

COMP 3721

Tutorial 6

Context-Free Language

- Pumping Theorem
- Closure Property

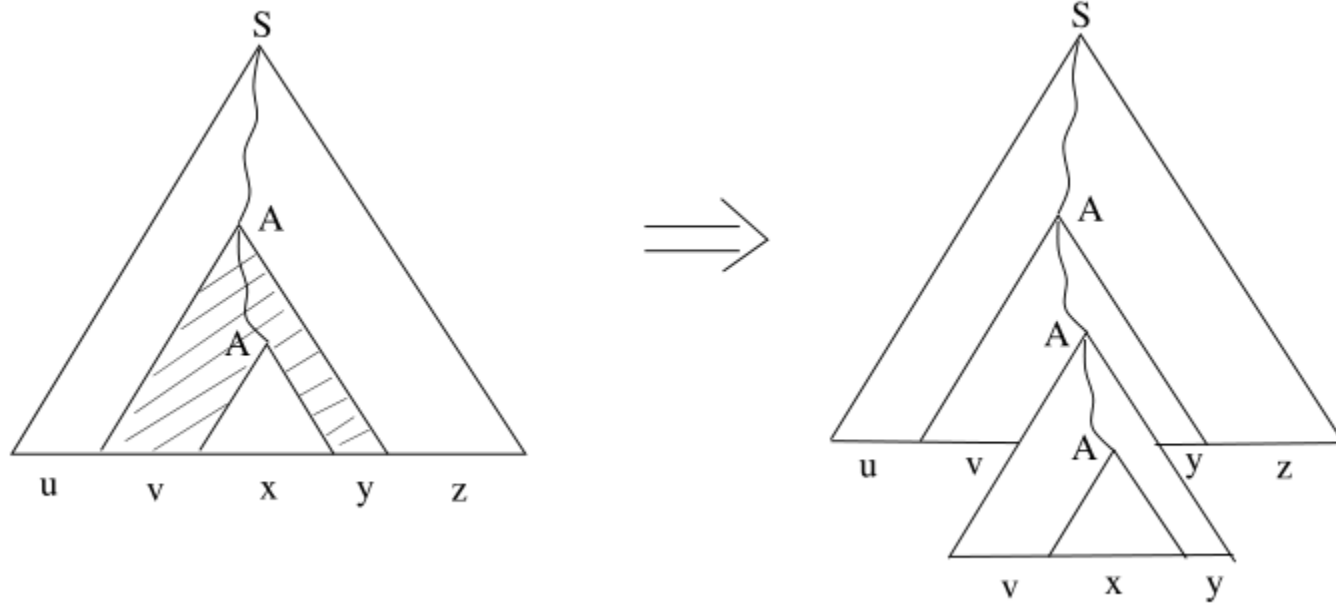
Pumping Theorem

The pumping theorem for CFLs

Theorem 1 *Let L be a context free language. Then there is an integer $N \geq 1$ such that for every $w \in L$ and $|w| \geq N$, w can be split into five parts $w = uvxyz$ such that*

1. $vy \neq \epsilon$ (i.e. v and y cannot both be ϵ)
2. $|vxy| \leq N$
3. $uv^i xy^i z \in L$ for $i = 0, 1, 2, \dots$

Pumping Theorem



Pumping Theorem

1. For each of the following languages L , state whether L is context-free or not.

a) $\{a^i b^j c^k : i + j = k, i, j, k \geq 0\}$

b) $\{a^i b^i c^{2i} : i \geq 0\}$

c) $\{a^i b^j c^k : j = \max(i, k), i, j, k \geq 0\}$

Justify your answer. If context free show a PDA or CFG, if not apply Pumping Lemma for CFL

Pumping Theorem

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Context-free.

$$K = \{a, b, c, S\}$$

$$\Sigma = \{a, b, c\}$$

$$R = \{S \rightarrow aSc, S \rightarrow bSc, S \rightarrow e\} \text{ NOT CORRECT!}$$

$$R = \{S \rightarrow e \mid A, A \rightarrow aAc \mid B, B \rightarrow bBc \mid e\}$$

Pumping Theorem

1. For each of the following languages L, state whether L is context-free or not.
 - a) $\{a^i b^j c^k : i + j = k, i, j, k \geq 0\}$ Context-Free.
 - b) $\{a^i b^j c^{2i} : i \geq 0\}$ Not Context Free.

Pumping Theorem

1. For each of the following languages L , state whether L is context-free or not.

b) $L = \{a^i b^j c^{2i} : i \geq 0\}$ Not Context Free.

Proof.

Assume L is context-free. Let N be the integer in the Pumping Theorem.

$w = a^N b^N c^{2N} = uvxyz$. $|vy| > 0$, $|vxy| \leq N$.

vxy contains at most 2 alphabet (a or b or c or a,b or b,c).

Show that,

For all cases, can find i such that $uv^i xy^i z$ is not in L .

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Closure Property of CFL

Theorem 1 *CFLs are closed under*

1. *union,*
2. *concatenation,*
3. *Kleene Star.*

Properties of CFLs

Theorem 2 *The intersection of a CFL and a regular language is a CFL.*

Closure Property of CFL

1. Use closure property of CFL to show that the following languages are context-free.
 - a) $\{a^i b^j : i \neq j\}$
 - b) $\{xx^R yy^R zz^R : x, y, z \in \{a, b\}^*\}$
 - c) $L - R$, where L is context-free and R is regular.

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a) $L = \{a^i b^j : i \neq j\}$

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c) $L - R$, where L is context-free and R is regular.

L is the union of:

$$L_1 = \{a^i b^j : i > j\}$$

$$L_2 = \{a^i b^j : i < j\}$$

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$$L' = \{ww^R : w \in \{a, b\}^*\}$$

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$$L - R$$

$$L \cap \bar{R}$$