

Interactive Evolutionary Multi-Objective Optimization via Learning-to-Rank Neural Network

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APPENDIX A COUNTER EXAMPLE OF ORDINAL REGRESSION

When using Ordinal Regression, it is quite possible that even though the DM provided accurate and consistent preference information, there is no additive value function that can represent all the preference information given by the DM [1]. That is to say, the additive preference model is not flexible enough to represent the DM's preferences as discussed in [1]. In order to enable representation of the most recently given preference information, some pairwise preferences given by the DM are removed, starting from the oldest as done in [1].

We give a simple example here, there are four solutions, called a_1, a_2, a_3 and a_4 , evaluated by two objective functions g_1 and g_2 to be minimized (cost-type criteria). The objective values and scores evaluated by the DM of solutions are shown in Table I. In this example, when the DM provided the preference information in the following order: 1) $a_1 \succ a_2$; 2) $a_3 \succ a_4$. In order to obtain one value function $U(\cdot)$ representing these preferences, one has to solve the following linear programming (LP) problem [preference representing value function (PRVF)]:

$$\begin{aligned} \max \quad & \epsilon \\ \text{s.t.} \quad & u_i(g_i(a_h)) \geq u_i(g_i(a_k)) \\ & \text{if } g_i(a_h) < g_i(a_k) \quad i, h, k = 1, 2, 3, 4; h \neq k \\ & U(a_1) \geq U(a_2) + \epsilon \\ & U(a_3) \geq U(a_4) + \epsilon \\ & u_i(4) = 0 \quad i = 1, 2, 3, 4 \\ & u_1(1) + u_2(1) + u_3(1) + u_4(1) = 1 \end{aligned}$$

whose unknown variables are $u_i(g_i(a_h)), i = 1, 2$ and $h = 1, 2, 3, 4$, and ϵ . And $U(a_h) = u_1(g_1(a_h)) + u_2(g_2(a_h))$ for $h = 1, 2, 3, 4$. According to the additive preference model, $U(a_i) > U(a_j)$ indicates a_i is preferred to a_j . While $U(a_i) = U(a_j)$ indicates a_i is indifferent with a_j . If the value of ϵ given by the solution of PRVF is positive there exist value functions $U(\cdot)$ representing the DM's preferences. However, for above LP problem, we obtain $\max \epsilon = 0$. To get $\max \epsilon > 0$, we have to remove the pairwise preference comparisons: 1) $a_1 \succ a_2$. Then we can obtain $\max \epsilon = 1$ for the new LP problem. And the obtained value function only represents the remaining pairwise comparisons: 2) $a_3 \succ a_4$. The obtained value function $U(\cdot)$ is presented in Table II while Table III gives the values assigned by the value function $U(\cdot)$ to solutions a_1, a_2, a_3 and a_4 .

As shown in Table III, the preferences relation given by the value function representing the remaining pairwise compar-

isons is $a_2 \succ a_3 \sim a_1 \sim a_4$ which seems to violate the first pairwise preference comparison: 1) $a_1 \succ a_2$. The reason for this is that the additive preference model is not flexible enough to represent the complete preferences of the DM (unable to obtain $\max \epsilon > 0$) thus some preference information is removed. As a result, the obtained value function is not able to represent all the preferences provided by the DM.

TABLE I: Performance matrix of four solutions.

	g_1	g_2
a_1	4	2
a_2	1	3
a_3	2	4
a_4	3	1

TABLE II: Value function representing the remaining pairwise comparisons.

$g_i(a)$	1	2	3	4
$u_1(\cdot)$	1	1	0	0
$u_2(\cdot)$	0	0	0	0

TABLE III: Values assigned to solutions by the value function representing the remaining pairwise comparisons.

	$u_1(g_1(a))$	$u_2(g_2(a))$	$U(a)$
a_1	0	0	0
a_2	1	0	1
a_3	1	0	1
a_4	0	0	0

We have two arguments here: 1) As a preference model, which interacts with real human, the cost of obtaining preference information is quite heavy. As a result, a preference model that cannot make full use of precious preference information from DM is somewhat defective. 2) It cannot be guaranteed to correctly represent preferences of the DM because of removing part of preference information.

APPENDIX B IMPACT OF INCONSISTENCY IN THE ELICITED PREFERENCE

In the previous experiments, the DM's preference information is assumed to be deterministic. However, it is not uncommon that practical decision-making and preference elicitation can be largely inconsistent. In other words, there exist certain level of noises to which the pairwise comparison results can be conflicting w.r.t. the ground truth. In this subsection, we plan to investigate the impact brought by the inconsistencies in the preference elicitation. To this end, our basic idea is to aggregate a random error into the pairwise comparison. Specifically, given a pair of selected solutions $\langle \mathbf{x}^i, \mathbf{x}^j \rangle$, we define the probability of flipping the comparison result as:

$$\mathbb{P}(\mathbf{x}^i, \mathbf{x}^j) = \exp(-\kappa \cdot \delta), \quad (1)$$

where κ determines the DM's ability to correctly express her preference information and $\delta = |\psi(\mathbf{x}^j) - \psi(\mathbf{x}^i)|$ measures the ‘similarity’ of a given pair of solutions. From the illustrative example shown in Fig. 1, we can infer that a smaller κ leads to a larger chance of eliciting a wrong preference information, i.e., the DM picks up an inferior solution as the winner from the pairwise comparison. Moreover, a larger δ indicates a more obvious difference between the given solution pair. In principle, a rationale DM is assumed to be less likely to make a wrong decision if the candidates are obviously different, and vice versa. In our experiment, we investigate different settings of $\kappa \in \{1, 10, 30, 50, 100, 200\}$. For proof-of-concept purposes, DTLZ2 and mDTLZ2 are chosen as the benchmark test problems and the parameters associated with our algorithm instances are kept the same as introduced in Section IV-B.

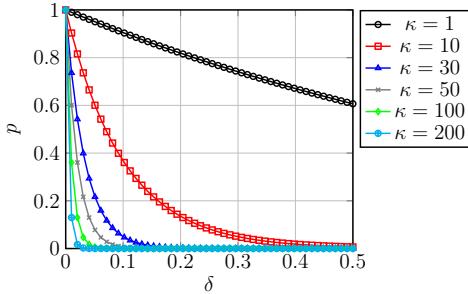


Fig. 1: An illustrative example of the impact of the noise level, i.e., different settings κ , on $\mathbb{P}(\mathbf{x}^i, \mathbf{x}^j)$.

From the comparison results shown in Fig. 2, we can see the performance of I-MOEA/D/LTR is influenced by the induced noise in the preference elicitation. Its approximation error w.r.t. the SOI is large when involving a large noise in the preference elicitation (i.e., having a small κ); while the approximation accuracy is very close to the noiseless case (denoted as the ‘oracle’ in Fig. 2) with the increase of κ (i.e., the noise in the preference elicitation becomes trivial). These observations indicate that I-MOEA/D/LTR has certain level of robustness w.r.t. mild inconsistencies during the preference elicitation. On the other hand, it is interesting to note that the induced noise in the preference elicitation does not pose significant impacts to the performance

of I-NSGA-II/LTR and I-R2-IBEA/LTR. In particular, since I-R2-IBEA/LTR shares the same preference elicitation method with I-MOEA/D/LTR, we infer the differences of robustness w.r.t. the noise are derived from the environmental selection in MOEA/D and R2-IBEA. This can be explained as the evolutionary search process is less dependent on the preference information for both I-NSGA-II/LTR and I-R2-IBEA/LTR. Therefore, both of them are able to drive the population towards the PF even with a large noise.

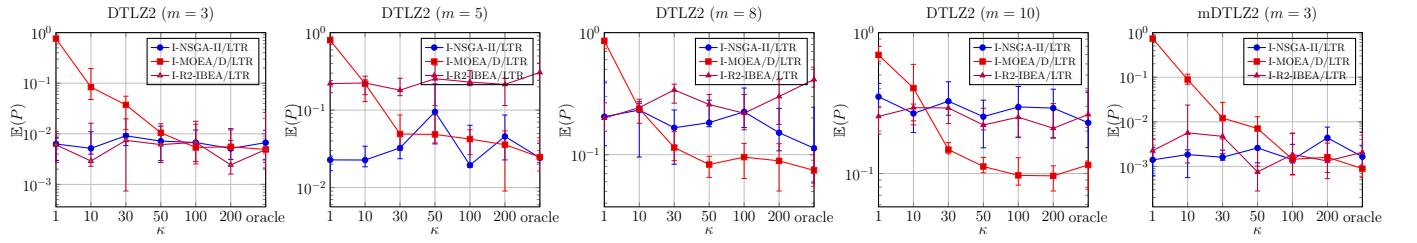


Fig. 2: Comparison results of the variance of the approximation errors with difference κ settings.

APPENDIX C
SOLVING MULTI-OBJECTIVE ROBOT CONTROL PROBLEM
VIA EVOLUTIONARY ALGORITHM

Mujoco¹ is a physics engine that allows users to model multi-joint robots and control their behavior. It has been widely applied in various domains including but not limited to animation, graphics, and robotics. Here we consider the Mujoco task Swimmer-v3 as the example in our empirical study. More specifically, Swimmer-v3 is a two-joint robot with an 8-dimensional vector $\mathbf{s} = (s_1, \dots, s_8)^\top$ defined by Mujoco to describe its current state. As a user, you are required to provide a 2-dimensional action vector $\mathbf{a} = (a_1, a_2)^\top$ according to the current state of the robot to control its behavior. To this end, we need to design a decision matrix (A, B) , as known as a policy $\pi(A, B)$, that defines the mapping from the current state to the corresponding action:

$$\pi(A, B) : a = As + B, \quad (2)$$

where $A \in \mathbb{R}^{2 \times 8}$ and $B \in \mathbb{R}^{2 \times 1}$. Based on the policy, the robot can interact with the environment and obtain its new state along with the rewards that constitute the objective functions as follows:

$$\begin{aligned} & \text{maximize } \mathbf{F}(\pi) = (f_1(s_t, \pi), f_2(s_t, \pi), f_3(s_t, \pi)) \\ & \text{subject to } \pi : \mathcal{S} \rightarrow \mathcal{A} \end{aligned} \quad (3)$$

where $f_1(s_t, \pi) = \sum_{t=0}^T r_1(s_t, \pi)$, $f_2(s_t, \pi) = \sum_{t=0}^T r_2(s_t, \pi)$, and $f_3(s_t, \pi) = \sum_{t=0}^T r_3(s_t, \pi)$. $r_1(s_t, \pi)$, $r_2(s_t, \pi)$, and $r_3(s_t, \pi)$ are rewards as mentioned above and T is the number of simulation steps (T is set to 500 in our simulations). More specifically, in each time step $t \in [0, T]$, $r_1(s_t, \pi)$ and $r_2(s_t, \pi)$ returns the current x-axis and y-axis velocity of the robot respectively, and $r_3(s_t, \pi)$ represents the current energy efficiency. To use an evolutionary algorithm to solve this three-objective optimization problem, (A, B) is treated as decision variables and the accumulated rewards as objective functions.

From the box plots of $\mathbb{E}(\mathcal{P})$ shown in Fig. 3 and the distribution of non-dominated solutions shown in Fig. 4, we can see that our proposed I-NSGA-II/LTR is the best algorithm to guide the MORL towards the DM preferred policies. It is interesting to note that NEMO-0 is the second best algorithm and is way better than the other three peer algorithms. This result supports on the observation in Section V-B and it also demonstrates that the ordinal regression is a competitive preference learning method as reported in Section V-C. To

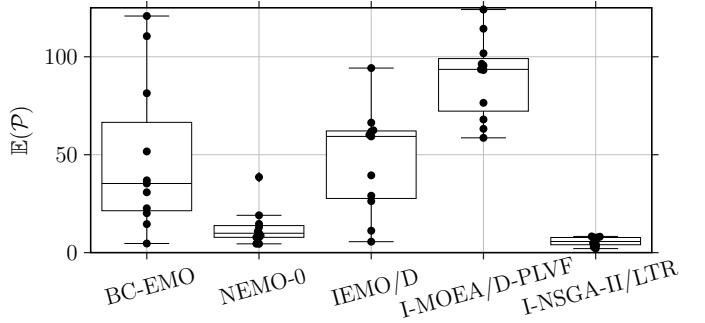


Fig. 3: Box plots of $\mathbb{E}(\mathcal{P})$ achieved by different algorithms.

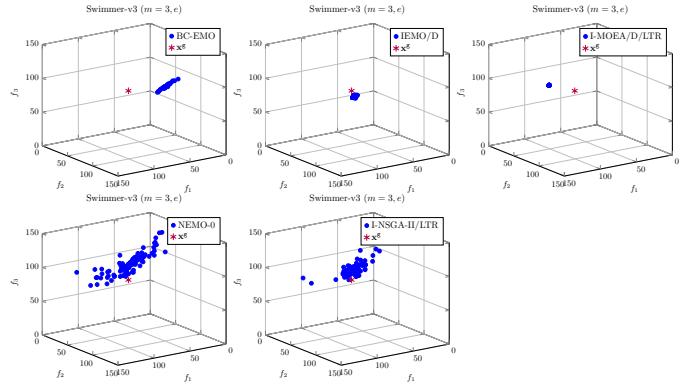


Fig. 4: Distribution of non-dominated solutions found by different algorithms with regard to the solution of interest \mathbf{x}^g .

have a better impression of the comparison of policies obtained by different algorithms, we present some video snapshots in our supplemental website.

¹<https://mujoco.readthedocs.io/en/latest/overview.html>

APPENDIX D
EXPERIMENTAL SETTINGS

TABLE IV: The number of function evaluations (FEs) and population size settings.

Test problem	N	$FEs \div N$
DTLZ1	32($m - 1$)	500 + 50($m - 2$)
DTLZ2		200 + 50($m - 2$)
DTLZ3		1000 + 50($m - 2$)
DTLZ4		200 + 50($m - 2$)
DTLZ5		200 + 50($m - 2$)
DTLZ6		200 + 50($m - 2$)
$DTLZ1^{-1}$		500 + 50($m - 2$)
$DTLZ2^{-1}$		200 + 50($m - 2$)
$DTLZ3^{-1}$		1000 + 50($m - 2$)
$DTLZ4^{-1}$		200 + 50($m - 2$)
mDTLZ1	300	
mDTLZ2		
mDTLZ3		
mDTLZ4		
WFG3	32($m - 1$)	1000 + 50($m - 2$)

TABLE V: The choice of w^* and x^* ($m=3$).

m	w^*	test problem	x^*
3	1,1,1	DTLZ1	(0.1667,0.1667,0.1667)
		DTLZ2-DTLZ6	(0.5774,0.5774,0.5774)
		$DTLZ1^{-1}$	(-183.7167,-183.7167,-183.7167)
		DTLZ2 & 4 $^{-1}$	(-2.0207,-2.0207,-2.0207)
		DTLZ3 $^{-1}$	(-1272.1913,-1272.1913,-1272.1913)
		mDTLZ1	(0.3333,0.3333,0.3333)
		mDTLZ2-mDTLZ4	(0.4226,0.4226,0.4226)
		WFG3	(1.4242,1.4242,1.4242)
3	1,2,1	DTLZ1	(0.125,0.25,0.125)
		DTLZ2-DTLZ4	(0.4082,0.8165,0.4082)
		DTLZ5 & DTLZ6	(0.5774,0.5774,0.5774)
		$DTLZ1^{-1}$	(-137.787,-275.574,-137.787)
		DTLZ2 & 4 $^{-1}$	(-1.4289,-2.8577,-1.4289)
		DTLZ3 $^{-1}$	(899.611,-1799.22,-899.611)
		mDTLZ1	(0.25,0.5,0.25)
		mDTLZ2-mDTLZ4	(0.3333,0.6667,0.3333)
		WFG3	(0.8590,1.7181,0.8590)

TABLE VI: The choice of w^* and x^* ($m=5$).

m	w^*	test problem	x^*
5	1,1,1,1,1	DTLZ1	(0.1,0.1,0.1,0.1,0.1)
		DTLZ2-DTLZ4	(0.4472,0.4472,0.4472,0.4472,0.4472)
		DTLZ5 & DTLZ6	(0.2887,0.2887,0.4082,0.5774,0.5774)
		$DTLZ1^{-1}$	(-110.228,-110.228,-110.228,-110.228,-110.228)
		DTLZ2 & 4 $^{-1}$	(-1.5653,-1.5653,-1.5653,-1.5653,-1.5653)
		DTLZ3 $^{-1}$	(-985.321,-985.368,-985.42,-985.318,-985.421)
		WFG3	(1.6359,1.6359,1.6359,1.6359,1.6359)
		DTLZ1	(0.07143,0.14286,0.07143,0.14286,0.07143)
5	1,2,1,2,1	DTLZ2-DTLZ4	(0.3015,0.603,0.3015,0.603,0.3015)
		DTLZ5 & DTLZ6	(0.4114,0.3323,0.4232,0.8464,0.4232)
		$DTLZ1^{-1}$	(-78.7336,-157.467,-78.734,-157.467,-78.7327)
		DTLZ2 & 4 $^{-1}$	(-1.0552,-2.1105,-1.0552,-2.1105,-1.0552)
		DTLZ3 $^{-1}$	(-663.776,-1328.69,-664.29,-1328.45,-664.183)
		WFG3	(1.2546,2.5091,1.2546,2.5091,1.2546)

TABLE VII: The choice of w^* and x^* ($m=8$).

m	w*	test problem	x*
8	1,1,1,1,1,1,1,1	DTLZ1	(0.0625,0.0625,0.0625,0.0625,0.0625,0.0625,0.0625,0.0625)
		DTLZ2-DTLZ4	(0.3536,0.3536,0.3536,0.3536,0.3536,0.3536,0.3536,0.3536)
		DTLZ5 & DTLZ6	(0.1021,0.1021,0.1443,0.2041,0.2887,0.4082,0.5774,0.5774)
		DTLZ1 $^{-1}$	(-64.0585,-64.2003,-63.2574,-64.1564,-63.9946,-64.0442,-62.6169,-64.2008)
		DTLZ2 & 4 $^{-1}$	(-1.2212,-1.2207,-1.2074,-1.2043,-1.2237,-1.22012,-1.2148,-1.2240)
		DTLZ3 $^{-1}$	(-743.132,-743.844,-744.054,-743.965,-732.654,-744.056,-743.765,-744.016)
	1,2,1,2,1,2,1,1	WFG3	(1.6264,1.62645,1.6265,1.6265,1.6264,1.6265,1.6265,1.6265)
		DTLZ1	(0.0455,0.0909,0.0455,0.0909,0.0455,0.0909,0.0455,0.0455)
		DTLZ2-DTLZ4	(0.2425,0.4851,0.2424,0.4851,0.2425,0.4851,0.2425,0.2426)
		DTLZ5 & DTLZ6	(0.1021,0.1021,0.1443,0.2041,0.2887,0.4082,0.5774,0.5774)
		DTLZ1 $^{-1}$	(-45.9371,-91.4772,-45.9887,-91.8348,-45.8489,-91.7987,-45.9261,-45.7479)
		DTLZ2 & 4 $^{-1}$	(-0.8394,-1.6818,-0.8257,-1.6797,-0.8258,-1.6745,-0.8422,-0.8423)
		DTLZ3 $^{-1}$	(-516.119,-1046.09,-520.381,-1047.03,-523.381,-1046.92,-522.189,-523.631)
		WFG3	(1.2644,2.5287,1.2644,2.5288,1.2644,2.5284,1.2644,1.2643)

TABLE VIII: The choice of w^* and x^* ($m=10$).

m	w*	test problem	x*
10	1,1,1,1,1,1,1,1,1,1	DTLZ1	(0.05,0.05,0.05,0.05,0.05,0.05,0.05,0.05,0.05,0.05)
		DTLZ2-DTLZ4	(0.3166,0.315,0.3164,0.3171,0.3173,0.312,0.3178,0.3164,0.316,0.3177)
		DTLZ5 & DTLZ6	(0.051,0.051,0.0722,0.1021,0.1443,0.2041,0.2887,0.4082,0.5773,0.5773)
		DTLZ1 $^{-1}$	(-48.5573,-48.5205,-49.4967,-49.4357,-46.3898,-49.0606,-48.8952,-49.414,-49.4251,-49.4761)
		DTLZ2 & 4 $^{-1}$	(-0.9337,-0.8979,-0.9216,-0.8211,-0.9356,-0.9373,-0.9257,-0.9414,-0.9347,-0.9491)
		DTLZ3 $^{-1}$	(-645.403,-649.683,-648.446,-648.985,-641.991,-648.539,-648.06,-649.631,-649.572,-648.334)
	1,2,1,3,1,2,1,1,2,1	WFG3	(1.6087,1.6081,1.6081,1.6087,1.6087,1.6087,1.6086,1.6086,1.6086,1.6087,1.6082)
		DTLZ1	(-32.3584,-65.5407,-33.0096,-98.4723,-32.8545,-65.8701,-33.0303,-32.7572,-66.0775,-32.9148)
		DTLZ2-DTLZ4	(0.1906,0.385,0.1866,0.5786,0.1928,0.3854,0.1928,0.1924,0.3857,0.1928)
		DTLZ5 & DTLZ6	(0.0774,0.0699,0.112,0.1383,0.1854,0.259,0.3684,0.4232,0.8464,0.4232)
		DTLZ1 $^{-1}$	(-45.9371,-91.4772,-45.9887,-91.8348,-45.8489,-91.7987,-45.9261,-45.7479)
		DTLZ2 & 4 $^{-1}$	(-0.6411,-1.2701,-0.6333,-1.9343,-0.6444,-1.2888,-0.6339,-0.479,-1.294,-0.6429)
		DTLZ3 $^{-1}$	(-394.669,-788.407,-392.618,-1182.62,-395.077,-790.088,-395.058,-395.159,-790.296,-394.875)
		WFG3	(1.1556,2.3112,1.1556,3.4667,1.1556,2.3112,1.1555,1.1556,2.3112,1.1555)

APPENDIX E
COMPARISON RESULTS OF $\mathbb{E}(\mathcal{P})$ VALUES

TABLE IX: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on DTLZ1 problem.

		DTLZ1		
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.0057(4.03e-03)	0.0387(4.55e-02) [†]	0.0153(3.67e-02) [†]
	<i>b</i>	0.0051(5.71e-03)	0.0366(2.26e-02) [†]	0.0499(8.13e-02) [†]
5	<i>e</i>	0.0167(1.69e-02)	0.0202(1.75e-02)	0.0923(6.09e-02) [†]
	<i>b</i>	0.0107(4.97e-03)	0.0242(1.06e-02) [†]	0.0823(5.85e-02) [†]
8	<i>e</i>	0.0696(1.97e-02) [†]	0.0207(9.43e-03)	0.0496(1.87e-02) [†]
	<i>b</i>	0.0301(1.87e-02)	0.0278(1.59e-02)	0.0661(2.73e-02) [†]
10	<i>e</i>	0.0446(5.28e-02) [†]	0.0194(1.27e-02)	0.0316(1.19e-02) [†]
	<i>b</i>	0.0512(3.23e-02) [†]	0.0353(1.47e-02)	0.0800(8.94e-03) [†]

TABLE X: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on DTLZ2 problem.

		DTLZ2		
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.0067(8.61e-03)	0.0049(4.38e-03)	0.0049(2.44e-02)
	<i>b</i>	0.0164(1.81e-02)	0.0564(2.15e-02) [†]	0.0834(5.08e-02) [†]
5	<i>e</i>	0.0242(1.64e-02)	0.0248(2.77e-02)	0.3060(1.62e-01) [†]
	<i>b</i>	0.0628(6.71e-02)	0.0823(4.79e-02)	0.1776(7.50e-02) [†]
8	<i>e</i>	0.1130(1.86e-01)	0.0744(5.34e-02)	0.4134(1.64e-01) [†]
	<i>b</i>	0.1572(1.32e-01) [†]	0.0701(2.87e-02)	0.3951(1.40e-01) [†]
10	<i>e</i>	0.2291(2.23e-01) [†]	0.1151(4.54e-02)	0.2627(1.74e-01) [†]
	<i>b</i>	0.3292(1.74e-01) [†]	0.1661(7.11e-02)	0.4979(7.83e-02) [†]

TABLE XI: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on DTLZ3 problem.

