

COMP 3721

Tutorial 5

October 11, 2017

Context-Free Grammar

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- ▶ $\Sigma = \{a, b\}$
- ▶ $R = \{S \rightarrow aSa, S \rightarrow bSb, S \rightarrow e, S \rightarrow a, S \rightarrow b\}$

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- ▶ $V = \{a, b, c, d, A, B, X, S\}$
- ▶ $\Sigma = \{a, b, c, d\}$
- ▶ $R = \{S \rightarrow aSd, S \rightarrow A, S \rightarrow B, A \rightarrow aAc, A \rightarrow X, B \rightarrow bBd, B \rightarrow X, X \rightarrow bXc, X \rightarrow e\}$

Context-Free Grammar

- (2) Let $G = (V, \Sigma, R, S)$, where $V = \{a, b, S\}$, $\Sigma = \{a, b\}$, and $R = \{S \rightarrow aSb, S \rightarrow aSa, S \rightarrow bSa, S \rightarrow bSb, S \rightarrow e\}$. Show that $L(G) = \{w \in \{a, b\}^* : w \text{ has even length}\}$.

Proof

- ▶ It is easy to see that any string generated by G have even length since every rule of G generates even number of terminals.
- ▶ Now it suffices to show that any string w of even length can be generated by G . We prove this by induction on the length of w .
 - (i) When $|w| = 0$, it is obvious that w can be generated by G .
 - (ii) Suppose that when $|w| = 2i$, w can be generated by G .
 - (iii) We show that when $w = 2(i+1)$, w can be generated by G .

Suppose w starts with a and ends with a (the other three cases can be handled in a similar way). Then $w = aya$ for some y with $|y| = 2i$. By the inductive hypothesis, w can be generated as follows.

$$S \Rightarrow aSa \Rightarrow^* aya = w$$

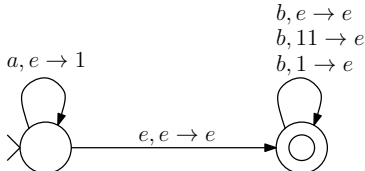
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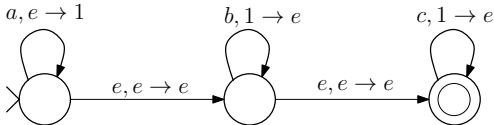
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Pushdown Automata

- (3) Construct a pushdown automaton that accepts the following language.

$$\{w \text{ in } \{a, b\}^* : w \text{ has twice as many } b\text{'s as } a\text{'s}\}$$

Solution

