Appendix of "Multi-Fidelity Methods for Optimization: A Survey"

ACM Reference Format:
. 2024. Appendix of "Multi-Fidelity Methods for Optimization: A Survey". 1, 1 (February 2024), 34 pages.

A MIND MAPS

Fig. A1 provides an overview of the structure of this survey. Fig. A2 depicts the structure of the applications related to multi-fidelity optimization covered in this survey.

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Fig. A1. Taxonomy of research strands for multi-fidelity optimization, and the research strands and structure of this survey.

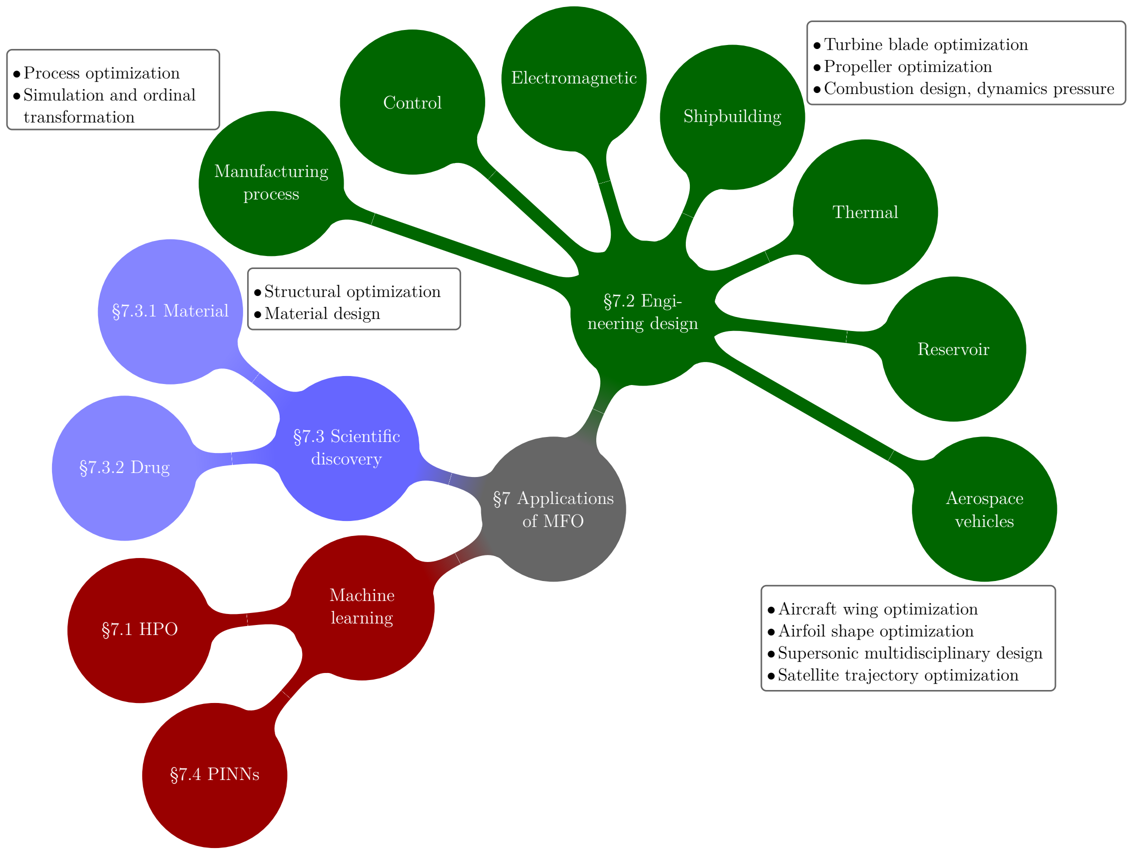

Fig. A2. Taxonomy of applications of multi-fidelity optimization acorss machine learning, engineering design and optimizaiton, and scientific discovery domains.

B SUPPLEMENTARY INFORMATION OF SECTIONS 2 TO 8

B.1 Supplementary information of Section 2

Tables A1 and A2 provide the tables complementary to the statistical diagrams presented in our manuscript.

Table A1. Statistics of the number of articles published in different countries (sorted in a descending order). Note that the articles are collected in our text mining process.

Country	# of articles	Country	# of articles	Country	# of articles
United States of America	372	Poland	12	Pakistan	3
China	288	Thailand	11	Norway	2
Germany	61	Iran	9	Czech Republic	2
United Kingdom	60	Turkey	9	Italy	2
Italy	41	Mexico	9	German	2
France	39	Portugal	8	Indonesia	2
Iceland	39	Sweden	6	Hungary	2
Canada	31	Ireland	6	United Arab Emirates	2
Japan	25	Greece	5	Colombia	2
India	22	Russia	4	Malaysia	2
Netherlands	22	Switzerland	4	Qatar	1
South Korea	20	Denmark	4	New Zealand	1
Australia	17	South Africa	4	Algeria	1
Spain	15	Egypt	3	Luxembourg	1
Singapore	14	Finland	3	Bangladesh	1
Belgium	12	Israel	3	NONE	23
Brazil	12	Vietnam	3		

Table A2. Statistics of the number of articles classfied into different topics.

Topic ID	# of articles	Topic key words	Topic ID	# of articles	Topic key words
1	180	Surrogate model and optimization	12	34	Neural network
2	74	Turbine blade optimization	13	34	Propeller optimization
3	59	Electromagnetic optimization	14	32	Material design
4	57	Aircraft wing optimization	15	29	Manufacturing process optimization
5	57	Evolutionary optimization	16	29	Supersonic multidisciplinary design
6	48	Hyperparameter optimization	17	28	Satellite trajectory optimization
7	45	Bayesian optimization	18	28	Image reconstruction
8	43	Reservoir optimization	19	23	Control model design
9	41	Composite, buckling, optimization	20	23	Aerodynamic optimization
10	37	Simulation and ordinal transformation	21	20	Combustor design
11	35	Airfoil shape optimization	22	19	Cooling hole design
			23	14	Drug discovery

B.2 Supplementary information of Section 3

Table A3 gives the number of articles categorized into different types of surrogate modeling methods. Table A4 provides a summary of the current works on multi-fidelity surrogate modeling, focusing on their native assumptions and relevant disadvantages.

Table A3. Statistics of the number of articles regarding the multi-fidelity surrogate modeling.

Models	# of articles	Models	# of articles	Models	# of articles
Autoregressive-based	30	Single model	8	Mapping	2
Correction-based	27	No	6	Nonlinear methods	1
MTGP	11	GP	2	Ensemble	1

Table A4. Summary of the current works on surrogate modeling for multi-fidelity data.

Methods	Assumptions	Disadvantages	References
Single model	Independent	Since each fidelity is modeled independently, samples from one fidelity cannot contribute to improving the others.	RSM [30], Kriging [25, 43, 63], RBF [42, 63, 114].
Space mapping	$\mathbf{x}_{\mathrm{l}} = P(\mathbf{x}_{\mathrm{h}})$	① It is hard to construct a good mapping model. ② It may be trapped in local minima if the LF is severely misaligned with the HF.	Linear [3, 5–7], nonlinear [4, 86]
Correction-based method	$f_{\mathrm{h}}(\mathbf{x}) = f_{\mathrm{l}}(\mathbf{x}) + \delta(\mathbf{x})$ $f_{\mathrm{h}}(\mathbf{x}) = \rho(\mathbf{x}) f_{\mathrm{l}}(\mathbf{x})$ $f_{\mathrm{h}}(\mathbf{x}) = \rho(\mathbf{x}) f_{\mathrm{l}}(\mathbf{x}) + \delta(\mathbf{x})$	① Since LF and HF are modeled independently, their correlations are neglected. ② It requires evaluating samples at all fidelities, thus incurring a significant amount of sample data. ③ It cannot model the nonlinear relationship between different fidelities.	First-order [14, 33, 51] and second-order Taylor series [23, 27, 29, 73, 74], NNs [20, 37, 47, 93, 100, 101, 106, 107, 113], SVR [15, 91, 94, 122, 124], RSM [49, 95, 103, 104], Kriging [28, 29, 60, 67, 109, 123, 125–127].
AR1	$f_{\mathrm{h}}(\mathbf{x}) = \rho f_{\mathrm{l}}(\mathbf{x}) + \delta(\mathbf{x})$ $f_{\mathrm{h}}(\mathbf{x}), f_{\mathrm{l}}(\mathbf{x}), \delta(\mathbf{x})$ $\sim \mathcal{N}\left(m, \sigma^2(x)\right)$	① It can only estimate the relationship between subsequent fidelities, thereby restricting information propagation across fidelities. ② It cannot model the nonlinear relationship between different fidelities.	co-Kriging [17, 26, 35, 39, 45, 81, 117, 118, 129], hierarchical models [10, 31, 34, 35, 65, 80, 120], non-hierarchical co-Kriging [119].
MTGP	$f_{\mathrm{h}}(\mathbf{x}) = \sum_{i=1}^{m} a_i f_i(\mathbf{x})$ $f_{\mathrm{l}}(\mathbf{x}) = \sum_{i=1}^{m} b_i f_i(\mathbf{x})$	① It treats all fidelities equally, thus neglecting the relative importance of each other. ② Its computational complexity is cubic to the number of training data.	Linear relationships [77, 79, 84, 96–98], nonlinear relationships [121].
Nonlinear hierarchical method	$f_{\mathrm{h}}(\mathbf{x}) = g(f_{\mathrm{l}}(\mathbf{x})) + \delta(\mathbf{x})$	① The inference is intractable. ② The number of hyperparameters is significantly larger than the other surrogate modeling methods.	GP [16, 82, 87], NNs [56, 57], others [110].
PINN	$f_{\mathrm{h}}(\mathbf{x}) = g(f_{\mathrm{l}}(\mathbf{x})) + \delta(\mathbf{x})$	① It is limited to problems with PDEs. ② The number of hyperparameters is huge.	[13, 36, 64, 75, 76].

