

# Scalable Logistic Regression

## XX-XX-XX

Shuo Zhou, PhD

COM6012 Scalable Machine Learning  
26.02.2025



University of  
**Sheffield**

# Contents (Check-in code: XX-XX-XX)

Logistic regression: cross-entropy error function

Optimization and efficient computation

First- and Second-order method

Quasi-Newton methods

Stochastic gradient descent

Beyond basic logistic regression

Regularization

Multi-class logistic regression

Logistic regression in PySpark

References and recommended reading

Acknowledgment

## Learning objectives

- Explain logistic regression using the Bernoulli (binary) and Multinomial (multi-class) distributions and define the cross-entropy loss function.
- Differentiate between first-order and second-order optimization methods and derive the gradient and Hessian for logistic regression.
- Describe Stochastic Gradient Descent (SGD) and Quasi-Newton methods (e.g., BFGS), and apply them to improve training efficiency.
- Analyze the impact of  $\ell_1$ ,  $\ell_2$ , and Elastic Net regularization.
- Describe the implementation of logistic regression in PySpark, including optimization and regularization, and train classifiers on real datasets.

# Contents (Check-in code: XX-XX-XX)

Logistic regression: cross-entropy error function

Optimization and efficient computation

- First- and Second-order method

- Quasi-Newton methods

- Stochastic gradient descent

Beyond basic logistic regression

- Regularization

- Multi-class logistic regression

Logistic regression in PySpark

References and recommended reading

Acknowledgment

## Probabilistic classifier

- A logistic regression model is an example of a probabilistic classifier.
- Let  $\mathbf{x} \in \mathbb{R}^p$  represents a feature vector and  $y$  the target value.
- $y \in \{0, 1\}$  or  $y \in \{-1, +1\}$  for a binary classification problem.
- We model the relationship between  $y$ , and  $\mathbf{x}$  using a Bernoulli distribution.

## Bernoulli distribution

- ❑ A Bernoulli random variable can only take two possible values.
- ❑ A Bernoulli distribution is a probability distribution for  $Y$ , expressed as

$$p(Y = y) = \text{Ber}(y|\mu) = \begin{cases} \mu & y = 1, \\ 1 - \mu & y = 0, \end{cases}$$

where  $\mu = P(Y = 1)$ .

- ❑ The expression above can be summarized in one line using

$$p(Y = y) = \text{Ber}(y|\mu) = \mu^y(1 - \mu)^{1-y},$$

## How are $y$ and $\mathbf{x}$ related in logistic regression?

- The target value  $y$  follows a Bernoulli distribution

$$p(y|\mathbf{x}) = \text{Ber}(y|\mu(\mathbf{x})).$$

- Notice how the probability  $\mu = P(y = 1)$  explicitly depends on  $\mathbf{x}$ .
- In logistic regression, the probability  $\mu(\mathbf{x})$  is given as

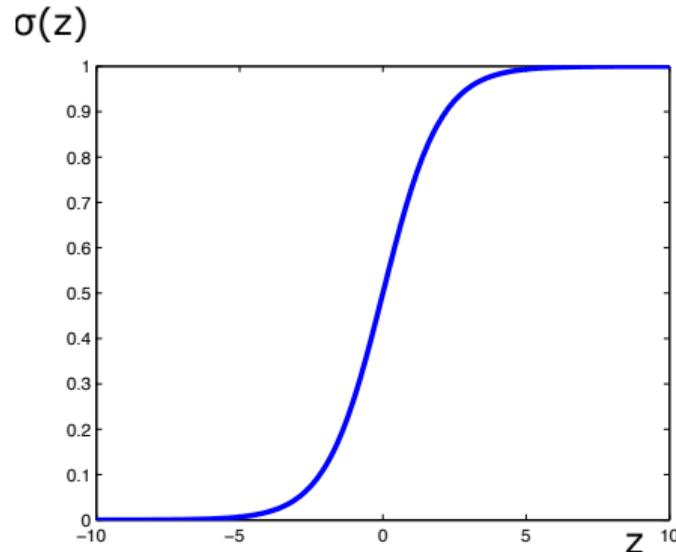
$$\mu(\mathbf{x}) = \frac{1}{1 + \exp(-\mathbf{w}^\top \mathbf{x})} = \sigma(\mathbf{w}^\top \mathbf{x}),$$

where  $\sigma(z)$  is known as the *sigmoid function*, and  $\mathbf{w}$  is the coefficient vector to learn.

- We then have

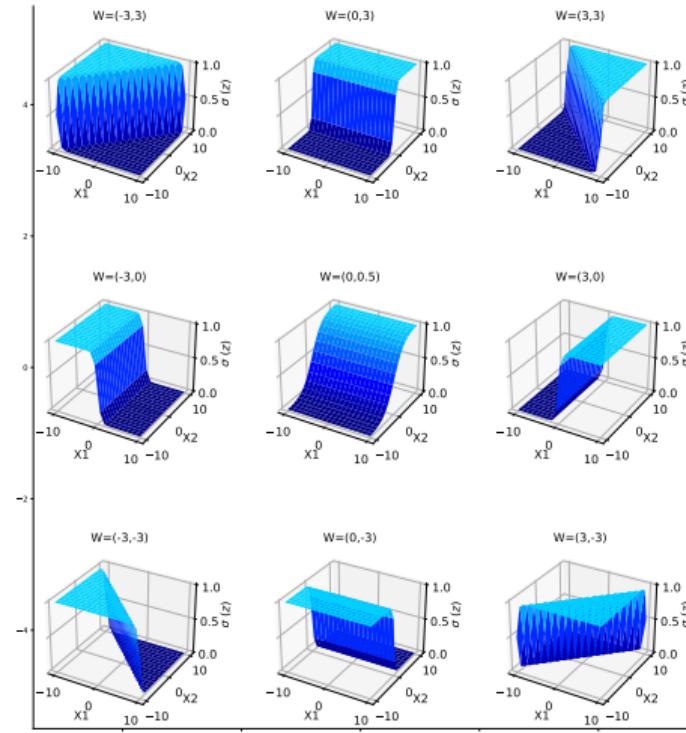
$$p(y|\mathbf{w}, \mathbf{x}) = \text{Ber}(y|\sigma(\mathbf{w}^\top \mathbf{x})).$$

# The sigmoid function $\sigma(z)$



- Recall  $\sigma(z) = \frac{1}{1 + \exp(-z)}$ .
- If  $z \rightarrow \infty$ ,  $\sigma(z) \rightarrow 1$ . If  $z \rightarrow -\infty$ ,  $\sigma(z) \rightarrow 0$ . If  $z = 0$ ,  $\sigma(z) = 0.5$ .
- Let  $z = \mathbf{w}^\top \mathbf{x}$  in logistic regression.

