隐函数的求导方法

/* Implicit Differentiation*/

1) 方程在什么条件下才能确定隐函数.

例如, 方程 $x^2+\sqrt{y}+C=0$ $\begin{cases} C<0 \text{ 时, 能确定隐函数,} \\ C>0 \text{ 时, 不能确定隐函数.} \end{cases}$

2) 方程能确定隐函数时, 研究其连续性,可微性及求导方法问题.

本节讨论:

一、一个方程所确定的隐函数及其导数

二、方程组所确定的隐函数组及其导数

一、一个方程所确定的隐函数及其导数

定理1. 设函数 $F(x,y)$ 在点 $P(x_0,y_0)$ 的某邻域内满足:

① 具有连续偏导数;

② $F(x_0,y_0)=0$;

③ $F_y(x_0,y_0)\neq 0$,

则方程 $F(x,y)=0$ 在点 (x_0,y_0) 的某邻域内可唯一确定一个连续函数 $y=f(x)$, 满足条件 $y_0=f(x_0)$, 并有连续导数

$$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{F_x}{F_y}$$ (隐函数求导公式)

求导公式推导如下:

设 $y=f(x)$ 为方程 $F(x,y)=0$ 所确定的隐函数, 则

$$F(x,f(x))\equiv 0$$

两边对 x 求导

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}=0$$

在 (x_0,y_0) 的某邻域内 $F_y\neq 0$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{F_x}{F_y}$$

记忆：隐函数导数这样算
负号符号对角线

若$F(x,y)$的二阶偏导数也都连续，则还可求隐函数的二阶导数:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{F_x}{F_y}$$

$x \quad y$

$\quad\quad x$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\partial}{\partial x}\left(-\frac{F_x}{F_y}\right)+\frac{\partial}{\partial y}\left(-\frac{F_x}{F_y}\right)\cdot\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$=-\frac{F_{xx}F_y-F_{yx}F_x}{F_y^2}-\frac{F_{xy}F_y-F_{yy}F_x}{F_y^2}\left(-\frac{F_x}{F_y}\right)$$

$$=-\frac{F_{xx}F_y^2-2F_{xy}F_xF_y+F_{yy}F_x^2}{F_y^3}$$

$F_x(x,y), F_y(x,y)$

例1. 验证方程 $\sin y + e^x - xy - 1 = 0$ 在点(0,0)某邻域可确定一个可导隐函数 $y = f(x)$, 并求

$$\left.\frac{dy}{dx}\right|_{x=0},\quad \left.\frac{d^2y}{dx^2}\right|_{x=0}$$

解: 令 $F(x,y) = \sin y + e^x - xy - 1$, 则

① $F_x = e^x - y,\ F_y = \cos y - x$ 连续;

② $F(0,0) = 0$;

③ $F_y(0,0) = 1 \neq 0$,

由定理1 可知, 在 $x = 0$ 的某邻域内方程存在可导的隐函数 $y = f(x)$, 且

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}=-\left.\frac{F_x}{F_y}\right|_{x=0}=-\left.\frac{\mathrm{e}^x-y}{\cos y-x}\right|_{x=0,\,y=0}=-1$$

$$\left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right|_{x=0}=-\frac{\mathrm{d}}{\mathrm{d}x}\left.\left(\frac{\mathrm{e}^x-y}{\cos y-x}\right)\right|_{x=0,\ y=0,\ y'=-1}$$

$$=-\left.\frac{(\mathrm{e}^x-y')(\cos y-x)-(\mathrm{e}^x-y)(-\sin y\cdot y'-1)}{(\cos y-x)^2}\right|_{\substack{x=0\\ y=0\\ y'=-1}}$$