		DTLZ3		
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.0289(8.12e-02)	0.3088(5.55e-01) [†]	0.5708(1.76e-01) [†]
	<i>b</i>	0.0200(1.43e-02)	0.1178(2.14e-01) [†]	0.3878(1.95e-01) [†]
5	<i>e</i>	0.1631(2.48e-01) [†]	0.0707(4.05e-02)	0.6280(1.12e-01) [†]
	<i>b</i>	0.0481(4.34e-02)	0.0950(8.81e-02) [†]	0.6126(1.63e-01) [†]
8	<i>e</i>	0.2138(3.25e-01) [†]	0.1017(3.73e-02)	0.6222(1.56e-01) [†]
	<i>b</i>	0.3114(2.92e-01) [†]	0.1478(1.19e-01)	0.6401(2.36e-01) [†]
10	<i>e</i>	0.4423(2.84e-01) [†]	0.1225(5.97e-02)	0.3985(1.04e-01) [†]
	<i>b</i>	0.4165(4.29e-01) [†]	0.2522(1.61e-01)	0.5879(2.11e-01) [†]

TABLE XII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on DTLZ4 problem.

		DTLZ4		
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.0389(5.83e-01)	0.0153(6.00e-01)	0.3853(7.75e-01)
	<i>b</i>	0.4176(8.95e-01)	0.0884(5.02e-01)	0.4174(7.81e-01)
5	<i>e</i>	0.4602(3.61e-01)	0.4597(2.89e-01)	0.4871(1.44e-01)
	<i>b</i>	0.4377(2.28e-01)	0.3666(4.21e-01)	0.6520(1.52e-01)
8	<i>e</i>	0.7668(1.44e-01) [†]	0.5191(2.84e-01)	0.4102(1.86e-01)
	<i>b</i>	0.7383(2.01e-01) [†]	0.3946(3.22e-01)	0.4966(1.74e-01)
10	<i>e</i>	0.7773(9.23e-02) [†]	0.4628(2.44e-01)	0.5656(1.30e-01) [†]
	<i>b</i>	0.7478(1.42e-01) [†]	0.4940(2.37e-01)	0.7387(1.29e-01) [†]

TABLE XIII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on DTLZ5 problem.

DTLZ5				
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.0007(2.64e-03)	0.0006(5.90e-03)	0.0002(3.67e-04)
	<i>b</i>	0.0005(3.16e-03) [†]	0.0009(1.83e-03) [†]	0.0002(3.17e-04)
5	<i>e</i>	0.0031(5.80e-03) [†]	0.0006(4.60e-03)	0.0001(2.95e-04)
	<i>b</i>	0.0933(9.13e-02)	0.2282(8.16e-03) [†]	0.2272(1.22e-02) [†]
8	<i>e</i>	0.0110(9.83e-03) [†]	0.0046(9.72e-03)	0.0104(2.42e-02) [†]
	<i>b</i>	0.0043(5.26e-03)	0.0030(9.52e-03)	0.0043(1.66e-02)
10	<i>e</i>	0.0027(5.65e-03)	0.0080(2.20e-02) [†]	0.0083(1.63e-02) [†]
	<i>b</i>	0.1102(1.48e-01)	0.2231(1.31e-01)	0.1670(7.95e-02)

TABLE XIV: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on DTLZ6 problem.

DTLZ6				
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.0901(3.70e-02)	0.1361(5.49e-02) [†]	0.1008(3.10e-02)
	<i>b</i>	0.1304(4.81e-02)	0.1381(5.10e-02)	0.1570(4.54e-02) [†]
5	<i>e</i>	0.2326(2.50e-01)	0.1576(3.65e-02)	0.1584(5.59e-02)
	<i>b</i>	0.1084(1.76e-02)	0.1086(5.86e-02)	0.2188(1.22e-01)
8	<i>e</i>	10.0651(2.59e+00) [†]	0.1581(5.70e-02)	0.1738(1.26e-01)
	<i>b</i>	1.1651(1.74e+00) [†]	0.1683(9.88e-02)	0.1998(1.72e-01) [†]
10	<i>e</i>	10.1961(2.24e-01) [†]	0.1629(8.90e-02)	0.1989(1.13e-01)
	<i>b</i>	9.9811(3.25e+00) [†]	0.1059(8.28e-02)	0.1262(3.37e-02)

TABLE XV: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on minus-DTLZ1 problem.

DTLZ1 ⁻¹				
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	2.1997(1.84e+00)	2.9519(7.65e+00)	0.2992(2.41e+00)
	<i>b</i>	3.2531(2.30e+00)	15.0412(1.21e+01) [†]	0.1729(8.72e+00)
5	<i>e</i>	13.2100(2.66e+01)	48.8735(7.02e+01) [†]	173.9680(5.93e+01) [†]
	<i>b</i>	9.1162(2.81e+00)	118.9956(4.36e+01) [†]	182.2714(6.08e+01) [†]
8	<i>e</i>	19.0630(9.78e+00)	102.3462(3.44e+01) [†]	190.4664(4.47e+01) [†]
	<i>b</i>	17.7467(6.88e+00)	140.7540(2.86e+01) [†]	198.2268(3.79e+01) [†]
10	<i>e</i>	27.3027(7.12e+00)	119.7411(3.25e+01) [†]	202.3099(2.60e+01) [†]
	<i>b</i>	26.9898(8.29e+00)	168.6769(8.81e+01) [†]	216.8463(2.18e+01) [†]

TABLE XVI: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on minus-DTLZ2 problem.

DTLZ2 ⁻¹				
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	8.8459(8.01e+00)	96.8799(1.74e+02) [†]	152.4524(1.43e+02) [†]
	<i>b</i>	23.2576(2.24e+01)	164.6484(9.98e+01) [†]	115.5498(1.19e+02) [†]
5	<i>e</i>	43.5592(2.00e+01)	231.1682(1.71e+02) [†]	177.7289(1.54e+02) [†]
	<i>b</i>	79.4876(4.23e+01)	555.8218(3.35e+02) [†]	556.8309(9.89e+01) [†]
8	<i>e</i>	350.9760(2.47e+02)	338.4022(1.89e+02)	404.8937(5.85e+01)
	<i>b</i>	222.3641(1.23e+02)	568.3700(2.48e+02) [†]	820.6048(1.14e+02) [†]
10	<i>e</i>	325.0416(1.63e+02)	438.6370(1.91e+02) [†]	731.1537(1.85e+02) [†]
	<i>b</i>	310.8917(7.92e+01)	787.1734(2.82e+02) [†]	1013.6206(9.36e+01) [†]

TABLE XVII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on minus-DTLZ3 problem.

TABLE XVIII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on minus-DTLZ4 problem.

DTLZ4 ⁻¹						
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR		
3	<i>e</i>	0.0684(6.51e-02) [†]	0.0263(2.54e-02)	0.0094(3.12e-02)		
	<i>b</i>	0.0701(9.53e-02) [†]	0.1598(1.11e-01) [†]	0.0161(2.55e-02)		
5	<i>e</i>	0.1853(1.33e-01) [†]	0.1093(1.60e-01) [†]	0.0515(8.78e-02)		
	<i>b</i>	0.2307(1.84e-01)	1.1260(4.63e-01) [†]	0.4707(1.95e-01) [†]		
8	<i>e</i>	0.5600(1.75e-01) [†]	0.7317(5.13e-01) [†]	0.3530(1.56e-01)		
	<i>b</i>	0.6707(2.62e-01)	1.2443(3.57e-01) [†]	0.9233(2.31e-01) [†]		
10	<i>e</i>	0.9778(4.36e-01)	0.9653(3.31e-01)	0.8794(1.87e-01)		
	<i>b</i>	0.9801(2.59e-01)	1.3484(3.79e-01) [†]	1.5077(1.88e-01) [†]		

TABLE XIX: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on mDTLZ1 problem.

mDTLZ1						
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR		
3	<i>e</i>	0.0046(5.95e-03)	0.0308(1.29e-01) [†]	0.2018(1.33e-01) [†]		
	<i>b</i>	0.0008(2.16e-03)	0.0075(5.35e-02) [†]	0.2050(2.14e-01) [†]		

TABLE XX: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on mDTLZ2 problem.

mDTLZ2						
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR		
3	<i>e</i>	0.0016(1.81e-03) [†]	0.0009(1.12e-03)	0.0021(4.80e-03) [†]		
	<i>b</i>	0.0026(3.01e-03)	0.0018(1.15e-03)	0.0016(9.78e-03)		

TABLE XXI: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on mDTLZ3 problem.

mDTLZ3						
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR		
3	<i>e</i>	0.0069(1.13e-02) [†]	0.0019(3.08e-03)	0.1444(1.06e-01) [†]		
	<i>b</i>	0.0072(5.96e-03)	0.0020(2.83e-03)	0.4026(2.81e-01) [†]		

TABLE XXII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on mDTLZ4 problem.

mDTLZ4						
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR		
3	<i>e</i>	0.0032(3.14e-03) [†]	0.0013(1.46e-03)	0.0013(2.27e-03)		
	<i>b</i>	0.0019(1.87e-03)	0.0014(1.48e-03)	0.0013(6.74e-03)		

TABLE XXIII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms on WFG3 problem.

WFG3						
<i>m</i>	ROI	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR		
3	<i>e</i>	0.0116(2.05e-02)	0.1076(1.45e-01) [†]	0.3100(1.46e-01) [†]		
	<i>b</i>	0.0721(4.52e-02)	0.0936(2.10e-01)	0.0641(7.38e-02)		
5	<i>e</i>	0.2843(9.86e-02)	0.2506(3.92e-01)	0.6851(2.50e-01) [†]		
	<i>b</i>	0.4674(2.47e-01)	0.3891(2.16e-01)	1.2546(2.35e-01) [†]		
8	<i>e</i>	0.4545(4.16e-01)	0.4741(5.97e-02)	0.7184(2.70e-01) [†]		
	<i>b</i>	0.4196(2.71e-01)	1.2394(4.30e-01) [†]	1.9540(3.36e-01) [†]		
10	<i>e</i>	0.6789(3.08e-01) [†]	0.4922(1.64e-01)	0.8892(8.85e-02) [†]		
	<i>b</i>	0.9458(1.89e+00)	1.6038(5.16e-01)	2.6890(1.57e-01) [†]		

TABLE XXIV: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on DTLZ1 problem.

<i>m</i>	ROI	BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.3495(2.63e-01)	0.0013(2.21e-03)	0.1045(5.50e-02)	0.0025(2.59e-03)	0.0057(4.03e-03)	0.0387(4.52e-02)	0.0153(6.02e-02)
	<i>b</i>	0.2924(1.08e-01)	0.0013(2.25e-03)	0.1688(2.30e-01) [†]	0.0020(6.17e-03)	0.0051(7.16e-03)	0.0366(2.26e-02)	0.0198(3.13e-02)
5	<i>e</i>	0.4847(1.08e+00)	0.0165(2.91e-02)	0.1199(8.84e-03) [†]	0.0171(4.56e-03)	0.0107(4.97e-03)	0.0342(1.08e-02)	0.082(3.53e-02)
	<i>b</i>	0.6828(2.86e+00)	0.0129(3.20e-02)	0.0800(3.47e-02)	0.0391(1.76e-03)	0.0207(9.43e-03)	0.0496(4.87e-02)	0.0278(1.59e-02)
8	<i>e</i>	0.8459(4.70e-01)	0.0937(7.55e-02)	0.0800(3.47e-02)	0.0364(1.86e-03)	0.0207(9.43e-03)	0.0496(4.87e-02)	0.0278(1.59e-02)
	<i>b</i>	0.7698(2.86e+00)	0.1295(3.20e-02)	0.0889(1.53e-02)	0.0301(1.87e-02)	0.0278(1.59e-02)	0.0661(2.73e-02)	0.0316(1.73e-02)
10	<i>e</i>	1.0735(4.12e-02)	0.0746(2.54e-02)	0.0514(1.53e-02)	0.0446(2.82e-02)	0.0194(1.27e-02)	0.0316(1.99e-02)	0.0353(1.47e-02)
	<i>b</i>	2.2371(2.11e+02)	\	0.1101(3.64e-02)	0.0462(2.14e-02)	0.0512(2.32e-02)	0.0800(8.94e-03)	\

TABLE XXV: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on DTLZ2 problem.

<i>m</i>	ROI	BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.2073(5.09e-01)	0.0076(5.90e-03)	0.1920(2.16e-01)	0.0040(3.57e-03)	0.0067(6.16e-03)	0.0349(3.86e-03)	0.0049(4.38e-03)
	<i>b</i>	0.3277(3.42e-01)	0.0159(1.56e-02)	0.4135(4.54e-01)	0.0059(5.71e-03)	0.0164(1.81e-02)	0.0364(4.15e-02)	0.0083(4.08e-02)
5	<i>e</i>	0.7520(4.70e-01)	0.0343(9.98e-02)	0.3241(1.90e-01)	0.0042(1.27e-03)	0.0163(1.52e-02)	0.0324(1.04e-02)	0.0083(1.05e-02)
	<i>b</i>	1.2362(2.04e-01)	0.0370(2.52e-01)	0.3321(1.19e-01)	0.0045(1.27e-03)	0.0163(1.52e-02)	0.0324(1.05e-02)	0.0083(1.06e-02)
8	<i>e</i>	2.3109(2.05e+00)	0.3938(2.71e-01)	0.3307(7.64e-02)	0.0303(1.83e-01)	0.1130(1.86e-01)	0.0544(5.34e-02)	0.0229(1.23e-01)
	<i>b</i>	2.8686(2.00e+00)	0.4386(1.63e-01)	0.3747(1.48e-01)	0.0301(1.80e-01)	0.1572(1.32e-01)	0.0501(2.87e-02)	0.0267(1.14e-01)
10	<i>e</i>	2.9471(1.43e-01)	0.3677(2.71e-01)	0.3677(2.71e-01)	0.0301(1.80e-01)	0.1572(1.32e-01)	0.0501(2.87e-02)	0.0267(1.14e-01)
	<i>b</i>	3.1078(1.75e+00)	\	0.5136(1.80e-01)	0.0363(0.54e-02)	0.1536(0.54e-02)	0.0529(1.16e-02)	0.0497(9.78e-02)

TABLE XXVI: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on DTLZ3 problem.

<i>m</i>	ROI	BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	<i>e</i>	0.7519(3.79e-01)	0.0161(5.86e-02)	0.3347(4.52e-01)	0.0050(4.64e-02)	0.02098(1.26e-03)	0.03088(5.53e-01)	0.0708(1.76e-01)
	<i>b</i>	0.5154(1.88e-01)	0.0109(10.49e-02)	0.4006(4.41e-01)	0.0055(2.44e-02)	0.0209(4.34e-02)	0.0317(8.50e-01)	0.0708(1.78e-01)
5	<i>e</i>	8.4009(5.61e-01)	0.0137(9.93e-02)	0.3321(1.19e-01)	0.0053(1.27e-02)	0.0163(1.52e-02)	0.0313(1.26e-01)	0.0708(1.79e-01)
	<i>b</i>	8.4009(5.61e-01)	0.0137(9.93e-02)	0.3321(1.19e-01)	0.0053(1.27e-02)	0.0163(1.52e-02)	0.0313(1.26e-01)	0.0708(1.79e-01)
8	<i>e</i>	2.4520(7.94e-01)	0.7447(2.32e-02)	0.5134(0.15e-01)	0.0177(3.53e-02)	0.1748(2.32e-01)	0.3768(1.75e-01)	0.1748(1.75e-01)
	<i>b</i>	2.4520(7.94e-01)	0.7447(2.32e-02)	0.5134(0.15e-01)	0.0177(3.53e-02)	0.1748(2.32e-01)	0.3768(1.75e-01)	0.1748(1.75e-01)
10	<i>e</i>	2.9145(1.55e+00)	0.5807(5.51e-01)	0.5807(5.51e-01)	0.0100(1.65e-04)	0.4423(2.84e-01)	0.4628(2.44e-01)	0.5656(1.30e-01)
	<i>b</i>	2.6635(6.90e-01)	\	0.6353(3.58e-02)	0.0100(1.65e-04)	0.5234(1.78e-01)	0.4748(1.4	

TABLE XXXII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on minus-DTLZ3 problem.

m	ROI	DTLZ3 ⁻¹						
		BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	e	308.1581(4.47e+02) [†]	15.6558(2.01e+01) [†]	268.0881(0.05e+02) [†]	159.7220(9.55e+02) [†]	8.8459(8.00e+00) [†]	96.8799(1.74e+02) [†]	152.4524(4.3e+02) [†]
3	b	514.9006(5.25e+02) [†]	9.0195(8.18e+00) [†]	931.7620(4.92e+02) [†]	680.9276(2.34e+01) [†]	23.2576(2.34e+01) [†]	164.6848(9.96e+01) [†]	115.5498(1.19e+02) [†]
5	e	831.3506(5.13e+02) [†]	54.0371(4.22e+01) [†]	1145.7254(7.54e+02) [†]	893.2118(5.30e+02) [†]	43.5592(2.00e+01) [†]	231.1682(1.71e+02) [†]	247.7289(1.54e+02) [†]
5	b	1164.4468(5.25e+02) [†]	143.7532(7.23e+01) [†]	1262.5265(2.00e+02) [†]	1079.8845(2.16e+02) [†]	350.9760(2.47e+02) [†]	338.4023(1.89e+02) [†]	404.8937(5.85e+01) [†]
8	e	1647.2067(8.83e+02) [†]	143.7532(7.23e+01) [†]	1262.5265(2.00e+02) [†]	1079.8845(2.16e+02) [†]	350.9760(2.47e+02) [†]	338.4023(1.89e+02) [†]	404.8937(5.85e+01) [†]
8	b	1146.4468(5.25e+02) [†]	209.9532(1.22e+02) [†]	1640.9975(3.78e+02) [†]	1188.7661(3.38e+02) [†]	222.3641(1.23e+02) [†]	568.3700(2.48e+02) [†]	820.6048(1.14e+02) [†]
10	e	1665.3151(4.62e+02) [†]	\	1460.4353(3.10e+02) [†]	1276.3342(2.70e+02) [†]	32.5541(6.16e+02) [†]	438.6370(1.91e+02) [†]	731.1537(1.85e+02) [†]
10	b	1587.4906(9.87e+02) [†]	\	1668.0383(2.17e+02) [†]	1299.3664(3.59e+02) [†]	310.8913(7.92e+01) [†]	787.1734(2.83e+02) [†]	1013.6266(9.36e+01) [†]

TABLE XXXIII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on minus-DTLZ4 problem.

m	ROI	DTLZ4 ⁻¹						
		BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	e	0.3801(2.74e-01) [†]	0.0986(2.02e-01) [†]	0.4250(1.44e-01) [†]	0.0686(0.51e-02) [†]	0.0263(2.45e-02) [†]	0.0094(3.12e-02) [†]	
3	b	0.7720(4.94e-01) [†]	0.0812(9.55e-02) [†]	0.0408(0.44e-01) [†]	0.6450(0.05e-01) [†]	0.0701(9.53e-02) [†]	0.1598(1.11e-01) [†]	0.0161(2.55e-02) [†]
5	e	0.6760(5.25e-01) [†]	0.4343(2.17e-01) [†]	1.0373(3.30e-01) [†]	1.4943(1.09e-01) [†]	0.1853(1.33e-01) [†]	0.1093(1.60e-01) [†]	0.0515(8.78e-02) [†]
5	b	1.5918(7.58e-01) [†]	0.5050(2.76e-01) [†]	1.5106(4.08e-01) [†]	1.5493(6.97e-01) [†]	0.2309(1.84e-01) [†]	1.1260(4.63e-01) [†]	0.4707(1.95e-01) [†]
8	e	1.5082(1.58e+00) [†]	0.7169(3.30e-01) [†]	1.9221(3.24e-01) [†]	0.5600(1.75e-01) [†]	0.7317(1.53e-01) [†]	0.3530(1.56e-01) [†]	
8	b	1.8319(1.16e+00) [†]	0.6659(3.32e-02) [†]	1.9518(3.86e-01) [†]	2.1421(4.67e-01) [†]	0.6707(0.62e-01) [†]	1.2443(3.57e-01) [†]	0.9233(2.31e-01) [†]
10	e	2.1902(1.43e+00) [†]	\	1.8069(4.87e-01) [†]	2.0372(3.04e-01) [†]	0.9778(4.36e-01) [†]	0.9653(3.31e-01) [†]	0.8794(1.87e-01) [†]
10	b	1.6908(6.75e-01) [†]	\	2.2000(4.51e-01) [†]	2.1666(6.82e-01) [†]	0.9801(2.59e-01) [†]	1.3484(3.79e-01) [†]	1.5077(1.88e-01) [†]

TABLE XXXIV: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on mDTLZ1 problem.

m	ROI	mDTLZ1						
		BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	e	1.5495(4.31e+00) [†]	0.0009(7.09e-02)	0.2149(6.69e-02) [†]	0.0033(1.68e-02) [†]	0.0046(5.95e-03) [†]	0.0308(1.29e-01) [†]	0.2018(1.33e-01) [†]
3	b	0.3553(2.58e+00) [†]	0.0024(2.58e-03)	0.3458(2.38e-01) [†]	0.0021(1.63e-03) [†]	0.0008(2.16e-03) [†]	0.0075(5.35e-02) [†]	0.2050(2.14e-01) [†]

TABLE XXXV: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on mDTLZ2 problem.

m	ROI	mDTLZ2						
		BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	e	0.2077(1.47e-01) [†]	0.0009(7.00e-04)	0.2555(1.66e-01) [†]	0.0024(1.58e-03) [†]	0.0016(1.51e-03) [†]	0.0009(1.12e-03) [†]	0.0021(4.80e-03) [†]
3	b	0.1584(4.10e-01) [†]	0.0010(1.24e-03) [†]	0.3120(3.41e-01) [†]	0.0025(4.18e-03) [†]	0.0026(3.04e-03) [†]	0.0038(1.15e-03) [†]	0.0016(9.78e-03) [†]

TABLE XXXVI: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on mDTLZ3 problem.

m	ROI	mDTLZ3						
		BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	e	0.8547(0.02e+001) [†]	0.0259(5.23e-02) [†]	0.3821(3.18e-01) [†]	0.0037(5.21e-03) [†]	0.0069(1.13e-02) [†]	0.0019(3.08e-03) [†]	0.1444(1.06e-01) [†]
3	b	0.7148(4.87e+00) [†]	0.0010(1.24e-03) [†]	0.4197(3.14e-01) [†]	0.0045(3.25e-03) [†]	0.0072(5.96e-03) [†]	0.0020(2.83e-03) [†]	0.4026(2.81e-01) [†]

TABLE XXXVII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on mDTLZ4 problem.

m	ROI	mDTLZ4						
		BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	e	0.1363(2.57e-01) [†]	0.0028(4.11e-03) [†]	0.2197(1.42e-01) [†]	0.0036(2.97e-03) [†]	0.0003(2.14e-03) [†]	0.0013(1.46e-03) [†]	0.0014(2.27e-03) [†]
3	b	0.1862(2.34e-01) [†]	0.0043(6.68e-03)	0.3299(2.62e-01) [†]	0.0050(5.64e-03) [†]	0.0019(1.87e-03) [†]	0.0014(1.48e-03) [†]	0.0012(6.74e-03) [†]

TABLE XXXVIII: Comparison results of $\mathbb{E}(\mathcal{P})$ values obtained by our proposed three algorithms along with four peer algorithms on WFG3 problem.

m	ROI	WFG3						
		BC-EMOA	NEMO-0	I-MOEA/D-PLVF	IEMO/D	I-NSGA-II/LTR	I-MOEA/D/LTR	I-R2-IBEA/LTR
3	e	0.7176(1.19e+000) [†]	0.0015(3.34e-03) [†]	1.0927(0.54e+01) [†]	0.0204(2.03e-02) [†]	0.0162(0.05e-02) [†]	0.1076(1.45e+01) [†]	0.3100(1.66e+01) [†]
3	b	0.9129(7.19e-01) [†]	0.0923(8.32e-02) [†]	1.1207(4.93e+00) [†]	0.0159(2.88e-02) [†]	0.0273(4.52e-02) [†]	0.0936(2.08e-01) [†]	0.0664(7.38e-02) [†]
5	e	1.8650(4.39e+000) [†]	0.0489(5.16e-02) [†]	1.1787(1.95e+01) [†]	0.2182(1.06e-01) [†]	0.2843(9.86e-02) [†]	0.2506(3.92e-01) [†]	0.6851(2.50e-01) [†]
5	b	2.9455(7.33e-01) [†]	0.0599(5.64e-02) [†]	1.5234(3.10e-01) [†]	0.3484(2.92e-01) [†]	0.4674(2.47e-01) [†]	0.3891(2.16e-01) [†]	0.2546(2.33e-01) [†]
8	e	3.8975(1.03e+01) [†]	0.5549(1.60e+00) [†]	2.1691(6.25e-01) [†]	0.7582(1.42e-01) [†]	0.4545(4.16e-01) [†]	0.4741(5.19e-02) [†]	0.7184(2.70e-01) [†]
8	b	2.8963(7.86e+00) [†]	1.8892(1.66e+00) [†]	2.7181(0.25e+00) [†]	0.8347(4.64e-01) [†]	0.4662(7.10e-01) [†]	1.2940(4.30e-01) [†]	1.9540(3.36e-01) [†]
10	e	8.6840(9.39e+00) [†]	\	1.7357(1.14e+01) [†]	1.0947(3.30e-01) [†]	0.6789(3.06e-01) [†]	0.4401(1.64e-01) [†]	0.8392(8.83e-02) [†]
10	b	2.4774(1.42e+01) [†]	\	2.9276(9.36e-01) [†]	1.3994(2.76e-01) [†]	0.9428(1.89e+00) [†]	2.6890(1.27e-01) [†]	

