Review

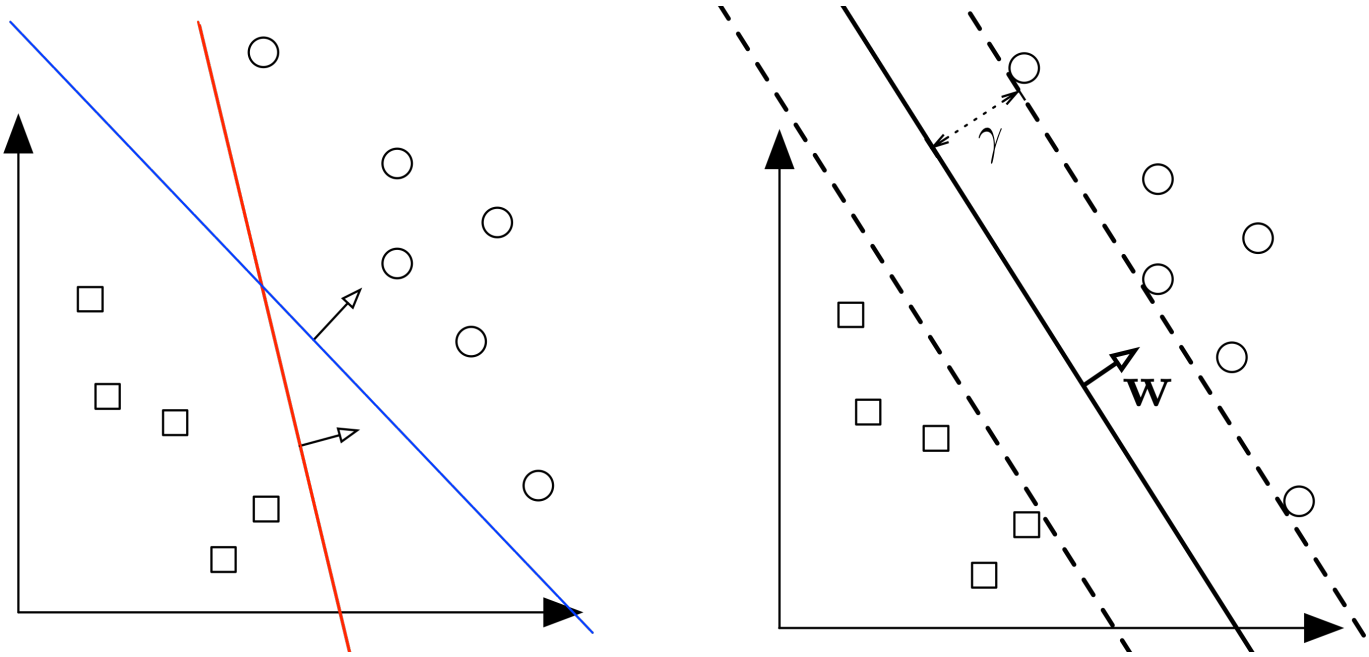
Multi-class classification

- One-vs-Rest (`sklearn.multiclass.OneVsRestClassifier`)
  - N-classes:
    - N binary classifiers
    - N-split of the training set, each includes all data points but divided into class i v.s. others)
  - Prediction: `argmax` of scores produced by a bunch of binary classifiers
- One-vs-One (`sklearn.multiclass.OneVsOneClassifier`)
  - N classes:
    - $\binom{N}{2}$ binary classifiers
    - $\binom{N}{2}$ split of the training set, each includes only data points with the label i and j
  - Prediction:
    - majority votes
    - Or the class with the most sum score is taken as the class label
- Compare:
  - the number of classifiers?
  - data used for training each classifier?
  - what if there is a specific class dominates the training set (i.e. imbalanced classes)?
  - when N is large, what will be the greatest issues for the two algorithms respectively?

SVM:

Concepts:

- difference between SVM and logistic regression?
- what is a hyperplane? what is an 'optimal' hyperplane?