# The sigmoid function in 2d



Plot of  $\sigma(w_1 x_1 + w_2 x_2)$ . Here  $\mathbf{w} = [w_1 \ w_2]^\top$ . Generated by code at [this link](#).

## Decision boundary

- After the training phase, we will have an estimation for  $\mathbf{w}$ .
- For a test input vector  $\mathbf{x}_*$ , we compute  $p(y = 1 | \mathbf{w}, \mathbf{x}_*) = \sigma(\mathbf{w}^\top \mathbf{x}_*)$ .
- This will give us a value between 0 and 1.
- We define a threshold of 0.5 to decide to which class we assign  $\mathbf{x}_*$ .
- With this threshold we induce a linear decision boundary in the input space.

## Cross-entropy error function

- Let  $\mathcal{D} = \{\mathbf{x}_n, y_n\}_{n=1}^N$ .
- We write  $\mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_N]^\top$ , and  $\mathbf{y} = [y_1 \cdots y_N]^\top$ .
- Assuming IID observations

$$p(\mathbf{y}|\mathbf{w}, \mathbf{X}) = \prod_{n=1}^N p(y_n|\mathbf{w}, \mathbf{x}_n) = \prod_{n=1}^N \text{Ber}(y_n|\sigma(\mathbf{w}^\top \mathbf{x}_n)).$$

- The cross-entropy function or negative log-likelihood is given as

$$\begin{aligned} NLL(\mathbf{w}) &= -\log p(\mathbf{y}|\mathbf{w}, \mathbf{X}) \\ &= -\sum_{n=1}^N \{y_n \log[\sigma(\mathbf{w}^\top \mathbf{x}_n)] + (1 - y_n) \log[1 - \sigma(\mathbf{w}^\top \mathbf{x}_n)]\}, \end{aligned}$$

which can be minimized with respect to  $\mathbf{w}$ .

# Contents (Check-in code: XX-XX-XX)

Logistic regression: cross-entropy error function

## Optimization and efficient computation

First- and Second-order method

Quasi-Newton methods

Stochastic gradient descent

Beyond basic logistic regression

Regularization

Multi-class logistic regression

Logistic regression in PySpark

References and recommended reading

Acknowledgment

## General problem

- We are given a function  $\mathcal{L}(\mathbf{w})$ , where  $\mathbf{w} \in \mathbb{R}^p$ .
- Aim: to find a value for  $\mathbf{w}$  that minimizes  $\mathcal{L}(\mathbf{w})$ .
- Use an iterative procedure

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \eta \mathbf{d}_k,$$

where  $\mathbf{d}_k$  is known as the search direction in the  $k$ th step and it is such that

$$\mathcal{L}(\mathbf{w}_{k+1}) < \mathcal{L}(\mathbf{w}_k).$$

- The parameter  $\eta$  is known as the **step size** or **learning rate**.

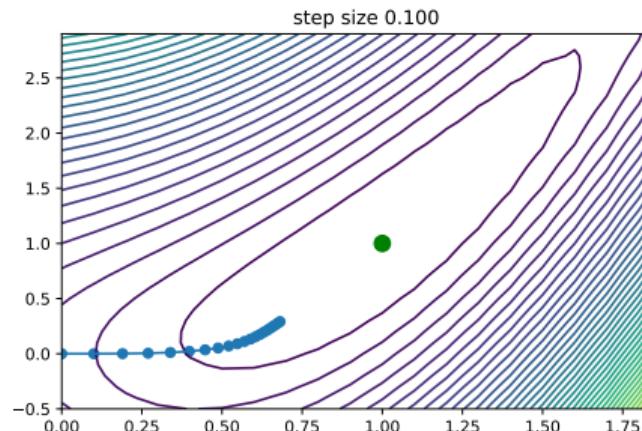
## First-order method: gradient descent

- ❑ Perhaps, the simplest algorithm for unconstrained optimization.
- ❑ It assumes that  $\mathbf{d}_k = -\mathbf{g}_k$ , where  $\mathbf{g}_k = \nabla \mathcal{L}(\mathbf{w}_k)$ .
- ❑ It can be written like

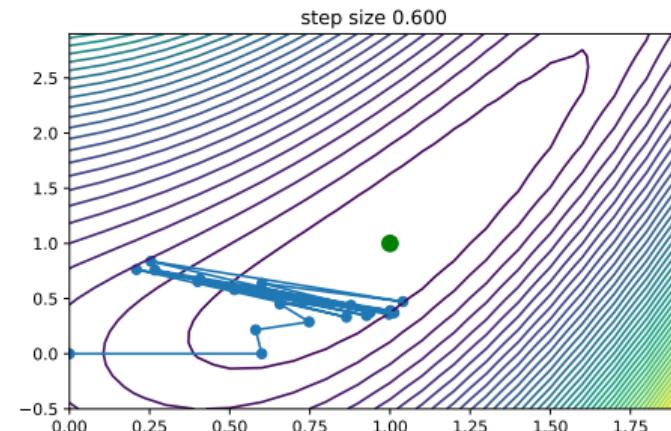
$$\mathbf{w}_{k+1} = \mathbf{w}_k - \eta \mathbf{g}_k.$$

# Step size

- Main issue: If it is too small, convergence will be very slow. If it is too large, the method can fail to converge at all.