APPENDIX F CONVERGENCE PLOTS

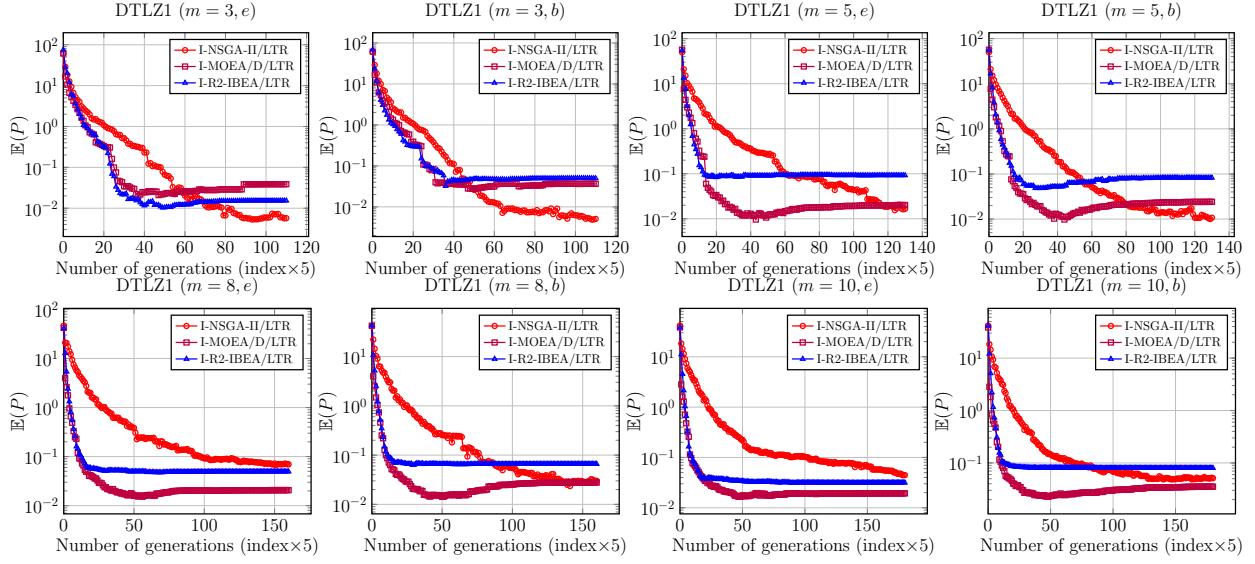


Fig. 5: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on DTLZ1 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

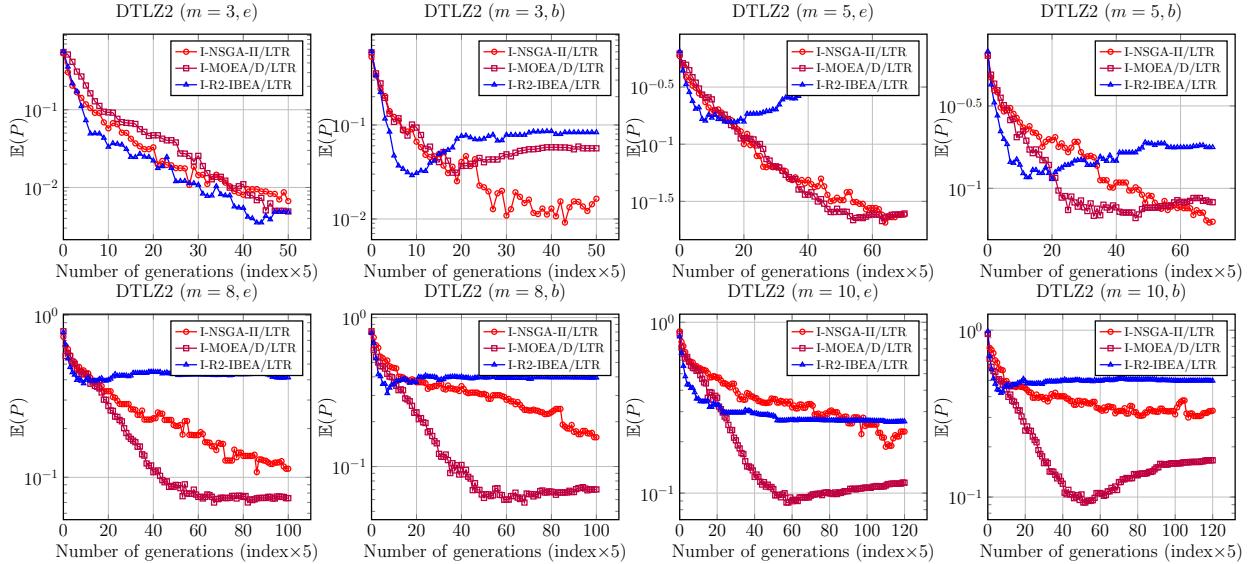


Fig. 6: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on DTLZ2 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

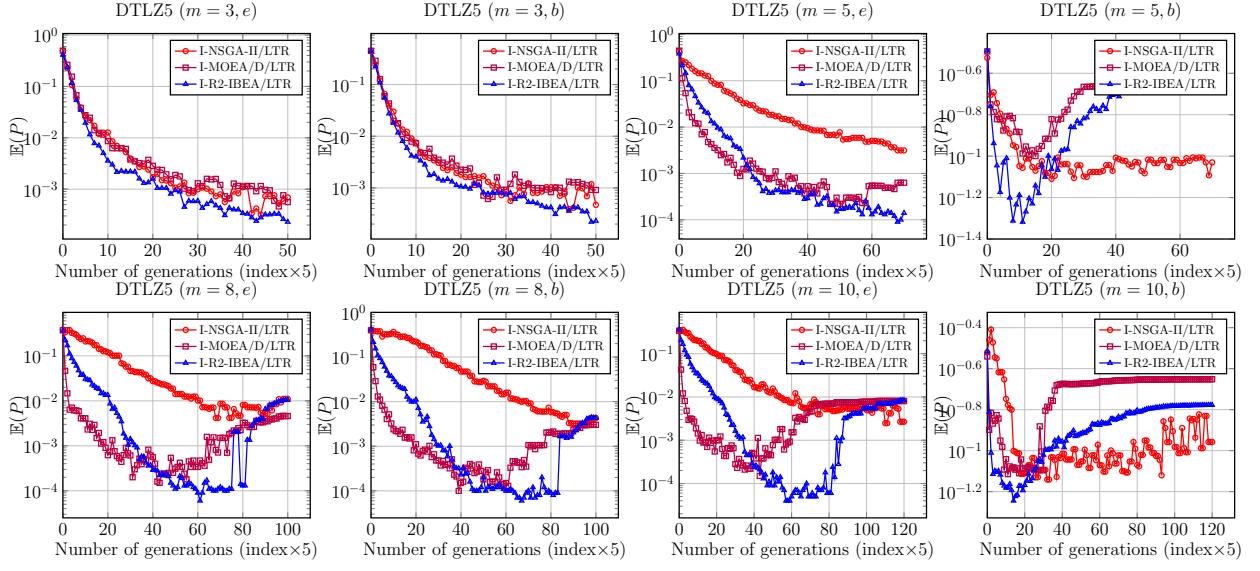


Fig. 7: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on dtlz5 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

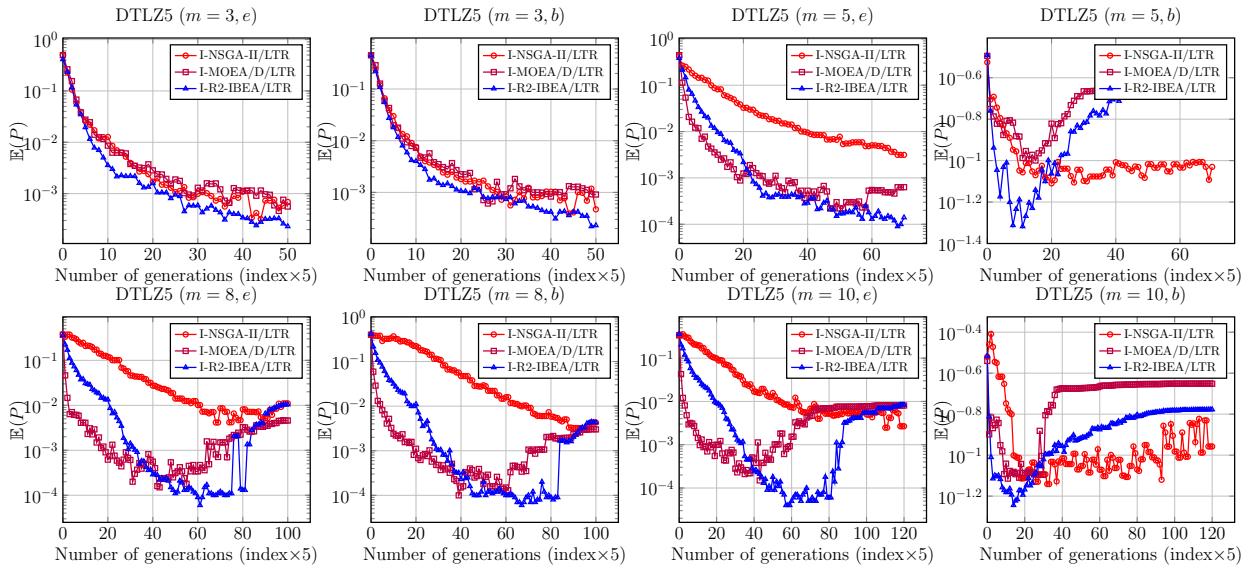


Fig. 8: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on dtlz5 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

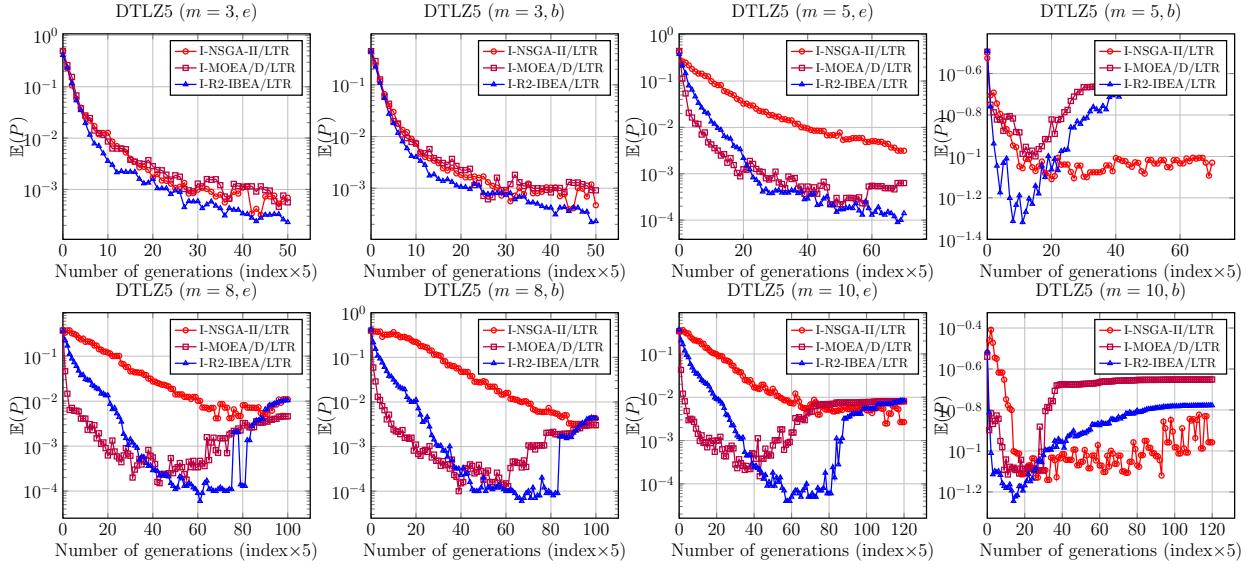


Fig. 9: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on DTLZ5 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

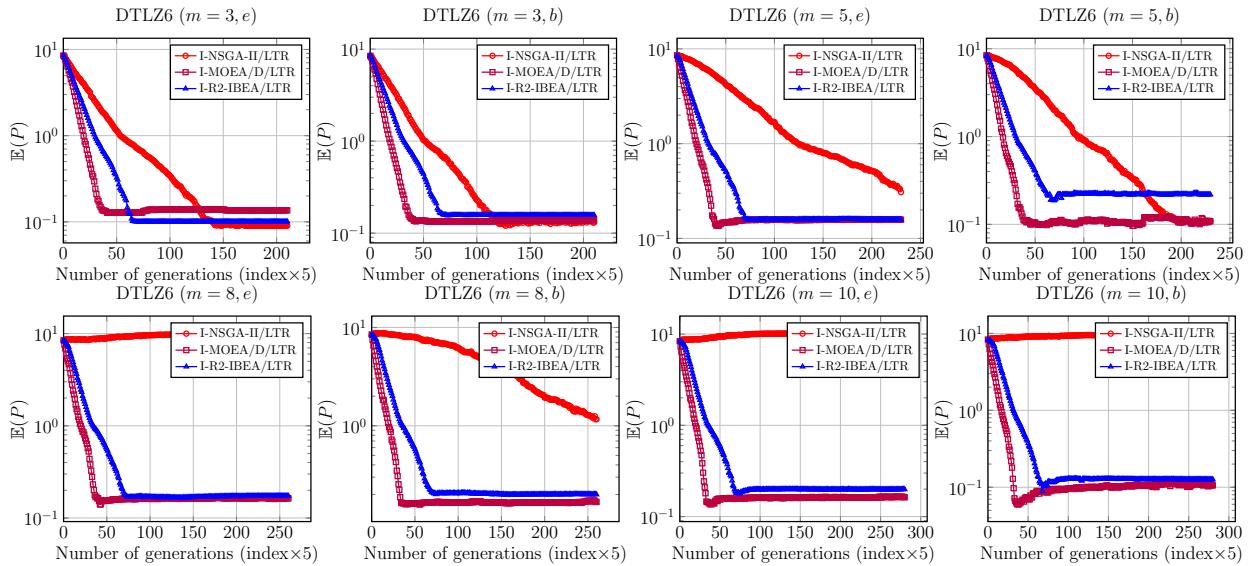


Fig. 10: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on DTLZ6 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

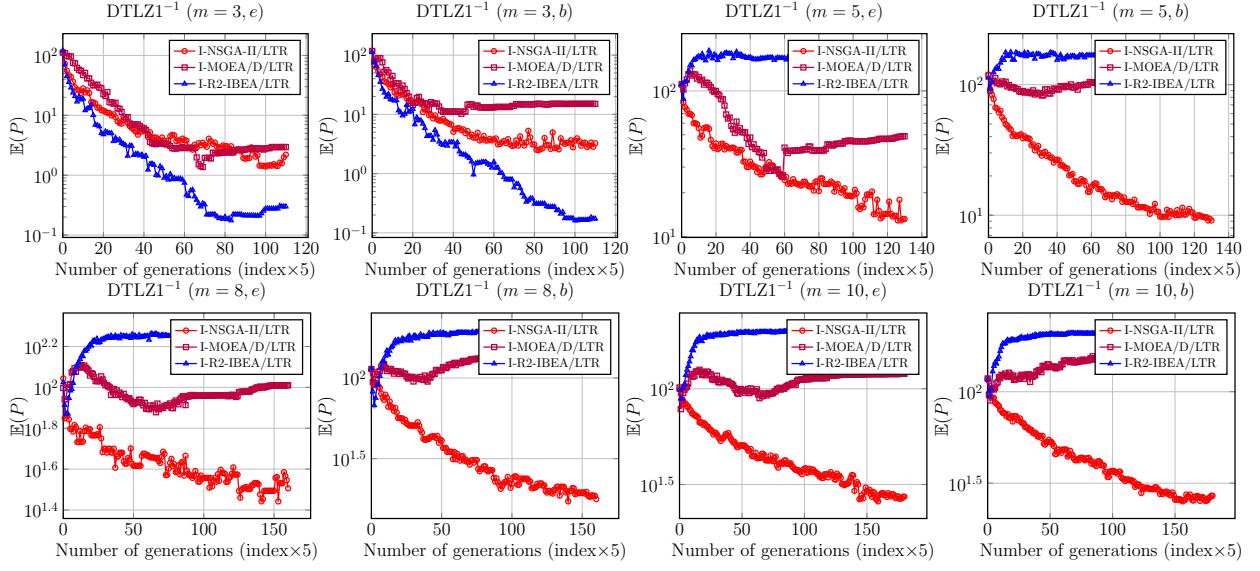


Fig. 11: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on minus-DTLZ1 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

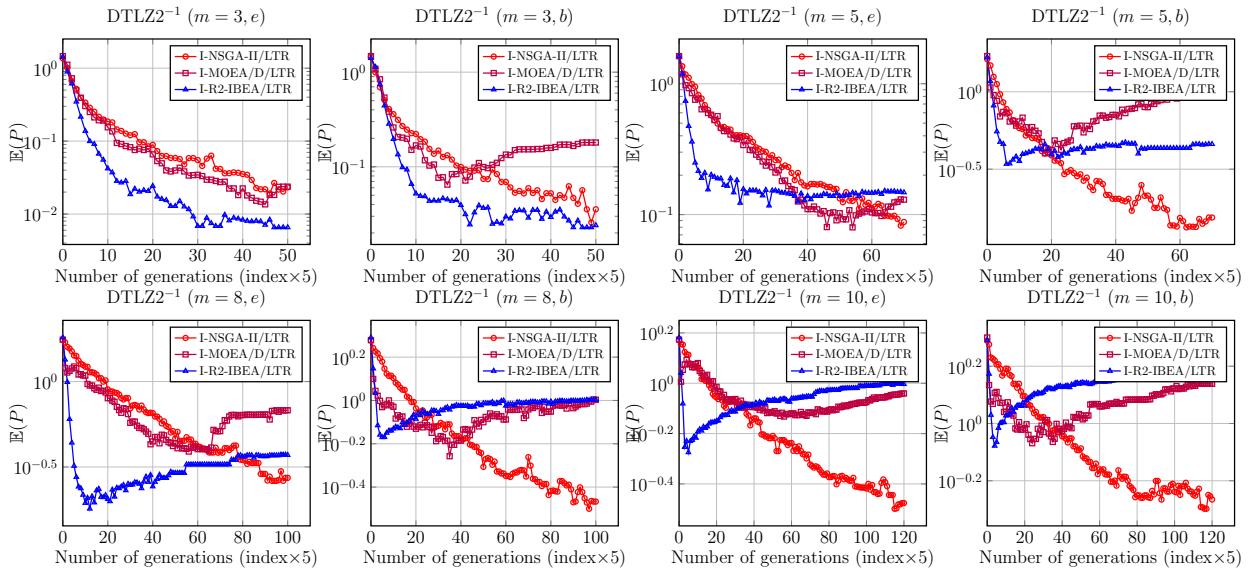


Fig. 12: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on minus-DTLZ2 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

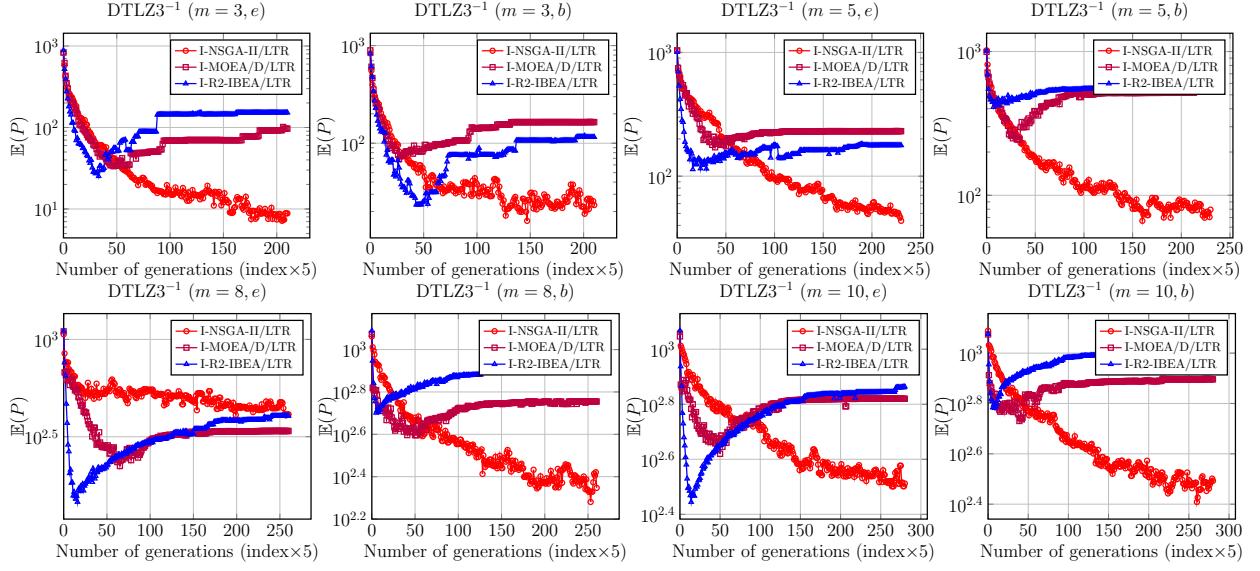


Fig. 13: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on minus-dtlz3 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

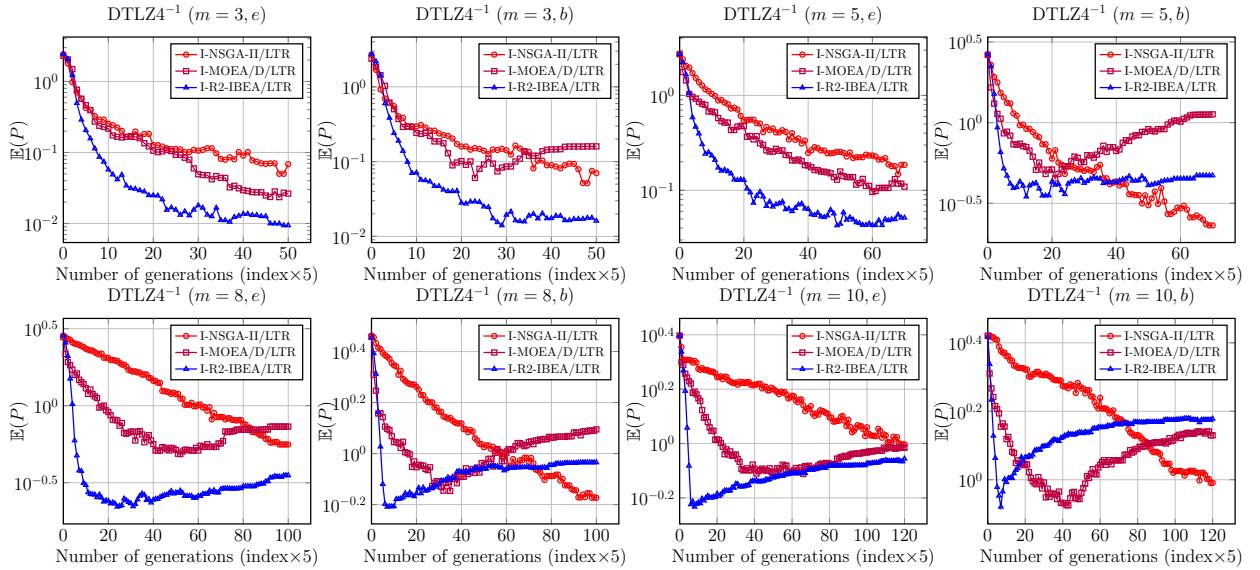


Fig. 14: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on minus-dtlz4 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

