COMP 3721

Tutorial 6

Context-Free Language

- Pumping Theorem

- Closure Property

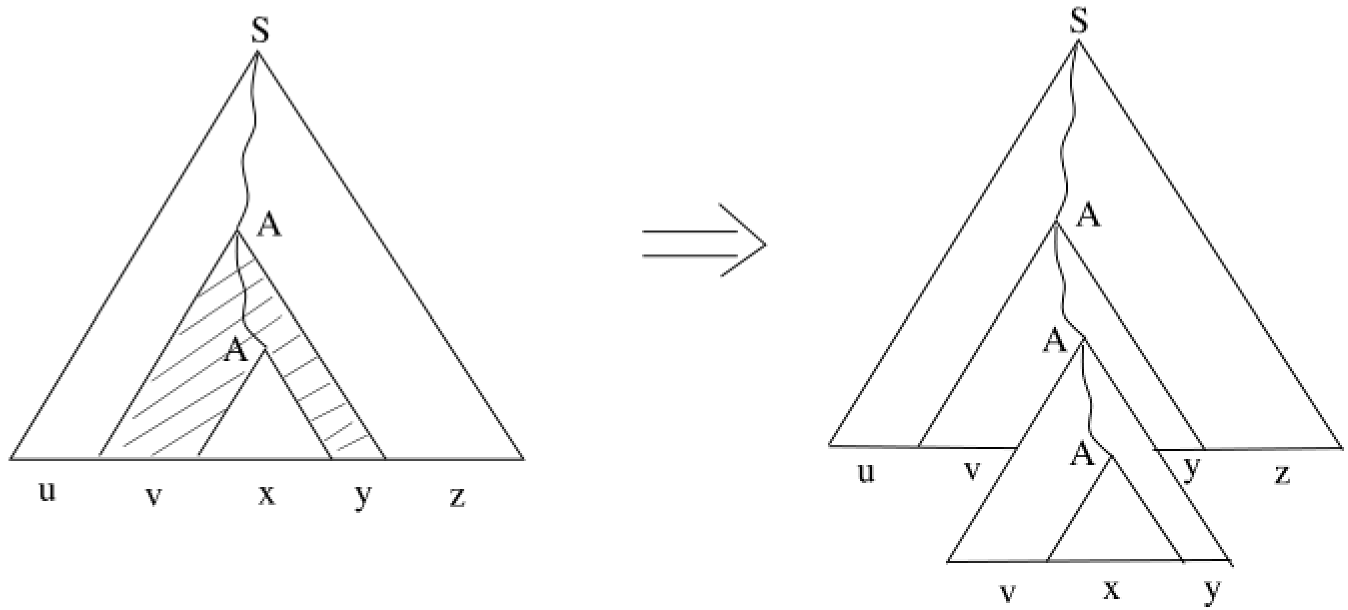
Pumping Theorem

The pumping theorem for CFLs

Theorem 1 *Let L be a context free language. Then there is an integer $N \geq 1$ such that for every $w \in L$ and $|w| \geq N$, w can be split into five parts $w = uvxyz$ such that*

1. $vy \neq e$ *(i.e. v and y cannot both be e)*
2. $|vxy| \leq N$
3. $uv^ixy^iz \in L$ *for* $i = 0, 1, 2, \ldots.$

Pumping Theorem



Pumping Theorem

1. For each of the following languages L, state whether L is context-free or not.
   a) $\{a^i b^j c^k : i + j = k, i, j, k \geq 0\}$
   b) $\{a^i b^i c^{2i} : i \geq 0\}$
   c) $\{a^i b^j c^k : j = \max(i, k), i, j, k \geq 0\}$

   Justify your answer. If context free show a PDA or CFG, if not apply Pumping Lemma for CFL

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   Context-free.

   K = {a, b, c, S}

   Σ = {a, b, c}

   R = { S -> aSc, S-> bSc, S -> e}

Pumping Theorem

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   a) $\{a^i b^j c^k : i + j = k, i, j, k \geq 0\}$ Context-Free.
   b) $\{a^i b^i c^{2i} : i \geq 0\}$ Not Context Free.

Pumping Theorem

1. For each of the following languages L, state whether L is context-free or not.

   b) $L = \{a^i b^i c^{2i} : i \geq 0\}$ Not Context Free.

   Proof.

   Assume L is context-free. Let N be the integer in the Pumping Theorem.

   $w = \mathrm{a}^N b^N c^{2N} = uvxyz$. $|vy| > 0$, $|vxy| \leq N$.

   vxy contains at most 2 alphebet(a or b or c or a,b or b,c).

   Show that,

   For all cases, can find i such that uv^ixy^iz is not in L.

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   Show that,

   For all cases, can find *i* such that $uv^i xy^i z$ is not in *L.*

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Closure Property of CFL

Theorem 1 *CFLs are closed under*

1. *union,*
2. *concatenation,*
3. *Kleene Star.*

Properties of CFLs

Theorem 2 *The intersection of a CFL and a regular language is a CFL.*

Closure Property of CFL

1. Use closure property of CFL to show that the following languages are context-free.
   a) $\{a^i b^j : i \neq j\}$
   b) $\{xx^R yy^R zz^R : x, y, z \in \{a, b\}^*\}$
   c) $L - R$, where L is context-free and R is regular.

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L is the union of:

$L_1 = \{a^i b^j : i > j\}$

$L_2 = \{a^i b^j : i < j\}$

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$L' = \{ww^R : w \in \{a, b\}^*\}$

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$L - R$

$L \cap \overline{R}$