Course Reminders

Due Sunday (11:59 PM)

- D4
- Q5
- Project Proposal
- Mid-course survey *(optional for EC, link also on Canvas assignment)*
- Weekly Project Survey (*optional, link also on Canvas assignment and homepage*)

Notes:

- A3 now available

Inferential Analysis

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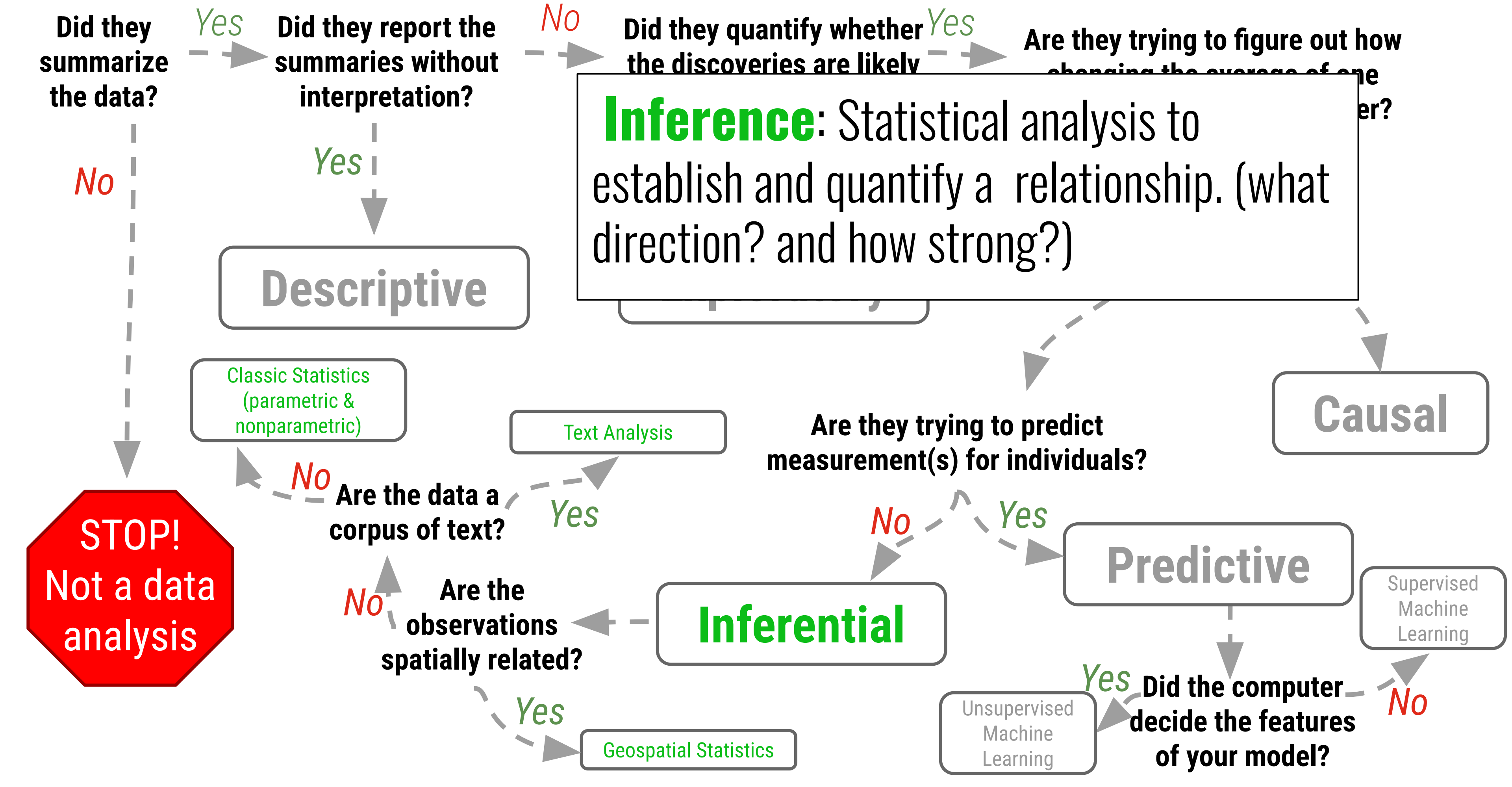
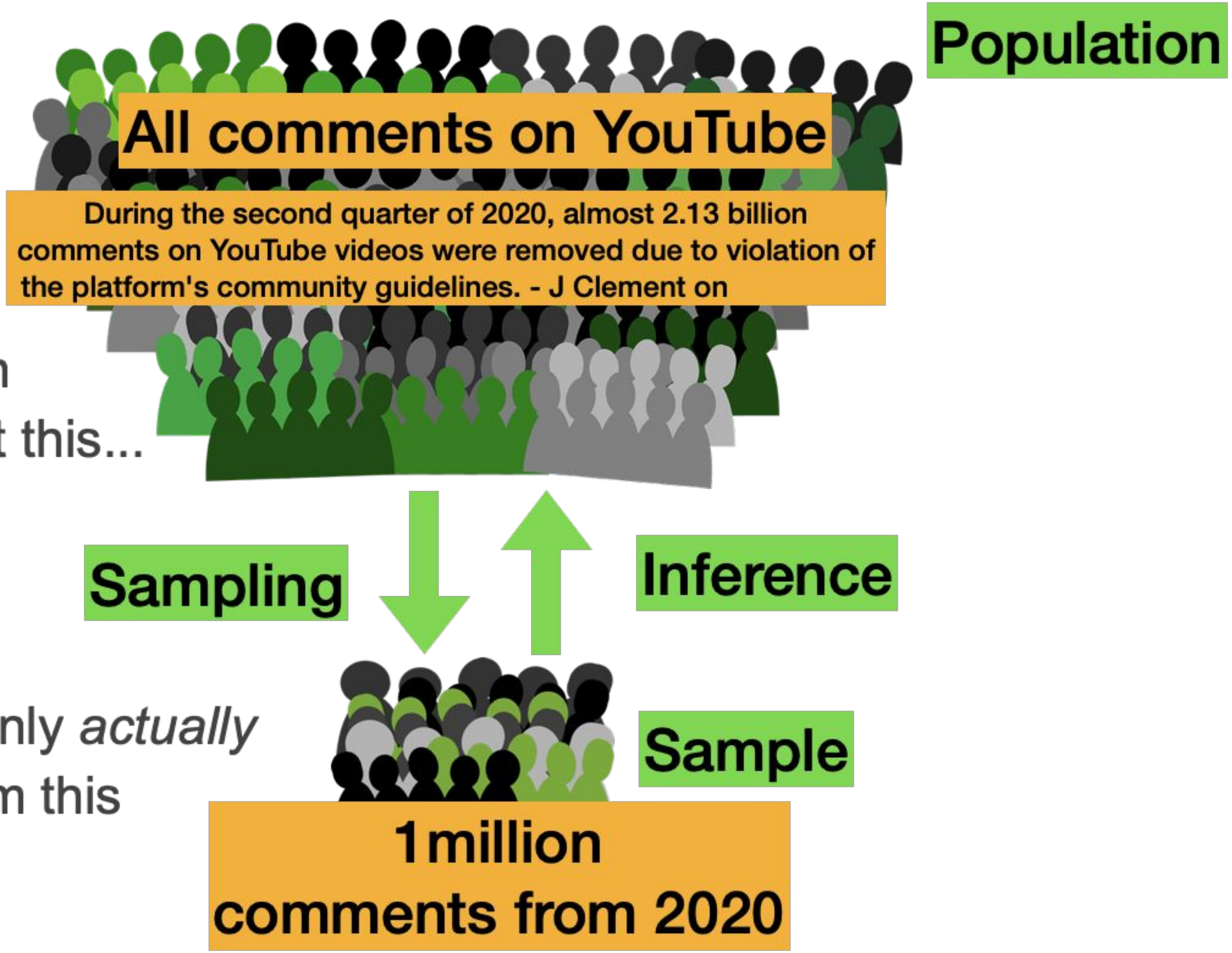

Did they summarize the data?
Yes
Did they report the summaries without interpretation?
No
Did they quantify whether the discoveries are likely
Yes
Are they trying to figure out how
Inference: Statistical analysis to establish and quantify a relationship. (what direction? and how strong?)
No
Yes
Descriptive
Classic Statistics (parametric & nonparametric)
Text Analysis
Are they trying to predict measurement(s) for individuals?
Causal
No
Are the data a corpus of text?
Yes
No
Yes
Predictive
STOP! Not a data analysis
No
Are the observations spatially related?
Inferential
Supervised Machine Learning
Yes
Did the computer decide the features of your model?
No
Yes
Unsupervised Machine Learning
Geospatial Statistics

We want to learn something about this...



....but we can only *actually* collect data from this

Air pollution control ?? Lifespan

What is the relationship between air pollution control and lifespan?

NIH Public Access
Author Manuscript
Epidemiology. Author manuscript; available in PMC 2014 January 01.

Published in final edited form as:
Epidemiology. 2013 January ; 24(1): 23–31. doi:10.1097/EDE.0b013e3182770237.

The Effect of Air Pollution Control on Life Expectancy in the United States: An Analysis of 545 US counties for the period 2000 to 2007

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Department of Economics, Brigham Young University, 142 Faculty Office Building, Provo, UT 84602

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A decrease of 10 µg/m3 in the concentration of $PM_{2.5}$ was associated with an increase in mean life expectancy of 0.35 years SD= 0.16 years, p = 0.033). This association was stronger in more urban and densely populated counties.

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3521092/

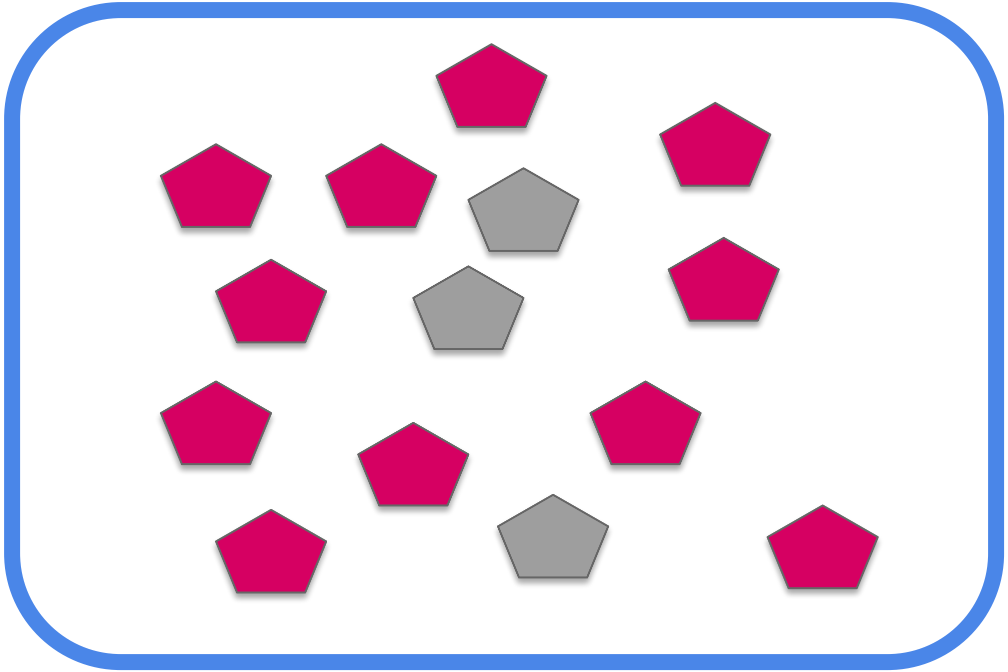
Establishing & Stating Your Null and Alternative Hypotheses Helps Guide Your Analysis

<u>Null Hypothesis</u>:

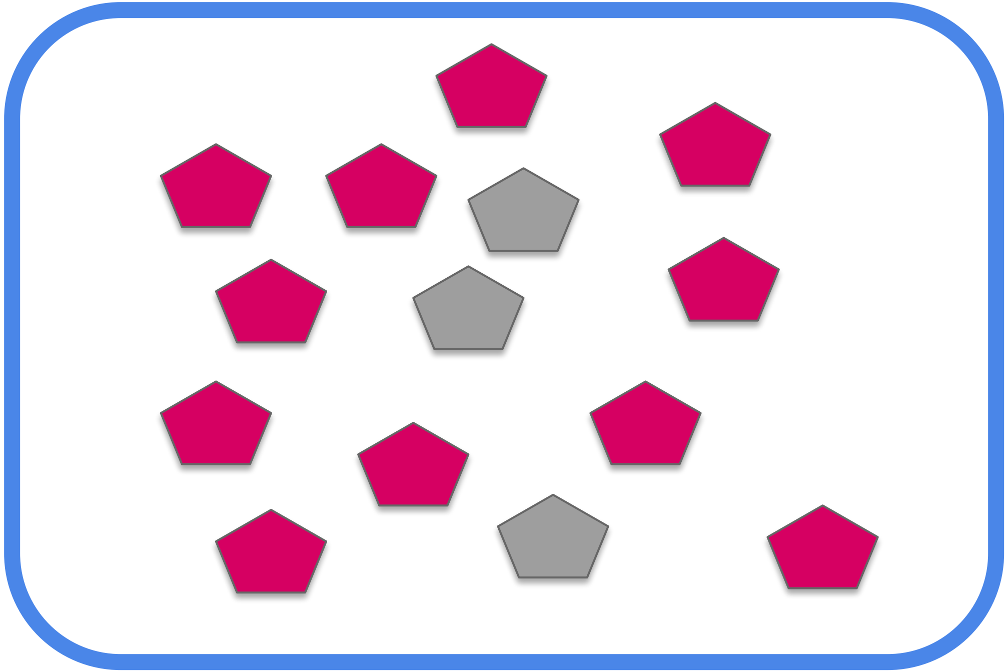
H_0: Air pollution has no effect on lifespan

<u>Alternative Hypothesis</u>:

H_a: Air pollution has an effect on lifespan

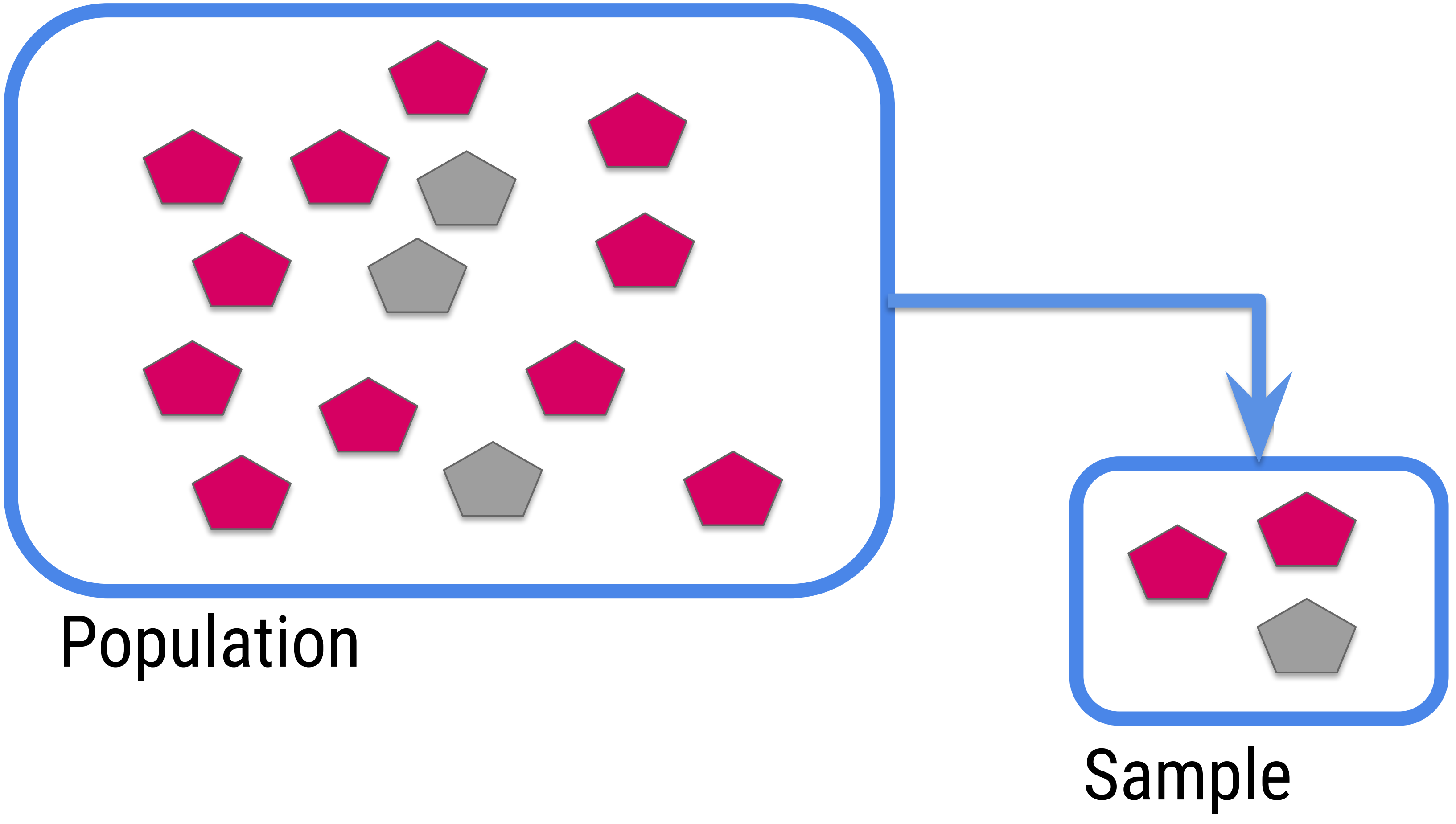


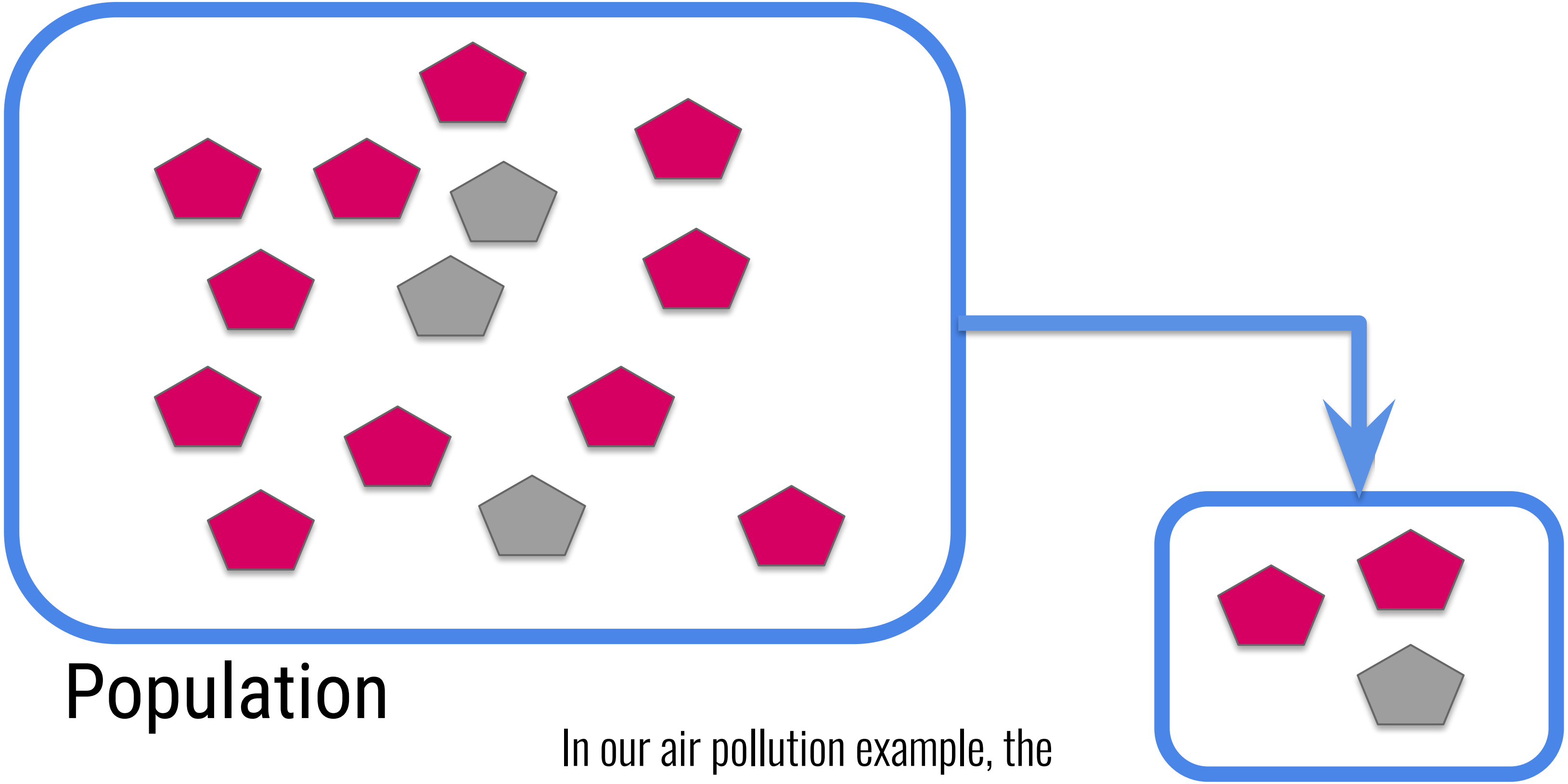
Population



Population

In our air pollution question, the population would be every individual in the US

