COM2031 Advanced Algorithms, Autumn Semester 2019

Lab 11: Complexity Classes Visualizations

Exercises Solutions:

1) a) Yes. One solution would be: Interval scheduling can be solved in polynomial time, and so it can be solved in polynomial time with access to a black box of vertex cover. Another solution is Yes, Interval Scheduling is NP, and anything in NP can be reduced to Vertex Cover. A third solution is: we have seen in the book that Interval Scheduling is $\leq_P$ Independent Set, and Independent Set is $\leq_P$ Vertex Cover, so the result follows by transitivity.

1) b) This is equivalent to P = NP. If P = NP, then Independent set can be solved in polynomial time, and so is Independent Set $\leq_P$ Interval Scheduling. Conversely, if Independent Set $\leq_P$ Interval Scheduling, then since Interval scheduling can be solved in in polynomial time, so could be Independent Set. But Independent Set is NP-Complete, so solving it in polynomial time would imply that P = NP.

2) a) The problem is in NP, since we can exhibit a set of k customers, and in polynomial time it can be checked that no two customers bought any product in common. Now we show that Independent Set $\leq_P$ Diverse Subset, given a graph G, and a number k, we construct a customer for each node in G, and a product for each edge in G. We then build an array, that says customer v bought product e if edge e in incident to node v. Finally, we ask, whether this array has a diverse subset of size k.

We claim that this holds if and only if G has independent set of size k. If there is a diverse subset of size k, then the corresponding set of nodes has the property that no two are incident to same edge, so it is an independent set of size k. Conversely, if there is an independent set of size k, then the corresponding set of customers has the property that no two bought the same product, so it is diverse.