(a)



(b)

The function to optimize is  $f(w_1, w_2) = 0.5(w_1^2 - w_2)^2 + 0.5(w_1 - 1)^2$ . The minimum is at (1, 1). In (a)  $\eta = 0.1$ . In (b)  $\eta = 0.6$ . Generated by code at [this link](#).

- Optimal step size can be found by **line search**.

## Second-order method: Newton's method

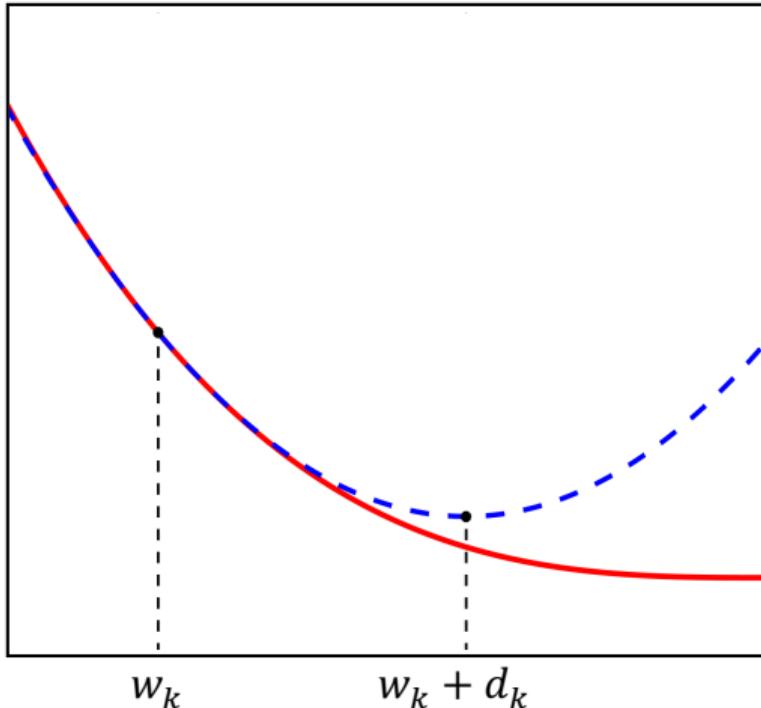
- Newton's algorithm

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \eta \mathbf{H}_k^{-1} \mathbf{g}_k,$$

where  $\mathbf{g}_k = \nabla \mathcal{L}(\mathbf{w}_k)$ , and  $\mathbf{H}_k = \nabla^2 \mathcal{L}(\mathbf{w}_k)$  is a Hessian matrix.

- Usually  $\eta = 1$ , and  $\mathbf{d}_k = -\mathbf{H}_k^{-1} \mathbf{g}_k$ .
- It derives a faster optimization algorithm by taking the curvature of the space (i.e., the Hessian) into account.

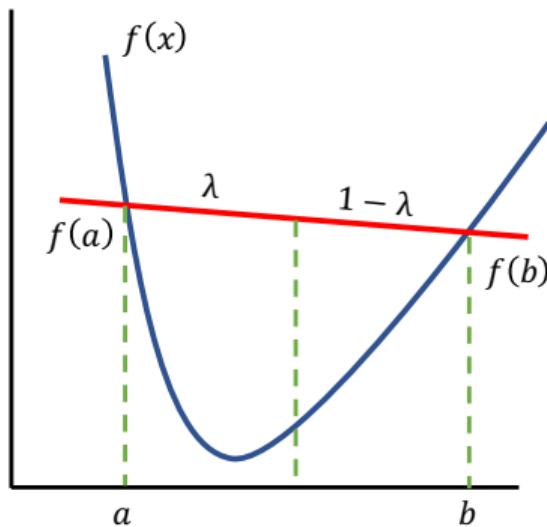
## Newton's method (II)



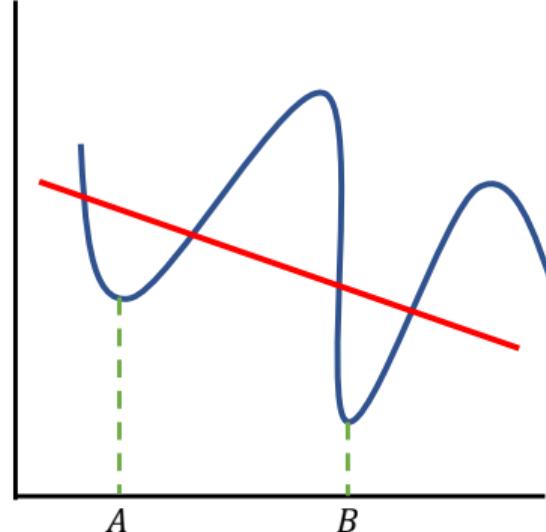
The solid curve is the function  $\mathcal{L}(x)$ . The dotted line  $\mathcal{L}_{\text{quad}}(w)$  is its second-order approximation at  $w_k$ . The Newton step  $d_k$  is what must be added to  $w_k$  to get to the minimum of  $\mathcal{L}_{\text{quad}}(w)$

## Newton's method and convex functions

- Newton's method requires that  $\mathbf{H}_k$  be **positive definite** (p.d.)
- The condition holds if the function is strictly convex.



(a)



(b)

In (a), an illustration of a convex function. The chord joining  $(a, f(a))$  and  $(b, f(b))$  lies above the function. In (b), a function that is neither convex nor concave.

## Gradient and Hessian of $NLL(\mathbf{w})$

- It can be shown that the gradient  $\mathbf{g}(\mathbf{w})$  of  $NLL(\mathbf{w})$  is given as

$$\mathbf{g}(\mathbf{w}) = \frac{d}{d\mathbf{w}} NLL(\mathbf{w}) = \sum_{n=1}^N [\sigma(\mathbf{w}^\top \mathbf{x}_n) - y_n] \mathbf{x}_n = \mathbf{X}^\top (\boldsymbol{\sigma} - \mathbf{y}),$$

where  $\boldsymbol{\sigma} = [\sigma(\mathbf{w}^\top \mathbf{x}_1) \cdots \sigma(\mathbf{w}^\top \mathbf{x}_N)]^\top$ .

