

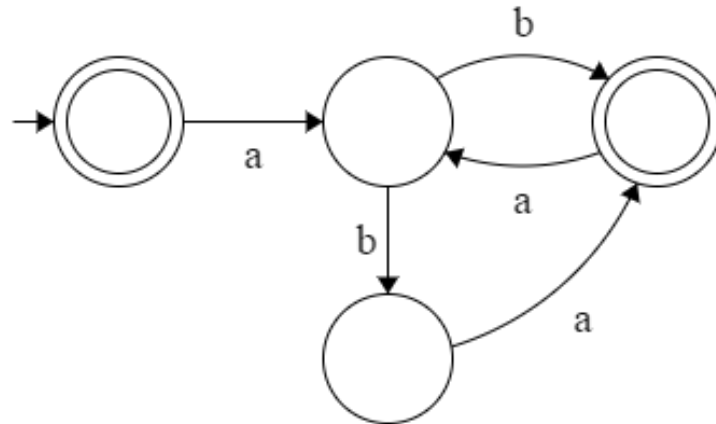
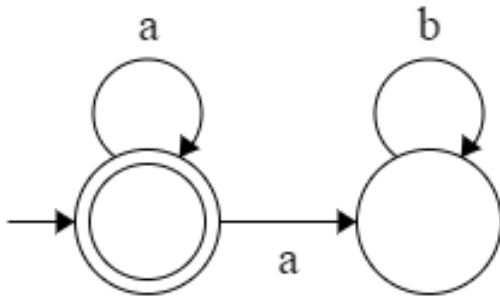
COMP 3721

Tutorial 4

- NFA \rightarrow Regular Expression
- NFA = DFA = Regular Expressions
- Pumping Theorem

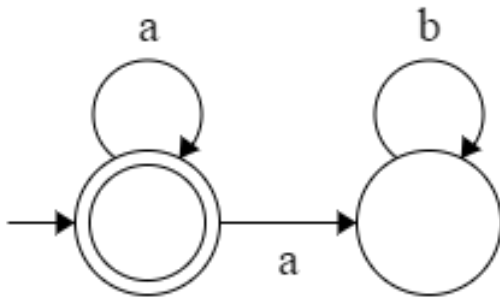
NFA \rightarrow Regular Expression

- 1) Write regular expressions for the languages accepted by the following NFA.

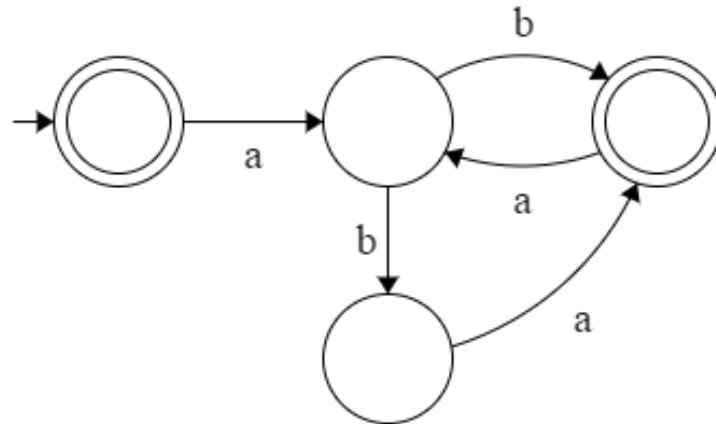


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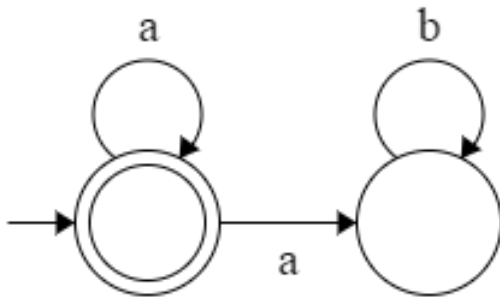