As discussed in Section 3.8 of our manuscript, the multi-fidelity surrogate modeling techniques can be represented as:

$$f_{\mathrm{h}}(\mathbf{x}) = \rho(\mathbf{x}) g(f_{\mathrm{l}}(\mathbf{x})) + \epsilon(\mathbf{x}), \tag{1}$$

except the single model method. While the format of multi-task Gaussian process (MTGP) shown in Table A4 looks different from the others, we argue that it can be reformulated as equation (1). In the following paragraphs, we use the linear model of coregionalization (LMC) [1] as the representative example of MTGP to derive the corresponding reformulation. Specifically, the equation of MTGP listed in Table A4 can be formulated as linear combinations of independent random functions:

$$\mathbf{f} = \begin{bmatrix} f_1(\mathbf{x}) \\ f_h(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \end{bmatrix}, \tag{2}$$

where $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are latent functions. The matrix coefficients $a_{i,j}$ are scalar coefficients, where $i, j \in \{1, 2\}$. Given a set of training data $X = \{\mathbf{x}^i\}_{i=1}^N$, we can split it into two parts $X = X_l \bigcup X_h$, where X_l and X_h are used for low- and high-fidelity evaluations, respectively. Accordingly, the outputs of X are denoted as $\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_h \end{bmatrix}$. For a testing input vector $\mathbf{x}^*$, the mean and variance of the target $f(\mathbf{x}^*)$ are predicted as:

$$\begin{aligned} \mu_h(\mathbf{x}^*) &= \left(\boldsymbol{k}_h^f \otimes \boldsymbol{k}_*^x\right)^{\mathrm{T}} \Sigma^{-1}\mathbf{y}, \Sigma = K^f \otimes K^x, \\ \mu_1(\mathbf{x}^*) &= \left(\boldsymbol{k}_1^f \otimes \boldsymbol{k}_*^x\right)^{\mathrm{T}} \Sigma^{-1}\mathbf{y}, \Sigma = K^f \otimes K^x, \end{aligned} \tag{3}$$

where $\otimes$ is the Kronecker product, and

$$K^f = \begin{bmatrix} {a_{11}}^2 + a_{21}^2 & a_{11}a_{21} + a_{12}a_{21} \\ a_{11}a_{21} + a_{12}a_{21} & a_{21}^2 + {a_{22}}^2 \end{bmatrix}, \tag{4}$$

$$\begin{aligned} \boldsymbol{k}_1^f \otimes \boldsymbol{k}_*^x &= \begin{bmatrix} \left({a_{11}}^2 + {a_{21}}^2\right) k\left(\mathbf{x}^*, X_1\right) \\ \left(a_{11}a_{21} + a_{12}a_{21}\right) k\left(\mathbf{x}^*, X_h\right) \end{bmatrix}, \\ \boldsymbol{k}_h^f \otimes \boldsymbol{k}_*^x &= \begin{bmatrix} \left(a_{11}a_{21} + a_{12}a_{21}\right) k\left(\mathbf{x}^*, X_1\right) \\ \left({a_{21}}^2 + {a_{22}}^2\right) k\left(\mathbf{x}^*, X_h\right) \end{bmatrix}. \end{aligned} \tag{5}$$

Substitute $K^f, \boldsymbol{k}_1^f \otimes \boldsymbol{k}_*^x$, and $\boldsymbol{k}_h^f \otimes \boldsymbol{k}_*^x$ into equation (3) and equation (4), we has:

$$\begin{aligned} \mu_1(\mathbf{x}^*) &= \begin{bmatrix} \left({a_{11}}^2 + {a_{21}}^2\right) k\left(\mathbf{x}^*, X_1\right) \\ \left(a_{11}a_{21} + a_{12}a_{21}\right) k\left(\mathbf{x}^*, X_h\right) \end{bmatrix}^{\mathrm{T}} \Sigma^{-1}\mathbf{y}, \\ \mu_h(\mathbf{x}^*) &= \begin{bmatrix} \left(a_{11}a_{21} + a_{12}a_{21}\right) k\left(\mathbf{x}^*, X_1\right) \\ \left({a_{21}}^2 + {a_{22}}^2\right) k\left(\mathbf{x}^*, X_h\right) \end{bmatrix}^{\mathrm{T}} \Sigma^{-1}\mathbf{y}, \end{aligned} \tag{6}$$

$$\mu_h(\mathbf{x}^*) = \mu_l(\mathbf{x}^*) \cdot \frac{\begin{bmatrix} \left(a_{11}a_{21} + a_{12}a_{21}\right) k\left(\mathbf{x}^*, X_1\right) \\ \left({a_{21}}^2 + {a_{22}}^2\right) k\left(\mathbf{x}^*, X_h\right) \end{bmatrix}^{\mathrm{T}} \Sigma^{-1}\mathbf{y}}{\begin{bmatrix} \left({a_{11}}^2 + {a_{21}}^2\right) k\left(\mathbf{x}^*, X_1\right) \\ \left(a_{11}a_{21} + a_{12}a_{21}\right) k\left(\mathbf{x}^*, X_h\right) \end{bmatrix}^{\mathrm{T}} \Sigma^{-1}\mathbf{y}} = \mu_l(\mathbf{x}^*) \cdot p(\mathbf{x}^*), \tag{7}$$

where $p(\mathbf{x}^*)$ is a composite function of $\mathbf{x}^*$. In this case, we can tell that equation (7) is a transformation of equation (1) with $\epsilon(\mathbf{x}) = 0$.

B.3 Supplementary information of Section 5

Table A5 gives the number of articles categorized into different types of fidelity management strategies. Table A6 provides a summary of the current works on fidelity management strategies, focusing on their basic ideas and relevant disadvantages.

Table A5. Statistics of the number of articles regarding the fidelity management strategies.

Methods	# of articles	Methods	# of articles	Methods	# of articles
Information and cost	32	Improvement and cost	10	Correlation	2
LF first, then HF	18	Uncertainty	10	Periodic	2
Simultaneously use HF and LF	14	Only use HF	8	Curve	1

Table A6. Summary of the current works on fidelity management strategies.

Methods	Description	Disadvantages	References
LF first, then HF	Allocate a fixed amount of computational budget to both LF and HF.	It is static and rigid to vibrate real-world black-box scenarios.	[22, 41, 53, 63, 66–68, 72, 92, 111, 112, 115]
Periodic strategy	Predefine the pattern used to periodically switch between different fidelities.	It is hard to determine an optimal frequency for switching fidelities.	[127]
Uncertainty	Select the fidelity level according to the uncertainty in the LF level.	① It can overly explor the LF when the LF and HF landscapes are significantly different. ② It is hard to define an appropriate uncertainty threshold.	[43]
Correlation estimation	Select the fidelity level according to the correlation between LF and HF.	It is hard to evaluate the correlation between LF and HF.	[11, 32, 61, 105]
Acquisition function	Design acquisition functions using the information gain associated with the evaluation cost as different fidelities.	It has the risk of overly exploit the fidelity with a low cost.	[38, 50, 56, 57, 69, 78, 79, 85, 96, 97, 120, 121]

B.4 Supplementary information of Section 8

Table A7 provides an overview of open-source software packages used for multi-fidelity optimization. Fig. A3 shows the temporal statistics of the packages. Further, Fig. A4 shows the proportion of the number of packages regarding different multi-fidelity optimizes and programming languages.

Table A7. Overview of multi-fidelity optimization software packages.

