COMP 3721

Tutorial 2

- Countability

- DFA

Countability

1) Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

Suppose we don't know the fact that the Cartesian product of two countable sets is countable.

Countability

- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

- Proof 1:
  - A way to enumerate all elements:
    - enumerate tuple (a, b) where:
    - a + b = 0
    - a + b = 1
    - a + b = 2
    - …
  - Every (a, b) will be reached in finite step..

(0,0)	(0,1)	(0,2)	(0,3)	…
(1,0)	(1,1)	(1,2)	(1,3)	
(2,0)	(2,1)	(2,2)	(2,3)	
(3,0)	(3,1)	(3,2)	(3,3)	
…				

Countability

- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

- Proof 2:
  - Let $A_i = \{(i, j): j \in \mathbb{N}\}\ for\ i \in \mathbb{N}$
  - Then each A_i is equinumerous with $\mathbb{N}$, so A_i is countably infinite.
  - $\mathbb{N}\times\mathbb{N} = A_0 \cup A_1 \cup \cdots$
  - Use the fact that countable union of countable sets is countable, we prove that $\mathbb{N}\times\mathbb{N}$ is countable.

Countability

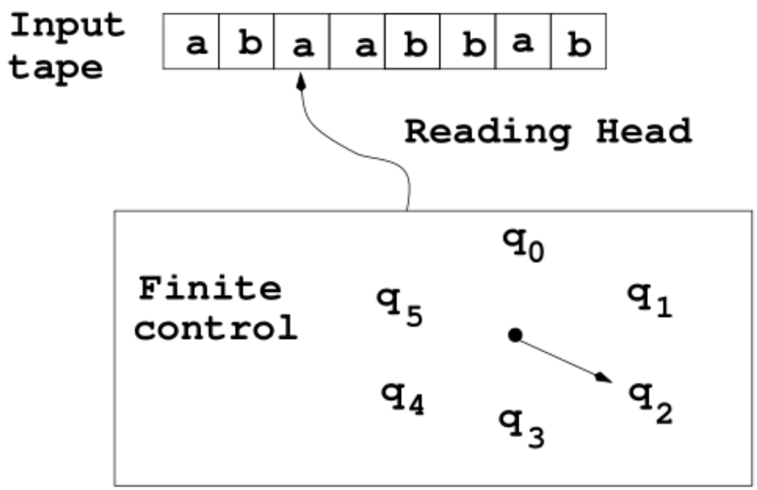
- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

- Proof 3(a little bit complicated):
  - Find a bijection between $\mathbb{N} \times \mathbb{N}$ and $\mathbb{N}$.

  - The following function is such a bijection:
  - f(a, b) = (a + b - 1)*(a + b - 2)/2 + a

  - Prove that it is a bijection….

DFA


Input
tape
a
b
a
a
b
b
a
b
Reading Head
Finite
control
q_0
q_5
q_1
q_4
q_3
q_2

DFA

1) Construct a DFA for accepting the language with regular expression $b^*a(a \cup b)^*$.

2) Construct a DFA for accepting each of the following languages:
   a) $\{w \in \{a, b\}^* : w$ contains the string $abbab\}$.
   b) $\{w \in \{a, b\}^*: w$ don't have abb as a substring$\}$.
   c) $\{w \in \{a, b\}^*: w$ has a number of a's divisible by 3$\}$.
   d) $\{w \in \{a, b\}^*: w$ has an odd number of a's and an even number of b's$\}$.