- It can also be shown that the Hessian  $\mathbf{H}(\mathbf{w})$  of  $NLL(\mathbf{w})$  is given as

$$\mathbf{H}(\mathbf{w}) = \frac{d}{d\mathbf{w}} \mathbf{g}(\mathbf{w})^\top = \sum_{n=1}^N \sigma(\mathbf{w}^\top \mathbf{x}_n) [1 - \sigma(\mathbf{w}^\top \mathbf{x}_n)] \mathbf{x}_n \mathbf{x}_n^\top = \mathbf{X}^\top \boldsymbol{\Sigma} \mathbf{X},$$

where  $\boldsymbol{\Sigma} = \text{diag}([\sigma(\mathbf{w}^\top \mathbf{x}_1)[1 - \sigma(\mathbf{w}^\top \mathbf{x}_1)], \dots, \sigma(\mathbf{w}^\top \mathbf{x}_N)[1 - \sigma(\mathbf{w}^\top \mathbf{x}_N)]])$ .

# Contents (Check-in code: XX-XX-XX)

Logistic regression: cross-entropy error function

## Optimization and efficient computation

First- and Second-order method

Quasi-Newton methods

Stochastic gradient descent

Beyond basic logistic regression

Regularization

Multi-class logistic regression

Logistic regression in PySpark

References and recommended reading

Acknowledgment

## Quasi-Newton methods

- The main drawback of the Newton direction is the need for the Hessian.
- Explicit computation of this matrix of second derivatives can sometimes be a cumbersome, error-prone, and expensive process.
- Quasi-Newton search directions provide an attractive alternative to Newton's method.
- They do not require computation of the Hessian and yet still attain a faster rate of convergence.
- In place of the true Hessian  $\mathbf{H}_k$ , they use an approximation  $\mathbf{B}_k$ , which is updated after each step to take account of the additional knowledge gained during the step.

## BFGS formula for $\mathbf{H}_k$

- In the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) formula

$$\mathbf{B}_{k+1} = \mathbf{B}_k + \frac{\mathbf{z}_k \mathbf{z}_k^\top}{\mathbf{z}_k^\top \mathbf{s}_k} - \frac{(\mathbf{B}_k \mathbf{s}_k)(\mathbf{B}_k \mathbf{s}_k)^\top}{\mathbf{s}_k^\top \mathbf{B}_k \mathbf{s}_k},$$

where  $\mathbf{s}_k = \mathbf{w}_k - \mathbf{w}_{k-1}$  and  $\mathbf{z}_k = \mathbf{g}_k - \mathbf{g}_{k-1}$ .

- This is a rank-two update to the matrix, and ensures that the matrix remains positive definite.
- Usually  $\mathbf{B}_0 = \mathbf{I}$ .
- The search direction is then  $\mathbf{d}_k = -\mathbf{B}_k^{-1}\mathbf{g}_k$ .

## BFGS formula for $\mathbf{H}_k^{-1}$

- BFGS can alternatively update  $\mathbf{C}_k = \mathbf{H}_k^{-1}$  using

$$\mathbf{C}_{k+1} = \left( \mathbf{I} - \frac{\mathbf{s}_k \mathbf{z}_k^\top}{\mathbf{z}_k^\top \mathbf{s}_k} \right) \mathbf{C}_k \left( \mathbf{I} - \frac{\mathbf{z}_k \mathbf{s}_k^\top}{\mathbf{z}_k^\top \mathbf{s}_k} \right) + \frac{\mathbf{s}_k \mathbf{s}_k^\top}{\mathbf{z}_k^\top \mathbf{s}_k}.$$

- The search direction is then  $\mathbf{d}_k = -\mathbf{C}_k \mathbf{g}_k$ .

**Broyden, Fletcher, Goldfarb, Shanno**



Image source: <https://aria42.com/blog/2014/12/understanding-lbfgs>

## Limited memory BFGS (L-BFGS)

- ❑ Since storing the Hessian takes  $O(p^2)$  space, for very large problems, one can use limited memory BFGS, or L-BFGS.
- ❑ In L-BFGS,  $\mathbf{H}_k$  or  $\mathbf{H}_k^{-1}$  is approximated by a diagonal plus low rank matrix.
- ❑ In particular, the product  $\mathbf{B}_k^{-1} \mathbf{g}_k$  can be obtained by performing a sequence of inner products with  $\mathbf{s}_k$  and  $\mathbf{z}_k$ , using only the  $m$  most recent  $(\mathbf{s}_k, \mathbf{z}_k)$  pairs, and ignoring older information.
- ❑ The storage requirements are therefore  $O(mp)$ . Typically,  $m \sim 20$  suffices for good performance.

## Exercise

An implementation of L-BFGS uses the 10 most recent values of differences in gradients,  $\mathbf{z}_k = \mathbf{g}_k - \mathbf{g}_{(k-1)}$ , and the 10 most recent values of differences in the weight vector,  $\mathbf{s}_k = \mathbf{w}_k - \mathbf{w}_{(k-1)}$ . If the number of features is 100, what percentage of storage is saved by L-BFGS compared to full BFGS? [Enter the integer percentage without the '%' sign]

## Algorithms in summation form

- When an algorithm sums over the data, we can easily distribute the calculations over multiple workers (e.g. cores).
- We just divide the dataset into as many pieces as there are workers, give each worker its share of the data to sum the equations over, and aggregate the result at the end.
- Both the gradient and Hessian in the optimization algorithms we saw before have a summation form.
- Notice that the summation form does not change the underlying algorithm: it is not an approximation but an exact implementation.

