Course Announcements

- Due Friday (11:59 PM)
  - D4
    - `assert(len(age_sub['year'].unique()) == 8)  # was erroneously 7`
  - Q4
  - A2
  - Weekly Project Survey (*optional*)

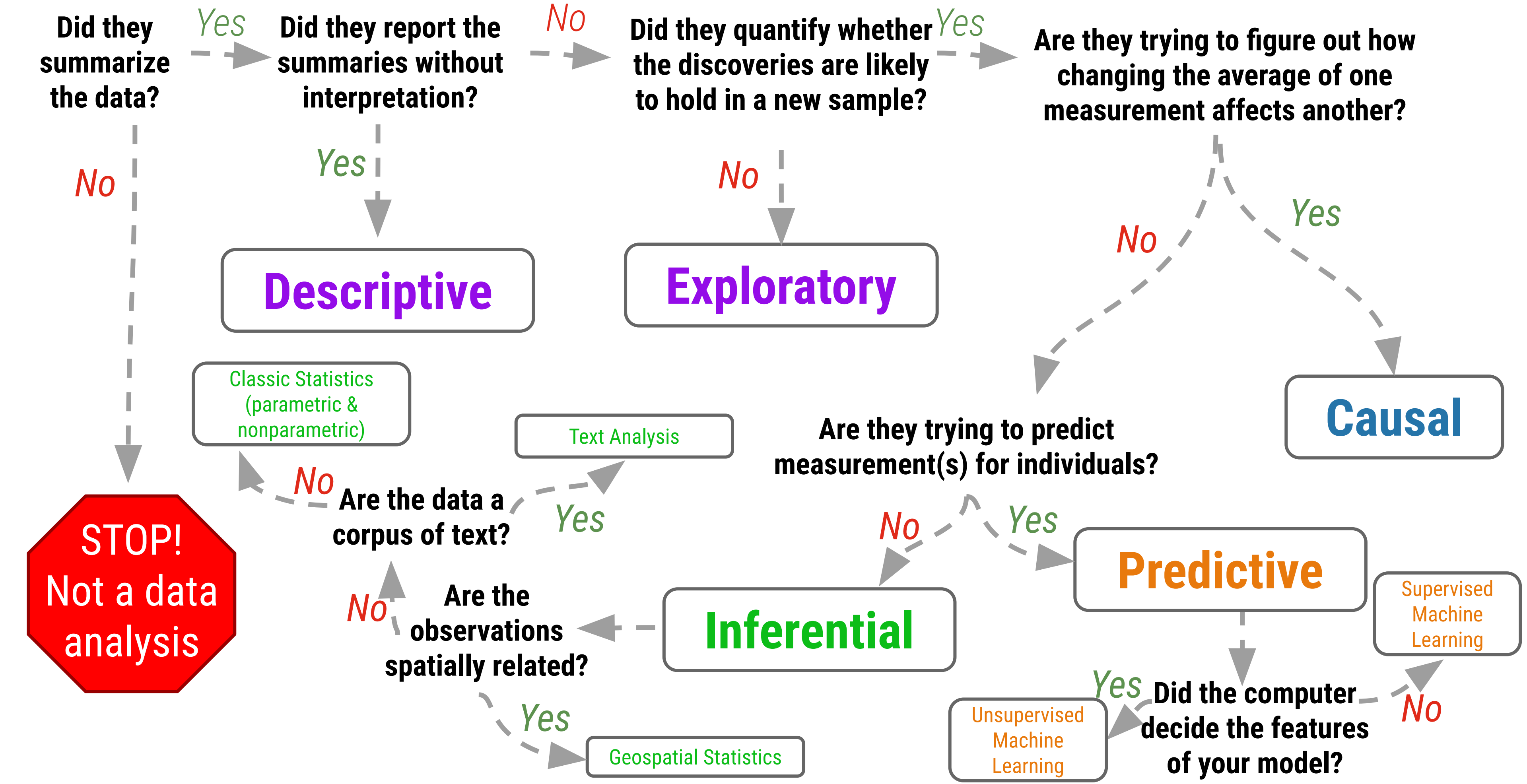
Grades posted: D3, Q3

Grading underway: Project Proposals

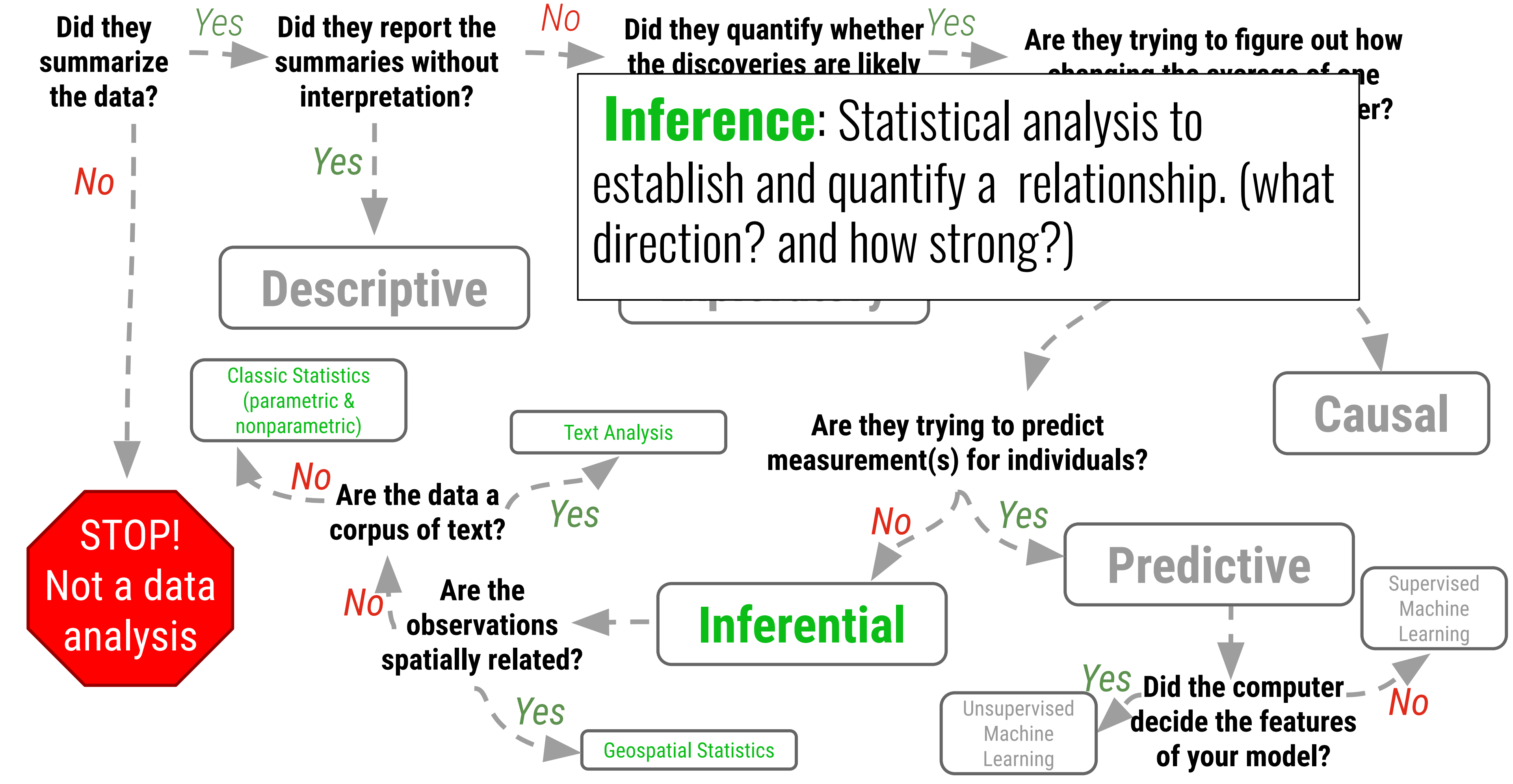
Inferential Analysis

Shannon E. Ellis, Ph.D
UC San Diego

Department of Cognitive Science
sellis@ucsd.edu



Did they summarize the data?
Yes
Did they report the summaries without interpretation?
No
Did they quantify whether the discoveries are likely to hold in a new sample?
Yes
Are they trying to figure out how changing the average of one measurement affects another?
No
Yes
No
Yes
No
Descriptive
Exploratory
No
Yes
Causal
STOP! Not a data analysis
Classic Statistics (parametric & nonparametric)
Text Analysis
No
Are the data a corpus of text?
Yes
Are they trying to predict measurement(s) for individuals?
No
Yes
Predictive
Supervised Machine Learning
No
Are the observations spatially related?
Inferential
Yes
Did the computer decide the features of your model?
No
Unsupervised Machine Learning
Yes
Geospatial Statistics


Did they summarize the data?
Yes
Did they report the summaries without interpretation?
No
Did they quantify whether the discoveries are likely
Yes
Are they trying to figure out how
Inference: Statistical analysis to establish and quantify a relationship. (what direction? and how strong?)
No
Yes
Descriptive
Classic Statistics (parametric & nonparametric)
Text Analysis
No
Are the data a corpus of text?
Yes
Are they trying to predict measurement(s) for individuals?
Causal
No
Yes
Predictive
Supervised Machine Learning
STOP! Not a data analysis
No
Are the observations spatially related?
Inferential
Yes
Did the computer decide the features of your model?
No
Unsupervised Machine Learning
Yes
Geospatial Statistics

- **Problem:** Does Sesame Street affect kids brain development?
- **Data science question:** What is the relationship between watching Sesame Street and test scores among children?
- **Type of analysis:** Inferential analysis

Sesame Street viewership

??

Test scores

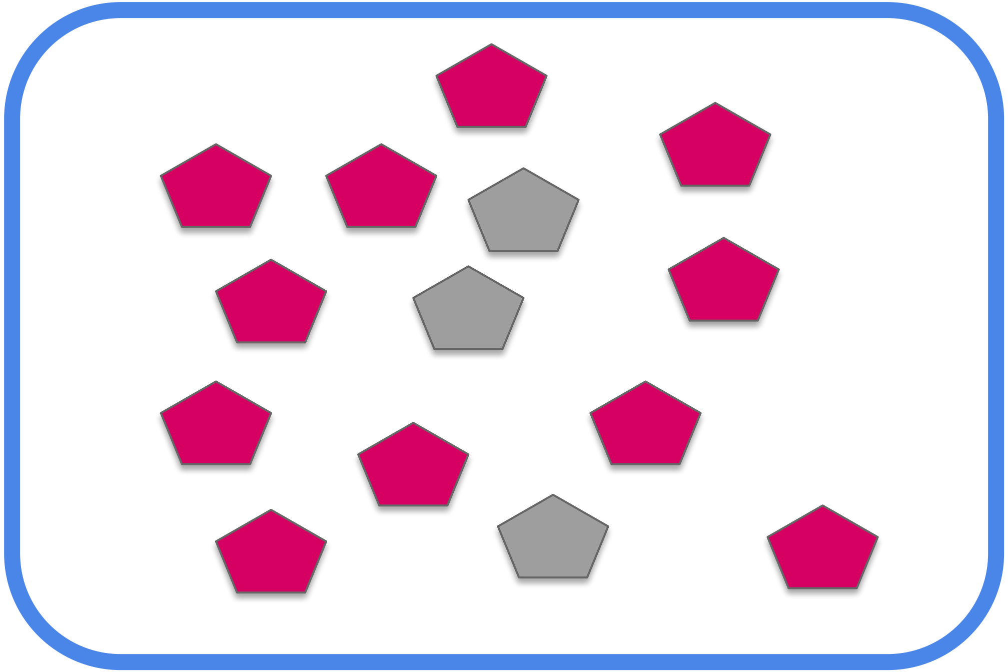
Establishing & Stating Your Null and Alternative Hypotheses Helps Guide Your Analysis

<u>Null Hypothesis</u>:

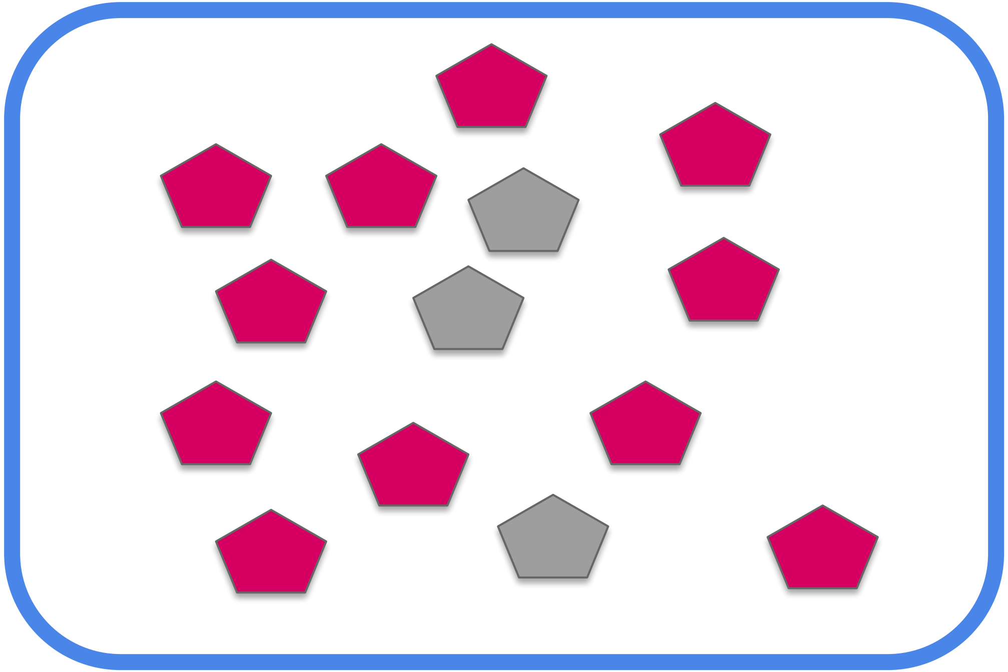
H_0: Sesame Street has *no effect* on kids brain development

<u>Alternative Hypothesis</u>:

H_a: Watching Sesame Street *has an effect* on kids' brain development

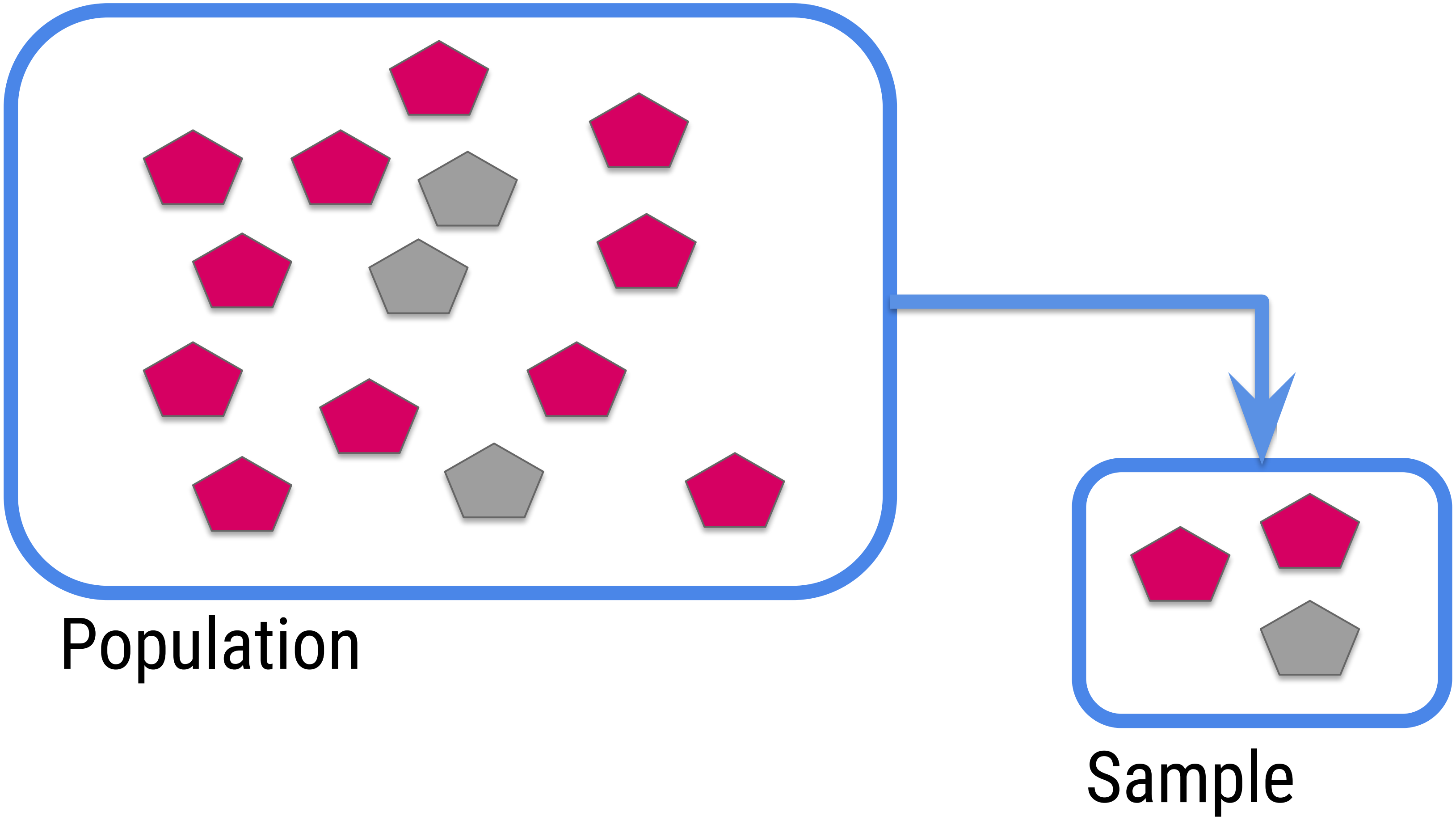


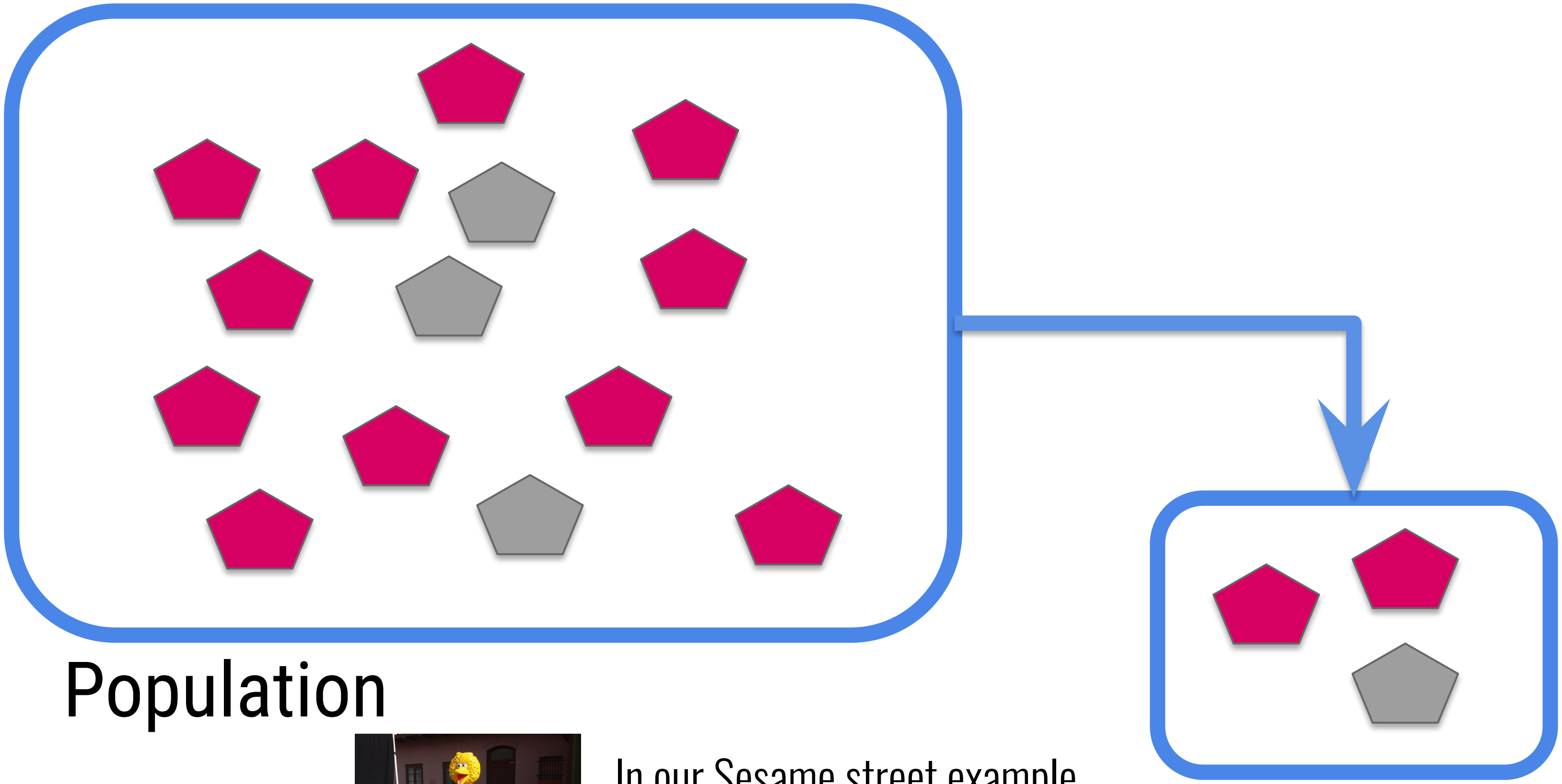
Population



Population

In our Sesame street example, the population would be all children

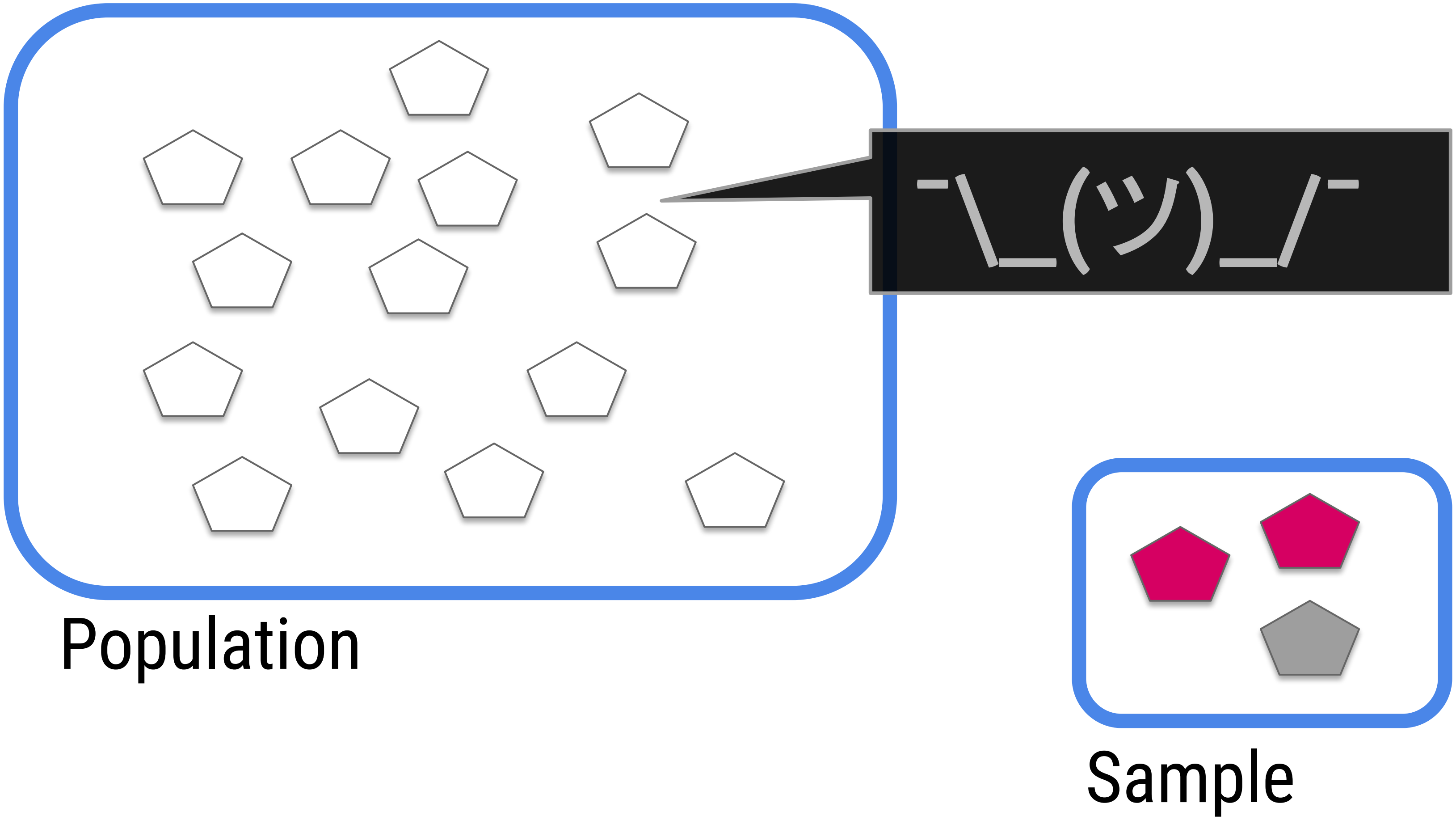

Population
Sample



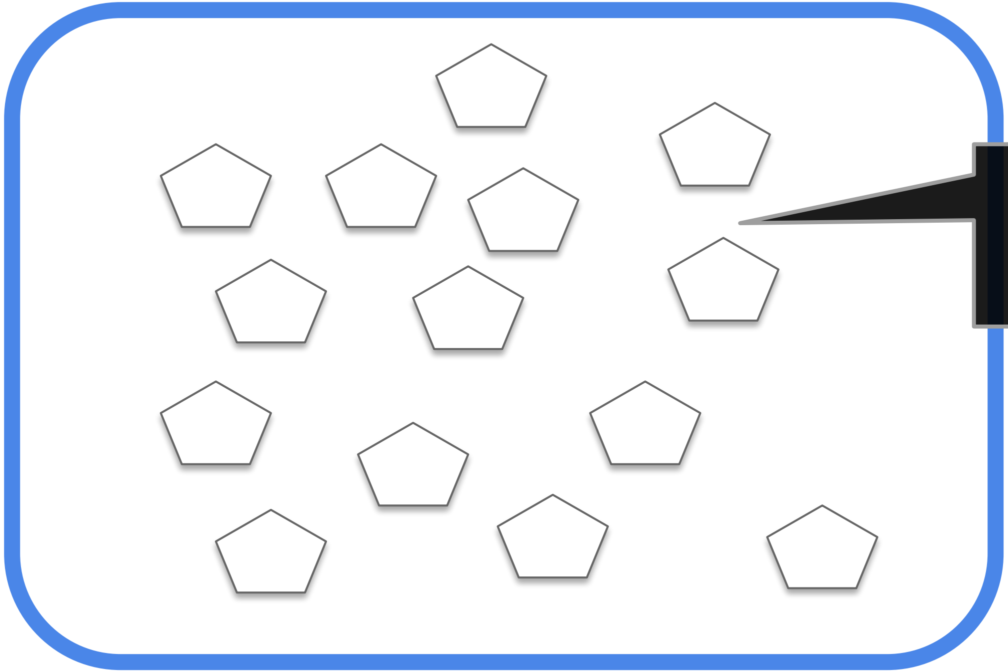
Population

Sample

In our Sesame street example, the sample would be the children included in the study

