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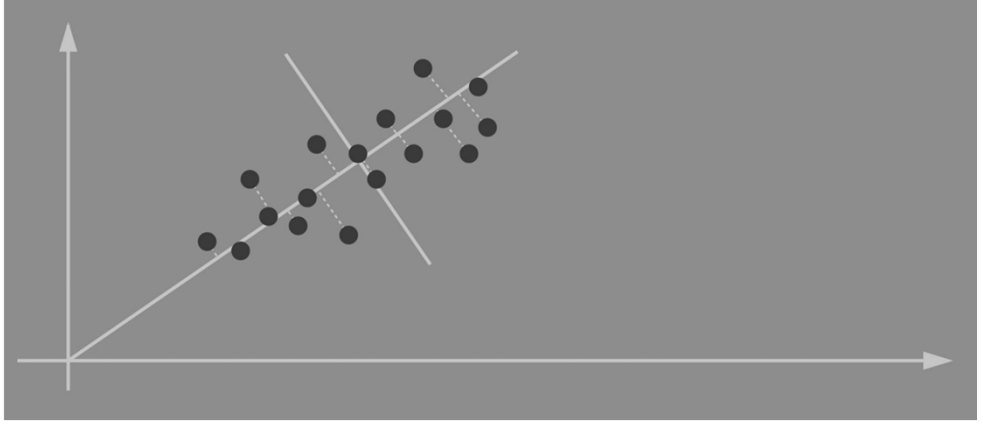

PCA-06-scaled.jpg (2560×1051) (perfectial.com)

Lecture 8: Scalable Matrix Factorisation for Collaborative Filtering in RecSys
&
Scalable PCA for Dimensionality Reduction

COM6012: Scalable ML by Haiping Lu

Week 8 Contents / Objectives

- Recommender Systems
- Collaborative Filtering
- Matrix Factorisation for Collaborative Filtering
- Scalable Collaborative Filtering in Spark

- Scalability Problem of PCA
- PCA via SVD
- Scalable PCA in Spark

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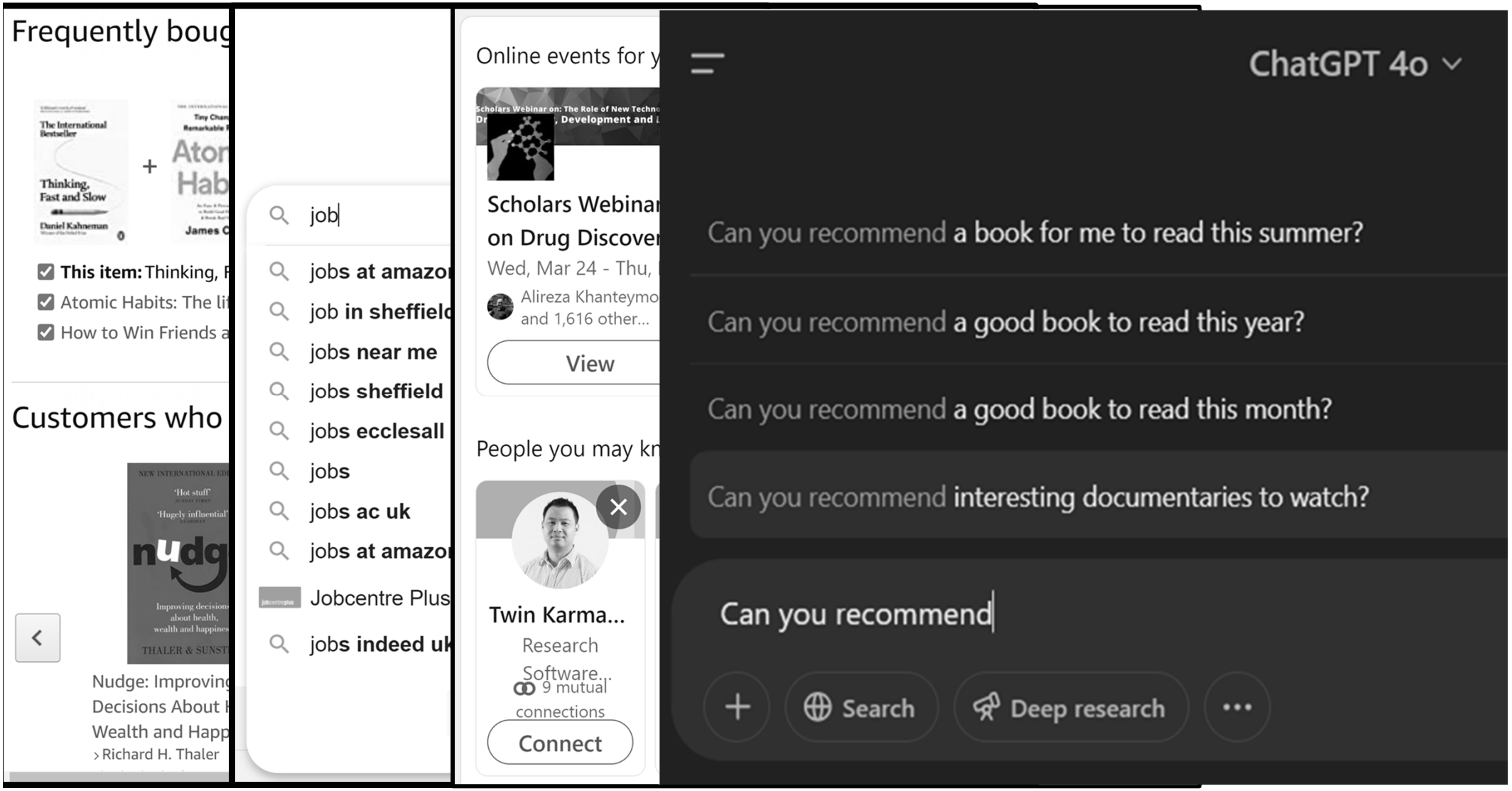
Many Decisions to Make

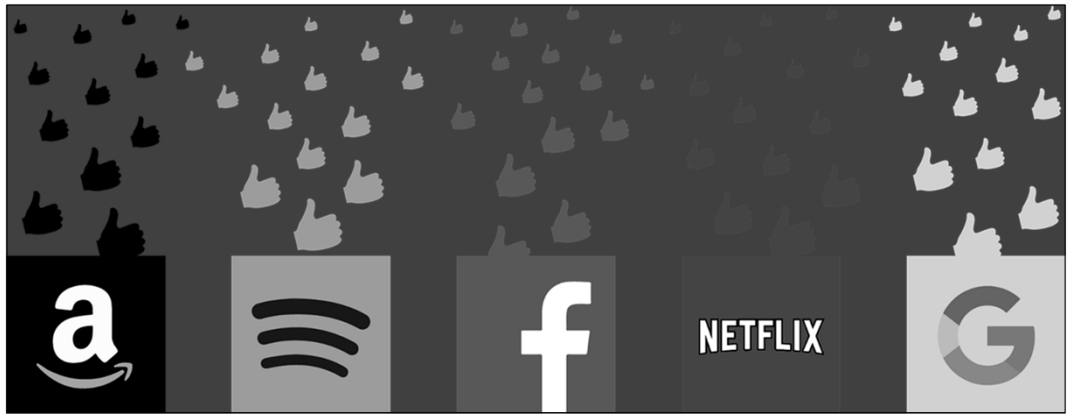
C https://blogs-image

Recommendations Everywhere



Recommender Systems (RecSys)