Repository Name	Algorithm Name	Year	Programming Language	Optimizer
MFB	PSO-AFAg [105]	2018	MATLAB	SAEA
TSA-BFEA	TSA-BFEA [106]	2020	MATLAB	SAEA
RoBO	Fabolas [48]	2017	Python	MFBO
dragonfly	BOCA [44]	2017	Python	MFBO
Spearmint	MTBO [96]	2012	Python	MFBO
mf-gp-ucb	MF-GP-UCB [43]	2019	MATLAB	MFBO
DNN-MFBO	DNN-MFBO [57]	2020	Python	MFBO
BMBO-DARN	BMBO-DARN [56]	2021	Python	MFBO
MF-DGP	MF-DGP [89]	2022	Python	MFBO
DyHPO	DyHPO [108]	2022	Python	MFBO
MF-MES	MF-MES [98]	2022	Python	MFBO
rmfbo	rMFBO [77]	2023	Python	MFBO
MFBoom	MFBoom [25]	2023	Python	MFBO
HpBandSter	Hyperband [54]	2017	Python	Bandit
	BOHB [24]	2018	Python	Bandit
	DEHB [2]	2021	Python	Bandit
	SMAC3 [62]	2022	Python	Bandit
darts_asha	ASHA [55]	2020	Python	Bandit
MFES-HB	MFES-HB [59]	2021	Python	Bandit
HyperTune	HyperTune [58]	2022	Python	Bandit
GPTune	GPTuneBand [128]	2022	Python	Bandit
pasha	PASHA [9]	2023	Python	Bandit
optuna	Successive Halving [40]	2019	Python	Bandit
	Hyperband [54]			

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Table A7 continued from previous page

Package	Reference	Year	Programming Language	Optimizer
mlr3hyperband	Successive Halving [40]	2020	R	Bandit
	Hyperband [54]			
syne-tune	Hyperband [54]	2022	Python	Bandit
	ASHA [55]			
	BOHB [24]			
	DEHB [2]			
	HyperTune [58]			
	DyHPO [108]			
	PASHA [9]			
open-box	Hyperband [54]	2023	Python	Bandit
	BOHB [24]			
	MFES-HB [59]			

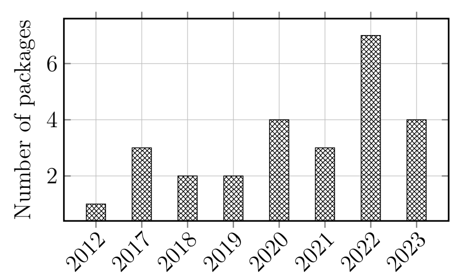
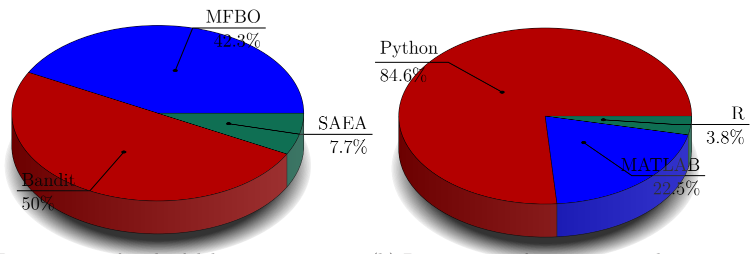


Fig. A3. The temporal statistics from 2017 to 2023 regarding the number of open source software packages used in multi-fidelity optimization.



(a) Proportions of multi-fidelity optimizers. (b) Proportions of programming language.

Fig. A4. Pie charts of the proportion of the number of software packages regarding (a) different multi-fidelity optimizes, and (b) programming languages considered in this survey.

B.5 Supplementary information of Section 6

Table A8 provides a comparison of key characteristics of existing benchmark test problems in multi-fidelity optimization. Table A9 provides an overview of multi-fidelity optimization benchmark software packages. Further, we provide lookup tables (Tables A10 to A18) of the mathematical definitions of the benchmark test problems examined in this survey. Additionally, we plot the 2D contours of the fitness landscapes in Figs. A5 to A20 to facilitate a visual understanding of the correlation between low- and high-fidelity levels.

Table A8. Comparison of key characteristics of existing benchmark test problems in MFO.

References	Scalability	Discrepancy	Fidelity correlations
[20, 52, 90]	✗	Linear	✗
[46, 99, 102]	✗	Linear	✓
[12, 88]	✓	Linear	✗
[105]	✓	Linear	✓
[16]	✗	Linear/Nonlinear	✗
[70]	✓	Linear/Nonlinear	✗

Table A9. Overview of multi-fidelity optimization benchmark software packages.

Repository Name	Reference	Year	Programming Language
MFB	[105]	2018	MATLAB
NAS-Bench-101	[116]	2019	Python
NAS-Bench-201	[19]	2020	Python
mf2	[102]	2020	Python
HPOBench	[21]	2021	Python
yahpo_gym	[83]	2022	Python
jahs_bench_201	[8]	2022	Python
mf-prior-bench	[71]	2023	Python
MOMF	[46]	2023	Python

Table A10. Benchmark test problems developed in [99].

PROBLEM NAME	MATHEMATICAL FORMULATION	DECISION SPACE
Branin	$f_h(x_1,x_2)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos(x_1)+10$ $f_l(x_1,x_2)=f_h(x_1,x_2)-(A_1+0.5)\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2, A_1\in[0,1]$	$x_1\in[-5,10], x_2\in[0,15]$
Paciorek	$f_h(x_1,x_2)=\sin\left(\frac{1}{x_1x_2}\right)$ $f_l(x_1,x_2)=f_h(x_1,x_2)-9A_2^2\cos\left(\frac{1}{x_1x_2}\right), A_2\in[0,1]$	$\{x_1,x_2\}\in[0.3,1.0]$
Hartmann H34	$f_h(x_1,x_2,x_3,x_4)=-\sum_{i=1}^{4}\alpha_i\exp\left[-\sum_{j=1}^{3}\beta_{ij}\left(x_j-P_{ij}\right)^2\right]$ $\alpha=\begin{bmatrix}1\\1.2\\3\\3.2\end{bmatrix}, \beta=\begin{bmatrix}3.0&10&30\\0.1&10&35\\3.0&10&30\\0.1&10&35\end{bmatrix}, P=\begin{bmatrix}3689&1170&2673\\4699&4387&7470\\1091&8732&5547\\381&5743&8828\end{bmatrix}\times10^{-4},$ $f_l(x_1,x_2,x_3,x_4)=-\sum_{i=1}^{4}\alpha_i\exp\left[-\sum_{j=1}^{3}\beta_{ij}\left(x_j-\frac{3}{4}P_{ij}(A_3+1)\right)^2\right], A_3\in[0,1]$	$\{x_1,\ldots,x_4\}\in[0.3,1.0]$
Trid	$f_h(x_1,\ldots,x_{10})=\sum_{i=1}^{10}(x_i-1)^2-\sum_{i=2}^{10}x_ix_{i-1}$ $f_l(x_1,\ldots,x_{10})=\sum_{i=1}^{10}(x_i-A_4)^2-(A_4-0.65)\sum_{i=2}^{10}ix_ix_{i-1}, A_4\in[0,1]$	$\{x_1,\ldots,x_{10}\}\in[-100,100]$

Table A11. Benchmark test problems developed in [18].

Problem name	Mathematical formulation	Decision space
Six-hump camel-back	$f_{\text{h}}(x_1, x_2) = 4x_1^2 - 2.1x_1^4 + \frac{x_1^6}{3} + x_1x_2 - 4x_2^2 + 4x_2^4$ $f_{\text{l}}(x_1, x_2) = f_{\text{h}}(0.7x_1, 0.7x_2) + x_1x_2 - 15$	$\{x_1, x_2\} \in [-2, 2]$
Branin	$f(x_1, x_2) = 10 + \left[x_2 - 5.1 \times \frac{x_1^2}{4\pi^2} + \frac{5x_1}{\pi} - 6\right] + 10\cos(x_1)\left[1 - \frac{1}{8\pi}\right]$ $f_{\text{h}}(x_1, x_2) = f(x_1, x_2) - 22.5x_2$ $f_{\text{l}}(x_1, x_2) = f(0.7x_1, 0.7x_2) - 15.75x_2 + 20(0.9 + x_1)^2 - 50$	$x_1 \in [-5, 10], x_2 \in [0, 15]$
Booth	$f_{\text{h}}(x_1, x_2) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ $f_{\text{l}}(x_1, x_2) = f_{\text{h}}(0.4x_1, x_2) - 1.7x_1x_2 - x_1 + 2x_2$	$\{x_1, x_2\} \in [-10, 10]$
Bohachevsky	$f_{\text{h}}(x_1, x_2) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) - 0.4\cos(4\pi x_2) + 0.7$ $f_{\text{l}}(x_1, x_2) = f_{\text{h}}(0.7x_1, x_2) + x_1x_2 - 12$	$\{x_1, x_2\} \in [-100, 100]$
Himmelblau	$f_{\text{h}}(x_1, x_2) = \left(x_1^2 + x_2 - 11\right)^2 + \left(x_2^2 + x_1 - 7\right)^2$ $f_{\text{l}}(x_1, x_2) = f_{\text{h}}(0.5x_1, 0.8x_2) + x_2^3 - (x_1 + 1)^2$	$\{x_1, x_2\} \in [-3, 3]$

Table A12. Benchmark test problems developed in [52].

