On the Evolution of Finite-Sized Complex Networks with Constrained Link Addition

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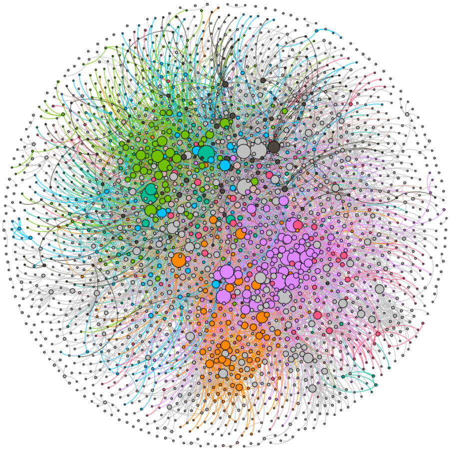
IEEE ANTS

December 19, 2018

Real-world network examples

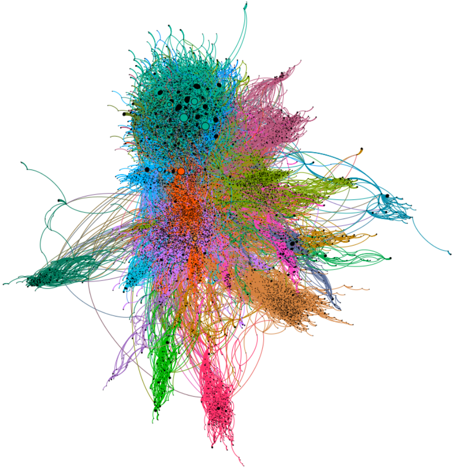
EuroSis network consists of 1285 nodes and 7586 links. The network represents web interaction of 12 European countries in the context of science community actors. The graph is generated with Gephi 0.9.1 and the network layout is ForceAtlas.

Real-world network examples (cont'd...)



The protein-protein biochemical interaction network of Yeast with 2361 nodes and 7182 edges. The protein-protein interaction networks show contacts established during the bio-chemical reaction in the body of yeast. The graph is generated with Gephi 0.9.1 and the network layout is Fruchterman-Reingold.

Real-world network examples (cont'd...)



The network of autonomous systems (ASs) in the Internet with 22,963 nodes and 48,434 links. A node in the network represents an AS and an edge represents interconnection between two ASs. The graph is generated with Gephi 0.9.1 and the network layout is ForceAtlas 2.

Key observation: Real-world network examples

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  - Size of the **network is growing** w.r.t. **new links**
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- Some real-world examples:
  - **Relationships in community networks**
  - **Transportation networks**
  - **Sensor networks**
  - **Social networks of closed community**
  - **Many more...**

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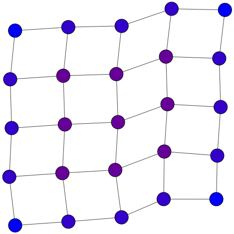
Our key contribution

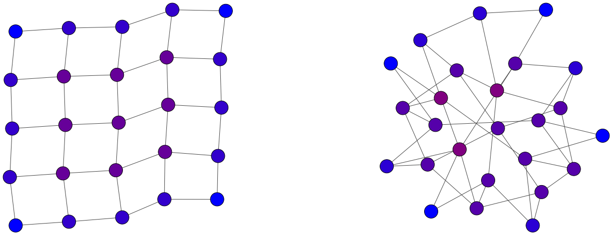
Study the **gradual evolution of finite-sized complex networks** with **constrained link addition**

Evolution of a finite-sized grid network



(a) A 5×5 regular grid network

Evolution of a finite-sized grid network



(a) A 5×5 regular grid network **(b) The grid transforms to a small-world network**

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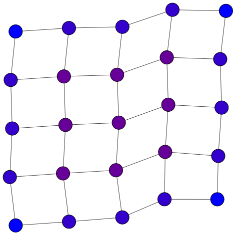

