COMP3721 Tutorial 11

CSE, HKUST

Problem (a)

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Solution: This problem is undecidable. Suppose it were solvable, then there exists some Turing machine M_A that solves it. It can be used to solve the halting problem:
M_H: on input "M""w"

1. Run M_A("M""w""h") where h is the halting state of M.
2. If M_A output y, M_H output y; If M_A output n, M_H output n.

Problem (b)

(b) To determine, given a Turing machine M and a symbol σ, does M ever write the symbol σ when started on the empty tape?

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Solution: This problem is undecidable. Suppose it were solvable, then there exists some Turing machine M_B that solves it. Then, it can be used to solve the problem of determining whether an arbitrary Turing machine halts on the empty tape.
M_E: On input "M",

1. Let a be a symbol that is not in the alphabet of M. Construct a Turing machine M^* that is identical to M except that whenever it halts it also writes an a. (Clearly, M^* writes an a when started on the empty tape if and only if M halts when started on the empty tape.)
2. Run M_B("M""a").
3. If M_B output y, M_E output y; If M_B output n, M_E output n.

Problem (c)

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Idea: Given any machine M and string w, we construct a machine M^* such that M^* use a finite amount of tape squares on input w if and only if M halts on w. Then we can conclude that this problem is undecidable, since otherwise we can use its solution to solve the halting problem.
Intuitively, M^* just runs M on w, and at the same time, M^* uses a unary counter to record the number of steps M have run so far. If M never halt, the this unary counter will use infinite number of tape squares. If M halts, then it is clear that M^* uses finite number of tape squares.