Determination of the Geodesic Sensor Nets' Average Electrode Positions and Their 10 – 10 International Equivalents

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This technical document describes a study conducted by EGI. The aims of this study are 1) to acquire average electrode positions for the 64- (adult and infant), 128- (adult and infant), and 256-channel GSN, and 2) to determine the approximate correspondence between the international 10-10 electrode positions with those sensors on the 64-, 128-, and 256-channel GSN.

Method

Subjects

Adults with head sizes fitting the small (n = 5), medium (n = 5), and large (n = 9) GSNs participated in the study. Five subjects also had 10 – 10 measurements taken for digitization (see below).

Materials

The 64-, 128-, 256-channel adult GSNs were used to determine the electrode position on the subjects head. The 64- and 128-channel infant GSNs were applied to infant head models obtained from Simuloids Inc. Three infant head models were used: small (35 cm circumference), medium (39.5 cm), and large (43.5 cm). Digitization of the GSN's sensor positions was obtained using the Fastrack digitizer (Polhemus) and the Locator software (Source Signal Imaging).

Proceedure

Application of the GSN was done by two trained technicians. This procedure involves identifying the vertex and applying the GSN with reference to this point. Once the net the nasion, and the left and right ears. Prior to the application of the GSN, Fastrack receivers were placed behind the left and right ear and on the nose. This was accomplished for each subject with the 64-, 128-, and 256-channel GSN. To obtain positions of the 10-10 electrode system, a registered EEG technician identified the 71 locations (minus the ear positions) on each of the five subjects. A similar procedure was used for the infant model heads.

Once the GSN has been applied or the 10-10 positions determined, each sensor position was digitized using the Fastrack hardware and Locator software. In addition to digitizing the sensor positions, the left and right preauricular points, nasion, and inion fiducials were also digitized.

Sensor Registration

The objective is to put all such electrode files in a common framework to compare how the electrode montages sit upon the human head on average. What follows is a step-by-step sequence for doing this. The choices made are not necessarily unique, but are sensible given the stated objective. Note that the resulting coordinate system is not simply related to any best-fit spheres approximation of the head, as would be used in bioelectric field modeling for example.

1. A Polhemus Tracker or Photogrammetry system returns the position of each electrode and the fiducial points in some arbitrary coordinate system

$$\vec{r}_i^{\,(p)} = \left(x_i^{(p)}, y_i^{(p)}, z_i^{(p)} \right)^T$$

where i=1,…,129,L,R,N,V. The first 129 points are the electrode locations. L and R represent the left and right pre-auricular points, N represents the nasion, and V represents the vertex. The vectors to each point each exist in 3-dimensional space in a coordinate independent sense, but the superscript (p) indicates that here these are represented in the Polhemus coordinate system. The superscript T stands for transpose, and reflects our preference toward working with column vectors to facilitate the matrix multiplications below.

2. We conceive a fiducial coordinate system, denoted with a superscript (f), with the following properties. First, its x-axis lies along the line connecting L and R with the positive direction pointing toward R. Second, its y-axis is perpendicular to the line LR, and passes through the nasion in the positive direction. The x-axis and y-axis intersect at a point, which forms the center of this coordinate system, but with the constraints that: i) the y-axis pass through the nasion, and ii) the x-axis and y-axis meet at a right angle, the intersection will not in general be at the midline of the line LR. As a matter of convention, we are placing more emphasis on the nasion than on the midpoint of the line LR.

3. It is a simple problem in 3-dimensional Euclidean geometry to determine the equation for the unique line passing through the nasion N and intersecting the line LR at a right angle. Let us denote the intersection of these lines with the vector

$$\vec{r}_C^{\,(p)} = \left(x_C^{(p)}, y_C^{(p)}, z_C^{(p)} \right)^T$$

where the subscript C stands for the center of the fiducial coordinate system in Polhemus coordinates (p). An expression for the location of the point C in a slightly simplifed case is given by Strang (1980). In terms of the fiducial position vectors in the Polhemus coordinates, we have

$$\vec{r}_C^{\,(p)} = \vec{r}_L^{\,(p)} + \frac{\left(\vec{r}_R^{\,(p)} - \vec{r}_L^{\,(p)} \right) \left(\vec{r}_N^{\,(p)} - \vec{r}_L^{\,(p)} \right)}{\left| \vec{r}_R^{\,(p)} - \vec{r}_L^{\,(p)} \right|^2} \left(\vec{r}_R^{\,(p)} - \vec{r}_L^{\,(p)} \right)$$