$$=-3$$

$$F(x,y)=\sin y+\mathrm{e}^x-xy-1,\ F_x=\mathrm{e}^x-y,\ \ F_y=\cos y-x$$

导数的另一求法 — 利用隐函数求导

$$\sin y + e^x - xy - 1 = 0,\quad y = y(x)$$

两边对 x 求导

$$\cos y \cdot y' + e^x - y - xy' = 0$$

$$y'\Big|_{x=0} = -\left.\frac{e^x - y}{\cos y - x}\right|_{(0,0)} = -1$$

两边再对 x 求导

$$-\sin y \cdot (y')^2 + \cos y \cdot y'' + e^x - y' - y' - xy'' = 0$$

令 $x = 0$, 注意此时 $y = 0,\ y' = -1$

$$\left.\frac{d^2 y}{dx^2}\right|_{x=0} = -3$$

定理2. 设函数 $F(x,y,z)$ 在点 $P(x_0,y_0,z_0)$ 某邻域内满足：①具有连续偏导数；

② $F(x_0,y_0,z_0)=0$；

③ $F_z(x_0,y_0,z_0)\neq 0$，

则方程 $F(x,y,z)=0$ 在点 (x_0,y_0,z_0) 某邻域内可唯一确定一个连续函数 $z=f(x,y)$，满足 $z_0=f(x_0,y_0)$，并有连续偏导数

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$

求导公式推导如下：

设 $z = f(x, y)$ 是方程 $F(x, y, z) = 0$ 所确定的隐函数，

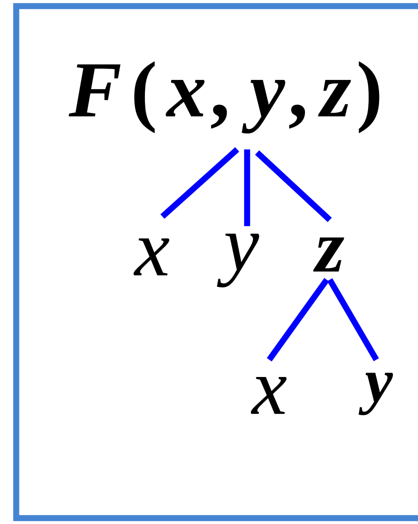
则 $$F(x, y, f(x, y)) \equiv 0$$

两边对 x 求偏导

$$F_x + F_z \frac{\partial z}{\partial x} = 0$$

在 (x_0, y_0, z_0) 的某邻域内 $F_z \neq 0$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$



同样可得 $$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

例2. 设 $x^2+y^2+z^2-4z=0$, 求 $\dfrac{\partial^2 z}{\partial x^2}$.

解法1 **利用公式**

设 $F(x,y,z)=x^2+y^2+z^2-4z$

则 $F_x=2x,\ F_z=2z-4$

$$\therefore\quad \frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{x}{z-2}=\frac{x}{2-z}$$

两边对 x 求偏导

$$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{x}{2-z}\right)=\frac{(2-z)+x\dfrac{\partial z}{\partial x}}{(2-z)^2}=\frac{(2-z)^2+x^2}{(2-z)^3}$$

例2. 设 $x^2+y^2+z^2-4z=0$, 求 $\dfrac{\partial^2 z}{\partial x^2}$.

解法2 **利用隐函数求导**

$$2x+2z\frac{\partial z}{\partial x}-4\frac{\partial z}{\partial x}=0 \longrightarrow \frac{\partial z}{\partial x}=\frac{x}{2-z}$$

再对 x 求导

$$2+2\left(\frac{\partial z}{\partial x}\right)^2+2z\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x^2}=0$$

$$\frac{\partial^2 z}{\partial x^2}=\frac{1+\left(\dfrac{\partial z}{\partial x}\right)^2}{2-z}=\frac{(2-z)^2+x^2}{(2-z)^3}$$

例3. 设$F(x,y)$具有连续偏导数,已知方程 $F(\frac{x}{z},\frac{y}{z})=0$, 求 $\mathrm{d}z$.