Problem name	Mathematical formulation	Decision space
Forrester	$f_{\mathrm{h}}(x) = (6x-2)^2 \sin(12x-4)\Big)$ $f_{\mathrm{l}}(x) = 0.5 f_{\mathrm{h}}(x) + 10(x-0.5) - 5$	$x \in [0, 1]$
Branin	$f(x_1, x_2) = 10 + \left[x_2 - 5.1 \times \frac{x_1^2}{4\pi^2} + \frac{5x_1}{\pi} - 6\right] + 10\cos(x_1)\left[1 - \frac{1}{8\pi}\right]$ $f_{\mathrm{h}}(x_1, x_2) = f(x_1, x_2) - 22.5x_2$ $f_{\mathrm{l}}(x_1, x_2) = f(0.7x_1, 0.7x_2) - 15.75x_2 + 20(0.9 + x_1)^2 - 50$	$x_1 \in [-5, 10]$, $x_2 \in [0, 15]$
Booth	$f_{\mathrm{h}}(x_1, x_2) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ $f_{\mathrm{l}}(x_1, x_2) = f_{\mathrm{h}}(0.4x_1, x_2) - 1.7x_1x_2 - x_1 + 2x_2$	$\{x_1, x_2\} \in [-10, 10]$
Six-hump camel-back	$f_{\mathrm{h}}(x_1, x_2) = 4x_1^2 - 2.1x_1^4 + \frac{x_1^6}{3} + x_1x_2 - 4x_2^2 + 4x_2^4,$ $f_{\mathrm{l}}(x_1, x_2) = f_{\mathrm{h}}(0.7x_1, 0.7x_2) + x_1x_2 - 15$	$\{x_1, x_2\} \in [-2, 2]$
3D Rosenbrock	$f_{\mathrm{h}}(x_1, x_2, x_3) = \sum_{i=1}^{2}\left[100\left(x_i^2 - x_{i+1}\right)^2 + (x_i - 1)^2\right]$ $f_{\mathrm{l}}(x_1, x_2, x_3) = 90\left(x_1^2 - x_2\right)^2 + 1.1(x_3 - 1)^2$ $+ 100\left(x_2^2 - x_3\right)^2 + (x_2 - 1)^2$	$\{x_1, x_2, x_3\} \in [-2.048, 2.048]$
Colville-Himmelblay	$f_{\mathrm{h}}(x_1, x_2, x_3, x_4) = 100\left(x_2 - x_1^2\right)^2 + (1 - x_1)^2$ $+ 90\left(x_4 - x_3^2\right)^2 + (1 - x_3)^2$ $+ 10.1\left((x_2 - 1)^2 + (x_4 - 1)^2\right)$ $+ 19.8(x_2 - 1)(x_4 - 1)$ $f_{\mathrm{l}}(x_1, x_2, x_3, x_4) = f_{\mathrm{h}}(x_1, x_2, x_3, x_4)$ $+ 0.1(1 - x_1)^2 + 0.1(1 - x_3)^2$ $- 0.1\left((x_2 - 1)^2 + (x_4 - 1)^2\right)$ $+ 0.2(x_2 - 1)(x_4 - 1)$	$\{x_1, \ldots, x_4\} \in [-10, 10]$
Hessen	$f_{\mathrm{h}}(x_1, \ldots, x_6) = 25(x_1 - 2)^2 + (x_2 - 2)^2 + (x_3 - 1)^2$ $+ (x_4 - 4)^2 + (x_5 - 1)^2 + (x_6 - 4)^2$ $f_{\mathrm{l}}(x_1, \ldots, x_6) = 0.6 f_{\mathrm{h}}(x_1, \ldots, x_6) + 0.25(x_2 - 2)^2 + 0.75(x_4 - 4)^2$	$\{x_1, \ldots, x_6\} \in [0, 10]$
Borehole	$f_{\mathrm{h}}(x_1, \ldots, x_8) = \frac{2\pi x_3(x_4 - x_6)}{\ln(x_2/x_1)\left[1 + 2x_7x_4/\ln(x_2/x_1)x_1^2x_8 + x_3/x_5\right]}$ $f_{\mathrm{l}}(x_1, \ldots, x_8) = 0.4 f_{\mathrm{h}}(x_1, \ldots, x_8) + 0.07x_1^2x_8$ $+ x_1x_7/x_3 + x_2x_6/x_2 + x_1^2x_4$	$x_1 \in [0.05, 0.15]$, $x_2 \in [100, 50000]$, $x_3 \in [63070, 115600]$, $x_4 \in [990, 1110]$, $x_5 \in [63.1, 116]$, $x_6 \in [700, 820]$, $x_7 \in [1120, 1680]$, $x_8 \in [9855, 12045]$

Table A13. Benchmark test problems developed in [90].

Problem name	Mathematical formulation	Decision space
SK	$f_{\mathrm{h}}(x) = x^4 + 3x^3 - 10x^2 - 10x - 10$ $f_{\mathrm{l}}(x) = f_{\mathrm{h}}(x) + 0.5x^4 + 2x^3 + 10x^2 + 10x + 5$	$x \in [-6, 4.5]$
SSFYY2	$f_h(x) = 10 + x^2 - 10\cos\left(\frac{x\pi}{2}\right)$ $f_{\mathrm{l}}(x) = f_h(x) - (x-4)^2$	$x \in [-16, 8]$
MLF1	$f_{\mathrm{h}}(x) = (1 + x/20)\sin(x)$ $f_{\mathrm{l}}(x) = f_{\mathrm{h}}(x) - \left(1 + \frac{x}{20}\right)\cos(x)$	$x \in [0, 20]$
Far1	$f_{\mathrm{h}}(x_1, x_2) = -2e^{\left(15\left(-(x_1-0.1)^2 - x_2^2\right)\right)} - e^{\left(20\left(-(x_1-0.6)^2-(x_2-0.6)^2\right)\right)}$ $+ e^{\left(20\left(-(x_1+0.6)^2-(x_2-0.6)^2\right)\right)} + e^{\left(20\left(-(x_1-0.6)^2-(x_2+0.6)^2\right)\right)}$ $+ e^{\left(20\left(-(x_1+0.6)^2-(x_2+0.6)^2\right)\right)}$ $f_{\mathrm{l}}(x_1, x_2) = f_{\mathrm{h}}(x_1, x_2) - 2e^{\left(20\left(-(x_1+0.1)^2-(x_2-0.3)^2\right)\right)}$ $- e^{\left(20\left(-(x_1-0.4)^2-(x_2-0.6)^2\right)\right)} + e^{\left(20\left(-(x_1+0.5)^2-(x_2-0.7)^2\right)\right)}$ $+ e^{\left(20\left(-(x_1-0.5)^2-(x_2+0.7)^2\right)\right)} - e^{\left(20\left(-(x_1+0.4)^2-(x_2+0.8)^2\right)\right)}$	$\{x_1, x_2\} \in [-0.4, 1.2]$

Table A14. Benchmark test problems developed in [88].

Problem name	Mathematical formulation	Decision space
Rosenbrock	$f_{\mathrm{h}}(\mathbf{x}) = \sum_{i=1}^{D-1} (1-x_i)^2 + 100\left(x_{i+1} - x_i^2\right)^2$ $f_{\mathrm{l}}(\mathbf{x}) = \frac{f_{\mathrm{h}}(\mathbf{x}) - 4.0 - \sum_{i=1}^{D} 0.5x_i}{10.0 + \sum_{i=1}^{D} 0.25x_i}$	$\mathbf{x} \in [-2, 2]$
exponential-sine	$f_{\mathrm{h}}(\mathbf{x}) = \frac{1}{D}\sum_{i=1}^{D} \exp(-0.1x_i)\sin\left(\frac{\pi}{2}x_i\right)$ $f_{\mathrm{l}}(\mathbf{x}) = \frac{1}{D}\sum_{i=1}^{D} \sin\left(\frac{\pi}{2}x_i\right)$	$\mathbf{x} \in [-2, 2]$
Heterogeneous-1	$f_{\mathrm{h}}(x) = \sin\left[30(x-0.9)^4\right]\cos[2(x-0.9)] + (x-0.9)/2$ $f_{\mathrm{l}}(x) = \frac{f_{\mathrm{h}}(x) - 1.0 + x}{1.0 + 0.25x}$	$x \in [0, 1]$
Heterogeneous-2	$f_{\mathrm{h}}(x_1, x_2) = \sin\left[21(x_1-0.9)^4\right]\cos\left[2(x_1-0.9)\right] + (x_1-0.7)/2 + 2x_2^2\sin(x_1x_2)$ $f_{\mathrm{l}}(x_1, x_2) = \frac{f_{\mathrm{h}}(x_1, x_2 - 2.0 + x_1 + x_2)}{5.0 + 0.25x_1 + 0.5x_2}$	$\{x_1, x_2\} \in [0, 1]$
Heterogeneous-3	$f_{\mathrm{h}}(x_1, x_2, x_3) = \sin\left[21(x_1-0.9)^4\right]\cos\left[2(x_1-0.9)\right] + (x_1-0.7)/2 + 2x_2^2\sin(x_1x_2) + 3x_3^3\sin(x_1x_2x_3)$ $f_{\mathrm{l}}(x_1, x_2, x_3) = \frac{f_{\mathrm{h}}(x_1, x_2, x_3) - 2.0 + x_1 + x_2 + x_3}{5.0 + 0.25x_1 + 0.5x_2 - 0.75x_3}$	$\{x_1, x_2, x_3\} \in [0, 1]$

Table A15. Benchmark test problems developed in [12].