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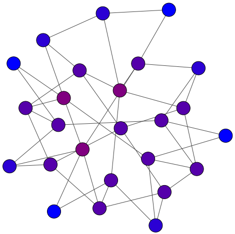
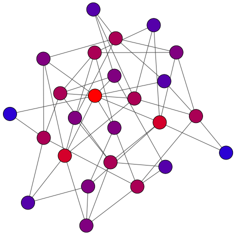


(b) The grid transforms to a small-world network



(c) It gradually transforms to a scale-free network

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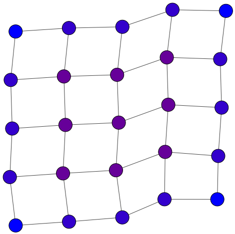

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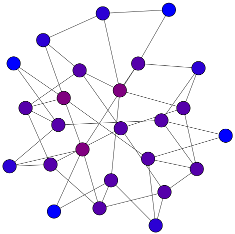

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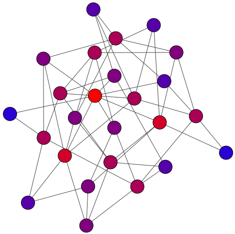

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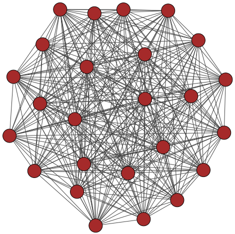

(d) **The grid becomes fully connected with unconstrained link addition**

Outline of the talk

1. Related work
2. Constrained link addition
3. Experimental results
4. Observations and conclusion

Related work

Evolution of real-world complex networks

- **Barabási *et al.***[1] first observed the **formation of real-world networks to be scale-free**
  - **Growth** and **preferential attachment**
  - **Fitness of nodes**[2]
- **Real-world scale-free networks** can also be **evolved with rewiring**[3] where $\gamma \simeq 1$
- **Papadopoulos *et al.***[4] claimed the formation of scale-free networks
  - **Involves no luck**[5]
  - Optimization of **popularity** and **similarity**

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[1] Albert-László Barabási and Réka Albert. "Emergence of scaling in random networks". In: *Science* 286.5439 (1999), pp. 509–512.

[2] Ginestra Bianconi and A-L Barabási. "Competition and multiscaling in evolving networks". In: *EPL (Europhysics Letters)* 54.4 (2001), p. 436.

[3] Gábor Timár, Sergey N Dorogovtsev, and José Fernando F Mendes. "Scale-free networks with exponent one". In: *Physical Review E* 94.2 (2016), p. 022302.

[4] Fragkiskos Papadopoulos et al. "Popularity versus similarity in growing networks". In: *Nature* 489.7417 (2012), pp. 537–540.

[5] Albert-László Barabási. "Network science: Luck or reason". In: *Nature* 489.7417 (2012), pp. 507–508.

Evolution of real-world networks (cont'd. . .)

- **We found**[6] that
  - **Greed** is one of the key reasons for **evolution of real-world scale-free networks**

[6] Abhishek Chakraborty and B. S. Manoj. "The reason behind the scale-free world". In: *IEEE Sensors Journal* 14.11 (2014), pp. 4014–4015.

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- Greedy decision-based approach **transforms a regular network to a scale-free network**
  - Hub nodes attract **unconstrained log-ranged links (LLs)** due to **long-ranged link affinity (LRA)**

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We study what happens when constrained LLs are added based on greedy decision making

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Observations on constrained LL addition

Constrained LL addition: Assumptions

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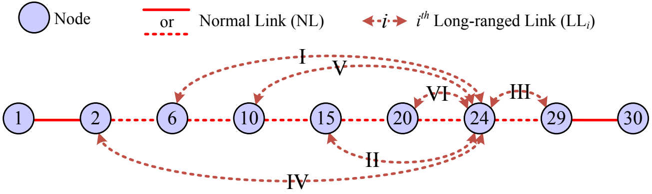
Addition of unconstrained LLs with greedy decision making:



An example unconstrained LL addition in a 30-node string

What do we mean by constrained LL addition?