解法1 利用偏导数公式. 设 $z=f(x,y)$ 是由方程 $F(\frac{x}{z},\frac{y}{z})=0$ 确定的隐函数, 则

$$\frac{\partial z}{\partial x}=-\frac{F_1'\cdot\frac{1}{z}}{F_1'\cdot(-\frac{x}{z^2})+F_2'\cdot(-\frac{y}{z^2})}=\frac{zF_1'}{xF_1'+yF_2'}$$

$$\frac{\partial z}{\partial y}=-\frac{F_2'\cdot\frac{1}{z}}{F_1'\cdot(-\frac{x}{z^2})+F_2'\cdot(-\frac{y}{z^2})}=\frac{zF_2'}{xF_1'+yF_2'}$$

故 $$\mathrm{d}z=\frac{\partial z}{\partial x}\mathrm{d}x+\frac{\partial z}{\partial y}\mathrm{d}y=\frac{z}{xF_1'+yF_2'}$$

解法2　微分法.

对方程两边求微分: $\mathrm{d}\left[F(\frac{x}{z},\frac{y}{z})\right]=0$

$$F_1'\cdot\mathrm{d}(\frac{x}{z})+F_2'\cdot\mathrm{d}(\frac{y}{z})=0$$

$$F_1'\cdot(\frac{z\,\mathrm{d}x-x\,\mathrm{d}z}{z^2})+F_2'\cdot(\frac{z\,\mathrm{d}y-y\,\mathrm{d}z}{z^2})=0$$

$$\frac{xF_1'+yF_2'}{z^2}\mathrm{d}z=\frac{F_1'\mathrm{d}x+F_2'\mathrm{d}y}{z}$$

$$\mathrm{d}z=\frac{z}{xF_1'+yF_2'}(F_1'\mathrm{d}x+F_2'\mathrm{d}y)$$

二、方程组所确定的隐函数组及其导数

隐函数存在定理还可以推广到方程组的情形.

以两个方程确定两个隐函数的情况为例，即

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \longrightarrow \begin{cases} u=u(x,y) \\ v=v(x,y) \end{cases}$$

由 F, G 的偏导数组成的行列式

$$J=\frac{\partial(F,G)}{\partial(u,v)}=\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$

称为F, G 的雅可比行列式.

雅可比，C.G.J.

补充:二阶行列式表示二元一次线性方程组的唯一解

考虑用消元法解二元一次方程组 $\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$，

用a_{22}和a_{12}分别乘以两个方程的两端，然后两个方程相减，

消去x_2得 $(a_{11}a_{22} - a_{12}a_{21})x_1 = b_1a_{22} - a_{12}b_2$

同理，消去x_1得

$$(a_{11}a_{22} - a_{12}a_{21})x_2 = a_{11}b_2 - b_1a_{21}$$

当 $a_{11}a_{22} - a_{12}a_{21} \neq 0$ 时，方程组的解为

$$x_1 = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}, \quad x_2 = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

当 $a_{11}a_{22}-a_{12}a_{21}\neq 0$ 时，方程组的解为

$$x_1=\frac{b_1a_{22}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{21}}=\frac{\boldsymbol{D_1}}{\boldsymbol{D}},\quad x_2=\frac{a_{11}b_2-b_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}}=\frac{\boldsymbol{D_2}}{\boldsymbol{D}}$$

为便于叙述和记忆， 引入二阶行列式符号

$$D=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{12}a_{21}$$

按照二阶行列式定义可得 $\boldsymbol{D_1}=\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}=b_1a_{22}-a_{12}b_2$

$$\boldsymbol{D_2}=\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}=a_{11}b_2-b_1a_{21}$$

于是，当$\boldsymbol{D}\neq 0$时，方程组的解为 $x_1=\dfrac{D_1}{D},\quad x_2=\dfrac{D_2}{D}.$

定理3. 设函数 $F(x,y,u,v)$, $G(x,y,u,v)$ 在点 $P(x_0,y_0,u_0,v_0)$ 的某邻域内满足:

① 具有连续偏导数;