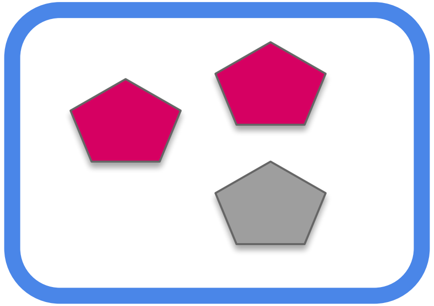

¯_(ツ)_/¯
Population
Sample

We don't know how much Sesame street was watched by or the tests scores of <u>all kids</u>



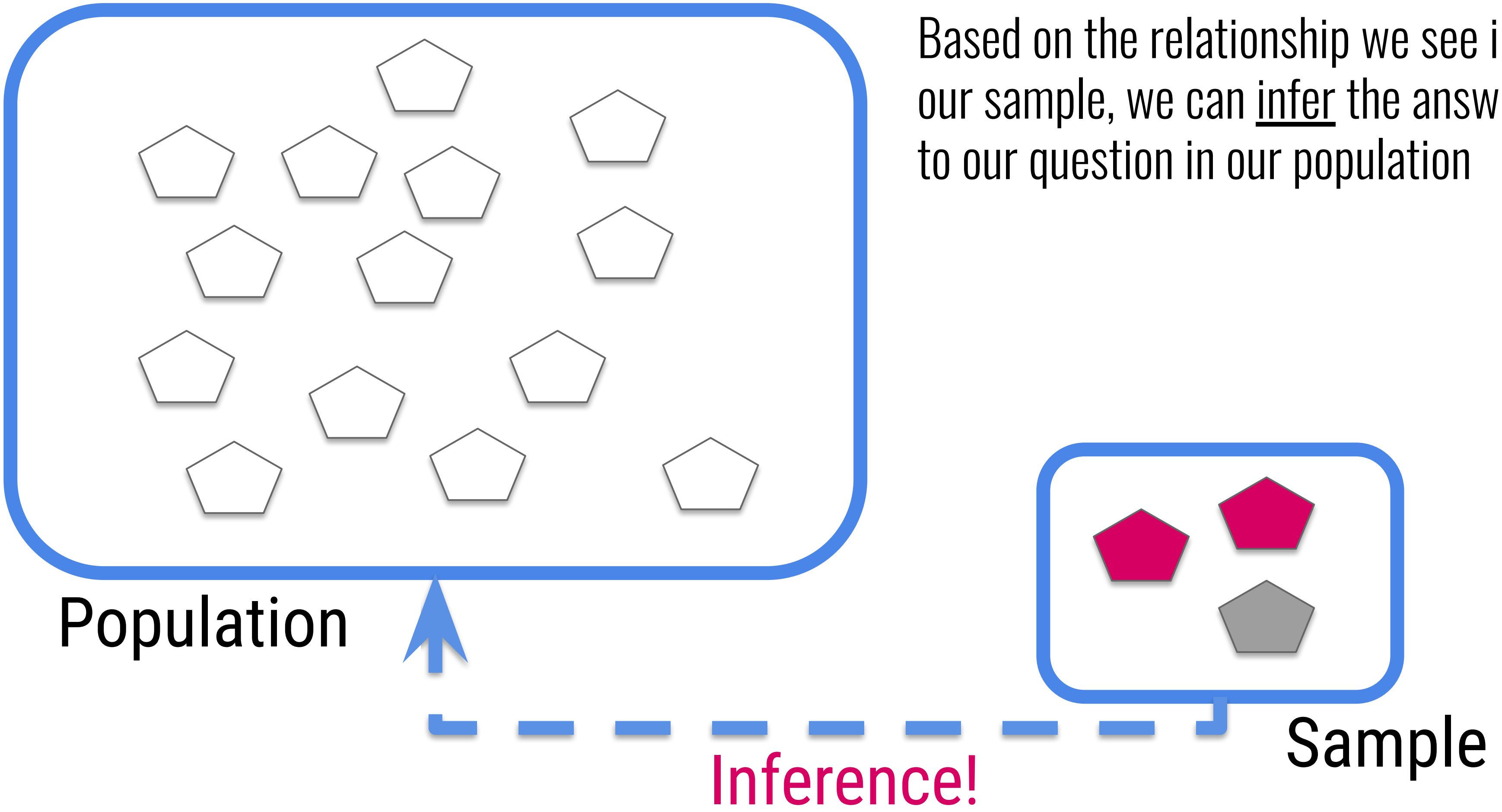
¯_(ツ)_/¯

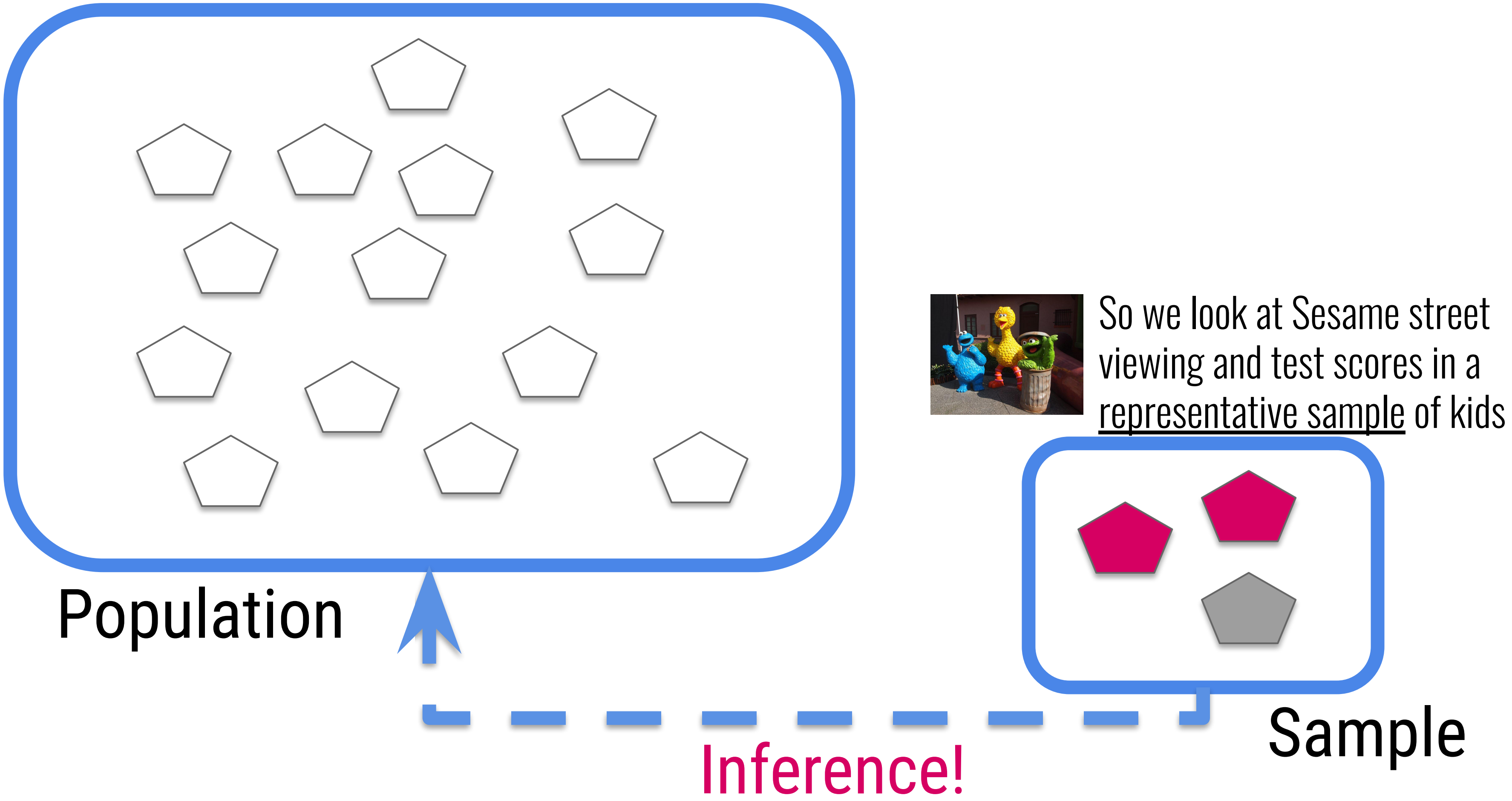
Population

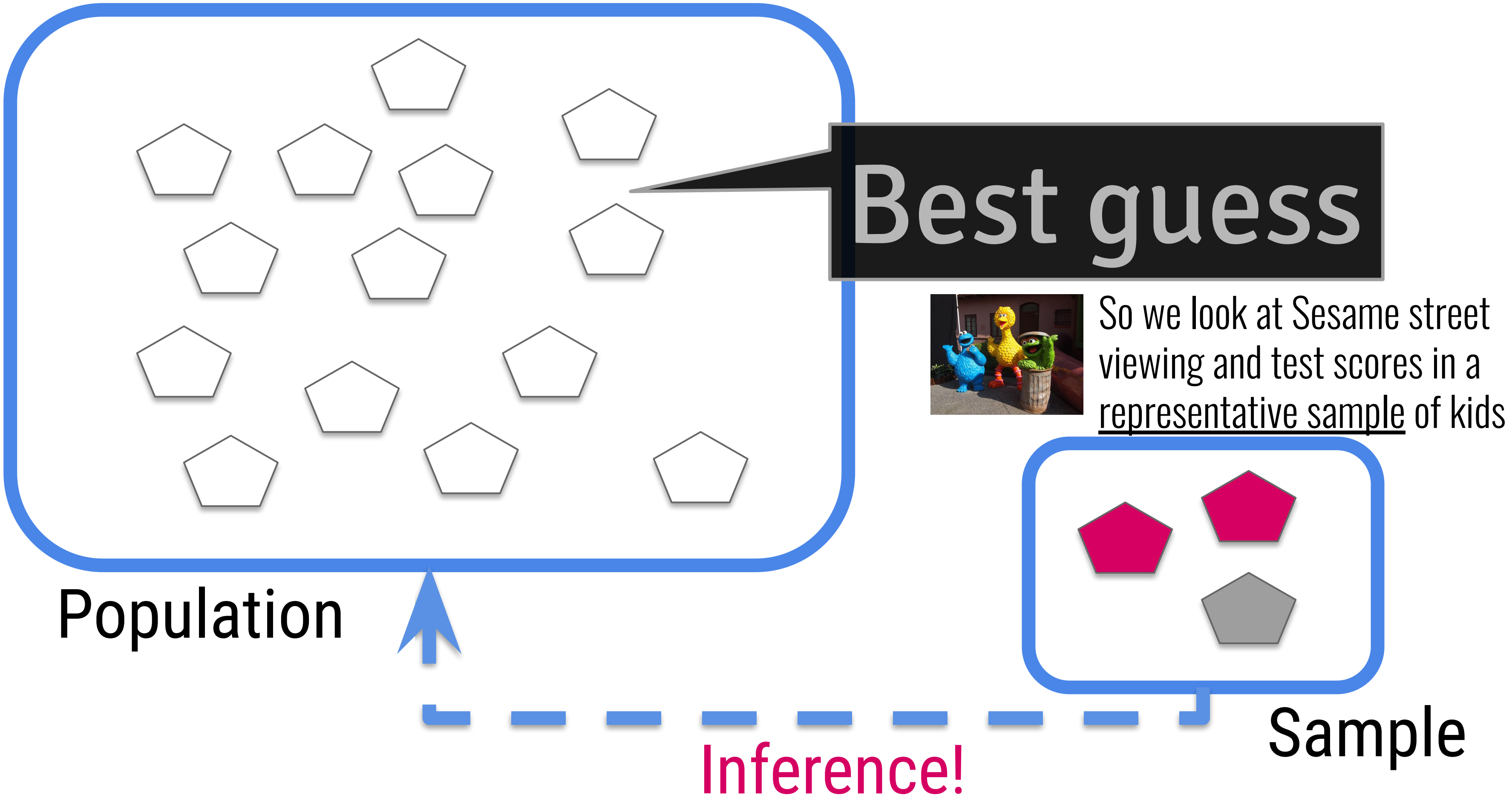


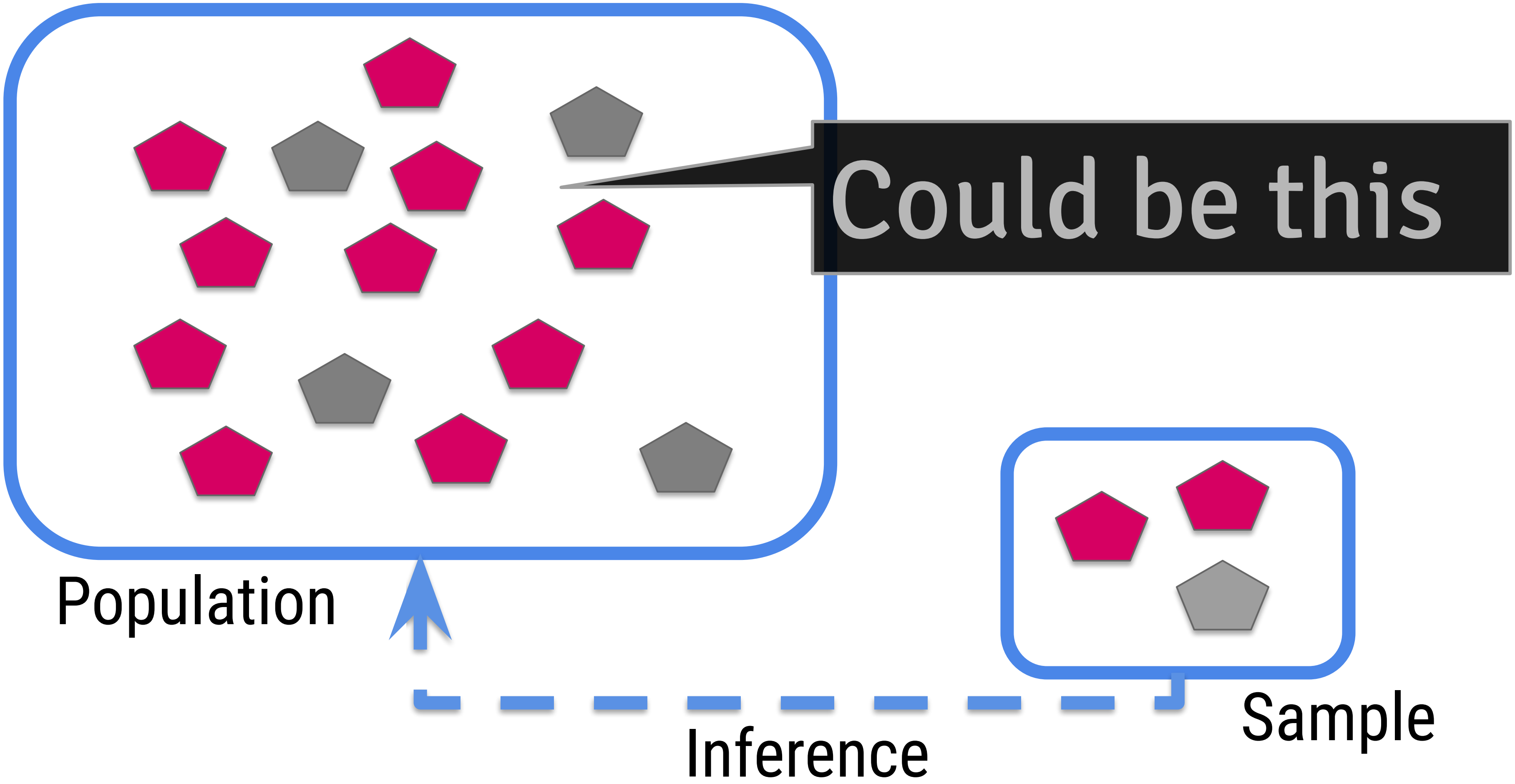
Sample

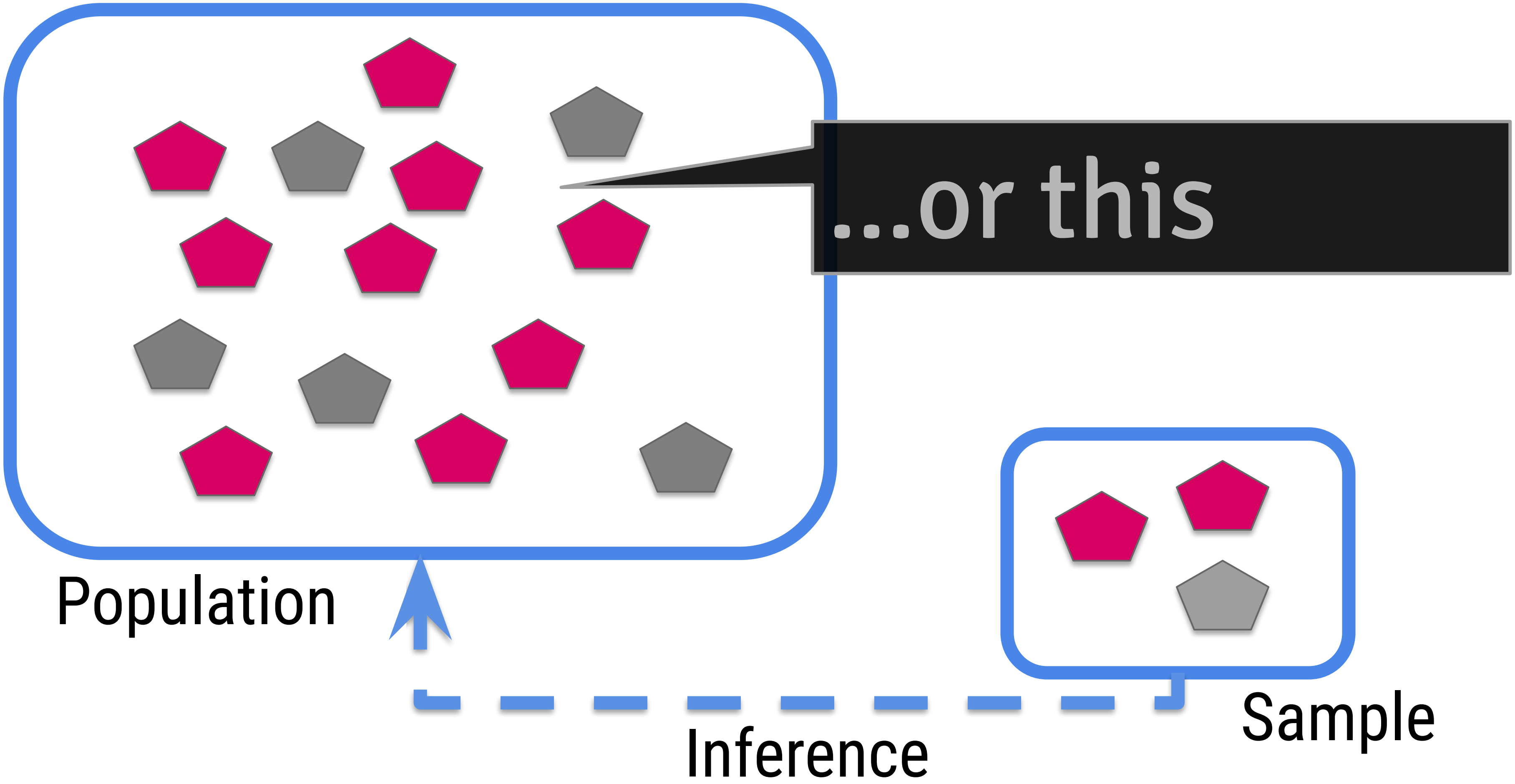
Based on the relationship we see in our sample, we can <u>infer</u> the answer to our question in our population

