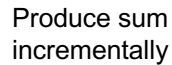
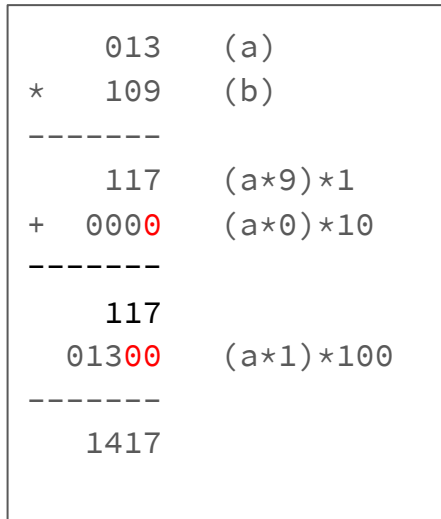


# Multiplication

- Consider:  $109 \times 13 = 1417$
- Approach: Successive Additions
  - Consider:  $9 + 9 + 9 \dots + 9$  (13 times) = ?
  - What is carry value for the 10's column?
- Approach: Long Multiplication



x13

# Algorithm for Decimal Multiplication

- Consider a number is an array:  
`int[] B = { 9, 0, 1 };`
- Base10 Algorithm:  
`sum = 0;`  
`for (d = 0 ; d < 3 ; d ++ ) {`  
    `sum += a * B[d];`  
    `a = a * 10 ; // Base 10 shift left`  
`}`
- Complexity:  $O(\text{\#digits})$ 
  - For  $2^{32}$ , at most 10 iterations

reframe:

	013	(a)	
*	109	(b)	
-----			
	117	(a*1)	* 9
+	000	(a*10)	* 0
-----			
	117		
	01300	(a*100)	* 1
-----			
	1417		

original:

	013	(a)
*	109	(b)
-----		
	117	(a*9)*1
+	000	(a*0)*10
-----		
	117	
	01300	(a*1)*100
-----		
	1417	

Note: commutative operation

# Algorithm for Binary Multiplication

- Base 2 Algorithm:

```
sum = 0;
for (d = 0 ; d < 3 ; d ++ ) {
    if (B[d] == 1) {
        sum += a * B[d];
    }
    a = a * 2; // Base 2 shift left
    a = a << 1;
}
```

- Complexity:  $O(\text{word\_size})$

- For MIPS, at most 32 iterations

original:

	0010	(a = 2)
*	1011	(b = 11)
-----		
	0000 0010	(a*2^0)* 1
+	0 0100	(a*2^1)* 1
-----		
	0000 0110	
+	00 0000	(a*2^4)* 0
-----		
	0000 0110	
+	001 0000	(a*2^8)* 1
-----		
	0001 0110	(a*b = 10)

	0010	(a = 2)
*	1011	(b = 11)
-----		
	0000 0010	(a*1)* 2^0
+	0 0100	(a*1)* 2^1
-----		
	0000 0110	
+	00 0000	(a*0)* 2^4
-----		
	0000 0110	
+	001 0000	(a*1)* 2^8
-----		
	0001 0110	(a*b = 22)

# Algorithm for Binary Multiplication

- Use the register as an stack
- Base 2 Algorithm:
 

```

sum = 0;
for (; b != 0; ) {
    bit = pop(b);
    if (bit == 1) {
        sum += A;
    }
    A << 1;
}

```
- Complexity:  $O(\text{word\_size})$

reframe:

	0010	(a = 2)
*	1011	(b = 11)
-----		
	0000 0010	(a*2 <sup>0</sup> )* 1
+	0 0100	(a*2 <sup>1</sup> )* 1
-----		
	0000 0110	
+	00 0000	(a*2 <sup>4</sup> )* 0
-----		
	0000 0110	
+	001 0000	(a*2 <sup>8</sup> )* 1
-----		
	0001 0110	(a*b = 10)

original:

	0010	(a = 2)
*	1011	(b = 11)
-----		
	0000 0010	(a*1)* 2 <sup>0</sup>
+	0 0100	(a*1)* 2 <sup>1</sup>
-----		
	0000 0110	
+	00 0000	(a*0)* 2 <sup>4</sup>
-----		
	0000 0110	
+	001 0000	(a*1)* 2 <sup>8</sup>
-----		
	0001 0110	(a*b = 22)