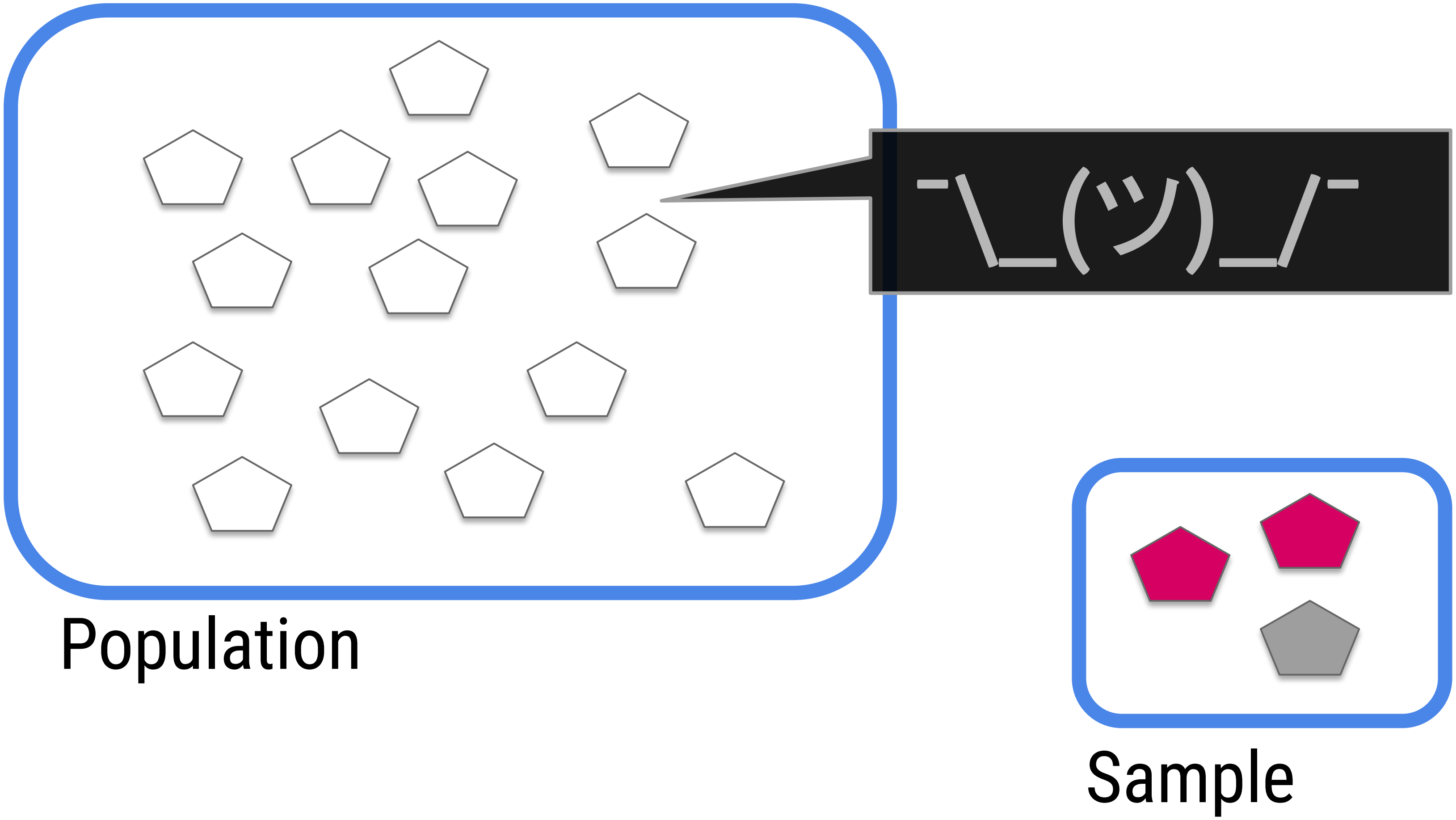

Population
Sample



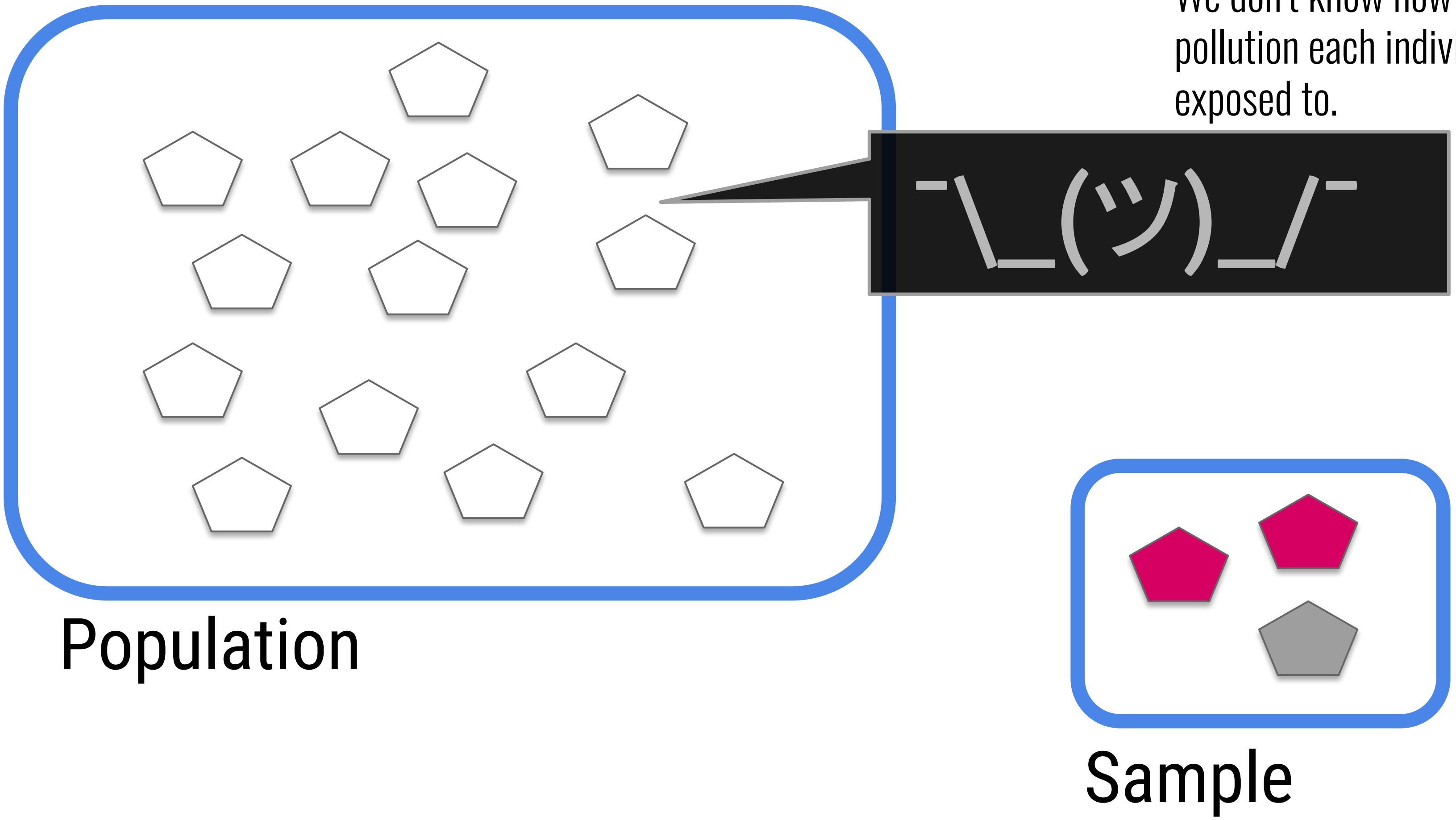
Population

In our air pollution example, the <u>sample</u> would be measurements for some of the US

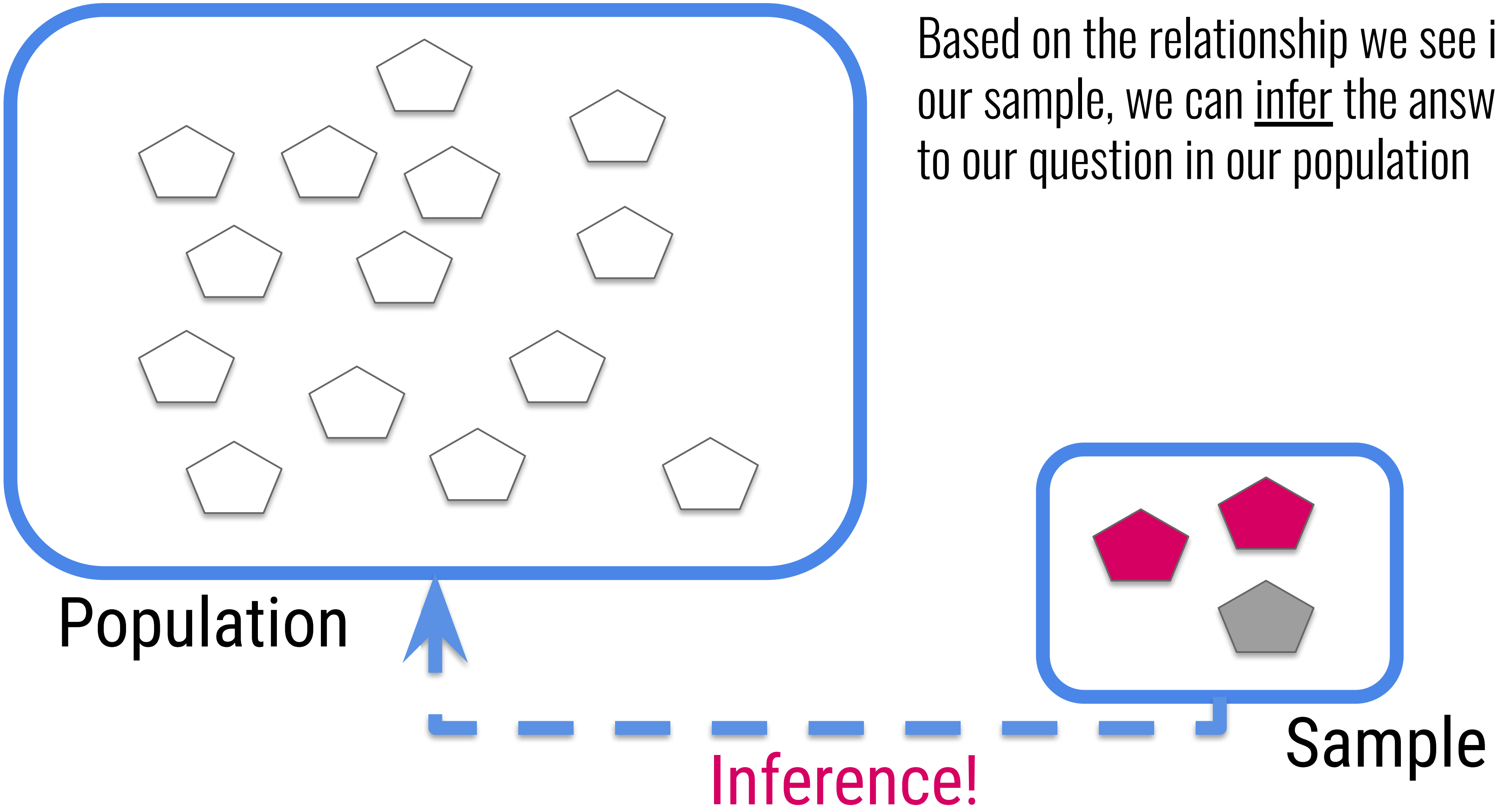
Sample


¯_(ツ)_/¯
Population
Sample

We don’t know how much air pollution each individual is exposed to.



Based on the relationship we see in our sample, we can infer the answer to our question in our population

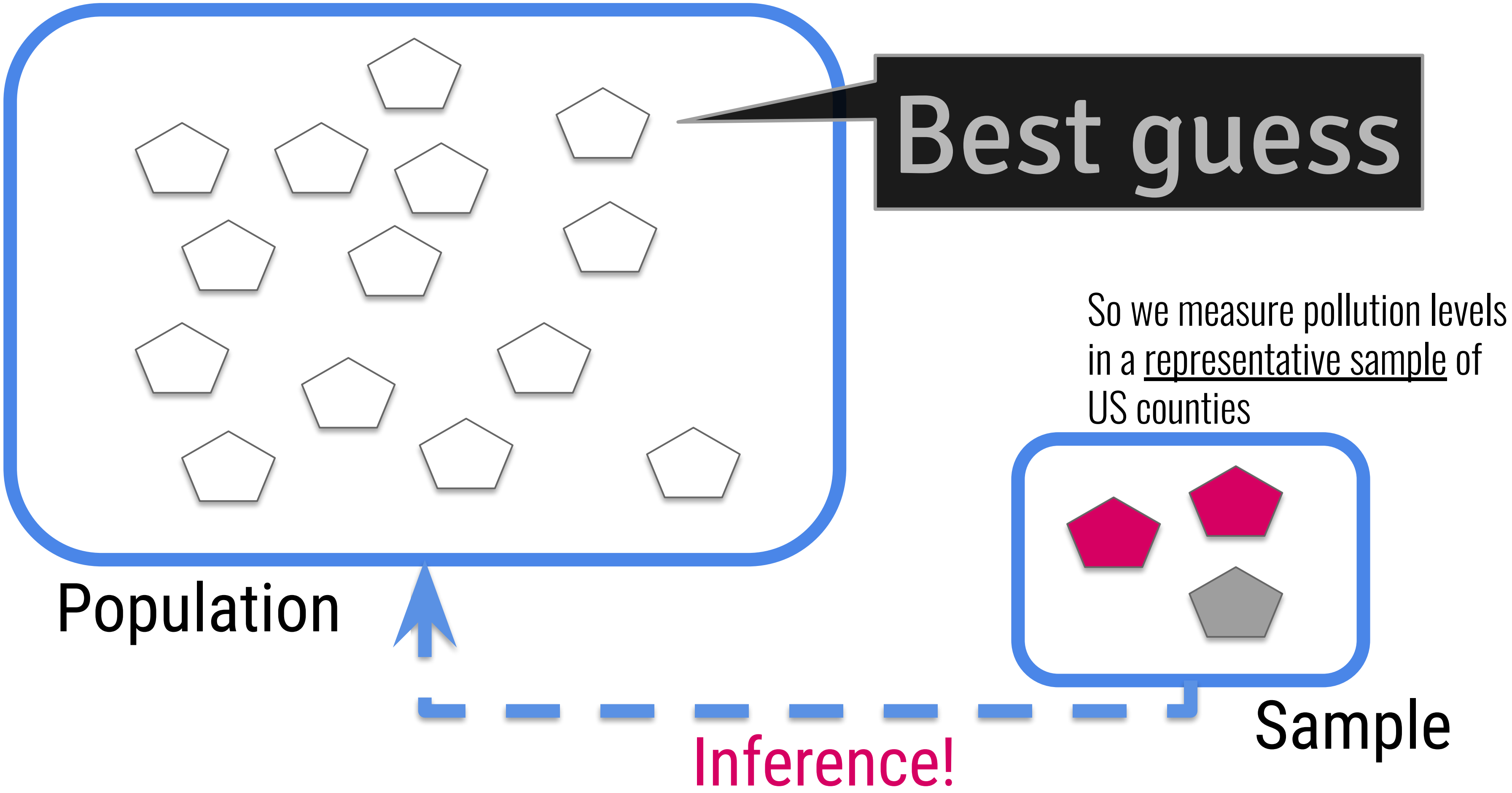


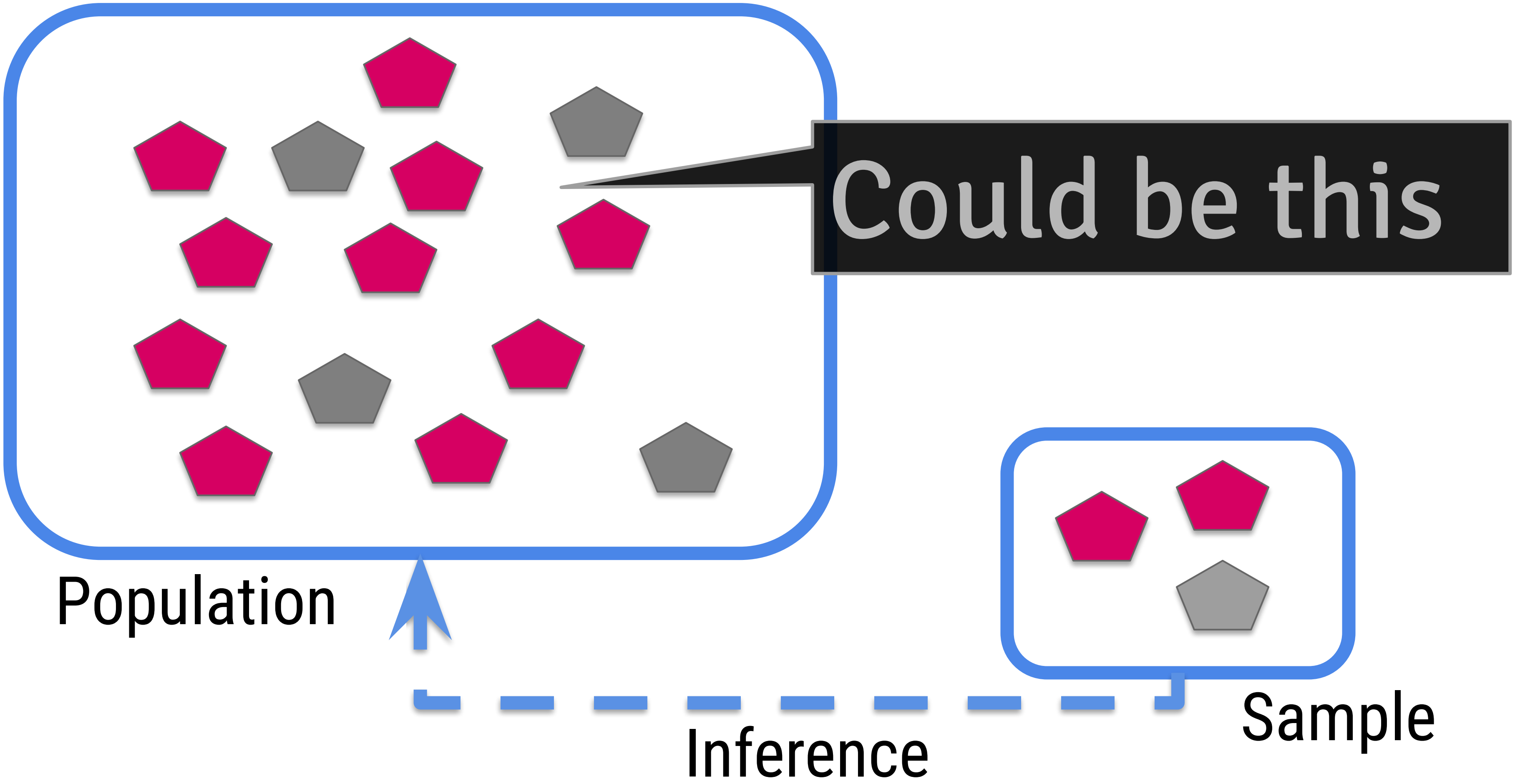
What would you need to consider when sampling air pollution in the US?

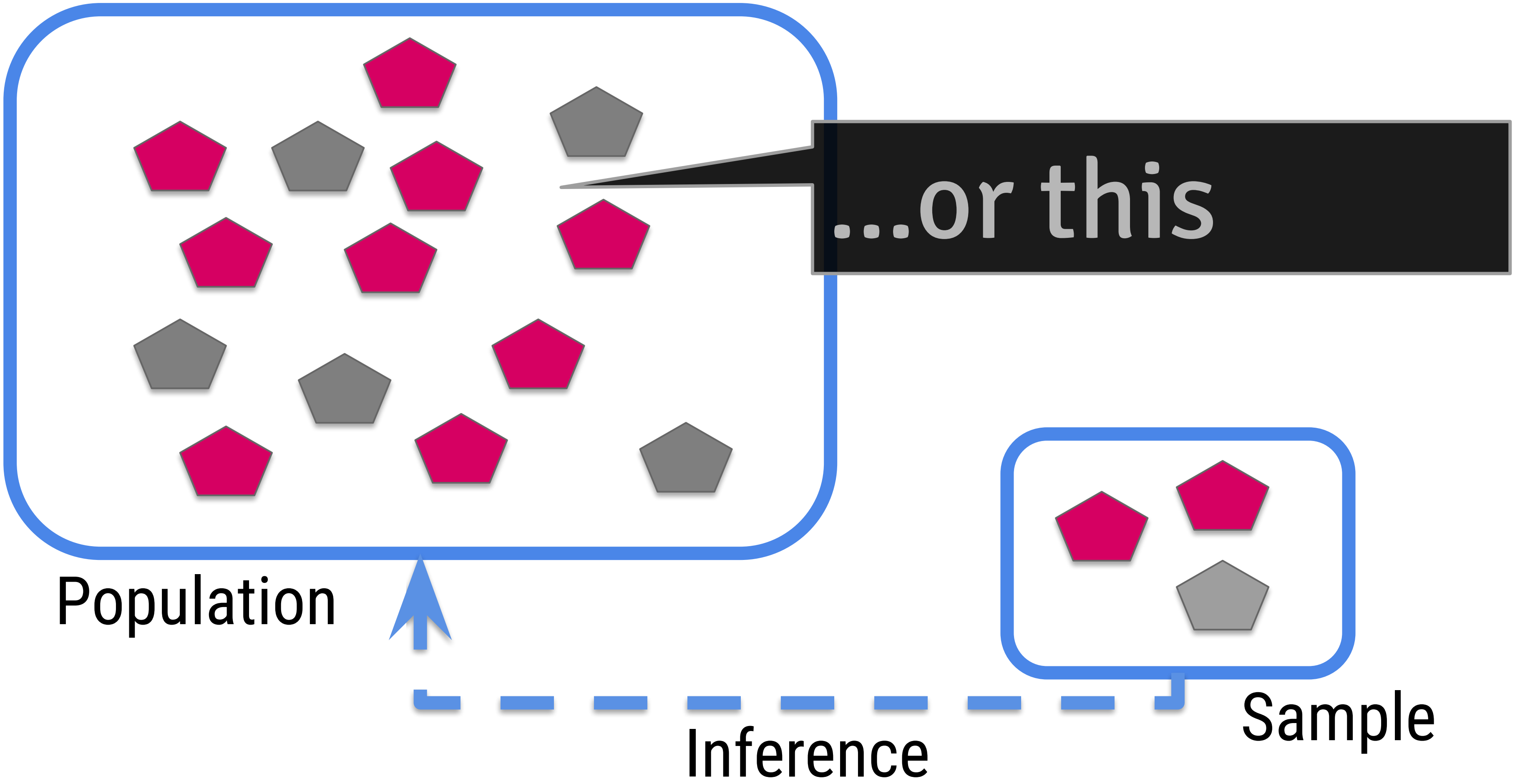
A
I have some ideas

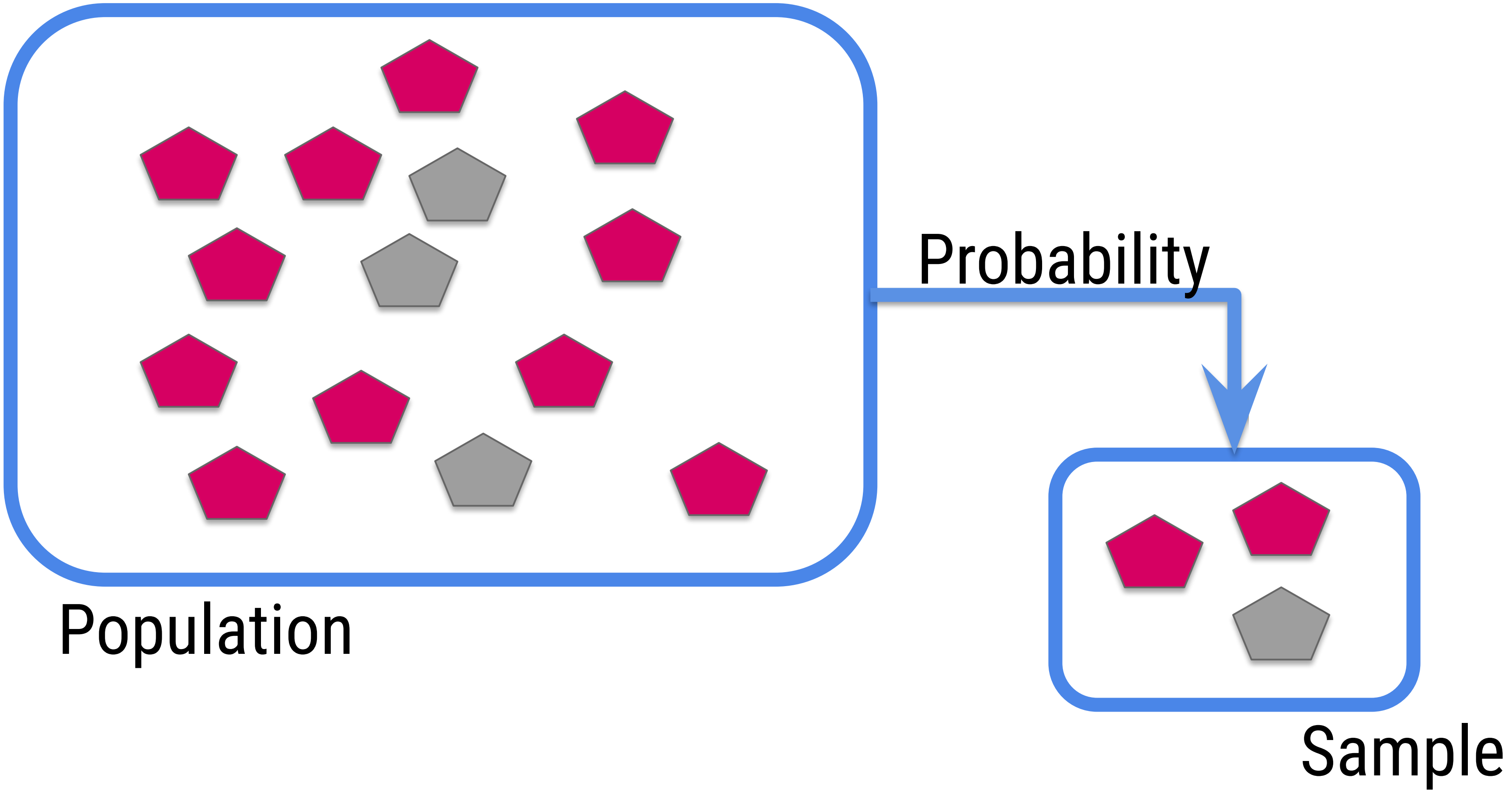
B
I've thought, but I don't know

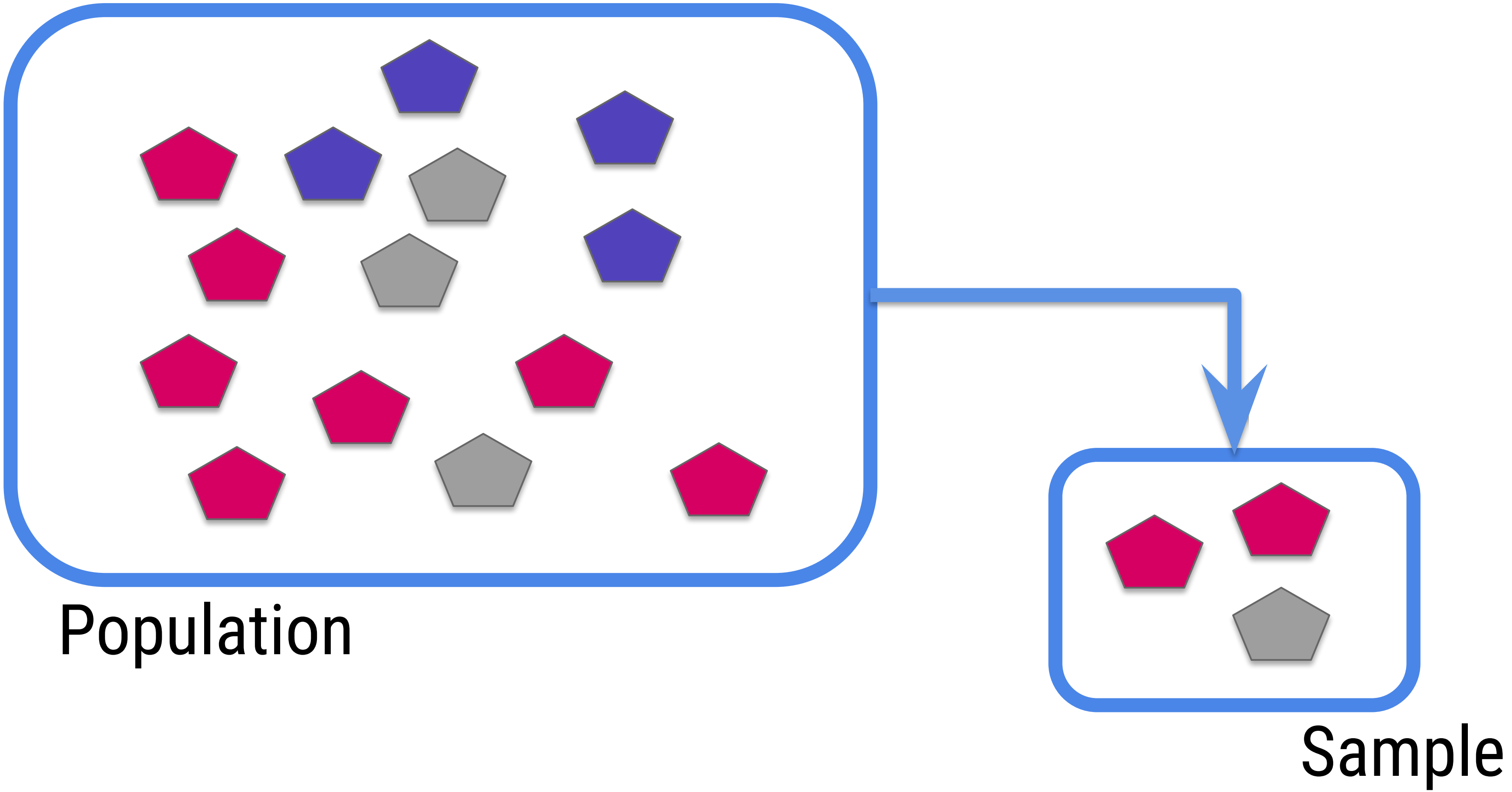
C
I don't understand the question


Best guess
So we measure pollution levels in a representative sample of US counties
Population
Sample
Inference!