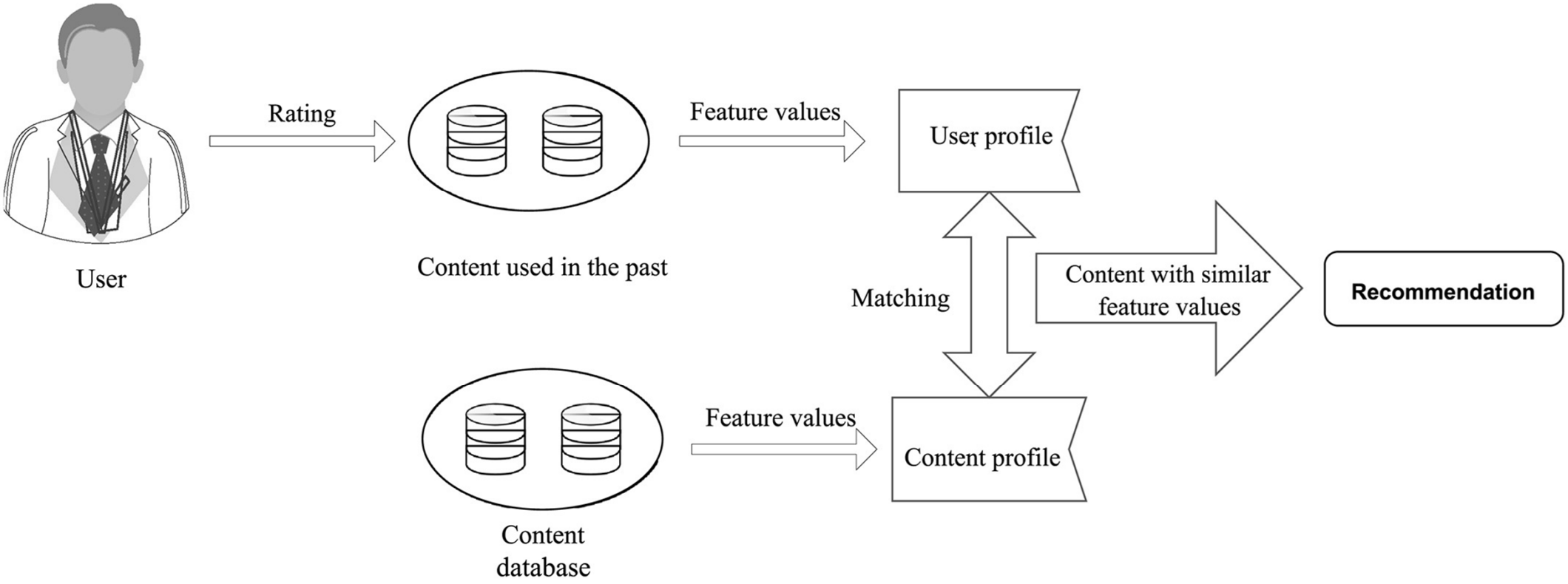
- Predict relevant items for a user, in a given context
- Predict to what extent these items are relevant
- A ranking task (searching as well)
- Implicit, targeted, **intelligent** advertisement
- Effective, popular marketing



Two Classes of RecSys

- Content-based recommender systems
- Collaborative filtering recommender systems

Content-based RecSys



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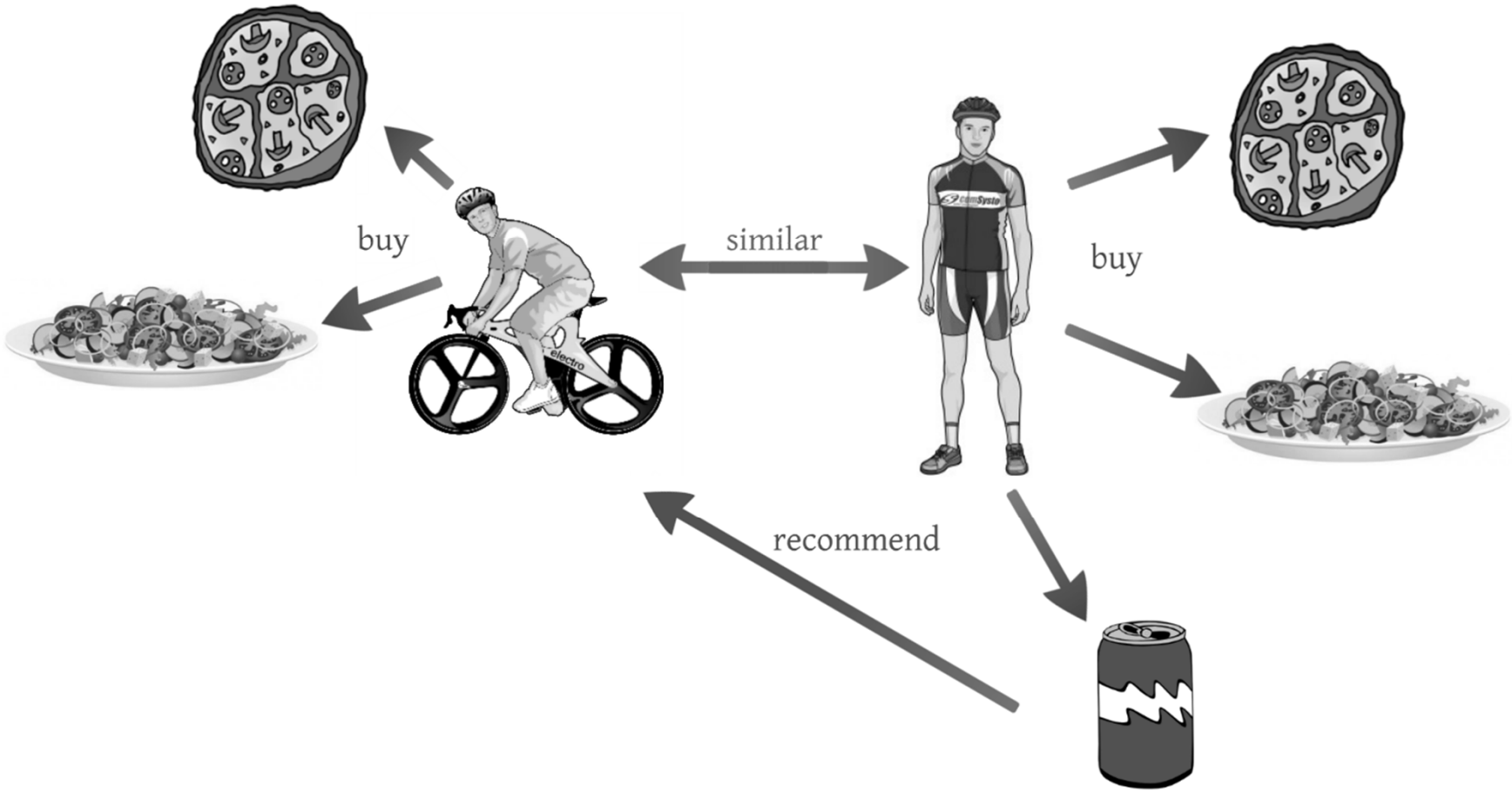
Collaborative Filtering RecSys