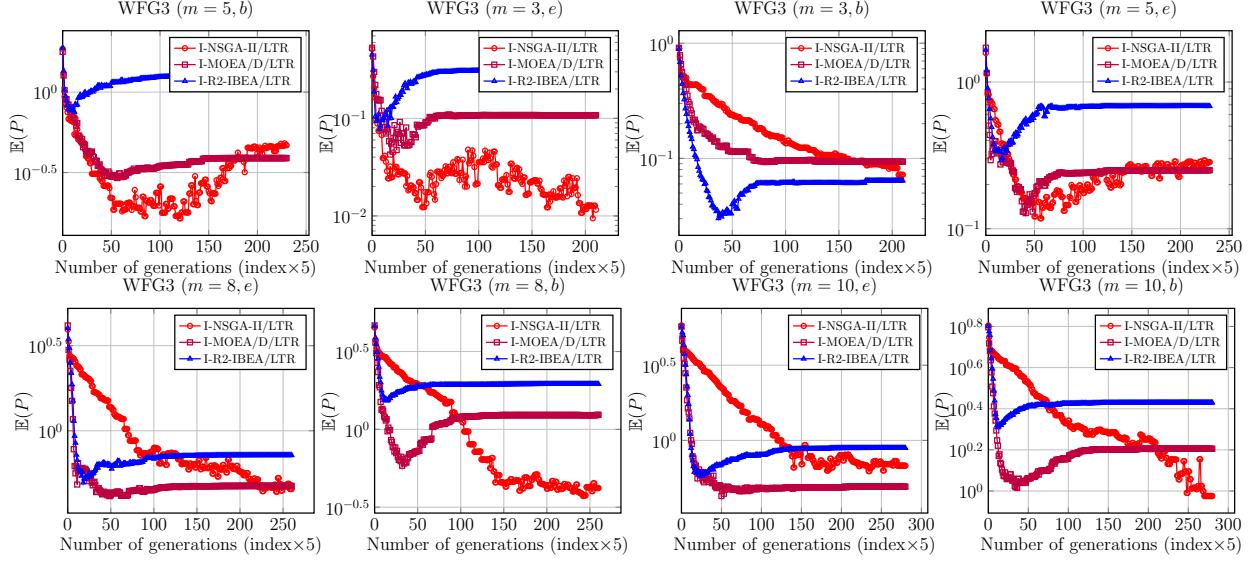


Fig. 15: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on WFG3 problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

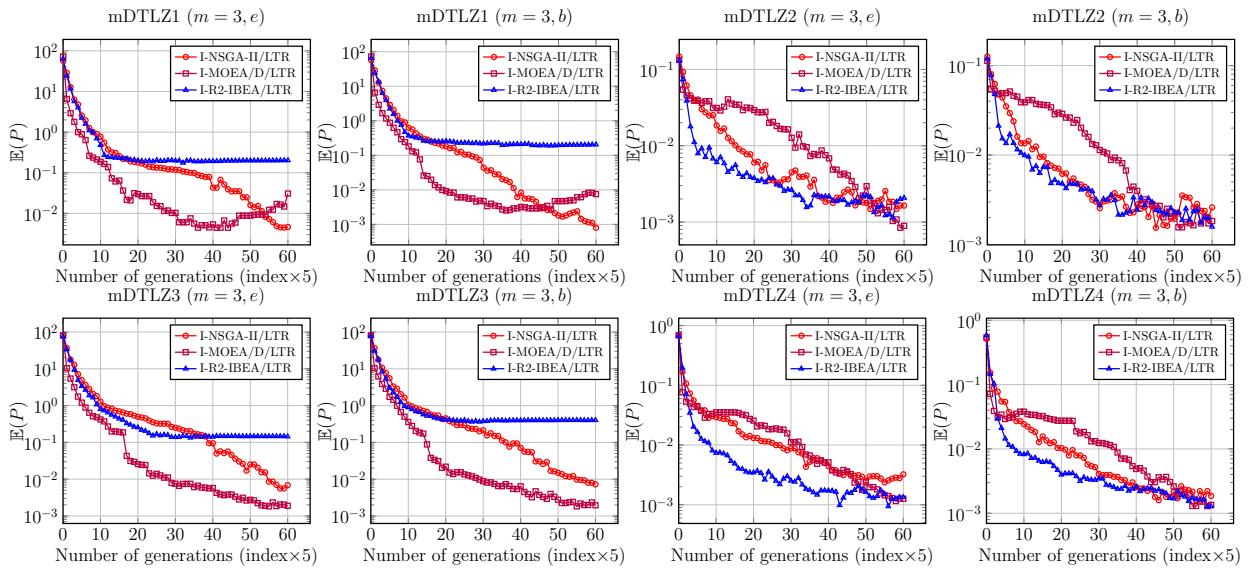


Fig. 16: Trajectories of the approximation error versus the number of generations by our proposed three algorithms on mDTLZ problems. (e) indicates an equal importance priority over all objectives, while (b) indicates the preference biased toward a particular side of the PF.

APPENDIX G POPULATION DISTRIBUTIONS

Fig. 17: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ1 when $m = 3$.

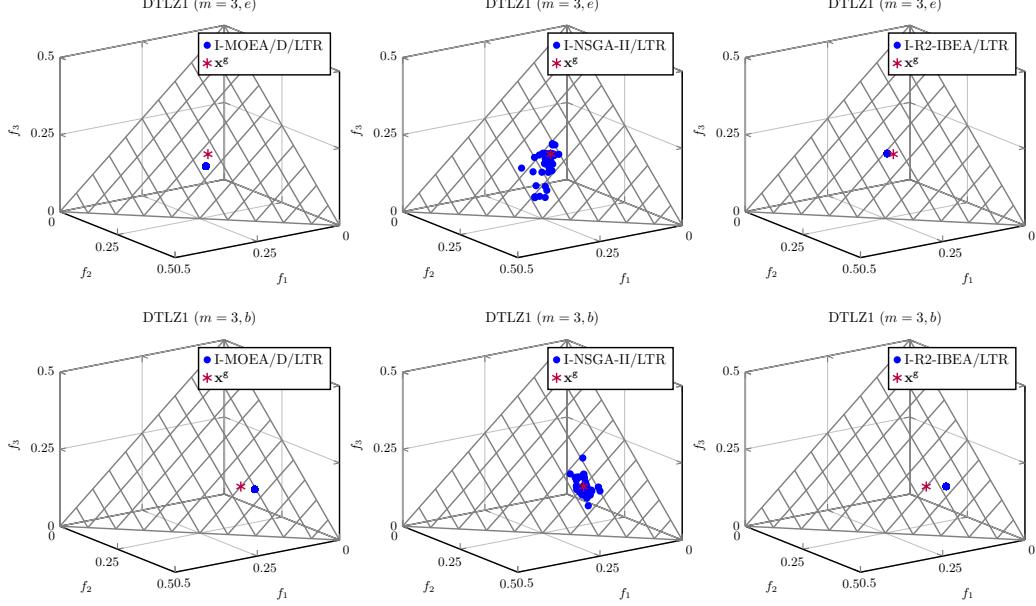


Fig. 18: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ2 when $m = 3$.

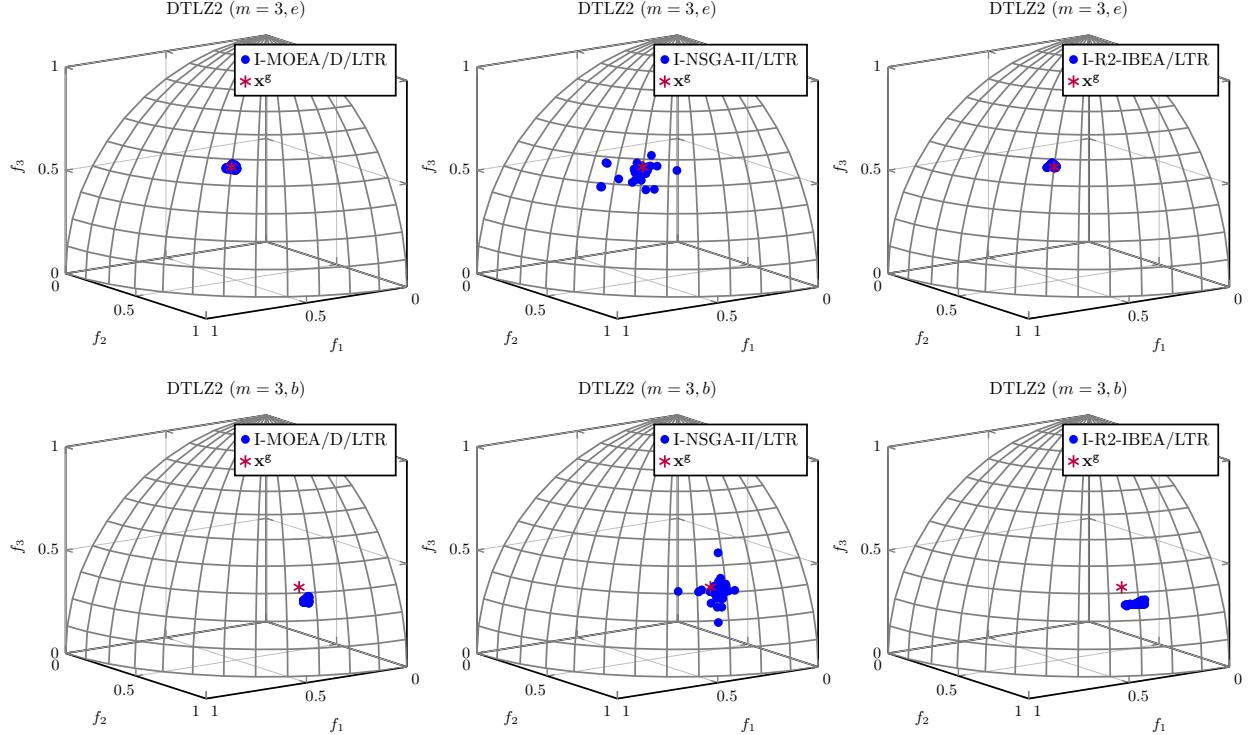


Fig. 19: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ3 when $m = 3$.

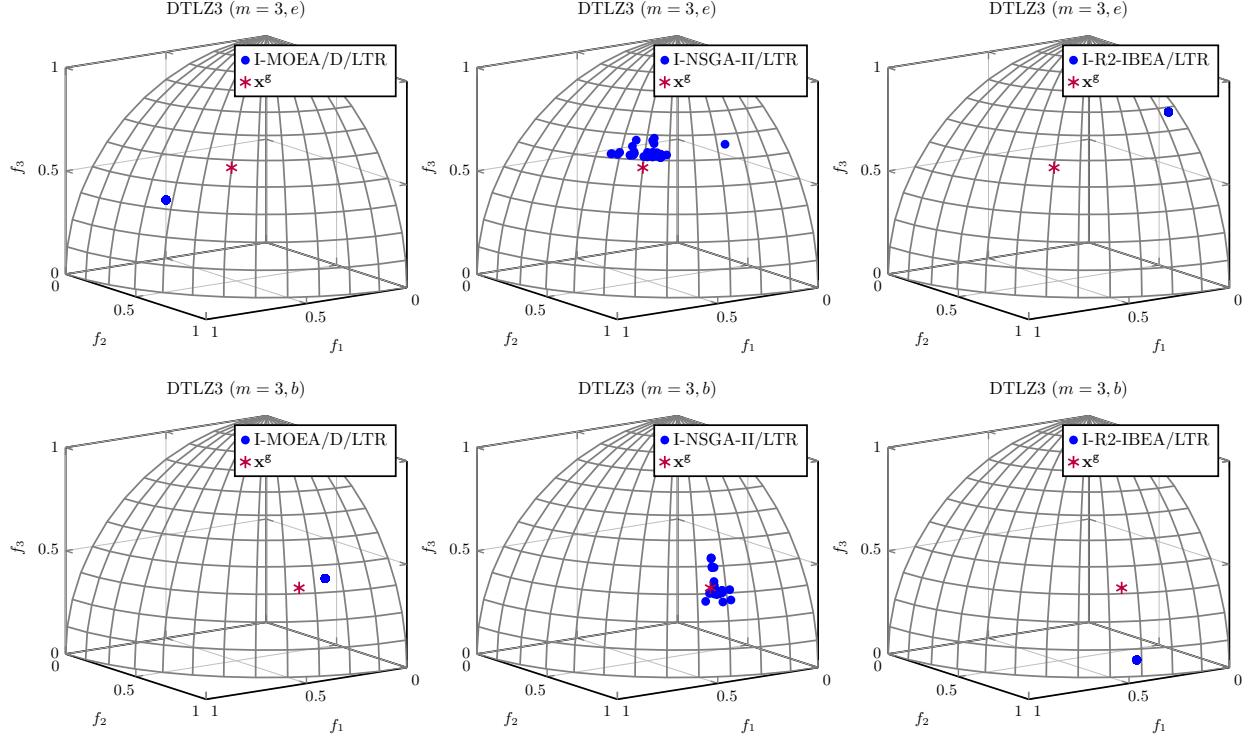


Fig. 20: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ4 when $m = 3$.

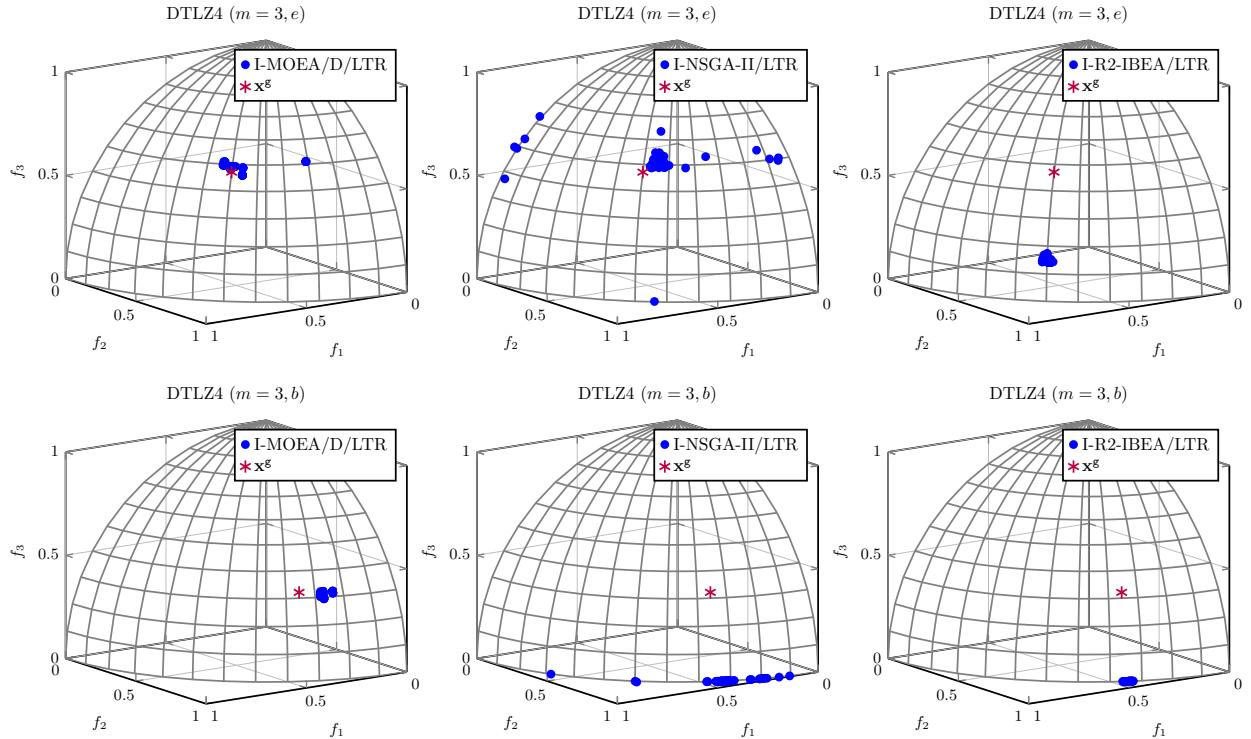


Fig. 21: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ5 when $m = 3$.

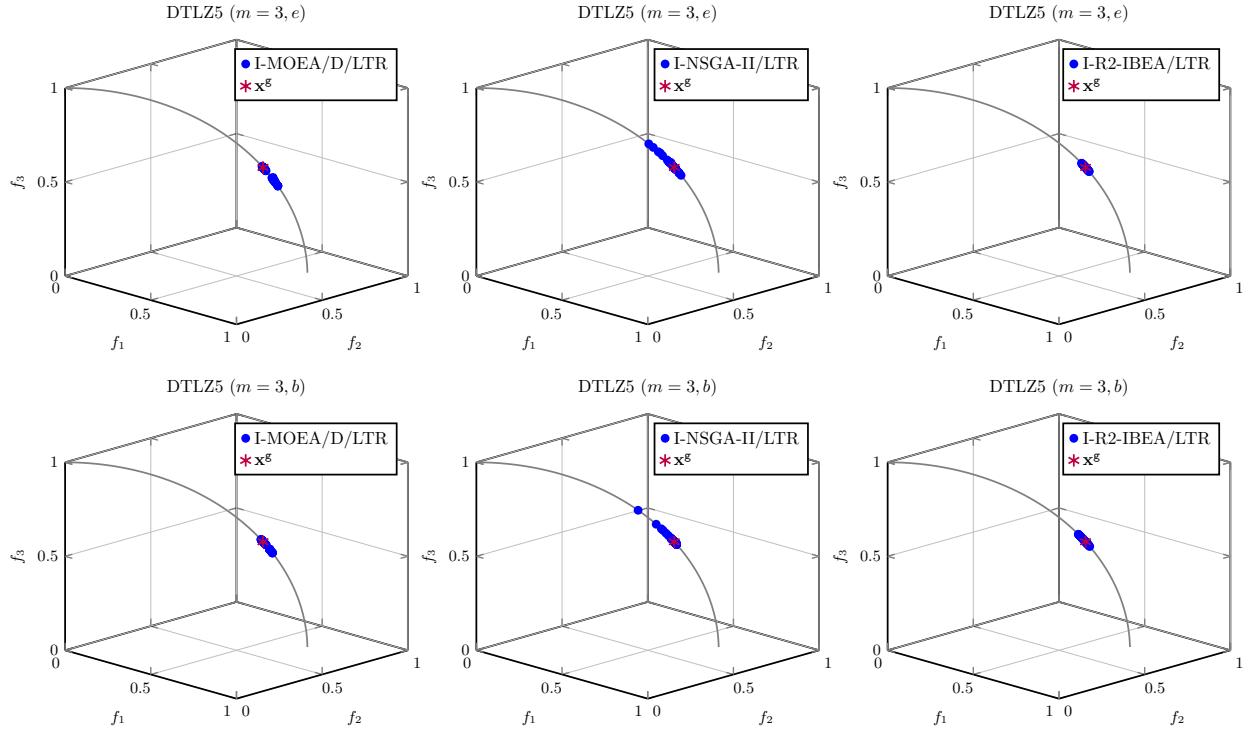


Fig. 22: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ6 when $m = 3$.

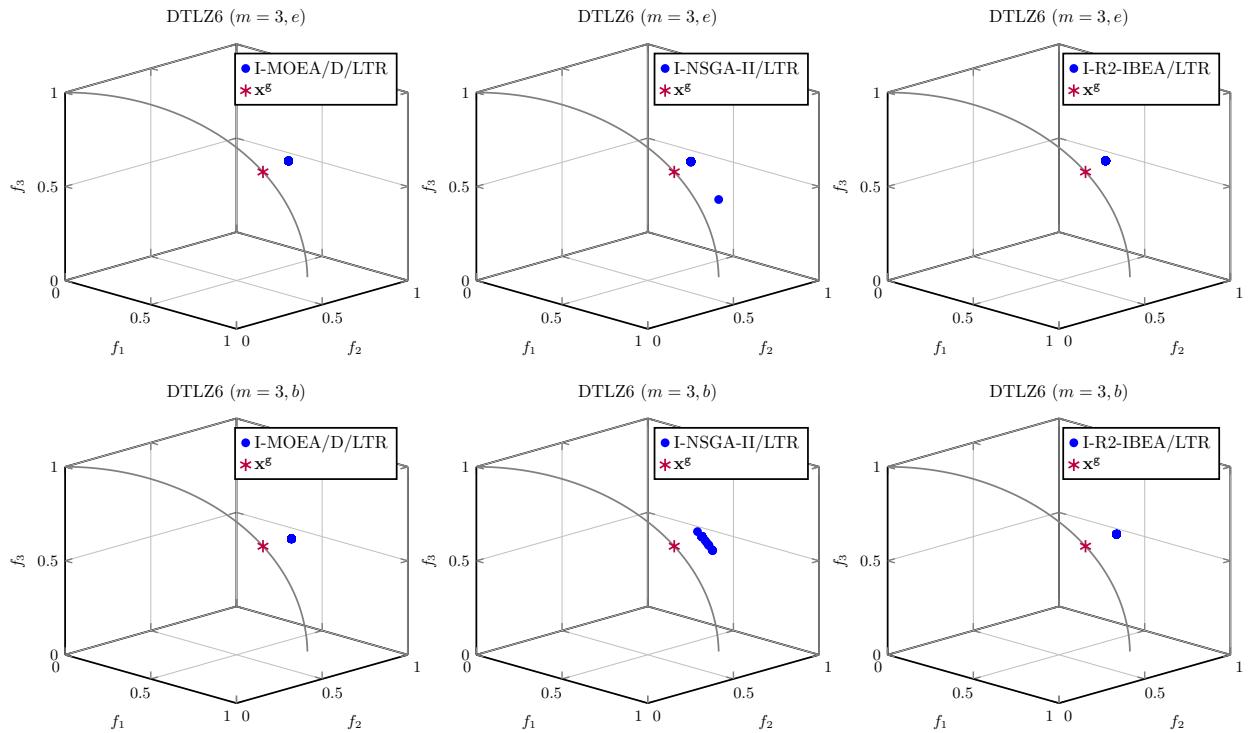


Fig. 23: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ1⁻¹ when $m = 3$.

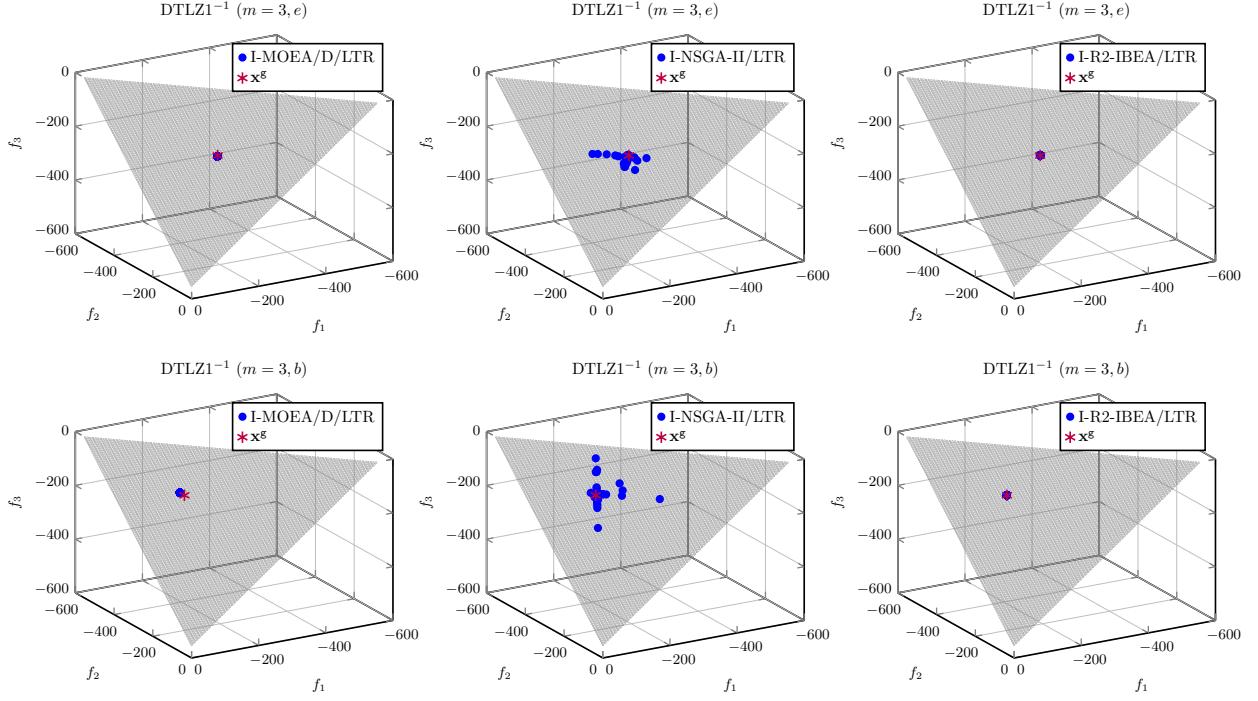


Fig. 24: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ2⁻¹ when $m = 3$.