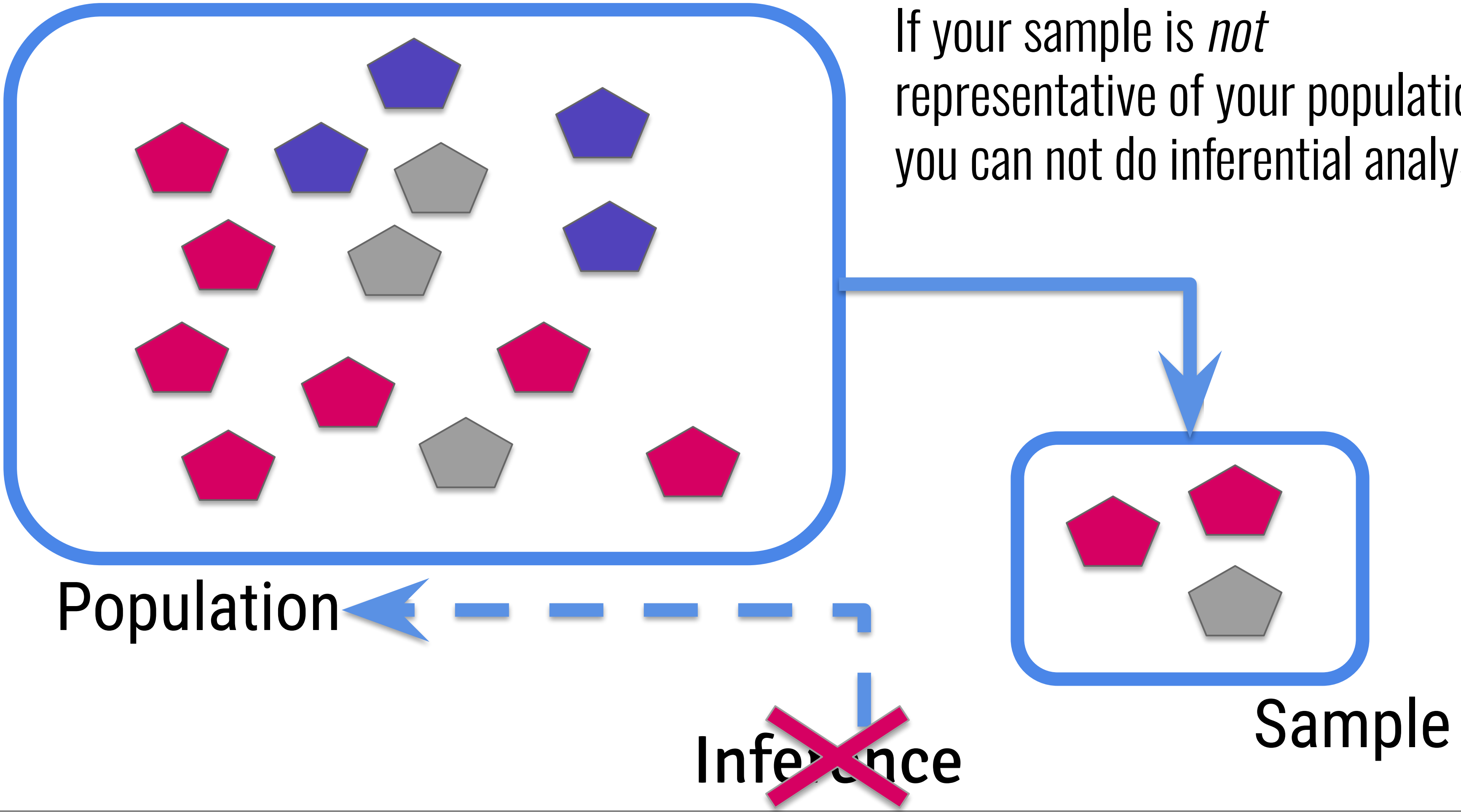

Could be this
Population
Sample
Inference

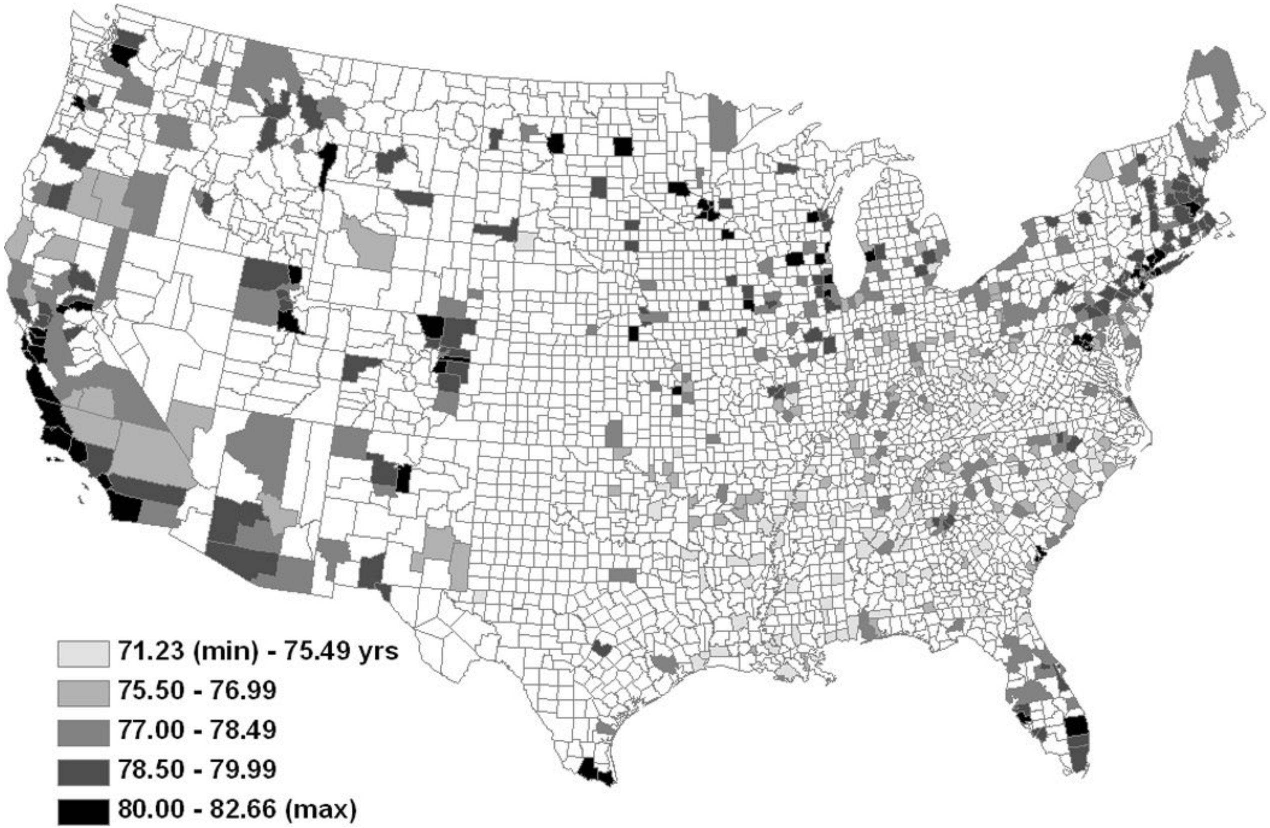

...or this
Population
Sample
Inference


Probability
Population
Sample

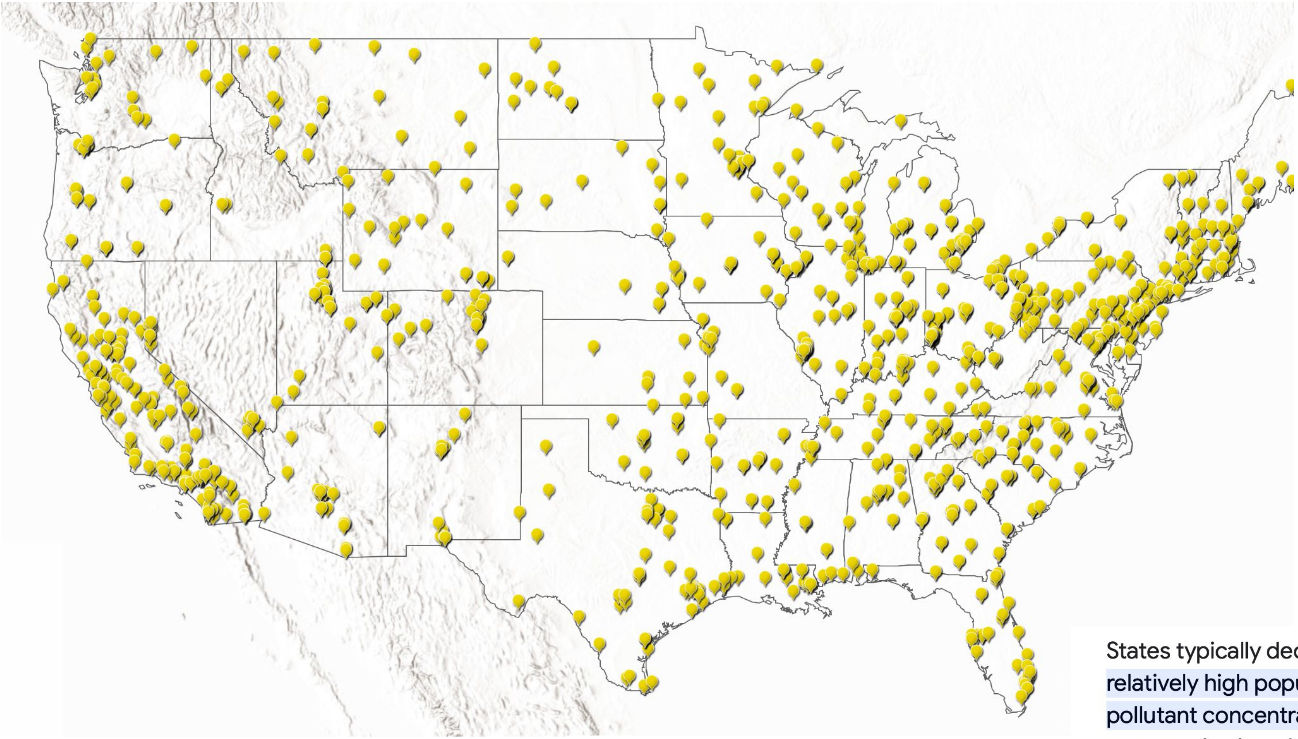

Population
Sample

If your sample is *not* representative of your population, you can not do inferential analysis.





All counties with with available matching PM2.5 data for 2000 and 2007 from the EPA's Air Quality System. Includes both metropolitan and non-metro counties



States typically decide where monitors are placed based on areas of relatively high population and/or areas believed to have relatively higher pollutant concentrations. Each state is responsible for developing its own monitoring plan, which is then reviewed and revised every five years. Aug 28, 2023

United States Environmental Protection Agency (.gov)
https://www.epa.gov › outdoor-air-quality-data › who-de...

Who decides where monitors get placed? | US EPA

Approaches to Inference

CORRELATION

ASSOCIATION BETWEEN VARIABLES

i.e. Pearson Correlation, Spearman Correlation, chi-square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS BETWEEN VARIABLES

i.e. t-test, ANOVA

REGRESSION

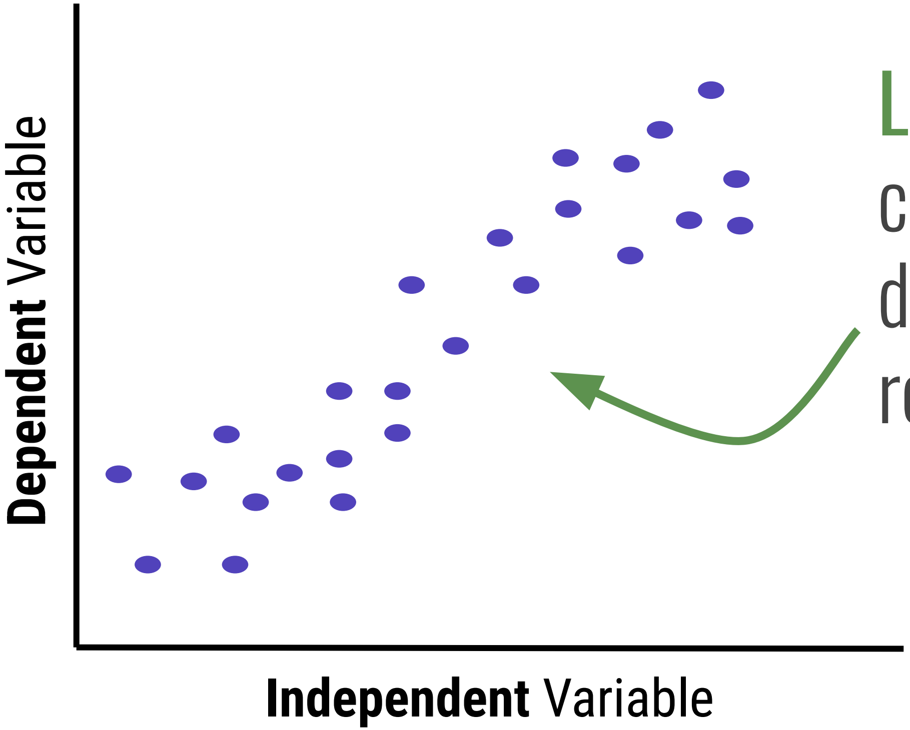
DOES CHANGE IN ONE VARIABLE MEAN CHANGE IN ANOTHER?

I.e. simple regression, multiple regression

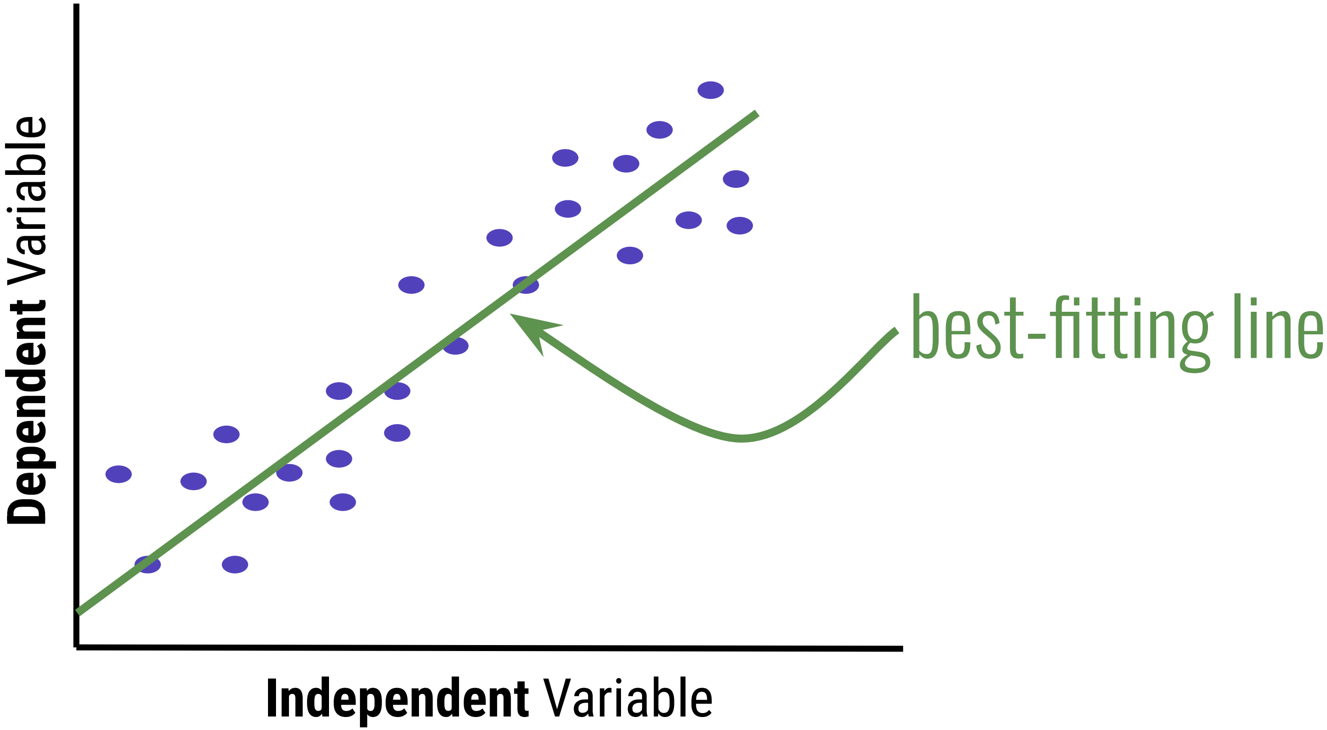
NON-PARAMETRIC TESTS

FOR WHEN ASSUMPTIONS IN THESE OTHER 3 CATEGORIES ARE NOT MET

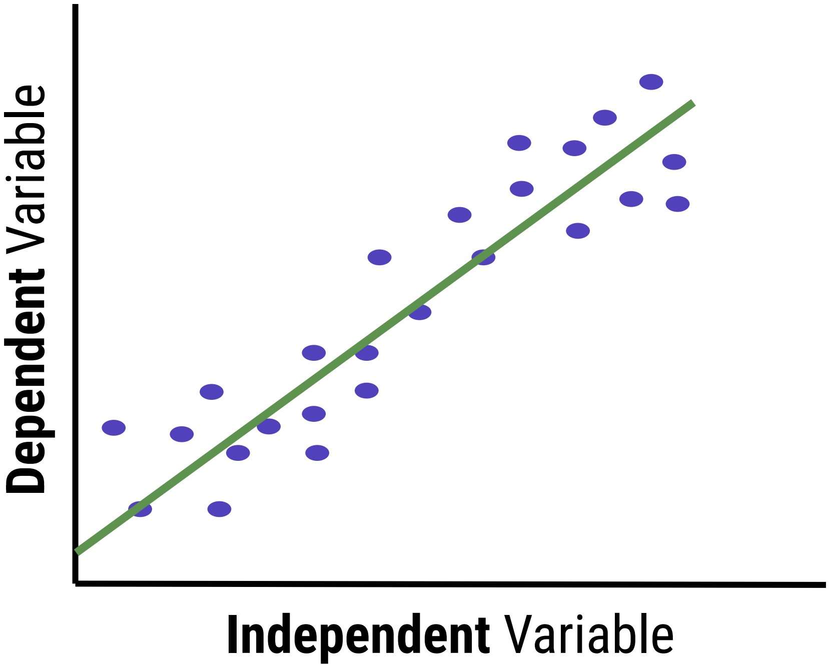
i.e. Wilcoxon rank-sum test, Wilcoxon sign-rank test, sign test



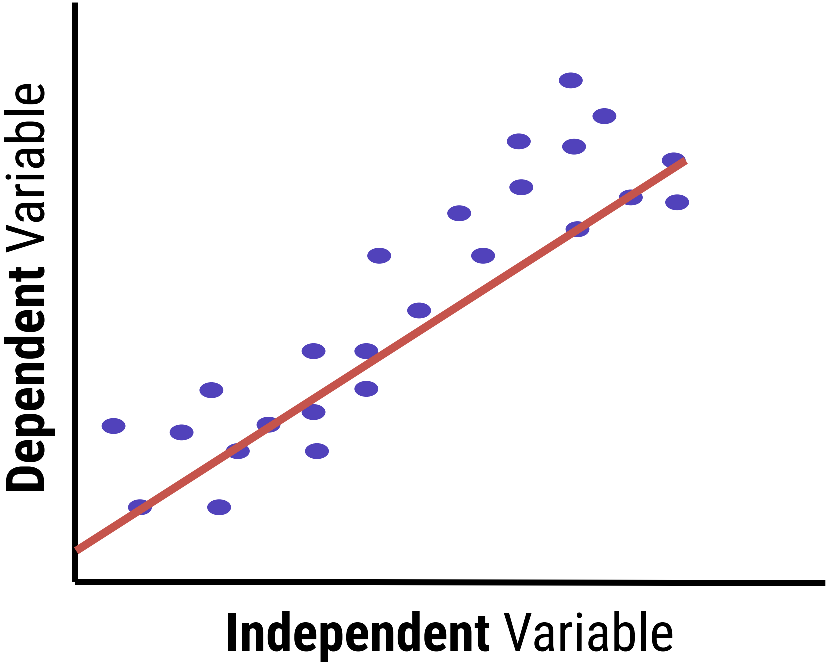
Linear regression can be used to describe this relationship

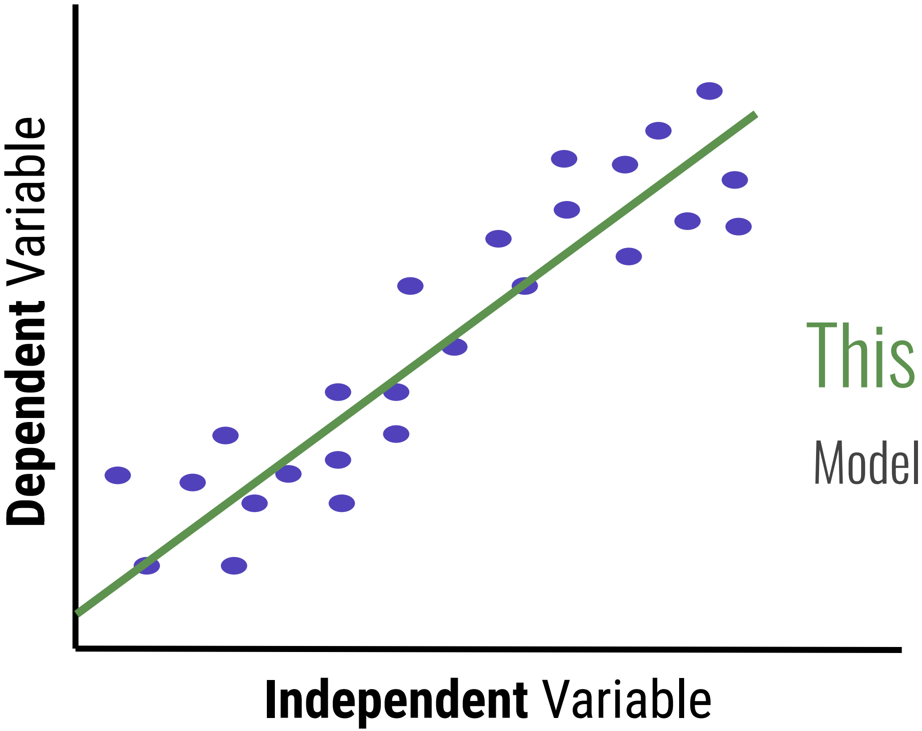

Dependent Variable
best-fitting line
Independent Variable

Best-fitting line



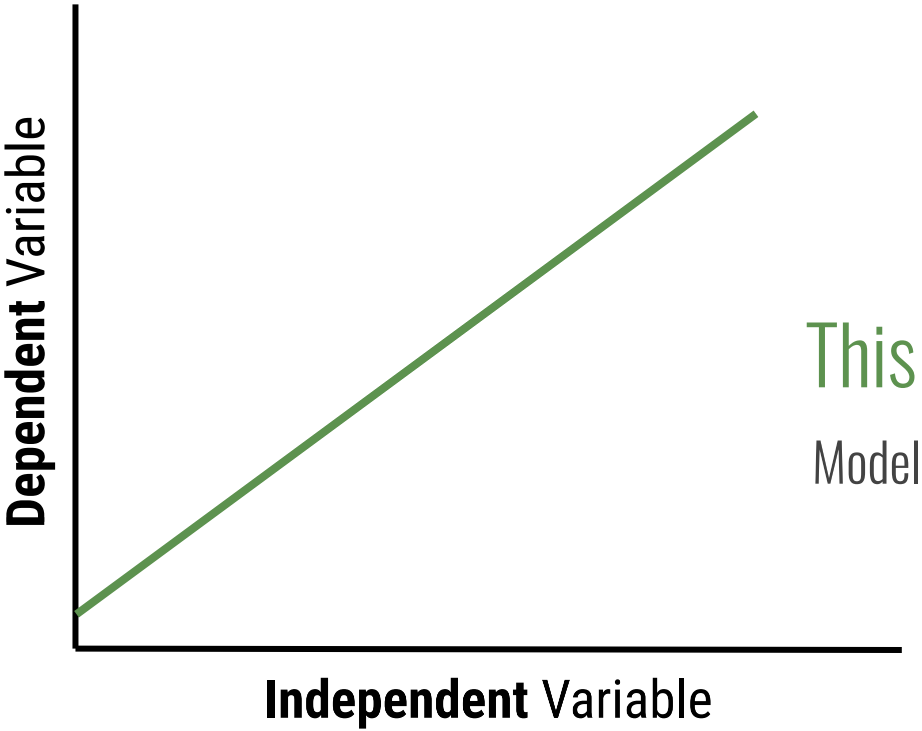
NOT a best-fitting line





This line is a **model** of the data

Models are mathematical equations generated to *represent* the real life situation