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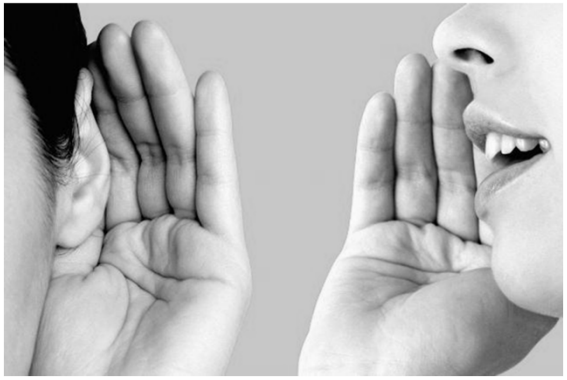
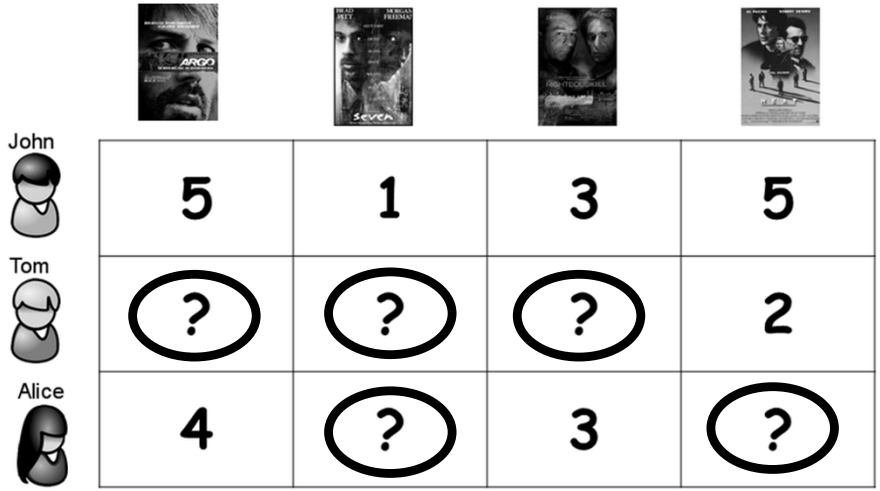
What is Collaborative Filtering?

- Information filtering based on past records
  - Electronic word of mouth marketing
  - Turn visitors into customers (e-Salesman)
- Components
  - Users (customers): who provide ratings
  - Items (products): to be rated
  - Ratings (interest): core data



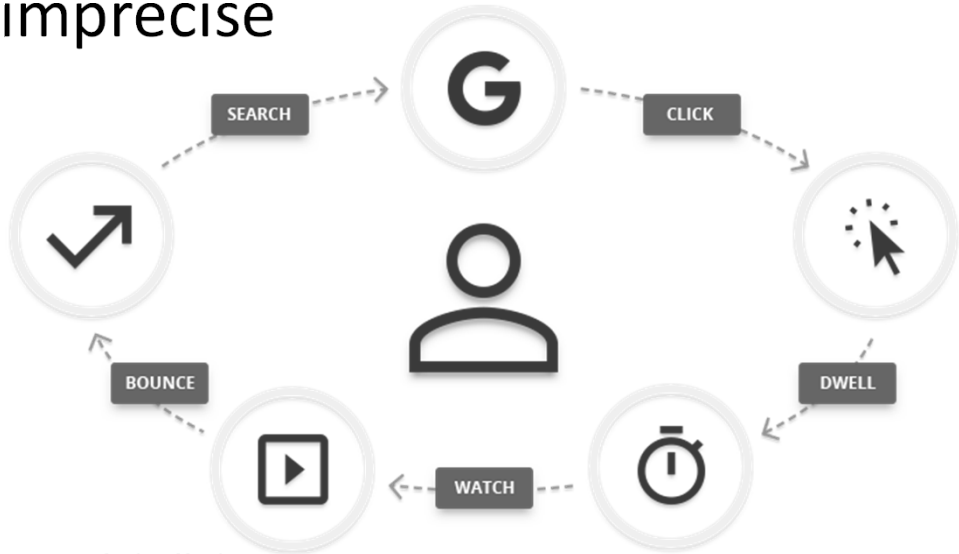


Collaborative Filtering (CF)

- Objective: predict how well a user will like an unrated item, given past ratings for a community of users
- How does CF work?
  - Input: many users' ratings for many items
  - Model: similar users ← ratings strongly correlate
  - Recommend items rated highly by similar users

Explicit vs Implicit Ratings

- Explicit (direct): users indicate levels of interest
  - Most accurate descriptions of a user's preference
  - Challenging in collecting data
- Implicit (indirect): observing user behavior
  - Can be collected with little or no cost to user
  - Ratings inference may be imprecise



Rating Scales

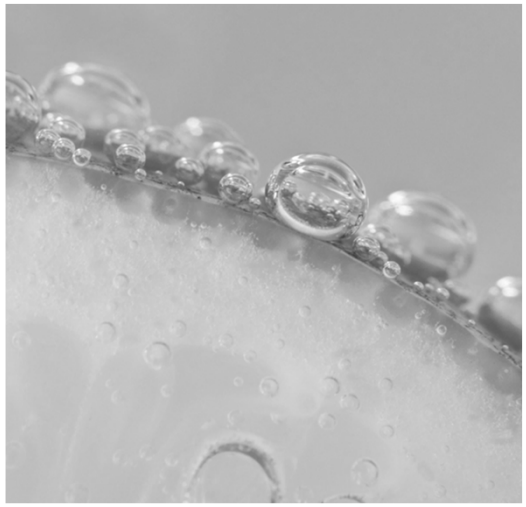
- Scalar ratings
  - Numerical scales
  - 1-5, 1-7, etc.

- Binary ratings
  - Agree/Disagree, Good/Bad, etc.

- Unary ratings
  - Presence/absence of an event, e.g., purchase/browsing history, search patterns, mouse movements
  - No info about the opposite ≠ 0

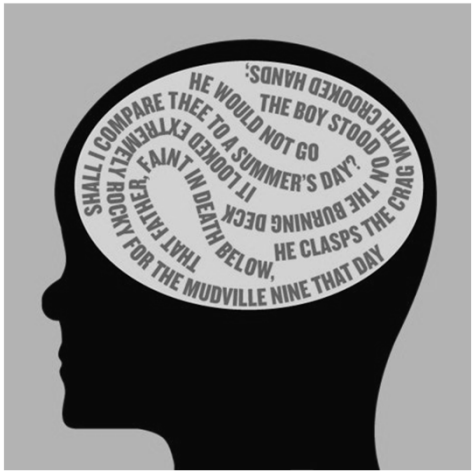
CF Preferences

- Many users, many items, many ratings
- Users rate multiple items
- Other users with similar needs/tastes
- Item evaluation requires personal taste
- Taste persists



CF Methods

- Memory-based: predict using past ratings directly
  - Weighted ratings given by other similar users
  - User-based & item-based (non-ML)
- Model-based: model users based on past ratings
  - Predict ratings using the learned model



iforget-465.jpg (465×465) (newyorker.com)

neural-header.jpg (756×503) (utsouthwestern.edu)

