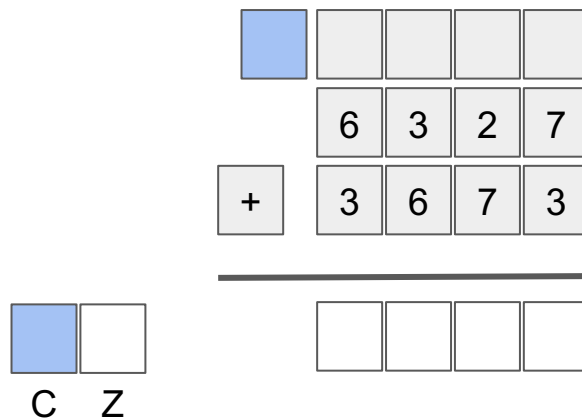
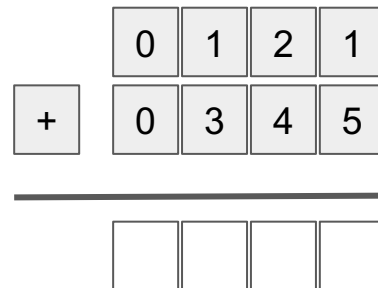
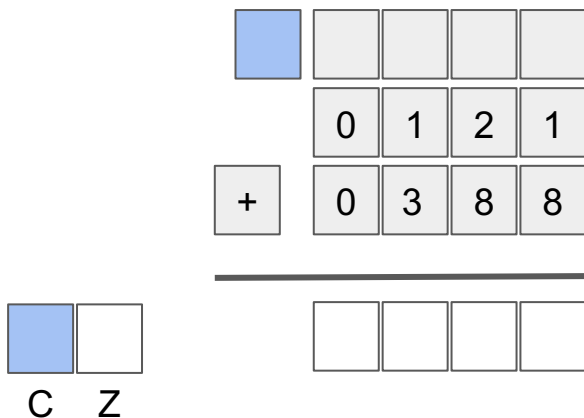


Mathematical Operations

- Base 10: our native base.
- Glyphs: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- The algorithms to evaluate various functions are the same, regardless of base
- On a computer, we are limited to a certain number of digits.
- We can summarize our results: 0 == FALSE, 1 == TRUE
 - For unsigned operations:
 - the final value is Zero (Z)
 - the calculation resulted in final carry (C)
 - For signed values
 - the final value is Negative (S)
 - the calculation resulted in an overflow ()

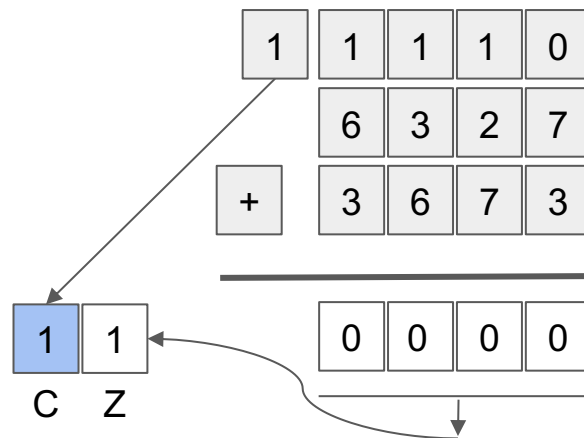
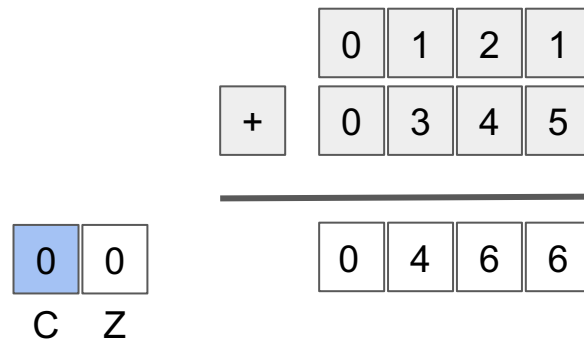
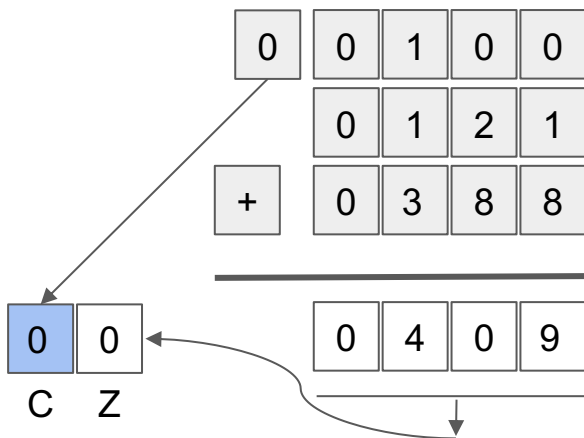
Addition: (Before)

- First, introduce some status values:
 - Zero, Carry, (Sign, Overflow)
- Assume a word size of 4
- Notice the notation of "to carry" a value



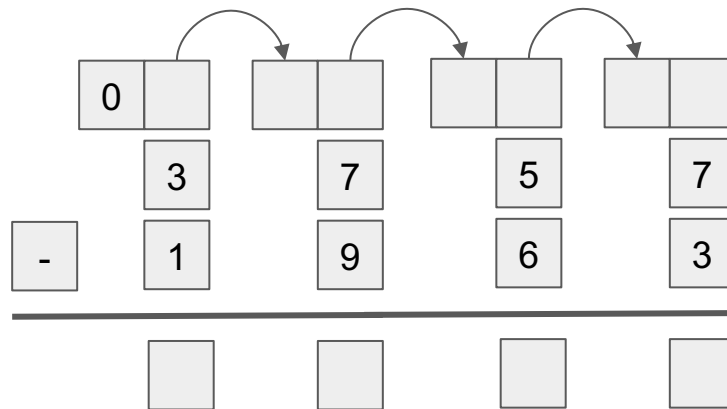
Addition: (After)