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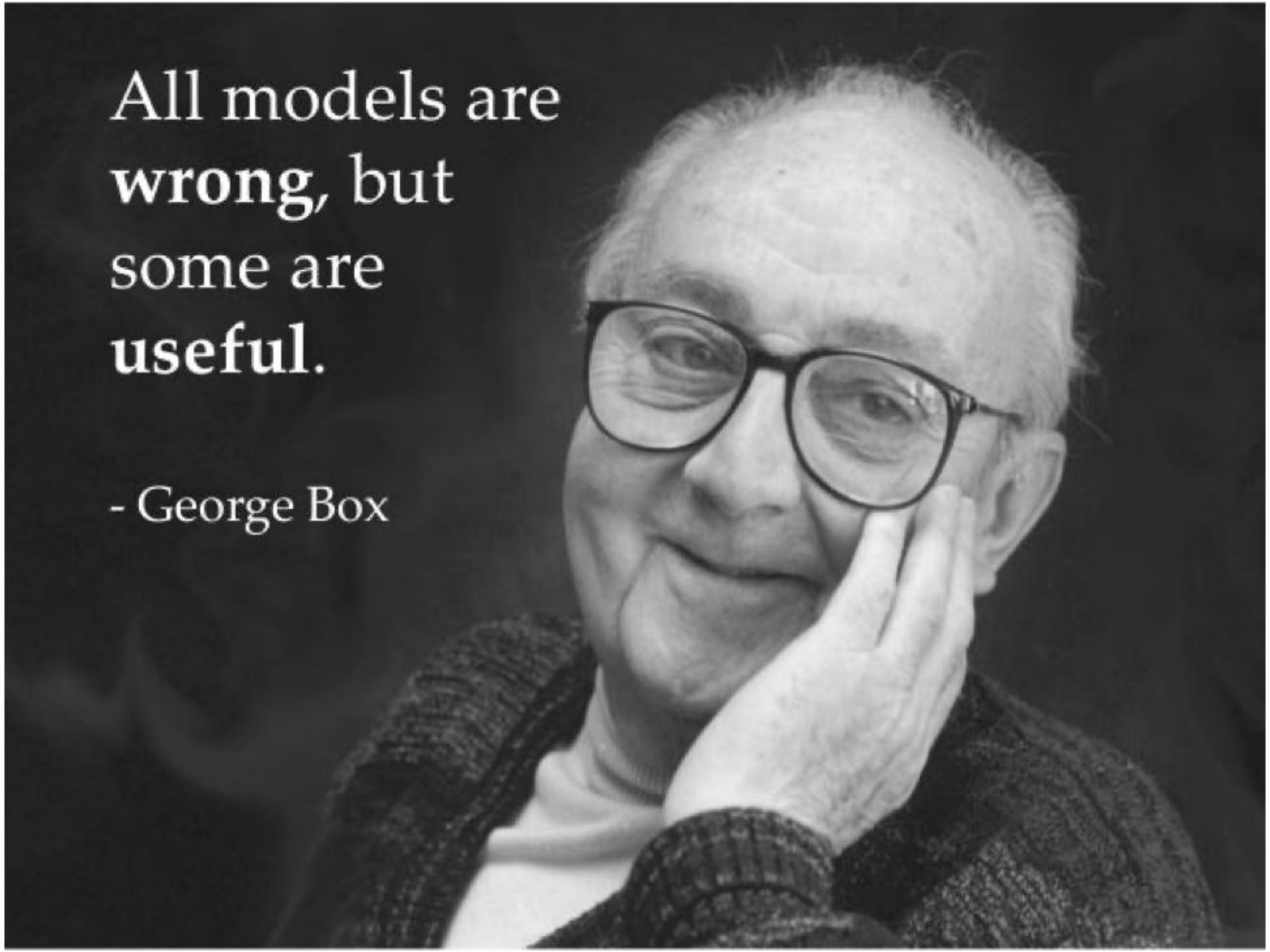
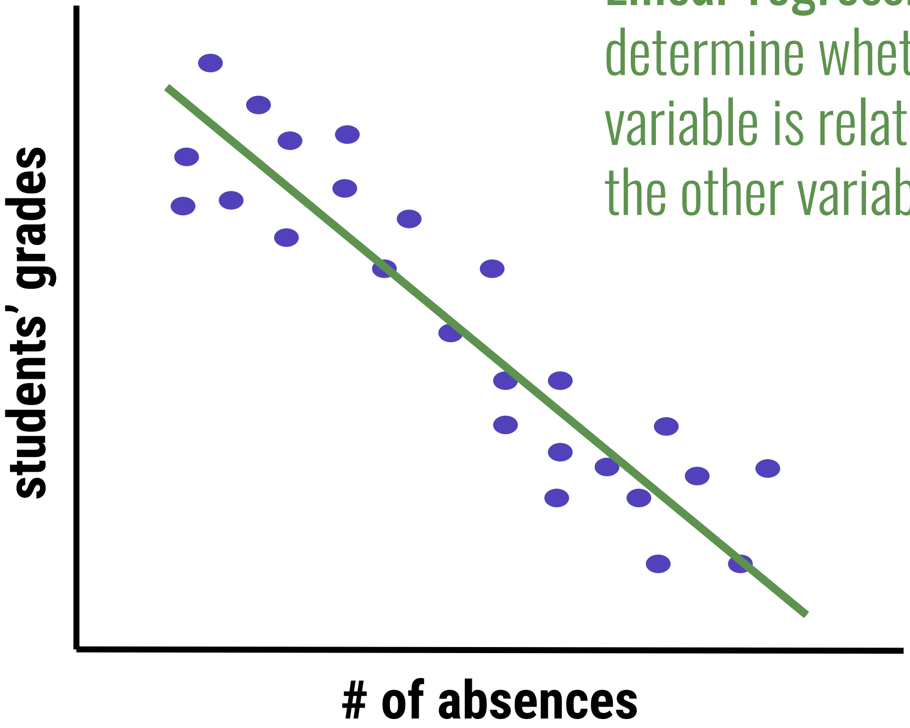
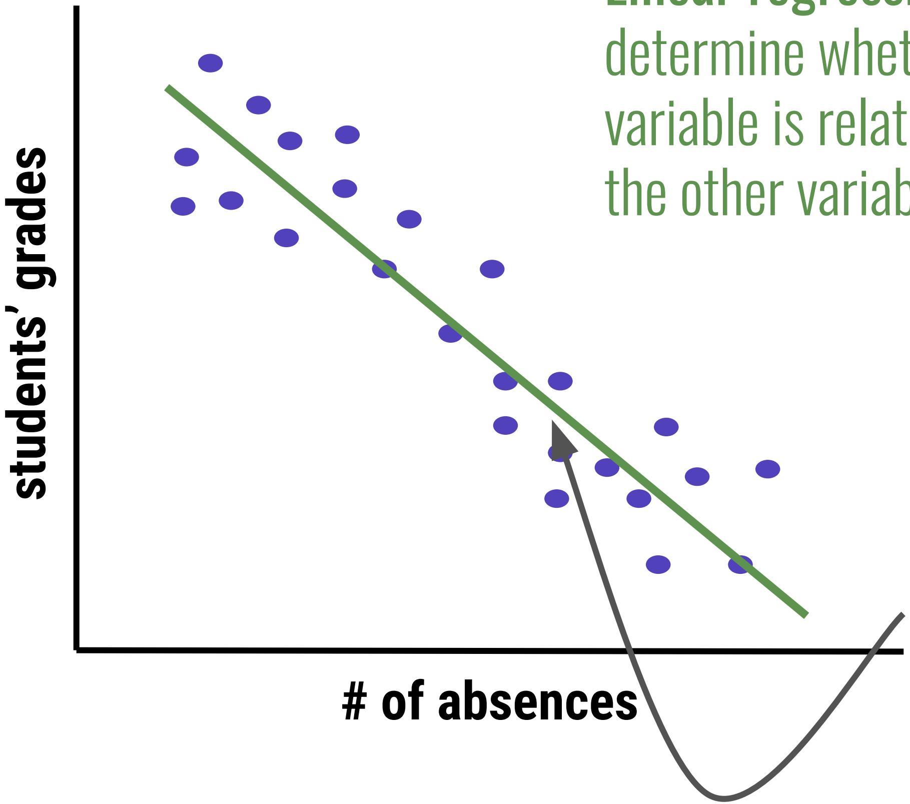

All models are **wrong**, but some are **useful**.
- George Box



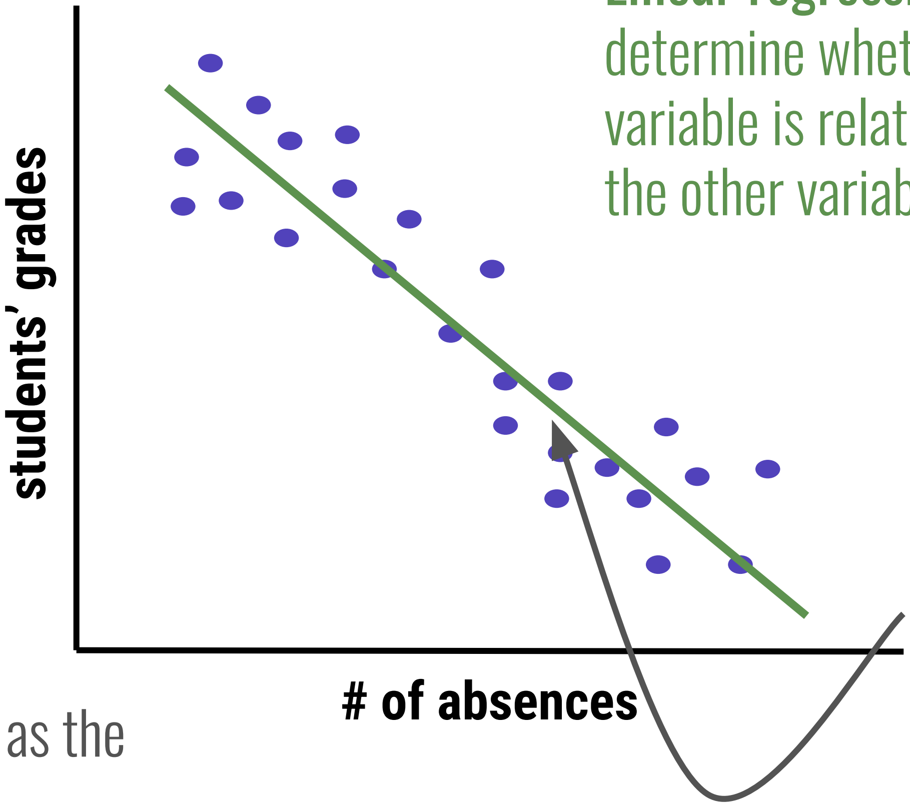
Linear regression can be used to determine whether a change in one variable is related to the change in the other variable

Linear regression can be used to determine whether a change in one variable is related to the change in the other variable



The magnitude of the relationship is measured by the slope of the line

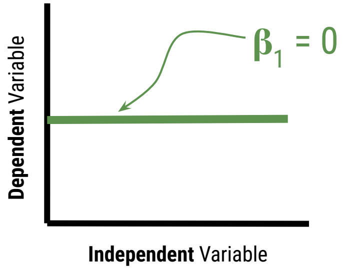
Linear regression can be used to determine whether a change in one variable is related to the change in the other variable



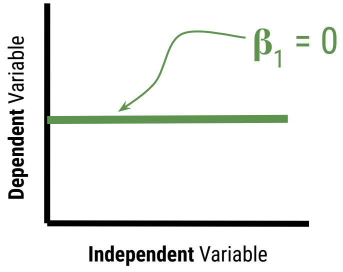
The magnitude of the relationship is measured by the slope of the line

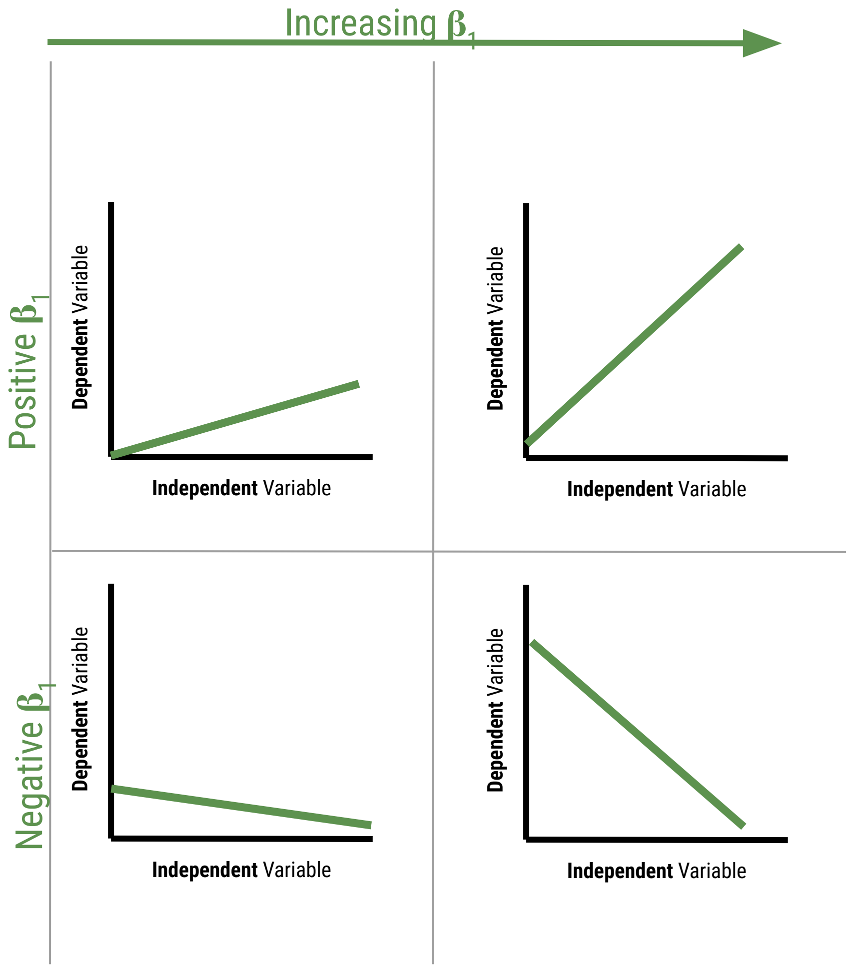
This is also referred to as the model's effect size (β_1)

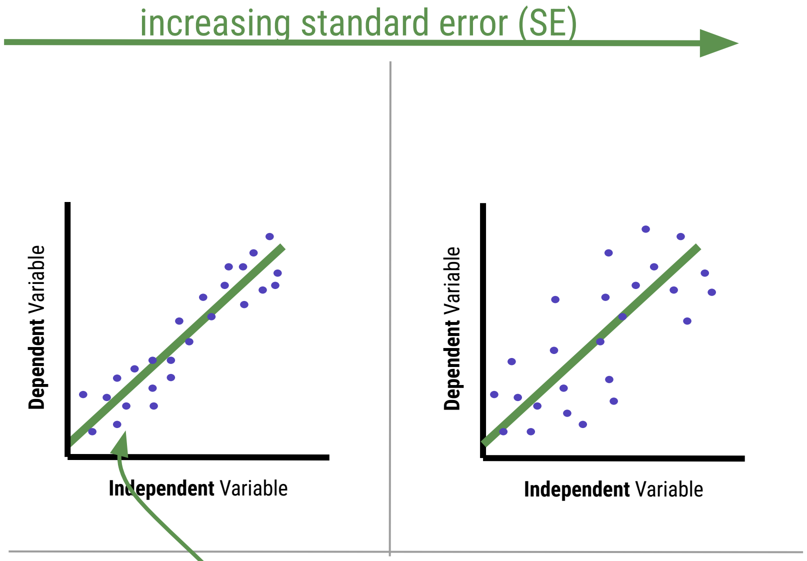
Effect size (β_1) can be estimated using the slope of the line



Effect size (β_1) can be estimated using the slope of the line



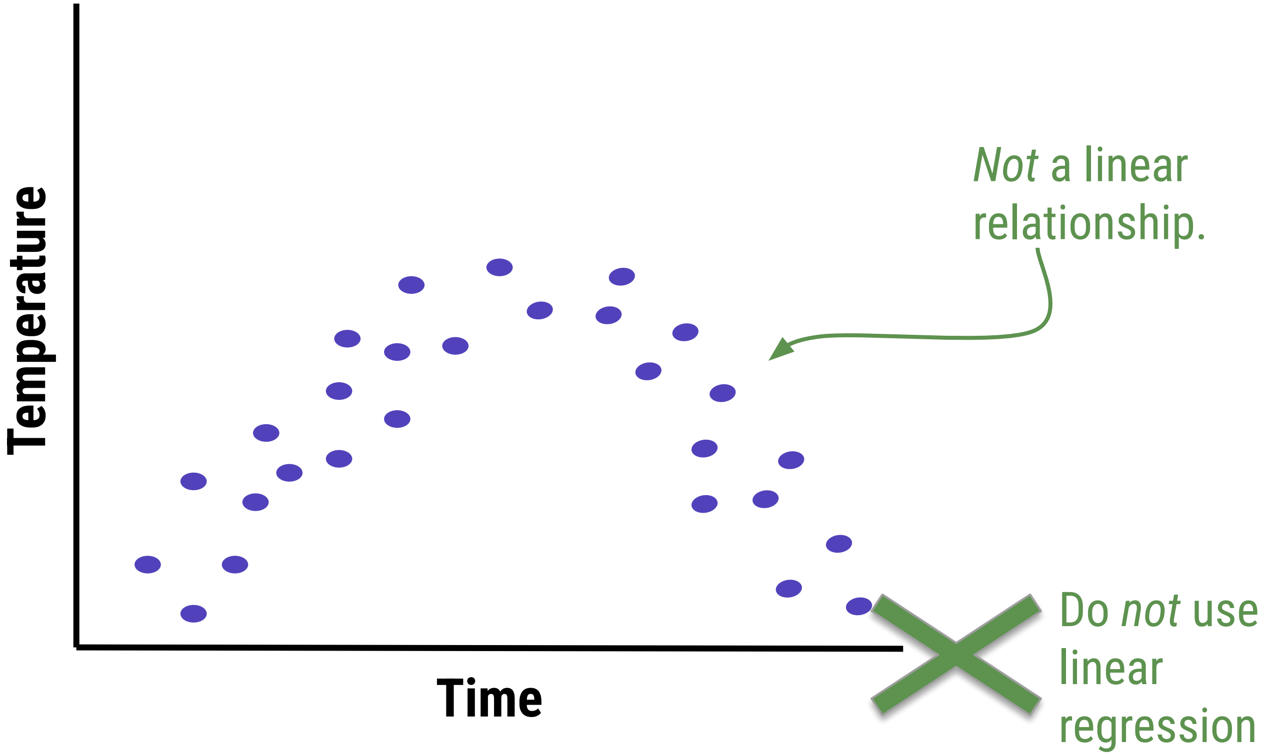




The *closer* the points are to the regression line, the *less uncertain* we are in our estimate

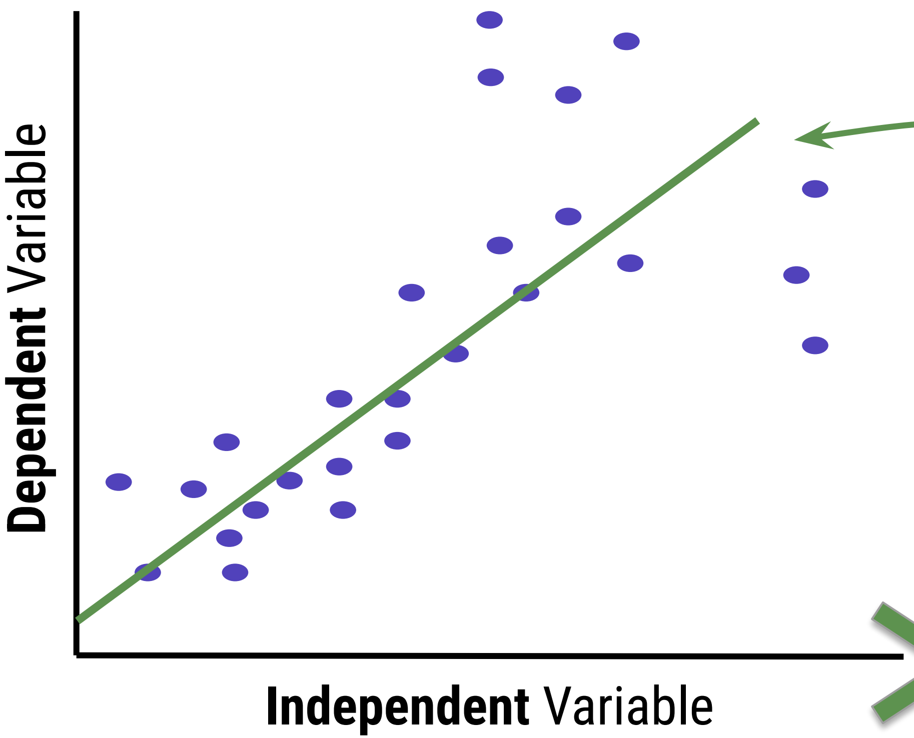
Assumptions of linear regression

1. Linear relationship
2. No multicollinearity
3. No auto-correlation
4. Homoscedasticity


Temperature
Time
Not a linear relationship.
Do *not* use linear regression

Linear regression assumes no multicollinearity. **Multicollinearity** occurs when the independent variables (in multiple linear regression) are too highly correlated with each other.

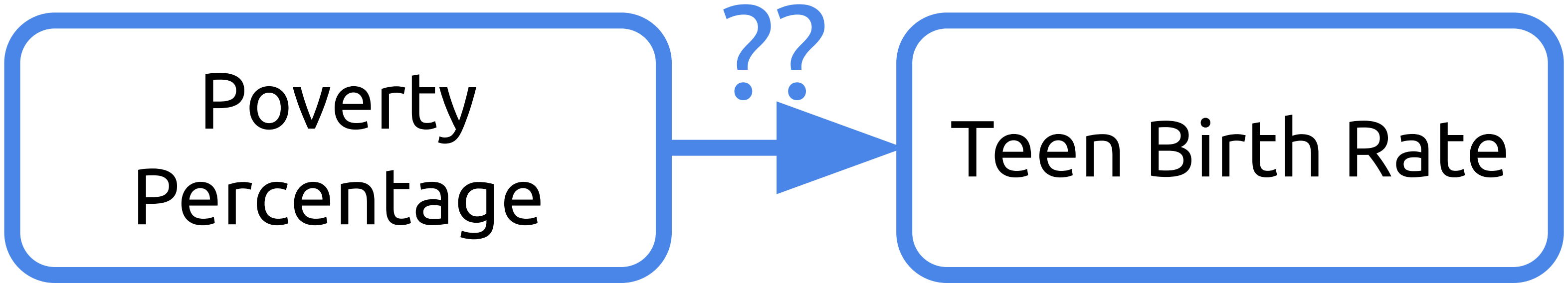
Autocorrelation occurs when the observations are *not* independent of one another (i.e. stock prices)



Not homoscedastic: points at this end are much further from the line than at the other end

Do *not* use linear regression

Does Poverty Percentage affect Teen Birth Rate?