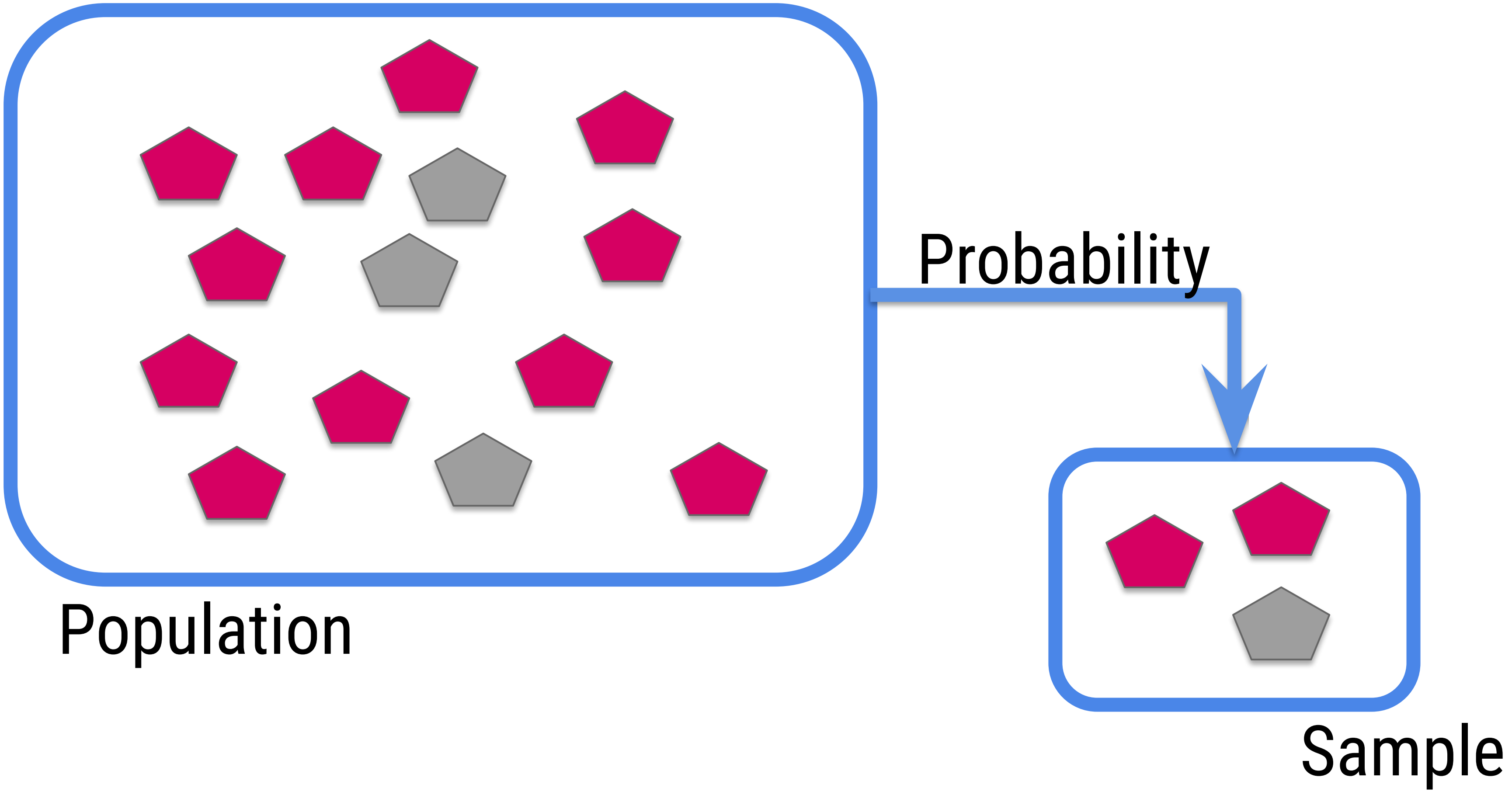


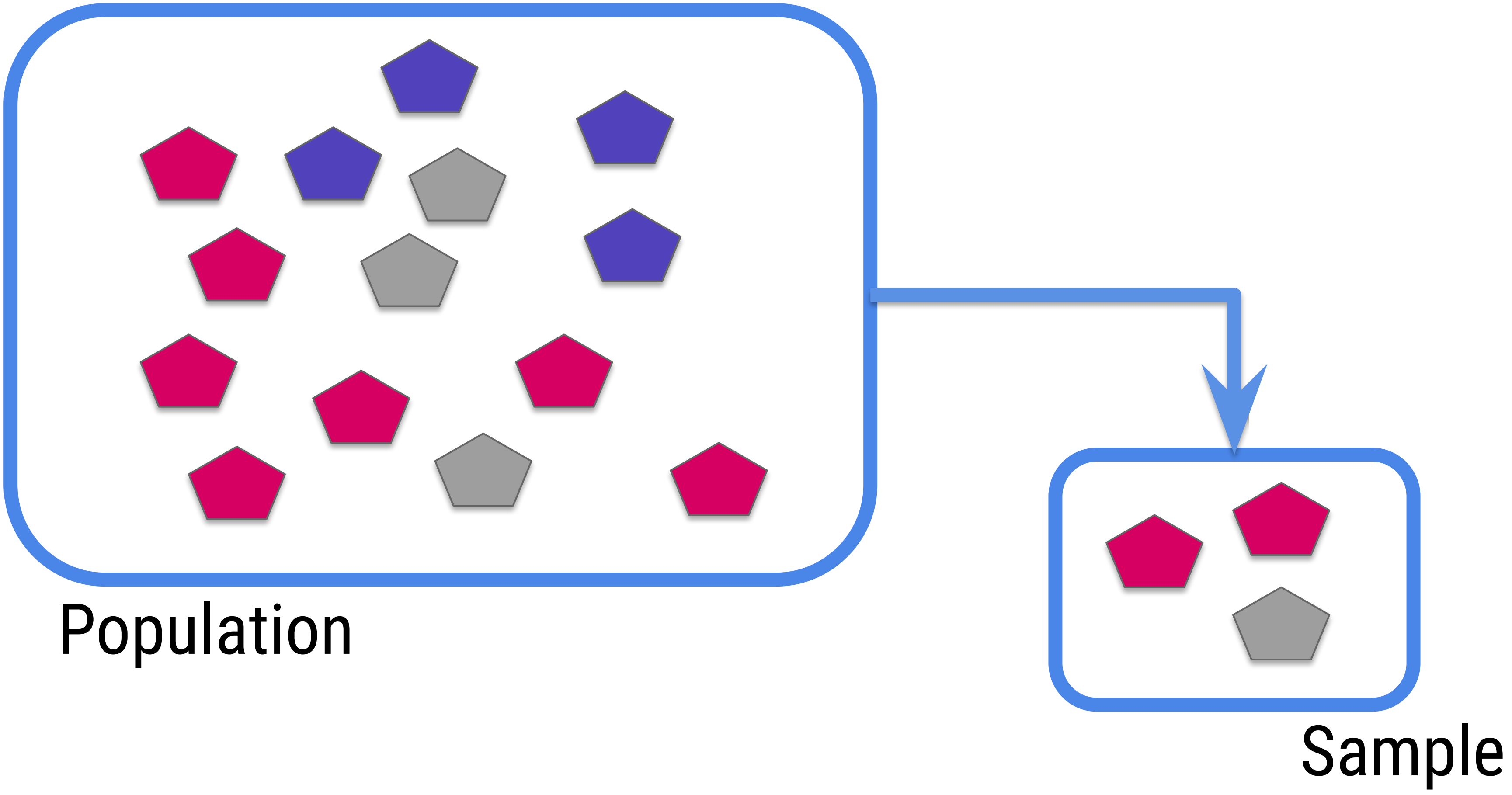

So we look at Sesame street viewing and test scores in a representative sample of kids
Population
Sample
Inference!


Best guess
So we look at Sesame street viewing and test scores in a representative sample of kids
Population
Sample
Inference!

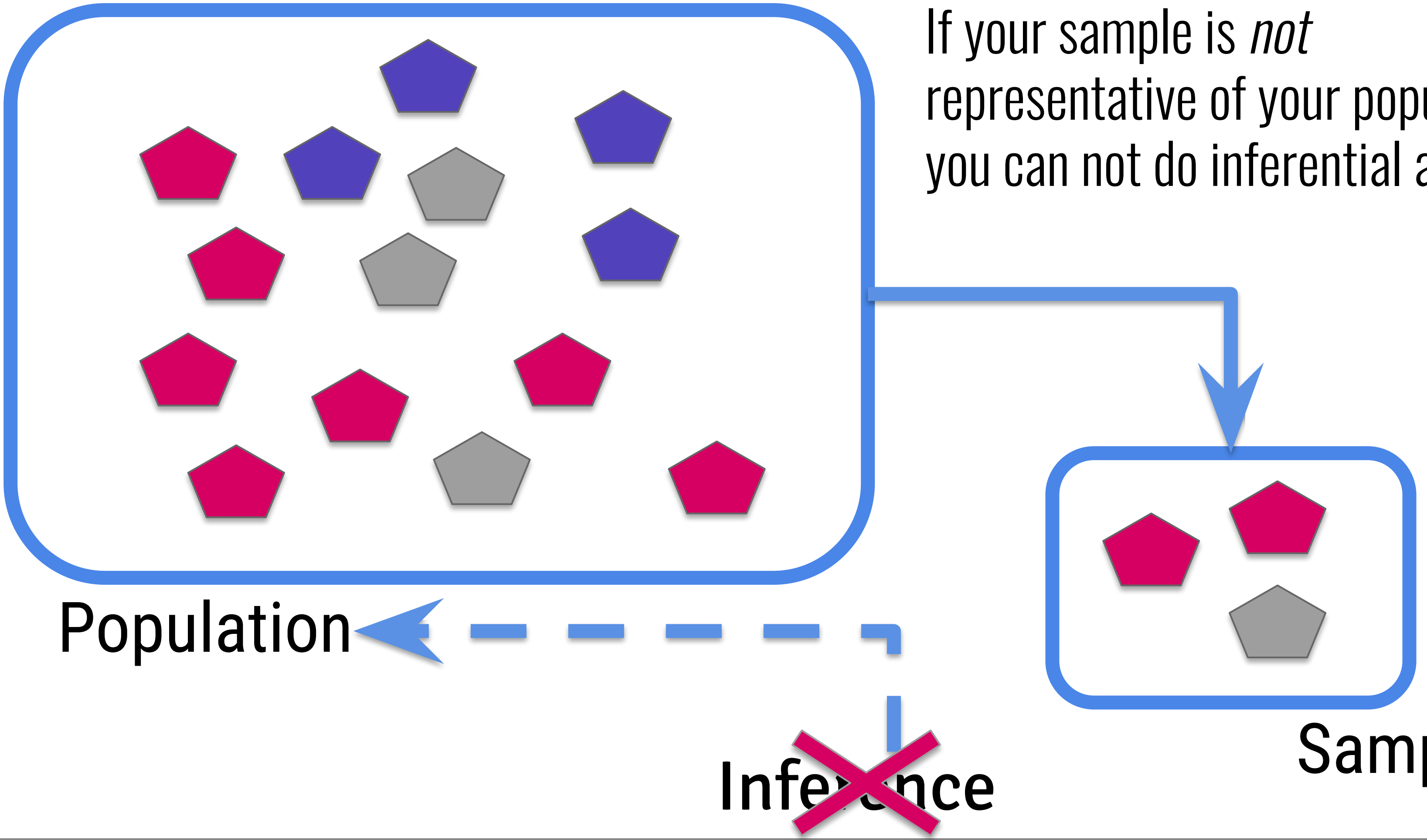

Could be this
Population
Sample
Inference


...or this
Population
Sample
Inference


Probability
Population
Sample


Population
Sample

If your sample is *not* representative of your population, you can not do inferential analysis.



Approaches to Inference

CORRELATION

ASSOCIATION BETWEEN VARIABLES

i.e. Pearson Correlation, Spearman Correlation, chi-square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS BETWEEN VARIABLES

i.e. t-test, ANOVA

REGRESSION

DOES CHANGE IN ONE VARIABLE MEAN CHANGE IN ANOTHER?

I.e. simple regression, multiple regression

NON-PARAMETRIC TESTS

FOR WHEN ASSUMPTIONS IN THESE OTHER 3 CATEGORIES ARE NOT MET

i.e. Wilcoxon rank-sum test, Wilcoxon sign-rank test, sign test

CORRELATION

ASSOCIATION BETWEEN VARIABLES

i.e. Pearson Correlation, Spearman Correlation, chi-square test

COMPARISON OF MEANS

DIFFERENCE IN MEANS BETWEEN VARIABLES

i.e. t-test, ANOVA

REGRESSION

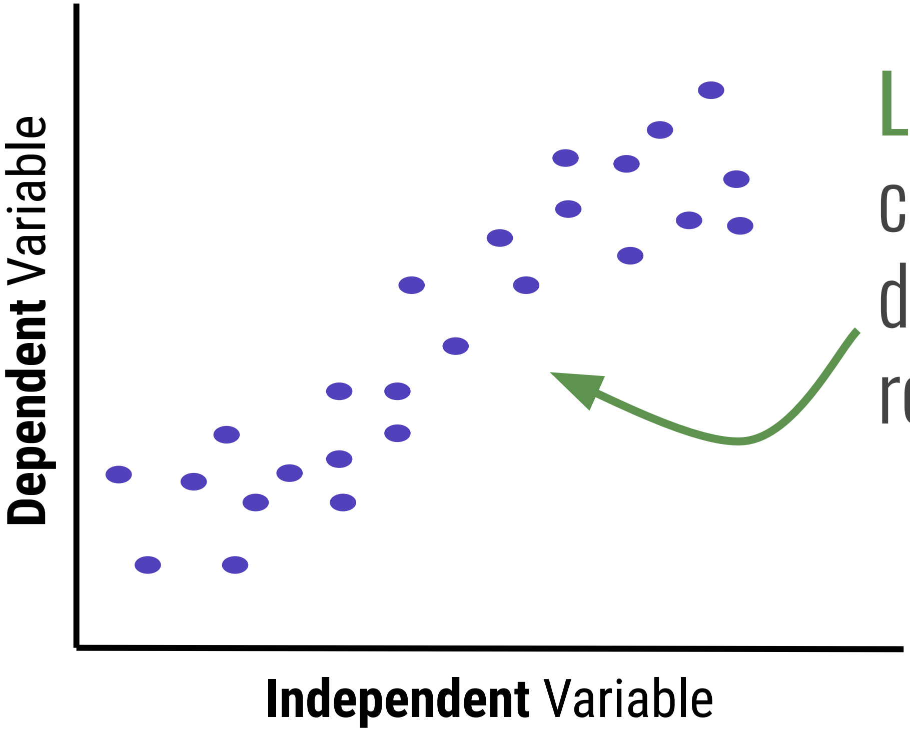
DOES CHANGE IN ONE VARIABLE MEAN CHANGE IN ANOTHER?

I.e. simple regression, multiple regression

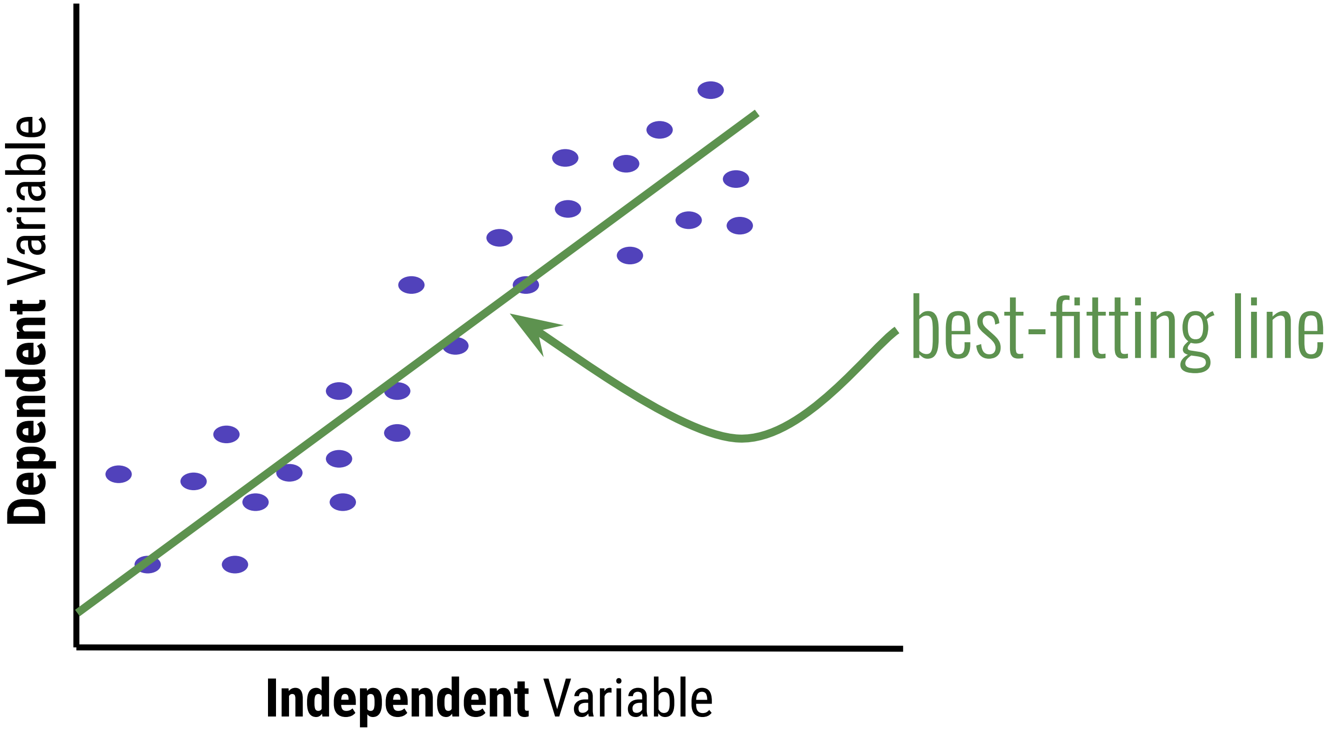
NON-PARAMETRIC TESTS

FOR WHEN ASSUMPTIONS IN THESE OTHER 3 CATEGORIES ARE NOT MET

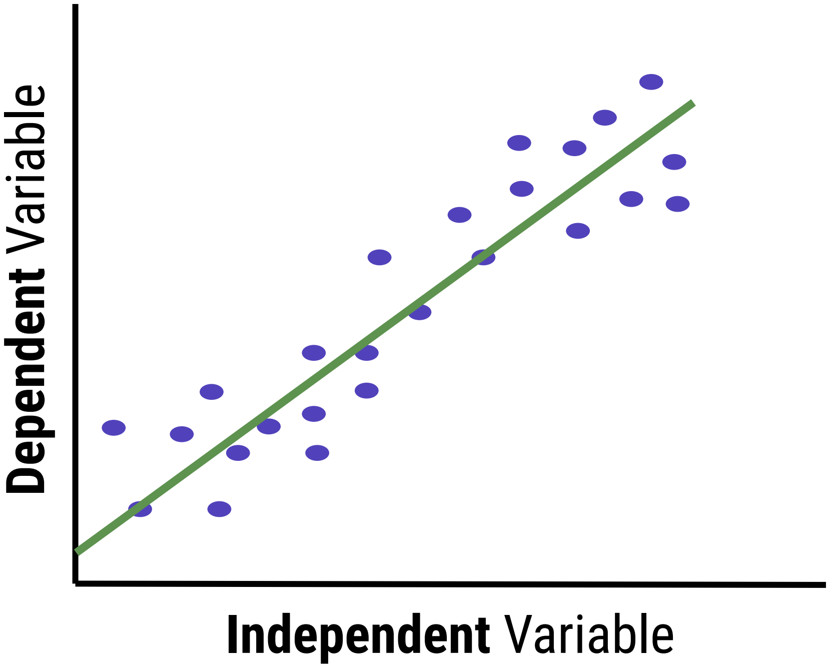
i.e. Wilcoxon rank-sum test, Wilcoxon sign-rank test, sign test



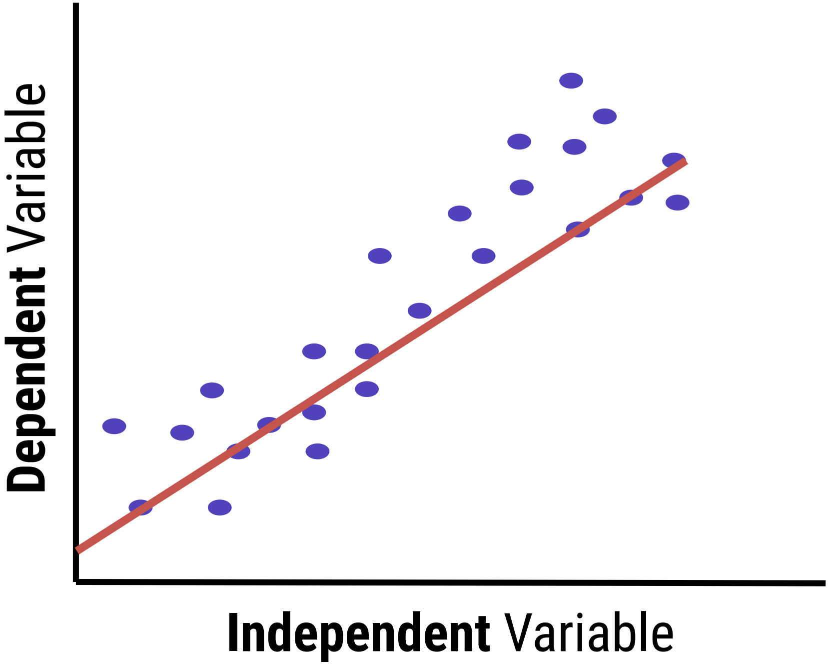
Linear regression can be used to describe this relationship

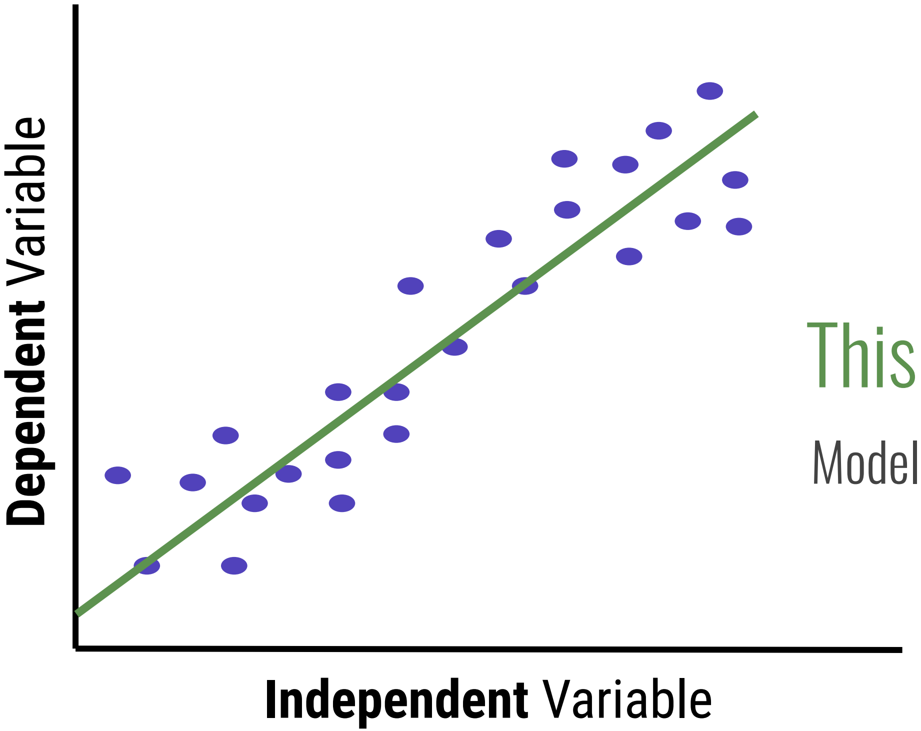

best-fitting line
Dependent Variable
Independent Variable

Best-fitting line



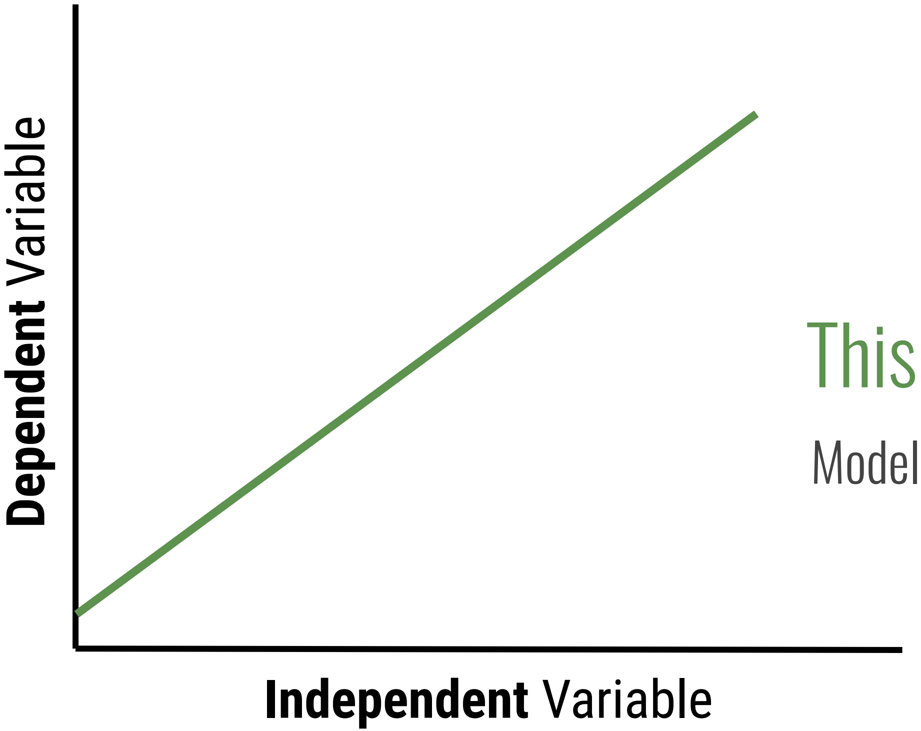
NOT a best-fitting line





This line is a **model** of the data

Models are mathematical equations generated to *represent* the real life situation