Problem name	Mathematical formulation	Decision space
Rosenbrock	$f_{\mathrm{h}}(x_1, x_2) = (1-x_1)^2 + 100\left(x_2 - x_1^2\right)^2$ $f_{\mathrm{l}}(\mathbf{x}) = \frac{f_{\mathrm{h}}(\mathbf{x}) - 4.0 - \sum_{i=1}^{2} 0.5x_i}{3.0 + \sum_{i=1}^{2} 0.25x_i}$	$\{x_1, x_2\} \in [-2, 2]$
Runge	$f_{\mathrm{h}}(x_1, x_2, x_3) = \frac{1}{1 + x_1^2 + x_2^2 + x_3^2}$ $f_{\mathrm{l}}(\mathbf{x}) = \frac{f_{\mathrm{h}}(\mathbf{x}) - 4.0 - \sum_{i=1}^{3} 0.5x_i}{3.0 + \sum_{i=1}^{3} 0.25x_i}$	$\{x_1, x_2, x_3\} \in [-2, 2]$

Table A16. Benchmark test problems developed in [102].

Problem name	Mathematical formulation	Decision space
PARK-1	$f_{\mathrm{h}}(\mathbf{x}) = \frac{x_1}{2}\left[\sqrt{1+\left(x_2+x_3^2\right)\frac{x_4}{x_1^2}}-1\right] + \left(x_1+3x_4\right)\exp\left[1+\sin\left(x_3\right)\right]$ $f_{\mathrm{l}}(\mathbf{x}) = \left[1+\frac{\sin\left(x_1\right)}{10}\right] f_{\mathrm{h}}(\mathbf{x}) - 2x_1 + x_2^2 + x_3^2 + 0.5$	$\{x_1, x_2, x_3, x_4\} \in [0, 1)$
PARK-2	$f_{\mathrm{h}}(\mathbf{x}) = \frac{2}{3}\exp\left(x_1+x_2\right) - x_4\sin\left(x_3\right) + x_3$ $f_{\mathrm{l}}(\mathbf{x}) = 1.2 f_{\mathrm{h}}(\mathbf{x})) - 1$	$\{x_1, x_2, x_3\} \in [-2, 2]$
Paciorek with noise	$f_{\mathrm{h}}(\mathbf{x}) = \sin\left(\prod_{i=1}^{D} x_i\right)^{-1} + N\left(0, \alpha_1\right)$ $f_{\mathrm{l}}(\mathbf{x}) = f_{\mathrm{h}}(\mathbf{x}) - 9A^2\cos\left(\prod_{i=1}^{D} x_i\right)^{-1} + N\left(0, \alpha_2\right), A \in [0, 1]$	$\mathbf{x} \in [0.3, 1.0]$

Table A17. Benchmark test problems developed in [70].

Problem name	Mathematical formulation	Decision space
Forrester	$f_{\mathrm{h}}(x) = (6x-2)^2 \sin(12x-4)$ $f_{\mathrm{l}}(x) = (5.5x-2.5)^2 \sin(12x-4)$ $f_{\mathrm{l}}(x) = 0.75 f_{\mathrm{h}}(x) + 5(x-0.5) - 2$ $f_{\mathrm{l}}(x) = 0.5 f_{\mathrm{h}}(x) + 10(x-0.5) - 5$	$x \in [0, 1]$
discontinuous Forrester	$f_{\mathrm{h}}(x) = \begin{cases} (6x-2)^2 \sin(12x-4), & 0 \le x \le 0.5 \\ (6x-2)^2 \sin(12x-4) + 10, & 0.5 < x \le 1 \end{cases}$ $f_{\mathrm{l}}(x) = \begin{cases} 0.5 f_{\mathrm{h}}(x) + 10(x-0.5) - 5, & 0 \le x \le 0.5 \\ 0.5 f_{\mathrm{h}}(x) + 10(x-0.5) - 2, & 0.5 < x \le 1 \end{cases}$	$x \in [0, 1]$
Rosenbrock	$f_{\mathrm{h}}(\mathbf{x}) = \sum_{i=1}^{D-1} 100 \left(x_{i+1} - x_i^2\right)^2 + (1-x_i)^2$ $f_{\mathrm{l}}(\mathbf{x}) = \sum_{i=1}^{D-1} 50 \left(x_{i+1} - x_i^2\right)^2 + (-2-x_i)^2 - \sum_{i=1}^{D} 0.5x_i$ $f_t(\mathbf{x}) = \frac{f_{\mathrm{h}}(\mathbf{x}) - 4 - \sum_{i=1}^{D} 0.5x_i}{10 + \sum_{i=1}^{D} 0.25x_1}$	$\mathbf{x} \in [-2, 2]$
Shifted-rotated Rastrigin	$f_{\mathrm{h}}(\mathbf{z}) = \sum_{i=1}^{D} \left(z_i^2 + 1 - \cos(10\pi z_i)\right)$ $\mathbf{z} = R(\theta)\left(\mathbf{x} - \mathbf{x}^{\star}\right), R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},$ $f_{\mathrm{l}}(\mathbf{z}) = f_i(\mathbf{z}, \phi_i) = f_{\mathrm{h}}(\mathbf{z}) + e_r(\mathbf{z}, \phi_i), i = 1, 2, 3$ $e_r(\mathbf{z}, \phi) = \sum_{i=1}^{D} a(\phi) \cos^2(w(\phi) z_i + b(\phi) + \pi)$ $a(\phi) = \Theta(\phi), w(\phi) = 10\pi\Theta(\phi),$ $b(\phi) = 0.5\pi\Theta(\phi), \Theta(\phi) = 1 - 0.0001\phi$ $\phi_1 = 10000, \phi_2 = 5000, \phi_3 = 2500$	$\mathbf{x} \in [-0.1, 0.2]$

Table A18. Benchmark test problems developed in [16].

Name	Problem	Parameter Domains
Linear-A	$f_{\mathrm{h}}(x) = (6x-2)^2 \sin(12x-4))$ $f_{\mathrm{l}}(x) = \frac{1}{2} f_{\mathrm{h}}(x) + 10\left(x - \frac{1}{2}\right) + 5$	$x \in [0, 1]$
Linear-B	$f_{\mathrm{h}}(x) = 5x^2 \sin(12x)$ $f_{\mathrm{l}}(x) = 2 f_{\mathrm{h}}(x) + \left(x^3 - \frac{1}{2}\right) \sin\left(3x - \frac{1}{2}\right) + 4\cos(2x)$	$x \in [0, 1]$
NonLinear-A	$f_{\mathrm{h}}(x) = (x - \sqrt{2})\,(f_{\mathrm{h}}(x))^2$ $f_{\mathrm{l}}(x) = \sin(8\pi x)$	$x \in [0, 1]$
NonLinear-B	$f_{\mathrm{h}}(x) = x e^{f_{\mathrm{l}}(2x-.2)} - 1$ $f_{\mathrm{l}}(x) = \cos(15x)$	$x \in [0, 1]$

B.6 Visualization benchmarks in Section 6

- Fig. A5 to Fig. A8 show the four multi-fidelity problems in [99].
- Fig. A9 to Fig. A13 show the five two-dimensional multi-fidelity problems in [18].
- Fig. A14 shows the two-dimensional multi-fidelity problem in [90].
- Fig. A15 to Fig. A17 show the three two-dimensional multi-fidelity problems in [88].
- Fig. A18 shows the two-dimensional multi-fidelity problem in [12].
- Fig. A19 shows the two-dimensional multi-fidelity problem in [70].
- Fig. A20 shows the one-dimensional multi-fidelity problems in [16, 52, 88, 90].