<u>Null Hypothesis</u>:

H_0: Poverty Rate does not affect Teen Birth Rate (β_1=0)

<u>Alternative Hypothesis</u>:

H_a: Poverty Rate affects Teen Birth Rate ($\beta_1 \neq 0$)

What is the relationship between Poverty Percentage & Teen Birth Rate?

What's your hypothesis?

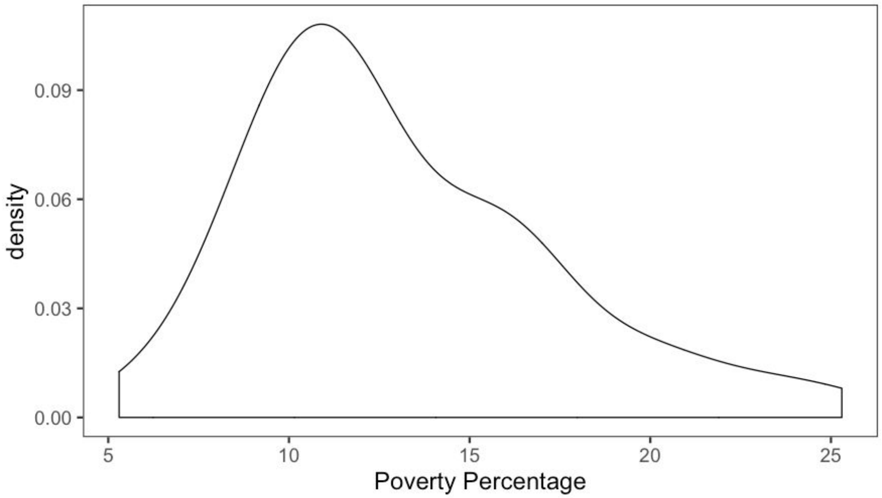
A
No relationship

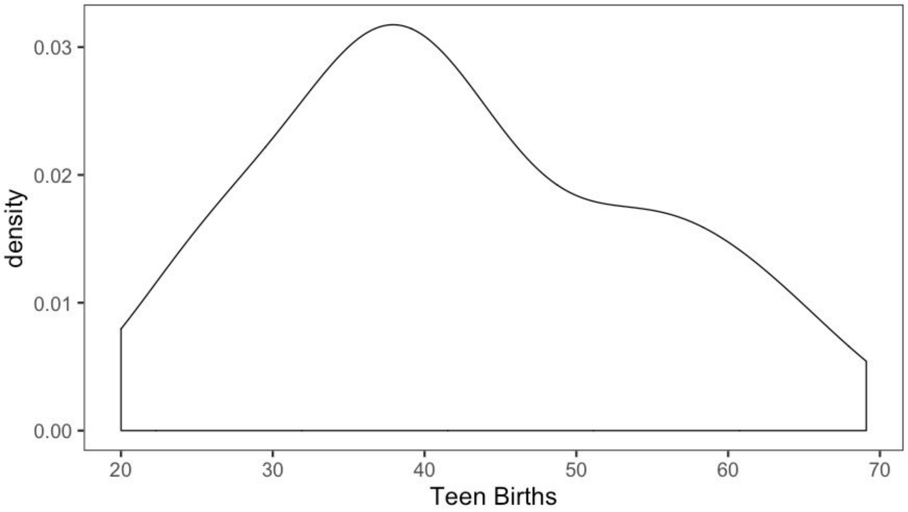
B
Positive relationship

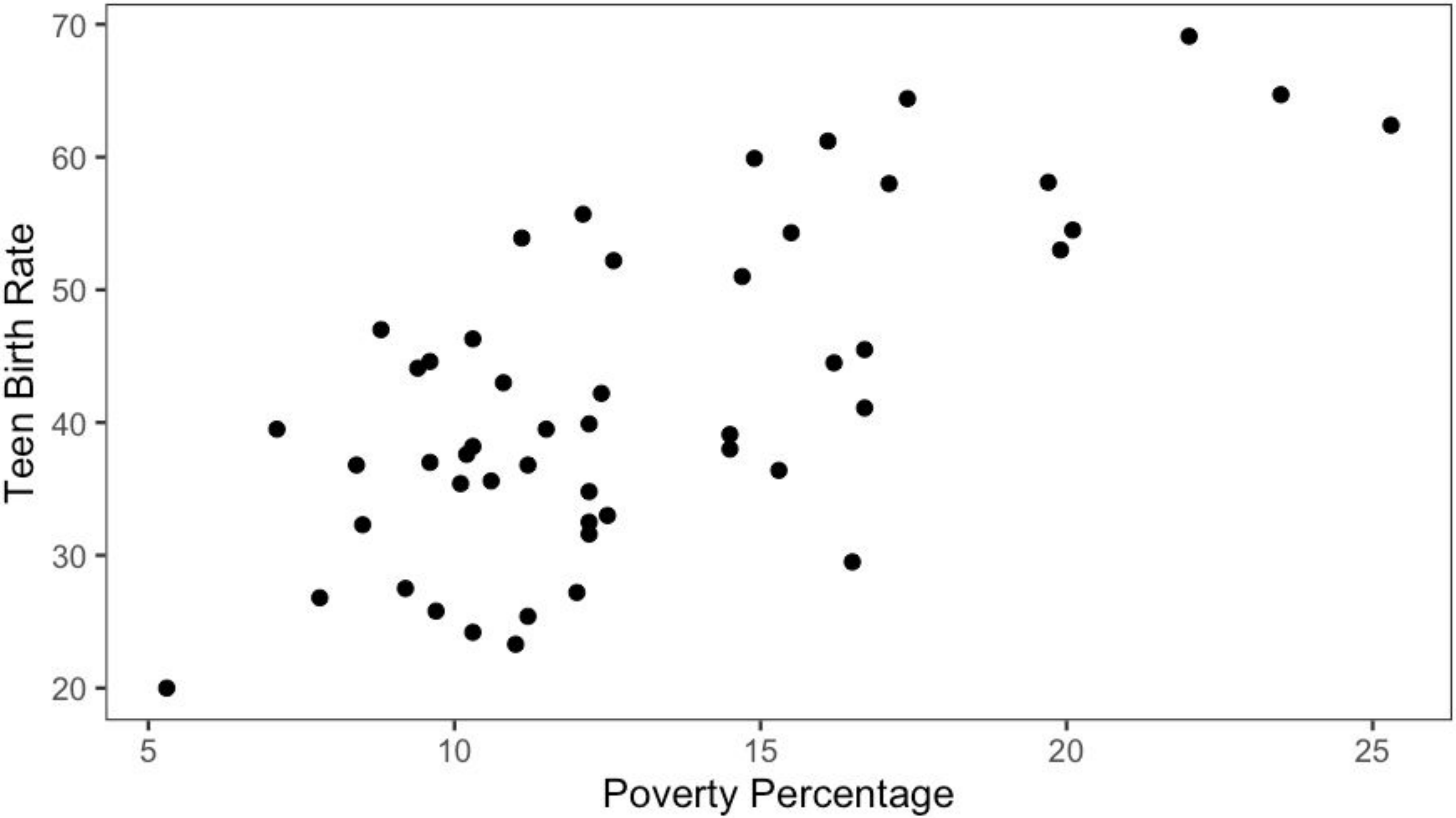
C
Negative relationship

	Location	PovPct	Brth15to17	Brth18to19	ViolCrime	TeenBrth
1	Alabama	20.1	31.5	88.7	11.2	54.5
2	Alaska	7.1	18.9	73.7	9.1	39.5
3	Arizona	16.1	35.0	102.5	10.4	61.2
4	Arkansas	14.9	31.6	101.7	10.4	59.9
5	California	16.7	22.6	69.1	11.2	41.1
6	Colorado	8.8	26.2	79.1	5.8	47.0
7	Connecticut	9.7	14.1	45.1	4.6	25.8
8	Delaware	10.3	24.7	77.8	3.5	46.3
9	District_of_Columbia	22.0	44.8	101.5	65.0	69.1
10	Florida	16.2	23.2	78.4	7.3	44.5
11	Georgia	12.1	31.4	92.8	9.5	55.7
12	Hawaii	10.3	17.7	66.4	4.7	38.2
13	Idaho	14.5	18.4	69.1	4.1	39.1
14	Illinois	12.4	23.4	70.5	10.3	42.2
15	Indiana	9.6	22.6	78.5	8.0	44.6
16	Iowa	12.2	16.4	55.4	1.8	32.5
17	Kansas	10.8	21.4	74.2	6.2	43.0

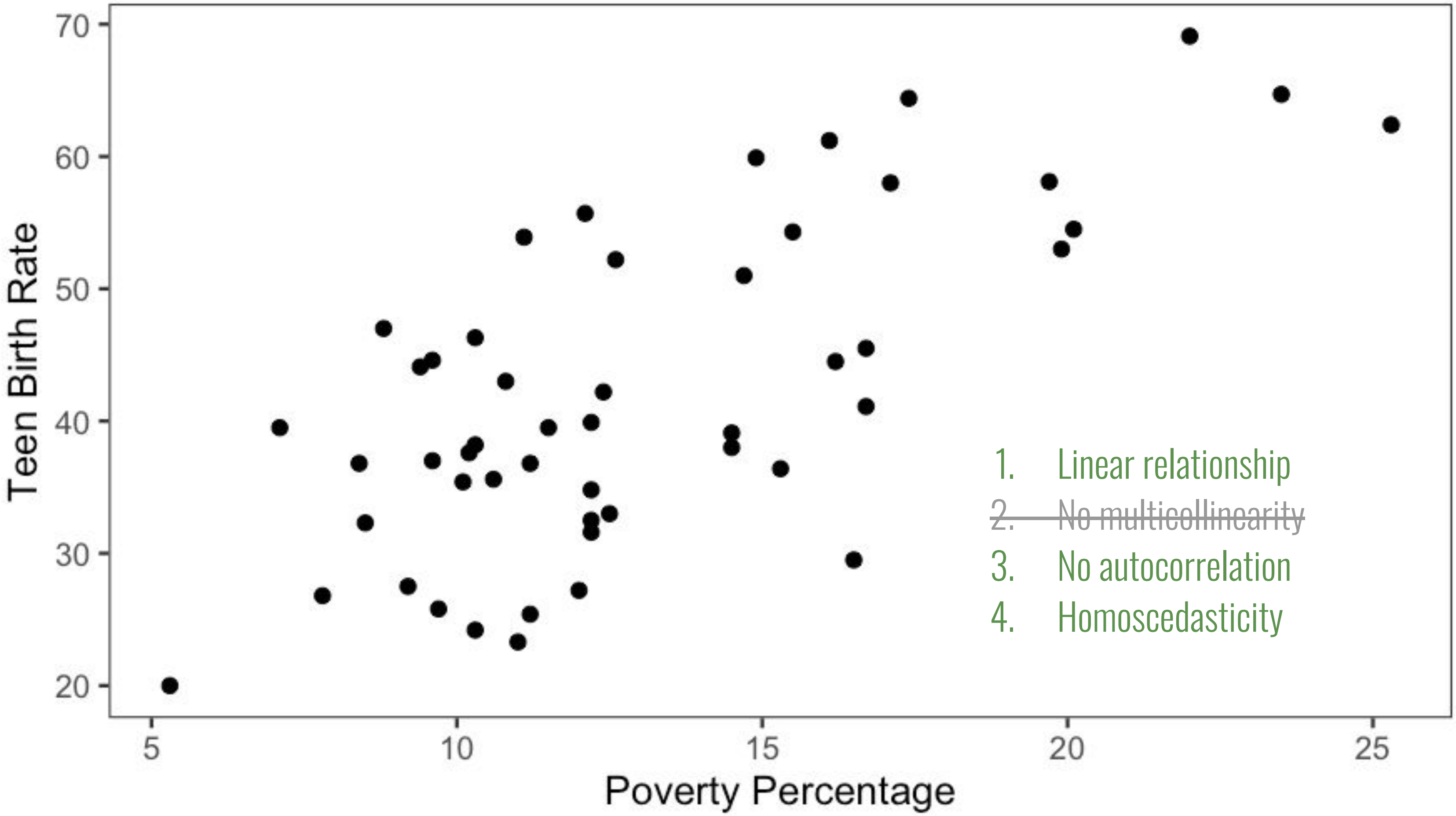
EDA: distributions







Data source: *Mind On Statistics*, 3rd edition, Utts and Heckard.



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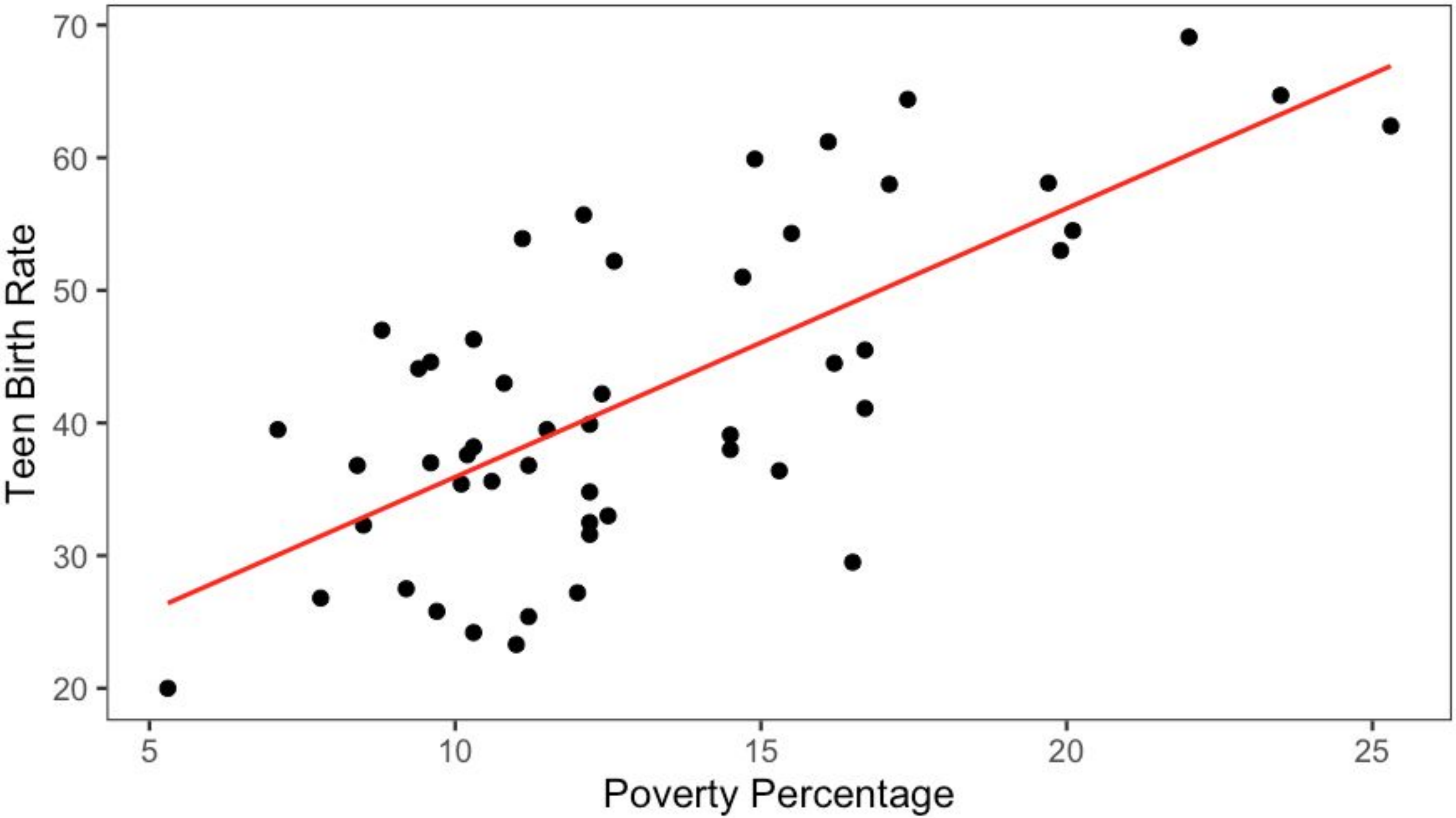
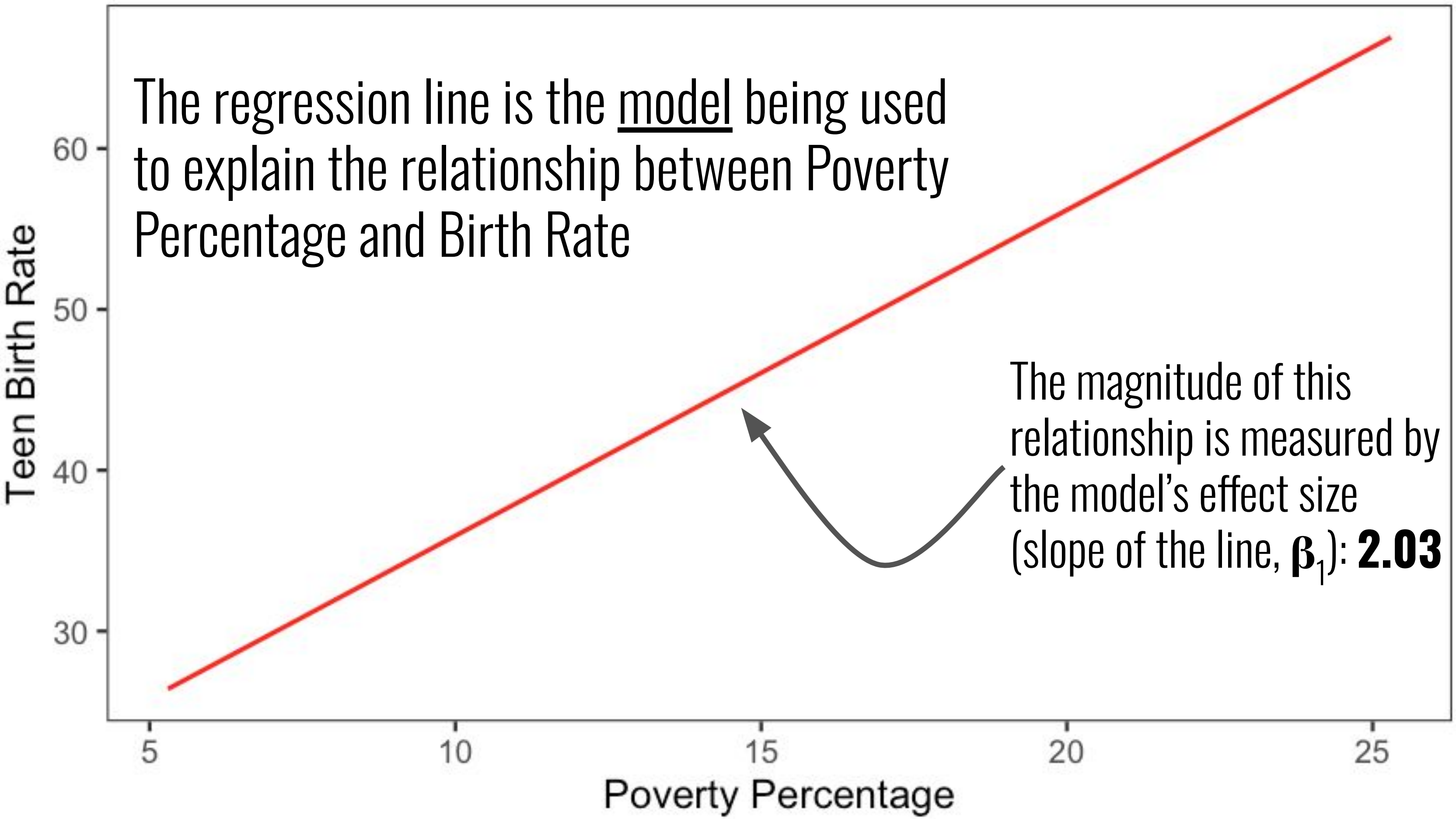
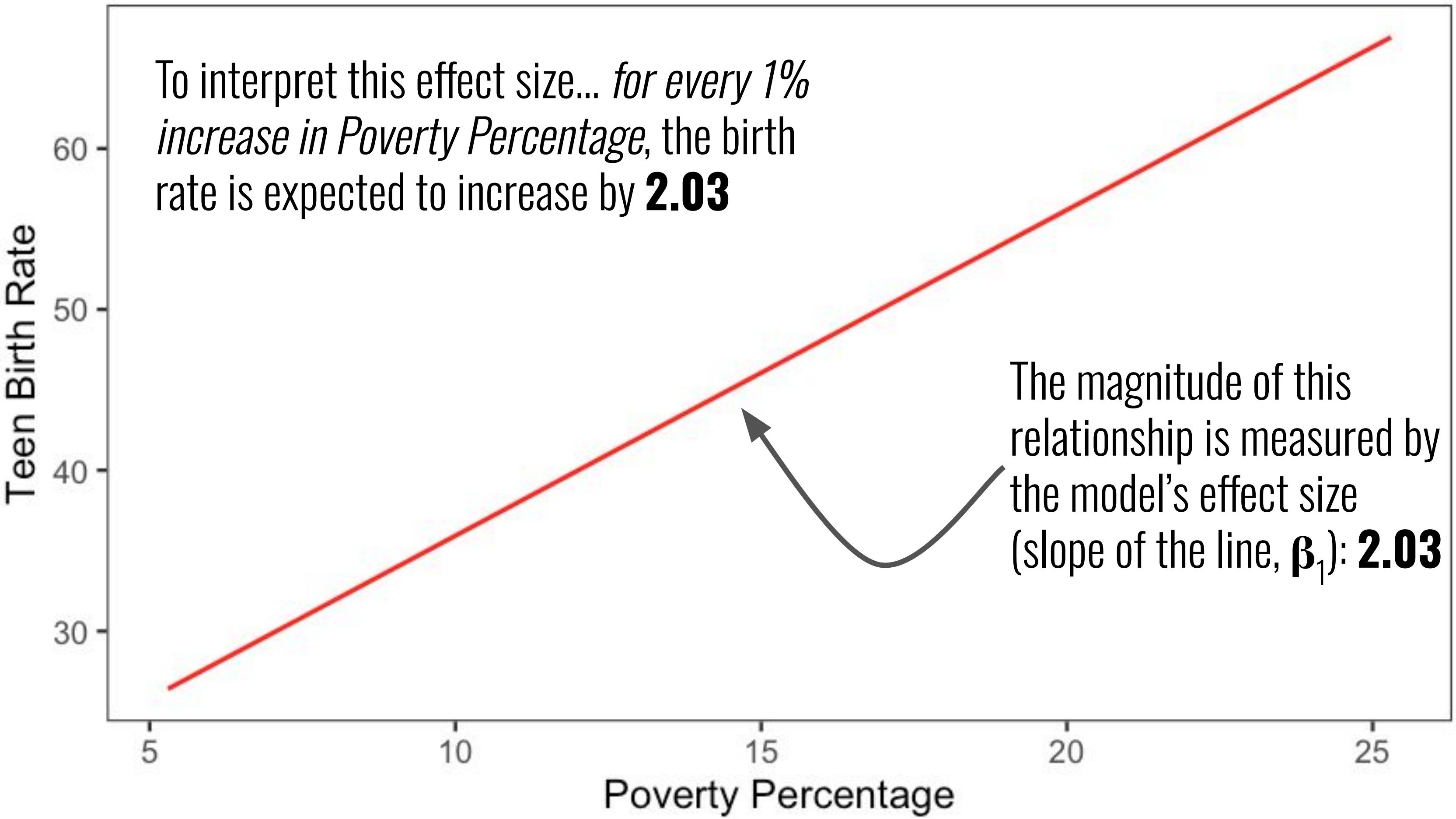


Teen Birth Rate
70
60
50
40
30
20
5
10
15
20
25
Poverty Percentage

The regression line is the <u>model</u> being used to explain the relationship between Poverty Percentage and Birth Rate