# Contents (Check-in code: XX-XX-XX)

Logistic regression: cross-entropy error function

## Optimization and efficient computation

First- and Second-order method

Quasi-Newton methods

Stochastic gradient descent

Beyond basic logistic regression

Regularization

Multi-class logistic regression

Logistic regression in PySpark

References and recommended reading

Acknowledgment

## Stochastic gradient descent (I)

- Traditionally in machine learning, the gradient  $\mathbf{g}_k$  and the Hessian  $\mathbf{H}_k$  are computed using the whole dataset  $\mathcal{D} = \{\mathbf{x}_n, y_n\}_{n=1}^N$ .
- Large datasets:** computing the exact values for  $\mathbf{g}_k$  and  $\mathbf{H}_k$  would be expensive.
- Stochastic gradient descent (SGD):**

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \eta_k \nabla \mathcal{L}(\mathbf{w}_k, \mathbf{x}_k) = \mathbf{w}_k - \eta_t \mathbf{g}_k.$$

- Online learning:** the instances  $(\mathbf{x}_n, y_n)$  appear one at a time.

## Stochastic gradient descent (II)

- In the stochastic setting, a better estimate can be found if the gradient is computed using

$$\mathbf{g}_k \approx \frac{1}{|S_k|} \sum_{i \in S_k} \nabla \mathcal{L}_i(\mathbf{w}_k),$$

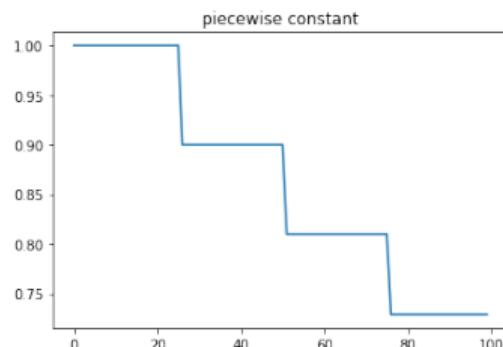
where  $S_k \in \mathcal{D}$ ,  $|S_k|$  is the cardinality of  $S_k$ , and  $\nabla \mathcal{L}_i(\mathbf{w}_k)$  is the gradient at iteration  $k$  computed using the instance  $(\mathbf{x}_i, y_i)$ .

- This setting is called Mini-batch gradient descent.
- For logistic regression,  $\nabla \mathcal{L}_i(\mathbf{w}_k)$  would be the gradient computed for  $-\log p(y_i | \mathbf{w}, \mathbf{x}_i)$ .

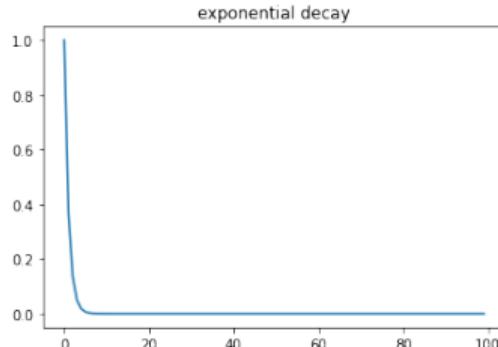
# Step size in SGD

- Choosing the value of  $\eta$  is important in SGD since there is no easy way to compute it.
- It should follow the **Robbins-Monro** conditions

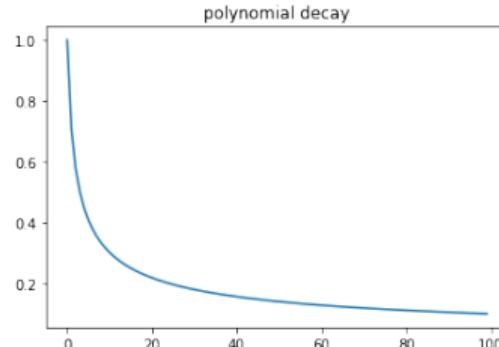
$$\eta_k \rightarrow 0, \frac{\sum_{k=1}^{\infty} \eta_k^2}{\sum_{k=1}^{\infty} \eta_k} \rightarrow 0.$$



(a)



(b)



(c)

Illustration of some common learning rate schedules. (a) Piecewise constant. (b) Exponential decay. (c) Polynomial decay. Generated by code at [this link](#).

# Contents (Check-in code: XX-XX-XX)

Logistic regression: cross-entropy error function

Optimization and efficient computation

First- and Second-order method

Quasi-Newton methods

Stochastic gradient descent

Beyond basic logistic regression

Regularization

Multi-class logistic regression

Logistic regression in PySpark

References and recommended reading

Acknowledgment

## What is regularization?

- It refers to a technique used for preventing overfitting in a predictive model.
- It consists in adding a term (a regularizer) to the objective function that encourages simpler solutions.
- With regularization, the objective function for logistic regression would be

$$\mathcal{L}(\mathbf{w}) = NLL(\mathbf{w}) + \lambda R(\mathbf{w}),$$

where  $R(\mathbf{w})$  is the regularization term and  $\lambda$  the regularization parameter.

- If  $\lambda = 0$ , we get  $\mathcal{L}(\mathbf{w}) = NLL(\mathbf{w})$ .

## $\ell_2$ regularization

- Prior of  $\mathbf{w}$ : zero-mean normal distribution

$$f(w|0, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(w-0)^2}{2\sigma^2}\right),$$

where  $\sigma$  is the standard deviation of the distribution.

- Likelihood:

$$p(\mathbf{y}|\mathbf{w}, \mathbf{X})p(\mathbf{w}) = \prod_{n=1}^N p(y_n|\mathbf{w}, \mathbf{x}_n) \prod_{m=1}^p \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{w_m^2}{2\sigma^2}\right)$$

- Negative log-likelihood:

$$NLL(\mathbf{w}) + \frac{1}{2\sigma^2} \sum_{m=1}^p w_m^2 \rightarrow NLL(\mathbf{w}) + \lambda \sum_{m=1}^p w_m^2$$

- Convex, smooth

## $\ell_1$ regularization

- Prior of  $\mathbf{w}$ : zero-mean Laplace distribution

$$f(\mathbf{w}|\mathbf{0}, b) = \frac{1}{2b} \exp\left(-\frac{|\mathbf{w} - \mathbf{0}|}{b}\right),$$

where  $b > 0$  is a scale parameter of the distribution.