Prediction Accuracy

- Mean absolute error (MAE)

$$MAE = \frac{\sum_{i,j} |p_{i,j} - r_{i,j}|}{n}$$

- Normalized MAE

$$NMAE = \frac{MAE}{r_{max} - r_{min}}$$

- Root mean squared error (RMSE)

$$RMSE = \sqrt{\frac{1}{n} \sum_{i,j} (p_{i,j} - r_{i,j})^2}$$

Challenges

- Cold Start
  - New user
    - Rate some initial items
    - Non-personalized rec.
    - Describe tastes
    - Demographic info
  - New item
    - Randomly selecting items
    - Content analysis, metadata (non-CF)
- Sparsity: sparse user-item matrix
- Scalability: millions of users and items

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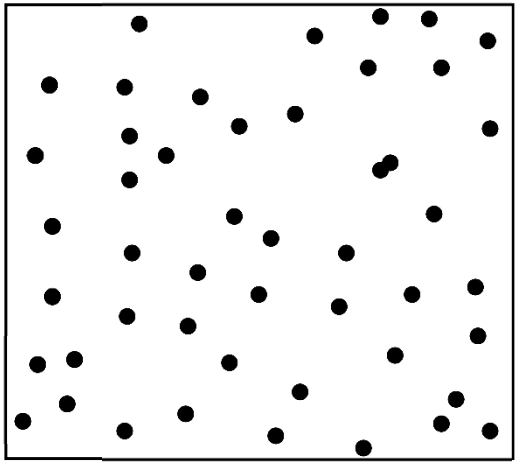

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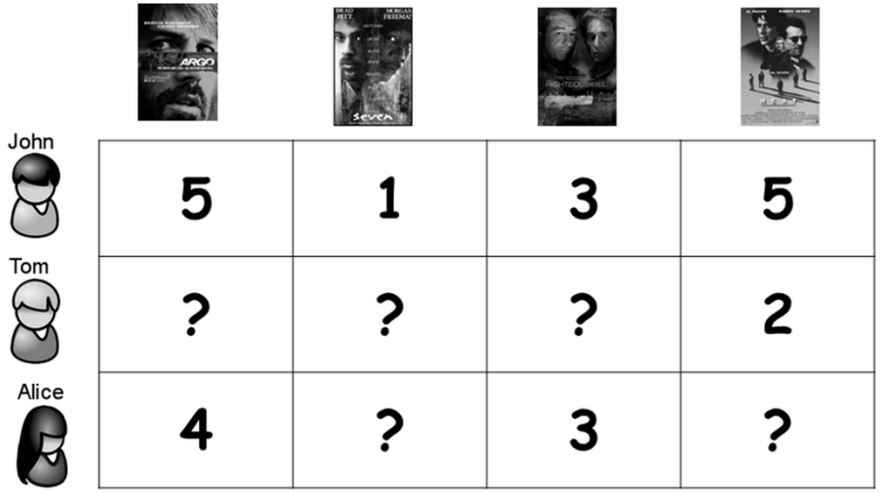
Matrix Factorisation (MF) for CF

- Characterise items/users by vectors of factors learned from the rating matrix user x item
- High correlation between item and user factors → good recommendation
- Flexibility: incorporate implicit feedback, temporal effects, and confidence levels



Basic MF Model

- Map users & items to a joint latent factor space of dimensionality k
  - Item i → vector q_i: the extent to which the item possesses those k factors
  - User u: vector p_u: the extent of interest the user has on those k factors
- User-item interactions: the user's overall interest in the item's characteristics
  - Inner product $q_i^T p_u$: predicted user u's rating of item i

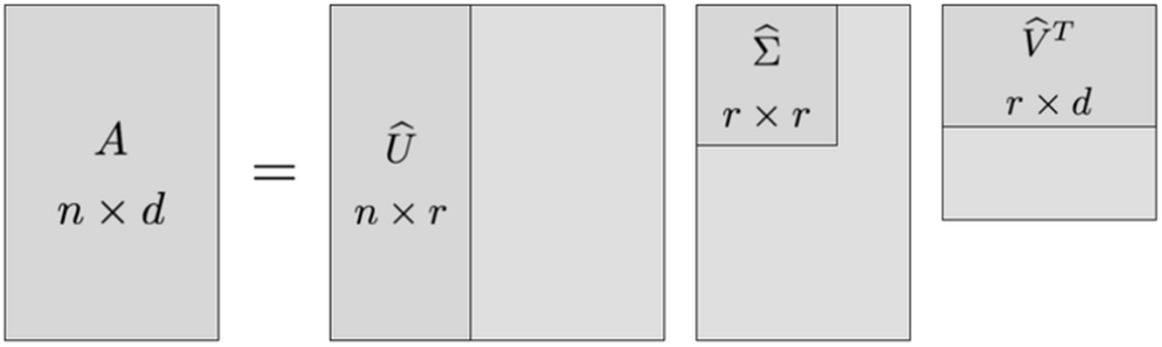
$$\hat{r}_{u,i} = q_i^T p_u$$

How to Learn the MF Model

- To learn: item factors $\{q_i\}$ and user factors $\{p_u\}$
- Factorisation assuming full rating matrix
  - Factorise rating matrix R using SVD to obtain P, S, Q

$$R = PSQ^T$$

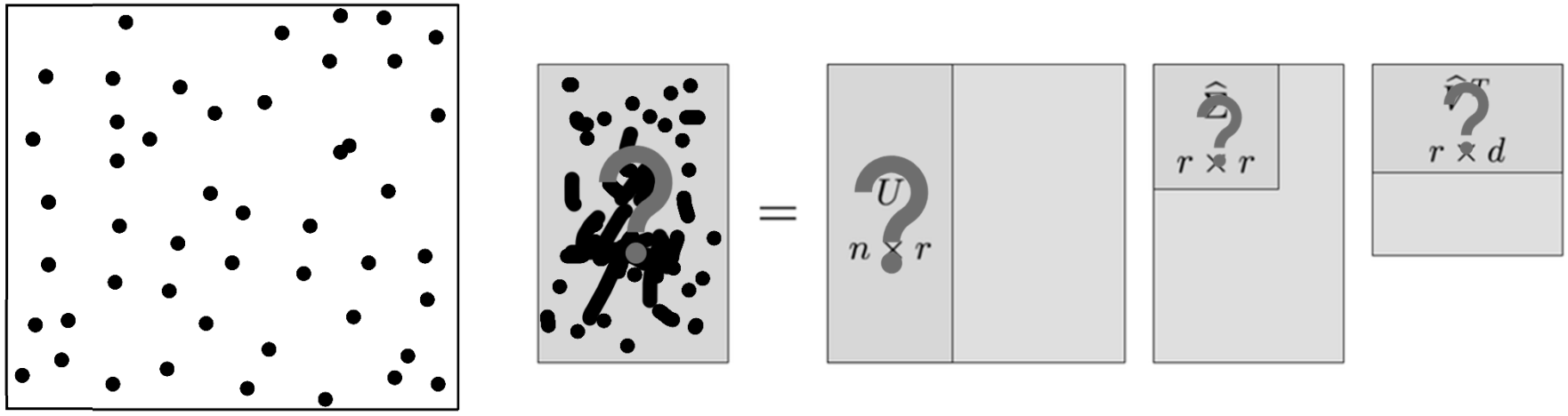
  - Reduce the matrix S to dimension k, i.e. S_k
  - $P \rightarrow P_k$ and $Q \rightarrow Q_k$: $P_k S_k \rightarrow \hat{P}$, and $S_k Q_k^T \rightarrow \hat{Q}^T$
  - uth row of $\hat{P} \rightarrow p_u$, ith column of $\hat{Q}^T \rightarrow q_i$