(ref: https://www.cs.cornell.edu/courses/cs4780/2018fa/lectures/images/svm/margin.png)

- what is a support vector?
- what is the margin M?
  - $\min_i \frac{2}{||w||_2}|w^T x_i + b|$
  - --> set the positive/negative plane to: $w^T x_i + b = \pm 1$
- The loss function to maximize the margin?
  - goal: $\max M(w, b)$ such that $\forall i, y_i(w^T x_i + b) \geq 0$
    - Or by definition of M: $\min w^T w$ such that $\forall i, y_i(w^T x_i + b) \geq 1$

Primal & Dual formulation

Kernel: a metric/weighting function on dot product:

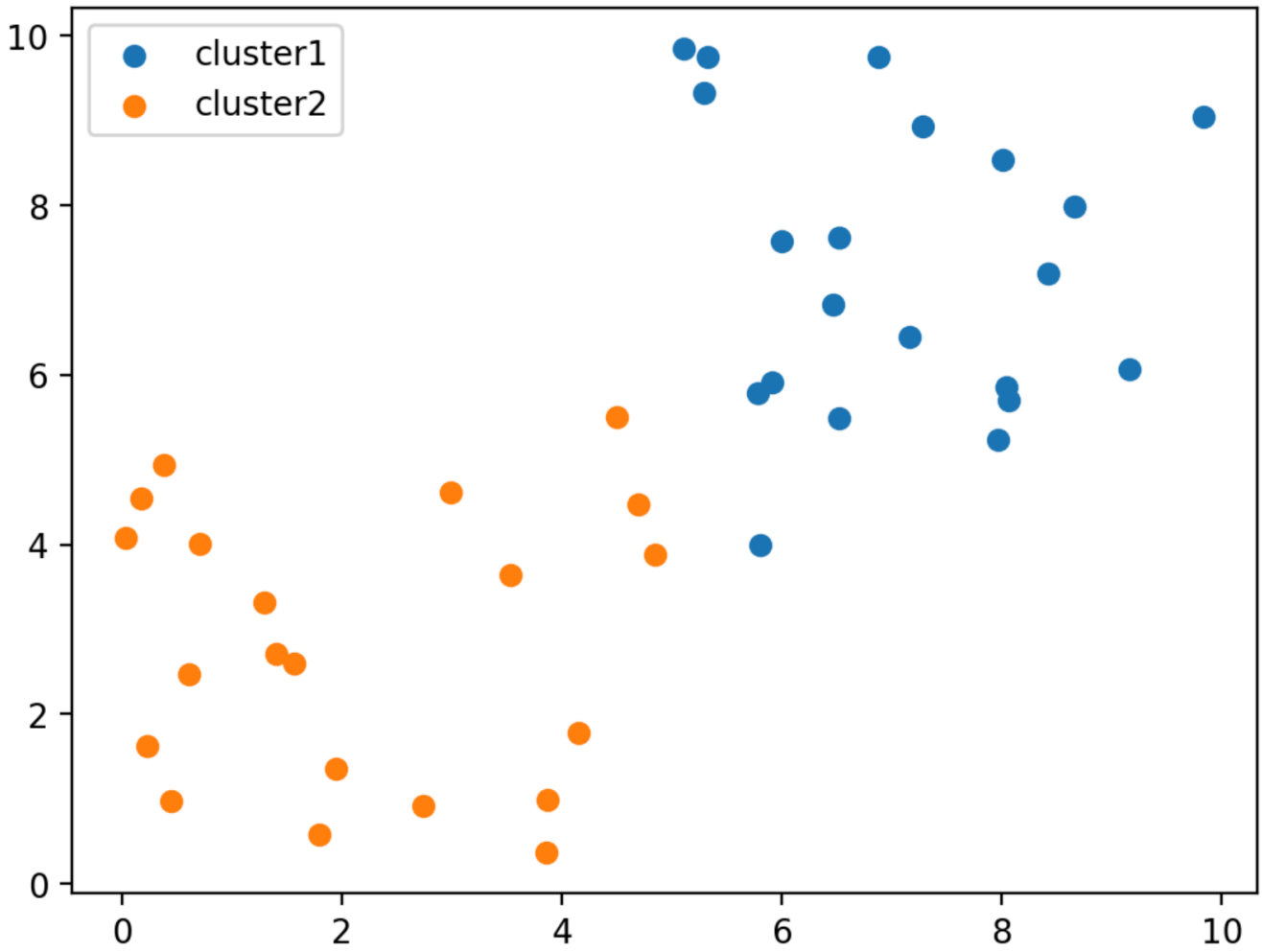
- $k(x, x') =< \phi(x), \phi(x') >$
  - linear: $K(x, x') = x^T x' + c$
  - polynomial: $K(x, x') = (ax^T x' + c)^d$
  - radial basis: $K(x, x') = exp(-\frac{||x-x'||^2}{2\sigma^2})$
  - Sigmoid: $K(x, x') = tanh(ax^T x' + c)$

Classifier:

- Primal: $f(x) = w^T x + b$
- Dual: $f(x) = \sum_i^N a_i y_i (x_i^T x) + b$
  - compare with KNN...
  - support vectors: $a_i \neq 0$

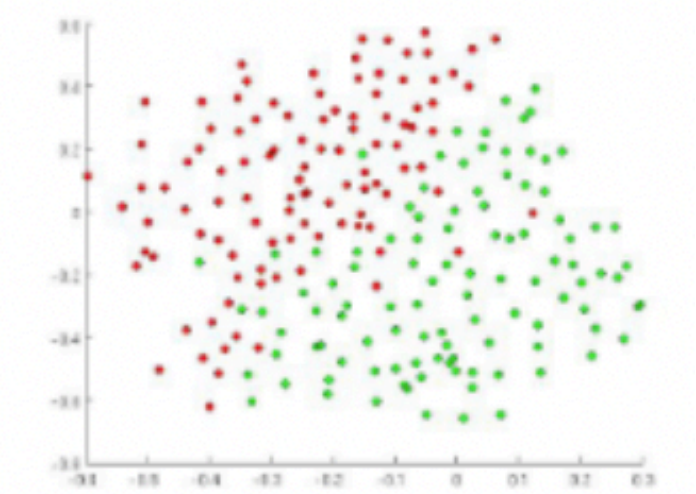
Discussion Questions

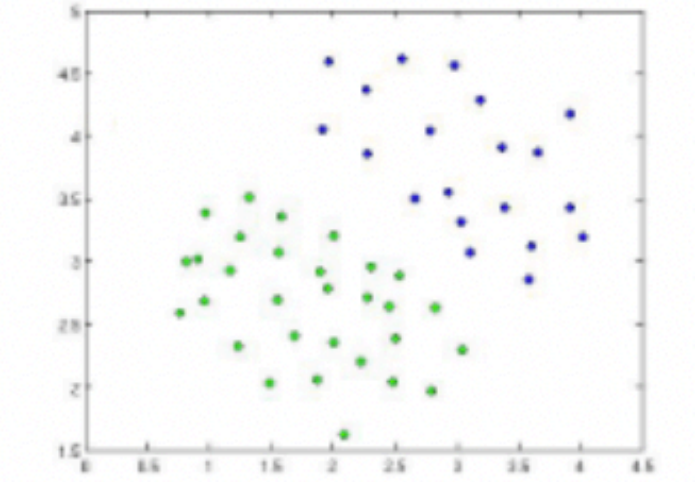
1. Consider the following dataset where the different colors indicate the different class labels a specific data point belongs to:

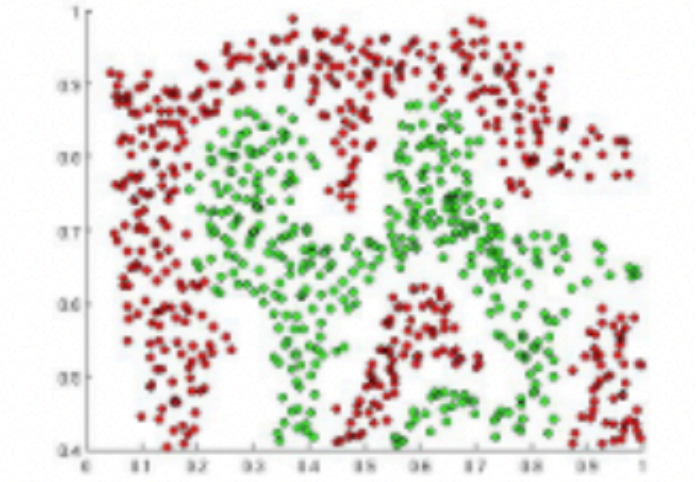


   - Is this dataset linearly separable?
   - Identify the support vectors and draw the positive margin, negative margin and decision boundary of a **hard**-SVM classifier. What is the training accuracy? Do you expect it to generalize well in the unseen dataset? What are some limitations of hard-SVM?
   - Identify the support vectors and draw the positive margin, negative margin and decision boundary of a **soft**-SVM classifier. How is that decision boundary differ from the hard-SVM classifier? What is
     the training accuracy? Do you expect it to generalize well in the unseen dataset?
   - Compare the margins of the hard SVM and soft SVM. Which of the two will will result in a weight vector of larger magnitude? Justify your reasoning

2. For the following feature space plots, identify the possible types of SVM (hard margin linear, soft margin linear, kernel) that we can use to classify the data.



