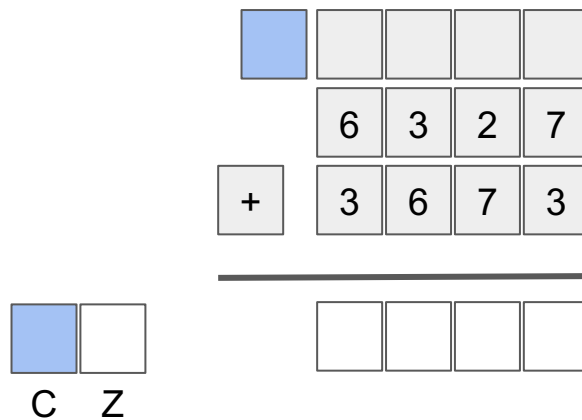
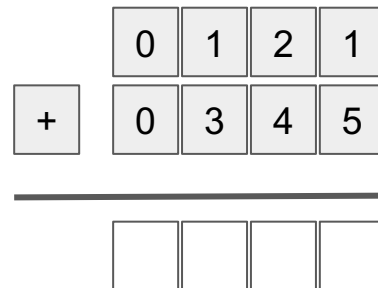
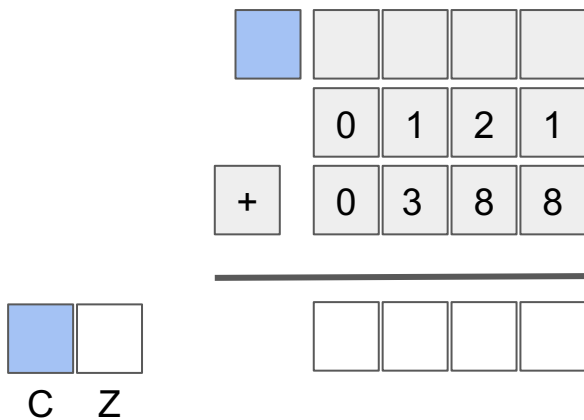


Mathematical Operations

- Base 10: our native base.
- Glyphs: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- The algorithms to evaluate various functions are the same, regardless of base
- On a computer, we are limited to a certain number of digits.
- We can summarize our results: 0 == FALSE, 1 == TRUE
 - For unsigned operations:
 - the final value is Zero (Z)
 - the calculation resulted in final carry (C)
 - For signed values
 - the final value is Negative (S)
 - the calculation resulted in an overflow ()

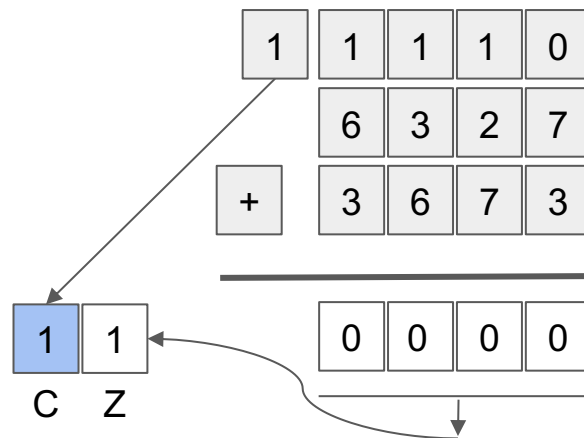
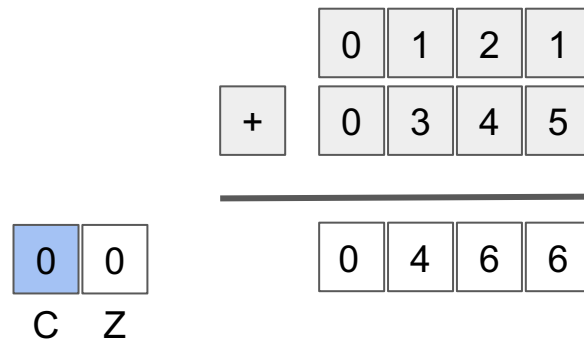
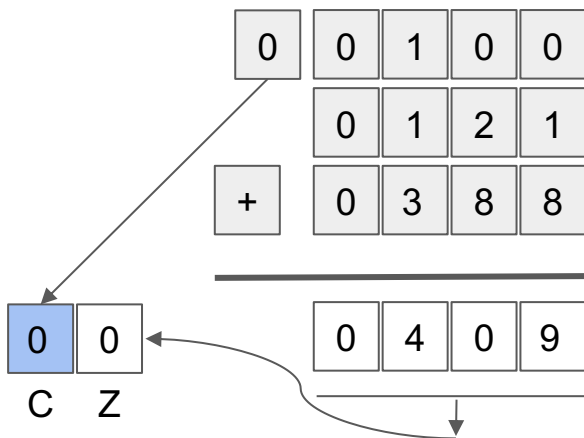
Addition: (Before)

- First, introduce some status values:
 - Zero, Carry, (Sign, Overflow)
- Assume a word size of 4
- Notice the notation of "to carry" a value