A: R
U: P
Σ: S
V: Q
r: k

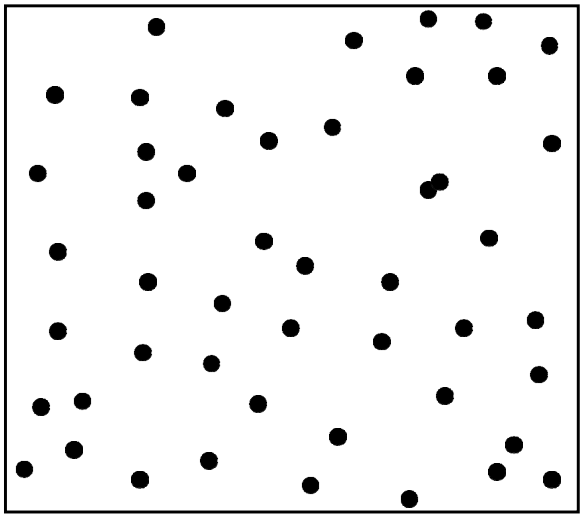
Challenges in MF for CF

- High portion of missing values caused by sparseness in the user-item rating matrix
- Conventional SVD is undefined when knowledge about the matrix is incomplete



How to Fill Missing Values

- Imputation: fill in missing ratings using the average ratings for user and item
- Problems
  - Expensive: significantly increases the amount of data
  - Inaccurate imputation might distort the data



MF with Missing Values

- Modelling directly the observed ratings only
  - Avoid overfitting through a regularised model
  - Minimize the regularised squared error on the set of known ratings to learn the factor vectors p_u and q_i

$$\min_{q^*,p^*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2 + \lambda(\| q_i \|^2 + \| p_u \|^2)$$

  - κ: the training set of the (u,i) pairs with known ratings
  - λ: the regularisation parameter

Alternating Least Squares for MF-CF

$$\min_{q^*,p^*} \sum_{(u,i)\in\kappa} (r_{ui} - q_i^T p_u)^2 + \lambda(\| q_i \|^2 + \| p_u \|^2)$$

- Both p_u and q_i are unknown (non-convex function)
- Fix one of them → quadratic with optimal solution
- Alternating Least Squares (ALS): alternate between fixing q_i s and fixing p_u s
  - Fix P (p_us) as $\hat{P}$ to recompute q_i s by solving a least-squares problem $\| R - PQ^T \|_F$ (Frobenius norm)

$$R = \hat{P}Q^T \Rightarrow Q^T = (\hat{P}^T\hat{P})^{-1}\hat{P}^T R$$

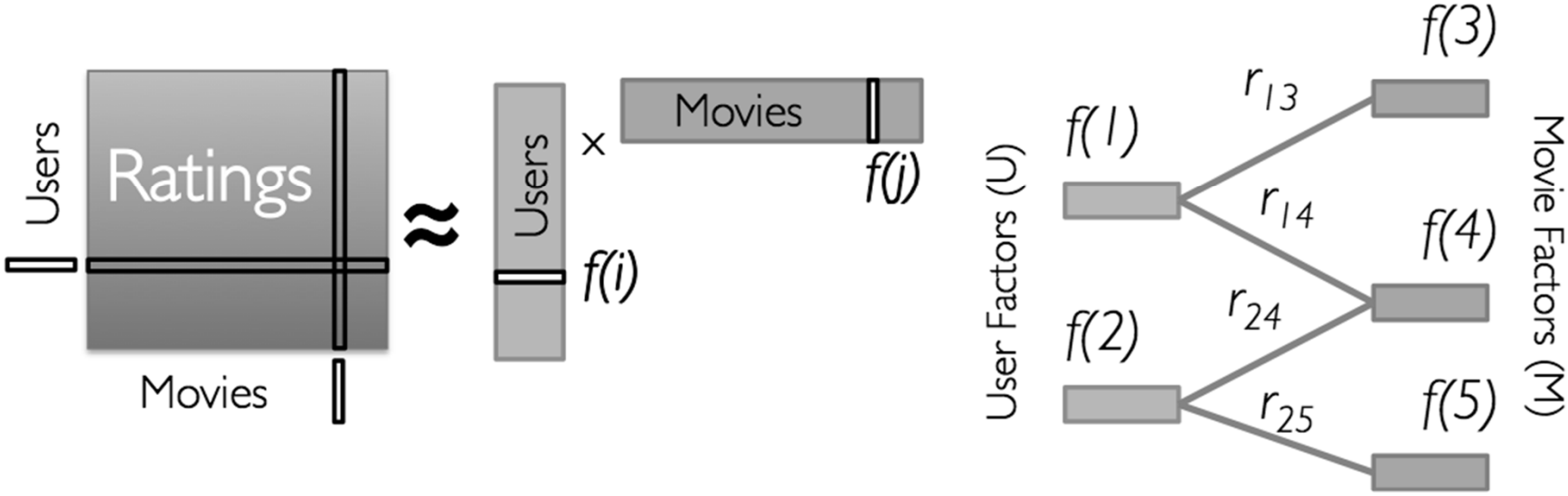
  - Fix Q as $\hat{Q}$, we have

$$P = R\hat{Q}(\hat{Q}^T\hat{Q})^{-1}$$

  - Random initialisation to start this iteration

MF for Movie Recommendation

Low-Rank Matrix Factorization:



Iterate:

$$f[i] = \arg\min_{w \in \mathbb{R}^d} \sum_{j \in \mathrm{Nbrs}(i)} \left(r_{ij} - w^T f[j]\right)^2 + \lambda ||w||_2^2$$

Figure frospark-training/matrix_factorization.png at master · databricks/spark-training (github.com)

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Key in Scalable ML

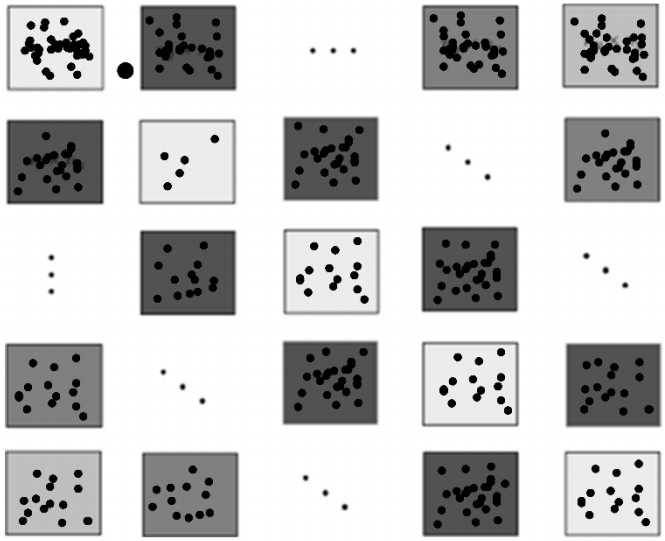
- Computation and storage should be linear (in n, d)
  → Low-cost computation (time + space)

- Perform parallel and in-memory computation
  → Many working + reduce disk I/O

- Minimise network communication → Reduce overhead in parallelisation, not the more the better