② $F(x_0,y_0,u_0,v_0)=0,\ \ G(x_0,y_0,u_0,v_0)=0;$

③ $J\Big|_P=\dfrac{\partial(F,G)}{\partial(u,v)}\Big|_P\neq 0,$

则方程组 $F(x,y,u,v)=0, G(x,y,u,v)=0$ 在点(x_0,y_0,u_0,v_0)的某邻域内可唯一确定一组连续函数 $u=u(x,y)$, $v=v(x,y)$, 满足 $u_0=u(x_0,y_0)$, $v_0=v(x_0,y_0)$, 且有连续偏导数

$$\frac{\partial u}{\partial x}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(\underline{x},v)}=-\frac{1}{\begin{vmatrix}F_u & F_v\\ G_u & G_v\end{vmatrix}}\begin{vmatrix}F_x & F_v\\ G_x & G_v\end{vmatrix}$$

$$\frac{\partial u}{\partial y}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(\underline{y},v)}=-\frac{1}{\begin{vmatrix}F_u & F_v\\ G_u & G_v\end{vmatrix}}\begin{vmatrix}F_y & F_v\\ G_y & G_v\end{vmatrix}$$

$$\boxed{J=\frac{\partial(F,G)}{\partial(u,v)}=\begin{vmatrix}F_u & F_v\\ G_u & G_v\end{vmatrix}}$$

$$\frac{\partial v}{\partial x} = -\frac{1}{J}\frac{\partial(F,G)}{\partial(u,\underline{x})} = -\frac{1}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}$$

$$\frac{\partial v}{\partial y} = -\frac{1}{J}\frac{\partial(F,G)}{\partial(u,\underline{y})} = -\frac{1}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}$$

定理证明略.
仅推导偏导数公式如下:

设方程组 $\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases}$ 有隐函数组 $\begin{cases} u=u(x,y) \\ v=v(x,y) \end{cases}$，则

$$\begin{cases} F(x,y,u(x,y),v(x,y))\equiv 0 \\ G(x,y,u(x,y),v(x,y))\equiv 0 \end{cases}$$

两边对 x 求导得
$$\begin{cases} F_x+F_u\cdot\dfrac{\partial u}{\partial x}+F_v\cdot\dfrac{\partial v}{\partial x}=0 \\ G_x+G_u\cdot\dfrac{\partial u}{\partial x}+G_v\cdot\dfrac{\partial v}{\partial x}=0 \end{cases}$$

两边对 x 求导得 $\begin{cases} F_x + F_u \cdot \dfrac{\partial u}{\partial x} + F_v \cdot \dfrac{\partial v}{\partial x} = 0 \\ G_x + G_u \cdot \dfrac{\partial u}{\partial x} + G_v \cdot \dfrac{\partial v}{\partial x} = 0 \end{cases}$

即得 $\begin{cases} F_u \cdot \dfrac{\partial u}{\partial x} + F_v \cdot \dfrac{\partial v}{\partial x} = -F_x \\ G_u \cdot \dfrac{\partial u}{\partial x} + G_v \cdot \dfrac{\partial v}{\partial x} = -G_x \end{cases}$

这是关于 $\dfrac{\partial u}{\partial x}, \dfrac{\partial v}{\partial x}$ 的二元一次线性方程组.

$$\begin{cases} F_u \cdot \dfrac{\partial u}{\partial x} + F_v \cdot \dfrac{\partial v}{\partial x} = -F_x \\ G_u \cdot \dfrac{\partial u}{\partial x} + G_v \cdot \dfrac{\partial v}{\partial x} = -G_x \end{cases}$$

这是关于 $\dfrac{\partial u}{\partial x}, \dfrac{\partial v}{\partial x}$ 的二元一次线性方程组，在点P 的某邻域内,系数行列式

$$J = \frac{\partial(F,G)}{\partial(u,v)} = \begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix} \neq 0,$$ 故得 $$\frac{\partial u}{\partial x} = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)}$$

$$\frac{\partial u}{\partial x}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)} \longleftarrow \partial(u,v)$$

$$\frac{\partial v}{\partial x}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,x)}$$

记忆要点:

偏 u 偏 x

变 u 为 x

依此类推

同样可得

$$\frac{\partial u}{\partial y}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(y,v)}$$

$$\frac{\partial v}{\partial y}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,y)}$$

例4. 设 $xu - yv = 0,\ yu + xv = 1$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$.

