COMP3721 Tutorial 5

1 Context-Free Grammars

1. Show that the following languages are context-free by exhibiting context-free grammars generating each.

   (a) $\{w \in \{a,b\}^* : w = w^R\}$

   (b) $\{a^m b^n : m \geq n \geq 0\}$

   (c) $\{a^m b^n c^p d^q : m + n = p + q\}$

2. Let $G = (V, \Sigma, R, S)$, where $V = \{a, b, S\}$, $\Sigma = \{a, b\}$, and $R = \{S \to aSb, S \to aSa, S \to bSa, S \to bSb, S \to e\}$. Show that $L(G) = \{w \in \{a,b\}^* : w \text{ has even length}\}$.

2 Pushdown Automata

1. Construct a pushdown automaton that accepts the language $\{a^i b^j : i \leq 2j\}$.

2. Construct a pushdown automaton that accepts the language $\{a^i b^j c^k : i = j + k\}$.

3. Construct a pushdown automaton that accepts the following language.

$$\{w \text{ in } \{a,b\}^* : w \text{ has twice as many } b\text{'s as } a\text{'s}\}$$