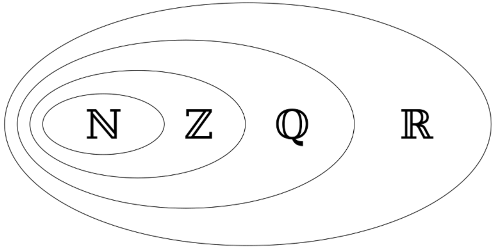
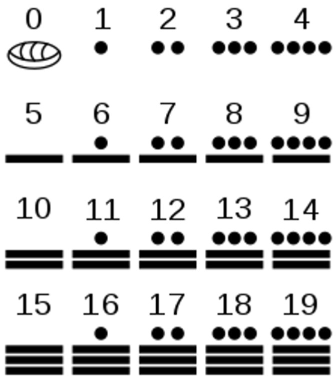
Numbering Systems Review



- Natural numbers: Things that we count and the base system
  - base 60: from the Sumerians: used for time, angles, geographical coordinates
  - base 20: e.g., the Mayan system
  - base 12: Jan, Feb, … Dec
  - base 10: Jan, Feb, … June, Sept, Oct, Nov, Dec!
  - base 7: Sun, Mon,Tues, … Sat, Sun
  - base 2, 8 (2^3), 16 (2^4)
  - unary: 110, 11110, 11111111111110, 111011011110=324

- Integers: Positive, Zero, and Negative
- Rational: e.g., ⅓, 2.3 but not… π
- Real Numbers: e.g., π
- Complex Numbers: e.g., 3 + 2í where í = √-1

- The concept of both $-\infty$ and ∞, x/0 == NaN



Different Representations for "42.25"

- Decimal:
  - 42.25
  - 10#42.25 (Bash shell)
  - 4.225 x10^ 2
  - 4.225E02 (calculators)
- Hexadecimal:
  - 0x2A.4
  - 16#2A.4
  - 2.A4 x16^ 1
- Octal:
  - 052.2 (Java, C, etc, but not Javascript)
  - 0o52.2 (Javascript)
  - 8#52.2
  - 5.22 x8^ 1
- Binary:
  - 0b101010.01
  - 2#101010.01
  - 1.0101001 x2^ 101

- In COMP122:
  - Use context
  - Allow spacing for clarity
  - Allow for separators
  - Allow for signs

- Examples:
  - - 101001.0100 x 2^ -101
  - 1010 1010 1101 1111
  - 10 1111, 10100, 00101, 10101, 10100, 10 1010

Computer Limitations and Representation

- Recall the Universal Computer
  - There is a limited tape size to perform calculation
- Recall the von Neumann and Harvard architecture
  - There is a predefined width to registers and memory
- Abstract representations with limited sizes for:
  - Natural Numbers & Zero: unsigned char, unsigned int
  - Integers: short int, int, long int, long long int
  - Rational/Real:
    - Fix Point ---
    - Floating Point float (singal), double

+ 4.225 x10^ +2
+1.010101 x2^ +101

- An encoding of each will include one or more of the following:

sign	whole	fractional	expon sign	exponent

Expanded Notation

- Recall from grade school

1234 =	1 x 10^3	1 * 1000 = 1000
	+ 2 x 10^2	2 * 100 = 200
	+ 3 x 10^1	3 * 10 = 30
	+ 4 x 10^0	4 * 1 = 4
		1234

thousands	hundreds	tens	ones
1	2	3	4

Expanded Notation for other Bases

Radix Point

BASE	Columns					
Base 16	4096's	256's	16's	1's	1/16	1/256
Base 10	1000's	100's	10's	1's	1/10	1/100
Base 8	512's	64's	8's	1's	1/8	1/64
Base 2	8's	4's	2's	1's	1/2	1/4

Expanded Notation

- Recall from grade school

154 =	1 x 10^2	100
	+ 5 x 10^1	50
	+ 4 x 10^0	4
		154

Expanded Notation

- Base 16
  16# 9A

Base 10 Value

`0x9A =`	`9 x 16^1`	9 x 16 = 144
	`+ A x 16^0`	10 x 1 = 10
		154

- Base 8
  0o232
  8# 232

Base 10 Value

`0232 =`	`2 x 8^2`	2 * 64 = 128
	`+ 3 x 8^1`	3 * 8 = 24
	`+ 2 x 8^0`	2 * 1 = 2
		154

10	A
11	B
12	C
13	D
14	E
15	F

Expanded Notation

- Base 2: 2# 1001 1010

1001 1010 =	1 x 2^7	128
	+ 0 x 2^6	0
	+ 0 x 2^5	0
	+ 1 x 2^4	16
	+ 1 x 2^3	8
	+ 0 x 2^2	0
	+ 1 x 2^1	2
	+ 0 x 2^0	0
		154

Real Numbers

- `      45.34`
- `   -  45.34`
- `  1011110.0100`
- `- 1011110.0100`

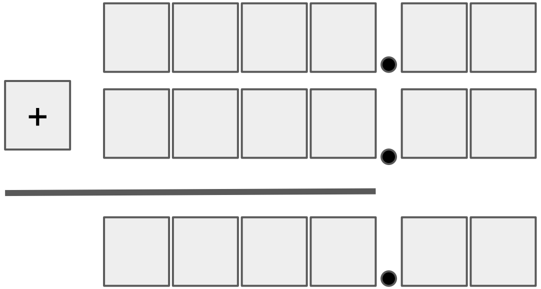
radix point

1000 ← 1000000.1011110 → 01

- But what about my limitations on the computer!

Fixed Point Numbers

- There is a need to represent rational numbers
- Consider operations on US currency. (dollars and cents)
  - Place the two digits after the decimal (radix) point
  - Provide a register of size 6



Note:
We can't represent standard price of a gallon of gas! $4.96 9/10

Scientific Notation

14.3×10^7

$- \quad 9.2 \times 10^7$

5.1

$\times 10^7$

- All numbers represented as: $m \times 10^N$
- Simplifies operations on large and small numbers.
  - Distance between sun and earth: 92,000,000 = 9.2×10^7
  - Distance between sun and mars: 143,000,000 = 1.43×10^8
- Floating point representation
  - a representation of scientific notation
  - introduces the notion of infinity, and NaN (0 / 0 = ?, 0 x infinity = ?)
- Representation of: $-1.1011101 \times 2^{-1001}$
  - Assume a size of 16
  - Note the whole part is alway "1", so I left it out!

sign	exponent part with sign					fractional part									
-	-	1	0	0	1	1	0	1	1	1	0	1	0	0	0

foreshadow

Floating Point Encoding

always 1: so we don't store it

Original number: - 0.000100101

Recall Scientific Notation: - 1.0100101 x $2^{-100\,(4)}$

- Components to Encode
  - sign: negative
  - significant or the mantissa: 0100101
  - exponent: - 100
    - Issue: negative exponents
    - Solution: store values with a bias
- Bias:
  - Shift all numbers along the number line by N
  - Typically it is half the range:
    - `3 bits ->               011 == 3`
    - `5 bits ->            0 1111 == 15`
    - `8 bits ->         0111 1111 == 127`
    - `11 bits ->   011 1111 1111 == 1023`

Symbol	Encoding
+	0
-	1

Number		Encoding (Bias: 4)
-4		000
-3		001
-2		010
-1		011
0	000	100
1	001	101
2	010	110
3	011	111