解2: 方程组两边对 x 求导, 并移项得

$$\begin{cases} x\frac{\partial u}{\partial x} - y\frac{\partial v}{\partial x} = -u \\ y\frac{\partial u}{\partial x} + x\frac{\partial v}{\partial x} = -v \end{cases}$$

由题设 $J = \begin{vmatrix} x & -y \\ y & x \end{vmatrix} = x^2 + y^2 \neq 0$

故有
$$\begin{cases} \frac{\partial u}{\partial x} = \frac{1}{J}\begin{vmatrix} -u & -y \\ -v & x \end{vmatrix} = -\frac{xu + yv}{x^2 + y^2} \\ \frac{\partial v}{\partial x} = \frac{1}{J}\begin{vmatrix} x & -u \\ y & -v \end{vmatrix} = -\frac{xv - yu}{x^2 + y^2} \end{cases}$$

练习: 求 $\frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$

答案:
$$\begin{cases} \frac{\partial u}{\partial y} = -\frac{yu - xv}{x^2 + y^2} \\ \frac{\partial v}{\partial y} = -\frac{xu + yv}{x^2 + y^2} \end{cases}$$

例5. 设函数 $x = x(u,v),\ y = y(u,v)$ 在点(u,v) 的某一邻域内有连续的偏导数，且 $\dfrac{\partial(x,y)}{\partial(u,v)} \neq 0$

1) 证明函数组 $\begin{cases} x = x(u,v) \\ y = y(u,v) \end{cases}$ 在与点 (u, v) 对应的点 (x, y) 的某一邻域内，唯一确定一组连续且具有连续偏导数的反函数 $u = u(x,y), v = v(x, y)$.

2) 求 $u = u(x,y), v = v(x, y)$ 对 x, y 的偏导数.

解: 1) 令 $F(x,y,u,v) \equiv x - x(u,v) = 0$

$$G(x,y,u,v) \equiv y - y(u,v) = 0$$

则有 $$J=\frac{\partial(F,G)}{\partial(u,v)}=\frac{\partial(x,y)}{\partial(u,v)}\neq 0,$$

分析 $$F(x,y,u,v)\equiv x-x(u,v)=0$$

$$G(x,y,u,v)\equiv y-y(u,v)=0$$

$$J=\frac{\partial(F,G)}{\partial(u,v)}=\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}=\begin{vmatrix} -x_u & -x_v \\ -y_u & -y_v \end{vmatrix}=\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

则有 $$J=\frac{\partial(F,G)}{\partial(u,v)}=\frac{\partial(x,y)}{\partial(u,v)}\neq 0,$$

由定理 3 可知结论 1) 成立.

2) 求反函数的偏导数.

$$\begin{cases} x\equiv x(u(x,y),v(x,y)) \\ y\equiv y(u(x,y),v(x,y)) \end{cases} \qquad ①$$

①式两边对 x 求导, 得

$$\begin{cases} 1=\dfrac{\partial x}{\partial u}\cdot\dfrac{\partial u}{\partial x}+\dfrac{\partial x}{\partial v}\cdot\dfrac{\partial v}{\partial x} \\ 0=\dfrac{\partial y}{\partial u}\cdot\dfrac{\partial u}{\partial x}+\dfrac{\partial y}{\partial v}\cdot\dfrac{\partial v}{\partial x} \end{cases} \qquad ②$$

注意 $J \neq 0$, 从方程组②解得

$$\frac{\partial u}{\partial x} = \frac{1}{J}\begin{vmatrix} 1 & \dfrac{\partial x}{\partial v} \\ 0 & \dfrac{\partial y}{\partial v} \end{vmatrix} = \frac{1}{J}\frac{\partial y}{\partial v}, \quad \frac{\partial v}{\partial x} = \frac{1}{J}\begin{vmatrix} \dfrac{\partial x}{\partial u} & 1 \\ \dfrac{\partial y}{\partial u} & 0 \end{vmatrix} = -\frac{1}{J}\frac{\partial y}{\partial u}$$