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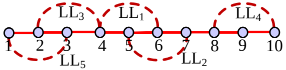

(a) LL addition with LL$_{MaxLen} = 2$

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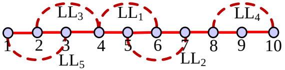

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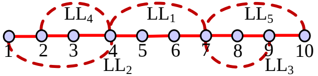

(b) **LL addition with LL$_{MaxLen} = 3$**

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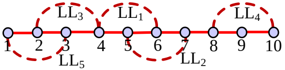

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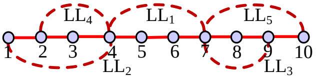

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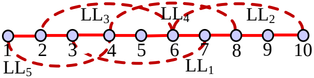

(c) **LL addition with $\mathbf{LL}_{MaxLen} = 4$**

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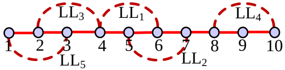

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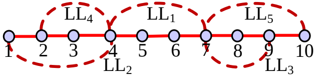

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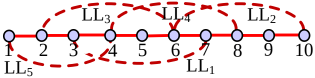

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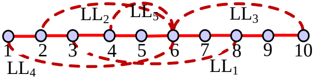

(d) **LL addition with LL**$_{MaxLen} = 5$

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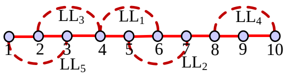

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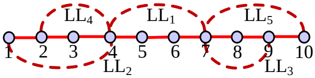

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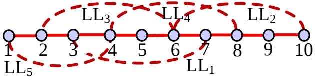

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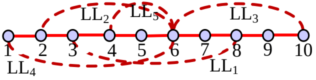

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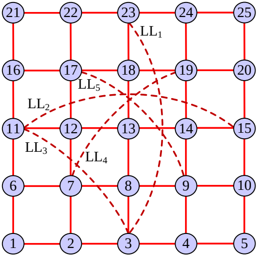

(e) **LL addition in grid with $LL_{MaxLen2D} = 4$**

Greedy decision-based constrained LL addition

Greedy Decision-based Constrained LL Addition

Initialization: LL_{Max} // ($\textbf{LL}_{MaxLen}$ **or** $\textbf{LL}_{MaxLen2D}$)

for $i = 1 \rightarrow k$ **do** // k: **Number of LLs to be added**
  for $p = 1 \rightarrow N$ **do**
    for $q = 1 \rightarrow N$ **do**
      if $2 \leqslant \text{LL}_{Len}(p,\ q) \leqslant \text{LL}_{Max}$ && $(p,\ q) \notin \mathcal{E}$ **then**
        Construct i^{th} LL between nodes p and q
        Estimate APL value of the network
        Save the APL value
        Remove the constructed LL
      end if
    end for
  end for
  Searches for optimal APL value to construct i^{th} LL
  if More than one optimal APL LL possibilities exist then
    Randomly select one node pair with the optimal value of APL
  end if
  Add i^{th} LL between selected node pair giving the lowest APL
  Update network graph
end for

Experimental results and discussion

Experimental setup

- A set of **constrained LLs are added with greedy decision to minimize APL**[8]

[8] B. S. Manoj, Abhishek Chakraborty, and Rahul Singh. *Complex networks: A networking and signal processing perspective*. Prentice Hall PTR, New Jersey, USA, 2018.