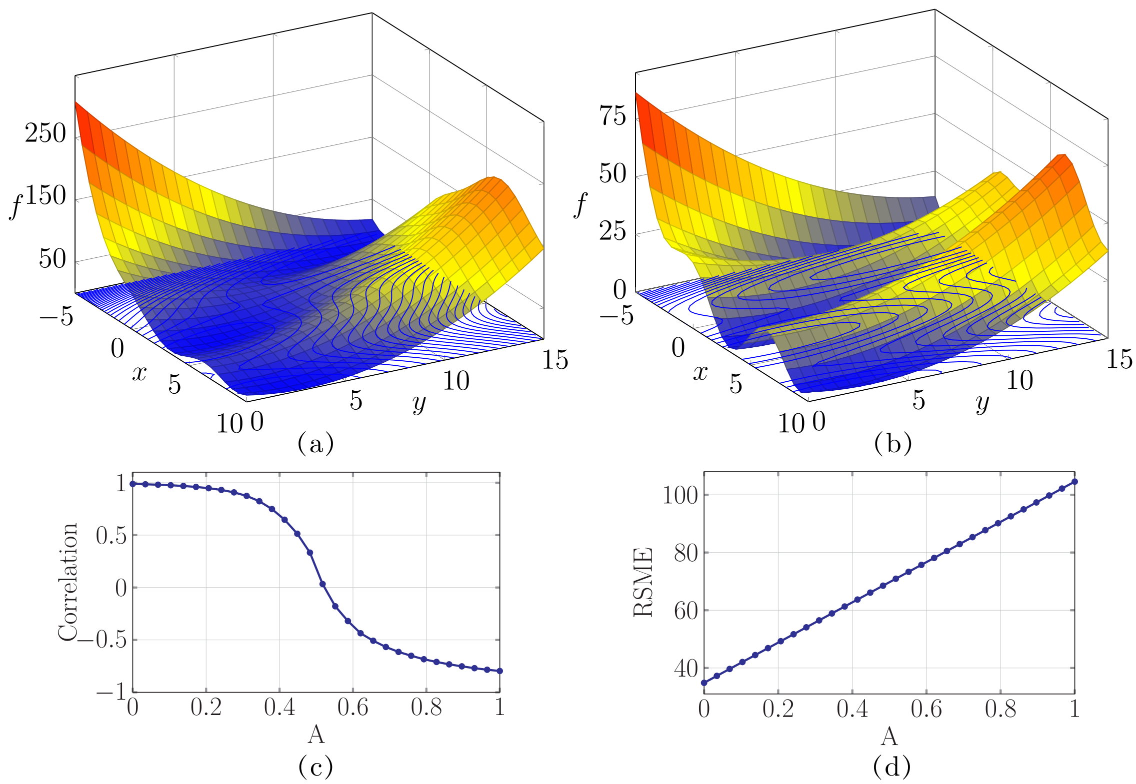


Fig. A5. With Branin representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively, with $A = 0.25$ for the LF problem; (c) and (d) show the Spearman correlation and RMSE between HF and LF problems, respectively.

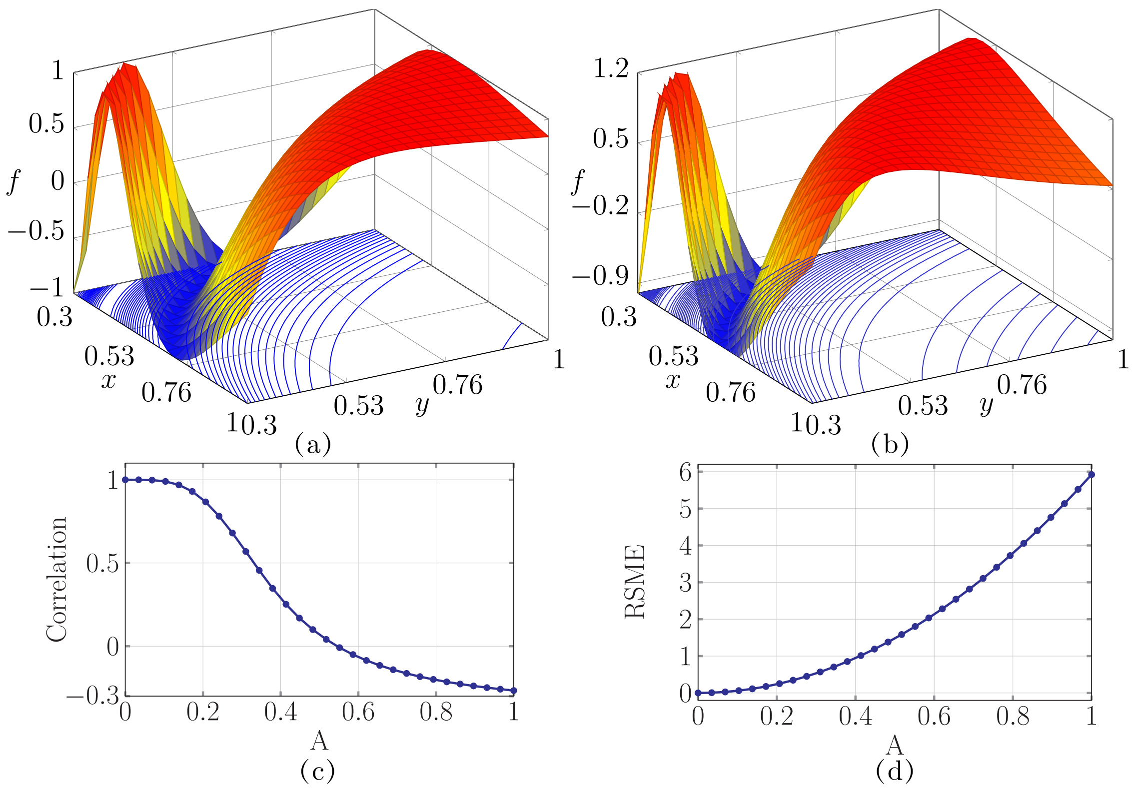


Fig. A6. With Paciorek representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively, with $A = 0.25$ for the LF problem; (c) and (d) show the Spearman correlation and RMSE between HF and LF problems, respectively.

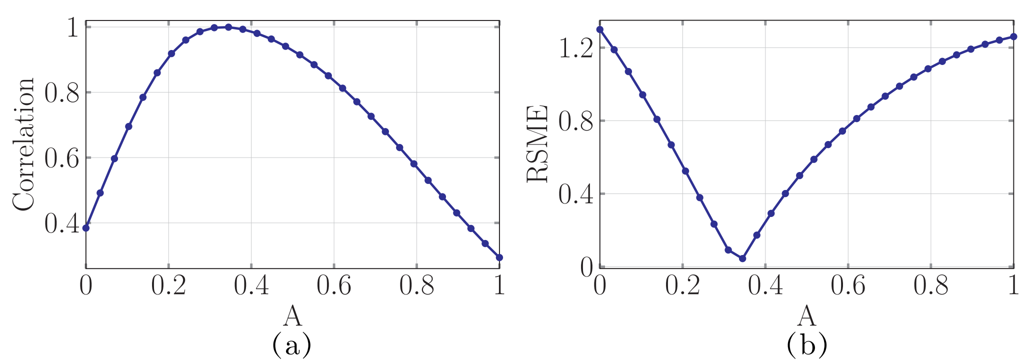


Fig. A7. With Hartmann3 representing the HF problem, the following are presented: (a) and (b) show the Spearman correlation and RMSE between HF and LF problems, respectively.

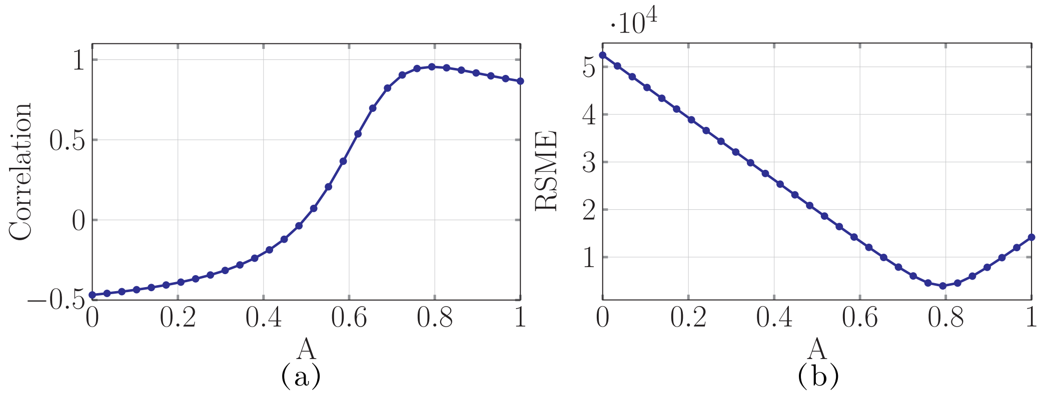


Fig. A8. With Trid representing the HF problem, the following are presented: (a) and (b) show the Spearman correlation and RMSE between HF and LF problems, respectively.