Blocked Implementation of ALS

- Group users and items into blocks
  - Reduce communication: only send one copy of each user vector to each item block on each iteration, and only for the item blocks that need that user's feature vector
  - Pre-compute info: out-links of each user (which blocks of items it will contribute to); in-links for each item (which of the feature vectors it receives from each user block it will depend on)



Color-online-A-symmetric-block-Toeplitz-matrix-Each-block-is-also-a-symmetric-Toeplitz.png (488×369) (researchgate.net)

Implicit Feedback Modelling

- Implicit feedback: views, clicks, purchases, likes, shares
  - Rating r = strength in observations of user actions (#clicks, viewing duration) → confidence level in observed user preference
  - Construct a preference matrix *P*: e.g. 1 if $r > 0$ and 0 if $r = 0$
  - Factorisation of *P* → latent factors to predict the preference of a user for an item (details in an ICDM08 paper)

The ALS API in Spark

- numUserBlocks/numItemBlocks: the number of blocks the users/items will be partitioned into to parallelize computation (defaults to 10)
- rank: the number of latent factors in the model (defaults to 10)
- regParam: the regularization parameter in ALS (defaults to 0.1)
- implicitPrefs: whether to use the explicit feedback ALS variant or one adapted for implicit feedback data (defaults to false: explicit ratings)
- alpha: the baseline confidence of implicit feedback (defaults to 1.0)
- nonnegative: whether to use nonnegative constraints (defaults to false)
- coldStartStrategy: “drop”→ drop any rows in the DataFrame of predictions that contain NaN values (defaults to "nan“: assign NaN to a user and/or item factor is not present in the model)
- blockSize: the size of the user/product blocks in the blocked implementation of ALS to reduce communication

CF in Spark ML

- Scala code (1800+ lines)
- Documentation: Collaborative Filtering in Spark
- DataBricks movie recommendations tutorial
- DataBricks: founded by the creators of Apache Spark
  - Their latest packages at their GitHub page
  - Databricks community edition: free with limited features (14-day free trial of the full platform).

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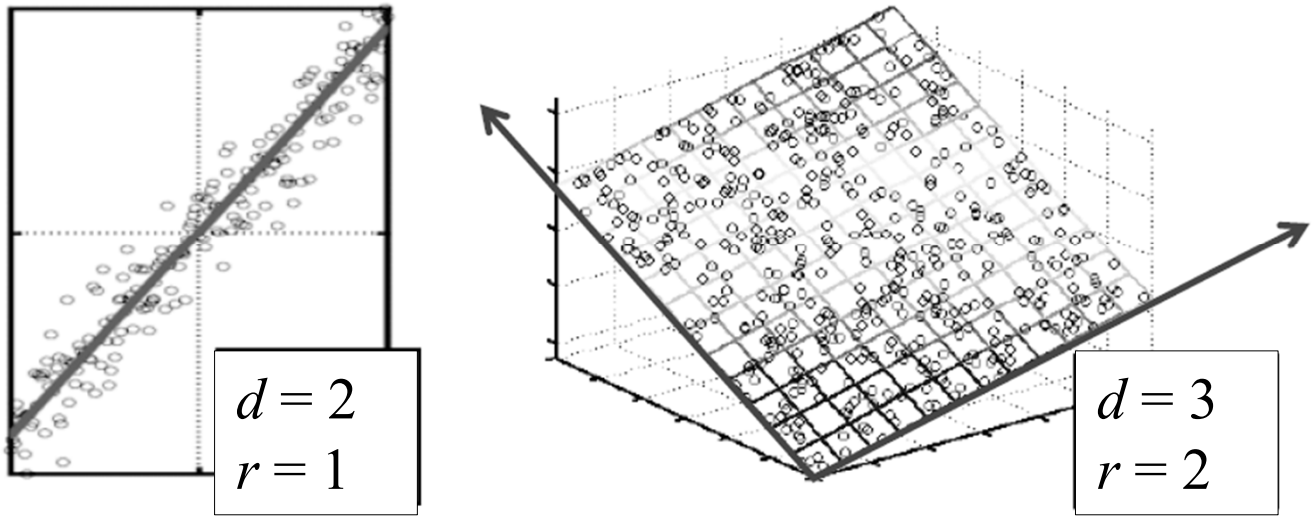
Motivation of Dimensionality Reduction

- Raw data: complex and high-dimensional
- Assumption: they lie on a low-dimensional subspace
  - Axes of this subspace→ representation of the data
  - Simpler, more compact, showing interesting patterns



Utilities of Dimensionality Reduction

- Discover hidden correlations/topics
- Remove redundant/noisy features
- Interpretation and visualisation
- Easier storage and processing of the data

owners-icebergs-blog-image-300x300.jpg (resettogrow.com)

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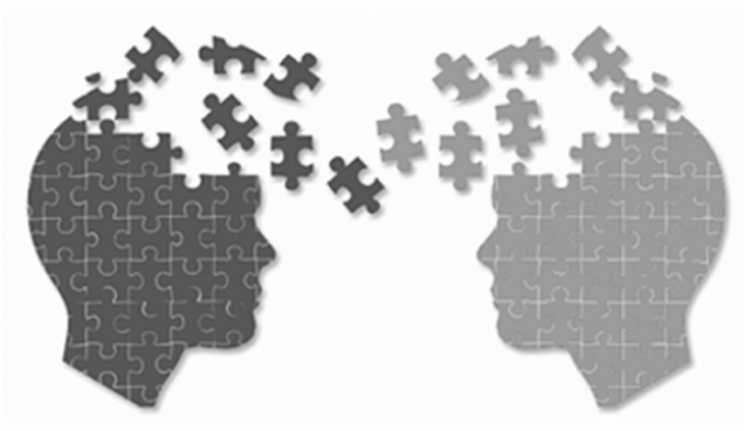

Interpreting and Translation Blog: Image (wordpress.com)

PCA → Variance Maximisation

- Input: n d-dimensional data points
- PCA algorithm
  - $\mathrm{X}_0 \leftarrow n \times d$ data matrix, data point → row vector x_i
  - X: subtract mean x from each row vector x_i in X_0
  - $\Sigma \leftarrow \mathrm{X}^\mathrm{T}\mathrm{X}$: Gramian/scatter matrix for X
  - Find eigenvectors and eigenvalues of Σ
  - U ($d \times r$) ← the top r eigenvectors (PCs)
- PCA features for y: $\mathrm{U}^\mathrm{T}\mathrm{y}$ (dimension: d → r)
  - Zero correlations, ordered by variance

Scalability Problem of PCA

- Input dimensionality → scatter matrix
  - Images: 100 x 100 → 10^4; 1000 x 1000 → 10^6
  - Scatter matrix Σ is of size d^2
    - $d = 10^4$ → Σ size 10^8
    - $d = 10^6$ → Σ size $= 10^{12}$

- Alternative: Singular Value Decomposition (SVD)
  - Efficient algorithms available
  - Often need just top r eigenvectors