- Likelihood:

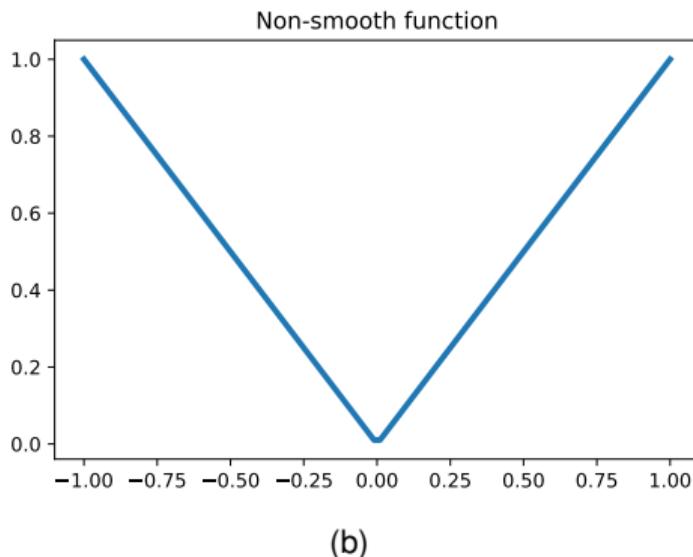
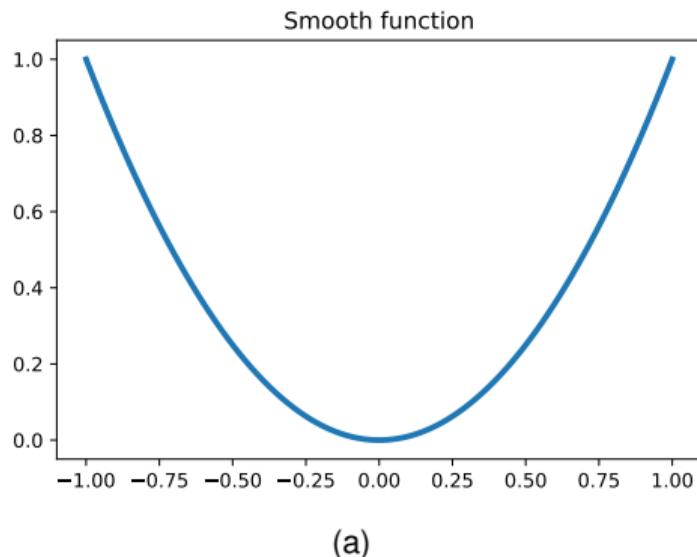
$$p(\mathbf{y}|\mathbf{w}, \mathbf{X})p(\mathbf{w}) = \prod_{n=1}^N p(y_n|\mathbf{w}, \mathbf{x}_n) \prod_{m=1}^p \frac{1}{2b} \exp\left(-\frac{|w_m|}{b}\right)$$

- Negative log-likelihood:

$$NLL(\mathbf{w}) + \frac{1}{b} \sum_{m=1}^p |w_m| \rightarrow NLL(\mathbf{w}) + \lambda \sum_{m=1}^p |w_m|$$

- Convex, nonsmooth, sparse

# Smooth vs nonsmooth optimization



(a) Smooth 1d function, (b) Nonsmooth 1d function. Generated by code at [this link](#).

## Elastic net regularization

- The objective function for logistic regression would be

$$\mathcal{L}(\mathbf{w}) = NLL(\mathbf{w}) + \lambda R(\mathbf{w}),$$

where  $R(\mathbf{w})$  follows as

$$R(\mathbf{w}) = \alpha \|\mathbf{w}\|_1 + (1 - \alpha) \frac{1}{2} \|\mathbf{w}\|_2^2,$$

where  $\|\mathbf{w}\|_1 = \sum_{m=1}^p |w_m|$ , and  $\|\mathbf{w}\|_2^2 = \sum_{m=1}^p w_m^2$ .

- If  $\alpha = 1$ , we get  $\ell_1$  regularization.
- If  $\alpha = 0$ , we get  $\ell_2$  regularization.
- If  $0 < \alpha < 1$ , we get the elastic net regularization.

# Contents (Check-in code: XX-XX-XX)

Logistic regression: cross-entropy error function

Optimization and efficient computation

- First- and Second-order method

- Quasi-Newton methods

- Stochastic gradient descent

Beyond basic logistic regression

- Regularization

- Multi-class logistic regression

Logistic regression in PySpark

References and recommended reading

Acknowledgment

## Categorical distribution

- Imagine a random experiment where the output can only take 1 of  $K$  outputs.
- The result of the experiment can be coded using a vector  $\mathbf{y}$  of dimension  $K$ , where only one of its entries is 1, and the rest is zero.
- Each of the  $K$  outputs of the experiment has a probability  $\mu_k$  with  $\sum_{k=1}^K \mu_k = 1$ .
- We say that the vector  $\mathbf{y}$  follows a categorical distribution

$$p(\mathbf{y}|\boldsymbol{\mu}) = \prod_{k=1}^K \mu_k^{y_k},$$

where  $\mathbf{y} = [y_1, y_2, \dots, y_K]^\top$ , and  $\boldsymbol{\mu} = [\mu_1, \mu_2, \dots, \mu_K]^\top$ .

## Multinomial logistic regression (I)

- The categorical distribution is a particular case of the multinomial distribution.
- The categorical distribution can be used for multi-class logistic regression.
- Each instance in a multi-class classification problem is made of  $(\mathbf{x}, \mathbf{y})$ .
- In multinomial logistic regression, each probability  $\mu_k$  is represented as

$$\mu_k = \frac{\exp(\mathbf{w}_k^\top \mathbf{x})}{\sum_{j=1}^K \exp(\mathbf{w}_j^\top \mathbf{x})},$$

where  $\{\mathbf{w}_k\}_{k=1}^K$  is a set of weights, that we jointly refer to as  $\mathbf{W}$ .