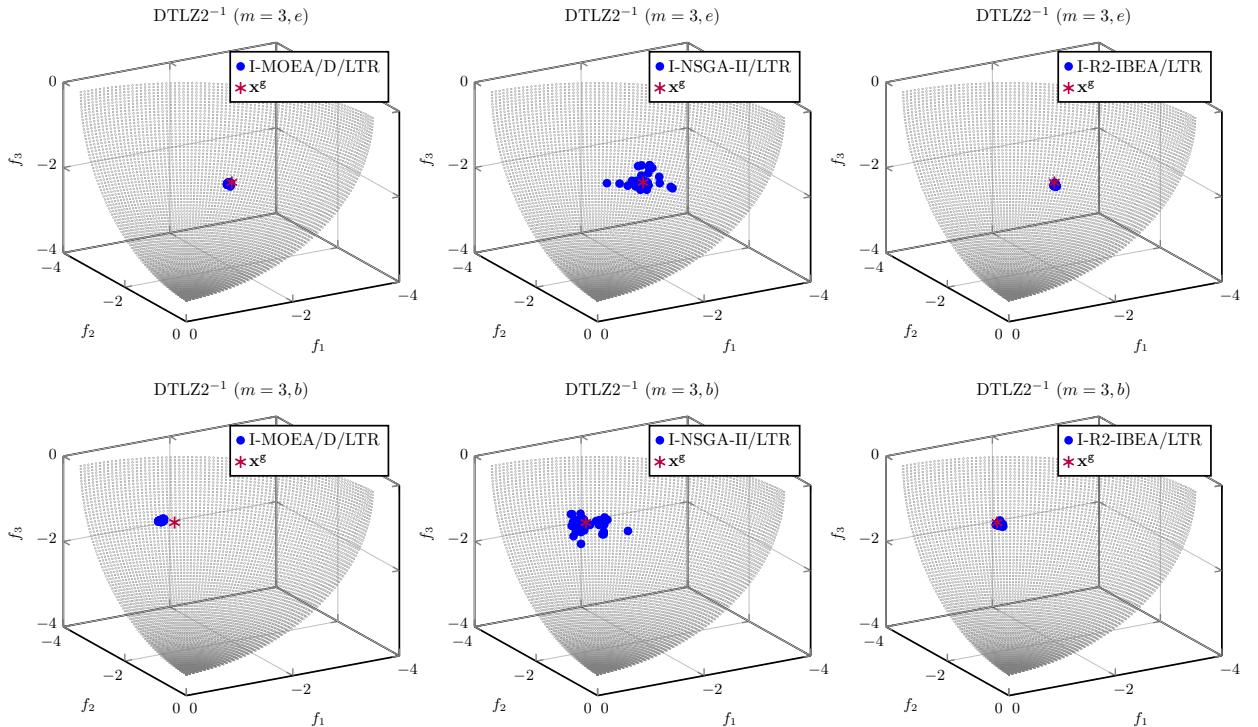


Fig. 25: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ3⁻¹ when $m = 3$.

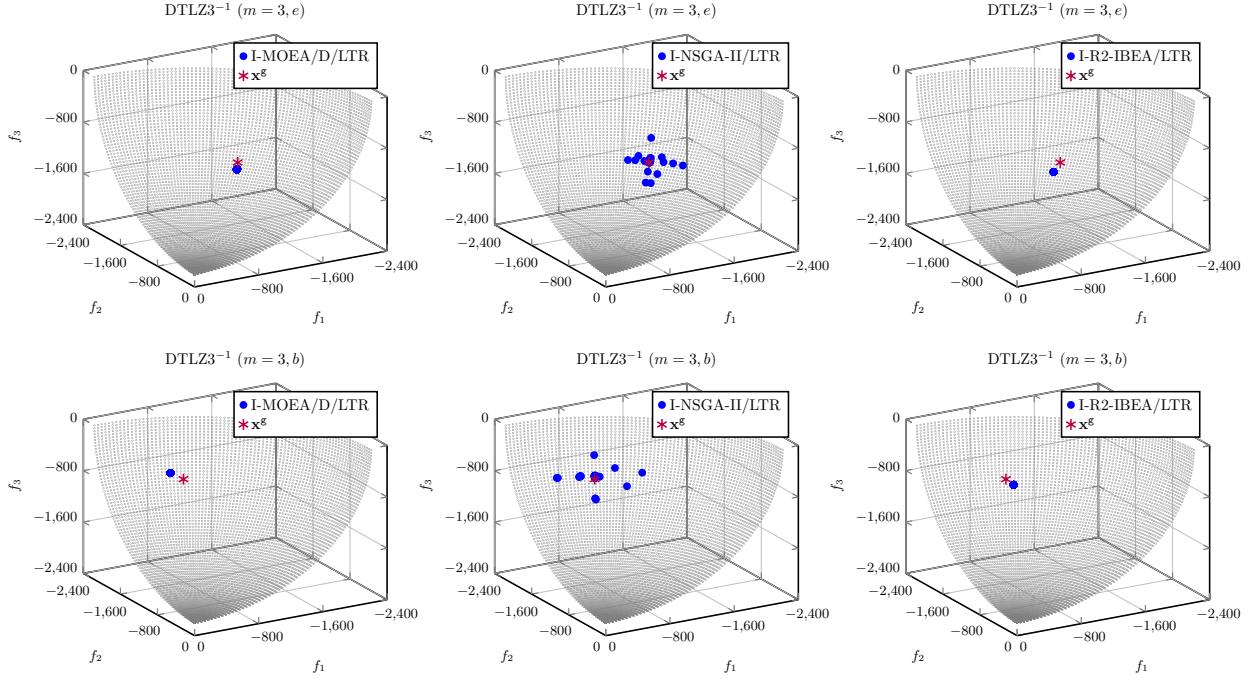


Fig. 26: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ4⁻¹ when $m = 3$.

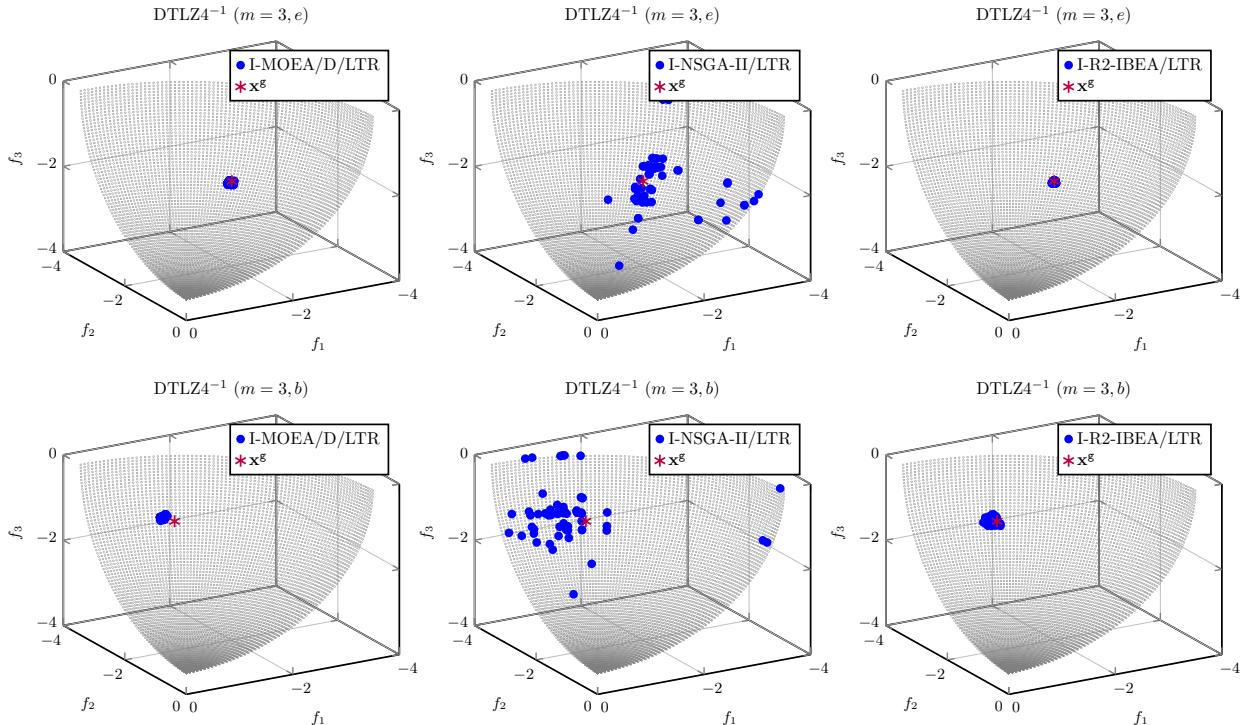


Fig. 27: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on mDTLZ1 when $m = 3$.

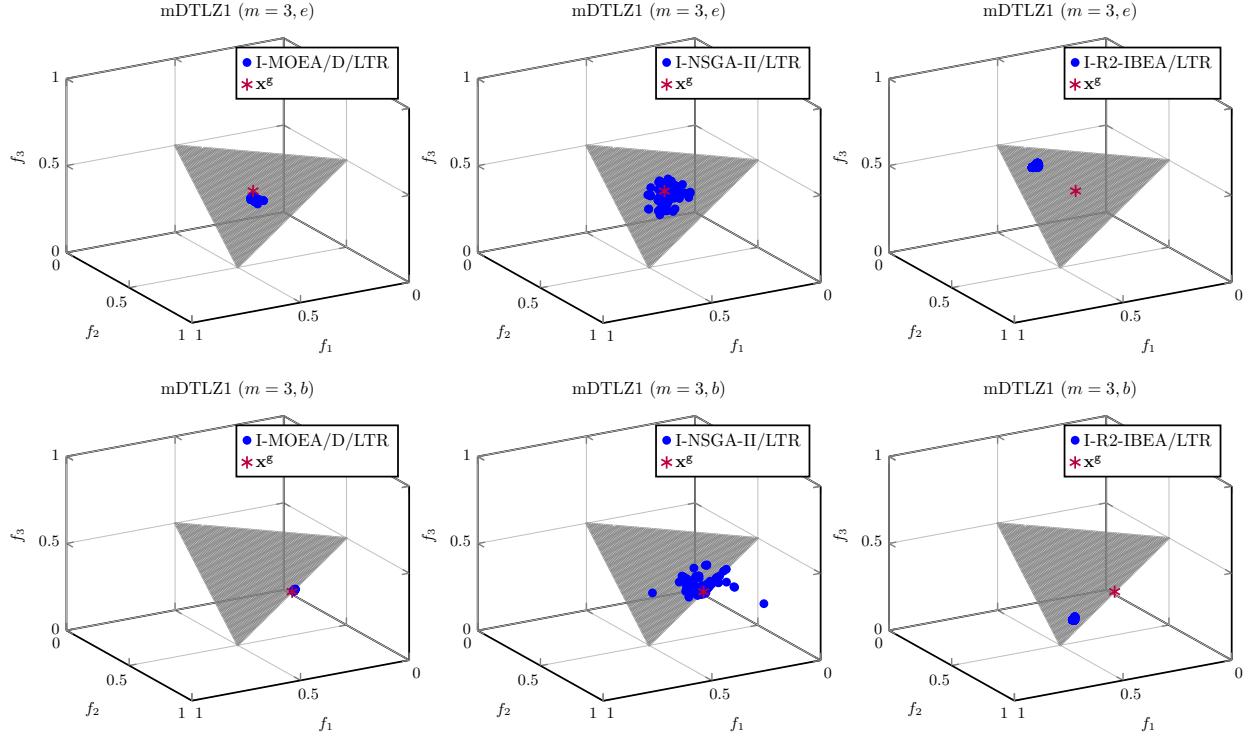


Fig. 28: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on mDTLZ2 when $m = 3$.

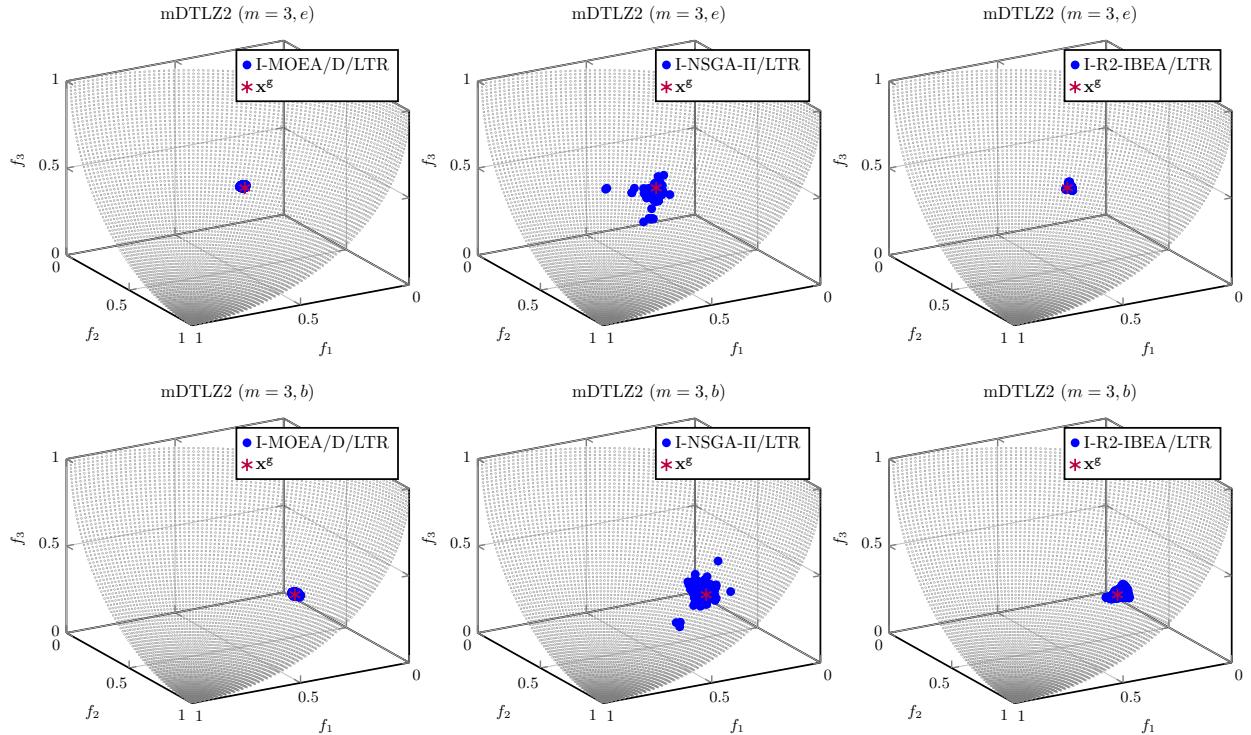


Fig. 29: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on mDTLZ3 when $m = 3$.

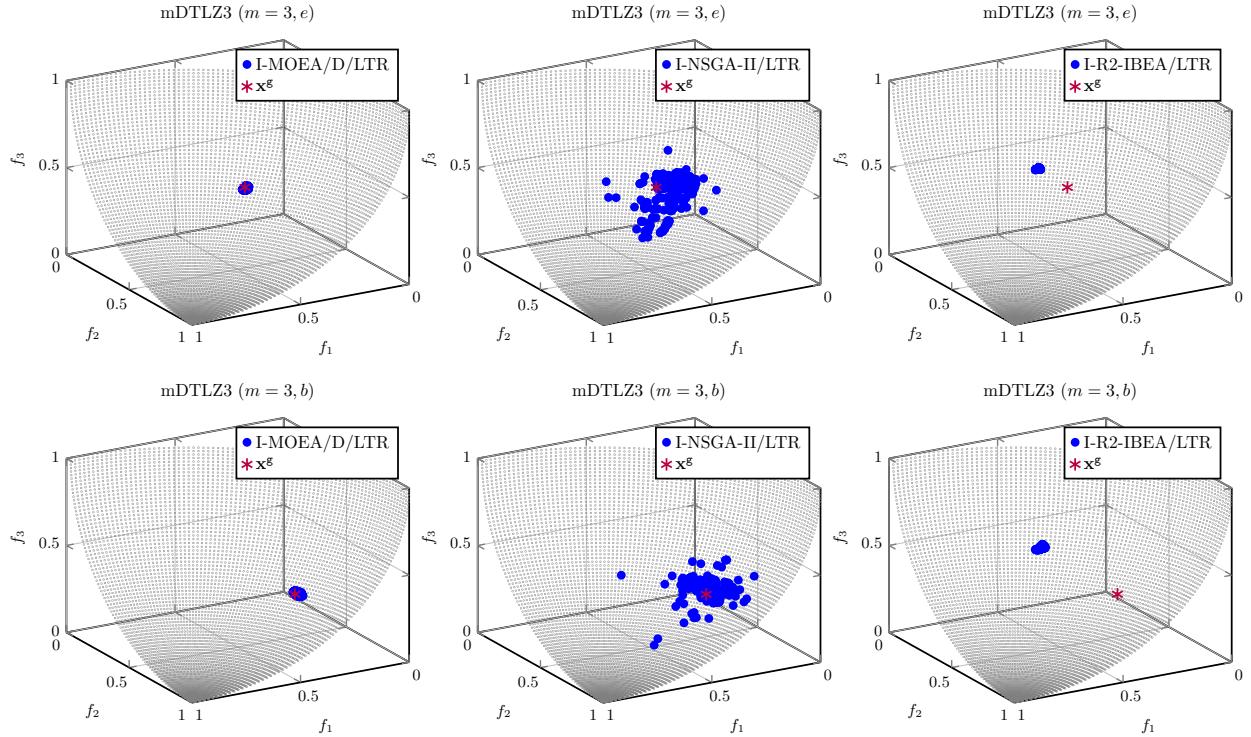


Fig. 30: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on mDTLZ4 when $m = 3$.

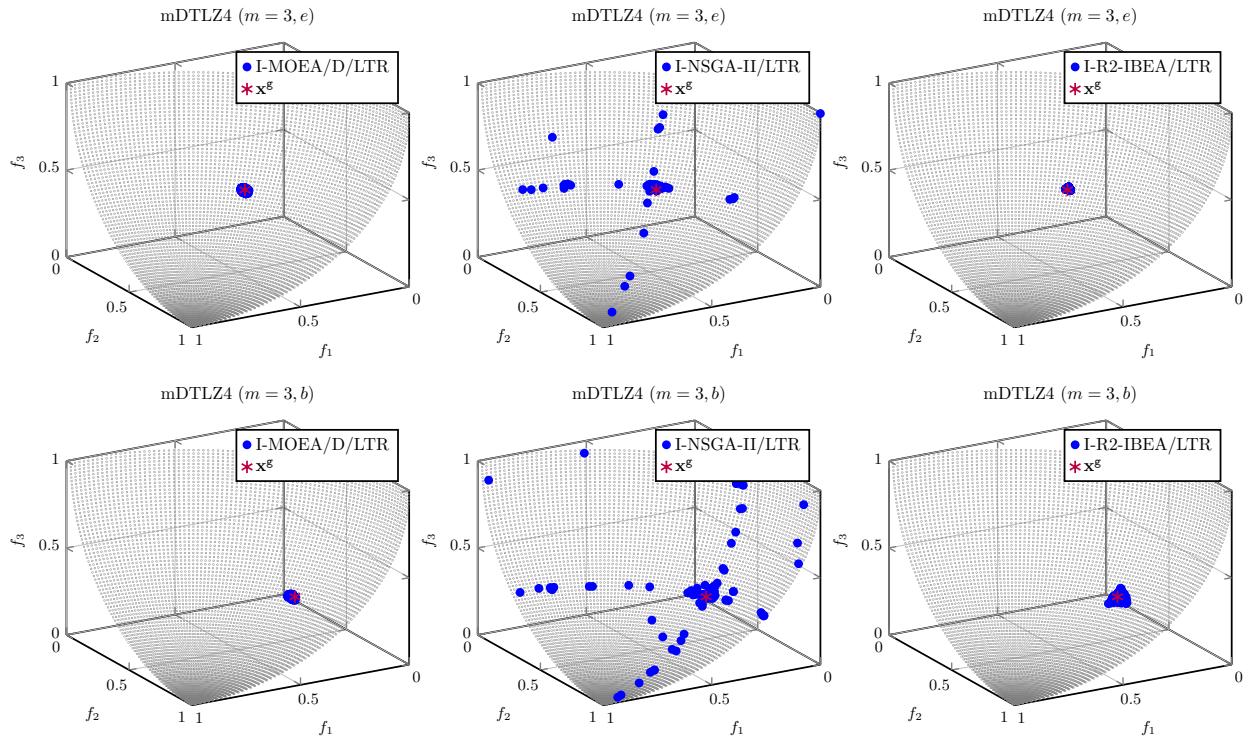


Fig. 31: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on WFG3 when $m = 3$.

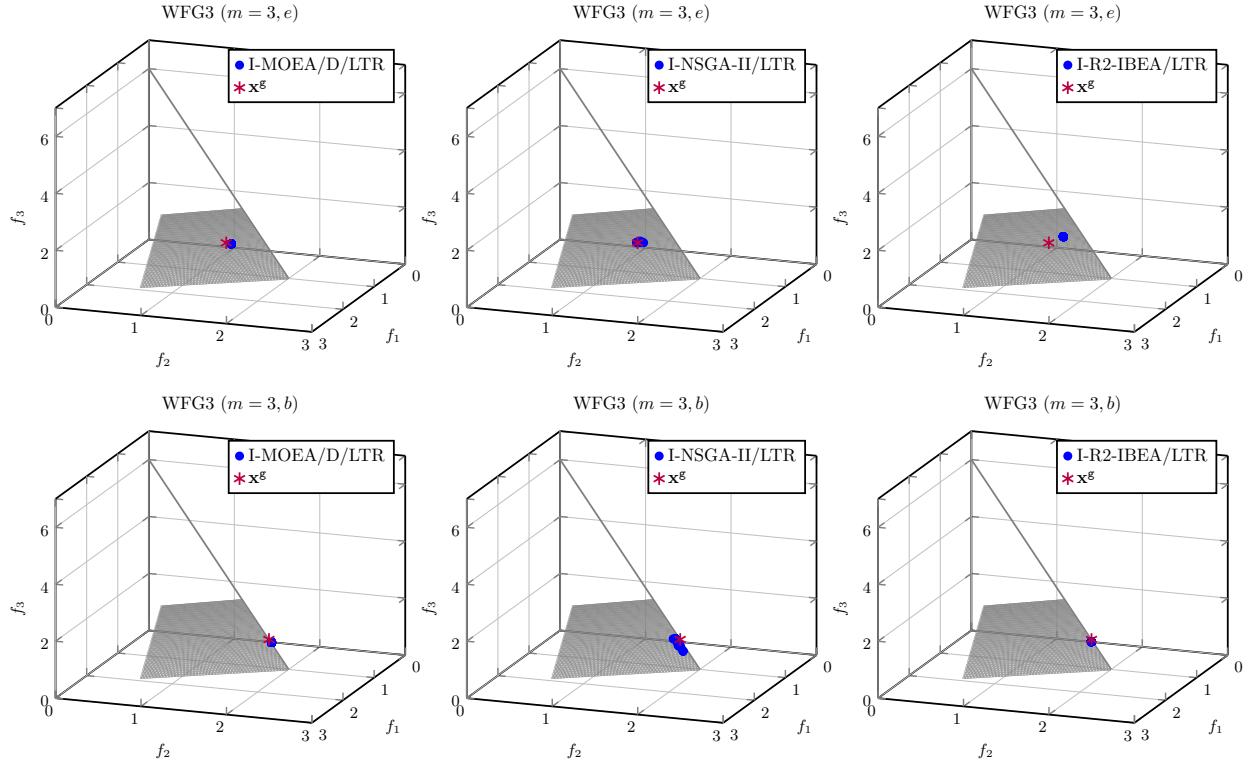


Fig. 32: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ1 when $m = 5$.

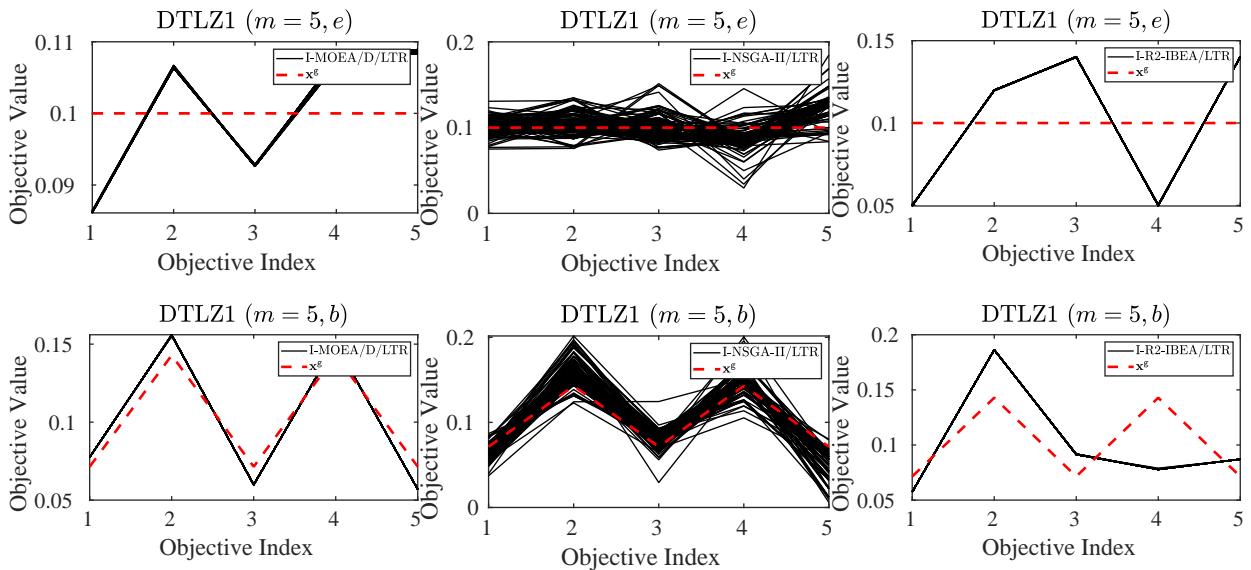


Fig. 33: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ2 when $m = 5$.