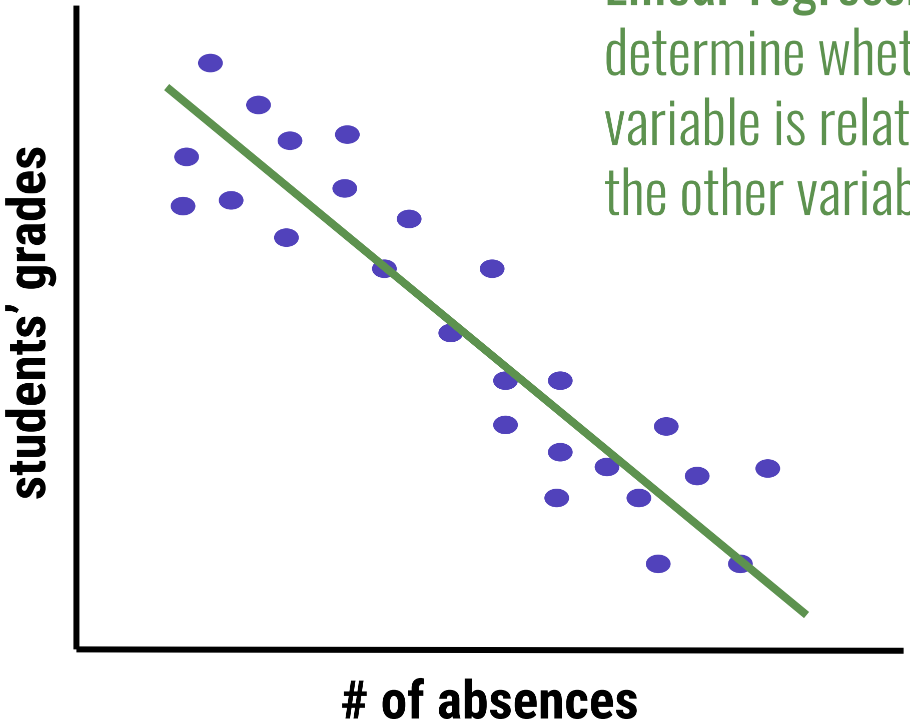


This line is a **model** of the data

Models are mathematical equations generated to *represent* the real life situation

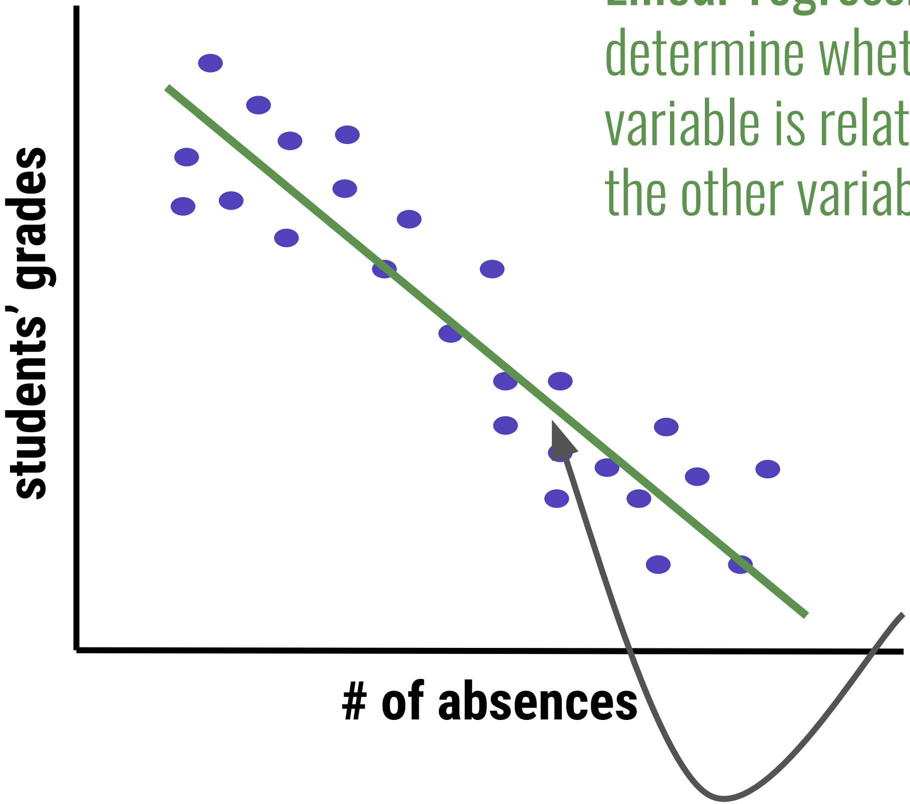
“All models are wrong, but some are useful”

-George Box (British Statistician, *JASA* 1976)

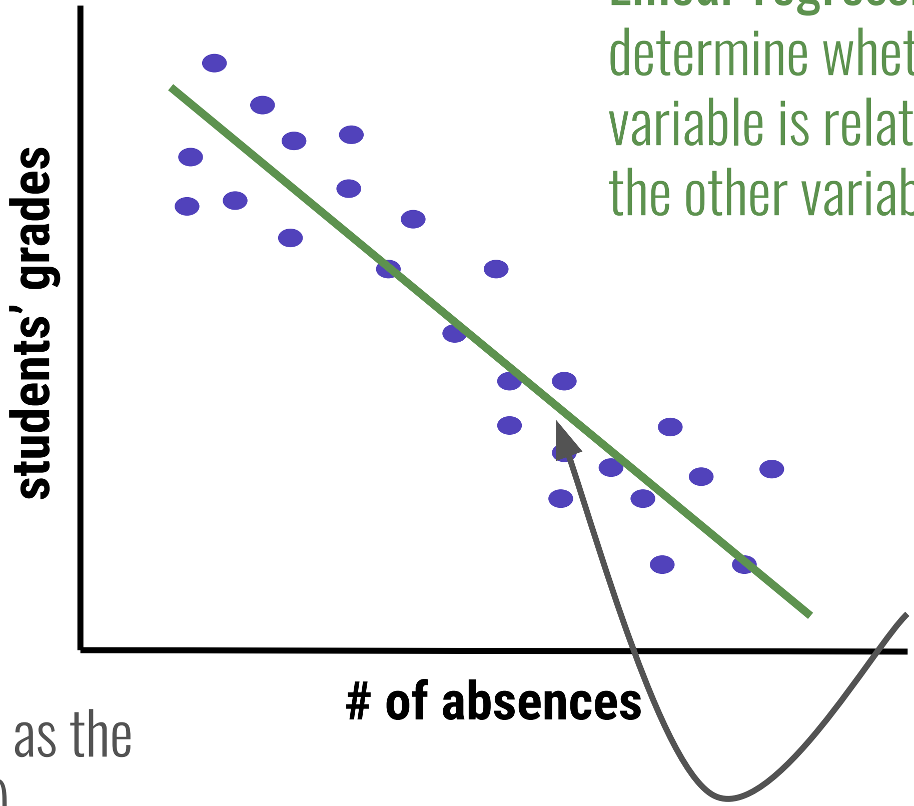


Linear regression can be used to determine whether a change in one variable is related to the change in the other variable

Linear regression can be used to determine whether a change in one variable is related to the change in the other variable



The magnitude of the relationship is measured by the slope of the line

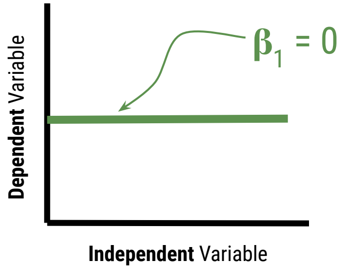


Linear regression can be used to determine whether a change in one variable is related to the change in the other variable

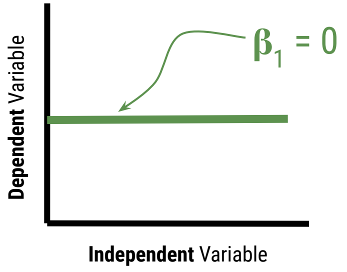
The magnitude of the relationship is measured by the slope of the line

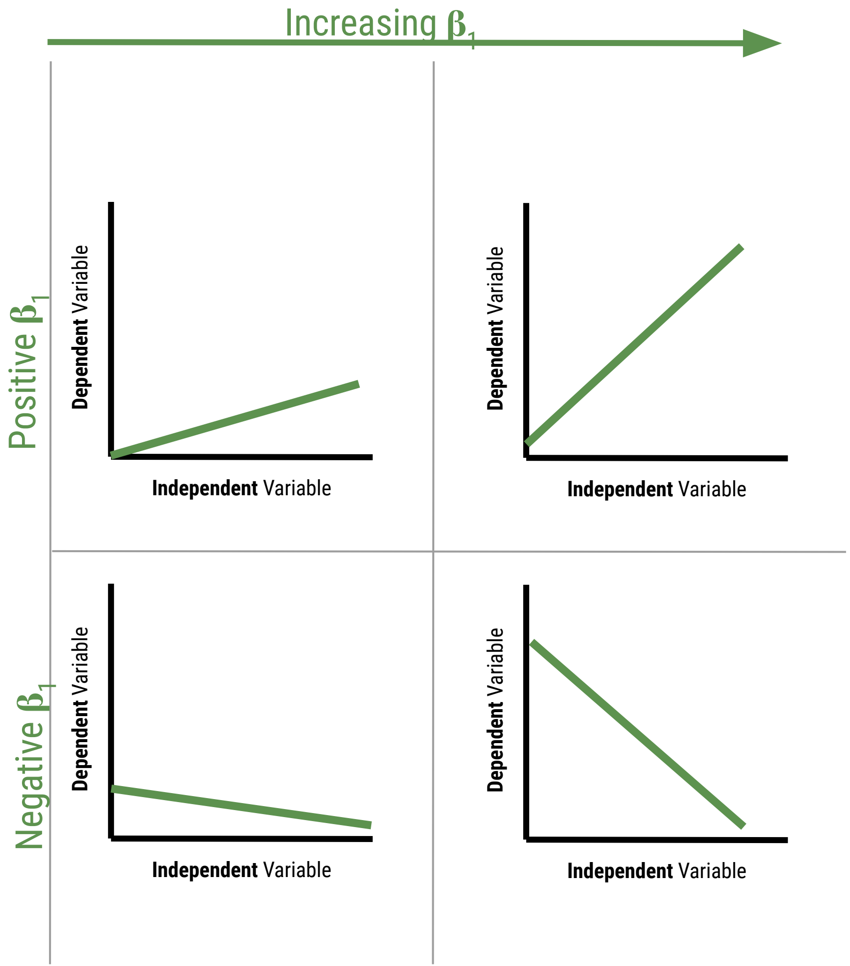
This is also referred to as the model's effect size (β_1)

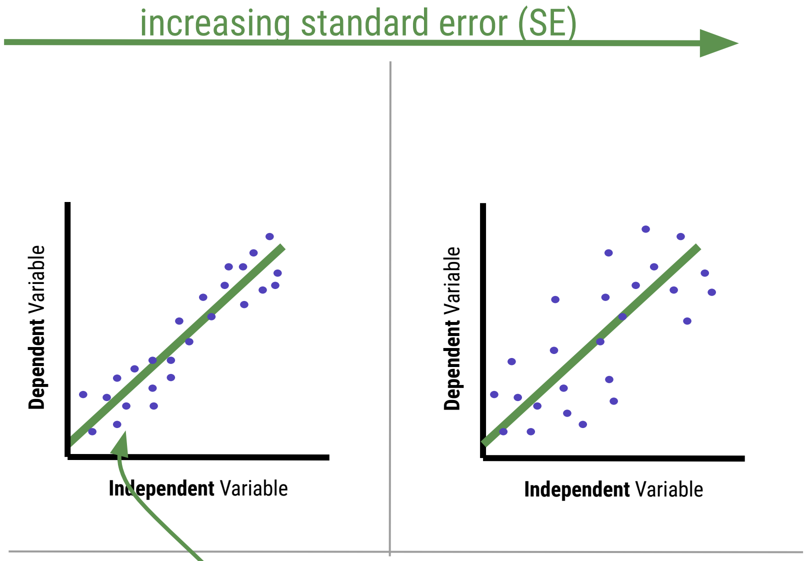
Effect size (β_1) can be estimated using the slope of the line



Effect size (β_1) can be estimated using the slope of the line



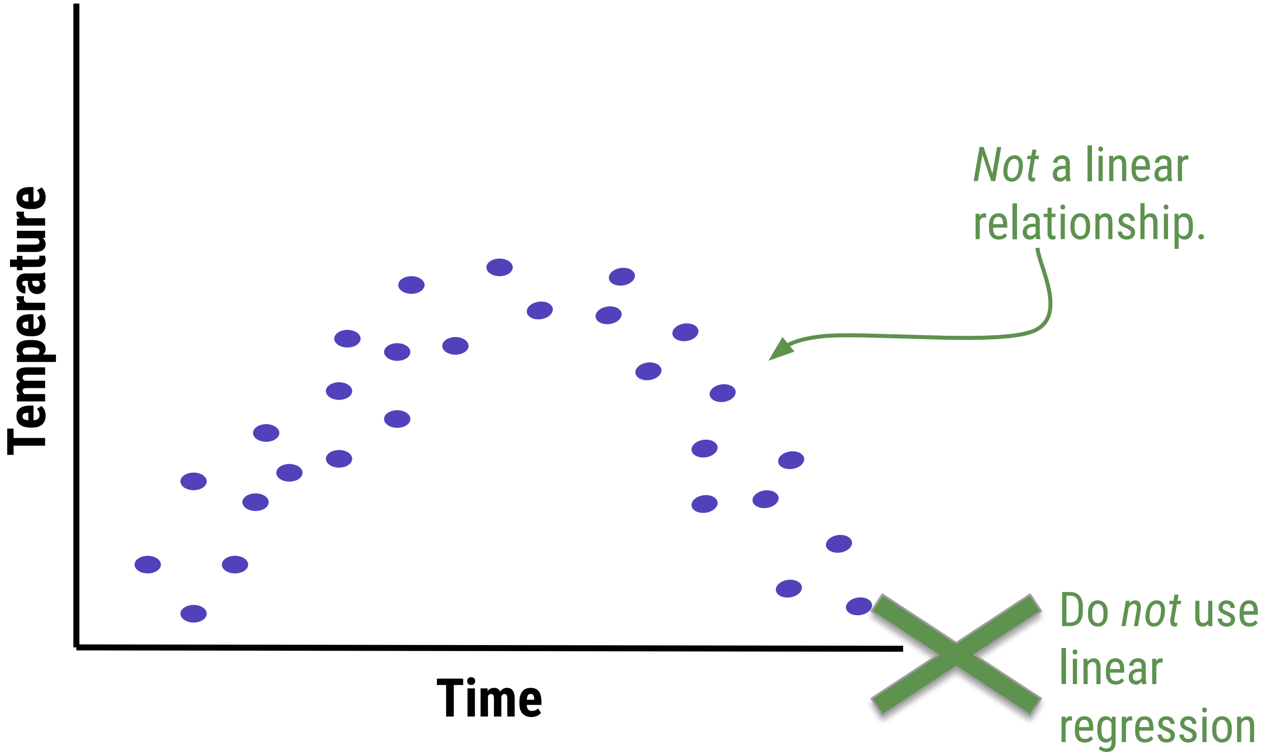




The *closer* the points are to the regression line, the *less uncertain* we are in our estimate

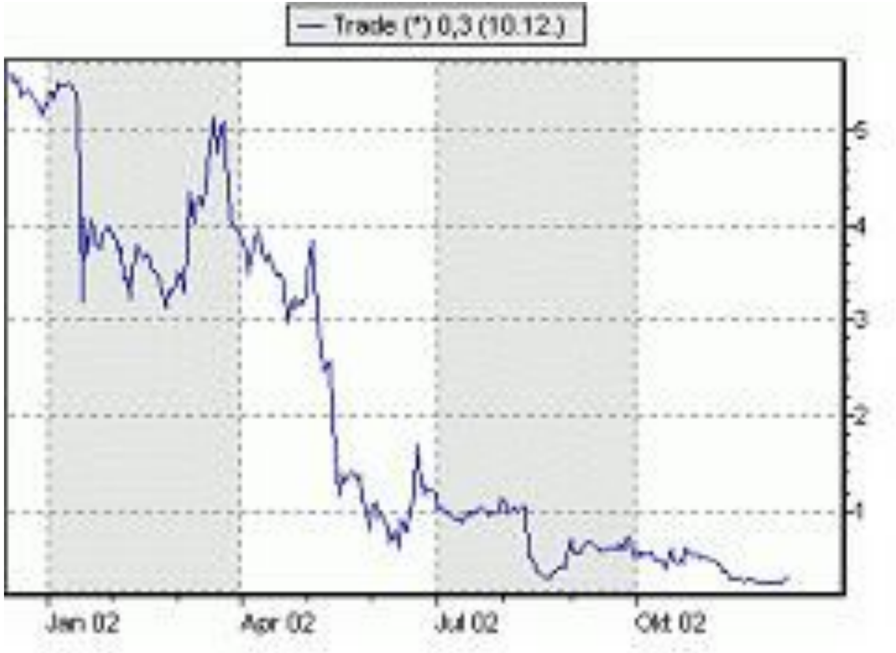
Assumptions of linear regression

1. Linear relationship
2. No multicollinearity
3. No auto-correlation
4. Homoscedasticity

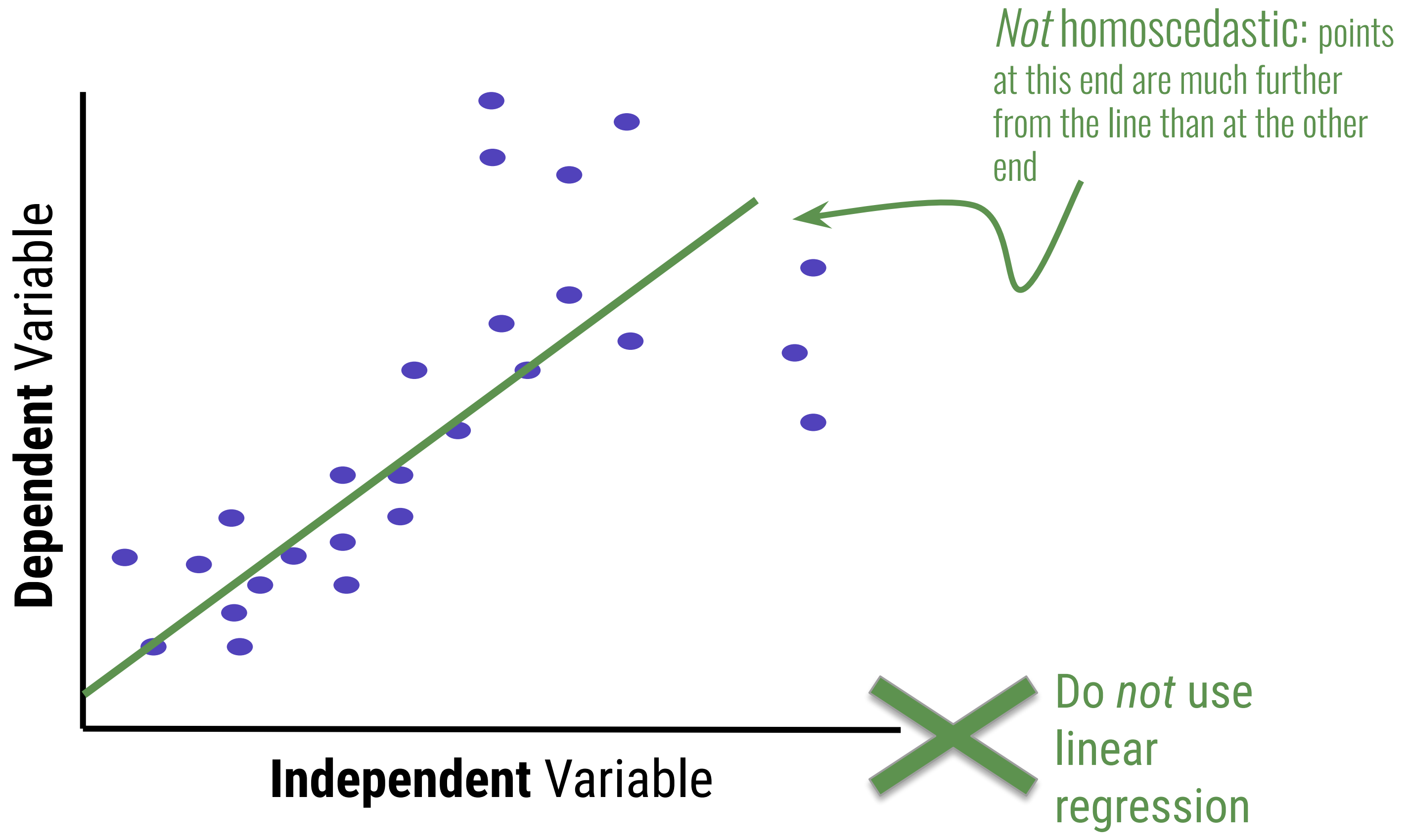

Temperature
Time
Not a linear relationship.
Do *not* use linear regression

Linear regression assumes no multicollinearity. **Multicollinearity** occurs when the independent variables (in multiple linear regression) are too highly correlated with each other.

Autocorrelation occurs when the observations are *not* independent of one another (i.e. stock prices)

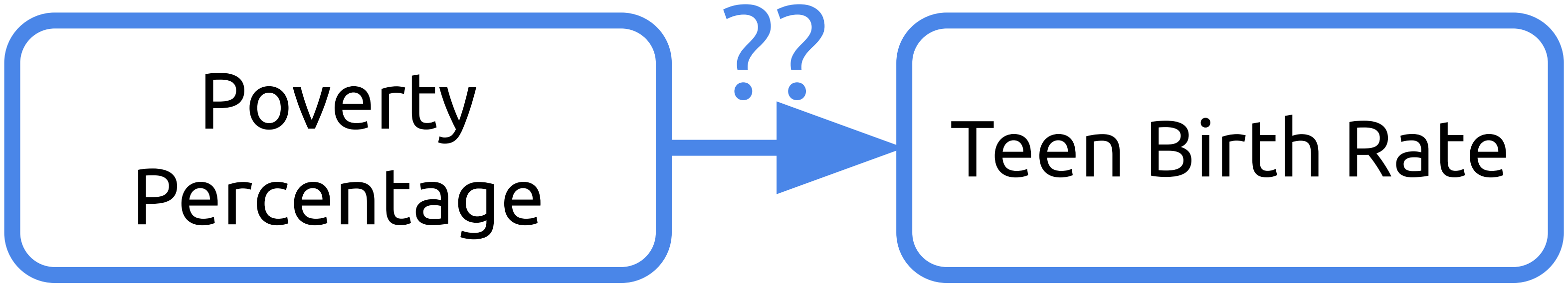


https://www.statisticssolutions.com/assumptions-of-linear-regression/



Dependent Variable
Independent Variable
Not homoscedastic: points at this end are much further from the line than at the other end
Do *not* use linear regression

Does Poverty Percentage affect Teen Birth Rate?



<u>Null Hypothesis</u>:

H_0: Poverty Rate does not affect Teen Birth Rate (β_1=0)

<u>Alternative Hypothesis</u>:

H_a: Poverty Rate affects Teen Birth Rate (β_1≠0)

What is the relationship between Poverty Percentage & Teen Birth Rate?

What's your hypothesis?