- First, introduce some status values:
 - Zero, Carry, (Sign, Overflow)
- Assume a word size of 4
- Notice the notation of "to carry" a value



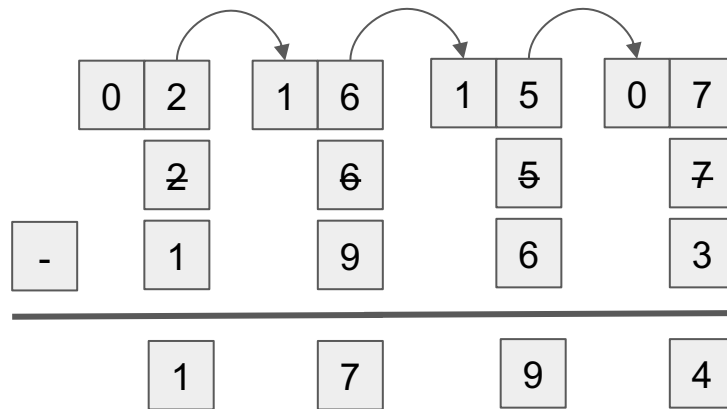
Subtraction (before)

- $3757 - 1963 = 1794$
- Traditional Method:
 - Notice the notation of "to borrow" a value
- Other Methods: (common core)
 - Left \rightarrow Right (Mental Math)
 - Singapore (No Borrow)
 - Counting Up (Giving Change)
- Via Method of Complements



Subtraction (after)

- $3757 - 1963 = 1794$
- Traditional Method:
 - Notice the notation of "to borrow" a value
- Other Methods: (common core)
 - Left \rightarrow Right (Mental Math)
 - Singapore (No Borrow)
 - Counting Up (Giving Change)
- Via Method of Complements



Method of Complements

- A technique to encode both positive and negative numbers
 - uses the same algorithm to perform addition, subtraction performed by the addition of complements
- Complement: *a thing that completes or brings to perfection:* $X + Y = 10^n$ (1 0...0)
- Radix 10:

- *10's complement*

- $7 + x = 10$

- : x is the 10s complements of 7

- $x = 3$

- $46 + y = 100$

- : y is the 10s complements of 46

- $y = 54$

- *9's complement*

- $7 + a = 9$

- : a is the 9s complements of 7

- $a = 2$

- $46 + b = 99$

- : b is the 9s complements of 46

- $b = 53$

- The math: 2nd Grade

$$\begin{array}{r} 45 \\ - 11 \\ \hline 34 \end{array}$$

- 10's complement

$$\begin{array}{r} 45 \\ + 89 \\ \hline 4 \ 34 \end{array}$$

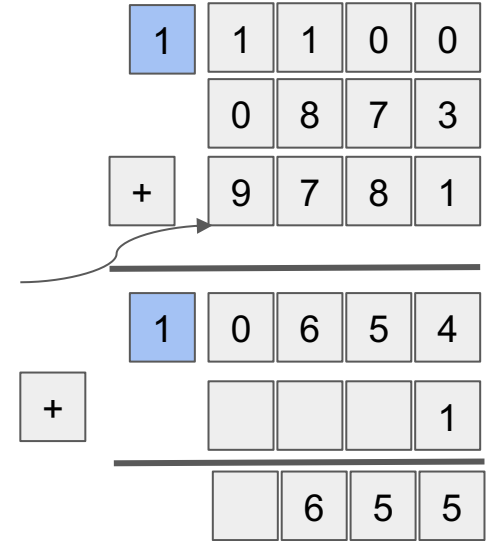
- 9's complement

$$\begin{array}{r} 45 \quad 33 \\ + 88 \quad + 1 \\ \hline 4 \ 33 \quad 34 \end{array}$$

Algorithm: Subtraction via 9's Complements

- Example: $873 - 218 \Rightarrow 0873 - 0218$
 - 4-digit register:
 - $10^5 = 10,000$

1. Take the nines complement of the subtrahend (0218)
 - With respect to 1 0000
2. Add the complement to the minuend (0873)
3. Drop the leading "1" (the Carry)
4. Add 1



Algorithm: Subtraction via 9's Complements

- Example: $873 - 218 \Rightarrow 0873 - 0218$

1. Take the nines complement of the subtrahend (0218)
2. Add the complement to the minuend (0873)
3. Drop the leading "1"
4. Add 1

- Optimization:

introduce initial carry in

			1
0	8	7	3
+	9	7	8
			1
<hr/>			
			5

1	1	1	0	0
	0	8	7	3
+	9	7	8	1
<hr/>				
4	0	6	5	4
+				1
<hr/>				
		6	5	5

Algorithm: Subtraction via 10's Complements

- Example: $13 - 9 \Rightarrow 0013 - 0009$

1. Take the 10s complement of the subtrahend (0009)
2. Add the complement to the minuend
3. Drop the leading "1".

1	1	1	0	0
	0	0	1	3
+	9	9	9	1
<hr/>				
1	0	0	0	4

- Optimization: Addition of adding one is baked in!