COMP3721 Question Bank #1

Sets, Relations, and Functions

Question 1

Determine whether each of the following is true or false.

a) $\emptyset \subseteq \emptyset$

b) $\emptyset \in \emptyset$

c) $\emptyset \in \{\emptyset\}$

d) $\emptyset \subseteq \{\emptyset\}$

e) $\{a, b\} \in \{a, b, c, \{a, b\}\}$

f) $\{a, b\} \subseteq \{a, b, \{a, b\}\}$

g) $\{a, b\} \subseteq 2^{\{a,b,\{a,b\}\}}$

h) $\{\{a, b\}\} \in 2^{\{a,b,\{a,b\}\}}$

i) $\{a, b, \{a, b\}\} - \{\text{a,b}\} = \{\text{a,b}\}$

Question 2

What are these sets? Write them using braces, commas, and numerals only:

a) $2^{\{7,8,9\}} - 2^{\{7,9\}}$

b) $2^{\emptyset}$

Question 3

Prove the following: $A \cap (A \cup B) = A$

Question 4

Write each of the following explicitly.

a) $\{1\} \times \{1, 2\} \times \{1, 2, 3\}$

b) $\emptyset \times \{1, 2\}$

Question 5

For each part, give an example of finite sets A and B with $|A|, |B| \geq 4$ and a function $f : A \mapsto B$ such that (a) f is neither one-to-one nor onto; (b) f is one-to-one but not onto; (c) f is onto but not one-to-one; (d) f is onto and one-to-one.

Question 6

Let R and S be the binary relations on $A = \{1, \ldots, 7\}$ with the graphical representations shown below:

a) Indicate whether each of R and S is symmetric, reflexive and transitive.

b) Repeat the previous part for the relation $R \cup S$.

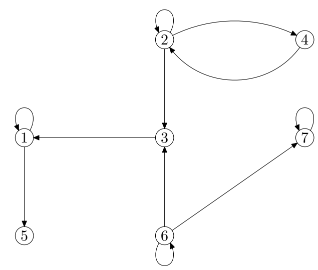

Figure 1: Graphical representation of relation R

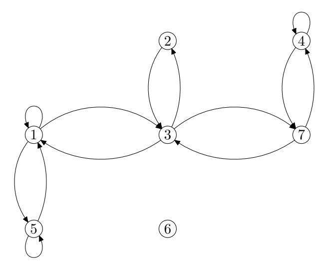

Figure 2: Graphical representation of relation S

Question 7

Let $f : A \mapsto B$. Show that the following relation R is an equivalence relation on A: $(a, b) \in R$ if and only if $f(a) = f(b)$.

Question 8

If $A = \{1, \ldots, 7\}$, define a relation R on A by $(x, y) \in R$ if and only if $x - y$ is a multiple of 3. (a) Show that R is an equivalence relation on A. (b) Draw a directed graph representing R

Languages and Regular Expressions

Question 1

Let $\Sigma = \{a, b\}$. Write regular expressions representing the following sets:

a) All strings in Σ^* with exactly one or two a's (but not zero a's, or more than two a's).

b) All strings in Σ^* with number of a's divisible by 3.

c) All strings in Σ^* with exactly one occurrence of abb.

Question 2

Which of the following are true? Explain.

a) $baa \in a^*b^*a^*b^*$

b) $b^*a^* \cap a^*b^* = a^* \cup b^*$

c) $a^*b^* \cap b^*c^* = \emptyset$

d) $abcd \in (a(cd)^*b)^*$

e) $\{a^nb^n : n \geq 0\}\{b^nc^n : n \geq 0\} = \{a^nb^{2n}c^n : n \geq 0\}$

Question 3

Let Σ be an alphabet.

a) Let $w \in \Sigma^*$. Prove by induction on the length of w that $\{w\}$ is a regular language.

b) Let $L \subset \Sigma^*$, L finite. Prove by induction on the size of L that L is a regular language.

Question 4

Write a regular expression representing the following language:

$$\{w \in \{a, b\}^* : w \text{ does not have } aab \text{ as a substring}\}.$$

Question 5

Show that $(L^*)^* = L^*$ for any language L.

Deterministic Finite Automata

Question 1

Consruct a DFA (and draw the corresponding state diagram) that accepts the language

$$\{w \in \{0, 1\}^* : w \text{ contains the string } 01101\}$$

Question 2

Construct a DFA for accepting the language with the following regular expression: $b^*a(a \cup b)^*$.