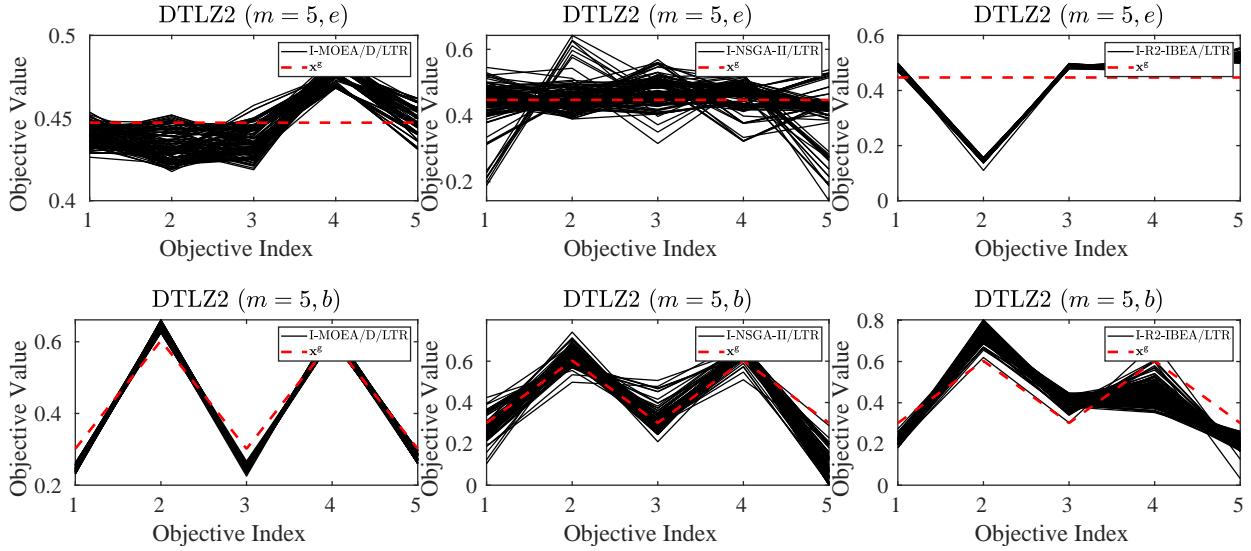


Fig. 34: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ3 when $m = 5$.

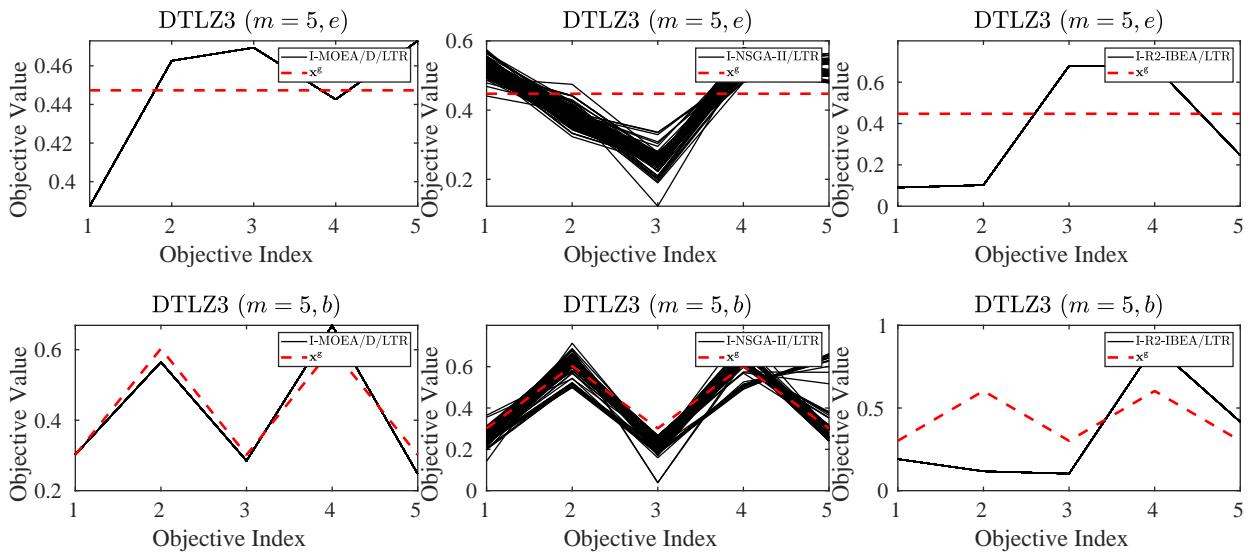


Fig. 35: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ4 when $m = 5$.

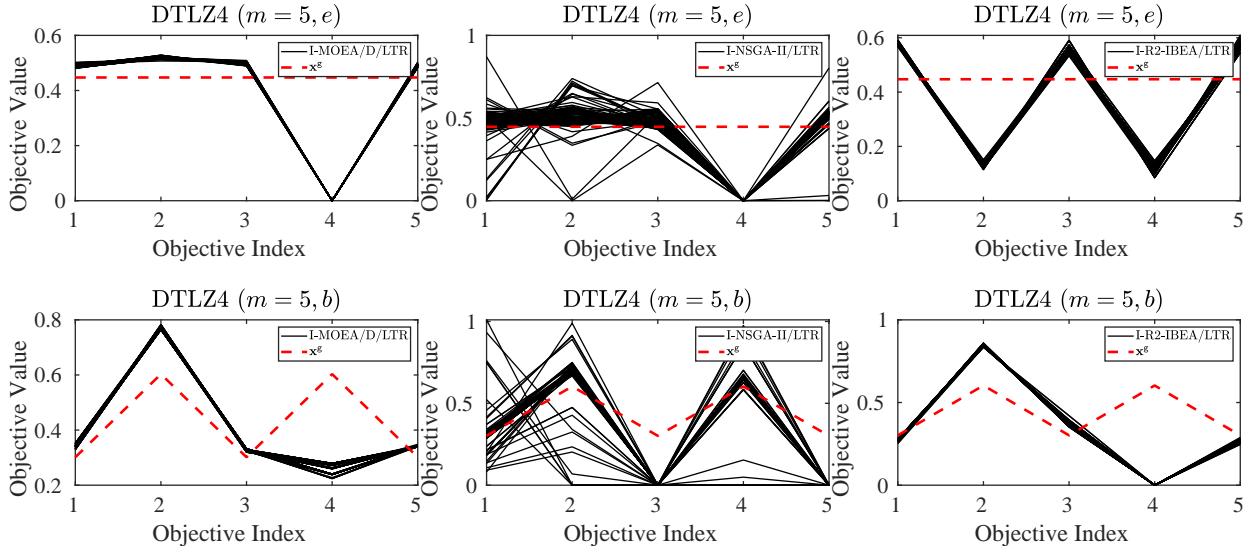


Fig. 36: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ5 when $m = 5$.

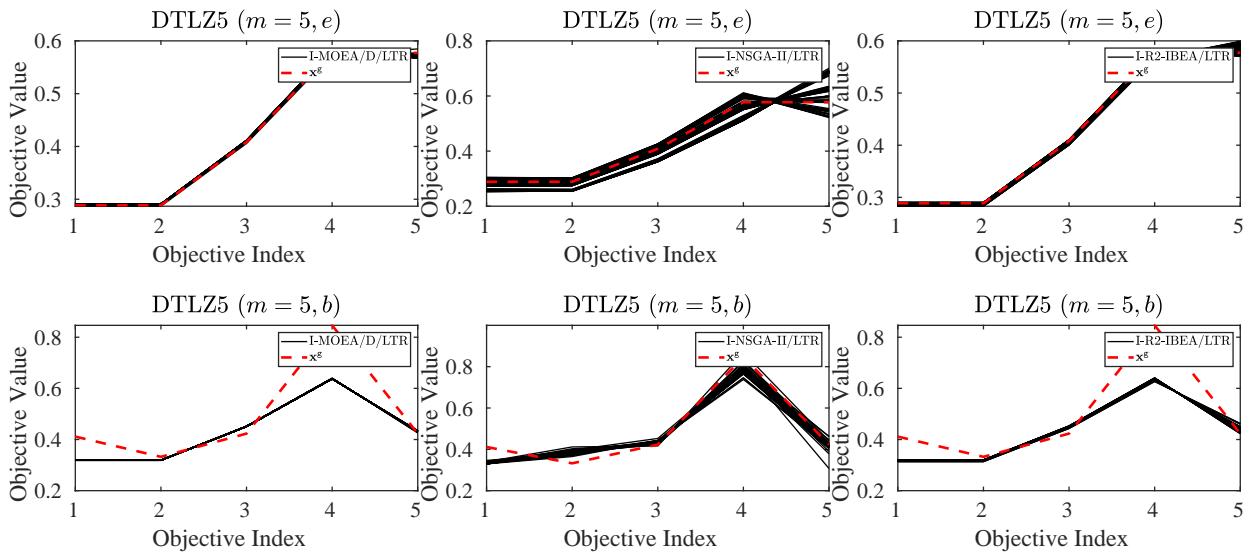


Fig. 37: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ6 when $m = 5$.

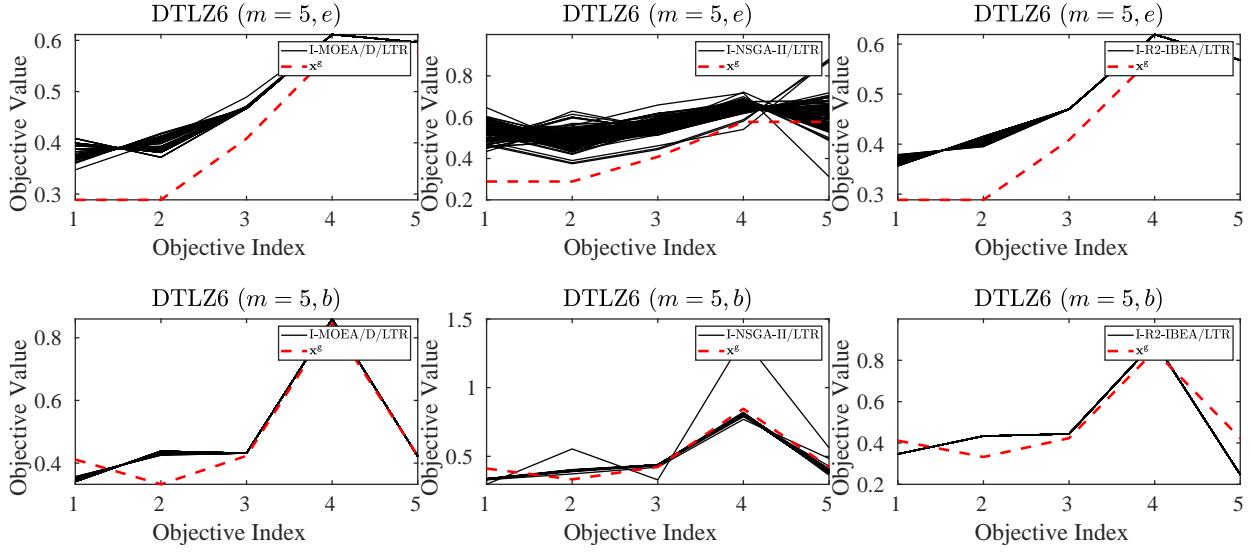


Fig. 38: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on $DTLZ1^{-1}$ when $m = 5$.

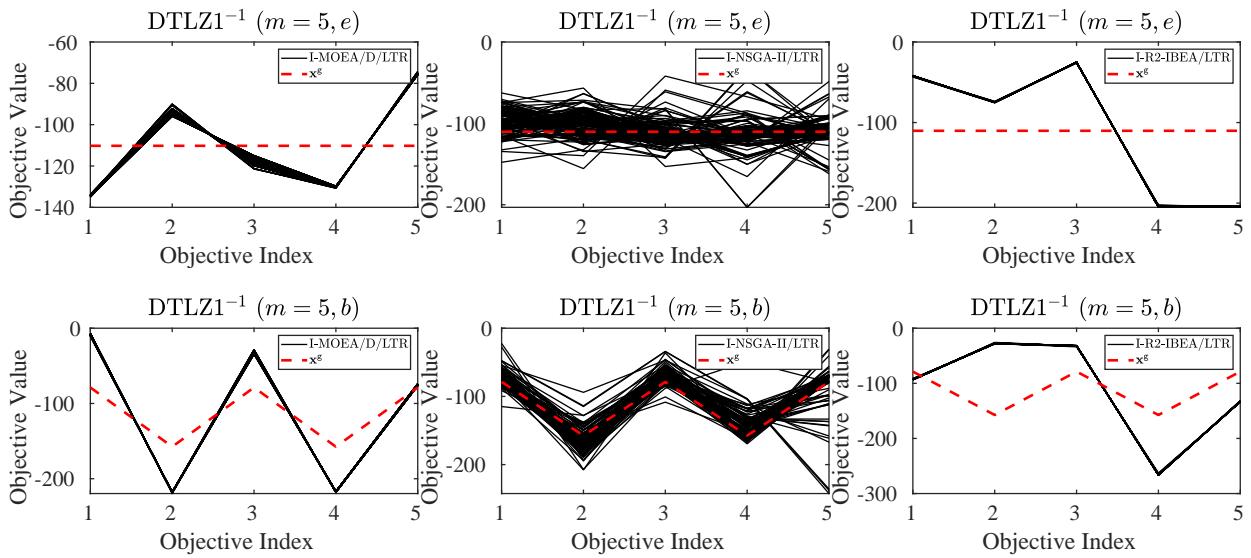


Fig. 39: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on $DTLZ2^{-1}$ when $m = 5$.

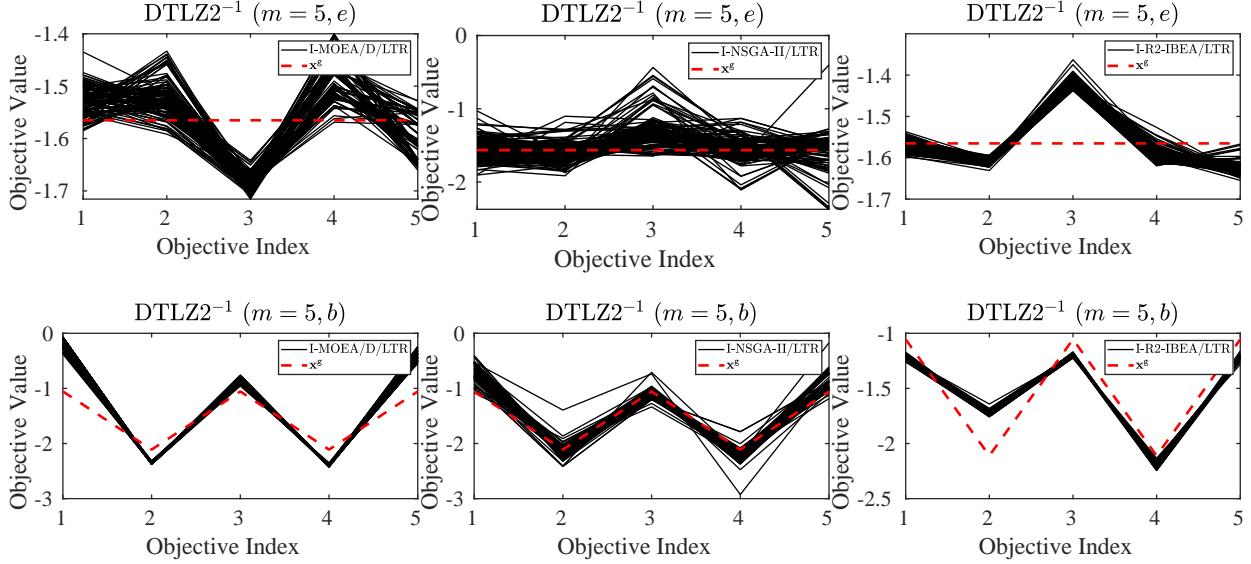


Fig. 40: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on $DTLZ3^{-1}$ when $m = 5$.

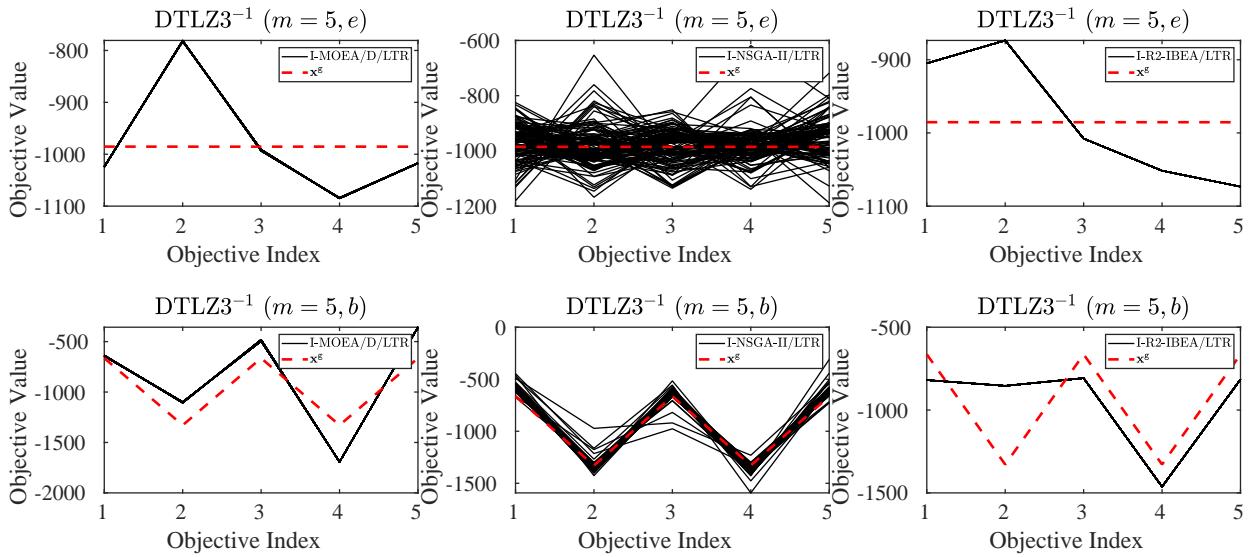


Fig. 41: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ4⁻¹ when $m = 5$.

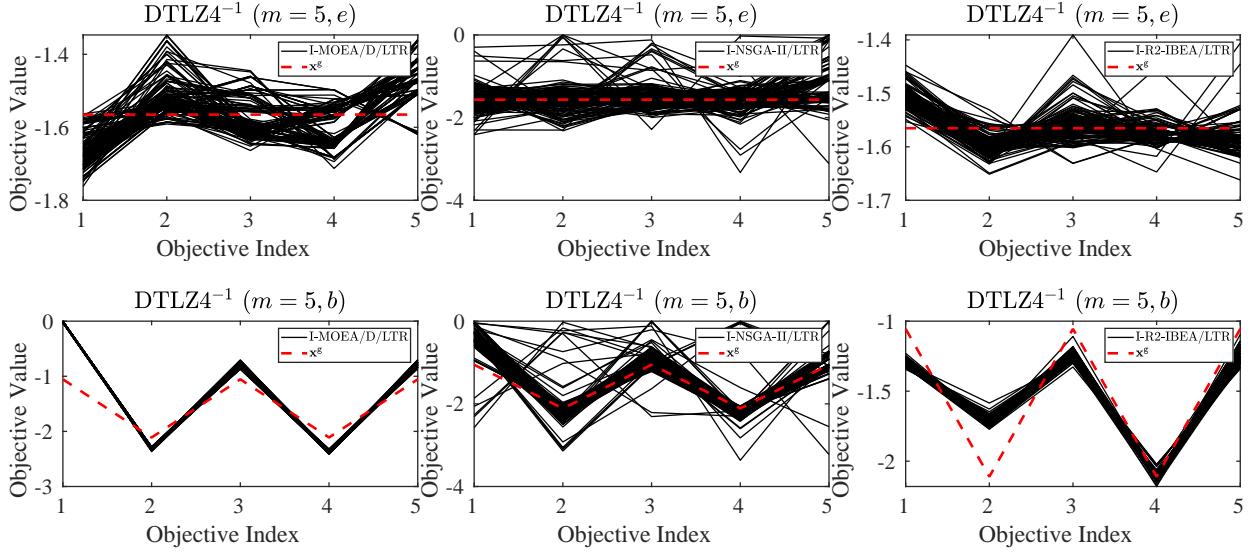


Fig. 42: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on WFG3 when $m = 5$.

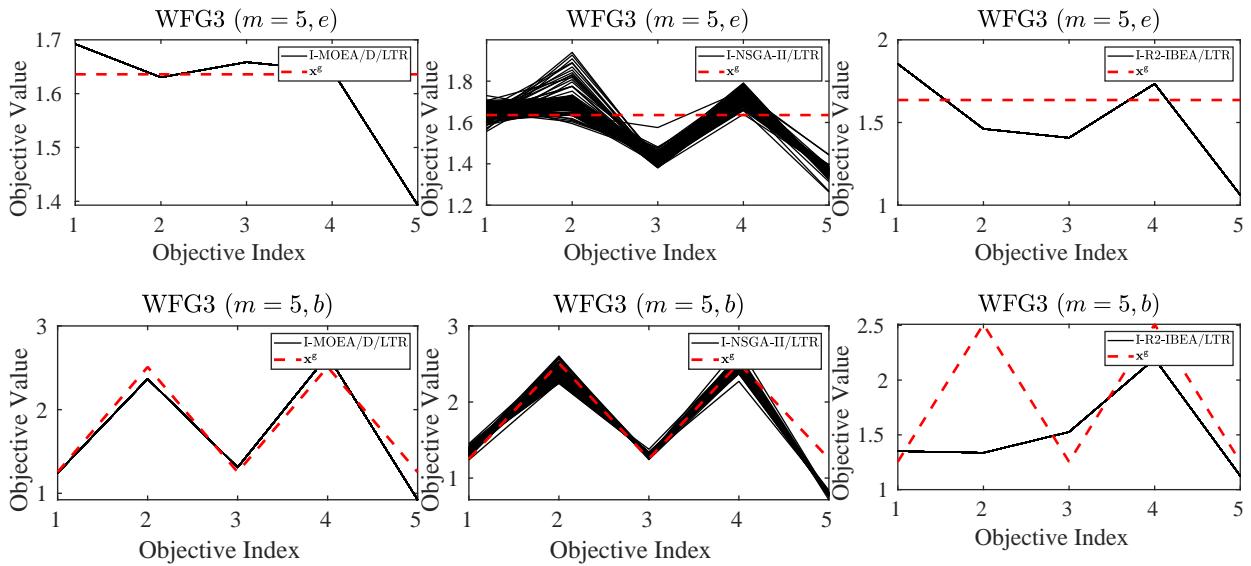


Fig. 43: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ1 when $m = 8$.

