COMP 3721 Question Bank #4

1 NP-completeness

1. Aliens from another world come to Earth and tell us that the 3-SAT problem is solvable in $O(n^8)$ time.

   Which of the following statements follow as a consequence? (List **all** that are true.)

   (i) All NP-complete problems are solvable in polynomial time.

   (ii) All NP-complete problems are solvable in $O(n^8)$ time.

   (iii) All problems in NP, even those that are not NP-complete, are solvable in polynomial time.

   (iv) No NP-complete problem can be solved faster than in $O(n^8)$ time in the worst case.

   (v) P = NP.

2. For each of the following assertions, indicate whether it is **True**: known to be true, **False**: known to be false, or **Unknown**: unknown based on our current scientific knowledge. In each case provide a short explanation for your answer.

   (i) No problems in NP can be solved in polynomial time.

   (ii) Every problem in NP can be solved in exponential time, that is in time $O(2^{p(n)})$ for some polynomial $p(n)$.

   (iii) Every NP-complete problem requires at least exponential time to be solved.

   (iv) X is in NP and X $\leq_P$ SAT. Then X is NP-complete.

3. Given an undirected graph $G = (V, E)$, a *feedback vertex set* is a subset of vertices such that every simple cycle in G passes through one or more of these vertices. The feedback vertex set problem (FVS) is: Given a graph G and an integer k, does G contain a feedback vertex set of size at most k?

   Show that FVS is in NP. That is, given a graph G that has a FVS of size k, give a certificate, and show how you would use this certificate to verify the presence of a FVS of size k in polynomial time.

4. The *subgraph isomorphism problem* takes two graphs G_1 and G_2 and asks whether G_1 is a subgraph of G_2. Prove that the subgraph isomorphism problem is NP-complete. (*Hint:* Reduce from a problem we have shown in class to be NP-complete.)

5. The *set cover problem* is: Given a finite set X and a collection of sets F whose elements are chosen from X, and given an integer k, does there exist a subset $C \subseteq F$ of k sets such that
$$X = \bigcup_{S \in C} S.$$
Prove that the set cover problem is NP-complete. (*Hint:* Reduce from a problem we have shown in class to be NP-complete.)

6. The *set partition problem* takes as input a set S of integers. The question is whether the integers can be partitioned into two sets A and $\overline{A} = S - A$ such that $\sum_{x \in A} x = \sum_{x \in \overline{A}} x$. Show that the set partition problem is NP-complete. You may use the fact that the subset sum problem is NP-complete.

7. Show that NP is closed under union, intersection, concatenation, and Kleene star.