Question 3

Construct a DFA for accepting each of the following languages.

(a) $\{w \in \{a, b\}^* : w$ has exactly one or two a's (but not zero a's or more than two a's)$\}$.

(b) $\{w \in \{a, b\}^* : w$ has number of a's divisible by 3$\}$.

(c) $\{w \in \{a, b\}^* : w$ does not have abb as a substring$\}$.

(d) $\{w \in \{a, b\}^* : w$ has one or more occurrences of abb as a substring$\}$.

(e) $\{w \in \{a, b\}^* : w$ has exactly one occurrence of abb as a substring$\}$.

(f) $\{w \in \{a, b\}^* : w$ does not have $abba$ as a substring$\}$.

Nondeterministic Finite Automata

Question 1

a)–c) Write regular expressions for the languages accepted by the non-deterministic finite automata of Figures 3 – 5.

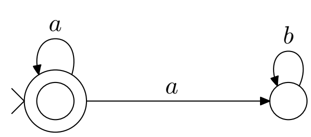

Figure 3: Non-deterministic finite automaton for (a)

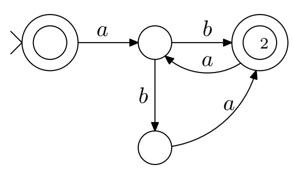

Figure 4: Non-deterministic finite automaton for (b)

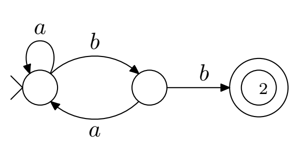

Figure 5: Non-deterministic finite automaton for (c)

Question 2

Draw state diagrams for non-deterministic finite automata that accepts these languages:

a) $((ab \cup aab)^* a^*)^*$

b) $((ab \cup aab)a^*)^*$

c) $(a^* b)^*$

d) $((a^* b^* a^*)^* b)^*$

Question 3

a) Find a simple non-deterministic finite automaton accepting $(ab \cup aab \cup aba)^*$.

b) Convert the non-deterministic finite automaton of part (a) to a deterministic finite automaton.

c) Try to understand how the machine constructed in part (b) operates.

d) Repeat parts (a) – (c) for the language $(a \cup b)^* aabab$.

Question 4

Let $L, L^{'} \subseteq \Sigma^*$. Define the following languages.

1. $\text{Pref}(L) = \{w \in \Sigma^* : x = wy \text{ for some } x \in L, y \in \Sigma^*\}$ (the set of prefixes of L).
2. $\text{Suf}(L) = \{w \in \Sigma^* : x = yw \text{ for some } x \in L, y \in \Sigma^*\}$(the set of suffixes of L).

Show that if L is accepted by some finite automaton, then so is each of the following.

(a) $\text{Pref}(L)$

(b) $\text{Suf}(L)$

The Fundamental Theorem

Question 1

Answer the following true and false questions.

(a) Regular expression is as powerful as DFA in expressing languages.

(b) DFA is less powerful than NFA in expressing languages.

(c) If α and β are regular expressions, then $L(\alpha) \cap L(\beta)$ can also be described by a regular expression.

(d) A string w must not be accepted by an NFA if on reading w there are some transitions leading to a non-final state.

(e) A string w must be accepted by a finite automaton if on reading w there are some transitions leading to a final state.

(f) The out-degree of a state in an NFA over some Σ must be greater than $|\Sigma|$.

(g) The in-degree of a state in a DFA over some Σ must be equal to $|\Sigma|$.

(h) The empty string is an element in every regular set.

(i) Every regular set can be described by an equivalent regular expression.

Question 2

Find regular expressions for the languages accepted by the finite automata shown below. Show your construct.

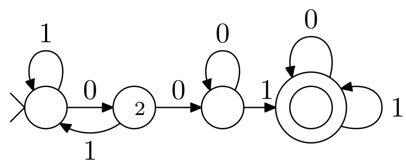

Figure 6: FA for Question 1(a)

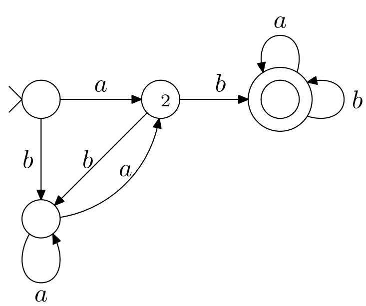

Figure 7: FA for Question 1(b)