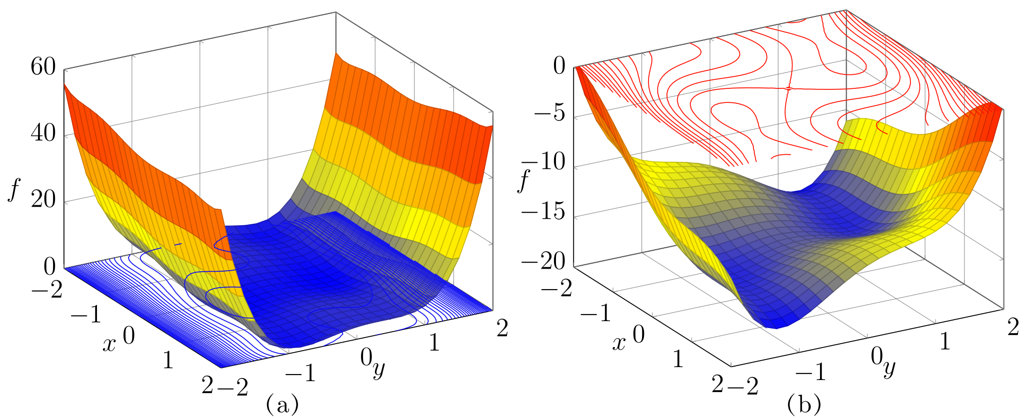


Fig. A9. With Six hump camel-back representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

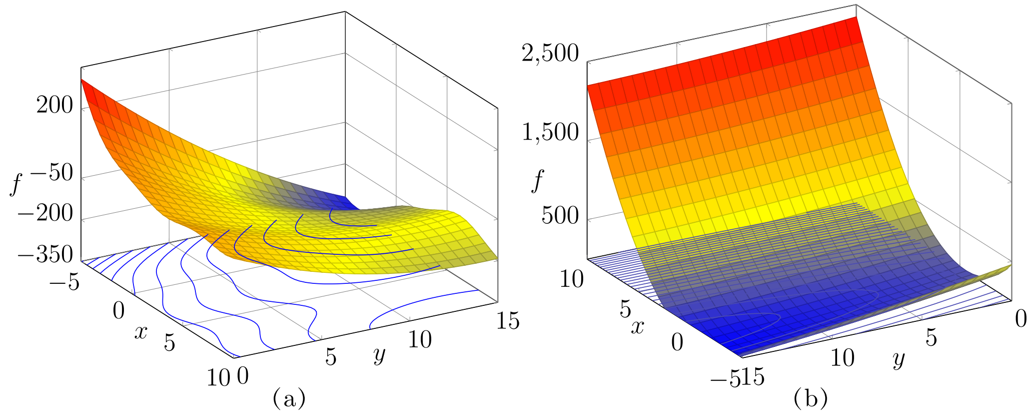


Fig. A10. With Branin representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

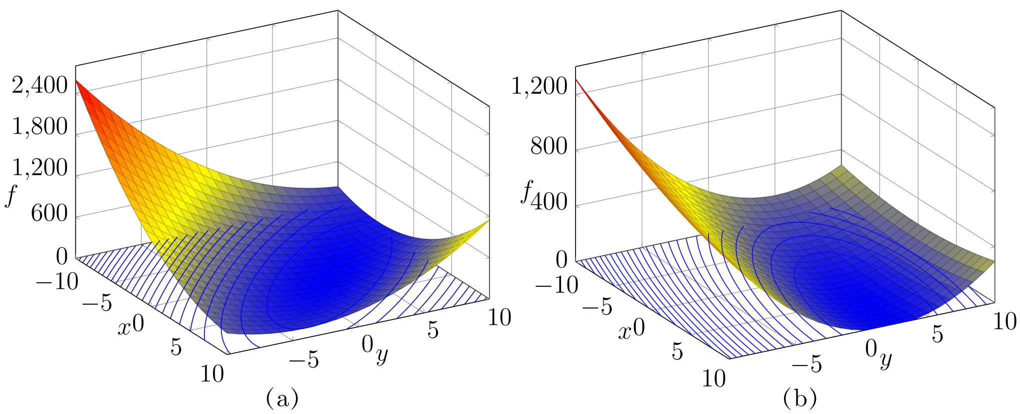


Fig. A11. With Booth representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

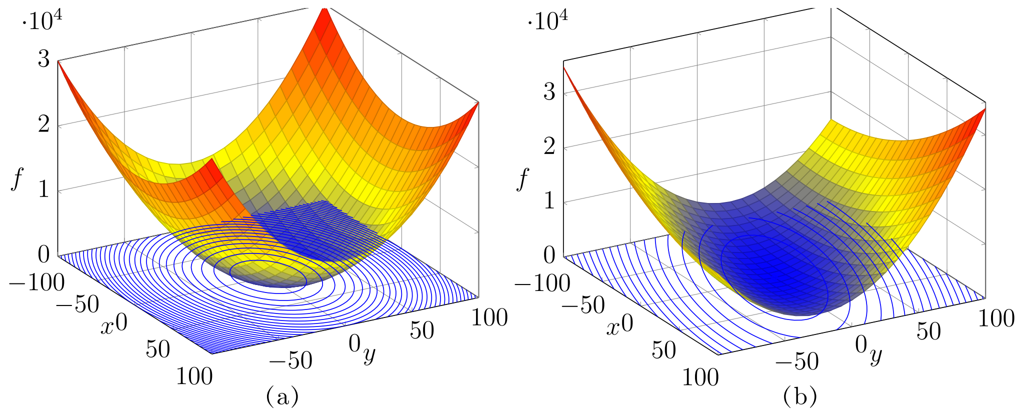


Fig. A12. With Bohachevsky representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

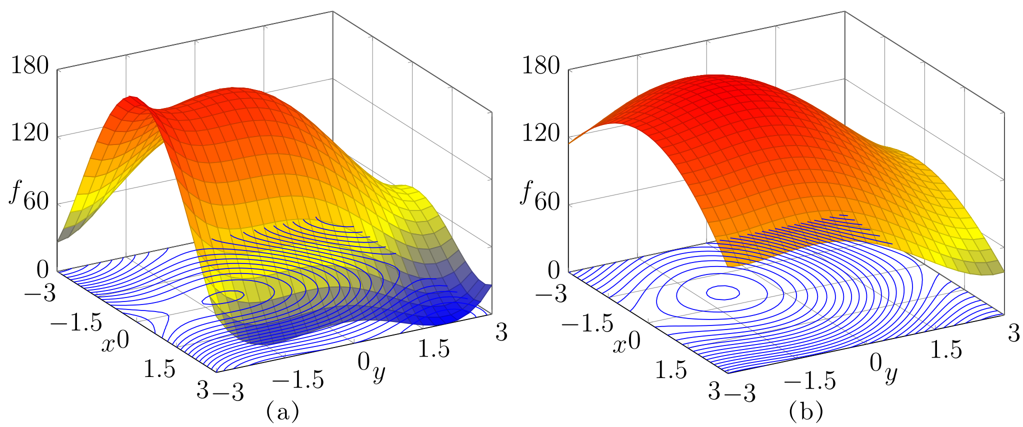


Fig. A13. With Himmelblau representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

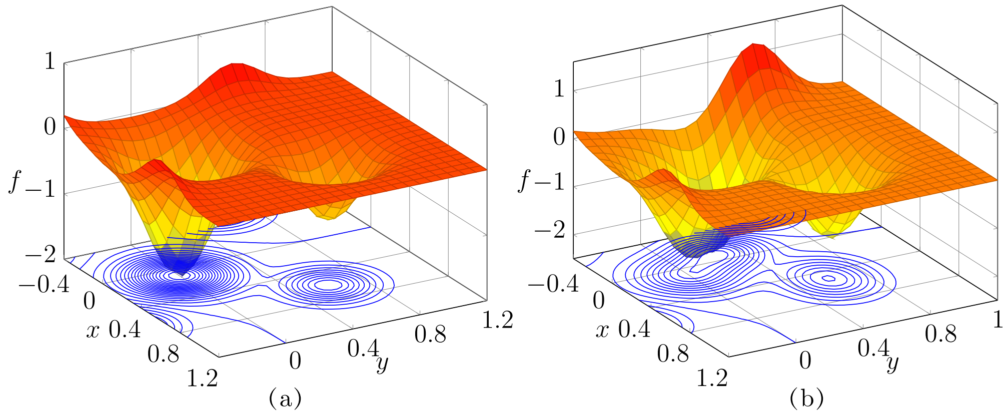


Fig. A14. With Far1 representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