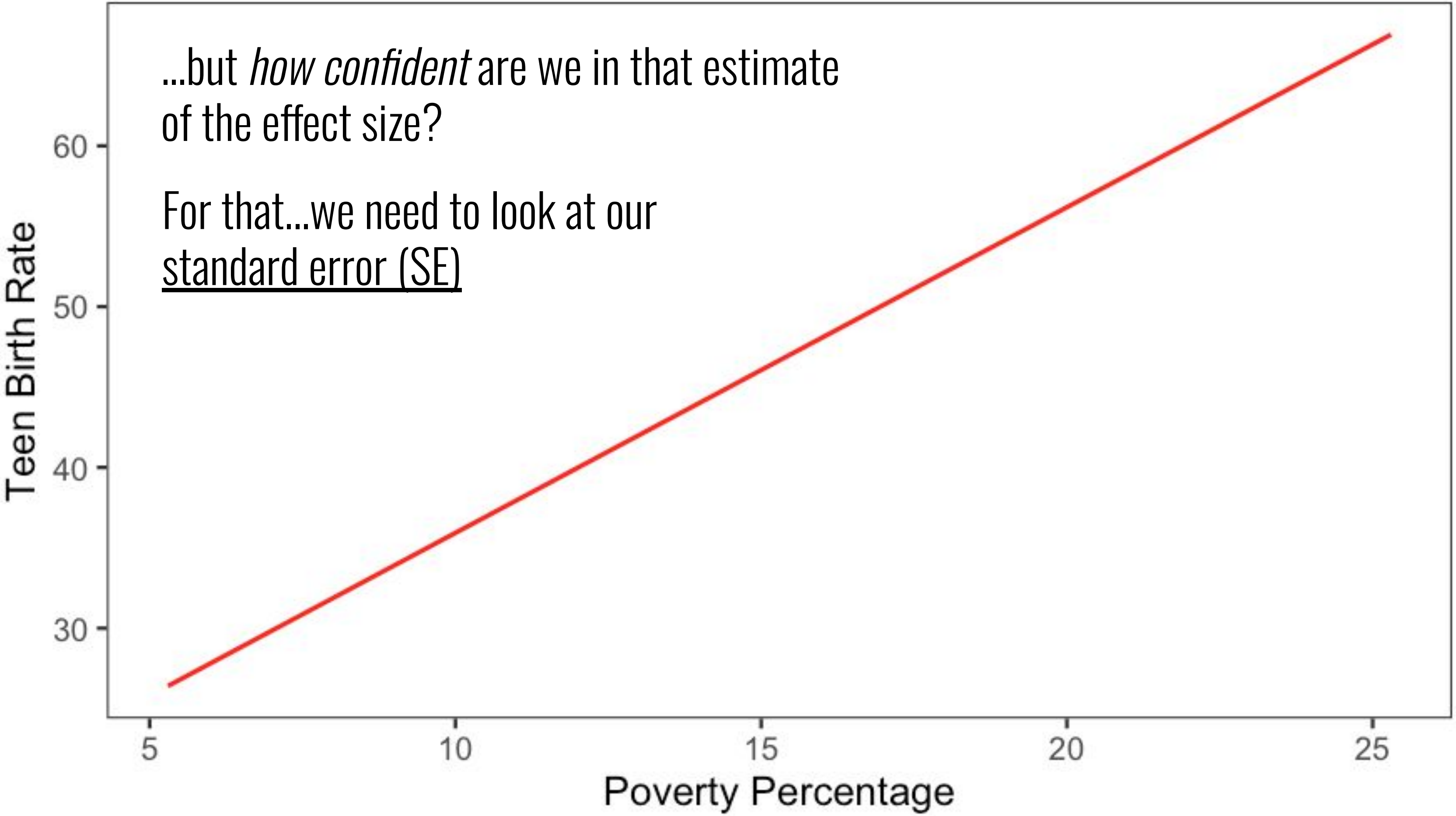


The magnitude of this relationship is measured by the model's effect size (slope of the line, $\boldsymbol{\beta}_1$): **2.03**

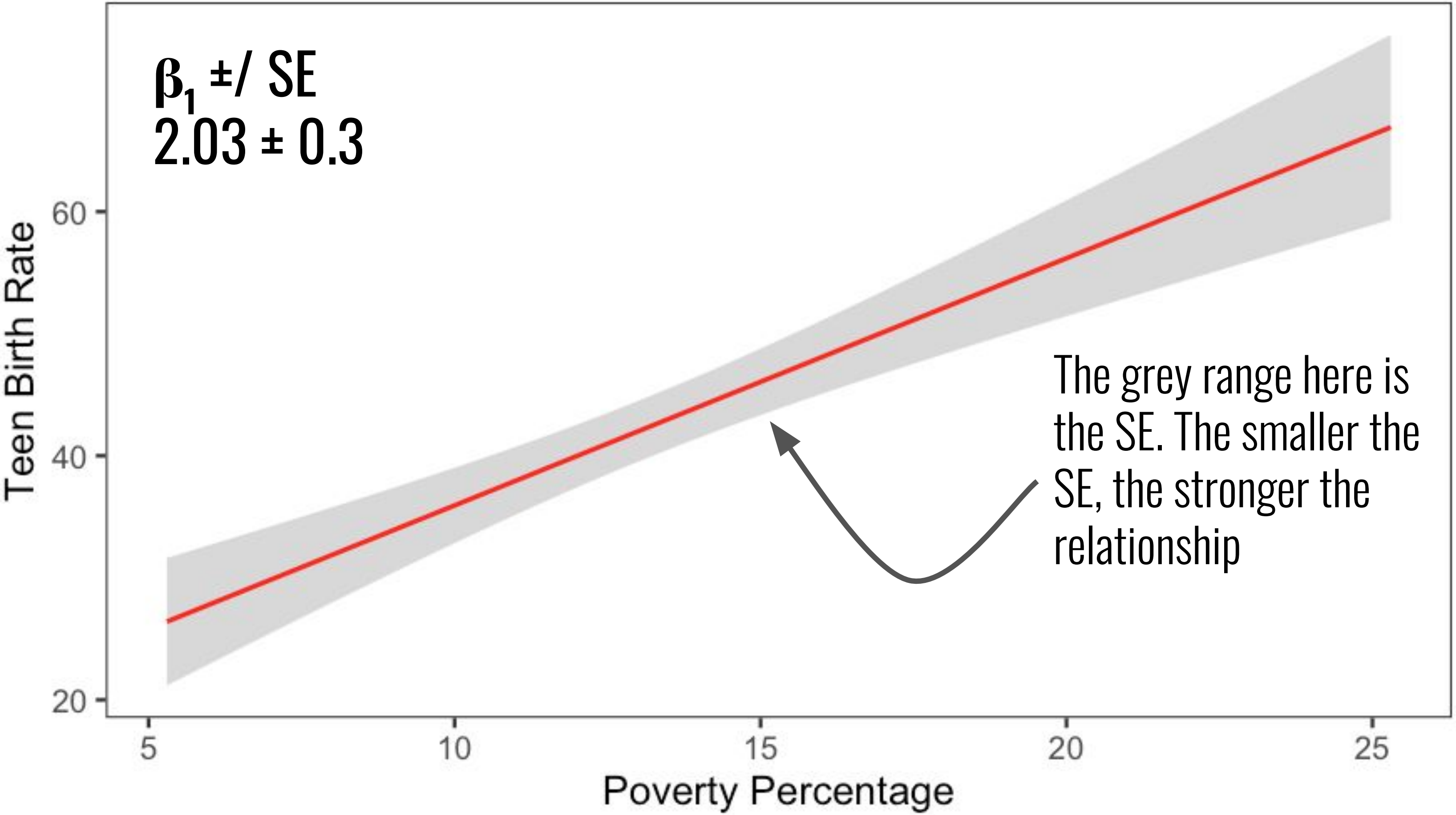

To interpret this effect size... *for every 1% increase in Poverty Percentage*, the birth rate is expected to increase by **2.03**
The magnitude of this relationship is measured by the model's effect size (slope of the line, β_1): **2.03**
Teen Birth Rate
60
50
40
30
5
10
15
20
25
Poverty Percentage

…but *how confident* are we in that estimate of the effect size?

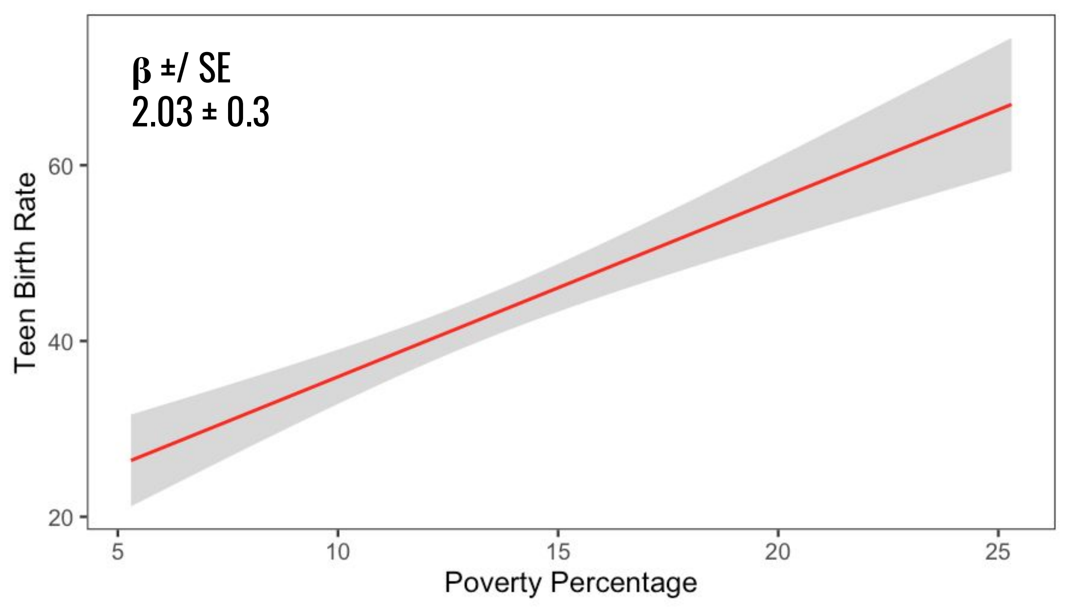
For that…we need to look at our standard error (SE)



β_1 ±/ SE
2.03 ± 0.3



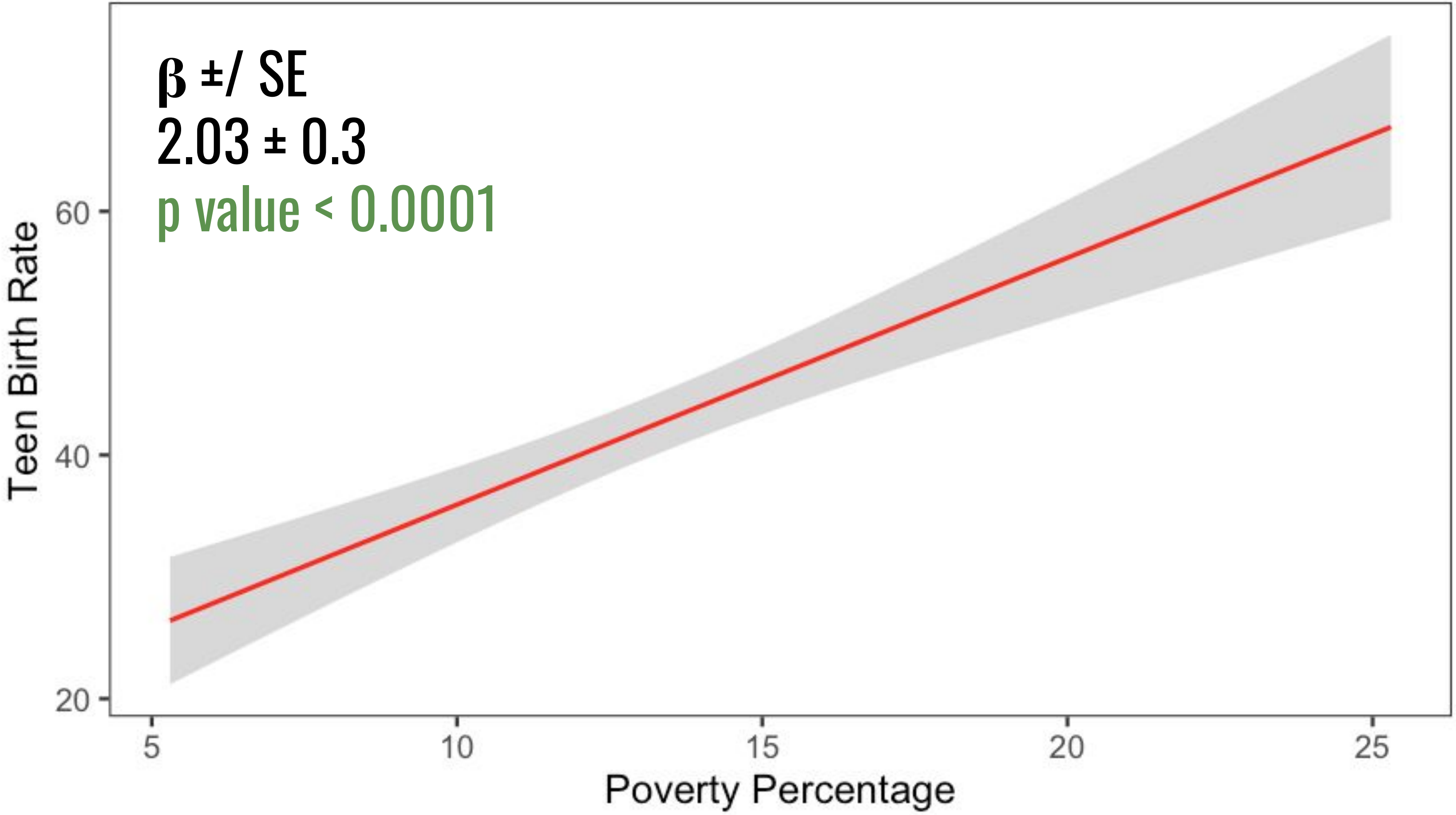
The grey range here is the SE. The smaller the SE, the stronger the relationship



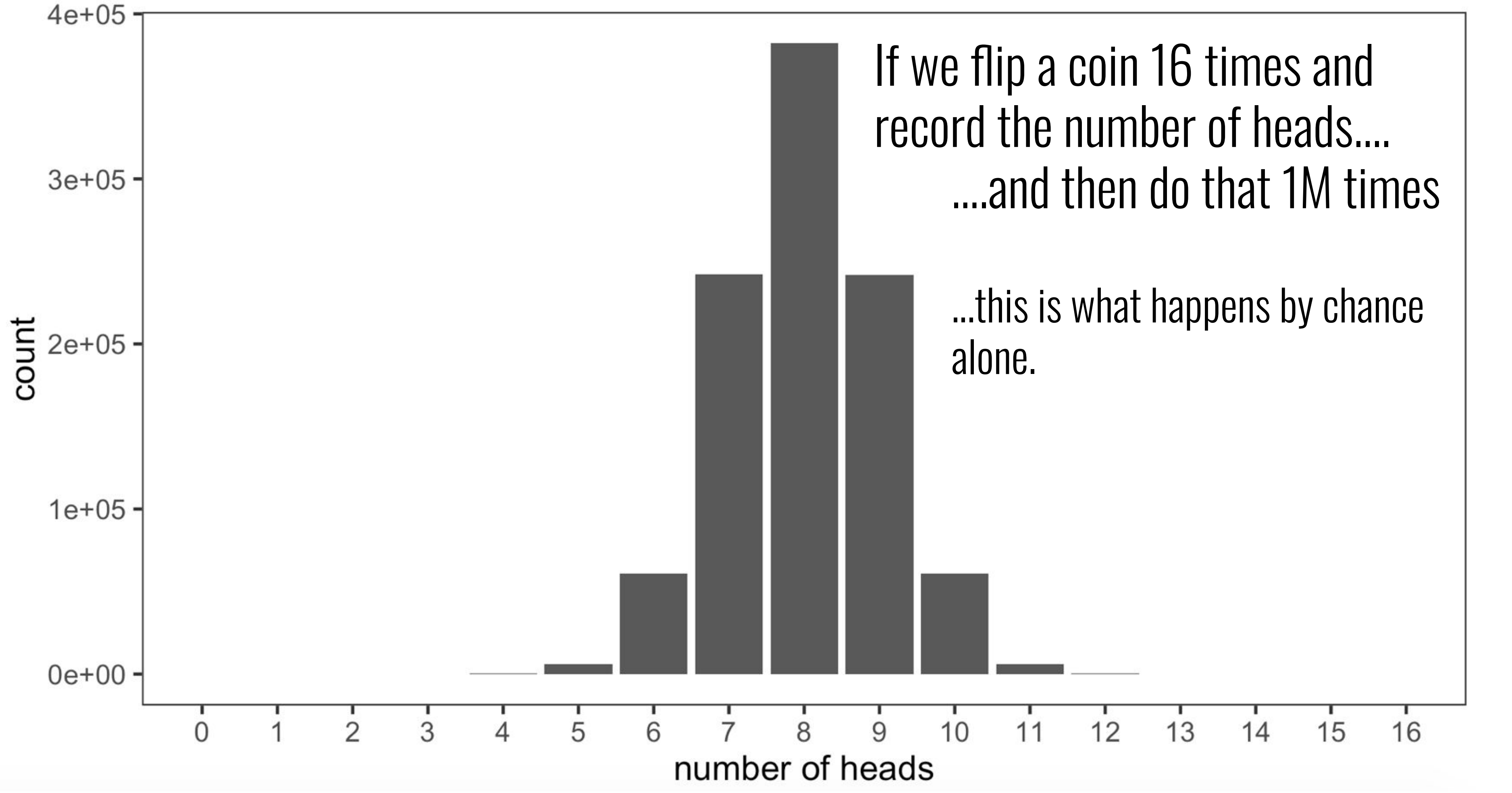
If there were a stronger effect of Poverty on Birth rate, what would β_1 be?

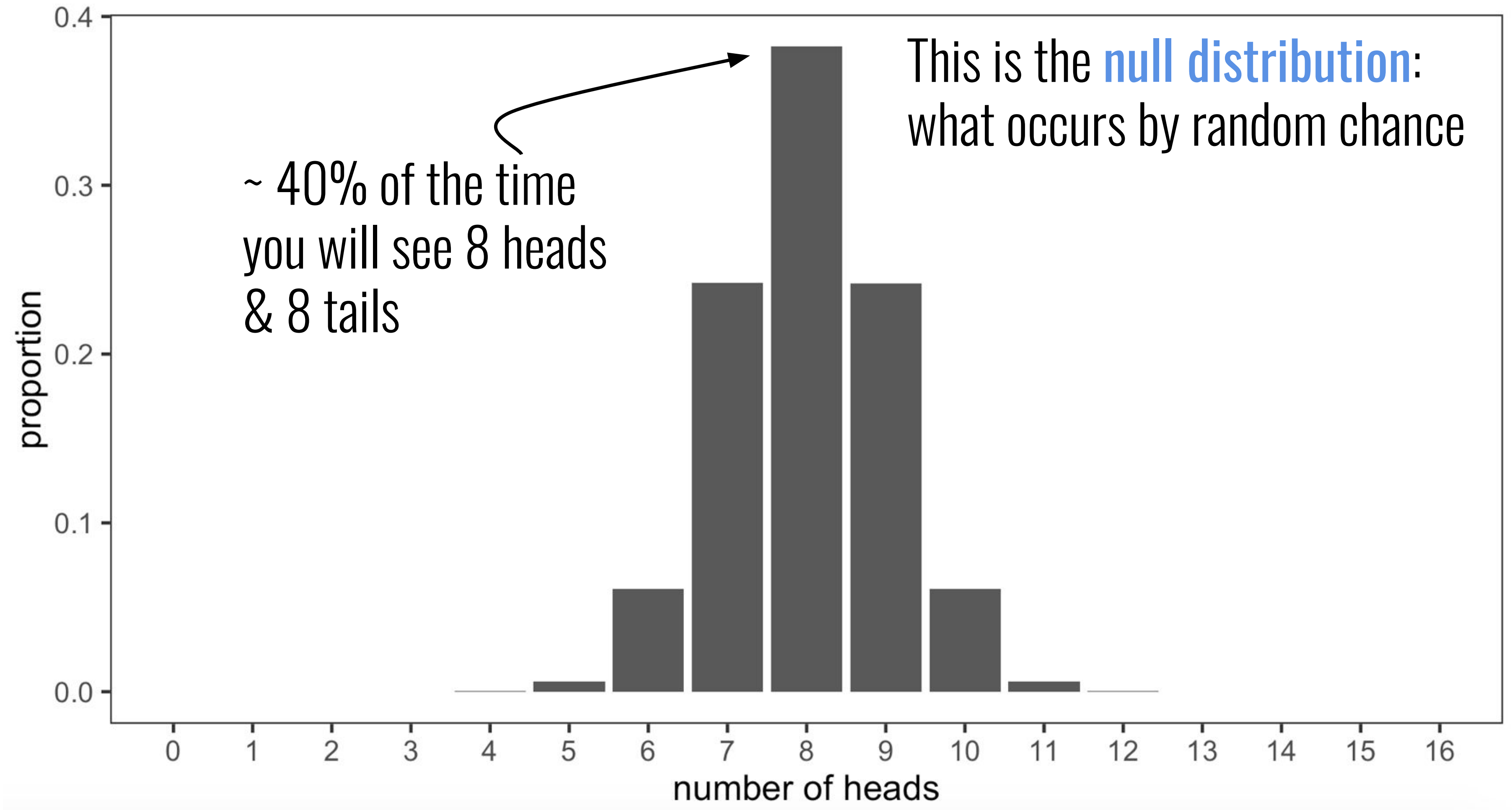
A
< 2.03

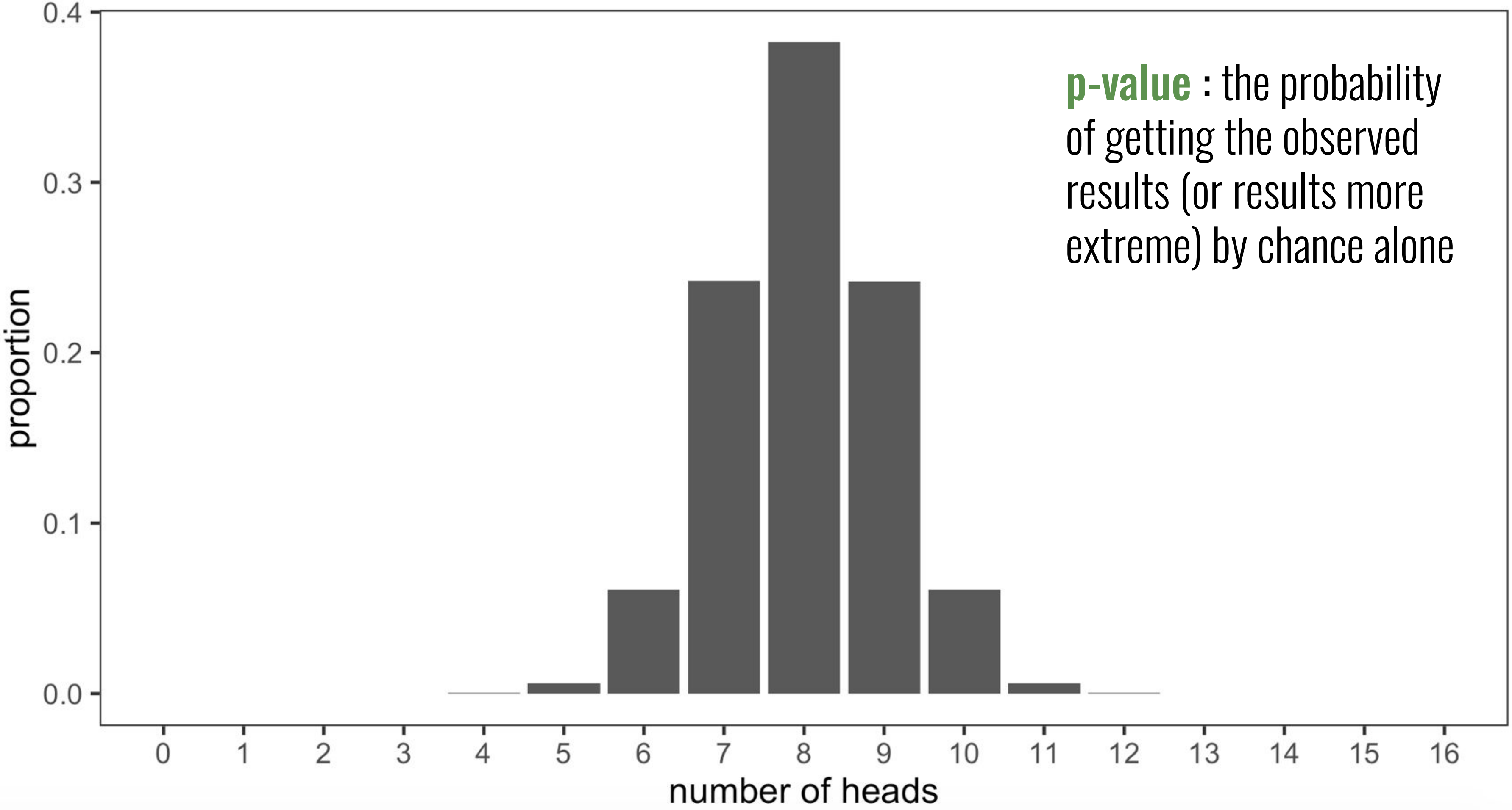
B
> 2.03


β ±/ SE
2.03 ± 0.3
p value < 0.0001
60
40
20
5
10
15
20
25
Teen Birth Rate
Poverty Percentage