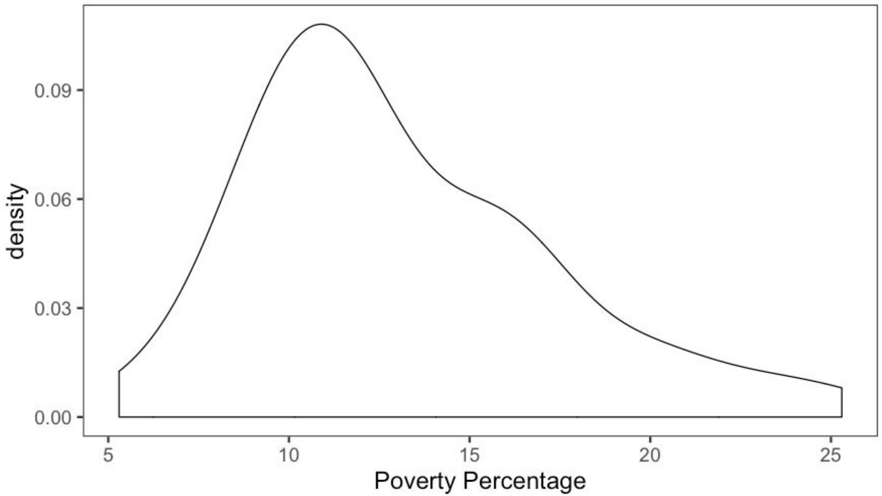
A
No
relationship

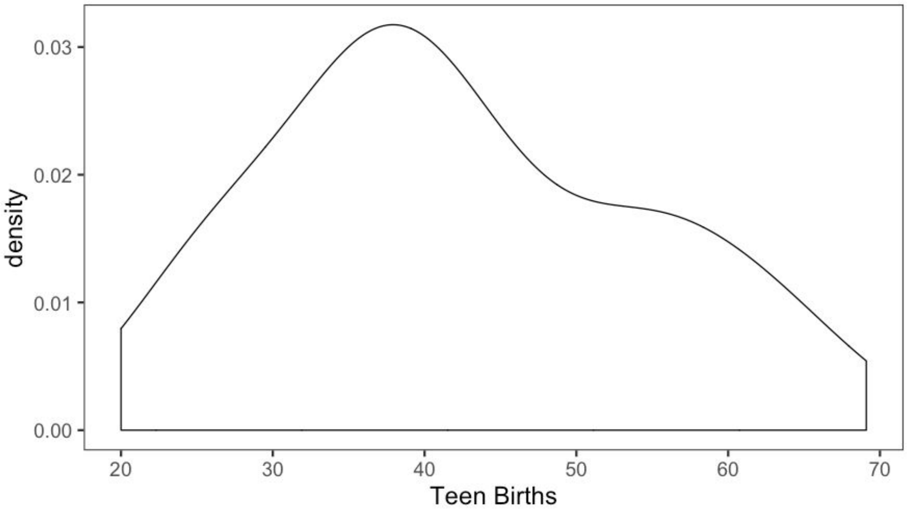
B
Positive
relationship

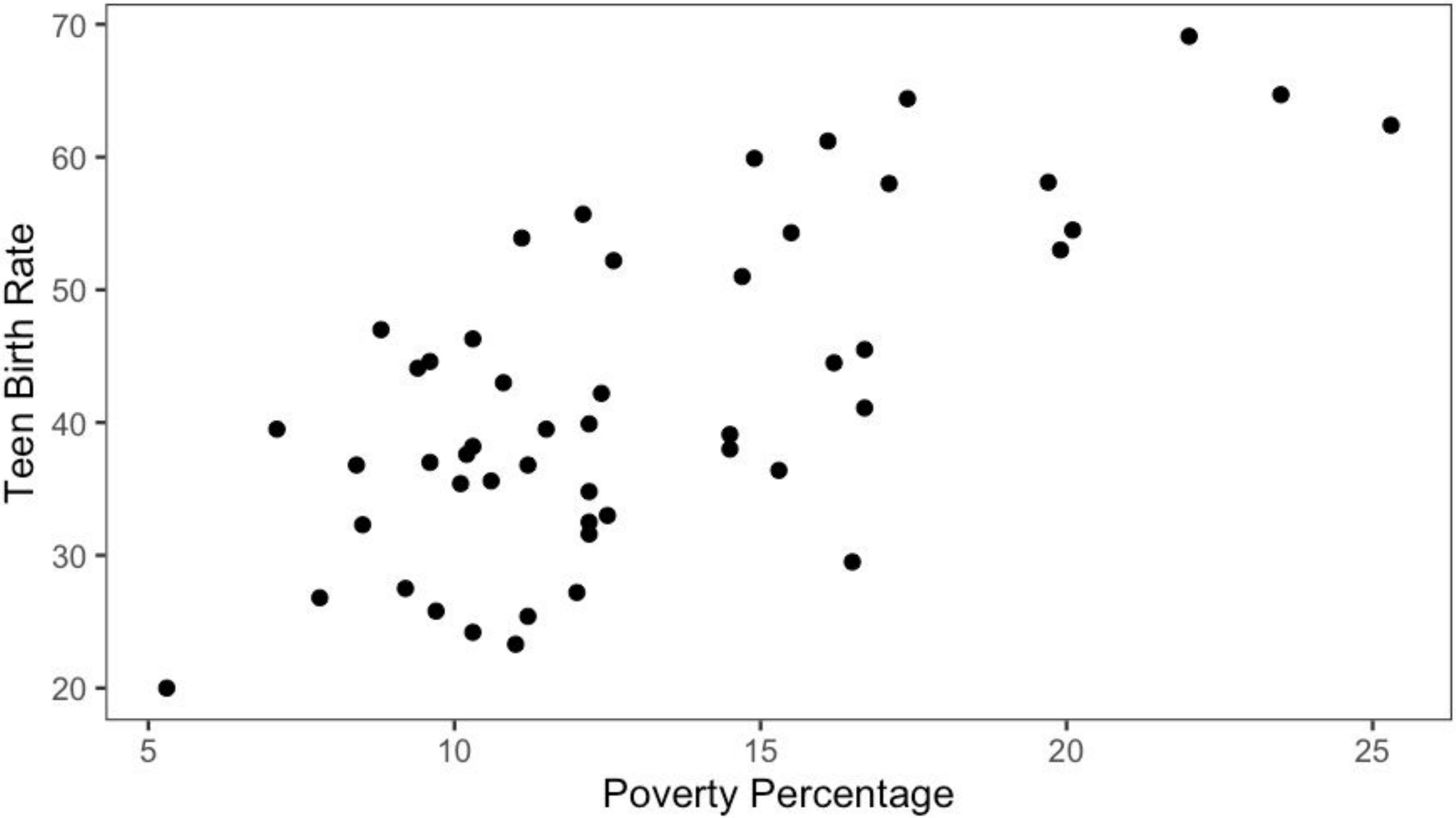
C
Negative
relationship

	Location	PovPct	Brth15to17	Brth18to19	ViolCrime	TeenBrth
1	Alabama	20.1	31.5	88.7	11.2	54.5
2	Alaska	7.1	18.9	73.7	9.1	39.5
3	Arizona	16.1	35.0	102.5	10.4	61.2
4	Arkansas	14.9	31.6	101.7	10.4	59.9
5	California	16.7	22.6	69.1	11.2	41.1
6	Colorado	8.8	26.2	79.1	5.8	47.0
7	Connecticut	9.7	14.1	45.1	4.6	25.8
8	Delaware	10.3	24.7	77.8	3.5	46.3
9	District_of_Columbia	22.0	44.8	101.5	65.0	69.1
10	Florida	16.2	23.2	78.4	7.3	44.5
11	Georgia	12.1	31.4	92.8	9.5	55.7
12	Hawaii	10.3	17.7	66.4	4.7	38.2
13	Idaho	14.5	18.4	69.1	4.1	39.1
14	Illinois	12.4	23.4	70.5	10.3	42.2
15	Indiana	9.6	22.6	78.5	8.0	44.6
16	Iowa	12.2	16.4	55.4	1.8	32.5
17	Kansas	10.8	21.4	74.2	6.2	43.0

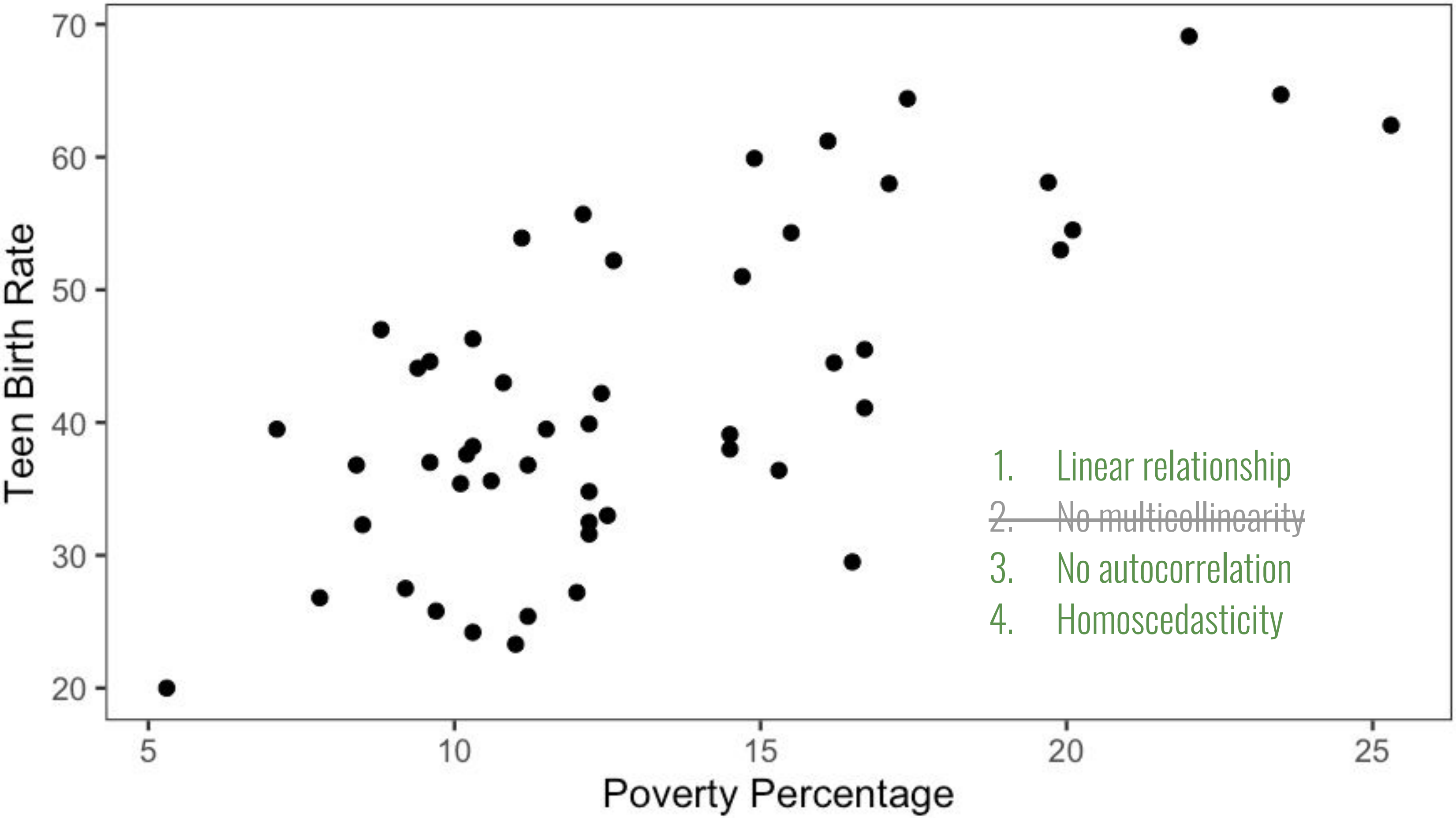
EDA: distributions





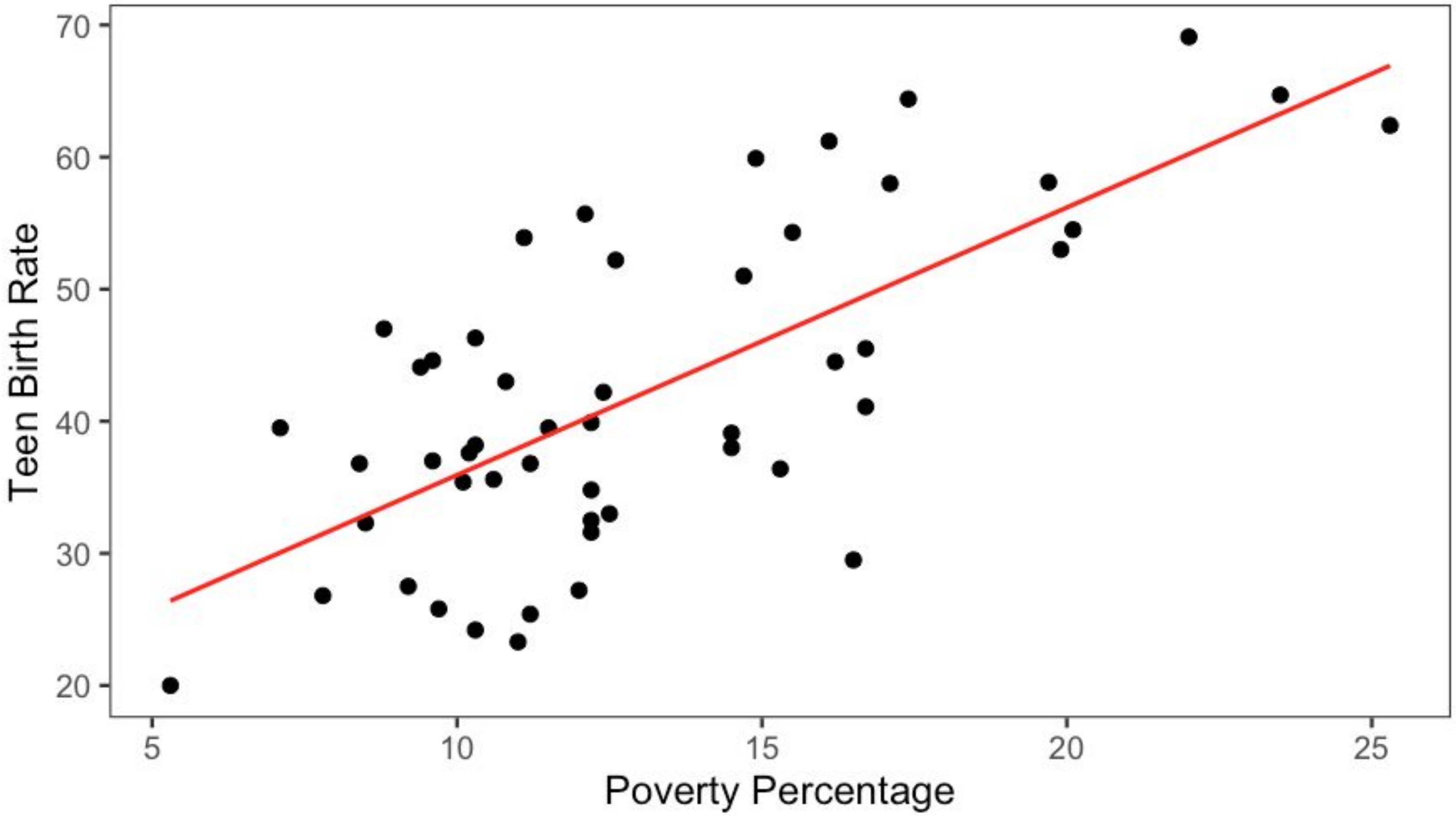


Data source: *Mind On Statistics*, 3rd edition, Utts and Heckard.



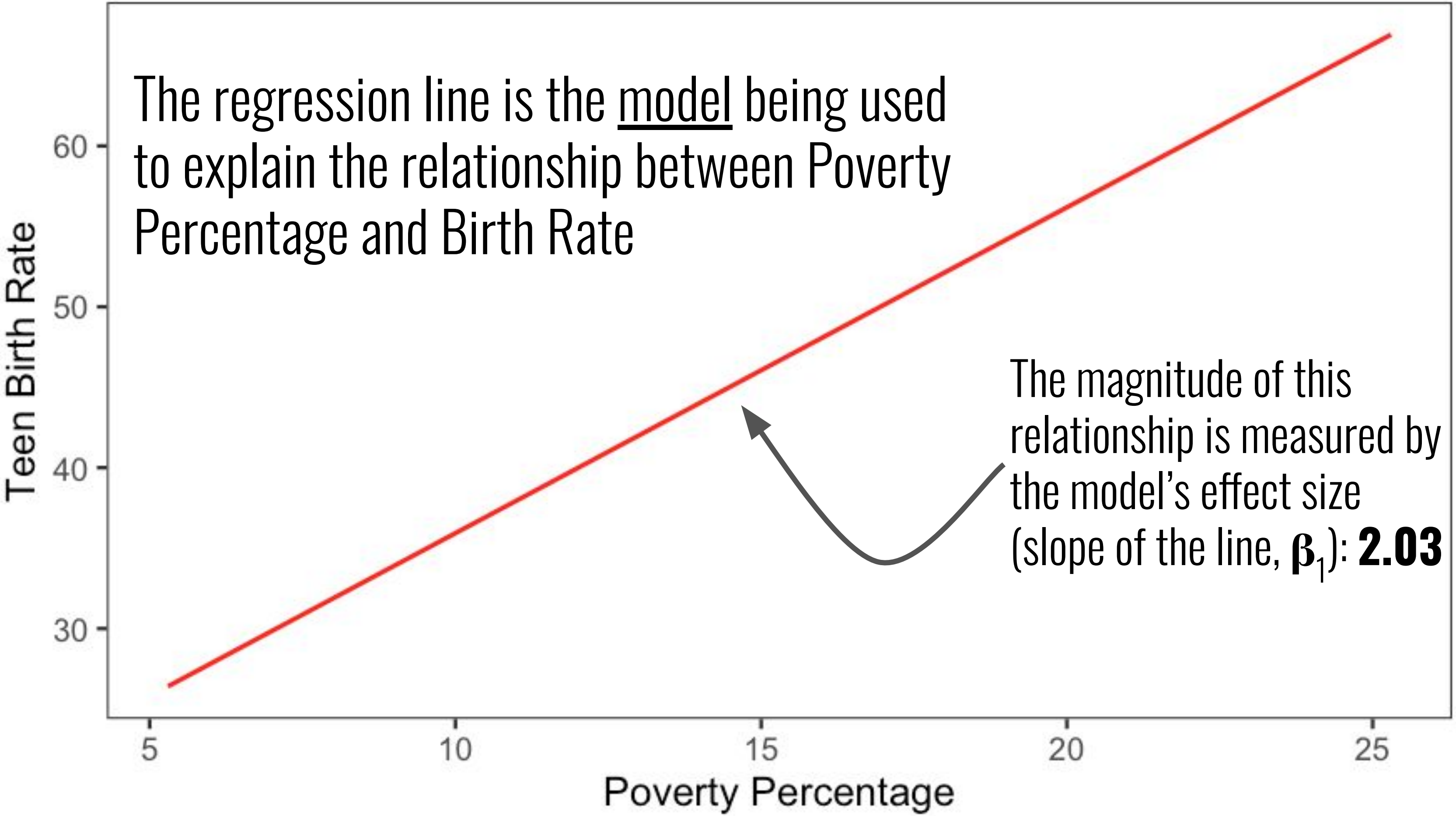
1. Linear relationship
2. ~~No multicollinearity~~
3. No autocorrelation
4. Homoscedasticity

Data source: *Mind On Statistics*, 3rd edition, Utts and Heckard.

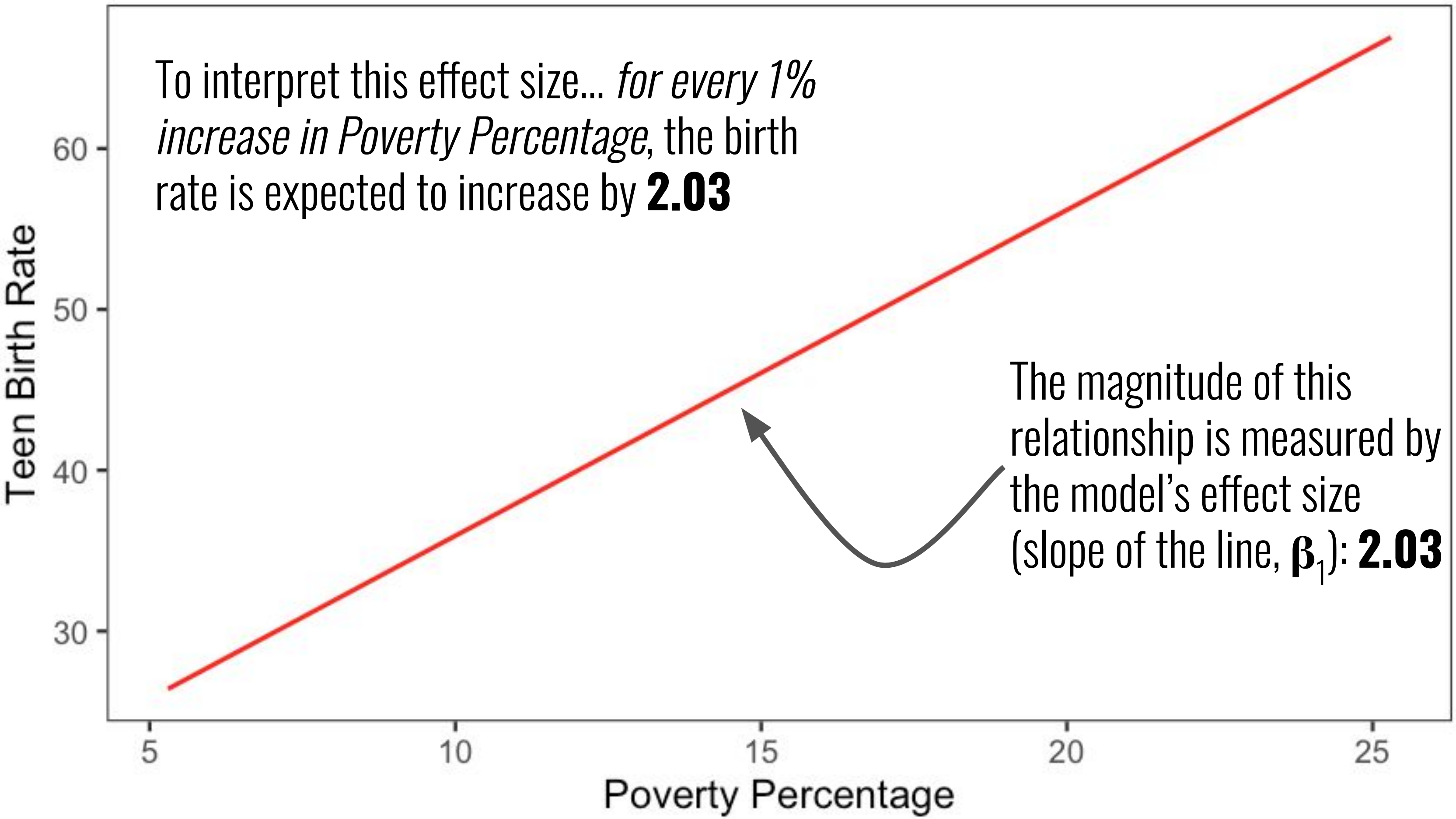


70
60
50
40
30
20
Teen Birth Rate
5
10
15
20
25
Poverty Percentage

The regression line is the <u>model</u> being used to explain the relationship between Poverty Percentage and Birth Rate