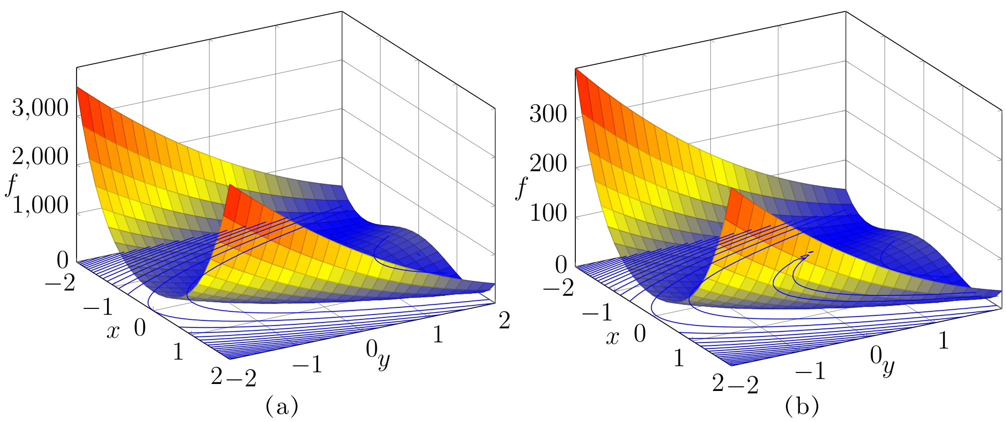


Fig. A15. With Rosenbrock representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

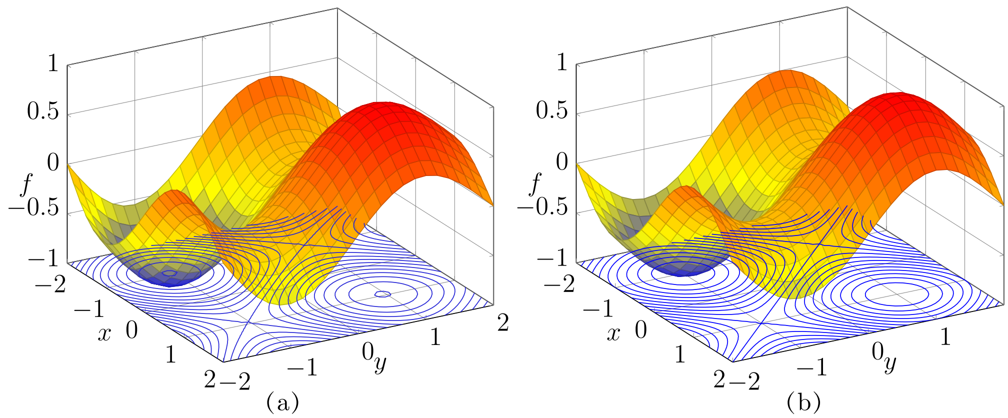


Fig. A16. With exponential-sine representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

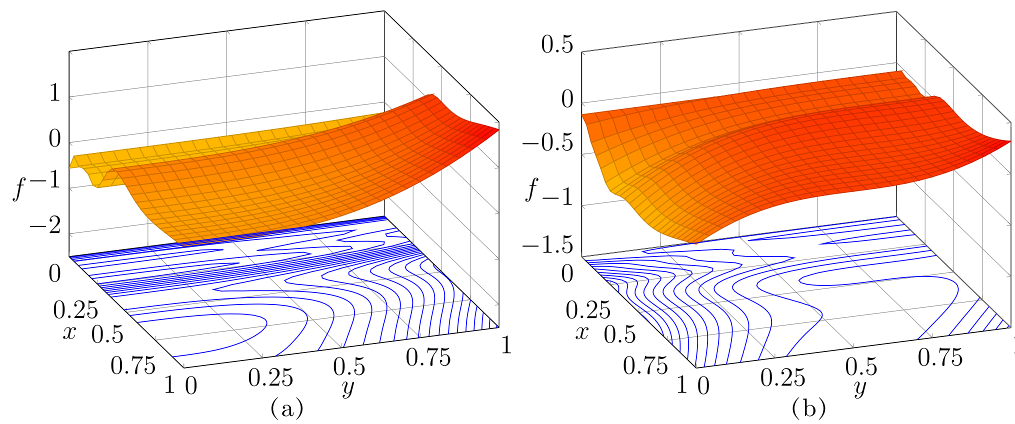


Fig. A17. With Heterogeneous−2 representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

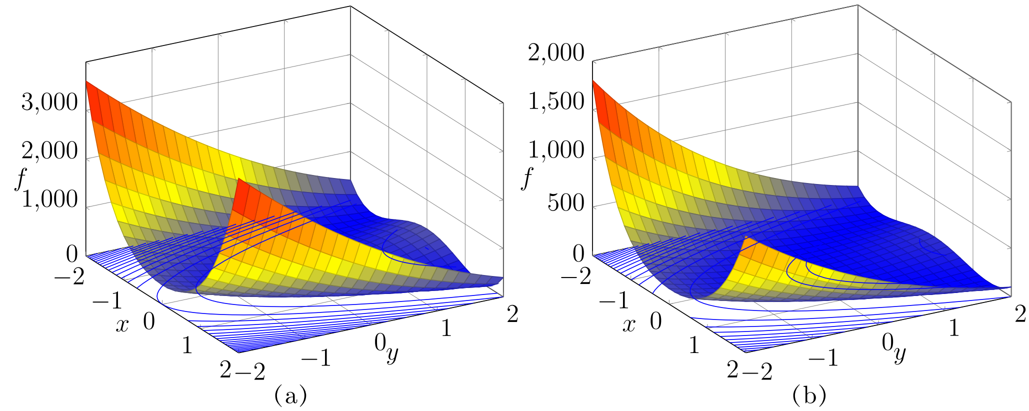


Fig. A18. With Rosenbrock representing the HF problem, the following are presented: (a) and (b) show the fitness landscape of HF and LF problems, respectively.

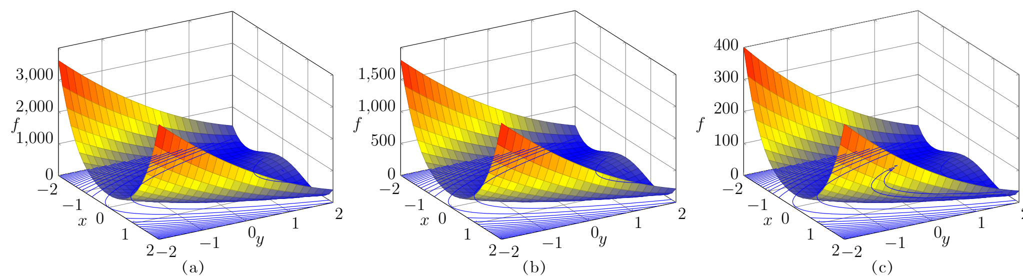


Fig. A19. With Rosenbrock representing the HF problem, the following are presented: (a), (b), and (c) show the fitness landscape of HF and, two LF problems, respectively.

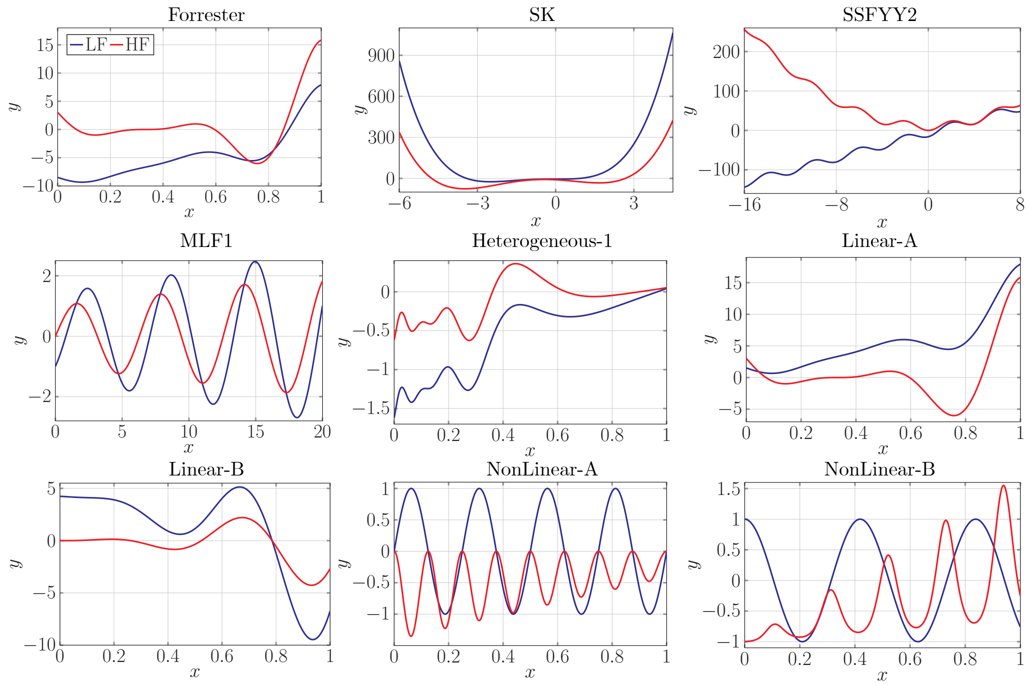


Fig. A20. This figure shows the HF and LF functions for one-dimensional benchmarks.

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