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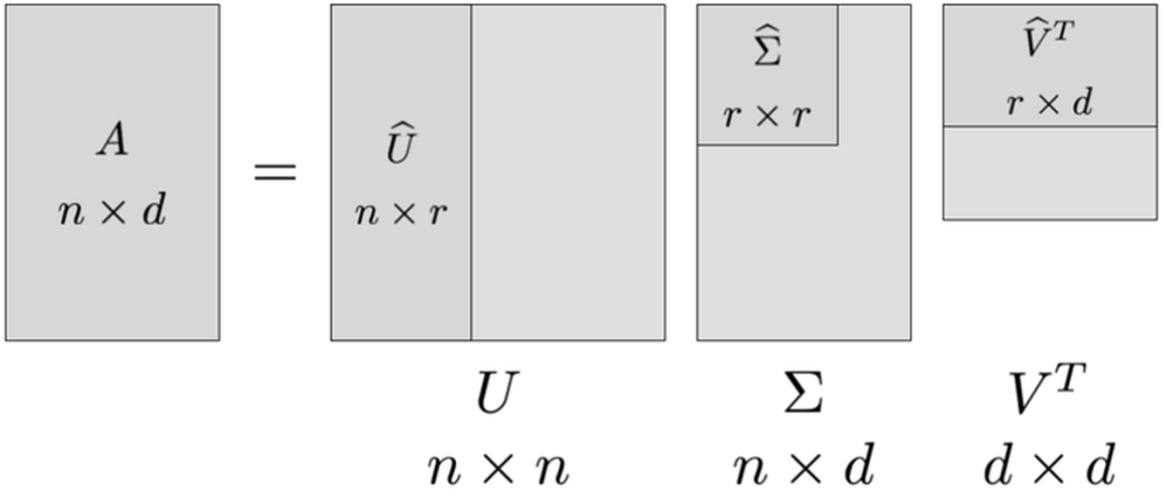
Singular Value Decomposition (SVD)

$$A_{[n \times d]} = U_{[n \times r]} \Sigma_{[r \times r]} (V_{[d \times r]})^T$$

- A: $n \times d$ matrix
- r: the rank of the matrix
- U: $n \times r$ matrix, column orthonormal, $U^T U = I$
- Σ : $r \times r$ diagonal matrix, strength of each factor
- V: $d \times r$ matrix, column orthonormal, $V^T V = I$



svd-matrices.png (800×339) (intoli.com)

SVD ←→Eigen-decomposition

- SVD gives
  - $X = U \Sigma V^T$
- Eigen-decomposition gives
  - $B = X^T X = W \Lambda W^T$
- U, V, W: orthonormal → $U^T U = I$, $V^T V = I$, $W^T W = I$
- Σ, Λ: diagonal
- Relationship:
  - $XX^T = U \Sigma V^T (U \Sigma V^T)^T = U \Sigma V^T (V \Sigma^T U^T) = U \Sigma^2 U^T$
  - $X^T X = V \Sigma^T U^T (U \Sigma V^T) = V \Sigma \Sigma^T V^T = V \Sigma^2 V^T$
  - $B = X^T X = W \Lambda W^T = V \Sigma^2 V^T$

PCA via SVD

- $X_0 \leftarrow n \times d$ data matrix, data point → row vector x_i
- X: subtract mean x from each row vector x_i in X_0
- $U \Sigma V^T \leftarrow$ SVD of X
- The top *r* right singular vectors V of X → the PCs
- The singular values in Σ = the square roots of the eigenvalues of X^TX

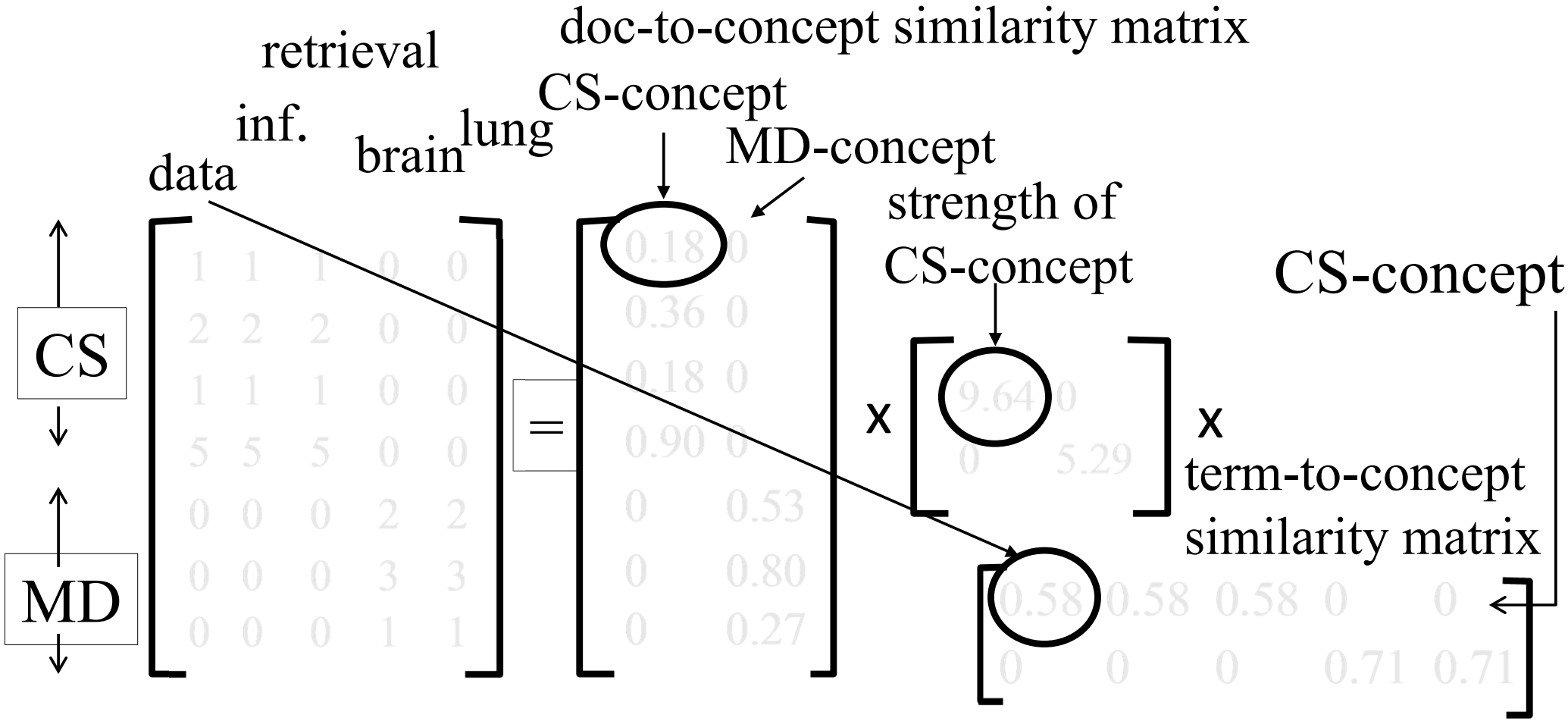
Example on a Document x Term

Term Document	data	information	retrieval	brain	lung
CS-TR1	1	1	1	0	0
CS-TR2	2	2	2	0	0
CS-TR3	1	1	1	0	0
CS-TR4	5	5	5	0	0
MED-TR1	0	0	0	2	2
MED-TR2	0	0	0	3	3
MED-TR3	0	0	0	1	1

- d = 5 but r=2 → two bases [1 1 1 0 0] & [0 0 0 1 1]
- U: document-to-concept similarity matrix
- V: term-to-concept similarity matrix
- Σ: its diagonal elements → strength of each concept

