COMP3721 Question Bank #2

Pumping Theorem for Regular Languages

Question 1

Are the following languages over alphabet $\Sigma = \{a, b\}$ regular? Prove your answers.

a) $\{a^i b a^{2i} : i \geq 1\}$

b) $\{(bab)^i (babbab)^i : i \geq 1\}$

c) $\{a^i b^j : i < j \ , \ i, j \geq 1\}$

d) $\{a^i b^j : i > j \ , \ i, j \geq 1\}$

Question 2

Using the pumping theorem, prove that the following languages are not regular:

a) $\{ww^R : w \in \{a, b\}^*\}$

b) $\{ww : w \in \{a, b\}^*\}$

Minimum-state DFA

Question 1

Find the minimum-state equivalent deterministic finite automaton for each of the following finite automata (deterministic in Figure 1 and nondeterministic in Figure 2).

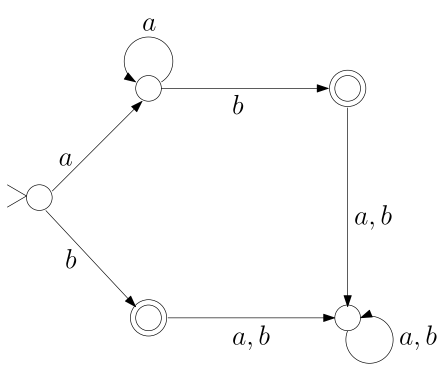

Figure 1: DFA

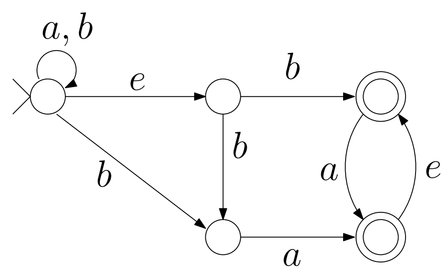

Figure 2: NFA

Uncountability, Finite representation

Question 1

Problem 1.5.7 in Lewis and Papadimitriou (2nd. Edition).
Suppose we try to prove, by an argument exactly parallel to the proof of Theorem 1.5.2, that the set of all finite subsets of $\mathcal{N}$ is uncountable. What goes wrong?

Question 2

Problem 1.5.8 in Lewis and Papadimitriou (2nd. Edition).
Give examples to show that the intersection of two countably infinite sets can be either finite or countably infinite, and that the intersection of two uncountable sets can be finite, countably infinite, or uncountable.

Question 3

Let $\mathcal{N}$={0, 1, 2,..} be the set of whole numbers and $\mathcal{Z}$ = {.., -2, -1, 0, 1, 2 ..} be the set of all integers.
Give a bijection between $\mathcal{N}$ and $\mathcal{Z}$.

Question 4

Show that $\mathcal{Z} \times \mathcal{Z}$ is countable.

Question 5

Prove that:

a) The set of rational numbers, Q, is countably infinite

(Hint: every rational number can be expressed as $\frac{p}{q}$, where $p \in Z$, $q \in N$, $q \neq 0$, and p, q are relatively prime)

b) (Q. 1.5.11 in textbook) The set of all real numbers in the interval $[0, 1]$ is uncountable

(Hint: refer to the hint in textbook)

Context-free Grammars

Question 1

Show that the following languages are context-free by exhibiting context-free grammars generating each.

(a) $\{a^m b^n : m \geq n\}$

(b) $\{a^m b^n c^p d^q : m + n = p + q\}$

(c) $\{w \in \{a, b\}^* : w$ has twice as many b's as a's$\}$

(d) $\{w \in \{a, b\}^* : w$ has equal number of a's and b's$\}$

(e) $\{w \in \{a, b\}^* : w = w^R\}$

Question 2

Let $G = (V, \Sigma, R, S)$, where $V = \{a, b, S\}, \Sigma = \{a, b\}$, and $R = \{S \rightarrow aSb, S \rightarrow aSa, S \rightarrow bSa, S \rightarrow bSb, S \rightarrow e\}$. Show that $L(G) = \{w \in \{a, b\}^* : w$ has even length$\}$.

Question 3

Prove that all regular languages are context-free.

(*Hint:* Recall that in class we showed how to construct the context-free grammar corresponding to a particular DFA. You need to prove that the construction always works correctly.)

Question 4

Construct a pushdown automaton that accepts the language $\{a^i b^j : i \leq 2j\}$

Question 5

Construct a pushdown automaton that accepts the language $\{a^i b^j c^k : i = j + k\}$

Pushdown Automata

Question 1

Consider the alphabet $\Sigma = \{a, b, (,), \cup, ^*, \emptyset\}$. Construct a context-free grammar which generates all strings in Σ^* that are regular expressions over $\{a, b\}$.

Question 2

Construct a pushdown automaton that accepts the following language:

$$\{w \in \{a, b\}^* : \text{w has twice as many } b\text{'s as } a\text{'s}\}$$

Question 3

Use closure under union to show that the following languages are context-free.

a) $\{a^m b^n : m \neq n\}$

b) $\{w \in \{a, b\}^* : w = w^R\}$

c) $\{a^m b^n c^p d^q : n = q \text{ OR } m \leq p \text{ OR } m + n = p + q\}$

d) $\{a, b\}^* - \{a^n b^n : n \geq 0\}$

Question 4

In the proof of Theorem 3.5.2, why did we assume that M_2 was deterministic?

Closure Properties & Pumping Theorem for CFLs

Question 1

Use the pumping theorem for CFLs to show that the following languages are not context free.

a) $\{a^{n^2} : n \geq 0\}$

b) $\{ww : w \in \{0, 1\}^*\}$

Question 2

You are told that the language $L = \{a^n b^n c^n : n \geq 0\}$ is not context-free. Prove that the following language is also not context-free:

$$\{w \in \{a, b, c\}^* : w \text{ has equal number of } a\text{'s, } b\text{'s, and } c\text{'s}\}$$

Question 3

Suppose that L is context-free and R is regular. Are the following languages necessarily context free? If yes, prove it; if not, give a counter-example.

a) $L - R$

b) $R - L$