

COMP 3721

Tutorial 2

- Countability

- DFA

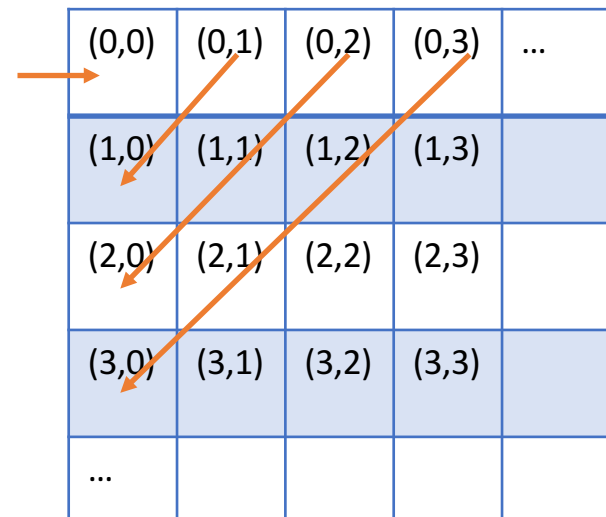
Countability

1) Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

Suppose we don't know the fact that the Cartesian product of two countable sets is countable.

Countability

- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.
- Proof 1:
 - A way to enumerate all elements:
 - enumerate tuple (a, b) where:
 - $a + b = 0$
 - $a + b = 1$
 - $a + b = 2$
 - ...
 - Every (a, b) will be reached in finite step..



The diagram shows a grid of cells representing pairs of natural numbers (a, b) . The cells are arranged in rows and columns. The first row contains $(0,0)$, $(0,1)$, $(0,2)$, $(0,3)$, and an ellipsis. The second row contains $(1,0)$, $(1,1)$, $(1,2)$, $(1,3)$, and an empty cell. The third row contains $(2,0)$, $(2,1)$, $(2,2)$, $(2,3)$, and an empty cell. The fourth row contains $(3,0)$, $(3,1)$, $(3,2)$, $(3,3)$, and an empty cell. The fifth row contains an ellipsis and four empty cells. Orange arrows indicate a path starting from $(0,0)$ and moving diagonally down and to the right, visiting $(1,0)$, $(2,0)$, $(3,0)$, and so on, illustrating how every element is reached in a finite step.

(0,0)	(0,1)	(0,2)	(0,3)	...
(1,0)	(1,1)	(1,2)	(1,3)	
(2,0)	(2,1)	(2,2)	(2,3)	
(3,0)	(3,1)	(3,2)	(3,3)	
...				

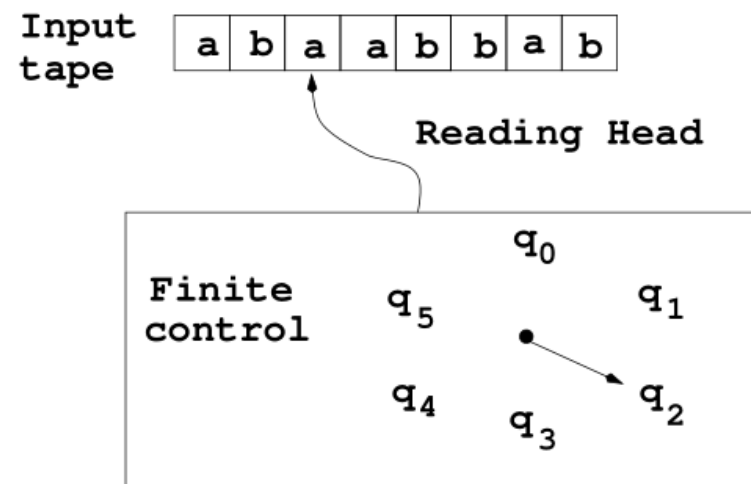
Countability

- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.
- Proof 2:
 - Let $A_i = \{(i, j) : j \in \mathbb{N}\}$ for $i \in \mathbb{N}$
 - Then each A_i is equinumerous with \mathbb{N} , so A_i is countably infinite.
 - $\mathbb{N} \times \mathbb{N} = A_0 \cup A_1 \cup \dots$
 - Use the fact that countable union of countable sets is countable, we prove that $\mathbb{N} \times \mathbb{N}$ is countable.

Countability

- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.
- Proof 3(a little bit complicated):
 - Find a bijection between $\mathbb{N} \times \mathbb{N}$ and \mathbb{N} .
 - The following function is such a bijection:
 - $f(a, b) = (a + b - 1) * (a + b - 2) / 2 + a$
 - Prove that it is a bijection....