同理, ①式两边对 y 求导, 可得

$$\frac{\partial u}{\partial y} = -\frac{1}{J}\frac{\partial x}{\partial v},$$

$$\frac{\partial v}{\partial y} = \frac{1}{J}\frac{\partial x}{\partial u}$$

例5的应用: 计算极坐标变换 $x = \underset{u}{r}\cos\underset{v}{\theta}\,,\ y = r\sin\theta$ **的反变换的导数.**

$$\boxed{\begin{aligned}\frac{\partial r}{\partial x} &= \frac{1}{J}\frac{\partial y}{\partial \theta}\\ \frac{\partial \theta}{\partial x} &= -\frac{1}{J}\frac{\partial y}{\partial r}\end{aligned}}$$

由于 $J = \dfrac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix}\cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta\end{vmatrix} = r$

所以 $\dfrac{\partial r}{\partial x} = \dfrac{1}{J}\dfrac{\partial y}{\partial \theta} = \dfrac{1}{r} r\cos\theta = \cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}$

$$\frac{\partial \theta}{\partial x} = -\frac{1}{J}\frac{\partial y}{\partial r} = -\frac{1}{r}\sin\theta = -\frac{y}{x^2+y^2}$$

同理 $\dfrac{\partial r}{\partial y} = \dfrac{y}{\sqrt{x^2+y^2}},\quad \dfrac{\partial \theta}{\partial y} = \dfrac{x}{x^2+y^2}$

内 容 小 结

1. 隐函数(组) 存在定理

2. 隐函数 (组) 求导方法

方法1. 利用复合函数求导法则直接计算;

方法2. 利用微分形式不变性;

方法3. 套用公式.

思考与练习

1.(6分)证明 $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^3y}{x^6+y^2}$ 不存在.

证明：令 $y=kx^3$，故而当 $x\to0$ 时 $y\to0$

$$\lim_{(x,y)\to(0,0)}\frac{x^3y}{x^6+y^2}=\lim_{\substack{x\to0\\y=kx^3}}\frac{x^3kx^3}{x^6+(kx^3)^2}$$

$$=\frac{k}{1+k^2}$$

结果与 k 有关,因此极限不存在.

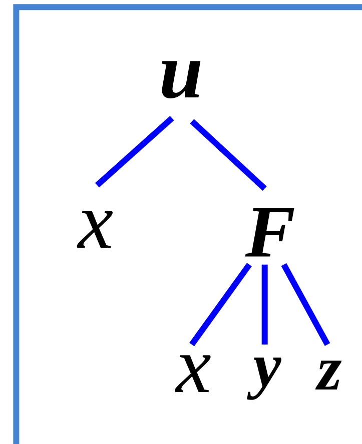
2.(6分)设函数$u = x^k F\left(\frac{z}{x}, \frac{y}{x}\right)$,其中$k$为常数,函数$F$具有一阶连续偏导数,试求 $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = kx^k F\left(\frac{z}{x}, \frac{y}{x}\right)$

解: $$\frac{\partial u}{\partial x} = kx^{k-1}F + x^k F_1' \cdot \left(-\frac{z}{x^2}\right) + x^k F_2' \cdot \left(-\frac{y}{x^2}\right)$$

$$= kx^{k-1}F - z x^{k-2}F_1' - yx^{k-2}F_2'$$

$$\frac{\partial u}{\partial y} = x^k F_2' \cdot \frac{1}{x} = x^{k-1}F_2'$$

$$\frac{\partial u}{\partial z} = x^k F_1' \cdot \frac{1}{x} = x^{k-1}F_1'$$



3． 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微的充分条件是〖 **D** 〗.

A. $f(x,y)$ 在 (x_0,y_0) 处连续；

B. $f'_x(x,y)$， $f'_y(x,y)$ 在 (x_0,y_0) 的某邻域内存在；

C. $\Delta z-f'_x(x_0,y_0)\Delta x-f'_y(x_0,y_0)\Delta y$ 当 $\sqrt{(\Delta x)^2+(\Delta y)^2}\to 0$ 时，是无穷小；

D. $\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\dfrac{\Delta z-f'_x(x_0,y_0)\Delta x-f'_y(x_0,y_0)\Delta y}{\sqrt{(\Delta x)^2+(\Delta y)^2}}=0$。