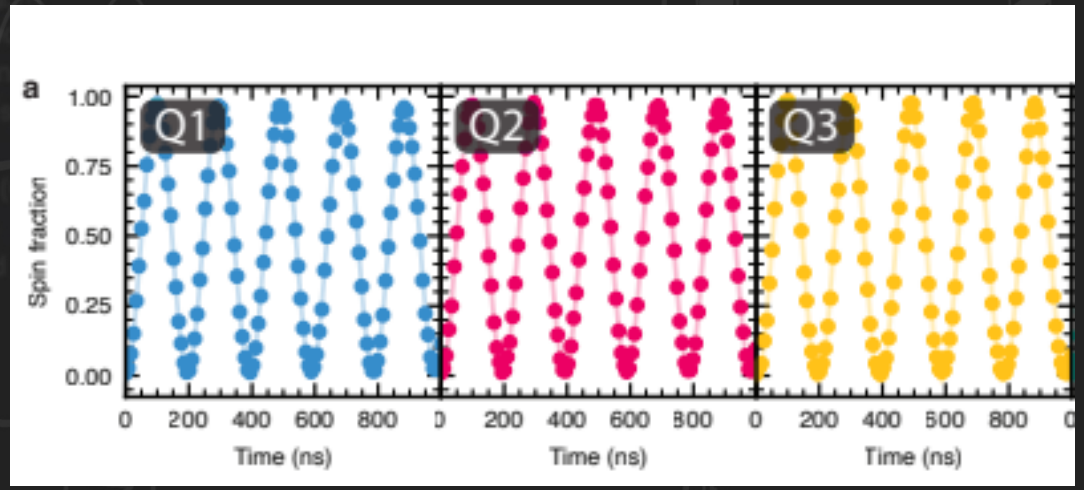


Quantum Resources

Rabi experiment - T_1

It is an experimental sequence composed by one long pulse that drive the probability populations of the qubit states.

This defines the T_1 time, that relates with the operation time.



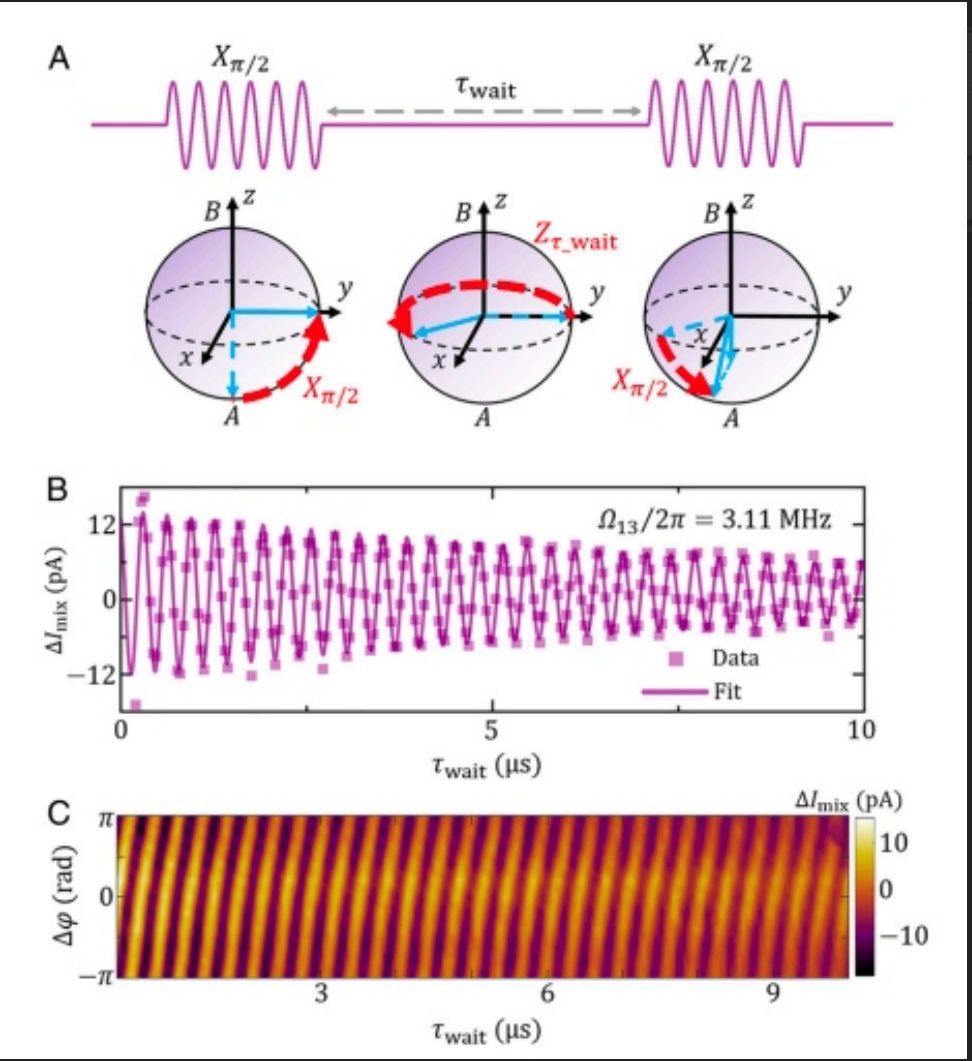
https://arxiv.org/abs/2202.09252

Quantum Resources

Ramsey experiment - T_2

It is an experimental sequence composed by two pi/2 pulses (¼ rotation in the Bloch sphere) separated by a time called Ramsey Time, where the qubit is free to dephase.