4. The next step is to translate all our coordinates to be centered on the above point. This is done trivially with the transformation

$$\vec{r}_i^{\,(c)} = \vec{r}_i^{\,(p)} - \vec{r}_C^{\,(p)}$$

where the superscript (c) denotes that the new vectors are in our so-called “centered” coordinate system. We of course must have the trivial result that

$$\vec{r}_C^{(c)} = (0, 0, 0)^T$$

that is, the center point C is at the origin of our centered coordinate system (c). We are still not done, however, because this centered coordinate system (c) is rotated arbitrarily relative to our desired fiducial coordinate system (f).

5. We can easily define unit vectors in these centered coordinates which lie along the axes of the desired fiducial coordinate system. First, the x-axis is defined to lie along the line connecting the center point C and the right pre-auricular point R:

$$\hat{x}^{(c)} = \frac{\vec{r}_R^{(c)}}{\left|\vec{r}_R^{(c)}\right|}$$

(Because the center point C is the origin of our centered coordinate system it does not appear explicitly in the above equation.) Second, the y-axis is defined to lie along the line connecting the center point C and the nasion N:

$$\hat{y}^{(c)} = \frac{\vec{r}_N^{(c)}}{\left|\vec{r}_N^{(c)}\right|}$$

A useful step in verifying the computer implementation is to test that these two vectors are indeed orthogonal. Since they are unit vectors, this may be considered satisfied of their dot product is small compared to unit. Finally, the z-axis is defined to be perpendicular to the resulting xy-plane, with its positive direction determined as usual by the right hand rule:

$$\hat{z}^{(c)} = \hat{x}^{(c)} \times \hat{y}^{(c)}$$

By definition, the fiducial unit vectors have trivial representations in the fiducial coordinate system (f).

$$\hat{x}^{(f)} = (1,0,0)^T$$
$$\hat{y}^{(f)} = (0,1,0)^T$$
$$\hat{z}^{(f)} = (0,0,1)^T$$

6. Our next goal is to determine a matrix which, when applied to the electrode positions in the centered coordinate system (c), will rotate them in to the fiducial coordinate system (f). This is most easily done as follows. We first construct a matrix M, whose columns are the fiducial unit vectors in centered coordinates:

$$M = \left[\hat{x}^{(c)}\ ,\ \hat{y}^{(c)}\ \ ,\hat{z}^{(c)}\right]$$

This matrix M has the obvious property that, when applied to the fiducial unit vectors in the fiducial coordinate system (f), it translates them into the centered coordinate system (c):

$$\hat{x}^{(c)} = M\,\hat{x}^{(f)}$$
$$\hat{y}^{(c)} = M\,\hat{y}^{(f)}$$
$$\hat{z}^{(c)} = M\,\hat{z}^{(f)}$$

Since any vector in the fiducial coordinate system can be represented as a linear sum of the fiducial unit vectors, the transformation matrix M actually rotates *any* vector from the fiducial coordinate system (f) to the centered coordinate system (c).

$$\vec{r}_i^{(c)} = M\,\vec{r}_i^{(f)}$$

Once this matrix is constructed it can be tested by applying it to each of the hat vectors in the (f) basis. It should be that the trivial hat vectors in the fiducial system get transformed to the nontrivial hat vectors in the centered coordinate sytstem.

7. By definition, the inverse of M rotates any vector in the centered coordinates (c) into the fiducial coordinates (f), that is

$$\vec{r}_i^{(f)} = M^{-1}\,\vec{r}_i^{(c)}$$

This is our desired result, and can be applied to each of the electrode and fiducial positions. Conveniently, it can be shown that the matrix M is an orthogonal matrix whose inverse is equal to its transpose (Arfken and Weber, 1995).

$$M^{-1} = M^{T}$$

so the inverse of M can be gotten trivially. The final result may be tested by applying the inverse (transpose) matrix to each of the hat vectors in the centered coordinates (c) and verifying that they are properly rotated into the fiducial coordinate axes.

As a final test of the entire set of transformations, one can verify that the distance between each pair of electrodes is preserved. Given the deterministic nature of each of the transformation, this can be expected to be satisfied to within the accuracy of single or double precision. If the distance between each pair of electrodes is preserved, and right pre-auricular point and nasion lie on the fiducial x and y axes respectively, then the coordinate transformation must be correct.