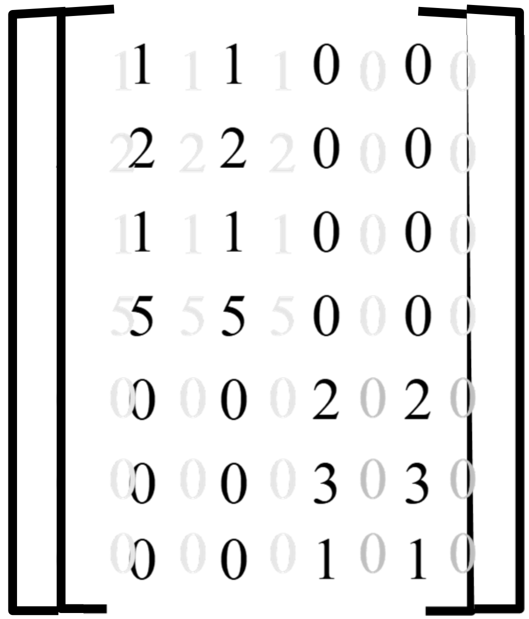
Interpretation

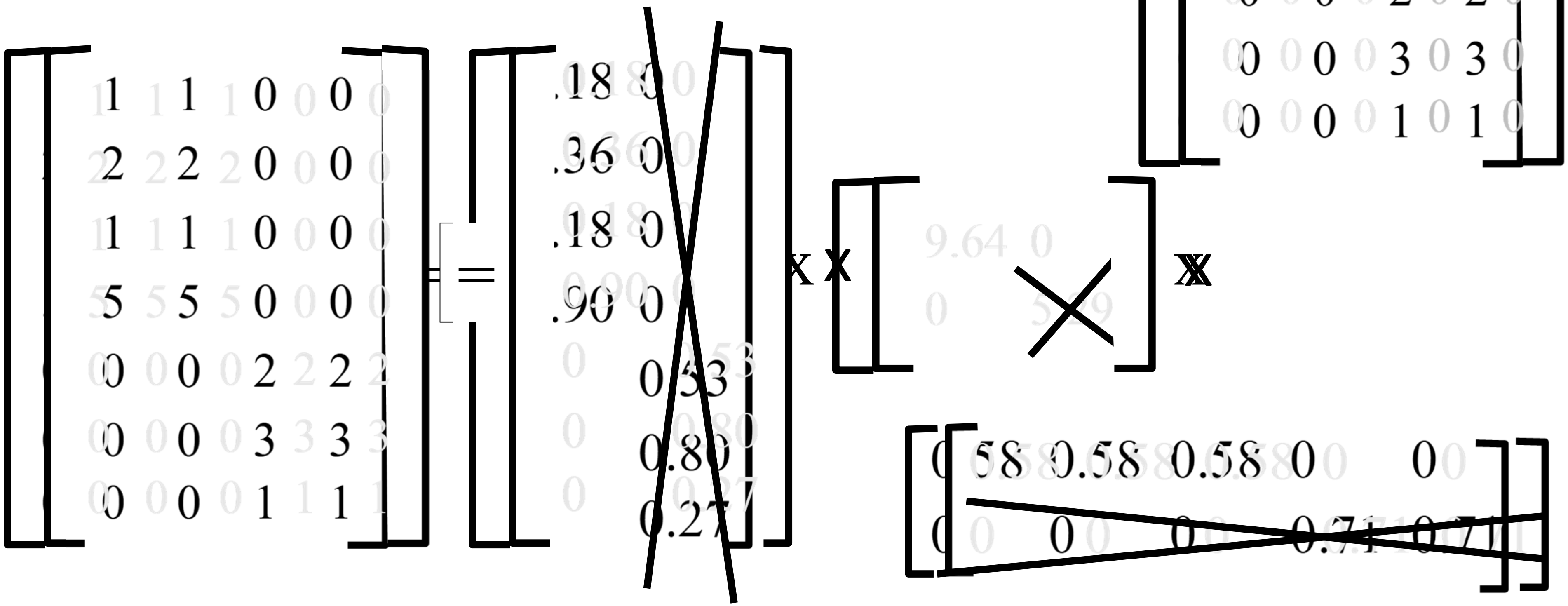
Term Document	data	information	retrieval	brain	lung
CS-TR1	1	1	1	0	0
CS-TR2	2	2	2	0	0
CS-TR3	1	1	1	0	0
CS-TR4	5	5	5	0	0
MED-TR1	0	0	0	2	2
MED-TR2	0	0	0	3	3
MED-TR3	0	0	0	1	1



SVD – Dimensionality Reduction

- To reduce the dimensionality further (3 zero singular values have already been removed)
    - Best rank-1 approximation →





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Three PCA APIs in Spark MLlib

- DataFrame-based API – PCA (source code, Scala doc)
  - `pyspark.ml.feature.PCA(k=None, inputCol=None, outputCol=None)`
- RDD-based API – RowMatrix (source code, Scala doc)
  - `computePrincipalComponents(k)`
  - Scalable: `computeSVD(k, computeU=False, rCond=1e-09)`

```
@Since("1.6.0")
def computePrincipalComponentsAndExplainedVariance(k: Int): (Matrix, Vector) = {
  val n = numCols().toInt
  require(k > 0 && k <= n, s"k = $k out of range (0, n = $n]")

  if (n > 65535) {
    val svd = computeSVD(k)
    val s = svd.s.toArray.map(eigValue => eigValue * eigValue / (n - 1))
    val eigenSum = s.sum
    val explainedVariance = s.map(_ / eigenSum)
```

SVD in Spark MLlib (RDD)

- U: $m \times k$; $\Sigma : k \times k$; V: $n \times k$
- Assumption: n (dimensionality) < m (# samples)
- Different methods based on computational cost
  - If n is small (n<100) or k is large compared with n (k>n/2), compute $\mathrm{A^TA}$ first and then compute its top eigenvalues and eigenvectors locally on the driver
  - Otherwise, compute $(\mathrm{A^TA})v$ in a distributive way and send it to ARPACK to compute $(\mathrm{A^TA})$'s top eigenvalues/eigenvectors on the driver node

Selection of SVD Computation

```
        if (n < 100 || (k > n / 2 && n <= 15000)) {
          // If n is small or k is large compared with n, we better compute the Gramian matrix first
          // and then compute its eigenvalues locally, instead of making multiple passes.
          if (k < n / 3) {
            SVDMode.LocalARPACK
          } else {
            SVDMode.LocalLAPACK
          }
        } else {
          // If k is small compared with n, we use ARPACK with distributed multiplication.
          SVDMode.DistARPACK
        }
      case "local-svd" => SVDMode.LocalLAPACK
      case "local-eigs" => SVDMode.LocalARPACK
      case "dist-eigs" => SVDMode.DistARPACK
      case _ => throw new IllegalArgumentException(s"Do not support mode $mode.")
```

Acknowledgement & References

- Acknowledgement
  - Some slides are adapted from the MMDS book slides.
- References
  - Yehuda Koren, Robert Bell, and Chris Volinsky. "Matrix factorization techniques for recommender systems." *Computer* 8 (2009): 30-37 (Yahoo & AT&T)
  - Yifan Hu, Yehuda Koren, and Chris Volinsky. "Collaborative filtering for implicit feedback datasets." *Eighth IEEE International Conference on Data Mining*, 2008
  - Charu C. Aggarwal, *Recommender Systems: The Textbook*, April 2016
  - Chapter 11 of the MMDS book