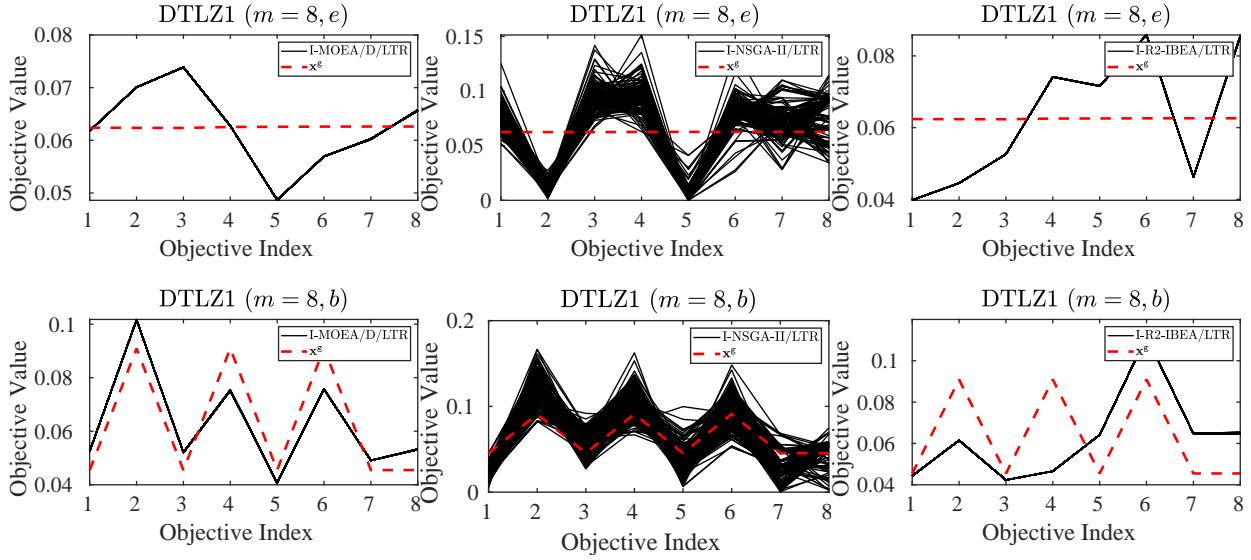


Fig. 44: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ2 when $m = 8$.

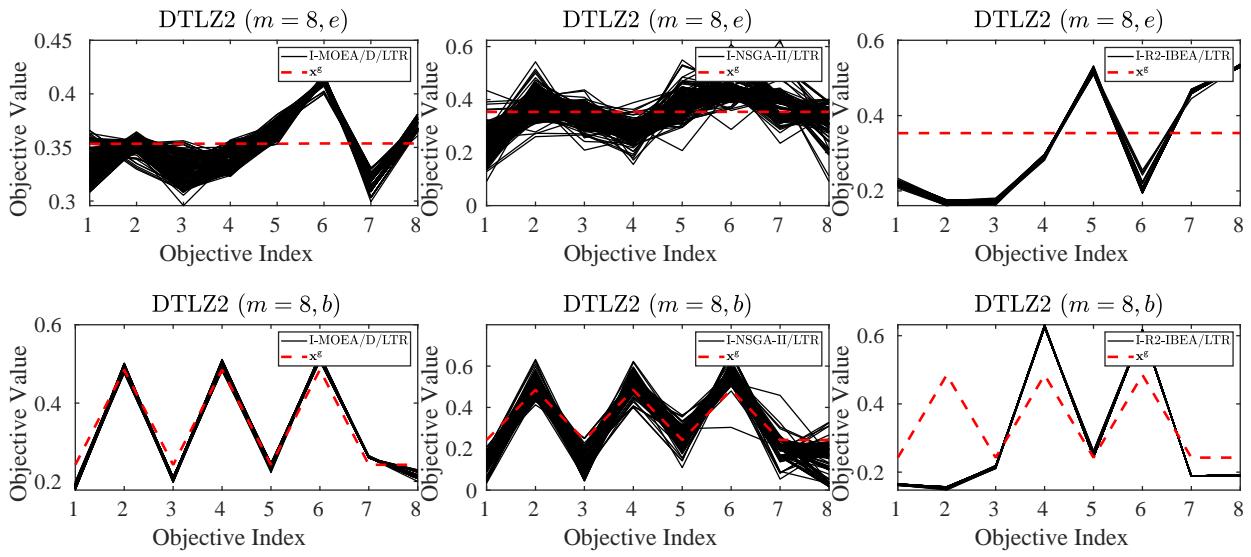


Fig. 45: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ3 when $m = 8$.

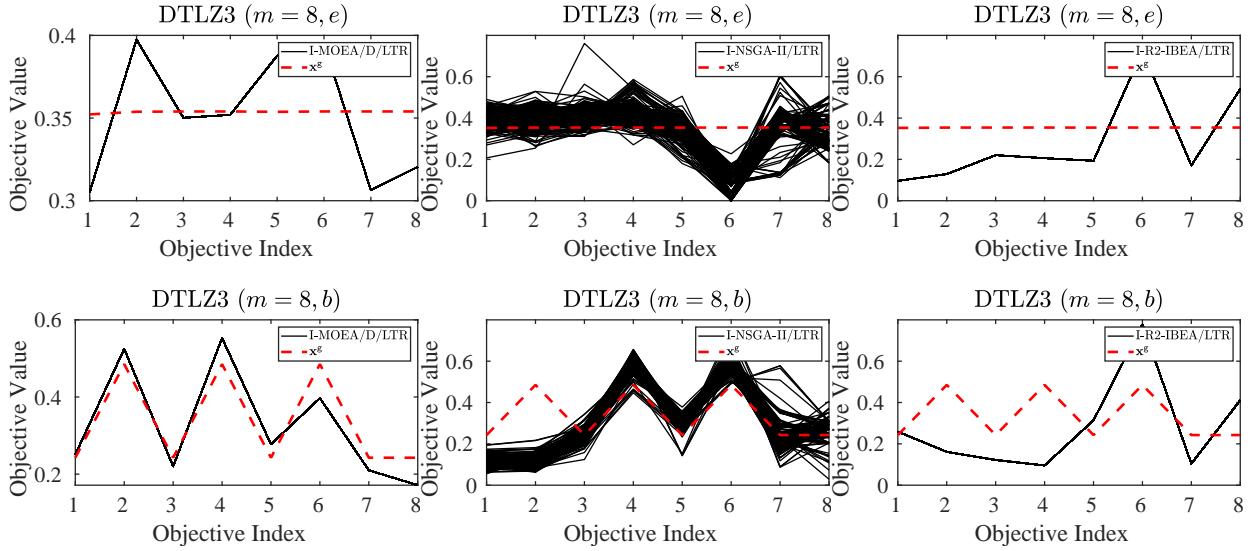


Fig. 46: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ4 when $m = 8$.

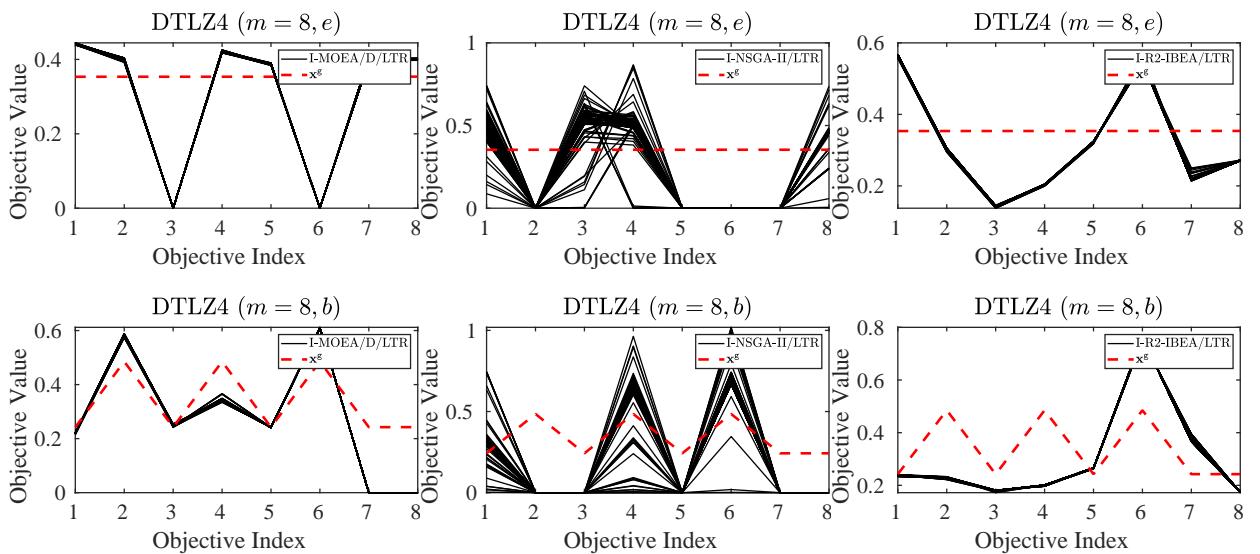


Fig. 47: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ5 when $m = 8$.

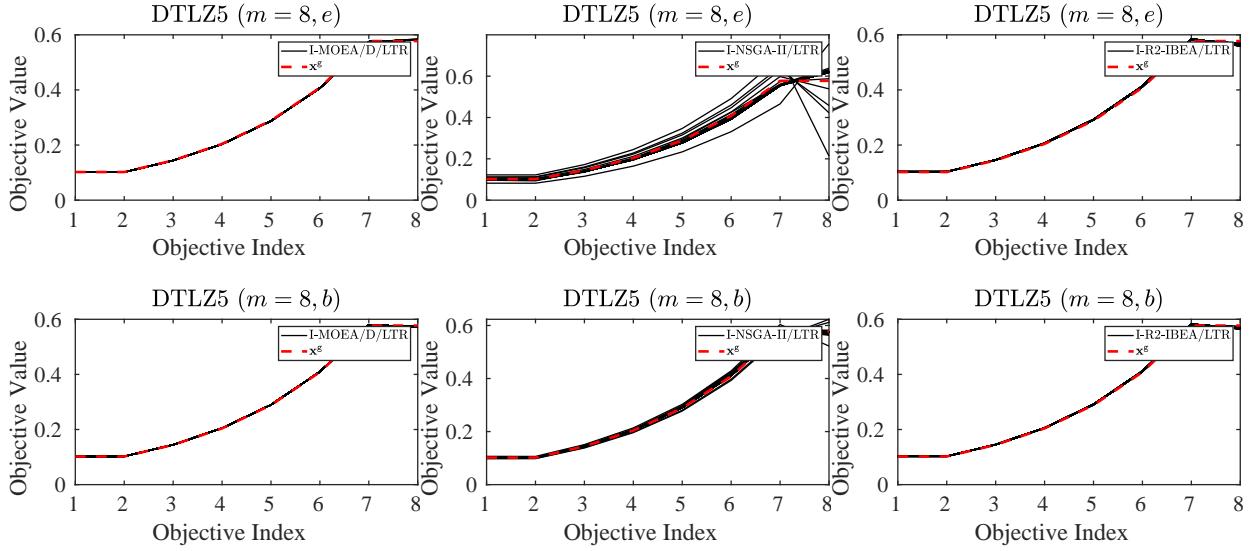


Fig. 48: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ6 when $m = 8$.

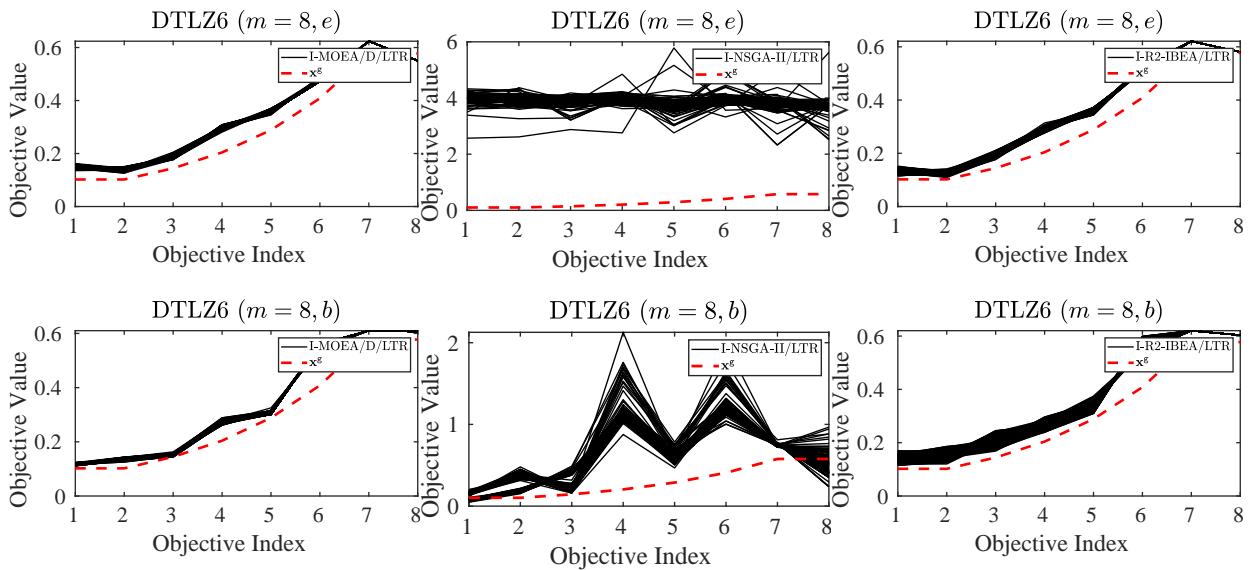


Fig. 49: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ1⁻¹ when $m = 8$.

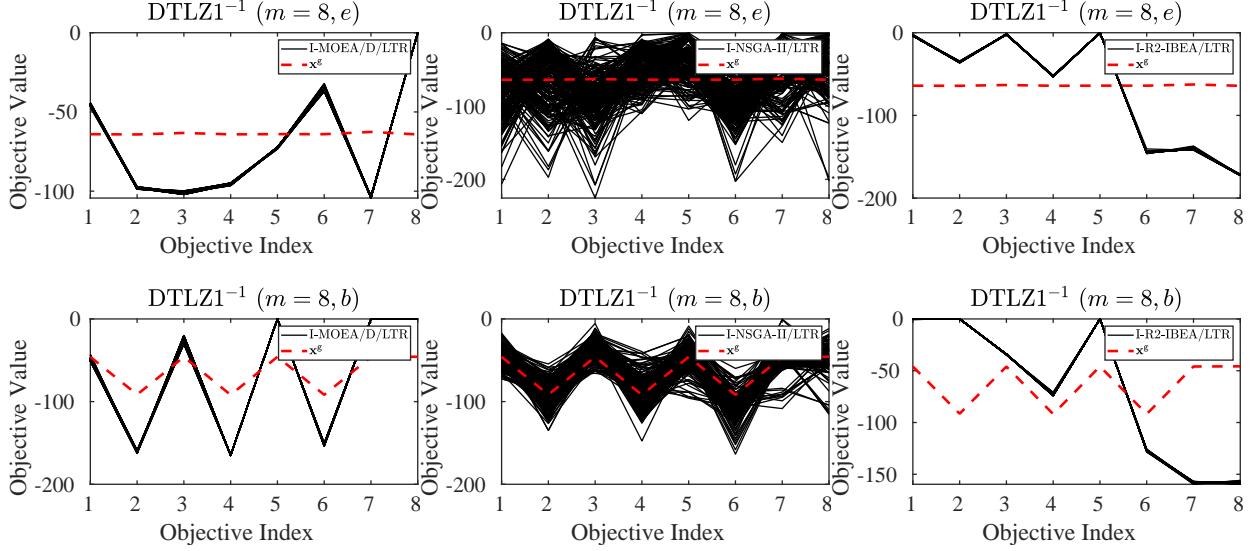


Fig. 50: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ2⁻¹ when $m = 8$.

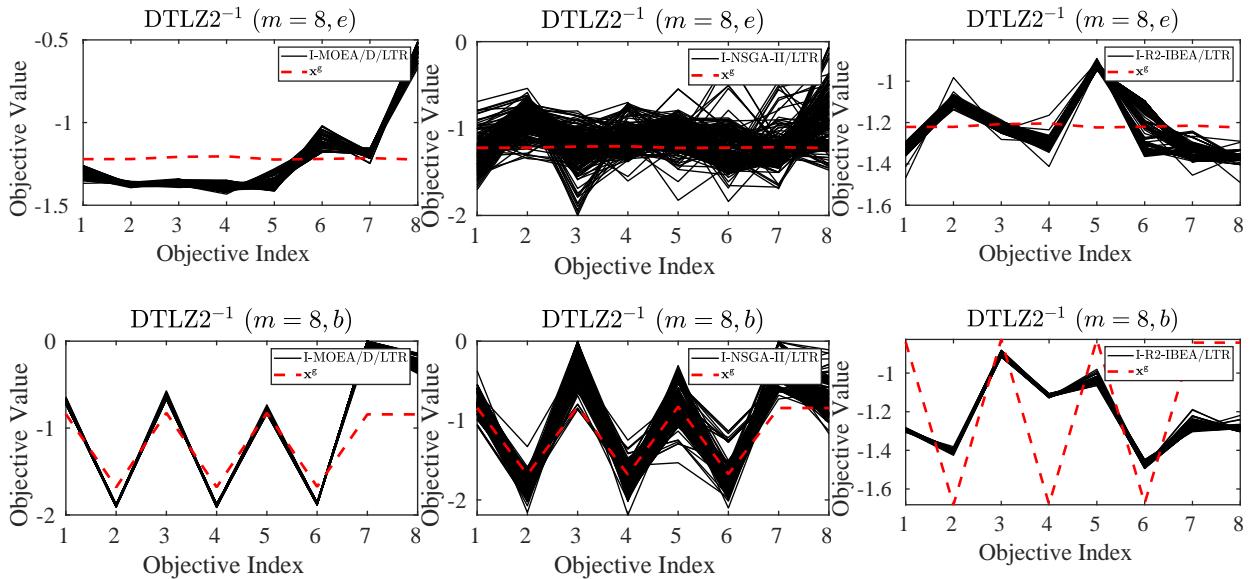


Fig. 51: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ3⁻¹ when $m = 8$.

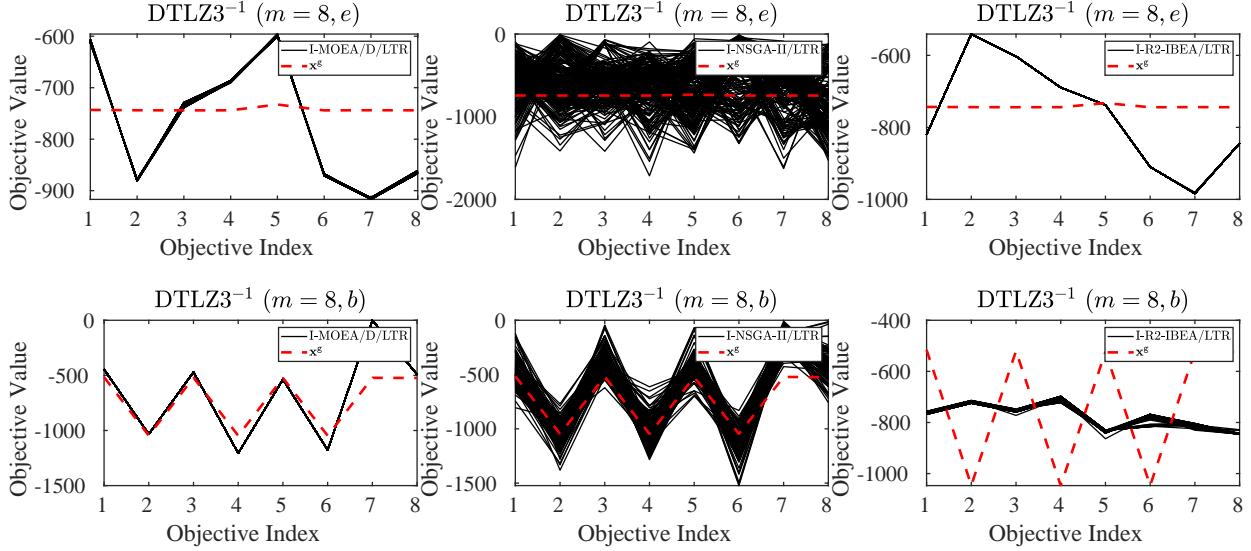


Fig. 52: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ4⁻¹ when $m = 8$.

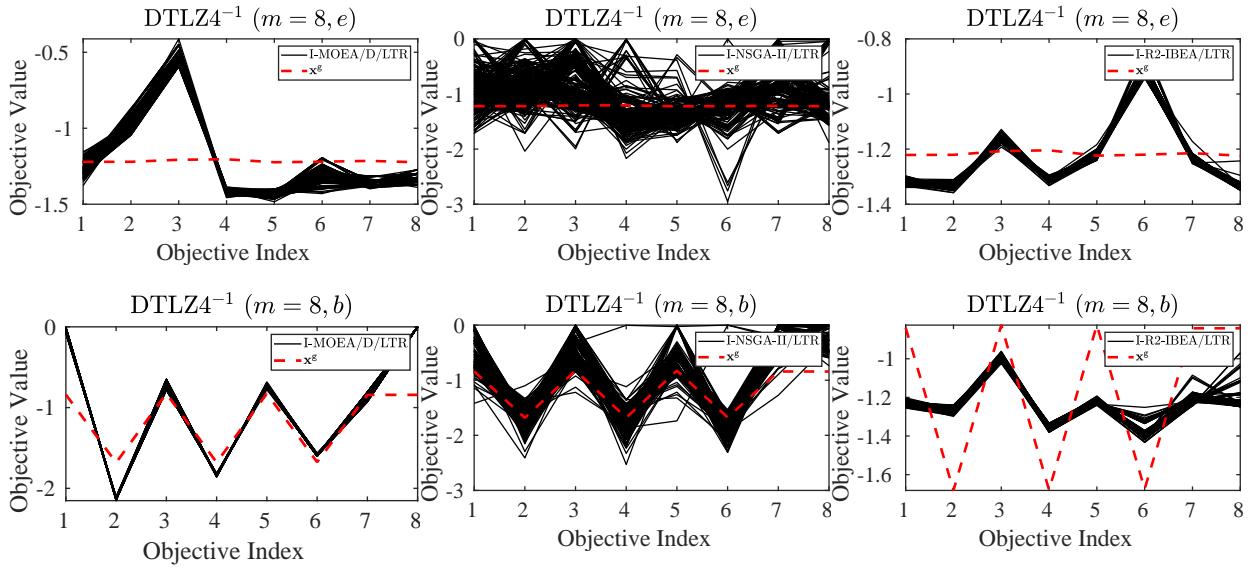


Fig. 53: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on WFG3 when $m = 8$.

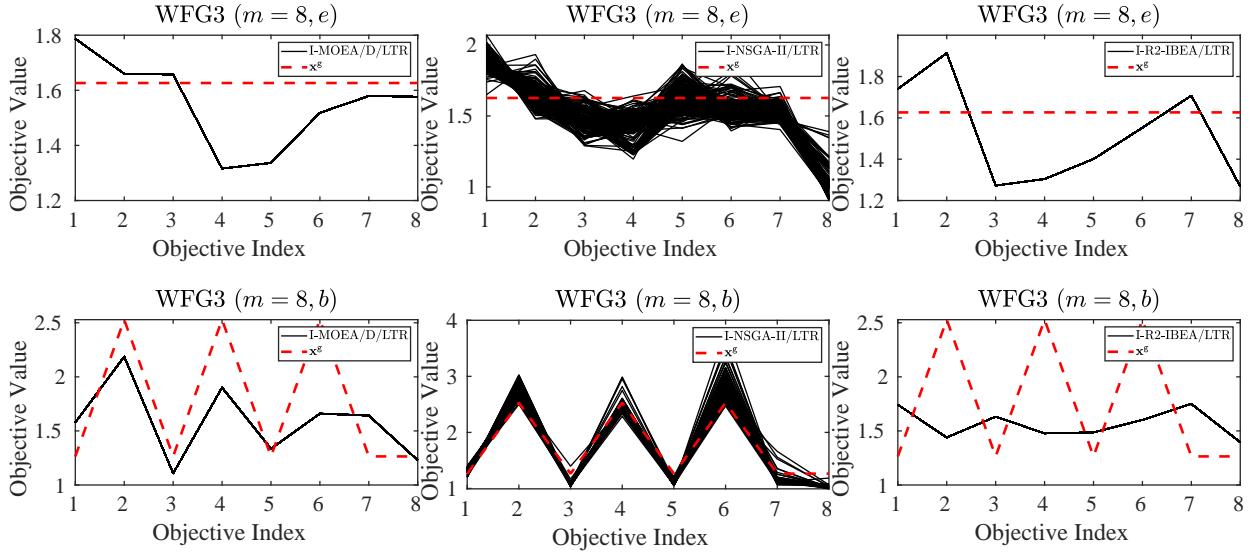


Fig. 54: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ1 when $m = 10$.