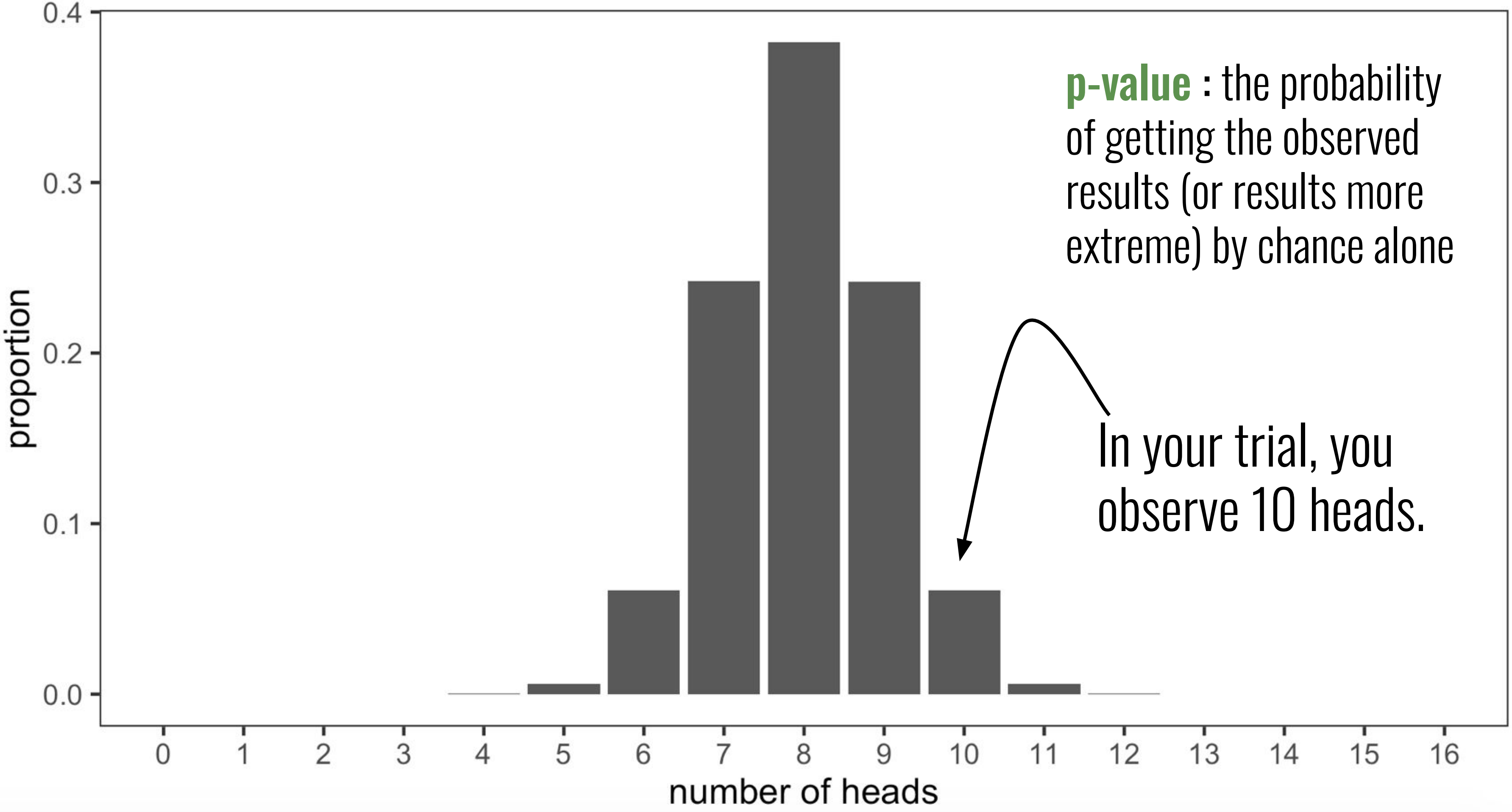
p-value : the probability of getting the observed results (or results more extreme) by chance alone


If we flip a coin 16 times and record the number of heads....
....and then do that 1M times
...this is what happens by chance alone.
count
4e+05
3e+05
2e+05
1e+05
0e+00
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads


This is the null distribution:
what occurs by random chance
~ 40% of the time
you will see 8 heads
& 8 tails
proportion
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads

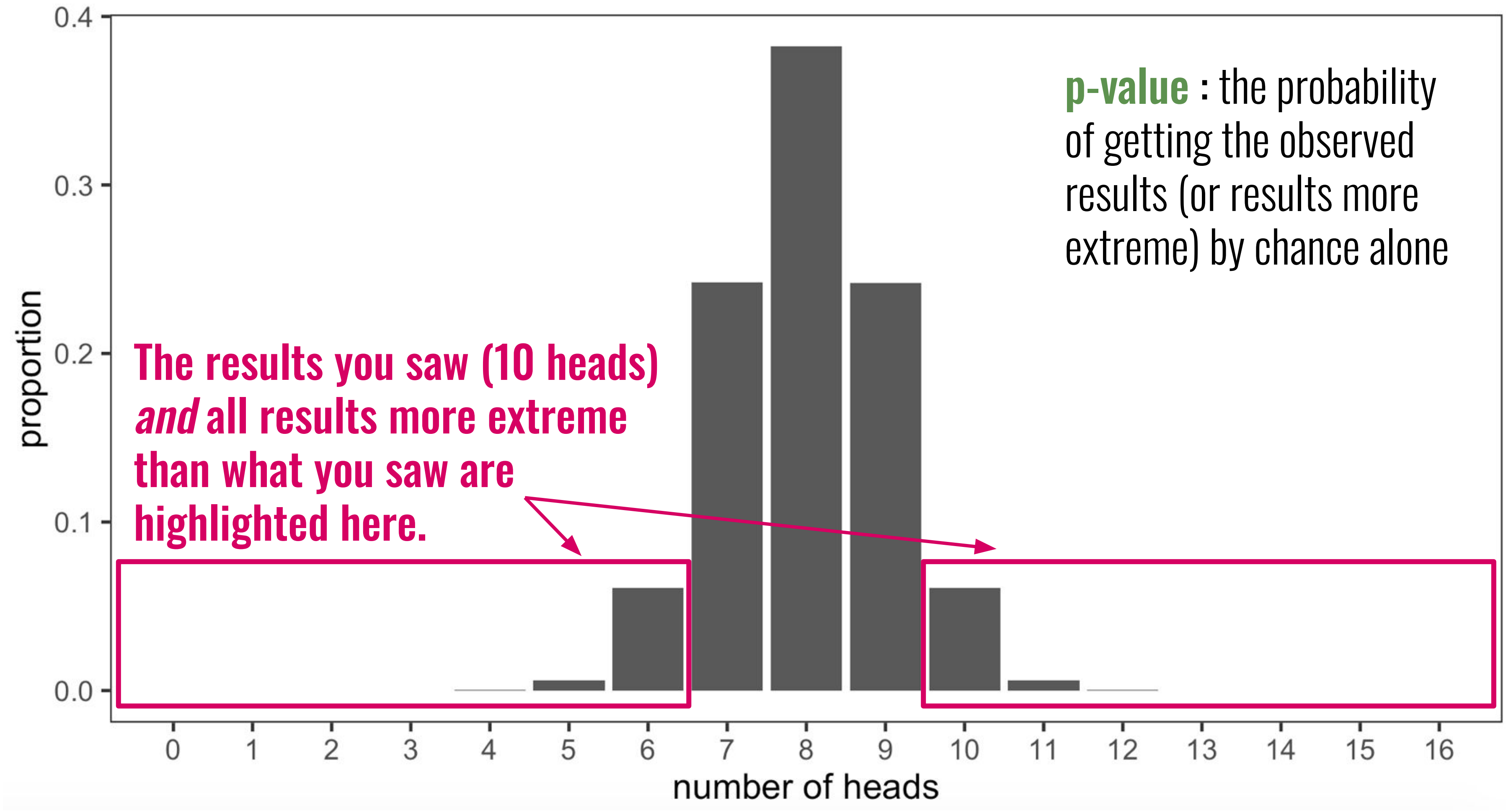


p-value : the probability of getting the observed results (or results more extreme) by chance alone

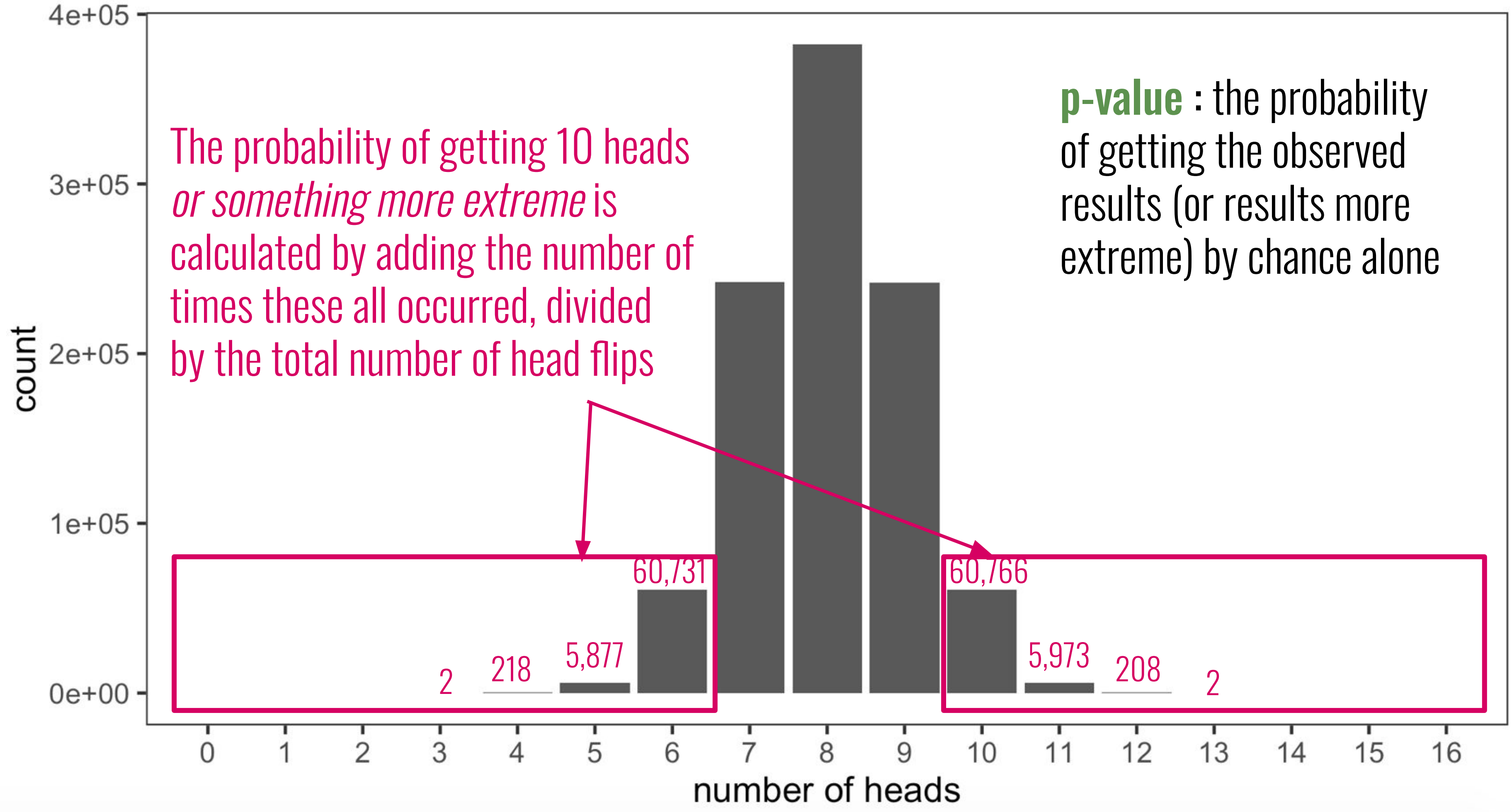


p-value : the probability of getting the observed results (or results more extreme) by chance alone

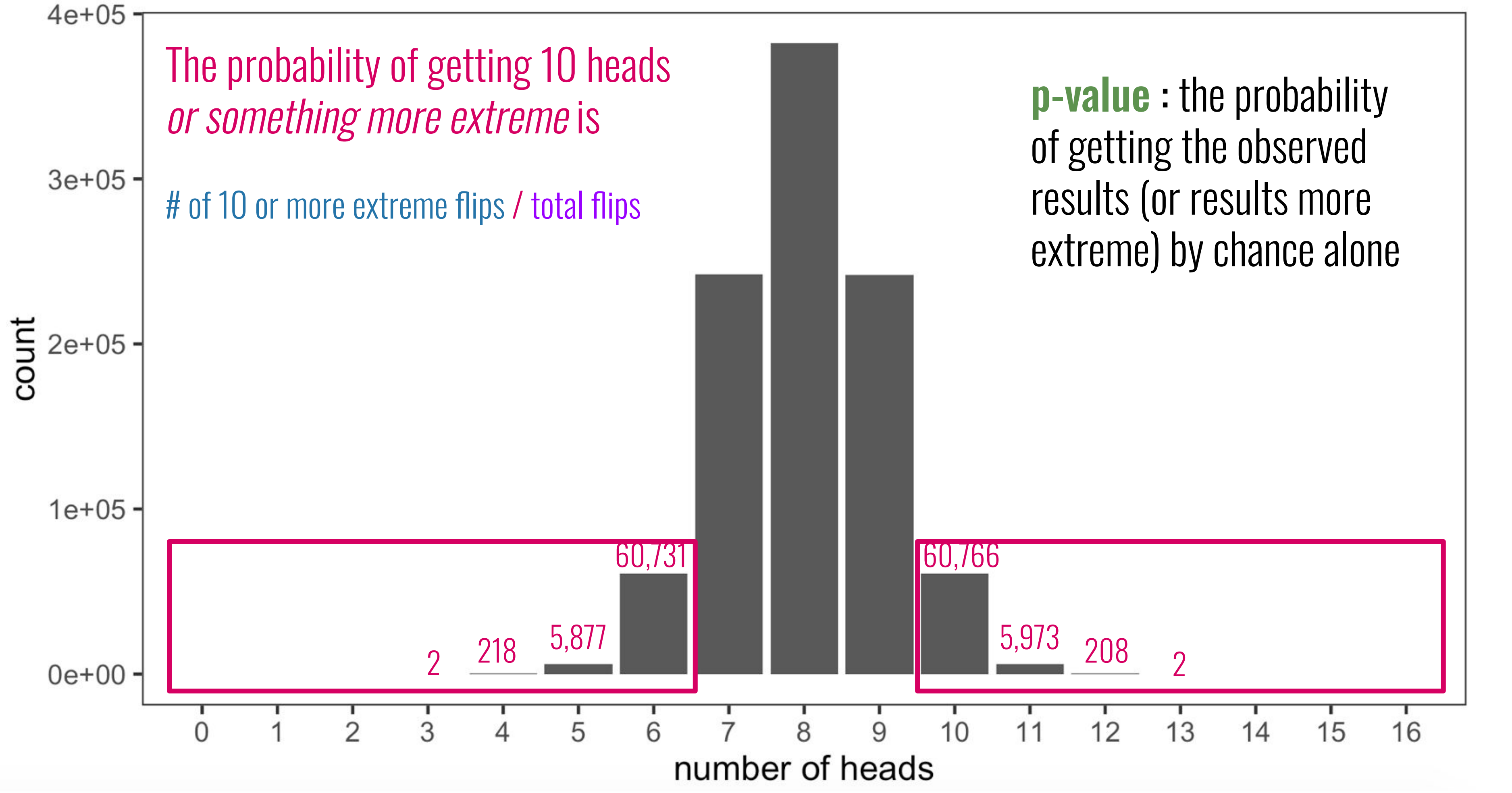
In your trial, you observe 10 heads.



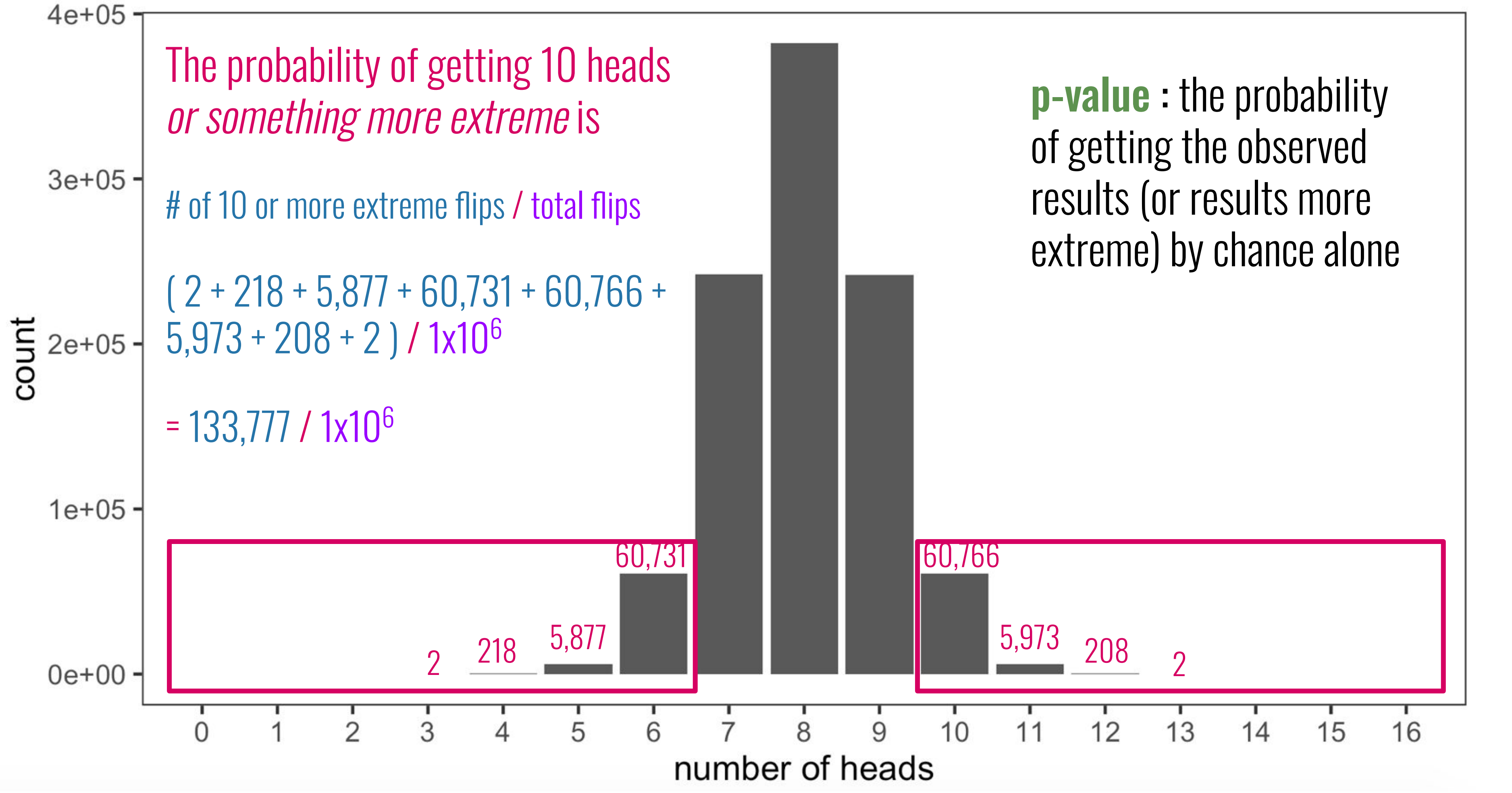
0.4
0.3
0.2
0.1
0.0
proportion
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads
The results you saw (10 heads) *and* all results more extreme than what you saw are highlighted here.
p-value : the probability of getting the observed results (or results more extreme) by chance alone



p-value : the probability of getting the observed results (or results more extreme) by chance alone



The probability of getting 10 heads
or something more extreme is
# of 10 or more extreme flips / total flips
p-value : the probability of getting the observed results (or results more extreme) by chance alone
4e+05
3e+05
2e+05
1e+05
0e+00
count
2
218
5,877
60,731
60,766
5,973
208
2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads



The probability of getting 10 heads
or something more extreme is
# of 10 or more extreme flips / total flips
(2 + 218 + 5,877 + 60,731 + 60,766 +
5,973 + 208 + 2) / 1x10^6
= 133,777 / 1x10^6
p-value : the probability
of getting the observed
results (or results more
extreme) by chance alone
4e+05
3e+05
2e+05
1e+05
0e+00
count
2
218
5,877
60,731
60,766
5,973
208
2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads

The probability of getting 10 heads *or something more extreme* is

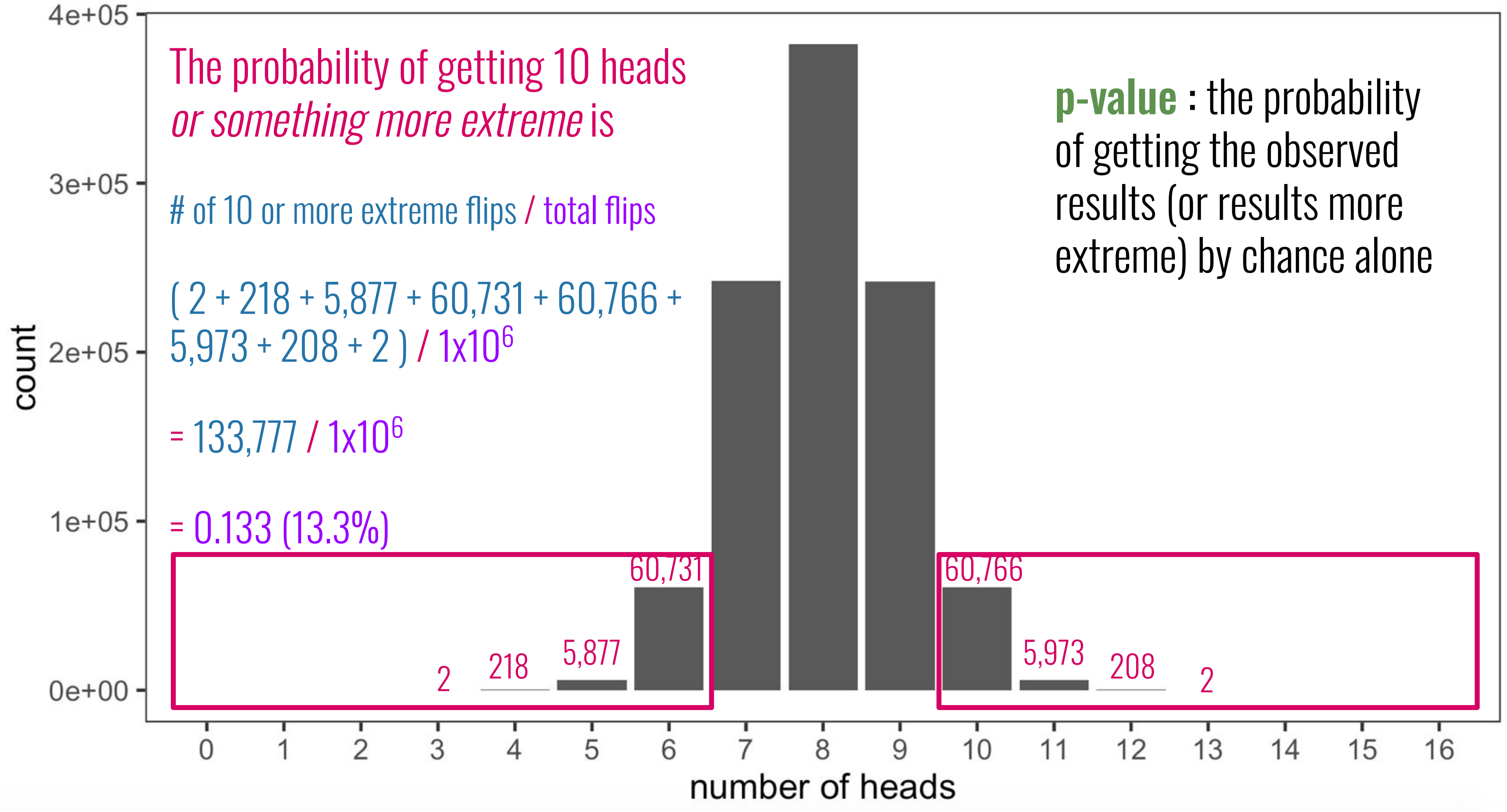
\# of 10 or more extreme flips / total flips