## Multinomial logistic regression (II)

- In this way, we can model  $p(\mathbf{y}|\mathbf{W}, \mathbf{x})$  using

$$p(\mathbf{y}|\mathbf{W}, \mathbf{x}) = \prod_{k=1}^K \mu_k^{y_k} = \prod_{k=1}^K \left[ \frac{\exp(\mathbf{w}_k^\top \mathbf{x})}{\sum_{j=1}^K \exp(\mathbf{w}_j^\top \mathbf{x})} \right]^{y_k}.$$

- Suppose we have a dataset  $\mathcal{D} = \{\mathbf{x}_n, \mathbf{y}_n\}_{n=1}^N$ .
- We write  $\mathbf{X} = [\mathbf{x}_1 \cdots \mathbf{x}_N]^\top$ , and  $\mathbf{Y} = [\mathbf{y}_1 \cdots \mathbf{y}_N]^\top$ .

## Multinomial logistic regression (III)

- The parameters  $\mathbf{W}$  could be estimated by minimizing

$$NLL(\mathbf{W}) = -\log p(\mathbf{Y}|\mathbf{W}, \mathbf{X}) = -\log \prod_{n=1}^N \prod_{k=1}^K \mu_{n,k}^{y_{n,k}} = -\sum_{n=1}^N \sum_{k=1}^K y_{n,k} \log \mu_{n,k},$$

where  $\mu_{n,k} = \frac{\exp(\mathbf{w}_k^\top \mathbf{x}_n)}{\sum_{j=1}^K \exp(\mathbf{w}_j^\top \mathbf{x}_n)}$ .

- We could also add a regularization term to  $NLL(\mathbf{W})$ , and minimize

$$f(\mathbf{W}) = NLL(\mathbf{W}) + \lambda R(\mathbf{W}),$$

with respect to  $\mathbf{W}$ .

# Contents (Check-in code: XX-XX-XX)

Logistic regression: cross-entropy error function

Optimization and efficient computation

- First- and Second-order method

- Quasi-Newton methods

- Stochastic gradient descent

Beyond basic logistic regression

- Regularization

- Multi-class logistic regression

## Logistic regression in PySpark

References and recommended reading

Acknowledgment

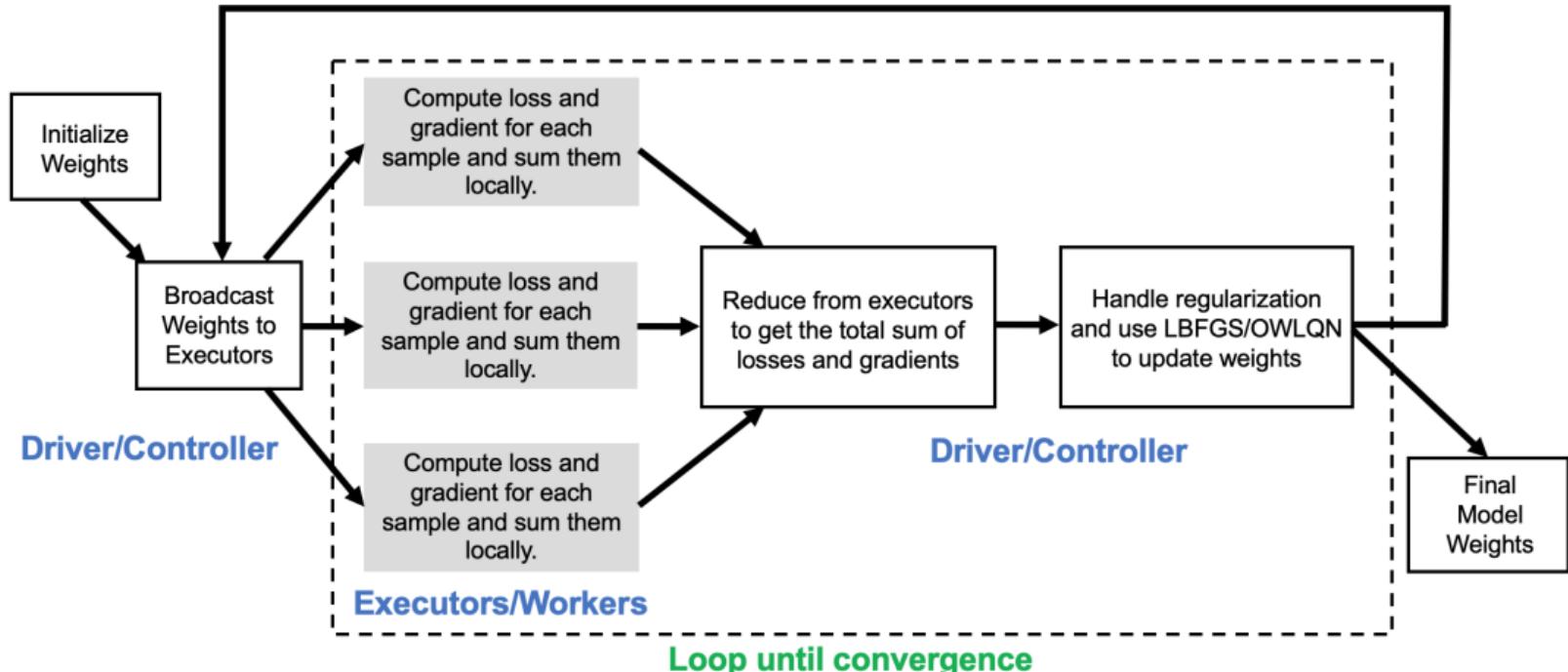
## LogisticRegression()

- ❑ LogisticRegression() works with DataFrames.
- ❑ It is possible to use regularization  $\ell_1$ ,  $\ell_2$  or Elastic Net.
- ❑ L-BFGS is used as a solver for LogisticRegression(), with  $\ell_2$ .
- ❑ When  $\ell_1$ , or Elastic Net is used, a variant of L-BFGS, known as Orthant-Wise Limited-memory Quasi-Newton (OWL-QN) is used instead.

## Parameters to adjust

- **maxIter**: max number of iterations.
- **regParam**: regularization parameter ( $\geq 0$ ).
- **elasticNetParam**: the ElasticNet mixing parameter, in range [0, 1]. For  $\alpha = 0$ , the penalty is an  $\ell_2$  penalty. For  $\alpha = 1$ , it is an  $\ell_1$  penalty.
- **family**: the name of family which is a description of the label distribution to be used in the model. Supported options: auto, binomial, multinomial.
- **standardization**: whether to standardize the training features before fitting the model. It can be true or false.