Averaging of Sensor Positions

Once the electrode positions have been co-registered, they were normalized to a unit sphere for all subjects. The positions of the sensor for each GSN and the 10 – 10 were then averaged across the five subjects.

The electrode positions should conform to certain standards if they were placed on an ideal head (i.e., perfectly symmetric). For example, midline electrodes should lie on the midline and all other electrodes should have a homologous site on the opposite hemisphere (the pair should be symmetric). Because real heads are not usually symmetric and because these asymmetries are not completely removed by averaging across the small number of subjects used in the present study, symmetry of electrode positions must be mathematically induced. To obtain symmetry for each version of the GSN and the 10-10, the following procedure was used. First, the known midline electrodes were made to lie in the sagital (Y) plane. Second, all electrodes on the left hemisphere were compared to their homologous counterpart by making their X (ear to ear) position positive. The difference between a pair in all three directions was taken and halved. This difference was then used to adjust the position of each of the electrode for that particular pair to make the two electrode positions symmetric.

Determining the 10-10 position equivalence for the GSN

Distance between a particular 10-10 position and all sensors for a particular GSN (i.e., 64-, 128-, 256-channel net) was computed by obtaining the arc length. The three nearest GSN sensor positions for that particular 10-10 position were used to determine the approximate 10-10 equivalent in the GSN sensor array. The following criteria were used to determine approximate equivalence:

1) Midline 10-10 positions must also be along the midline in the GSN array, even if the nearest GSN position did not lie on the midline.

2) The nearest GSN position was used as an approximate equivalent 10-10 position.

4) No GSN sensor equivalence for a 10-10 position could be obtained when the arc distance of all 3 nearest GSN position exceed .20 the sphere's radius.

Results

Average Positions of the GSNs

These are available from EGI by emailing us at support@EGI.com.

The GSNs' Approximate 10 – 10 Equivalents

Tables 1, 2, and 3 present the GSNs' approximate 10 – 10 equivalents, and Figures 1 and 2 presents approximate 10 – 10 position on the 64- and 128 channel GSN.

10-10 to 64 V 2.0

10-10	GSN	Arc Length	10-10	GSN	10-10
FPZ	11	0.161837	FC4	54	0.177195
	6	0.161837		57	0.177371
Afz	7	0.0634139		58	0.218897
Fz	3	0.154376	FC6	57	0.0696699
	8	0.154376	FT8	56	0.0715139
Fcz	4	0.0658022	T8	52	0.12636
CpZ	30	0.0916561	C6	53	0.0540593
Pz	34	0.132976	C4	54	0.126231
Poz	38	0.147266	C2	43	0.166758
Oz	38	0.169856		54	0.194738
	37	0.176516		55	0.214054
	40	0.176516	C1	18	0.166758
FP1	11	0.124448		17	0.194738
FP2	6	0.124448		5	0.214054
F9	19	0.0993434	C3	17	0.126231
F10	60	0.0993434	C5	21	0.0540593
Ft9	19	0.260501	T7	24	0.12636
	23	0.334643	Tp7	27	0.174719
	20	0.428871		25	0.212575
Ft10	60	0.260501		24	0.225688
	59	0.334643	Cp5	25	0.0855924
	56	0.428871	CP3	22	0.0661674
T9	23	0.408182	CP1	18	0.146967
	24	0.438815	Cp2	43	0.146967
	26	0.471063	CP4	47	0.0661674
T10	59	0.419279	CP6	50	0.0855924
	52	0.434167	TP8	49	0.174719
	51	0.453065		50	0.212575
TP9	26	0.0758048		52	0.225688
TP10	51	0.0758048	P8	49	0.142733
P9	31	0.142757	P6	46	0.0215042
P10	48	0.142757	P4	46	0.139266
AF7	14	0.115935		42	0.207309
AF3	12	0.103447	P2	42	0.0541376
AF4	2	0.103447	P1	29	0.0541376
AF8	1	0.115935	P3	28	0.139266
F8	61	0.0540516		29	0.207309
F6	62	0.141327	P5	28	0.0215042
	61	0.159185	P7	27	0.142733
F4	62	0.0933557	Po7	32	0.137045
F2	3	0.0755565	PO3	33	0.0805964
F1	8	0.0755565	PO4	41	0.0805964
F3	13	0.0933557	PO8	45	0.137045
F5	13	0.141327	O2	40	0.16958
	15	0.159185	O1	37	0.16958
F7	15	0.0540516			
FT7	20	0.0715139			
FC5	16	0.0696699			
FC3	17	0.177195			
	16	0.177371			
	9	0.218897			
FC1	9	0.090935			
FC2	58	0.090935			