DFA

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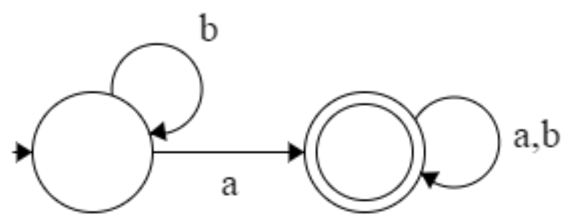

DFA

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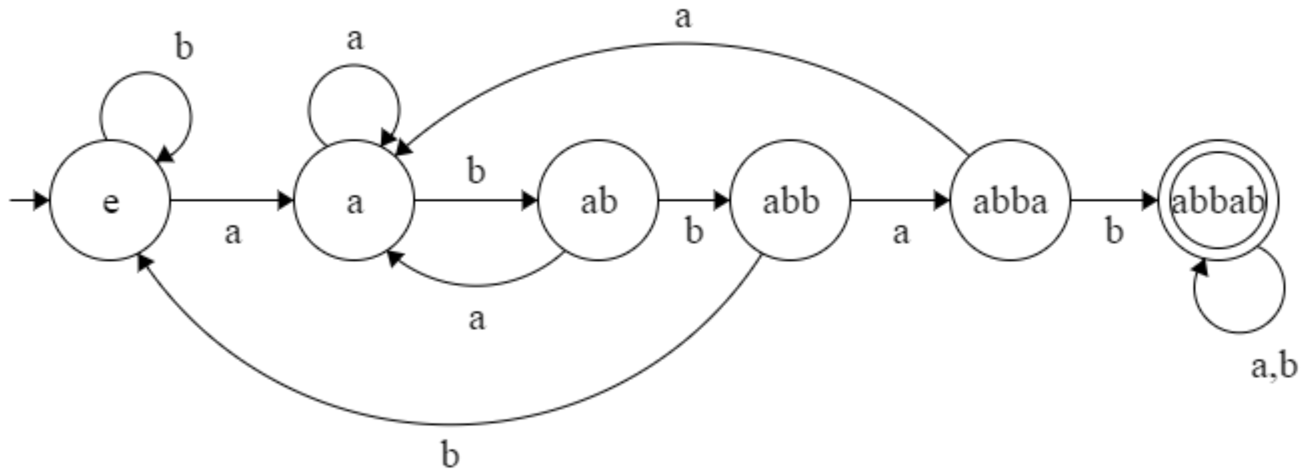

DFA

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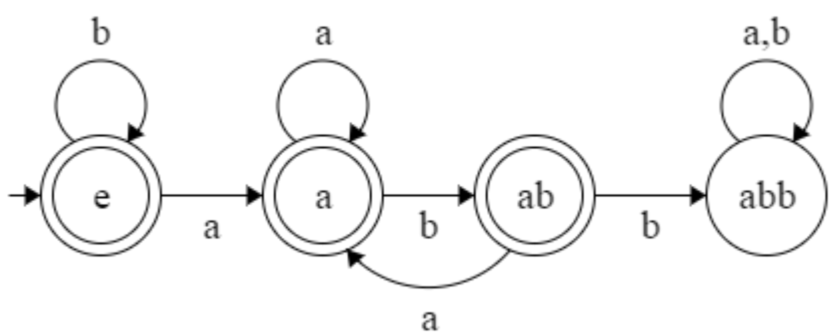

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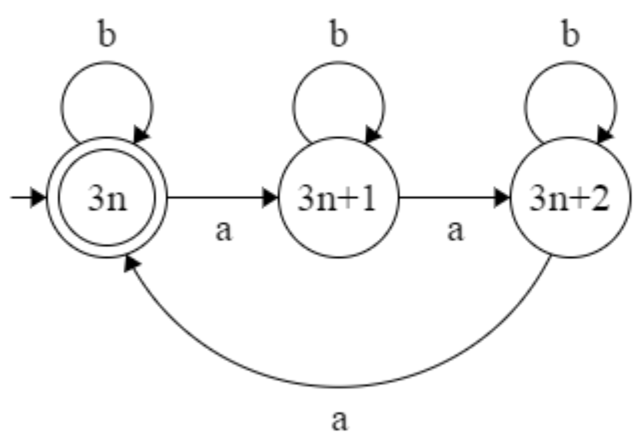

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   d) $\{w \in \{a, b\}^*: w$ has an odd number of *a*'s and an even number of *b*'s$\}$.