a^*

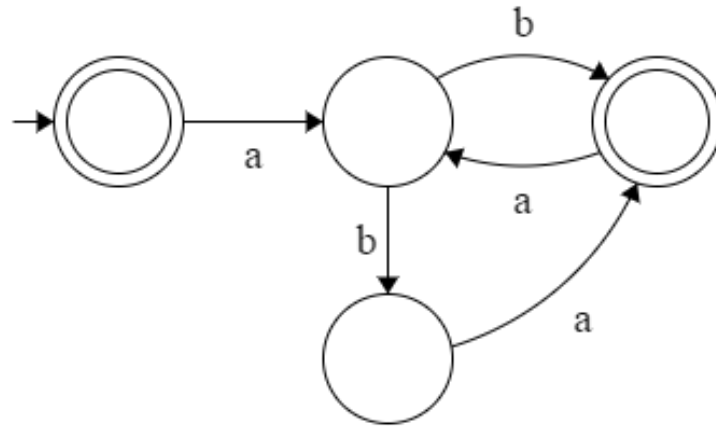


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$(a(ba \cup b))^*$

NFA = DFA = Regular Expressions

2) Prove that if L is regular, then the following languages are also regular

a) $Subseq(L) = \{w_1 \dots w_k : x = x_0 w_1 \dots w_k x_k \in L \text{ for some } x_0, \dots, x_k\}$.

b) $L^R = \{w : w^R \in L\}$.

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Proof:

Since L is regular, it is accepted by some NFA $M = \{K, \Sigma, \Delta, s, F\}$.

Next we will construct a NFA M' that accepts $Subseq(L)$.

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We claim that M' accepts $Subseq(L)$ and hence $Subseq(L)$ is regular. Details are omitted

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Next we will construct a NFA M' that accepts L^R by reversing all the transitions in $M.$

More precisely, we let $M' = (K, \Sigma, \Delta', s', f')$ where $f' = s,$ and $\Delta' = \{(p, a, q) : (q, a, p) \in \Delta\} \cup \{(s', e, f)\}.$

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Pumping Theorem

We use the pumping theorem to prove that a language is NOT regular. To prove that L is not regular, we show the following, which contradicts the P.T.:

$$\forall n \geq 1,$$

$$\exists w \in L, |w| \geq n,$$

$$\forall x, y, z \text{ such that } w = xyz \text{ and } y \neq e, |xy| \leq n,$$

$$\text{but } \exists i \geq 0, xy^i z \notin L.$$

Pumping Theorem

- Assume L is regular.
- Then, by Pumping Theorem, there exists an integer n such that *all* strings of length $\geq n$ in L can be pumped.
- Find *one* string w in L that has length $\geq n$ and is not pumpable no matter how you split it into 3 parts, i.e.,:
 - Choose *one* string $w \in L$ with $|w| \geq n$.
 - Consider *all* valid ways of splitting w into x, y, z .
 - For each way of splitting, demonstrate *one* value i such that $xy^iz \notin L$
- The existence of one string w that is not pumpable contradicts the Pumping Theorem.
- Hence L is not regular.

Pumping Theorem

3) Are the following languages on $\{a, b\}$ regular or not? Prove your answer.

- a) $L = \{a^i b^j : i > j \geq 1\}$
- b) $L = \{ww : w \in \{a, b\}^*\}$
- c) $L = \{(bab)^i (babbab)^i : i \geq 1\}.$

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Consider the string $w = a^{n+1}b^n \in L$.

Let x , y , and z be specified as in pumping theorem.

Then $x = a^i$, $y = a^j$, $z = a^k b^n$ for $i, k \geq 0$ and $j > 0$.

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Consider the string $u = a^{n+1}ba^{n+1}b \in L$.

Let x , y , and z be specified as in pumping theorem.

Then $x = a^i$, $y = a^j$, $z = a^kba^{n+1}b$ for $i, k \geq 0$ and $j > 0$.

By the theorem, xy^0z is in L .

But this is impossible since $a^{i+k}ba^{n+1}b$ can't be written as ww .

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Proof.

In fact,

$$L = (babbabbab)^+ = (babbabbab)(babbabbab)^*$$

Properties of R. L.

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False.

Counter example:

$\{a^n b^n : n \geq 0\}$ is a subset of $(a \cup b)^*$, but is not regular.

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- a) Every subset of a regular language is regular. False.
 - b) If L is regular, then so is $L' = \{xy : x \in L \text{ and } y \notin L\}$.

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Regular language is closed under *Concatenation* and *Complementation*.

$$L' = L\bar{L}$$

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False.
Can be proved by pumping theorem.

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True.
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 - d) If L is a regular language, then so is $\{w : w \in L \text{ and } w^R \in L\}$.

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True.

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True.

$L \cap L^R$, the closure properties of regular languages