10-10 to 128

10-10	GSN	Arc Length	10-10	GSN	10-10
FPZ	15	0.153127	C1	31	0.0613779
	18	0.153127	C3	37	0.0544276
Afz	16	0.000690534	C5	42	0.0584615
Fz	11	0.0245364	T7	46	0.117001
Fcz	6	0.143405	Tp7	51	0.0939897
CpZ	55	0.0787382	Cp5	48	0.0748337
Pz	62	0.0492064	CP3	43	0.115817
Poz	68	0.0505041	CP1	38	0.116055
Oz	72	0.136695	Cp2	88	0.116055
	77	0.136695	CP4	94	0.115817
	73	0.136763	CP6	99	0.0748337
FP1	22	0.0949401	TP8	98	0.0939897
FP2	14	0.0949401	P8	92	0.097913
F9	39	0.0518522	P6	93	0.065948
F10	121	0.0518522	P4	87	0.0816716
Ft9	45	0.125576	P2	79	0.108002
Ft10	115	0.125576	P1	61	0.108002
T9	45	0.23644	P3	53	0.0816716
	46	0.287369	P5	52	0.065948
	50	0.414399	P7	59	0.097913
T10	115	0.255352	Po7	66	0.177199
	109	0.290864	PO3	60	0.130532
	102	0.39966	PO4	86	0.130532
TP9	57	0.19289	PO8	85	0.177199
TP10	101	0.19289	O2	77	0.133263
P9	58	0.136955	O1	72	0.133263
P10	97	0.136955			
AF7	27	0.0732928			
AF3	24	0.0962136			
AF4	3	0.0962136			
AF8	2	0.0732928			
F8	122	0.0354637			
F6	123	0.0123527			
F4	124	0.0110054			
F2	4	0.0243193			
F1	20	0.0243193			
F3	25	0.0110054			
F5	28	0.0123527			
F7	34	0.0354637			
FT7	40	0.0994947			
FC5	35	0.104364			
FC3	30	0.11176			
FC1	21	0.0931028			
FC2	119	0.0931028			
FC4	112	0.11176			
FC6	117	0.104364			
FT8	116	0.0994947			
T8	109	0.117001			
C6	104	0.0859411			
C4	105	0.0544276			
C2	106	0.0613779			

10-10 to 256

10-10	GSN	Arc Length	10-10	GSN	10-10
FPZ	25	0.127039	C1	43	0.0545349
	31	0.127039	C3	58	0.0773437
Afz	21	0.0471089	C5	63	0.0986002
Fz	22	0.0956917	T7	68	0.111667
	14	0.0956917	Tp7	74	0.0284179
Fcz	8	0.0943687	Cp5	71	0.0601916
	16	0.0986135	CP3	59	0.114337
	7	0.0986135	CP1	78	0.0855478
CpZ	80	0.0198884	Cp2	144	0.0855478
Pz	100	0.0660303	CP4	156	0.114337
Poz	111	0.0617354	CP6	174	0.0601916
Oz	127	0.0437939	TP8	181	0.0284179
FP1	36	0.0802365	P8	172	0.101759
FP2	18	0.0802365	P6	163	0.0469759
F9	251	0.0929199	P4	154	0.0437817
F10	227	0.0929199	P2	143	0.0814536
Ft9	66	0.13644	P1	87	0.0814536
Ft10	221	0.13644	P3	86	0.0437817
T9	67	0.254216	P5	85	0.0469759
	68	0.34865	P7	84	0.101759
	66	0.368469	Po7	96	0.0981767
T10	211	0.270967	PO3	98	0.0157202
	203	0.35478	PO4	142	0.0157202
	221	0.385711	PO8	162	0.0981767
TP9	82	0.124214	O2	141	0.101554
TP10	192	0.124214	O1	109	0.101554
P9	106	0.0774123			
P10	170	0.0774123			
AF7	37	0.136724			
AF3	34	0.0627026			
AF4	4	0.0627026			
AF8	11	0.136724			
F8	223	0.0741851			
F6	225	0.083295			
F4	226	0.0694117			
F2	5	0.0543498			
F1	28	0.0543498			
F3	35	0.0694117			
F5	39	0.083295			
F7	54	0.0741851			
FT7	61	0.0674646			
FC5	56	0.103291			
FC3	41	0.0761363			
FC1	24	0.0222052			
FC2	208	0.0222052			
FC4	207	0.0761363			
FC6	205	0.103291			
FT8	212	0.0674646			
T8	194	0.0943491			
C6	183	0.0696528			
C4	184	0.0773437			
C2	186	0.0545349			