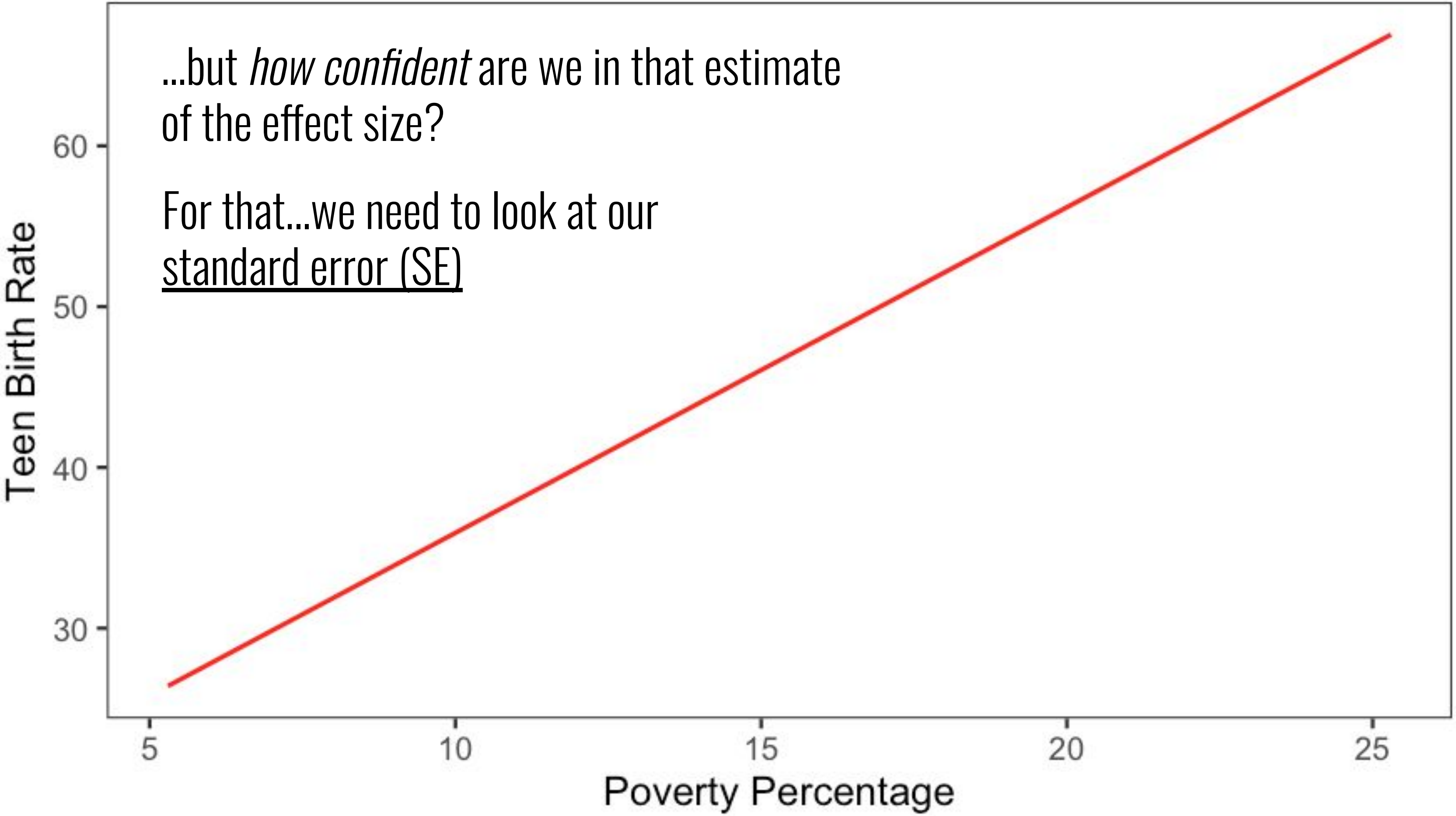


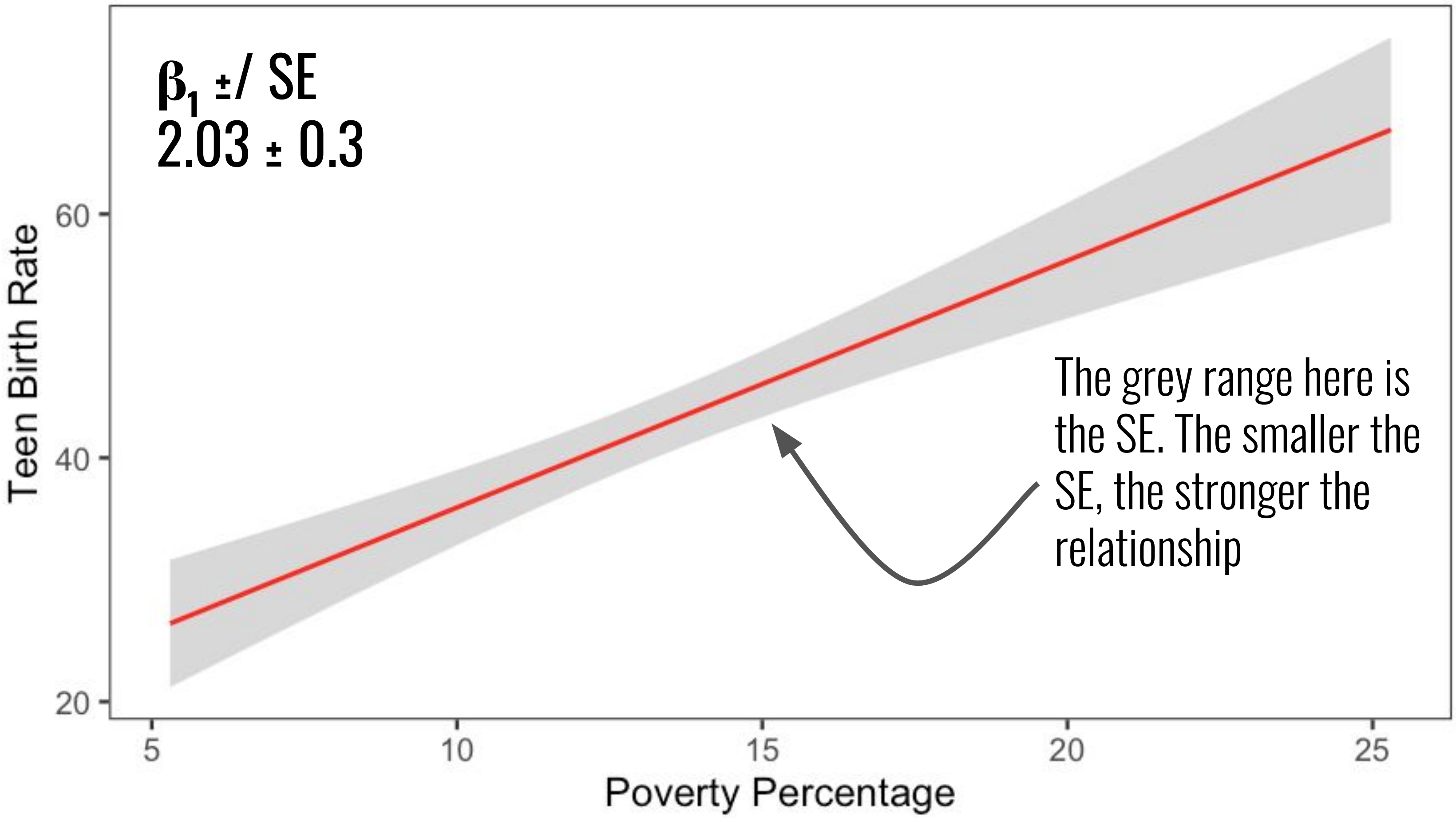
The magnitude of this relationship is measured by the model's effect size (slope of the line, $\boldsymbol{\beta}_1$): **2.03**

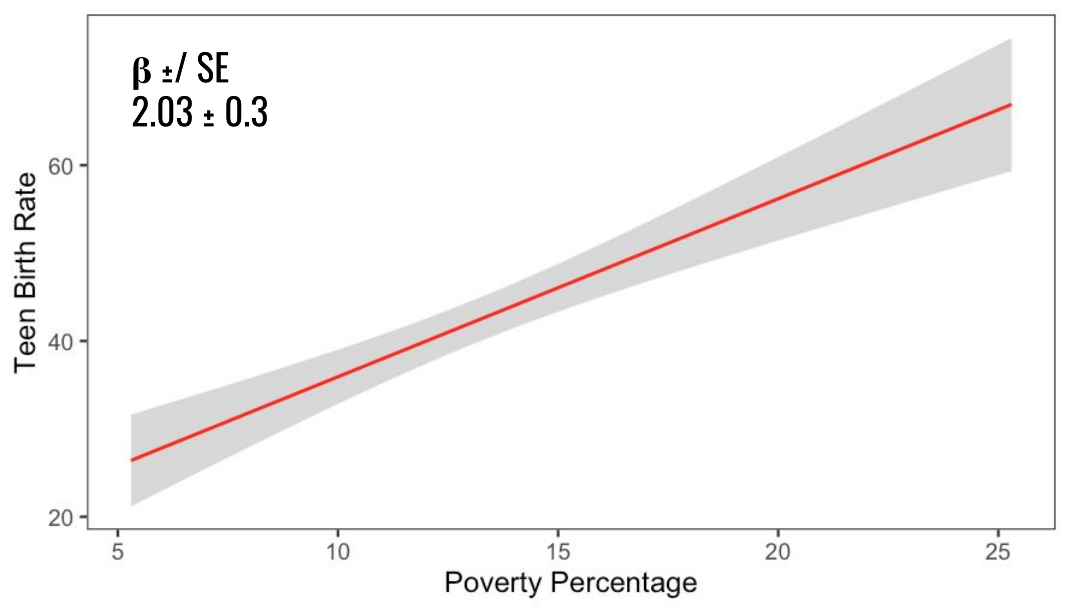

To interpret this effect size… *for every 1% increase in Poverty Percentage*, the birth rate is expected to increase by **2.03**
The magnitude of this relationship is measured by the model’s effect size (slope of the line, β_1): **2.03**
Teen Birth Rate
60
50
40
30
5
10
15
20
25
Poverty Percentage

...but *how confident* are we in that estimate of the effect size?

For that...we need to look at our <u>standard error (SE)</u>



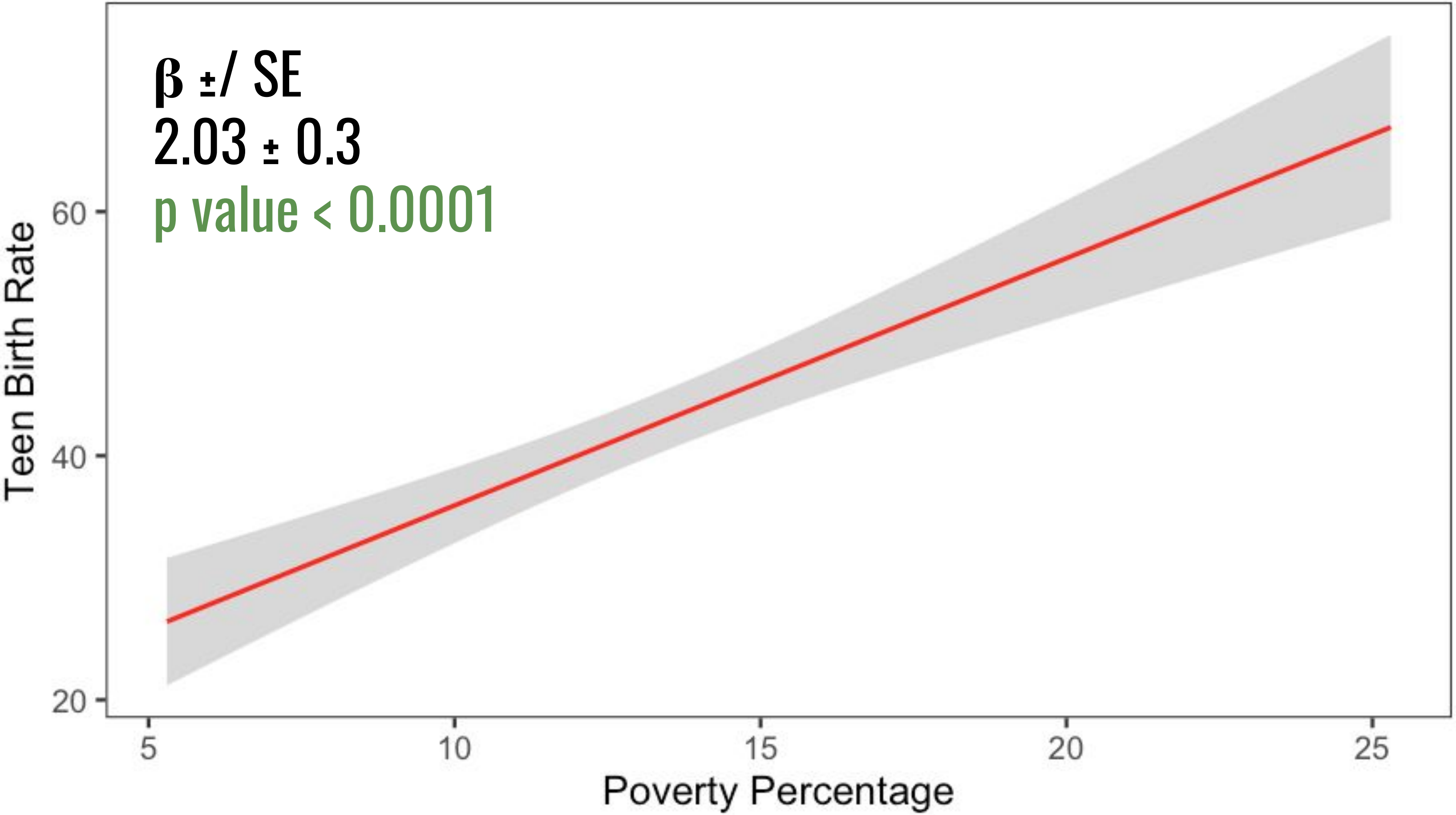

β1 ±/ SE
2.03 ± 0.3
The grey range here is the SE. The smaller the SE, the stronger the relationship
Teen Birth Rate
20
40
60
5
10
15
20
25
Poverty Percentage



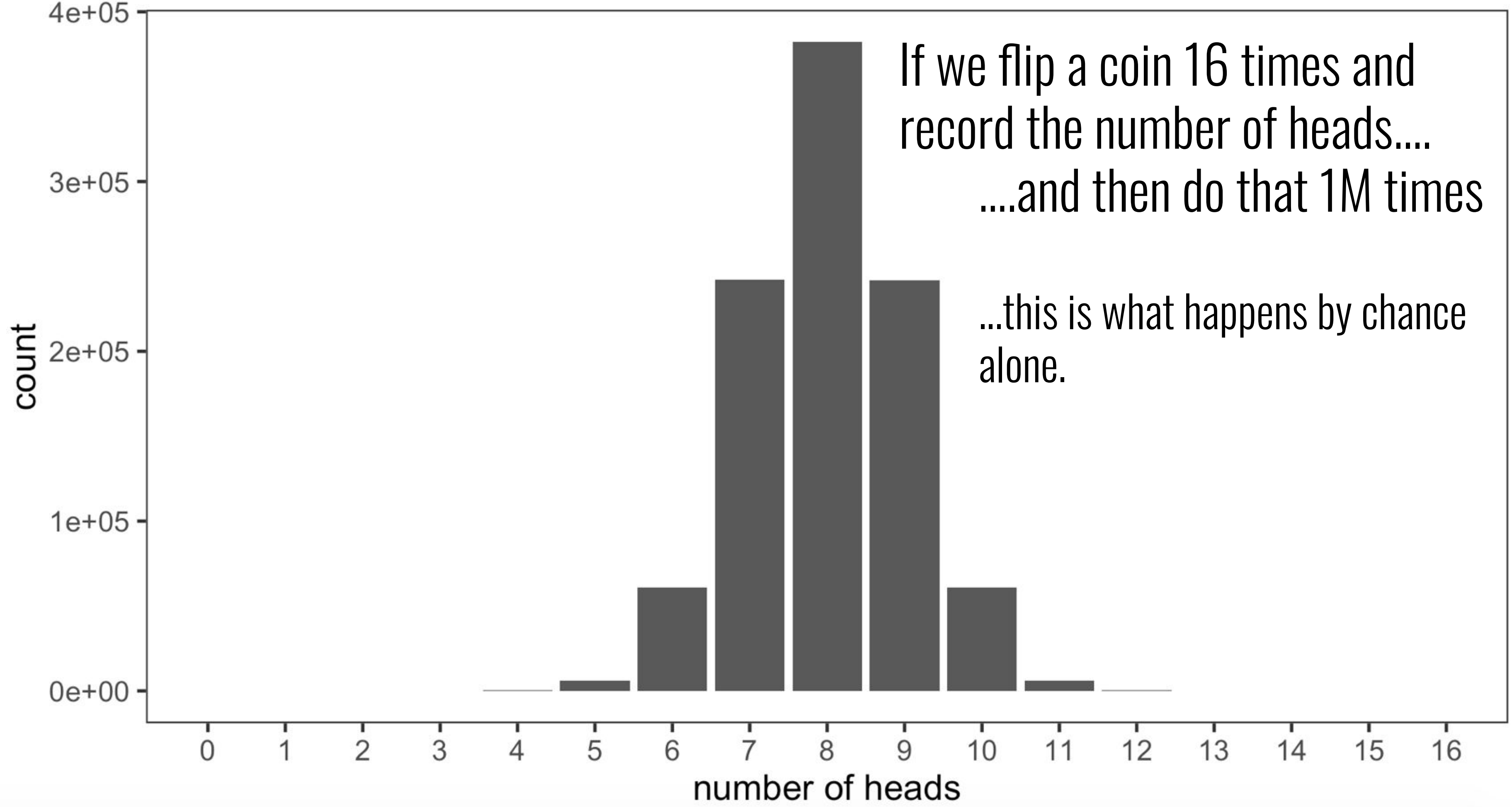
If there were a stronger effect of Poverty on Birth rate, what would β_1 be?

A
< 2.03

B
> 2.03


β ±/ SE
2.03 ± 0.3
p value < 0.0001
60
40
20
5
10
15
20
25
Teen Birth Rate
Poverty Percentage

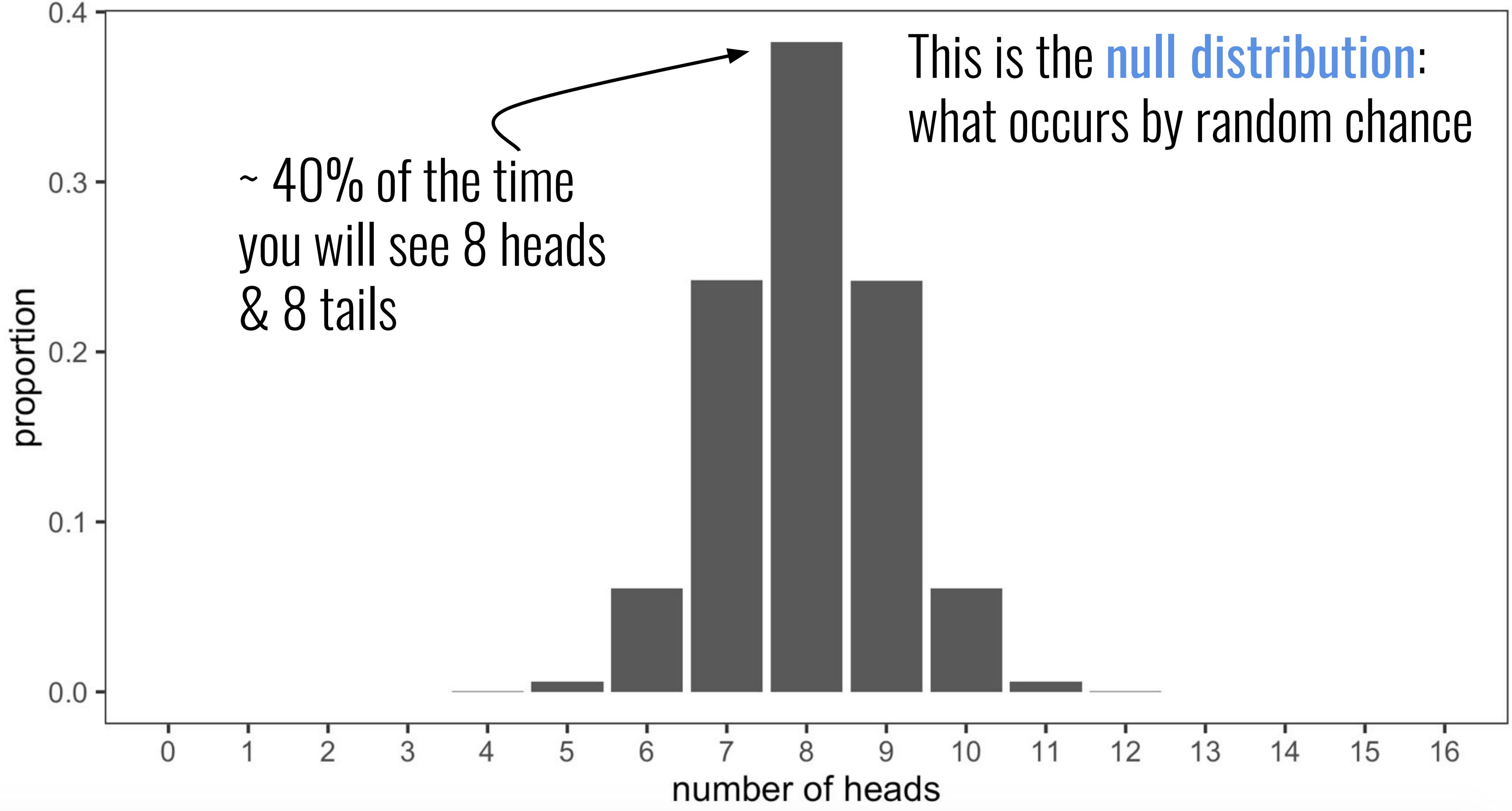
p-value : the probability of getting the observed results (or results more extreme) by chance alone



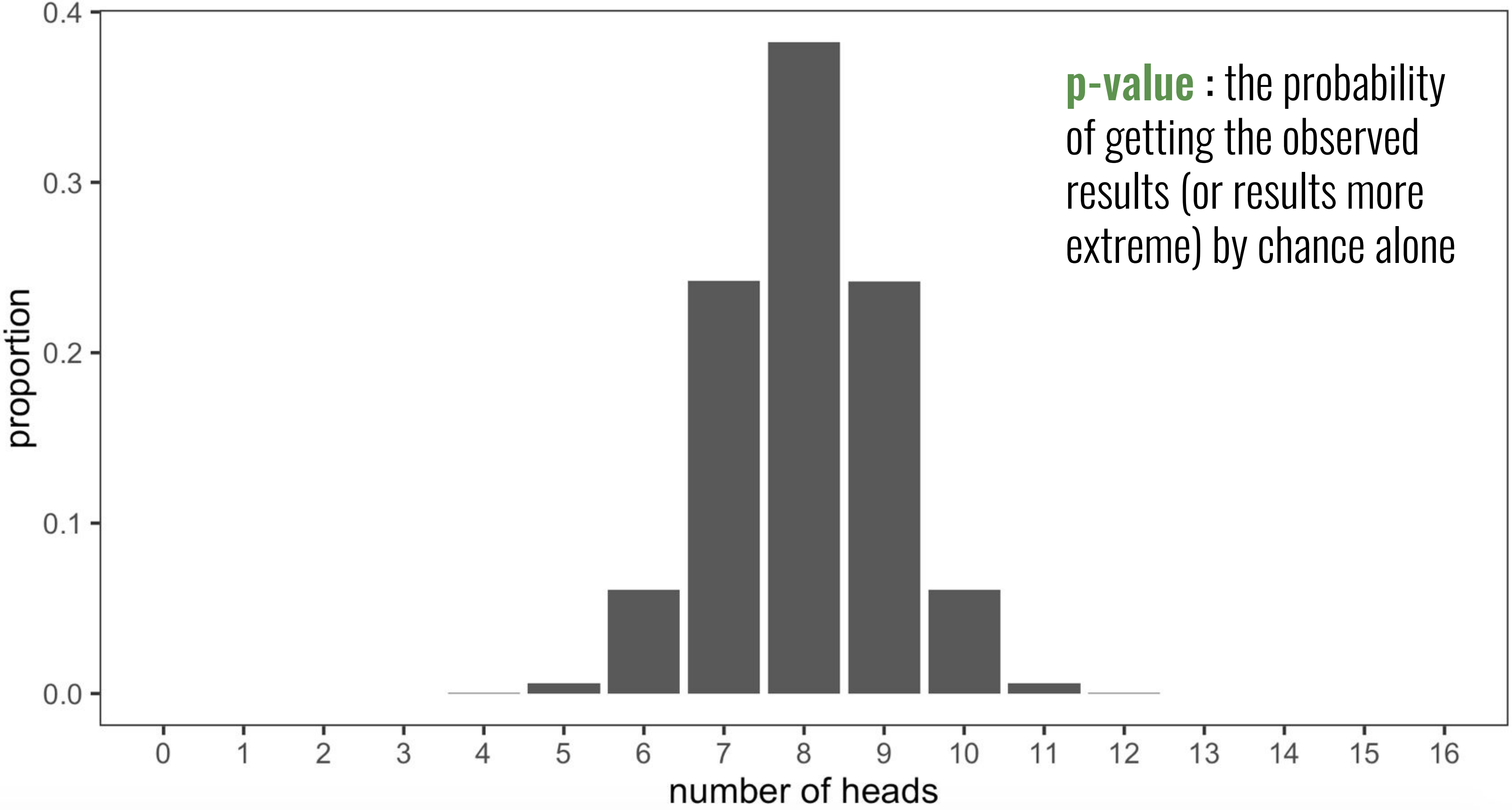
If we flip a coin 16 times and record the number of heads....