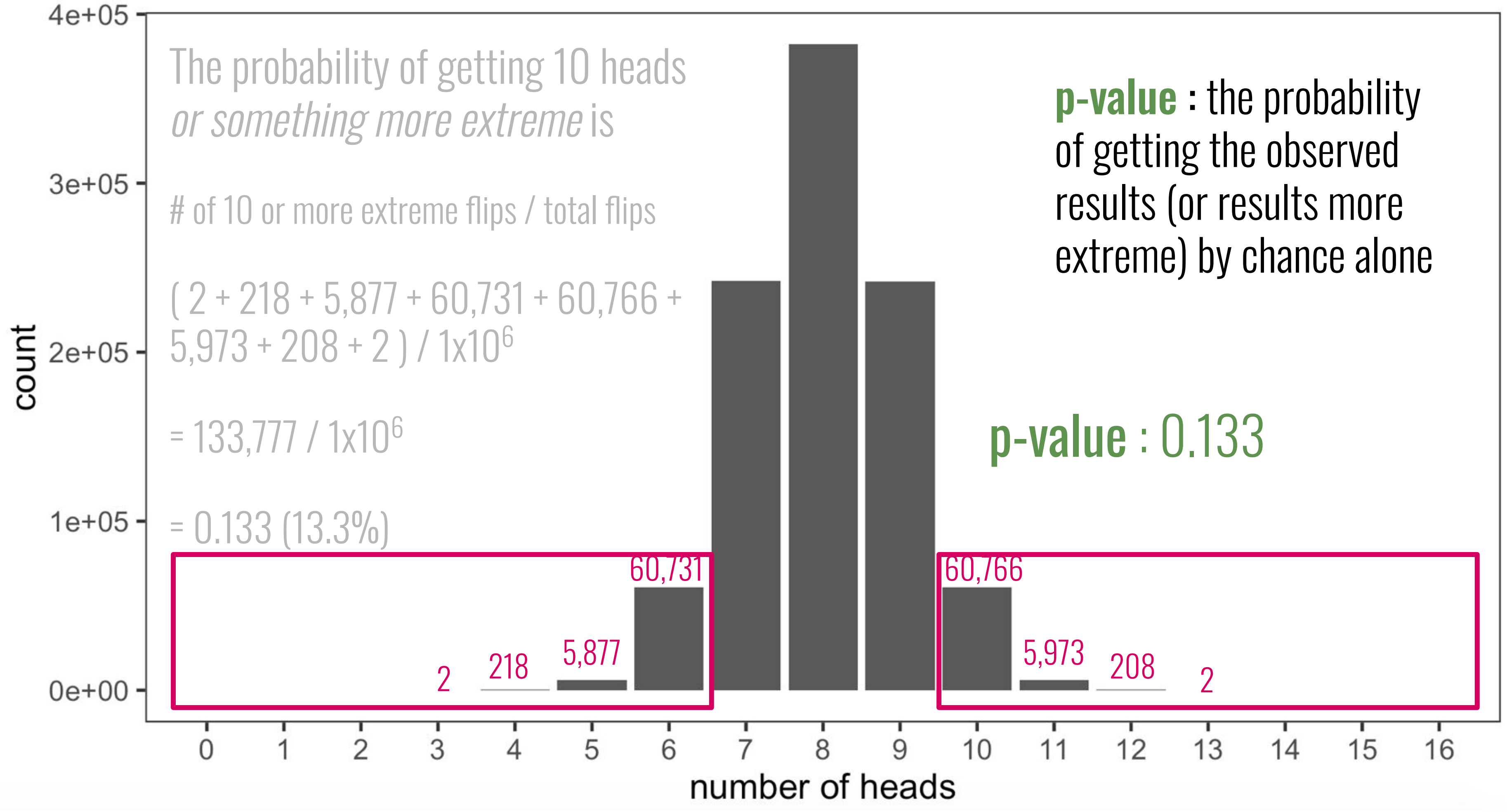
(2 + 218 + 5,877 + 60,731 + 60,766 + 5,973 + 208 + 2) / 1×10^6

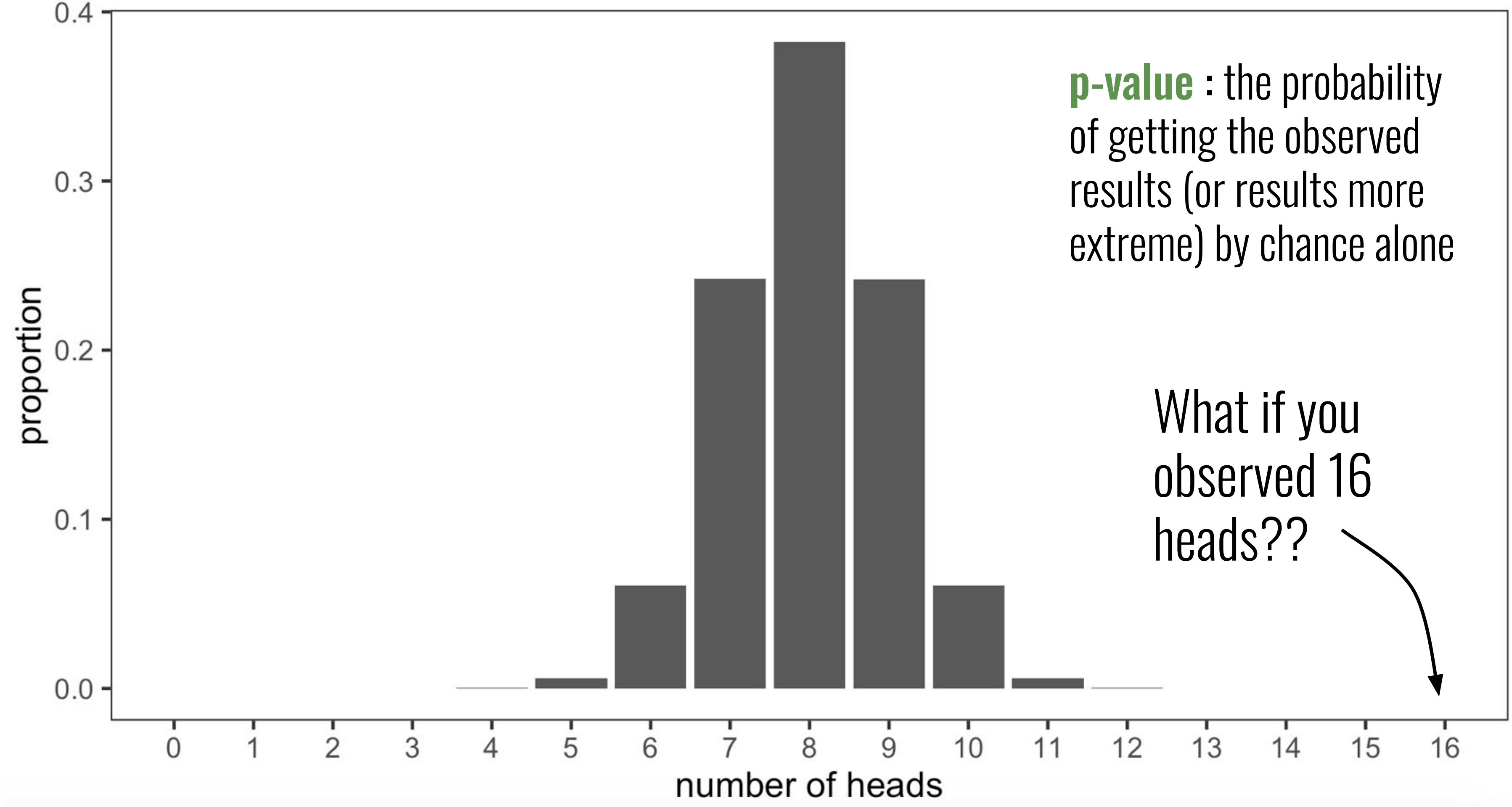
= 133,777 / 1×10^6

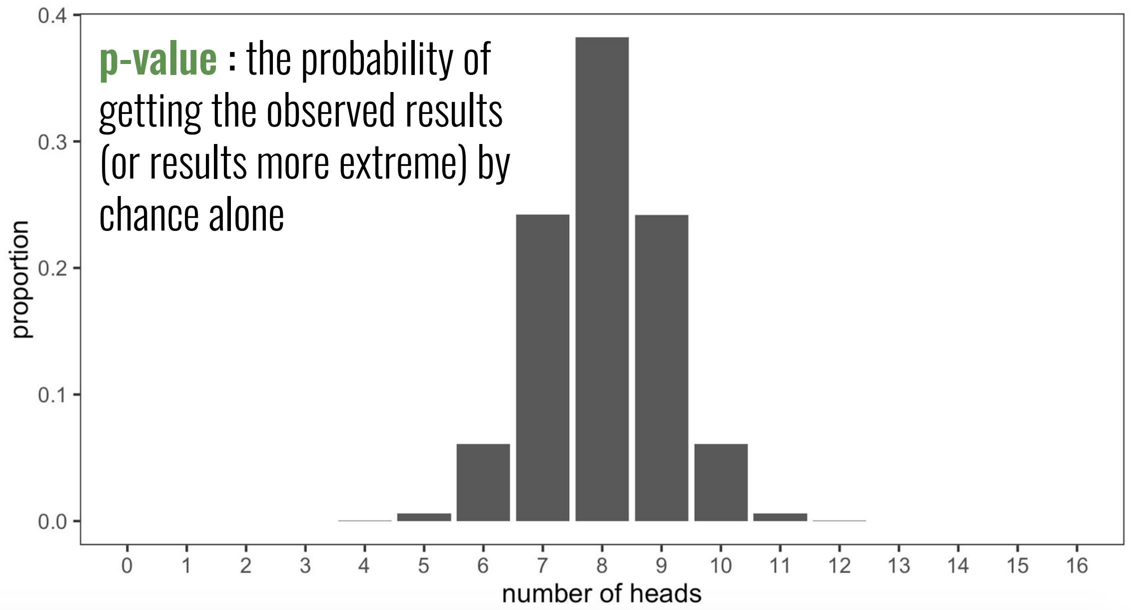
= 0.133 (13.3%)

p-value : the probability of getting the observed results (or results more extreme) by chance alone




The probability of getting 10 heads
or something more extreme is
# of 10 or more extreme flips / total flips
(2 + 218 + 5,877 + 60,731 + 60,766 + 5,973 + 208 + 2) / 1x10^6
= 133,777 / 1x10^6
= 0.133 (13.3%)
p-value : the probability of getting the observed results (or results more extreme) by chance alone
p-value : 0.133
2
218
5,877
60,731
60,766
5,973
208
2
count
4e+05
3e+05
2e+05
1e+05
0e+00
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads

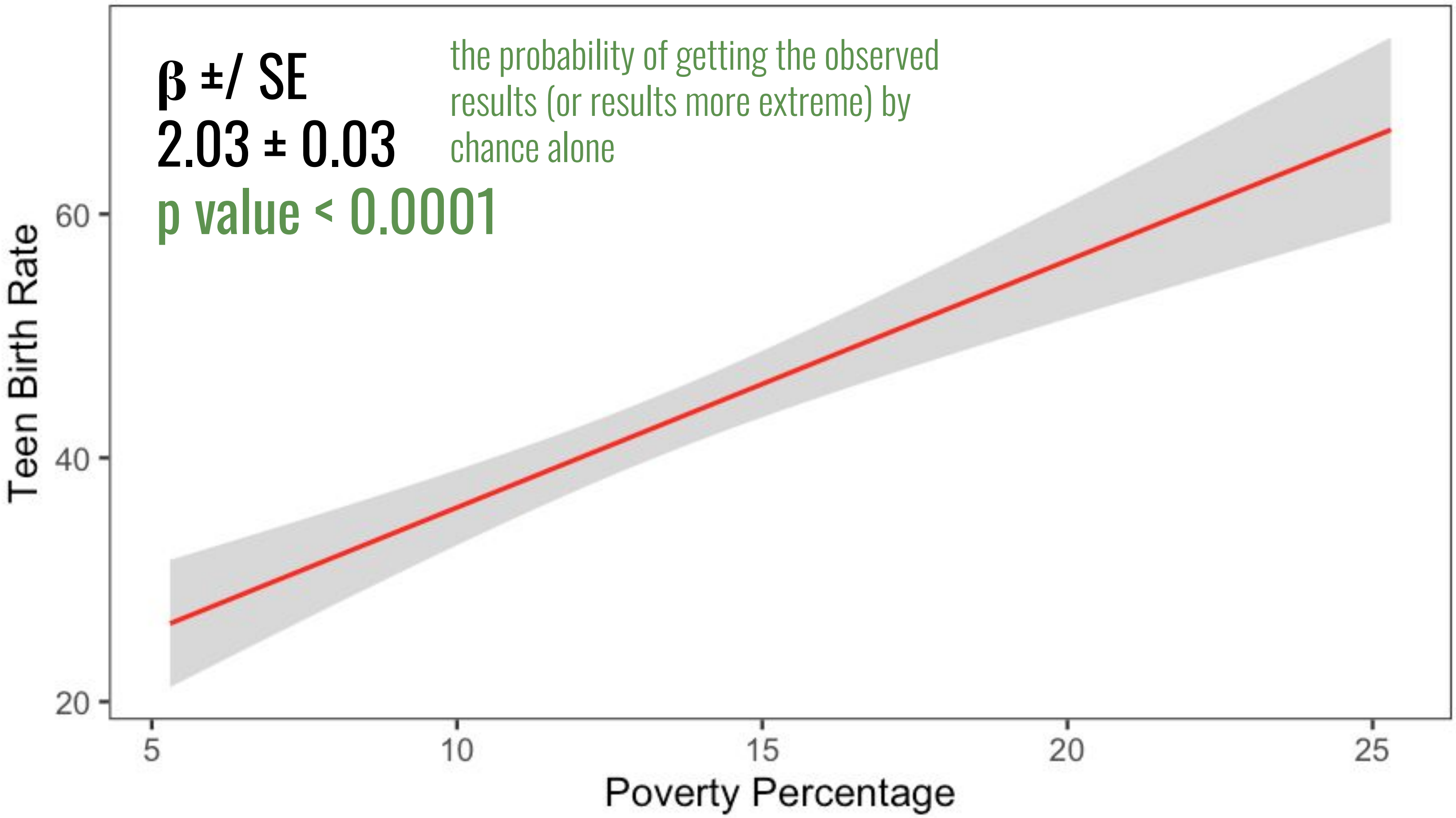

p-value : the probability of getting the observed results (or results more extreme) by chance alone
What if you observed 16 heads??
proportion
0.0
0.1
0.2
0.3
0.4
number of heads
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16



What would be the p-value of you flipping 16 heads?

A
< 0.13

B
> 0.13

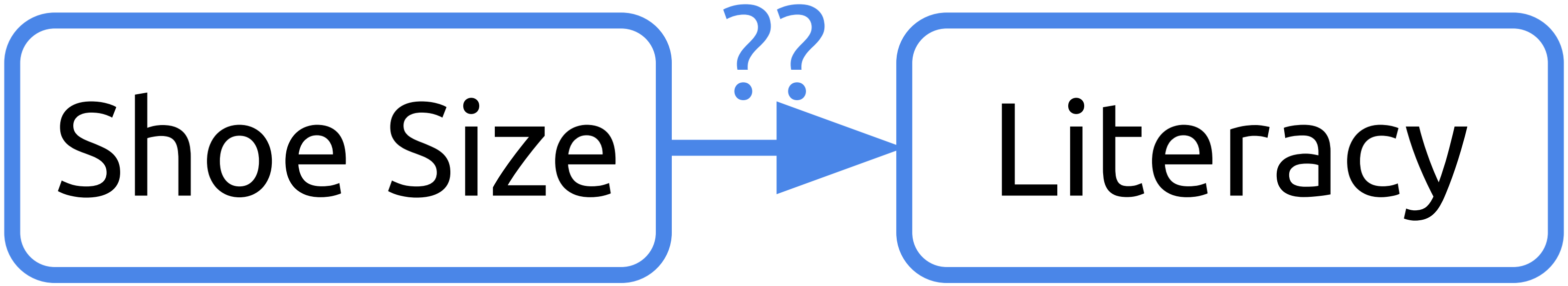

β ±/ SE
2.03 ± 0.03
p value < 0.0001
the probability of getting the observed results (or results more extreme) by chance alone
Teen Birth Rate
20
40
60
5
10
15
20
25
Poverty Percentage

Takes into account the effect size (β_1) and the SE

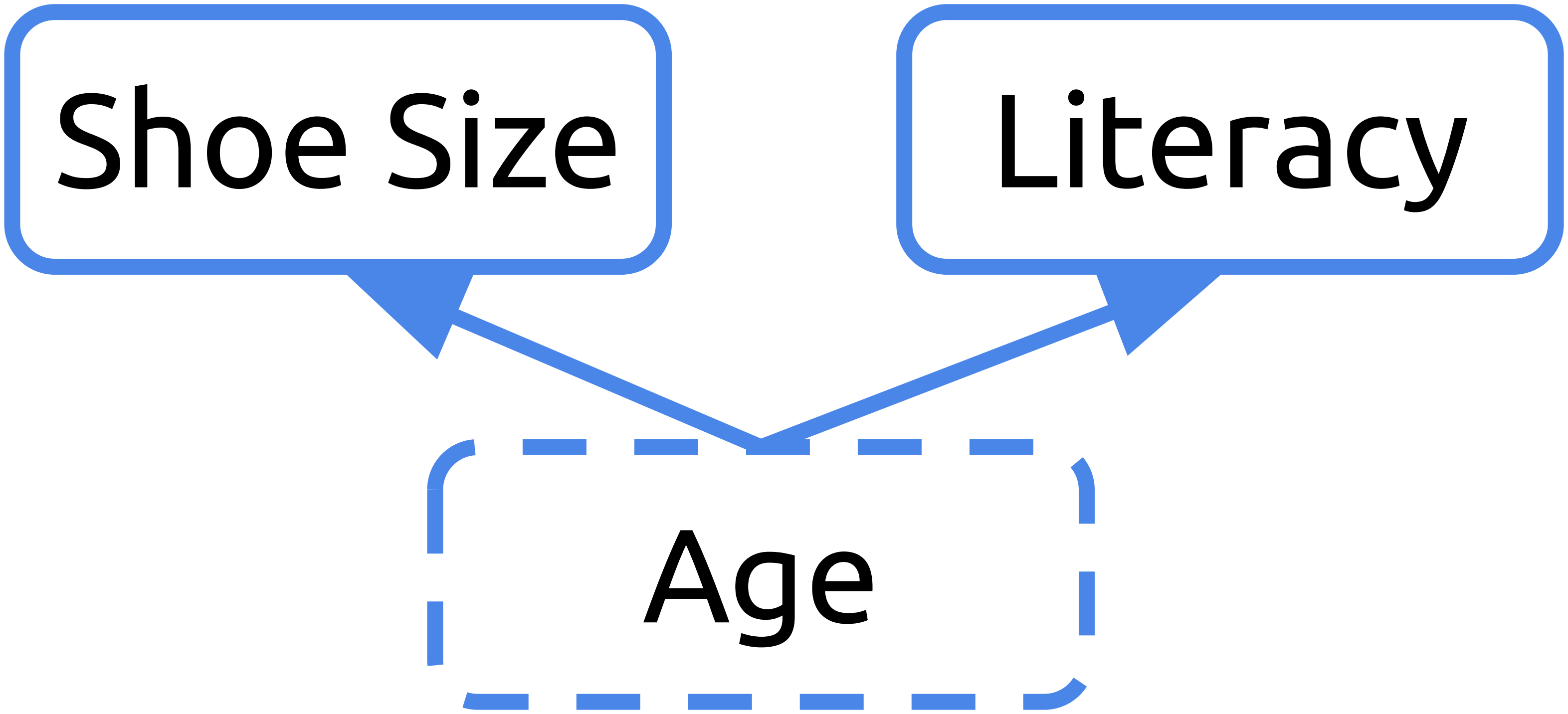
p-value : the probability of getting the observed results (or results more extreme) by chance alone

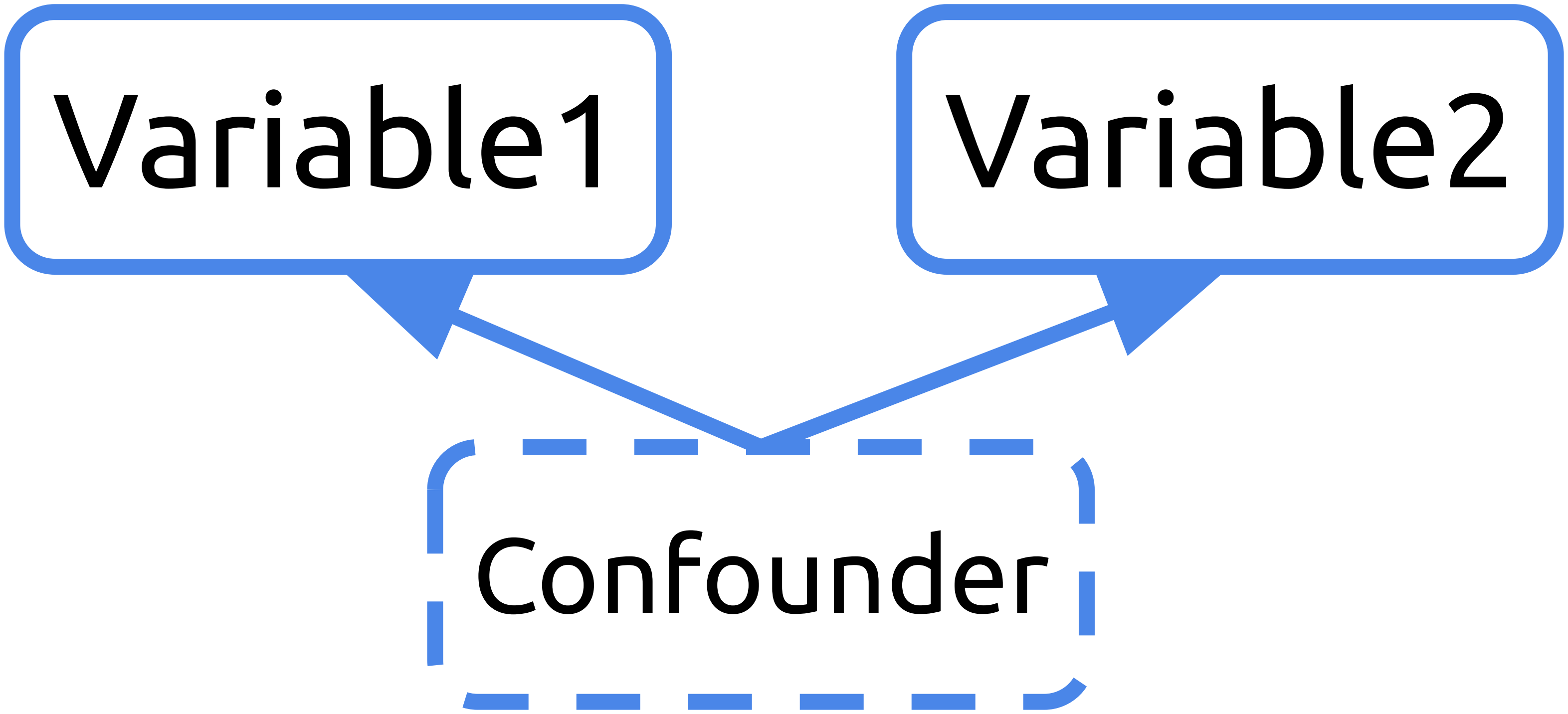
Confounding


Small shoes
Not literate
Big shoes
Literate


Shoe Size
??
Literacy


Small shoes
Not literate
Child
Big shoes
Literate
Adults


Shoe Size
Literacy
Age


Variable1
Variable2
Confounder

Confounding

popsicles → crime rate

Your analysis sees an increase in crime rate whenever popsicle sales increase. What could confound this analysis?

A popsicle preference

B new gun laws

C temperature

D changes in popsicle prices

E new law enforcement officers

We'll discuss additional approaches of how to account for confounding in your analysis in the next lecture.

Ignoring confounders will lead you to draw incorrect conclusions from your analyses

Spine Surgery Results

Sample: 400 patients with index vertebral fractures

Vertebroplasty	Conservative care	Relative risk (95% confidence interval)
30/200 (15%)	15/200 (7.5%)	2.0 (1.1–3.6)

subsequent fractures

Eek….looks like vertebroplasty was *way* worse for patients!

But wait...at time of initial fracture...

	Vertebroplasty N = 200	Conservative care N = 200
Age, y, mean ± SD	78.2 ± 4.1	79.0 ± 5.2
Weight, kg, mean ± SD	54.4 ± 2.3	53.9 ± 2.1
Smoking status, No. (%)	110 (55)	16 (8)

Age and weight are similar between groups. **Smoking Status** differs vastly.

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

So...let's stratify those results real quick

Smoke			No smoke		
Vertebroplasty	Conservative	RR (95% confidence interval)	Vertebroplasty	Conservative	RR (95% confidence interval)
23/110 (21%)	3/16 (19%)	1.1 (0.4, 3.3)	7/90 (8%)	12/184(7%)	1.2 (0.5, 2.9)

Risk of re-fracture is now similar within group

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

Confounding

What are possible confounders for our analysis of the effect of poverty on teen birth rate?

A
I have some ideas

B
Not sure