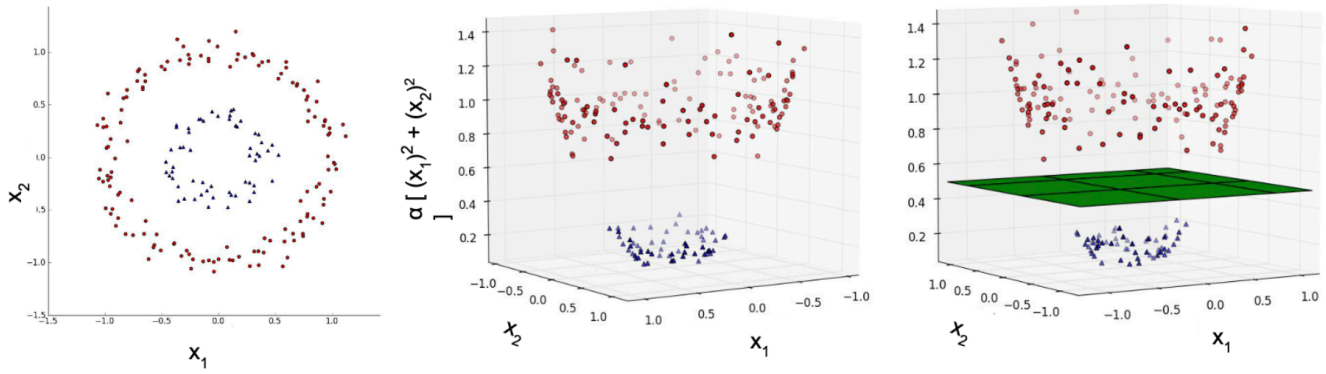




3.

Overview of SVM III: Kernel Trick Visualization

Polynomial Kernel (to the power 2)

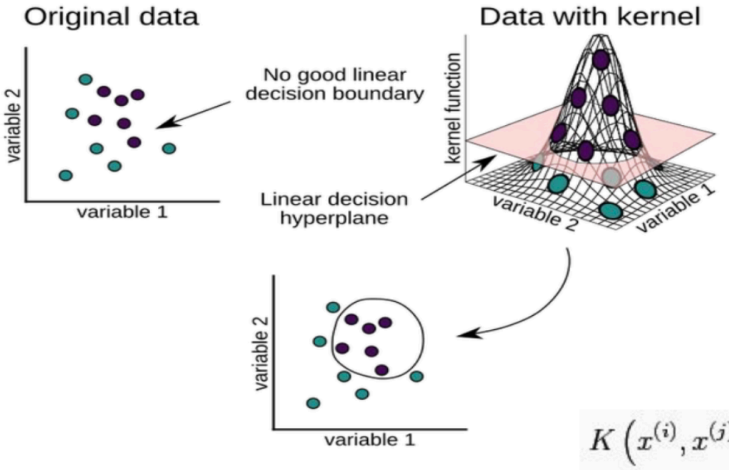


Important: You mic is set to mute. Use **"raise hand"** option/tab if you have a question.

4.