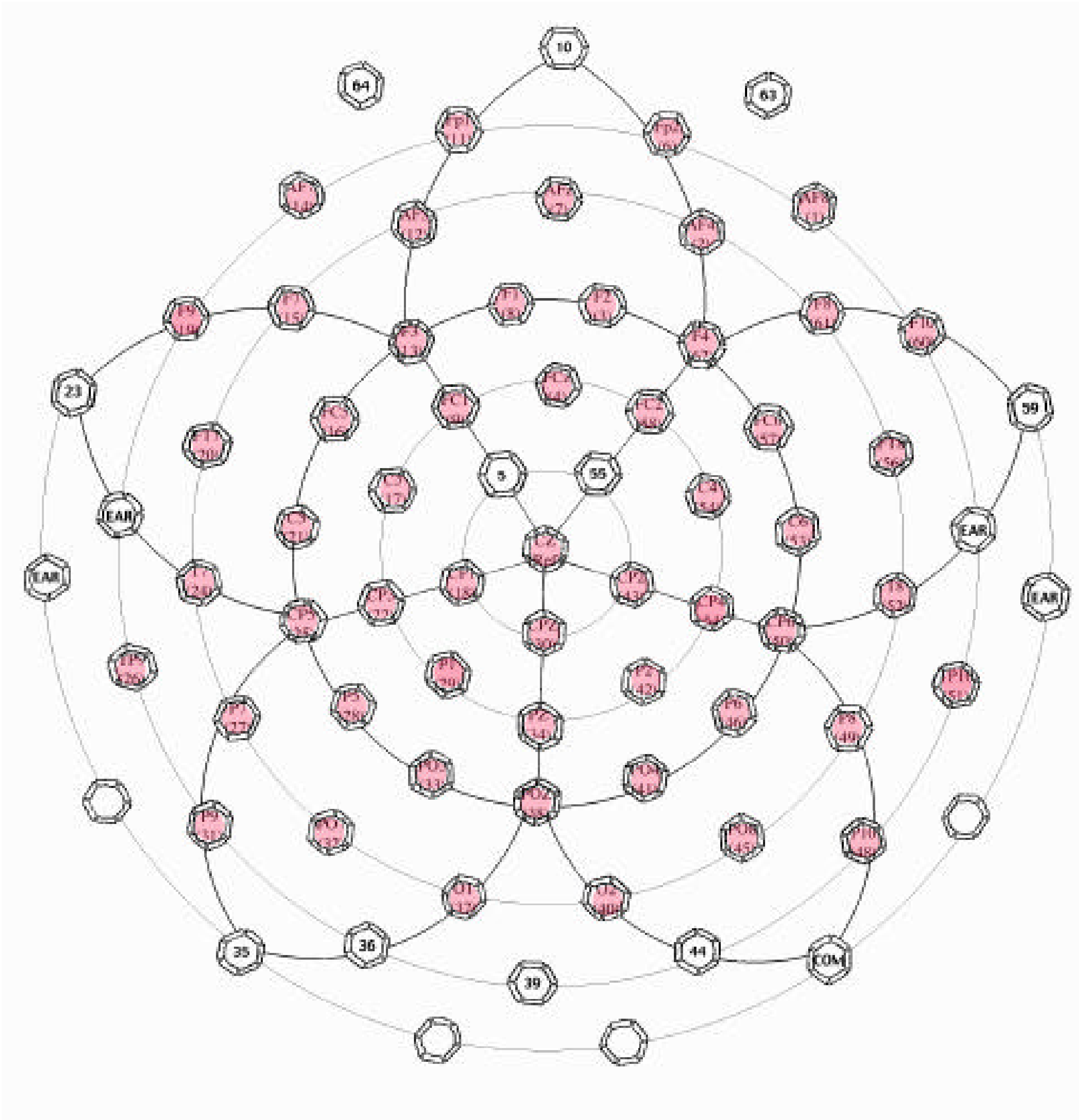

Figure 1. Layout illustrating the 10 – 10 equivalent on the 64-channel GSN.