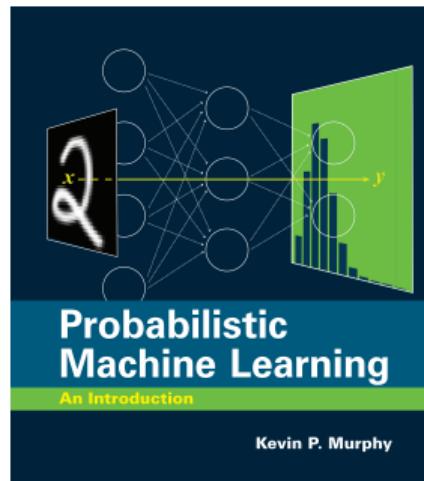
# Update of $w$ in Apache Spark



# References and recommended reading

Book: *Probabilistic Machine Learning: An Introduction* by Kevin P Murphy, 2022.

- Chapter 8 Optimization pp.217 -292
- Chapter 10 Logistic regression pp. 335-355



# References and recommended reading

Website: *Spark Python API Documentation*.



Q. Search the docs ...

Spark SQL  
Pandas API on Spark  
Structured Streaming  
MLlib (DataFrame-based)  
Spark Streaming  
MLlib (RDD-based)  
Spark Core  
Resource Management

## API Reference

This page lists an overview of all public PySpark modules, classes, functions and methods.

Pandas API on Spark follows the API specifications of pandas 1.3.

- **Spark SQL**
  - Core Classes
  - Spark Session
  - Configuration
  - Input/Output
  - DataFrame
  - Column
  - Data Types
  - Row
  - Functions
  - Window
  - Grouping
  - Catalog
  - Observation
  - Avro
- **Pandas API on Spark**
  - Input/Output
  - General Functions
  - Series
  - DataFrame
  - Index objects
  - Window
  - GroupBy
  - Machine Learning utilities
  - Extensions
- **Structured Streaming**
  - Core Classes
  - Input/Output
  - Query Management
- **MLlib (DataFrame-based)**
  - Pipeline APIs
  - Parameters
  - Feature
  - Classification
  - Clustering
  - Functions
  - Vector and Matrix
  - Recommendation
  - Regression
  - Statistics
  - Timing
  - Evaluation
  - Frequency Pattern Mining
  - Image
  - Utilities
- **Spark Streaming**
  - Core Classes
  - Streaming Management
  - Input and Output
  - Transformations and Actions
  - Kinesis
- **MLlib (RDD-based)**

# References and recommended reading

Paper: *Map-Reduce for Machine Learning on Multicore* by C-T Chu et al. (2006).

---

## Map-Reduce for Machine Learning on Multicore

---

Cheng-Tao Chu \*  
chengtao@stanford.edu

Sang Kyun Kim \*  
skkim38@stanford.edu

Yi-An Lin \*  
ianl@stanford.edu

YuanYuan Yu \*  
yuanyuan@stanford.edu

Gary Bradski †  
garybradski@gmail.com

Andrew Y. Ng \*  
ang@cs.stanford.edu

Kunle Olukotun \*  
kunle@cs.stanford.edu

\* CS. Department, Stanford University 353 Serra Mall,  
Stanford University, Stanford CA 94305-9025.

† Rexee Inc.

### Abstract

We are at the beginning of the multicore era. Computers will have increasingly many cores (processors), but there is still no good programming framework for these architectures, and thus no simple and unified way for machine learning to take advantage of the potential speed up. In this paper, we develop a broadly applicable parallel programming method, one that is easily applied to *many* different learning algorithms. Our work is in distinct contrast to the tradition in machine learning of designing (often ingenious) ways to speed up a *single* algorithm at a time. Specifically, we show that algorithms that fit the Statistical Query model [15] can be written in a certain “summation form,” which allows them to be easily parallelized on multicore computers. We adapt Google’s map-reduce [7] paradigm to demonstrate this parallel speed up technique on a variety of learning algorithms including locally weighted linear regression (LWLR), k-means, logistic regression (LR), naive Bayes (NB), SVM, ICA, PCA, gaussian discriminant analysis (GDA), EM, and backpropagation (NN). Our experimental results show basically linear speedup with an increasing number of processors.

# References and recommended reading

Paper: *Optimization Methods for Large Scale Machine Learning* by L. Bottou et al. (2018).

- Chapter 3-4 pp. 235 -258
- Chapter 6 pp. 270-284
- Chapter 8 pp. 293-298

SIAM REVIEW  
Vol. 60, No. 2, pp. 223–311

© 2018 Society for Industrial and Applied Mathematics

## Optimization Methods for Large-Scale Machine Learning\*

Léon Bottou<sup>†</sup>  
Frank E. Curtis<sup>‡</sup>  
Jorge Nocedal<sup>§</sup>

**Abstract.** This paper provides a review and commentary on the past, present, and future of numerical optimization algorithms in the context of machine learning applications. Through case studies on text classification and the training of deep neural networks, we discuss how optimization problems arise in machine learning and what makes them challenging. A major theme of our study is that large-scale machine learning represents a distinctive setting in which the stochastic gradient (SG) method has traditionally played a central role while conventional gradient-based nonlinear optimization techniques typically falter. Based on this viewpoint, we present a comprehensive theory of a straightforward, yet versatile SG algorithm, discuss its practical behavior, and highlight opportunities for designing algorithms with improved performance. This leads to a discussion about the next generation of optimization methods for large-scale machine learning, including an investigation of two main streams of research on techniques that diminish noise in the stochastic directions and methods that make use of second-order derivative approximations.

**Key words.** numerical optimization, machine learning, stochastic gradient methods, algorithm complexity analysis, noise reduction methods, second-order methods

# Acknowledgment

Thanks to [Dr. Mauricio Alvarez](#) for his contribution to this module from 2017 to 2022