This define the T_2 time, or coherence time.

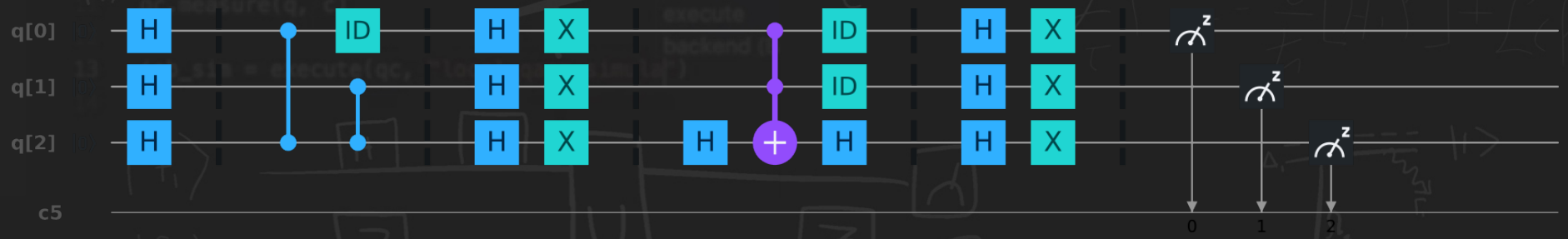


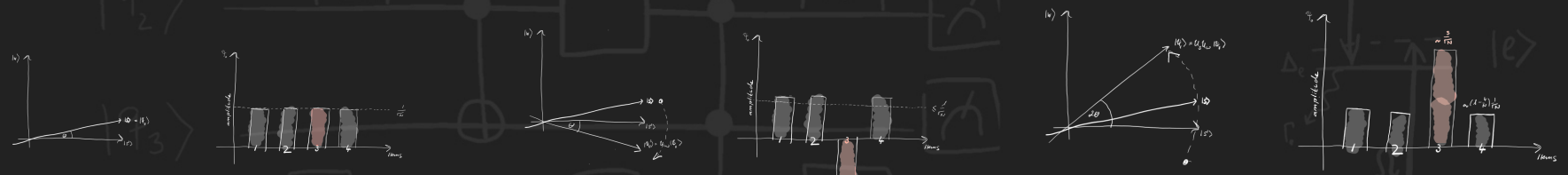
https://www.pnas.org/doi/pdf/10.1073/pnas.1916978117

Gate-based Quantum Computing

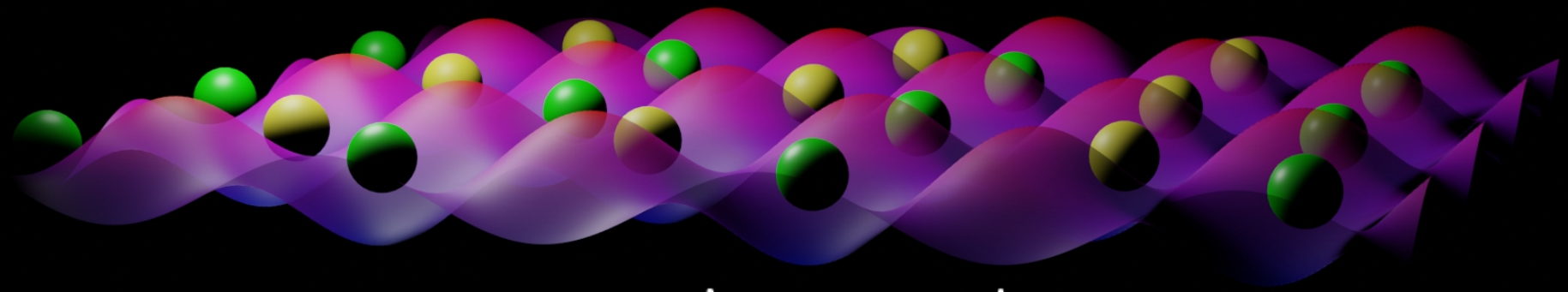
Quantum Algorithms

3-qubits Grover's Algorithm





THANK YOU FOR YOUR ATTENTION ;)



$$|\text{Questions}\rangle + |\text{answers}\rangle$$

Gather

Backup slides