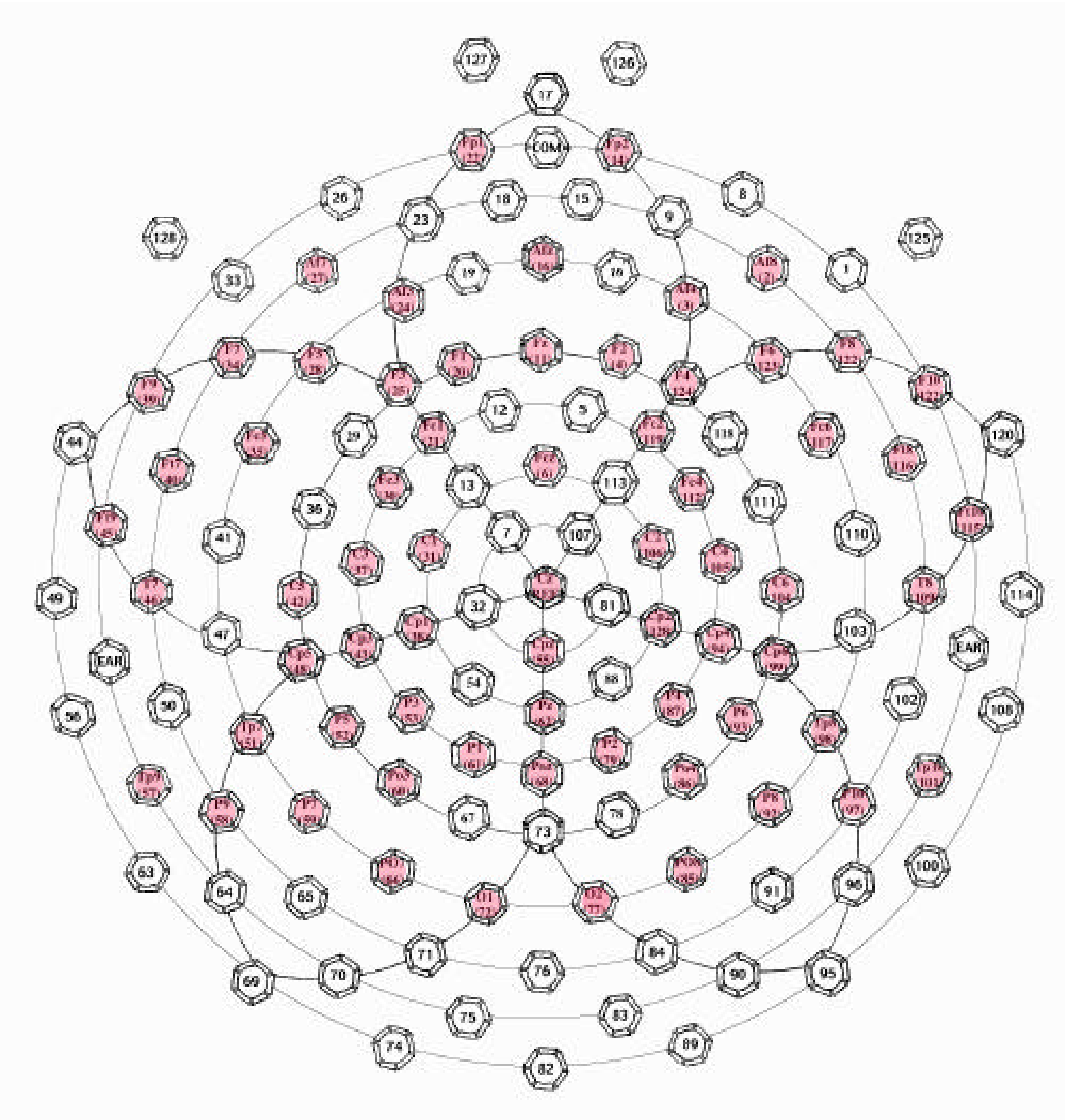

Figure 2. Layout illustrating the 10 – 10 equivalent on the 128-channel GSN.

References

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