Multiplication

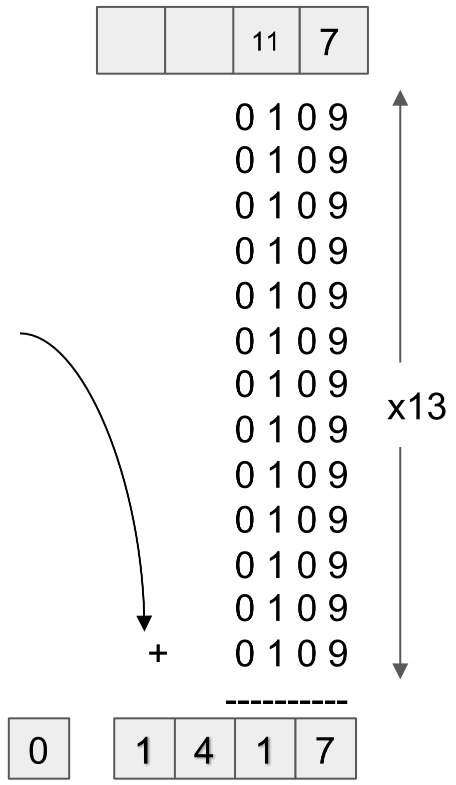
- Consider: 109 x 13 = 1417
- Approach: Successive Additions
    - Consider: 9 + 9 + 9 .. + 9 (13 times) = ?
    - What is carry value for the 10's column?
- Approach: Long Multiplication

Bookkeeping of N products

```
    013   (a)
*   109   (b)
--------
    117   (a*9)*1
   0000   (a*0)*10
+ 01300   (a*1)*100
--------
   1417
```

Produce sum incrementally

```
    013   (b)
*   109   (b)
-------
    117   (a*9)*1
+  0000   (b*0)*10
-------
    117
  01300   (a*1)*100
-------
   1417
```



Algorithm for Decimal Multiplication

- Consider a number is an array:

```
int[]  B = { 9, 0, 1 };
```

- Base10 Algorithm:

```
sum = 0;
for (d = 0 ; d < 3 ; d ++ ) {
    sum += A * B[d];
    A = A * 10 ;  // Base 10 shift left
}
```

- Complexity: O(#digits)
  - For 2^32, at most 10 iterations

reframe:

original:

```
     013    (a)
*    109    (b)
--------
     117    (a*9)*1
+    000    (b*0)*10
--------
     117
   01300    (A*1)*100
--------
    1417
```

Note: commutative operation

Algorithm for Binary Multiplication

- Base 2 Algorithm:

```
sum = 0;
for (d = 0 ; d < 3 ; d ++ ) {
    if (B[d]  == 1) {
        sum += A  ~~*  B[d]~~;
    }
    ~~A = A * 2 ;~~ // Base 2 shift left
    A = A << 1 ;
}
```

- Complexity: O(word_size)
    - For MIPS, at most 32 iterations

original:

```
      0010    (a =  2)
*     1011    (b = 11)
----------
 0000 0010    (a*1)* 2^0
+   0 0100    (a*1)* 2^1
-------
 0000 0110
+  00 0000    (a*0)* 2^4
--------
 0000 0110
+ 001 0000    (a*1)* 2^8
-------
 0001 0110    (a*b = 22)
```

Algorithm for Binary Multiplication

- Use the register as an stack
- Base 2 Algorithm:

```
sum = 0;
for (; b != 0; ) {
    bit = pop(b);
    if (bit  == 1) {
        sum += A;
    }
    A << 1 ;
}
```

- Complexity: O(word_size)

reframe:

```
      0010    (a =  2)
*     1011    (b = 11)
----------
 0000 0010    (a*2^0)* 1
+   0 0100    (a*2^1)* 1
-------
 0000 0110
+  00 0000    (a*2^4)* 0
---------
 0000 0110
+ 001 0000    (a*2^8)* 1
-------
 0001 0110    (a*b = 10)
```

original:

```
      0010    (a =  2)
*     1011    (b = 11)
----------
 0000 0010    (a*1)* 2^0
+   0 0100    (a*1)* 2^1
-------
 0000 0110
+  00 0000    (a*0)* 2^4
--------
 0000 0110
+ 001 0000    (a*1)* 2^8
-------
 0001 0110    (a*b = 22)
```