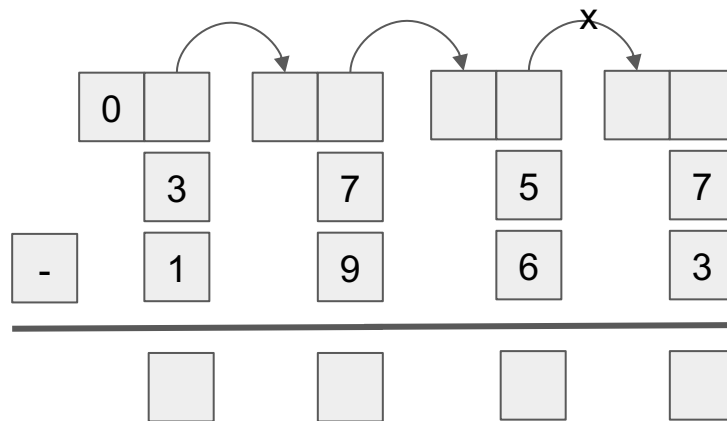
Addition: (After)

- First, introduce some status values:
 - Zero, Carry, (Sign, Overflow)
- Assume a word size of 4
- Notice the notation of "to carry" a value



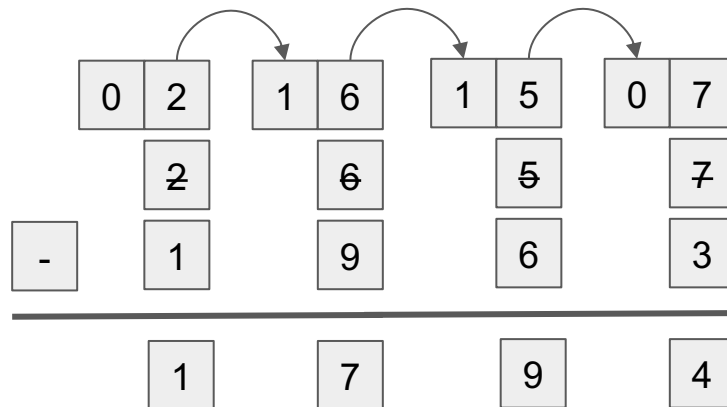
Subtraction (before)

- $3757 - 1963 = 1794$
- Traditional Method:
 - Notice the notation of "to borrow" a value
- Other Methods: (common core)
 - Left \rightarrow Right (Mental Math)
 - Singapore (No Borrow)
 - Counting Up (Giving Change)
- Via Method of Complements



Subtraction (after)

- $3757 - 1963 = 1794$
- Traditional Method:
 - Notice the notation of "to borrow" a value
- Other Methods: (common core)
 - Left → Right (Mental Math)
 - Singapore (No Borrow)
 - Counting Up (Giving Change)
- Via Method of Complements



Method of Complements

- A technique to encode both positive and negative numbers
 - uses the same algorithm to perform addition
 - subtraction perform my addition of complements
- Complement: *a thing that completes or brings to perfection*
- Radix 10: *(the radix or base is the number of unique digits to represent a number)*
 - 10's complement
 - $7 + x = 10$: x is the 10s complements of 7 $x = 3$
 - $46 + y = 100$: y is the 10s complements of 46 $y = 54$
 - 9's complement
 - $7 + a = 9$: a is the 9s complements of 7 $a = 2$
 - $46 + b = 99$: b is the 9s complements of 46 $b = 53$

- The math:

2nd Grade

$$\begin{array}{r} 45 \\ - 11 \\ \hline 34 \end{array}$$

10's complement

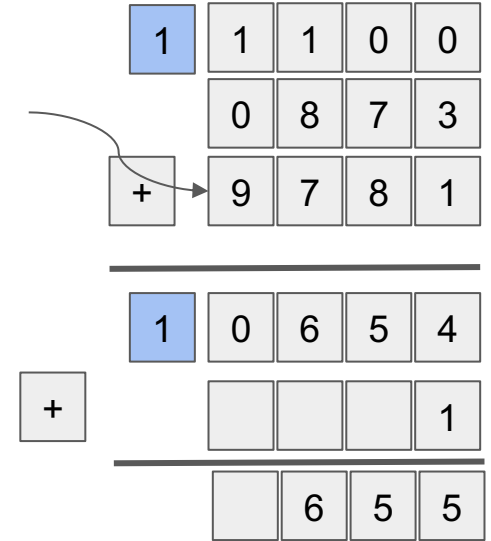
$$\begin{array}{r} 45 \\ + 89 \\ \hline 134 \end{array}$$

9's complement

$$\begin{array}{r} 45 \\ + 88 \\ \hline 133 + 1 = 34 \end{array}$$

Algorithm: Subtraction via 9's Complements

- Example: $873 - 218 \Rightarrow 0873 - 0218$
1. Take the nines complement of the subtrahend (0218)
 2. Add the complement to the minuend (0873)
 3. Drop the leading "1"
 4. Add 1



Algorithm: Subtraction via 9's Complements

- Example: $873 - 218 \Rightarrow 0873 - 0218$
 1. Take the nines complement of the subtrahend (0218)
 2. Add the complement to the minuend (0873)
 3. Drop the leading "1"
 4. Add 1

- Optimization:
introduce initial carry in

				1
	0	8	7	3
+	9	7	8	1
<hr/>				
				5

1	1	1	0	0
	0	8	7	3
+	9	7	8	1
<hr/>				
4	0	6	5	4
+				1
<hr/>				
		6	5	5

Algorithm: Subtraction via 10's Complements

- Example: $13 - 9 \Rightarrow 0013 - 0009$

1. Take the 10s complement of the subtrahend (0009)
2. Add the complement to the minuend
3. Drop the leading "1".

- ~~Optimization:~~ Addition of adding one is baked in!

1	1	1	0	0
	0	0	1	3
+	9	9	9	1
<hr/>				
1	0	0	0	4