# Mathematical Operations

- Base 10: our native base.
- Glyphs: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- The algorithms to evaluate various functions are the same, regardless of base
- On a computer, we are limited to a certain number of digits.
- We can summarize our results: 0 == FALSE, 1 == TRUE
  - For unsigned operations:
    - the final value is Zero (Z)
    - the calculation resulted in final carry (C)
  - For signed values
    - the final value is Negative (S)
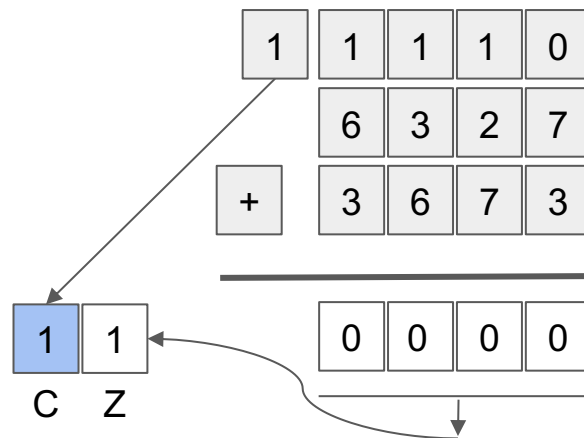   - the calculation resulted in an overflow ()

Addition: (Before)

	0	1	2	1
+	0	3	4	5

| | | | |

- First, introduce some status values:
  - Zero, Carry, (Sign, Overflow)
- Assume a word size of 4
- Notice the notation of "to carry" a value

	0	1	2	1
+	0	3	8	8

| | | | |

C Z

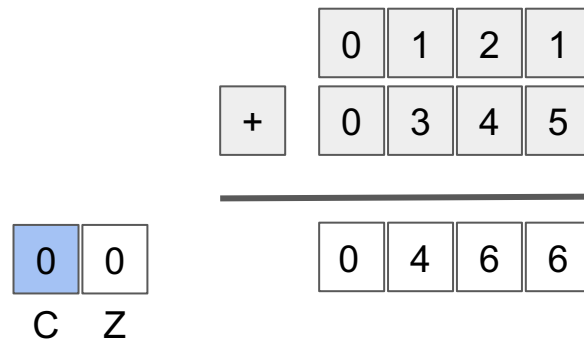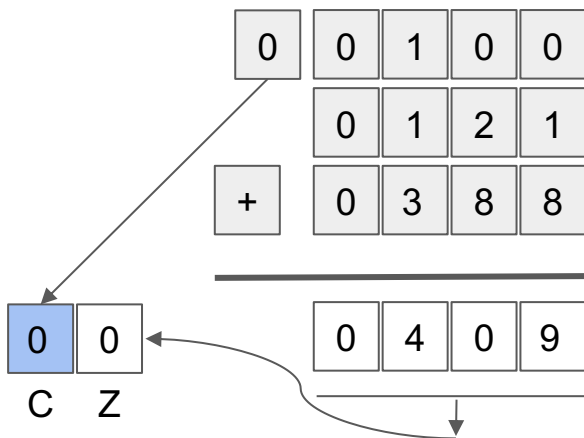|				
	6	3	2	7
+	3	6	7	3

| | | | |

C Z
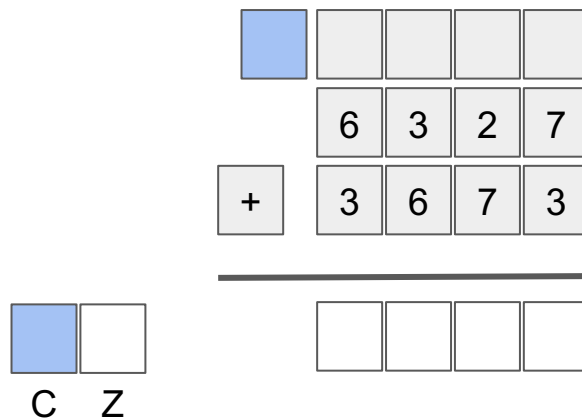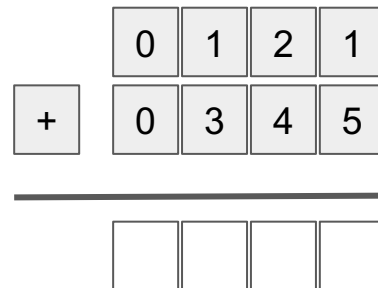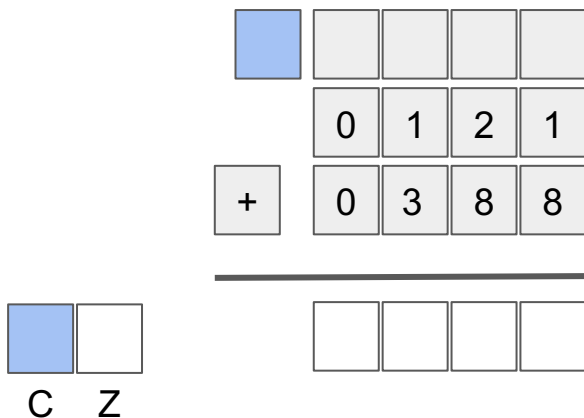
Addition: (After)

- First, introduce some status values:
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- Assume a word size of 4
- Notice the notation of "to carry" a value

0	1	2	1
+	0	3	4	5

0	4	6	6

0	0
C	Z

0
0	1	0	0
	0	1	2	1
+	0	3	8	8

0	4	0	9

0	0
C	Z

1
1	1	1	0
	6	3	2	7
+	3	6	7	3

0	0	0	0

1	1
C	Z

Subtraction (before)