DFA

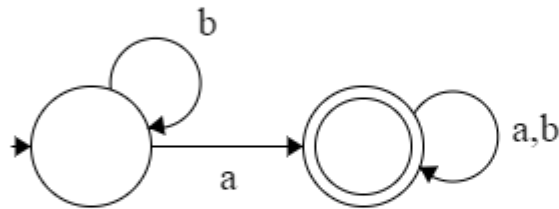


DFA

- 1) Construct a DFA for accepting the language with regular expression $b^*a(a \cup b)^*$.
- 2) Construct a DFA for accepting each of the following languages:
 - a) $\{w \in \{a, b\}^* : w \text{ contains the string } abbab\}$.
 - b) $\{w \in \{a, b\}^* : w \text{ don't have } abb \text{ as a substring}\}$.
 - c) $\{w \in \{a, b\}^* : w \text{ has a number of } a\text{'s divisible by } 3\}$.
 - d) $\{w \in \{a, b\}^* : w \text{ has an odd number of } a\text{'s and an even number of } b\text{'s}\}$.

DFA

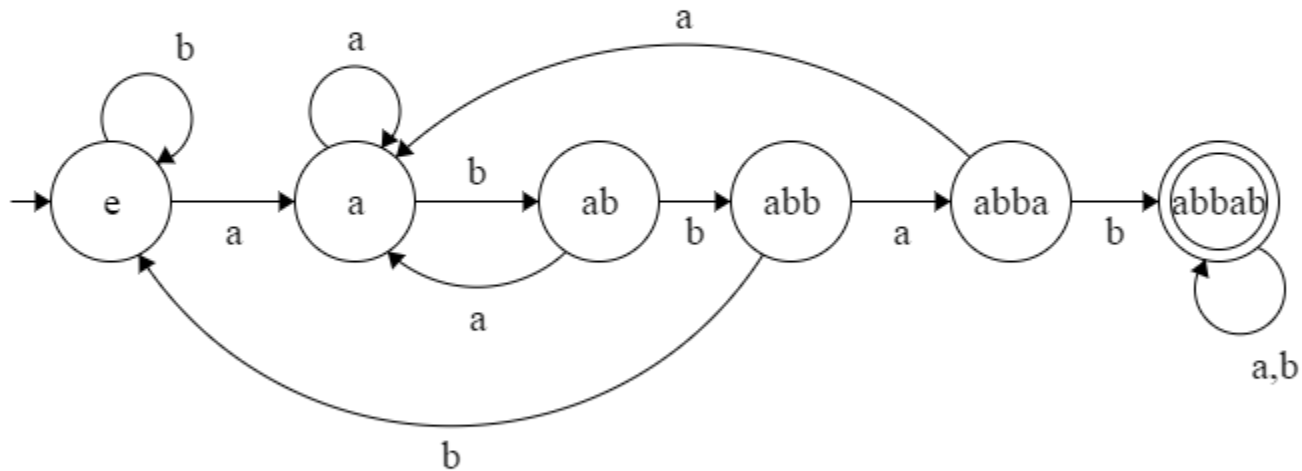
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DFA

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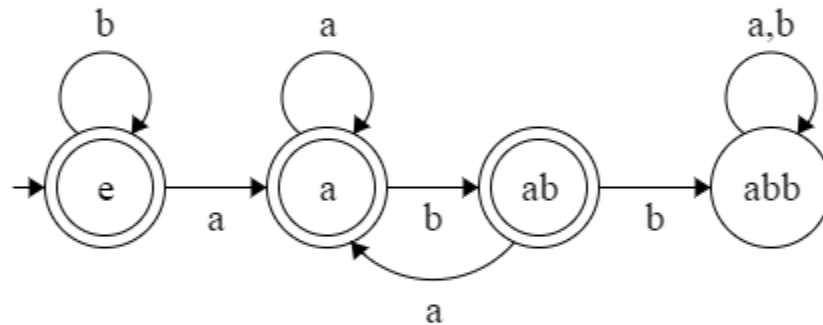


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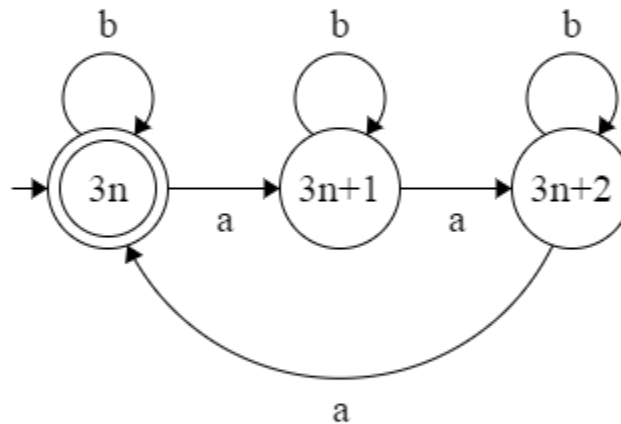
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