....and then do that 1M times

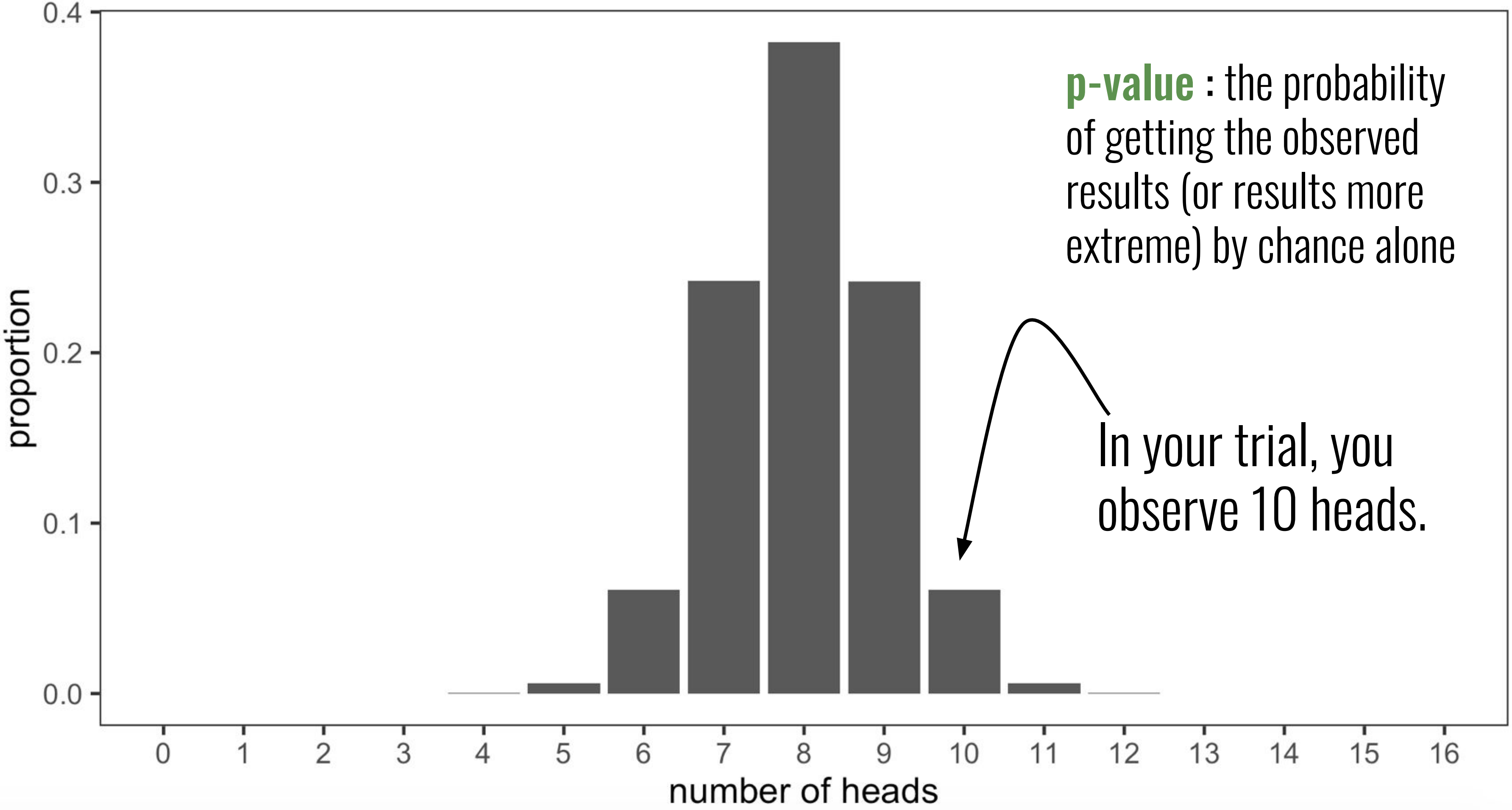
...this is what happens by chance alone.



This is the null distribution:
what occurs by random chance
~ 40% of the time
you will see 8 heads
& 8 tails
proportion
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads

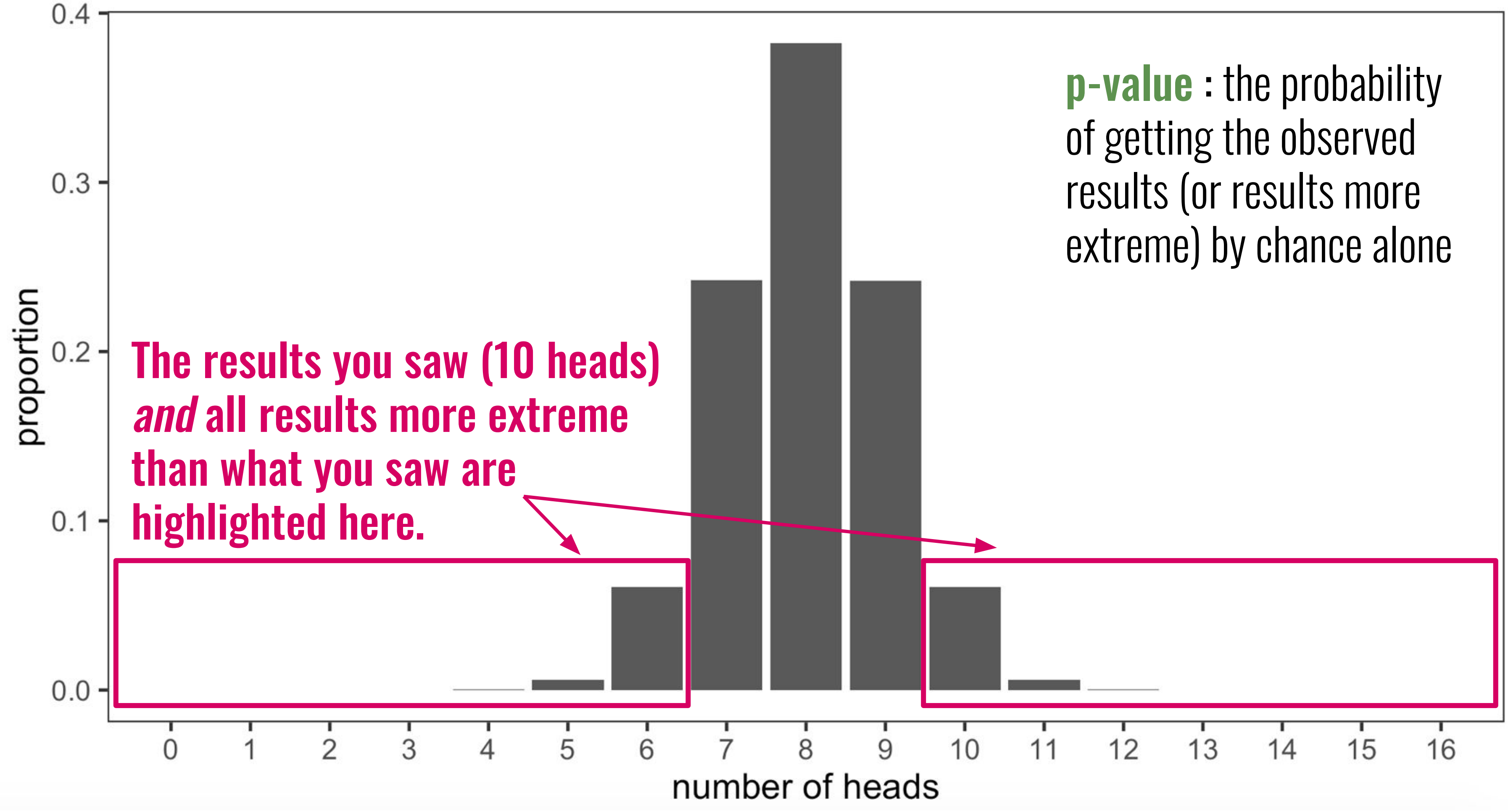


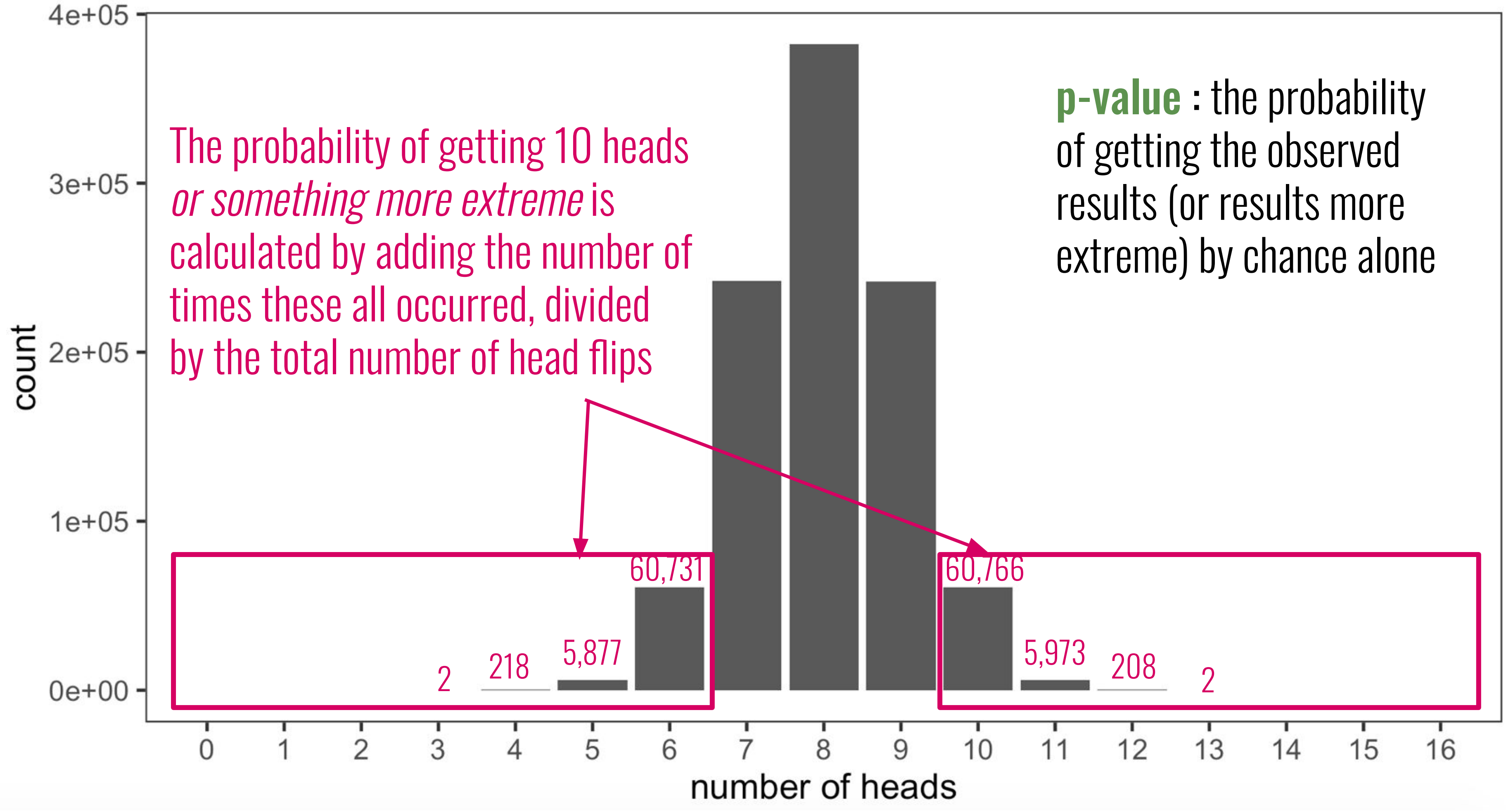
p-value : the probability of getting the observed results (or results more extreme) by chance alone

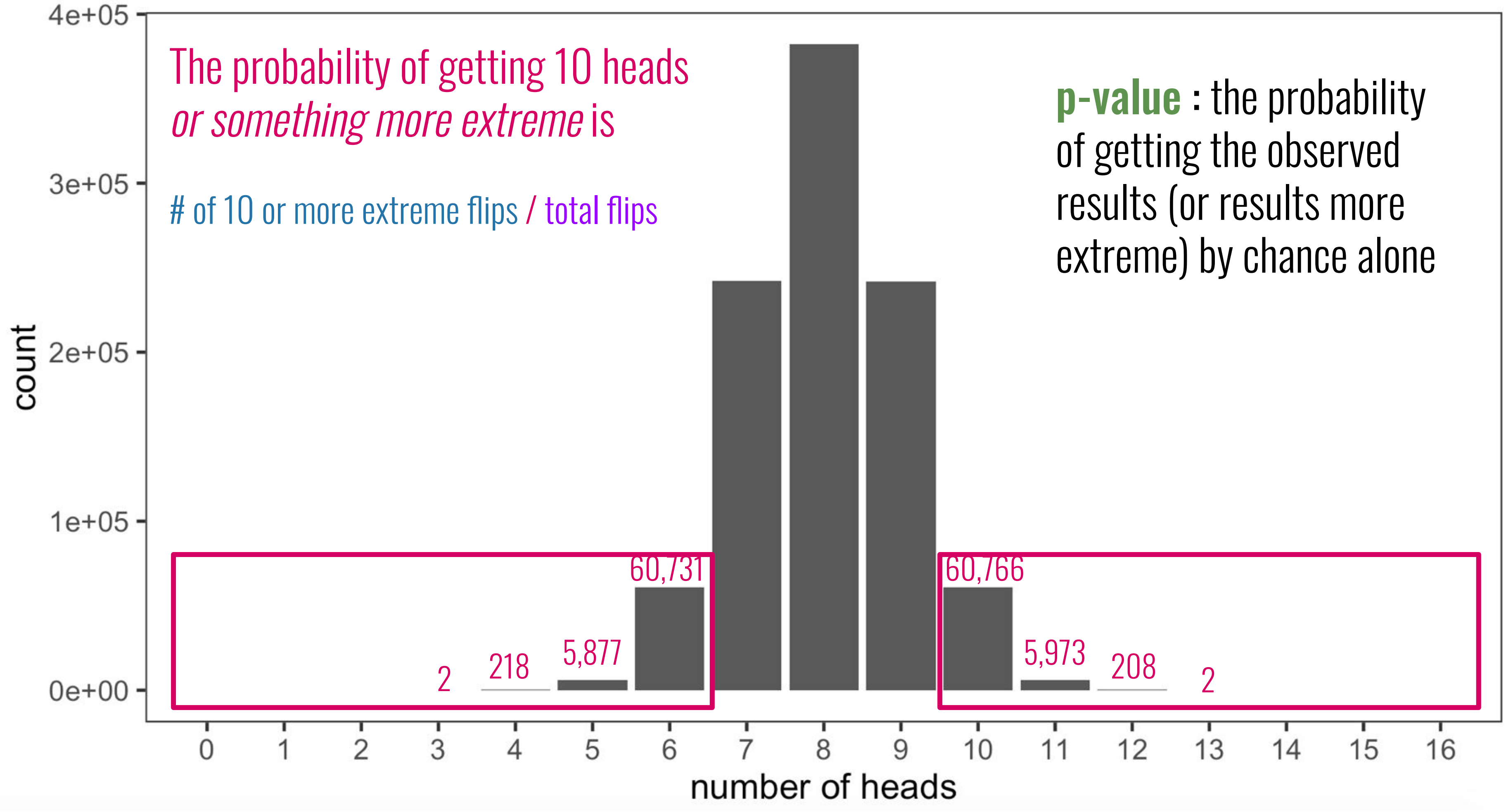


p-value : the probability of getting the observed results (or results more extreme) by chance alone

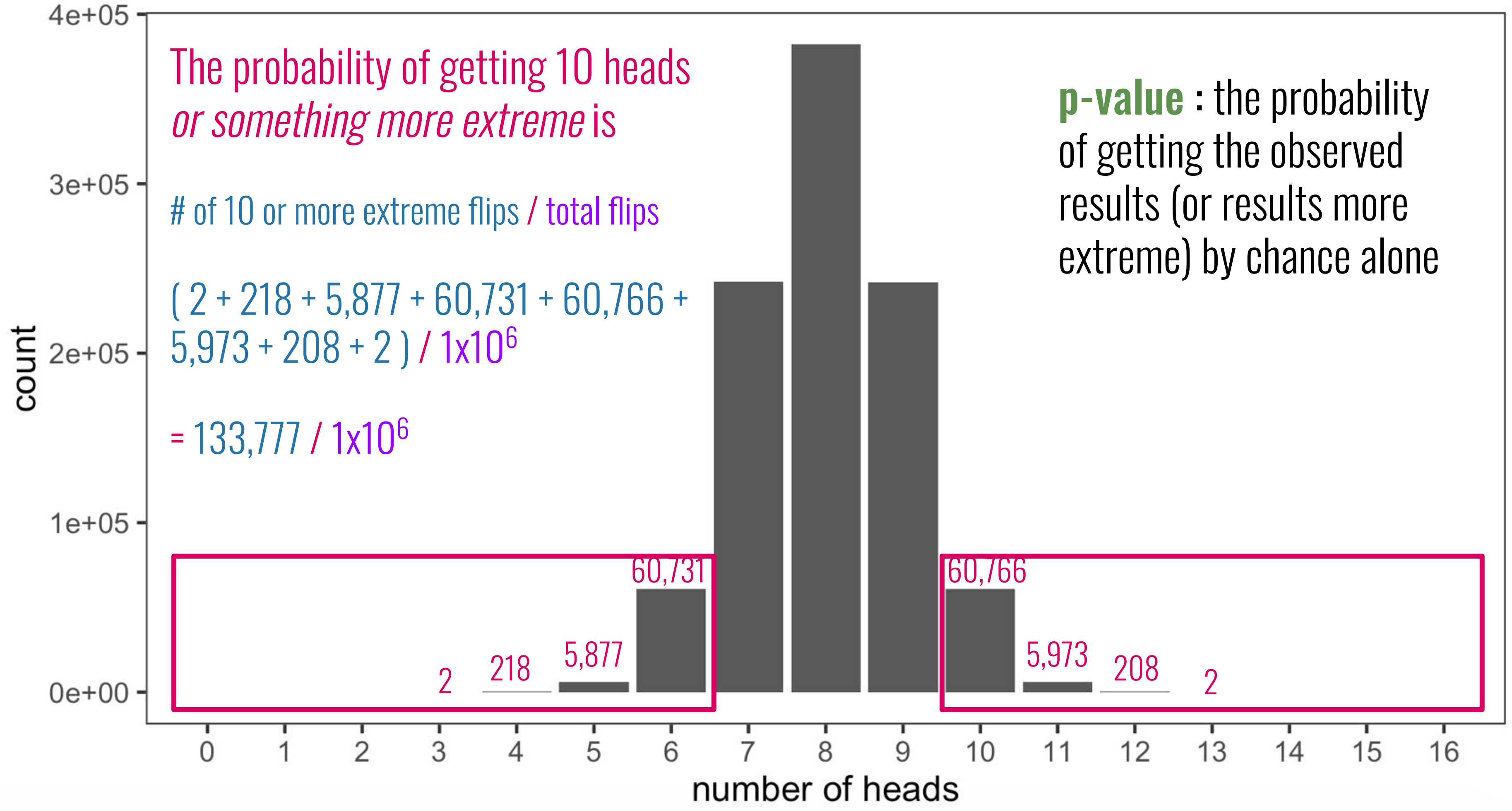
In your trial, you observe 10 heads.

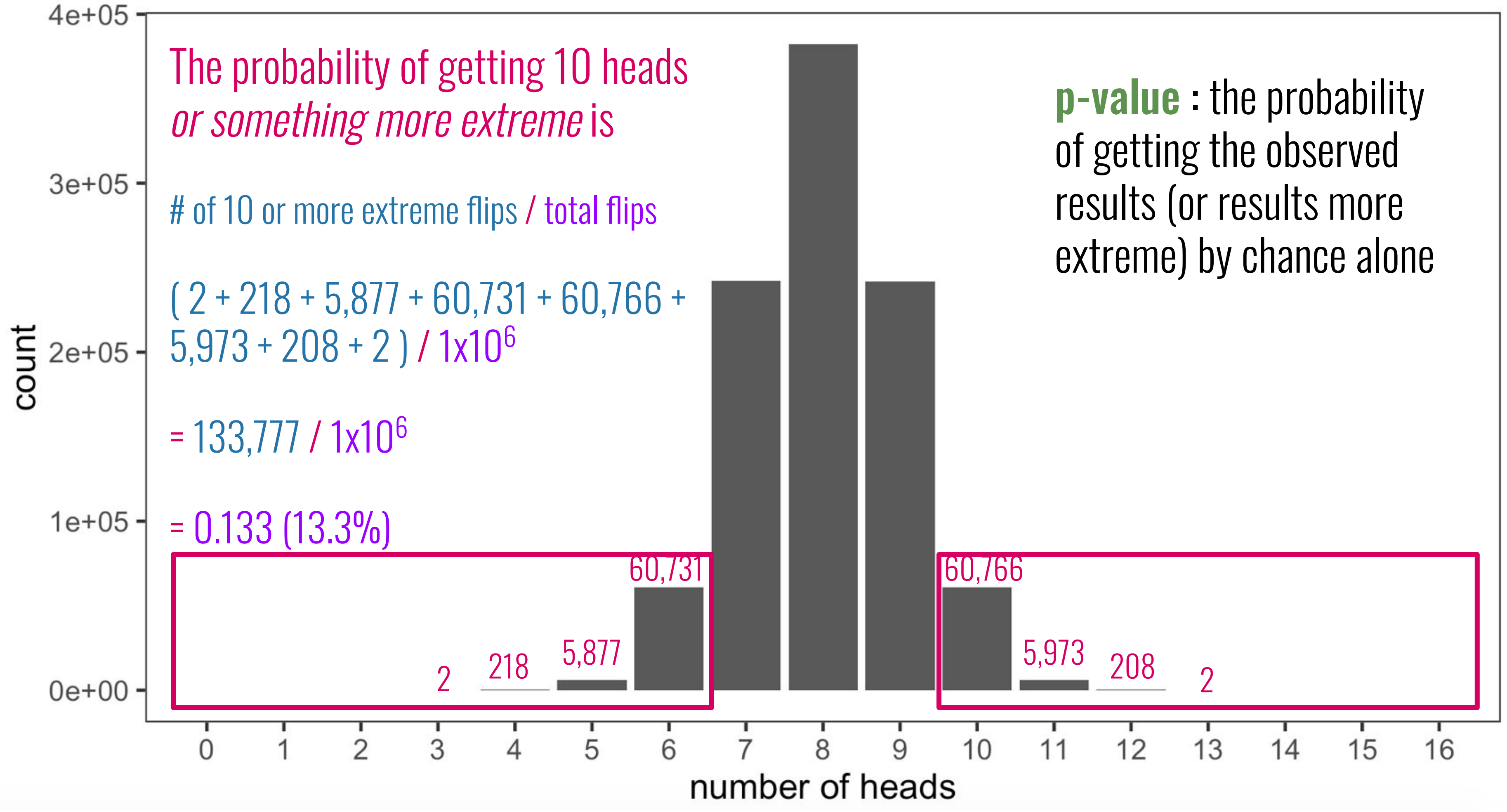

p-value : the probability of getting the observed results (or results more extreme) by chance alone
The results you saw (10 heads) *and* all results more extreme than what you saw are highlighted here.
0.4
0.3
0.2
0.1
0.0
proportion
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads


p-value : the probability of getting the observed results (or results more extreme) by chance alone
The probability of getting 10 heads *or something more extreme* is calculated by adding the number of times these all occurred, divided by the total number of head flips
60,731
60,766
2
218
5,877
5,973
208
2
count
4e+05
3e+05
2e+05
1e+05
0e+00
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads