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- A set of **constrained LLs are added with greedy decision to minimize APL**[8]
- $\left\lceil \frac{N}{2} \right\rceil$ **constrained LLs are deployed** in an N-node string
  - **Range of an LL (1D): $2 \leq$ LL $\leq \left\lceil \frac{N}{2} \right\rceil$**
  - With various sized networks (**from 50 to 200 nodes**)

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  - With various sized networks (**from 50 to 200 nodes**)
- $\left\lceil \frac{N^2}{2} \right\rceil$ **constrained LLs are deployed** in an $N \times N$-node grid
  - **Range of an LL (2D): $2 \leq \mathbf{LL} \leq \mathbf{N}$**
  - With various sized networks (**from** 10×10 **to** 25×25 **nodes**)

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  - With various sized networks (**from** 10×10 **to** 25×25 **nodes**)
- **Observations are made** for the following parameters
  - **Nodal degree** w.r.t. different **$\mathbf{LL}_{Max}$ values** (**$\mathbf{D}_{max}$, $\mathbf{D}_{max2}$, and $\mathbf{D}_{max3}$**)
  - **Average length of constrained LLs**

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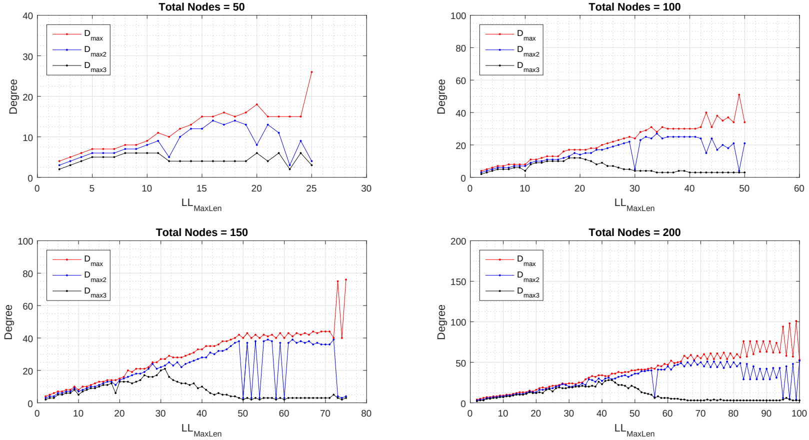
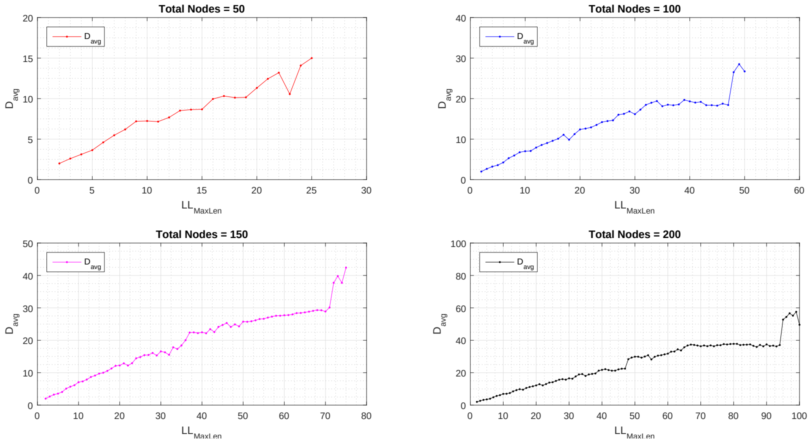
Observations on string networks



Plot of the maximum degree (D_{max}), the second maximum degree (D_{max2}), and the third maximum degree (D_{max3}) with respect to different LL_{MaxLen} values for 50-, 100-, 150-, and 200-node string networks

Observations on string networks (cont'd. . .)