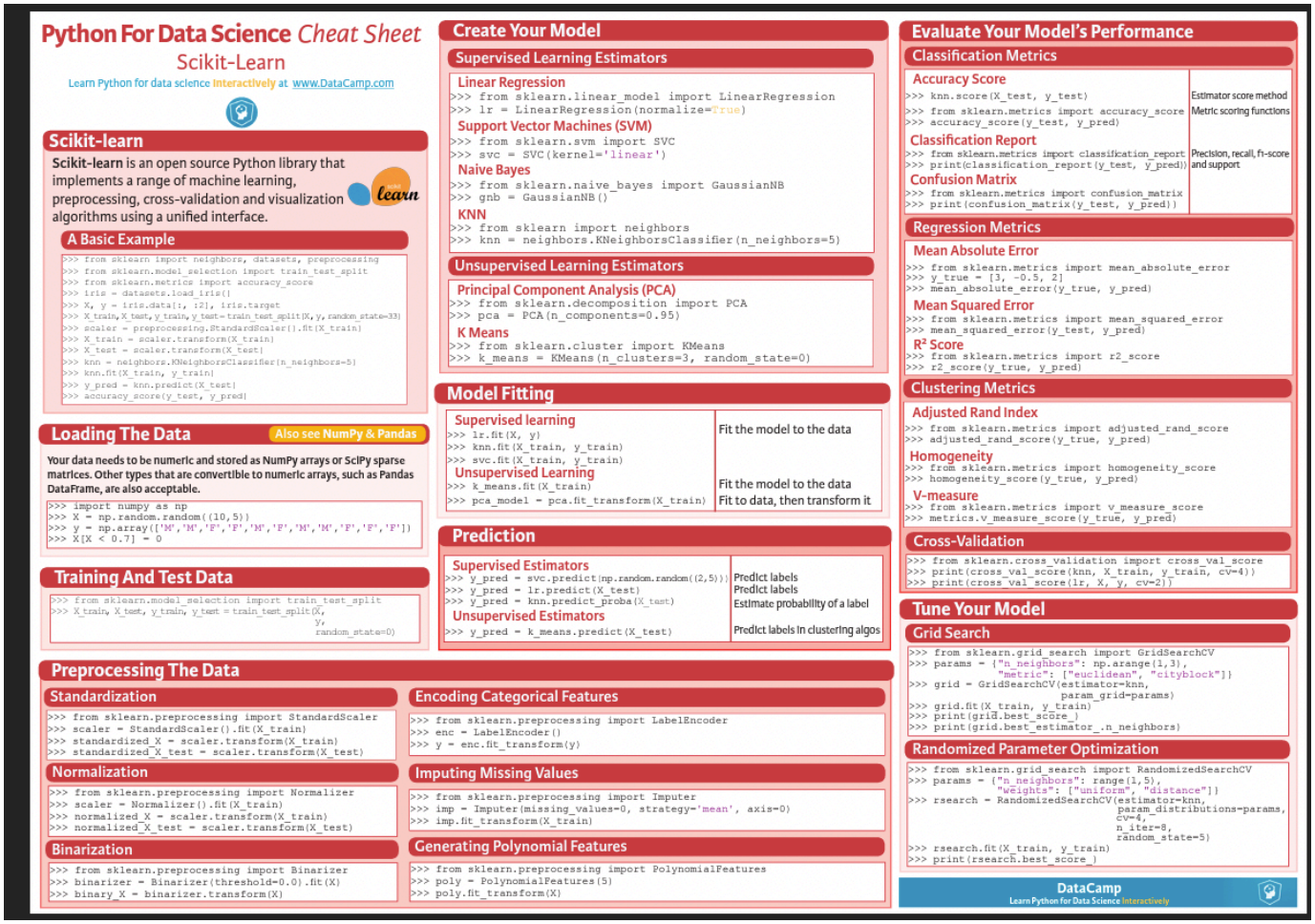
Overview of SVM III: Kernel Trick Visualization

Radial Basis Function Kernel (RBF)



$$K(\mathbf{x}, \mathbf{x}') = \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}'\|^2}{2\sigma^2}\right)$$

$$K\left(x^{(i)}, x^{(j)}\right) = \exp\left(-\gamma \left\|x^{(i)} - x^{(j)}\right\|^2\right)$$



Assignment 6

Package: `sklearn.model_selection.GridSearchCV`

`cv_results_`: (example: 4 sets of hyperparameters configuration to test)

```
{
'param_kernel': masked_array(data = ['poly', 'poly', 'rbf', 'rbf'],
                             mask = [False False False False]...)
'param_gamma': masked_array(data = [-- -- 0.1 0.2],
                            mask = [ True  True False False]...),
'param_degree': masked_array(data = [2.0 3.0 -- --],
                             mask = [False False  True  True]...),
'split0_test_score'  : [0.80, 0.70, 0.80, 0.93],
'split1_test_score'  : [0.82, 0.50, 0.70, 0.78],
'mean_test_score'    : [0.81, 0.60, 0.75, 0.85],
'std_test_score'     : [0.01, 0.10, 0.05, 0.08],
'rank_test_score'    : [2, 4, 3, 1],
'split0_train_score' : [0.80, 0.92, 0.70, 0.93],
'split1_train_score' : [0.82, 0.55, 0.70, 0.87],
'mean_train_score'   : [0.81, 0.74, 0.70, 0.90],
```

```
'std_train_score'    : [0.01, 0.19, 0.00, 0.03],
'mean_fit_time'      : [0.73, 0.63, 0.43, 0.49],
'std_fit_time'       : [0.01, 0.02, 0.01, 0.01],
'mean_score_time'    : [0.01, 0.06, 0.04, 0.04],
'std_score_time'     : [0.00, 0.00, 0.00, 0.01],
'params'             : [{'kernel': 'poly', 'degree': 2}, ...],
}
```