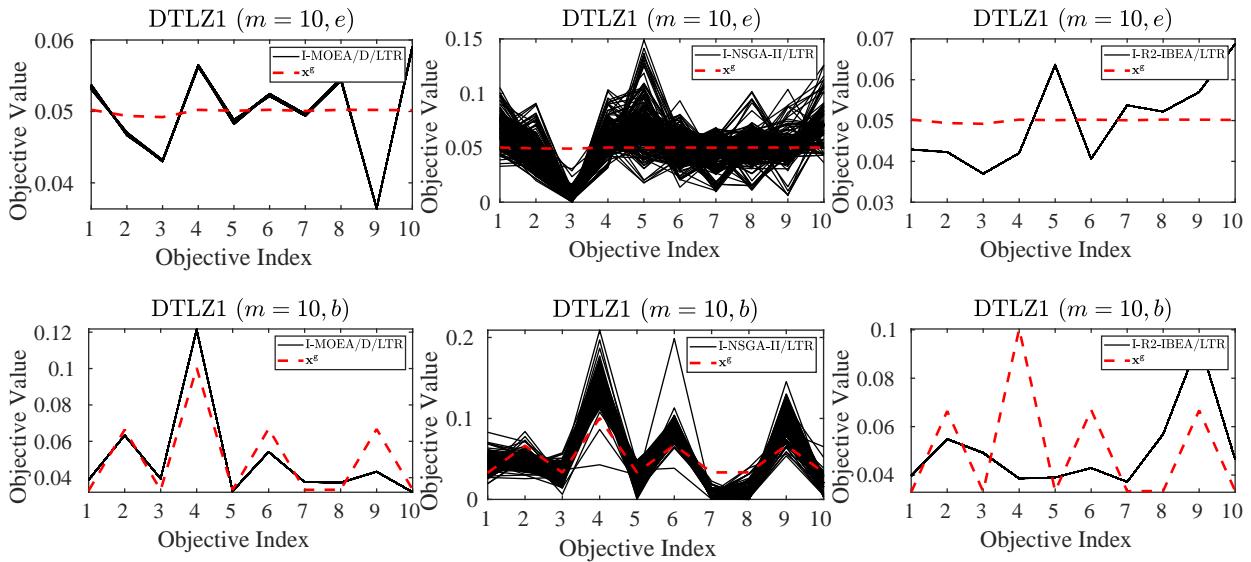


Fig. 55: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ2 when $m = 10$.

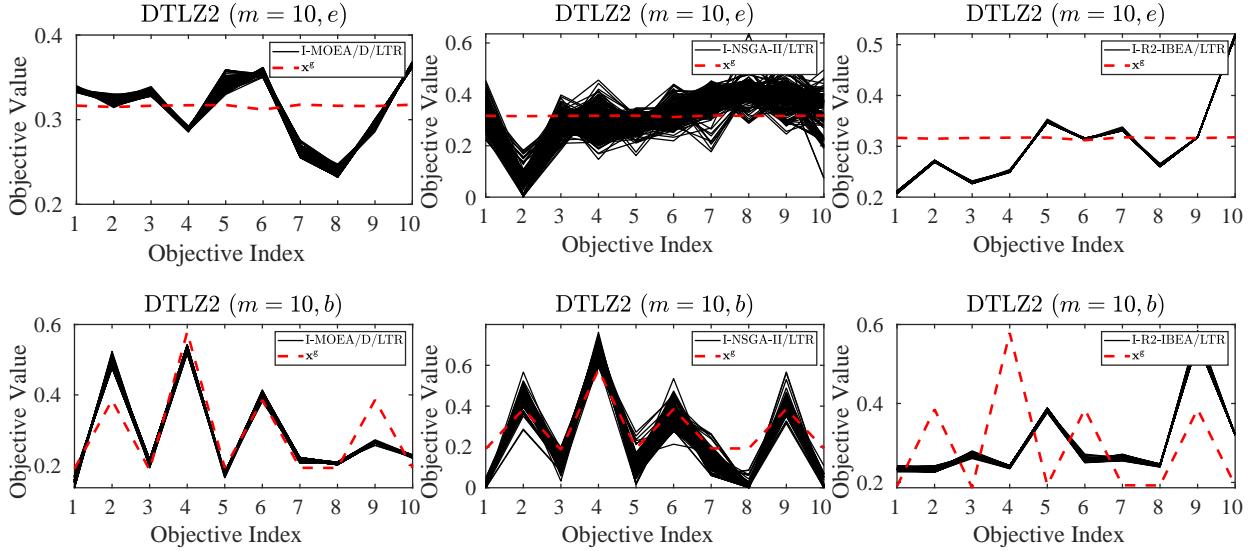


Fig. 56: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ3 when $m = 10$.

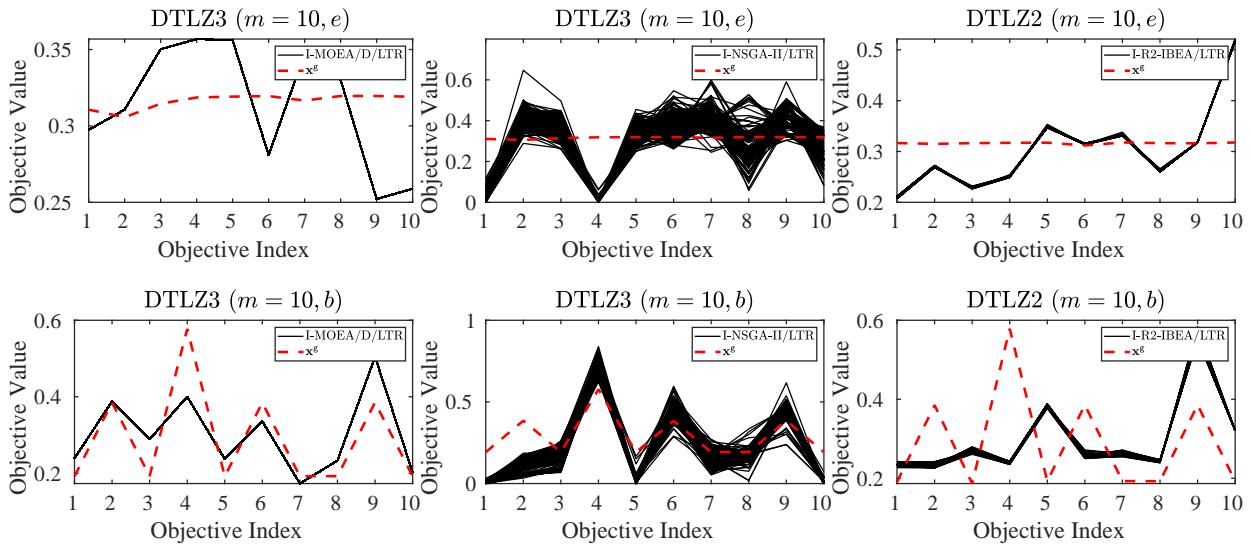


Fig. 57: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ4 when $m = 10$.

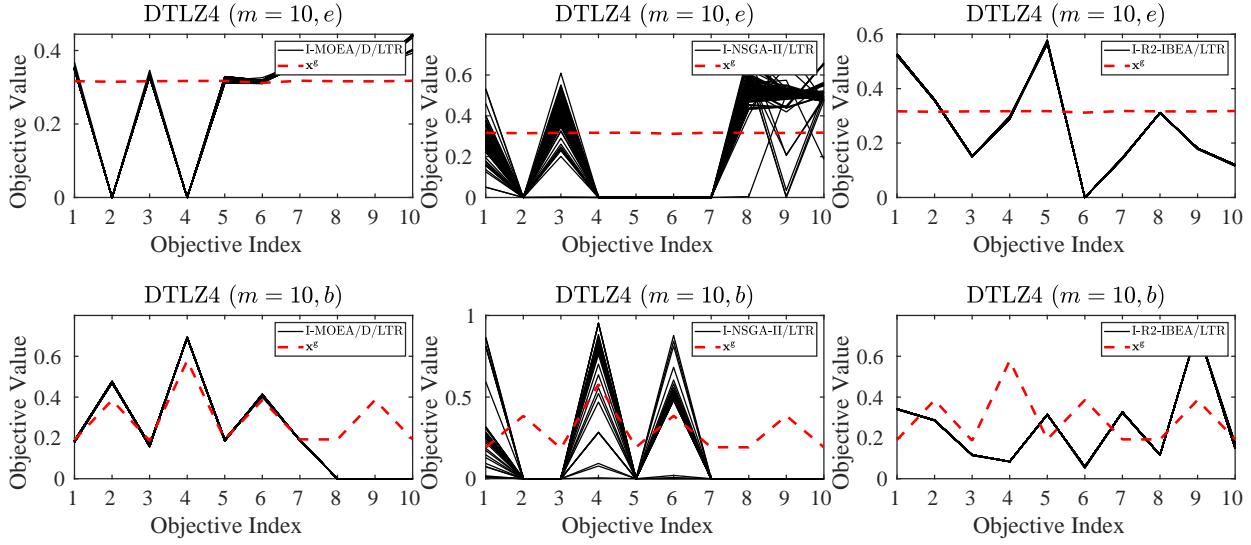


Fig. 58: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ5 when $m = 10$.

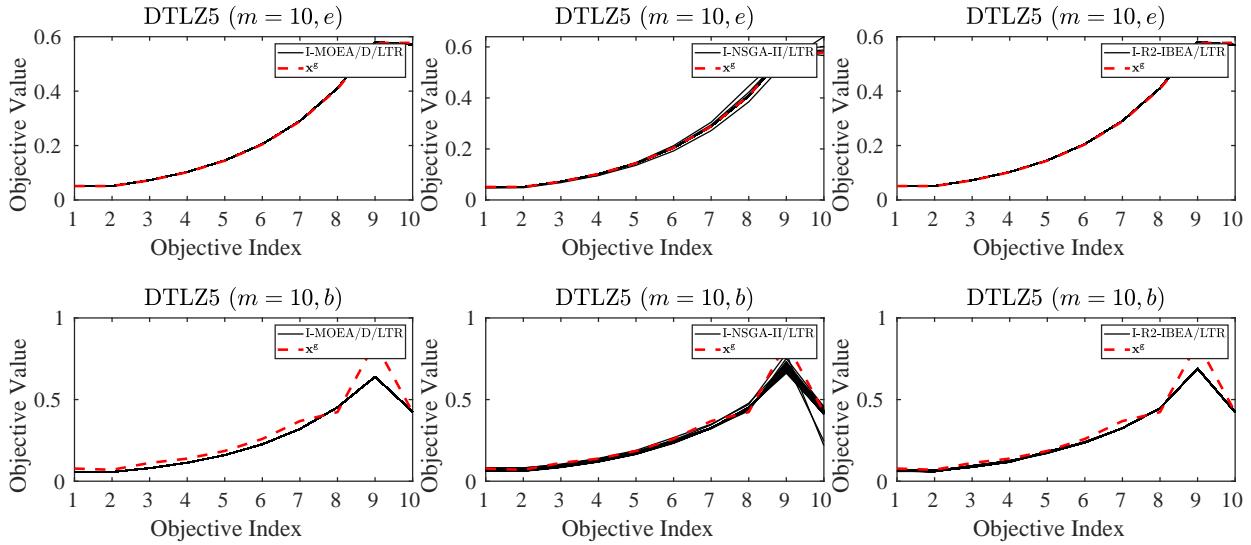


Fig. 59: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ6 when $m = 10$.

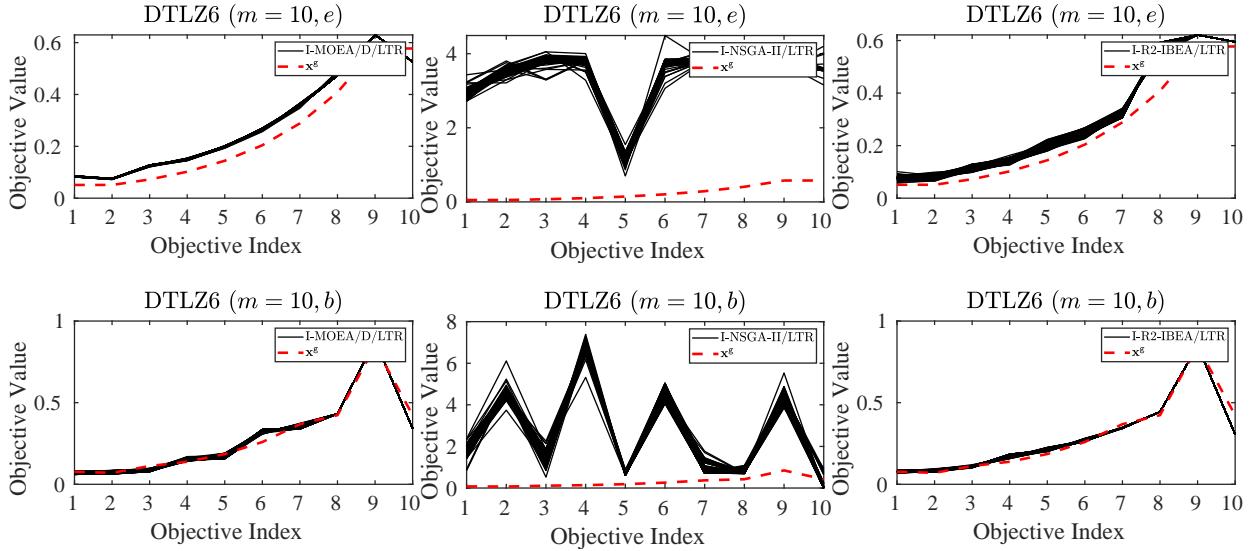


Fig. 60: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on $DTLZ1^{-1}$ when $m = 10$.

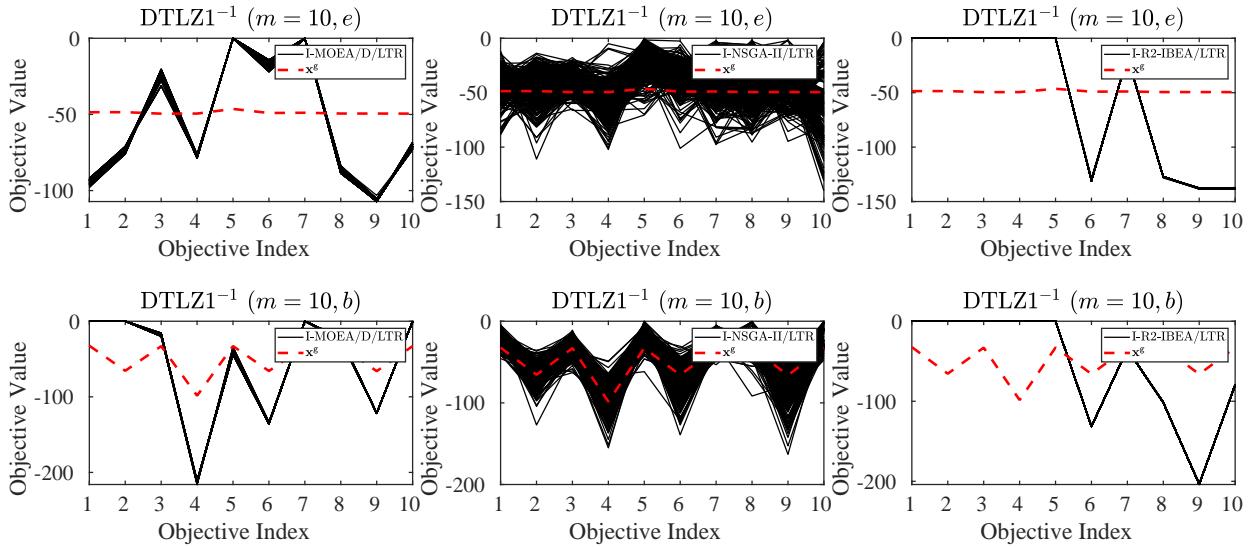


Fig. 61: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ2⁻¹ when $m = 10$.

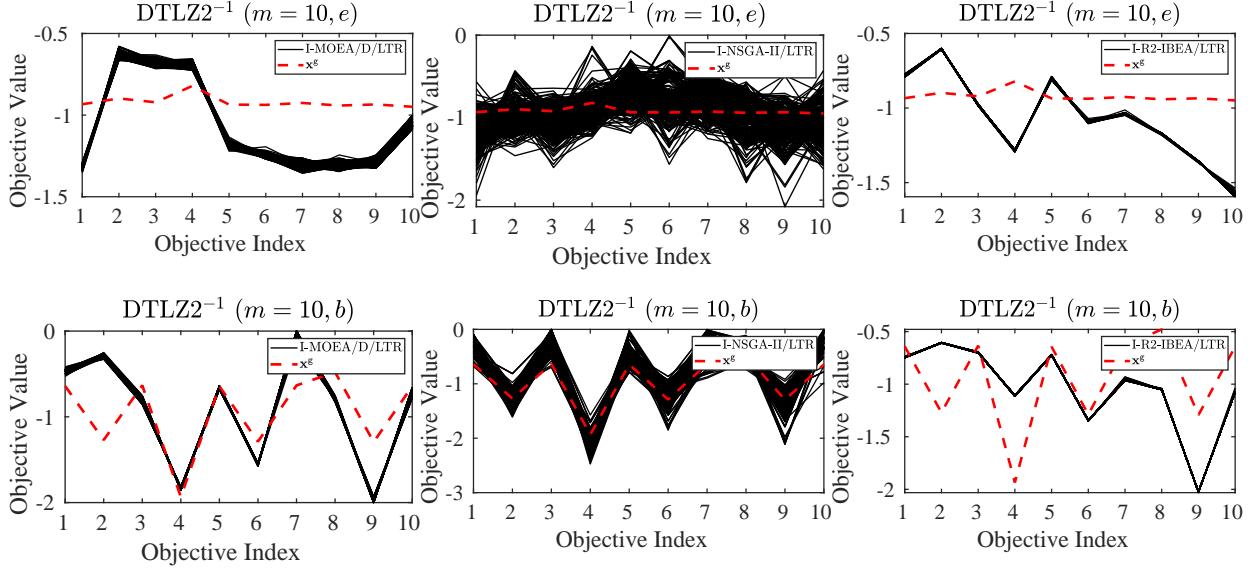


Fig. 62: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ3⁻¹ when $m = 10$.

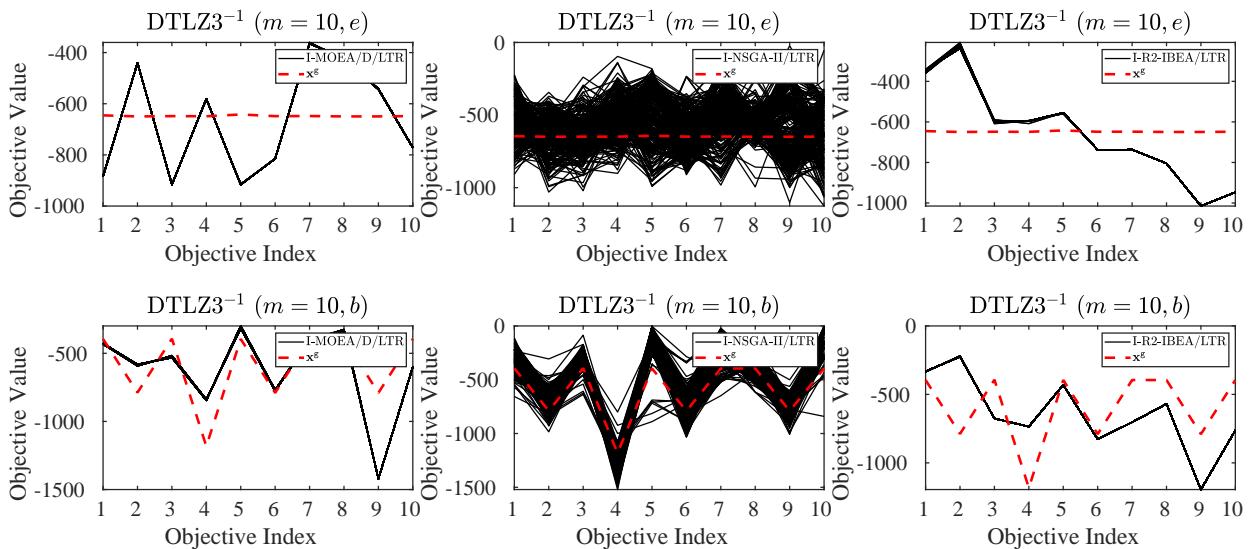


Fig. 63: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on DTLZ4⁻¹ when $m = 10$.

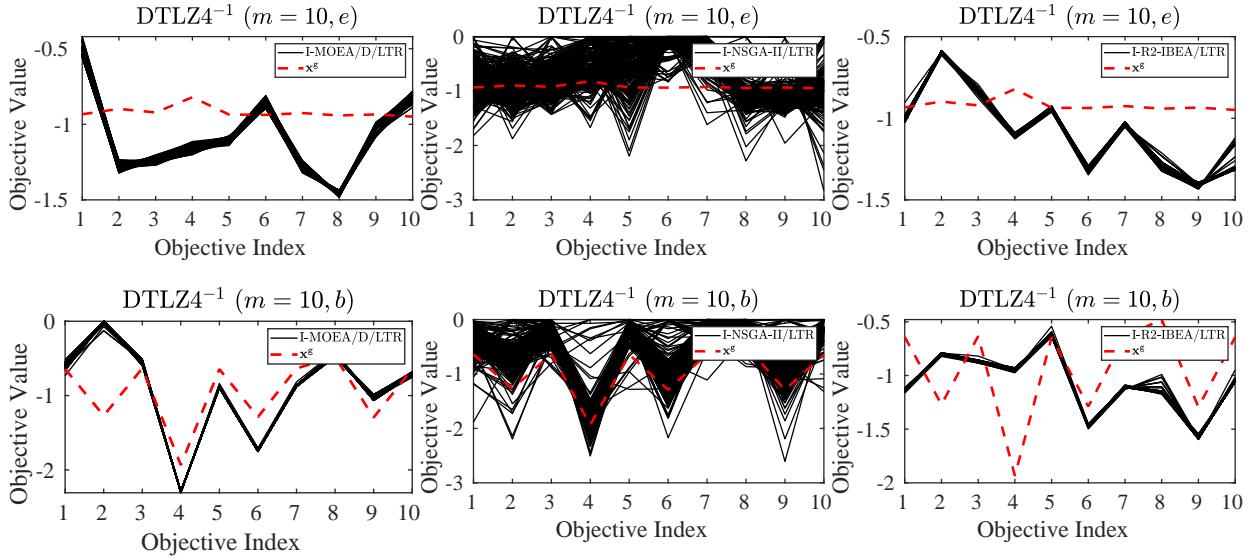
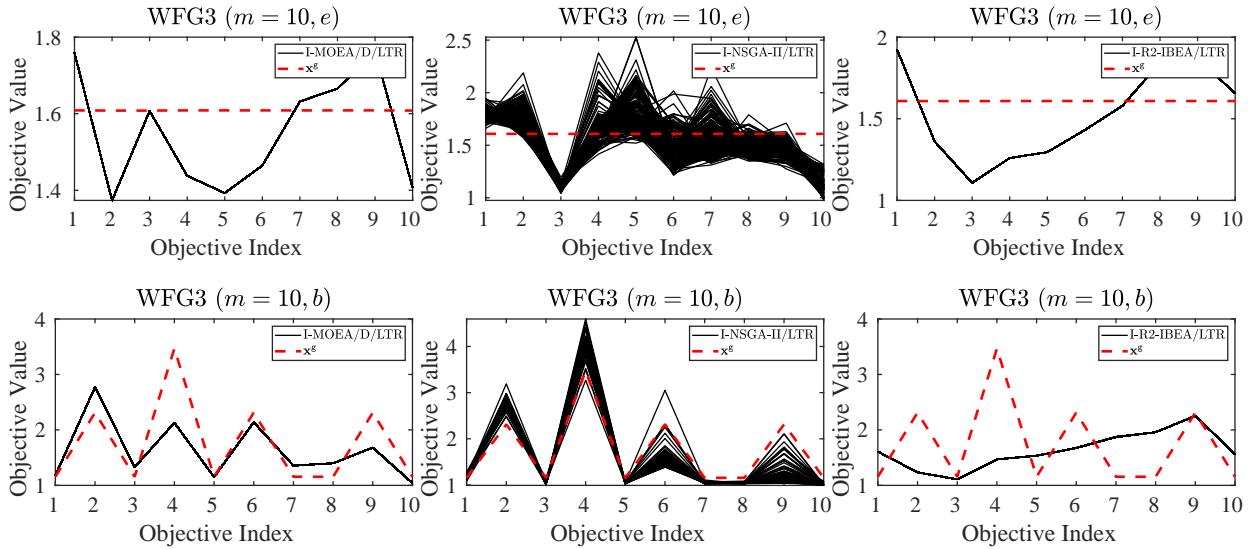


Fig. 64: Population distribution of non-dominated solutions with the medium $\mathbb{E}(\mathcal{P})$ value obtained by I-MOEA/D/LTR, I-NSGA-II/LTR and I-R2-IBEA/LTR on WFG3 when $m = 10$.



REFERENCES

- [1] J. Branke, S. Greco, R. Slowinski, and P. Zielniewicz, "Learning value functions in interactive evolutionary multiobjective optimization," *IEEE Trans. Evol. Comput.*, vol. 19, no. 1, pp. 88–102, 2015.