- 3757 - 1963 = 1794
- Traditional Method:
  - Notice the notation of "to borrow" a value
- Other Methods: (common core)
  - Left → Right (Mental Math)
  - Singapore (No Borrow)
  - Counting Up (Giving Change)
- Via Method of Complements

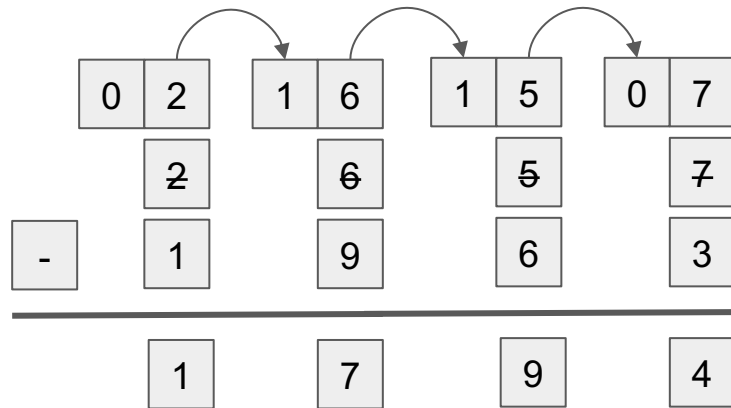|	0						
	3		7		5		7
-	1		9		6		3
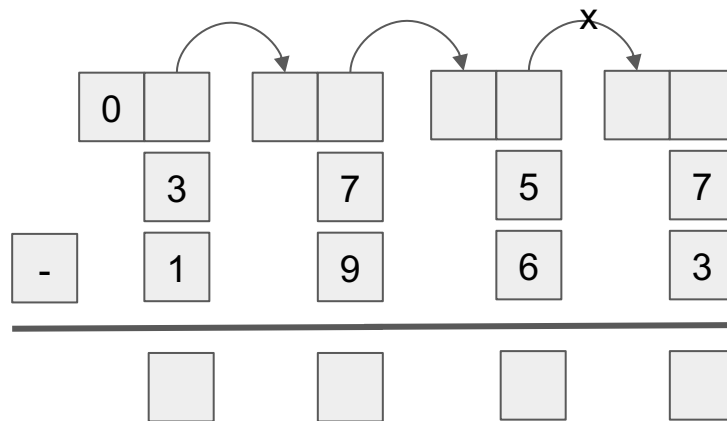
Subtraction (after)

- 3757 - 1963 = 1794
- Traditional Method:
  - Notice the notation of "to borrow" a value
- Other Methods: (common core)
  - Left → Right (Mental Math)
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- Via Method of Complements

0	2	1	6	1	5	0	7
	2		6		5		7
-	1		9		6		3

1	7	9	4

Method of Complements

- A technique to encode both positive and negative numbers
  - uses the same algorithm to perform addition
  - subtraction perform my addition of complements

- Complement: *a thing that completes or brings to perfection*
- Radix 10: *(the radix or base is the number of unique digits to represent a number)*
  - *10's complement*
    - $7 + x = 10$: x is the 10s complements of 7   x = 3
    - $46 + y = 100$: y is the 10s complements of 46   y = 54
  - 9's complement
    - $7 + a = 9$: a is the 9s complements of 7   a = 2
    - $46 + b = 99$: b is the 9s complements of 46   b = 53

- The math:

2nd Grade	10's complement	9's complement
45	45	45
- 11	+ 89	+ 88
34	~~1~~34	~~1~~33 + 1 = 34

Algorithm: Subtraction via 9's Complements

- Example: 873 - 218 ⇒ 0873 - 0218
1. Take the nines complement of the subtrahend (0218)
2. Add the complement to the minuend (0873)
3. Drop the leading "1"
4. Add 1

1	1	1	0	0
	0	8	7	3
+	9	7	8	1

1	0	6	5	4
+				1
		6	5	5
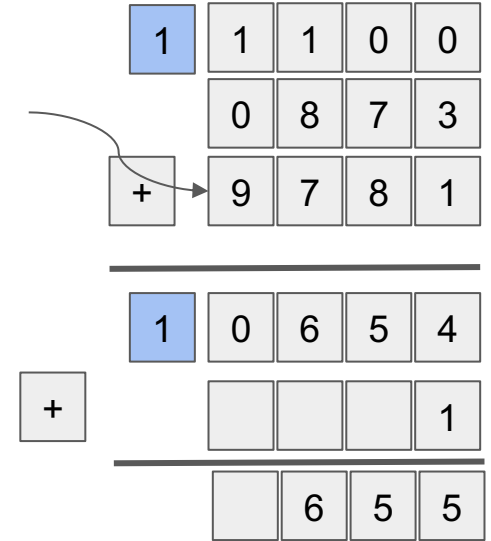
Algorithm: Subtraction via 9's Complements

- Example: 873 - 218 ⇒ 0873 - 0218
1. Take the nines complement of the subtrahend (0218)
2. Add the complement to the minuend (0873)
3. Drop the leading "1"
4. Add 1

- Optimization:
   introduce initial carry in

1	1	1	0	0
	0	8	7	3
+	9	7	8	1

1̶	0	6	5	4
+				1
		6	5	5

				1
	0	8	7	3
+	9	7	8	1
				5

Algorithm: Subtraction via 10's Complements

- Example: 13 - 9 ⇒ 0013 - 0009
1. Take the 10s complement of the subtrahend (0009)
2. Add the complement to the minuend
3. Drop the leading "1".

- ~~Optimization~~: Addition of adding one is baked in!

1	1	1	0	0
	0	0	1	3
+	9	9	9	1

1	0	0	0	4