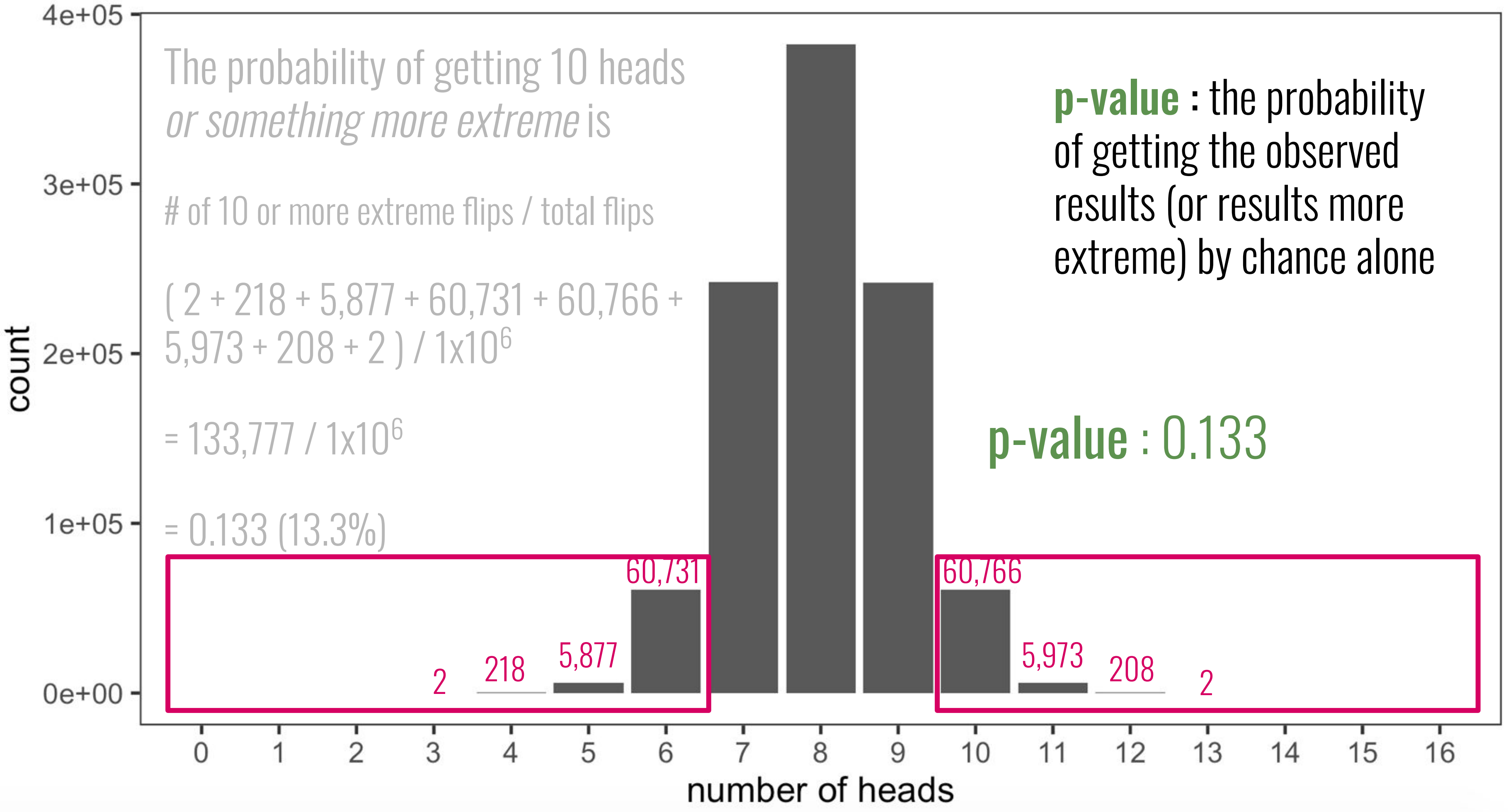


The probability of getting 10 heads
or something more extreme is
# of 10 or more extreme flips / total flips
p-value : the probability of getting the observed results (or results more extreme) by chance alone
count
4e+05
3e+05
2e+05
1e+05
0e+00
60,731
60,766
2
218
5,877
5,973
208
2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
number of heads

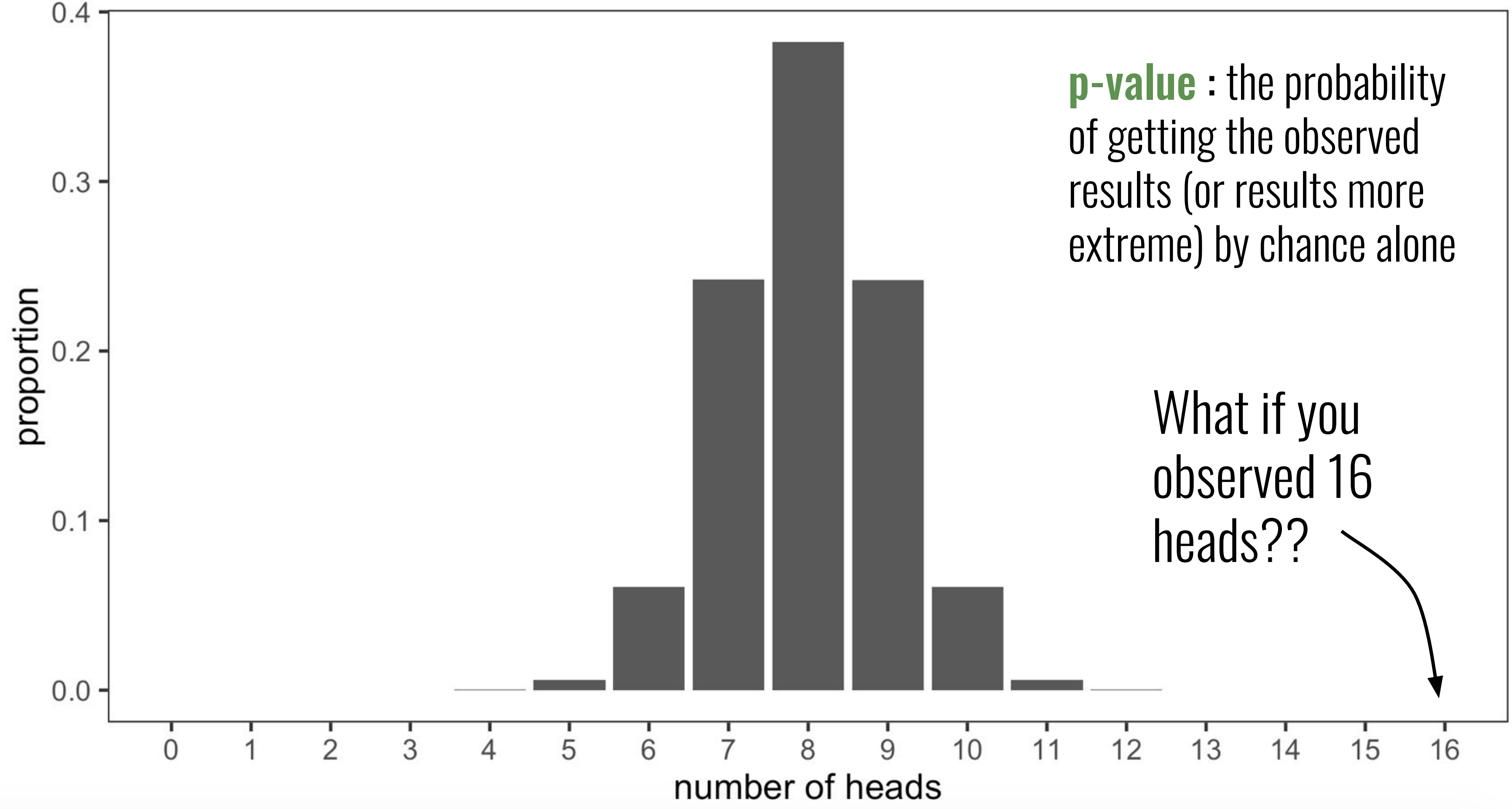

The probability of getting 10 heads
or something more extreme is
# of 10 or more extreme flips / total flips
(2 + 218 + 5,877 + 60,731 + 60,766 +
5,973 + 208 + 2) / 1x10^6
= 133,777 / 1x10^6
p-value : the probability
of getting the observed
results (or results more
extreme) by chance alone
4e+05
3e+05
2e+05
1e+05
0e+00
count
2
218
5,877
60,731
60,766
5,973
208
2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads



The probability of getting 10 heads
or something more extreme is
# of 10 or more extreme flips / total flips
(2 + 218 + 5,877 + 60,731 + 60,766 + 5,973 + 208 + 2) / 1x10^6
= 133,777 / 1x10^6
= 0.133 (13.3%)
p-value : the probability of getting the observed results (or results more extreme) by chance alone
4e+05
3e+05
2e+05
1e+05
0e+00
count
60,731
60,766
2
218
5,877
5,973
208
2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads

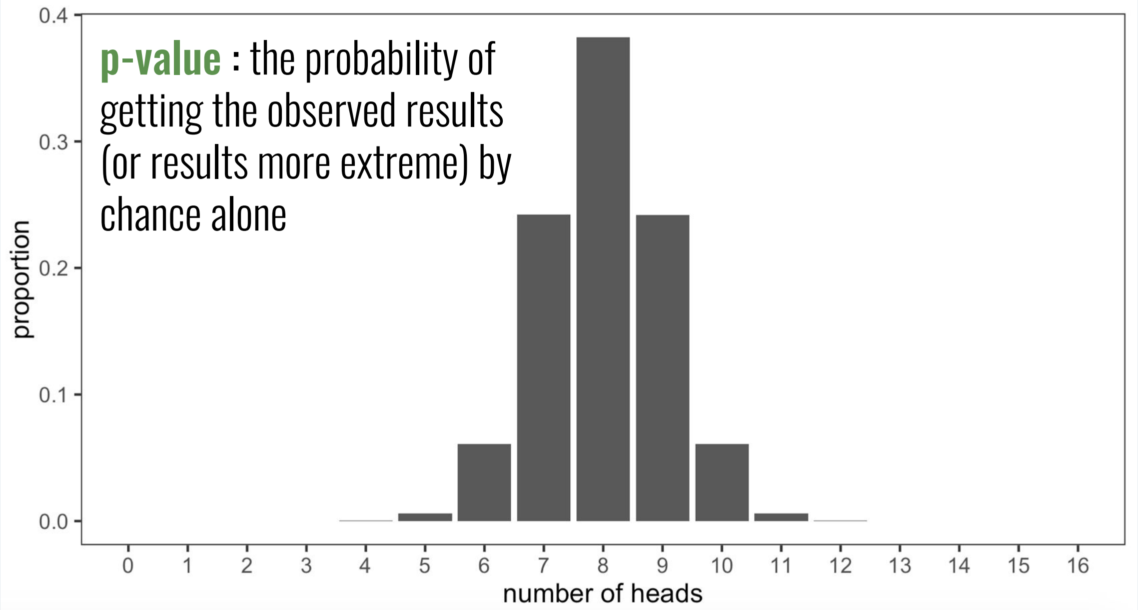


The probability of getting 10 heads
or something more extreme is
# of 10 or more extreme flips / total flips
(2 + 218 + 5,877 + 60,731 + 60,766 + 5,973 + 208 + 2) / 1x10^6
= 133,777 / 1x10^6
= 0.133 (13.3%)
p-value : the probability of getting the observed results (or results more extreme) by chance alone
p-value : 0.133
60,731
60,766
2
218
5,877
5,973
208
2
4e+05
3e+05
2e+05
1e+05
0e+00
count
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
number of heads



p-value : the probability of getting the observed results (or results more extreme) by chance alone

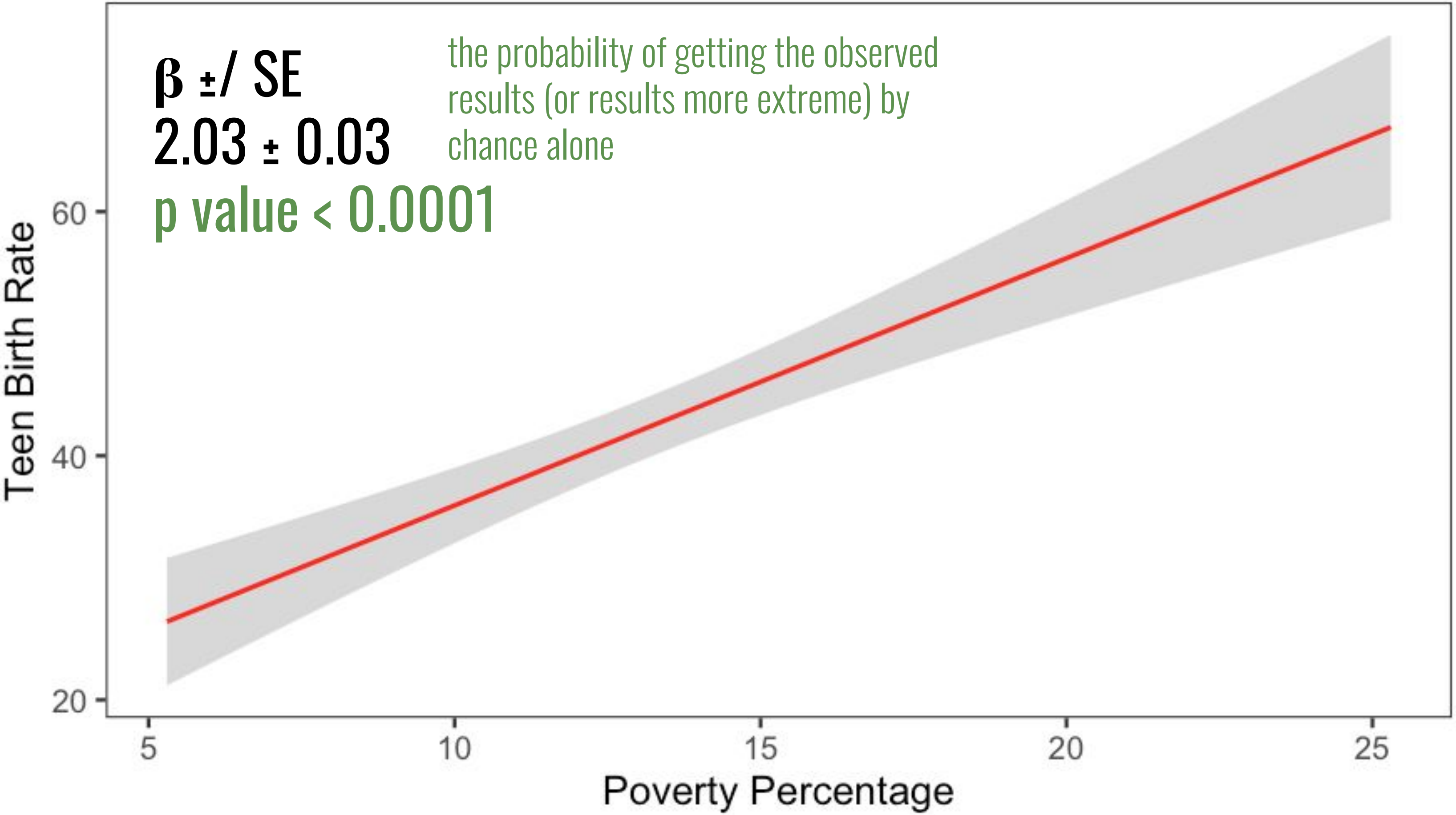
What if you observed 16 heads??



What would be the p-value of you flipping 16 heads?

A
< 0.13

B
> 0.13



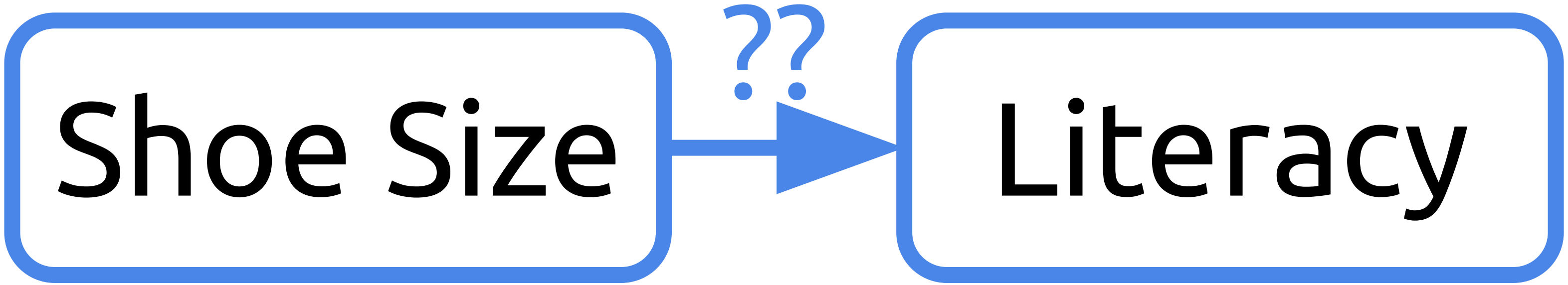
β ±/ SE
2.03 ± 0.03
p value < 0.0001
the probability of getting the observed results (or results more extreme) by chance alone
Teen Birth Rate
60
40
20
5
10
15
20
25
Poverty Percentage

Takes into account the effect size (β_1) and the SE

p-value : the probability of getting the observed results (or results more extreme) by chance alone

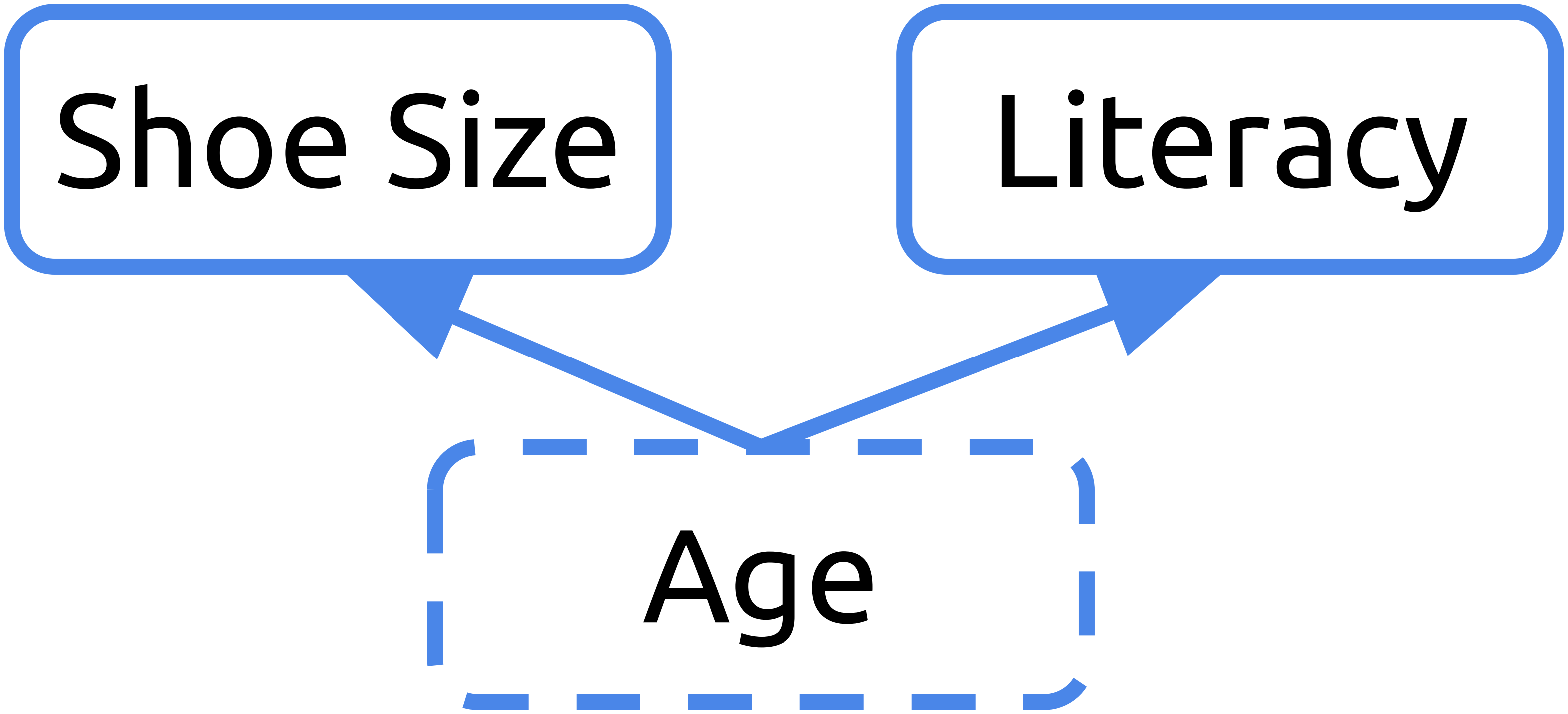
Confounding

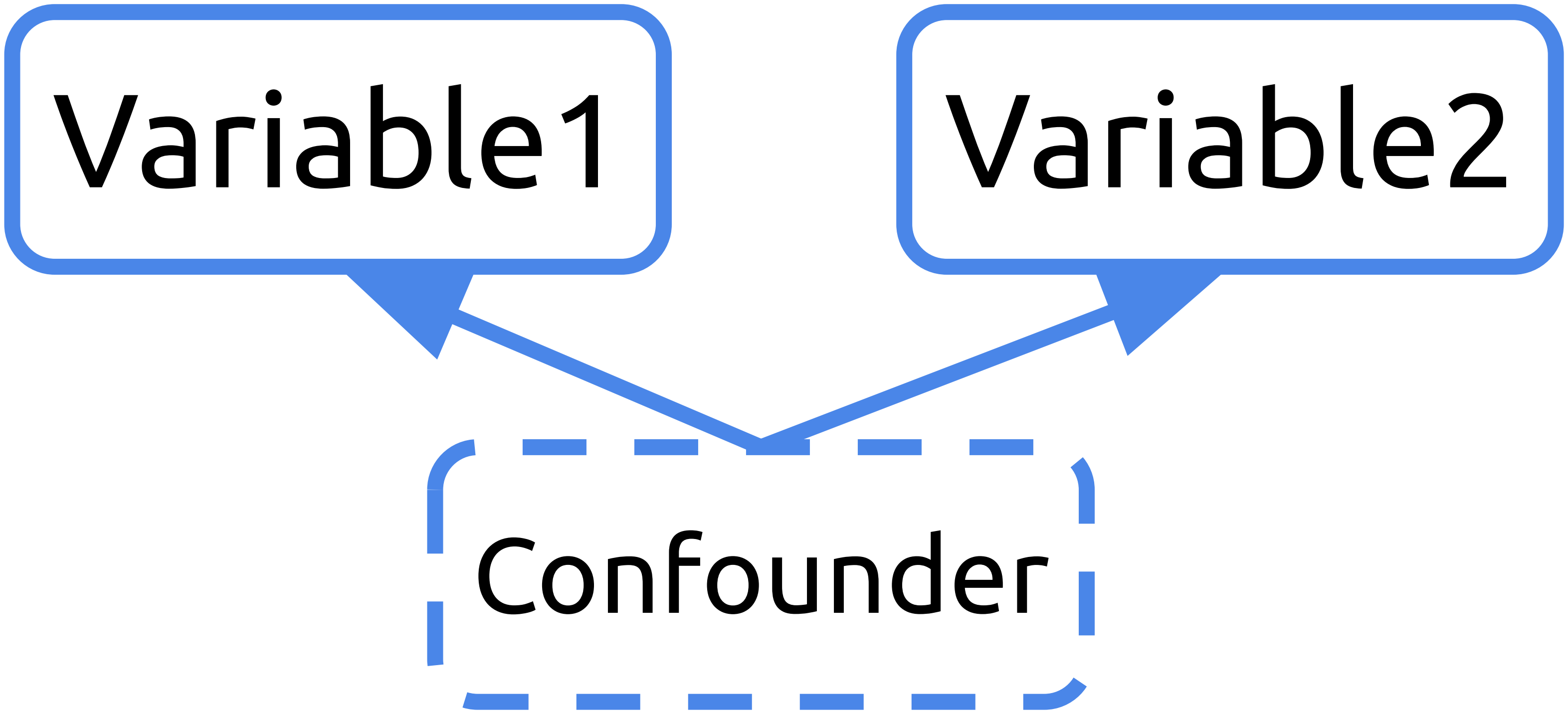

Small shoes
Not literate
Big shoes
Literate


Shoe Size
??
Literacy

Small shoes
Not literate
Child

Big shoes
Literate
Adult


Shoe Size
Literacy
Age


Variable1
Variable2
Confounder

Confounding

popsicles → crime rate

Your analysis sees an increase in crime rate whenever popsicle sales increase. What could confound this analysis?

- A popsicle preference
- B new gun laws
- C temperature
- D changes in popsicle prices
- E new law enforcement officers

We’ll discuss additional approaches of how to account for confounding in your analysis in the next lecture.

Ignoring confounders will lead you to draw incorrect conclusions from your analyses

Spine Surgery Results

Sample: 400 patients with index vertebral fractures

Vertebroplasty	Conservative care	Relative risk (95% confidence interval)
30/200 (15%)	15/200 (7.5%)	2.0 (1.1–3.6)

subsequent fractures

Eek....looks like vertebroplasty was *way* worse for patients!

But wait...at time of initial fracture...

	Vertebroplasty N = 200	Conservative care N = 200
Age, y, mean ± SD	78.2 ± 4.1	79.0 ± 5.2
Weight, kg, mean ± SD	54.4 ± 2.3	53.9 ± 2.1
Smoking status, No. (%)	110 (55)	16 (8)

Age and weight are similar between groups. **Smoking Status** differs vastly.

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

So...let's stratify those results real quick

Smoke			No smoke		
Vertebroplasty	Conservative	RR (95% confidence interval)	Vertebroplasty	Conservative	RR (95% confidence interval)
23/110 (21%)	3/16 (19%)	1.1 (0.4, 3.3)	7/90 (8%)	12/184(7%)	1.2 (0.5, 2.9)

Risk of re-fracture is now similar within group

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3503514/

Confounding

What are possible confounders for our analysis of the effect of poverty on teen birth rate?

A
I have some ideas

B
Not sure