Plot of the average length of an LL (D_{avg}) with respect to different LL_{MaxLen} values for 50-, 100-, 150-, and 200-node string networks

Key observations from simulation results

- **Observations on nodal degree**
  - Differences among D_{max}, D_{max2}, and D_{max3} values are more as $LL_{MaxLen} \rightarrow \left\lceil \frac{N}{2} \right\rceil$
    - With increasing LL_{MaxLen}, a single node becomes hub

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  - In $0.3N < LL_{MaxLen} < 0.4N$, **presence of multiple hub-nodes**

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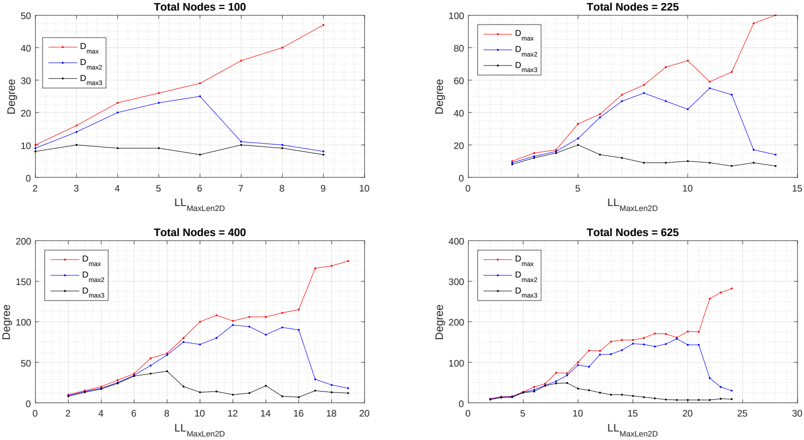
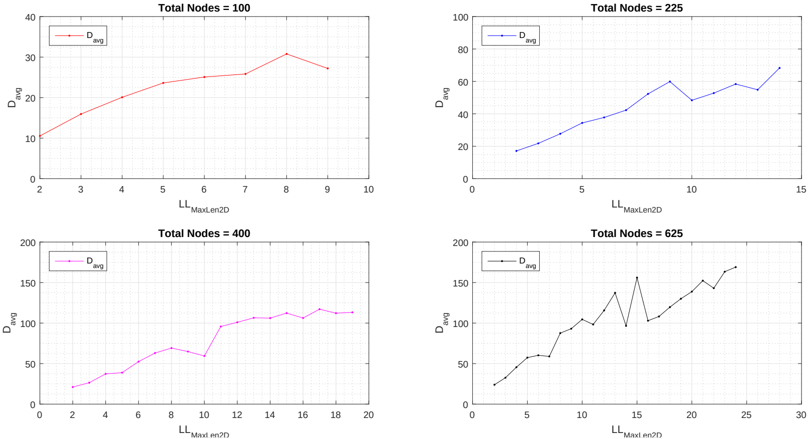
Observations on grid networks



Plot of the maximum degree (D_{max}), the second maximum degree (D_{max2}), and the third maximum degree (D_{max3}) with respect to different $LL_{MaxLen2D}$ values for 10×10-, 15×15-, 20×20-, and 25×25-node grid networks

Observations on grid networks (cont'd...)



Plot of the average length of an LL (D_{avg}) with respect to different $LL_{MaxLen2D}$ values for 10×10-, 15×15-, 20×20-, and 25×25-node grid networks

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Regular network $\rightarrow$ small-world network $\rightarrow$ scale-free network $\rightarrow$ scale-free network with truncated degree distribution[a]

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Various phases of network evolution and average length of an LL

Network Types	Phases of Evolution	Range of LL (w.r.t. N)
String	Small-world characteristics	$2 \leqslant \mathrm{LL}_{MaxLen} \leqslant 0.2N$
	Scale-free characteristics (multiple hub nodes)	$0.3N < \mathrm{LL}_{MaxLen} < 0.4N$
	Scale-free characteristics (one hub node emerges with most of the LLs)	$\mathrm{LL}_{MaxLen} \geqslant 0.4N$
	Average length of an LL $= 0.6 \times \mathbf{LL}_{MaxLen}$	
Grid	Small-world characteristics	$2 \leqslant \mathrm{LL}_{MaxLen2D} \leqslant 0.2N$
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Questions?

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Thank You.

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