COMP 3721

Tutorial 3

- NFA
- NFA = DFA = Regular Language

NFA

- In a DFA,
  - *each* symbol read causes a transition to the next state, which is *completely* determined by the current state and current symbol (i.e., there is exactly one next state).
- In an NFA,
  - some state may have more than one outgoing edge labeled with the same symbol
  - some edges may be labeled with e, the empty word.

NFA

1) Draw state diagrams for NFAs that accepts the following languages.
   a) $(ab)^*(ba)^* \cup aa^*$
   b) $((ab \cup aab)^*a^*)^*$

Hints: Use ideas from the proof which shows that every regular language is accepted by some NFA.

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NFA = DFA

Given an NFA $M = \{K, \Sigma, \Delta, s, F\}$.

We want to construct an equivalent DFA that accepts the same language:

$M' = \{K', \Sigma, \delta', s', F'\}$

- $K' = 2^K$
- $s' = E(s)$
- $F' = \{Q \in K' \mid Q \cap F \neq \emptyset\}$
- for all $Q \in K'$, $\sigma \in \Sigma$,
  $\delta'(Q, \sigma) = \cup_{q \in Q}\{E(p) : (q, \sigma, p) \in \Delta\}$.

NFA = DFA

2)

a) Find a simple NFA accepting $(ab \cup aab \cup aba)^*$.
b) Convert the NFA of (a) to DFA.

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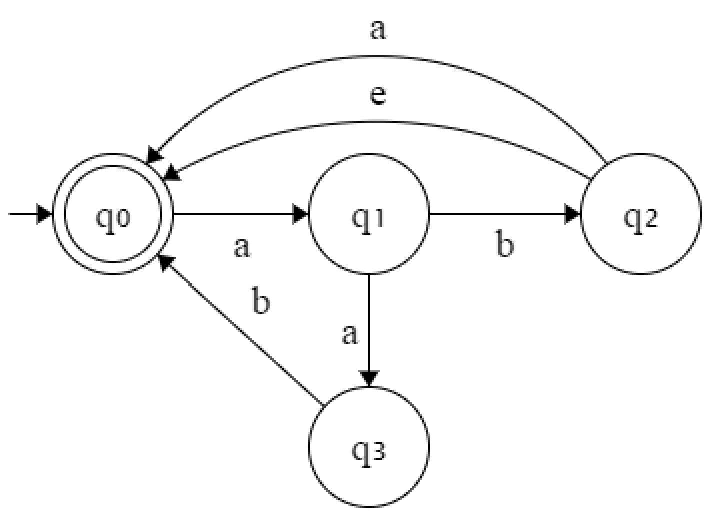

NFA = DFA

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a) Find a simple NFA accepting $(ab \cup aab \cup aba)^*$.

b) Convert the NFA of (a) to DFA.

Then, the states in the corresponding DFA are found as follows:

States in the DFA Q_i	$\delta'(Q_i, a)$	$\delta'(Q_i, b)$
$Q_0 = E(q_0) = \{q_0\}$	$E(q_1) = \{q_1\}$	$\emptyset$
$Q_1 = \{q_1\}$	$E(q_3) = \{q_3\}$	$E(q_2) = \{q_0, q_2\}$
$Q_2 = \{q_3\}$	$\emptyset$	$E(q_0) = \{q_0\}$
$Q_3 = \{q_0, q_2\}$	$E(q_1) \cup E(q_0) = \{q_0, q_1\}$	$\emptyset$
$Q_4 = \{q_0, q_1\}$	$E(q_1) \cup E(q_3) = \{q_1, q_3\}$	$E(q_2) = \{q_0, q_2\}$
$Q_5 = \{q_1, q_3\}$	$E(q_3) = \{q_3\}$	$E(q_2) \cup E(q_0) = \{q_0, q_2\}$
$Q_6 = \emptyset$	$\emptyset$	$\emptyset$

NFA = DFA

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a) Find a simple NFA accepting $(ab \cup aab \cup aba)^*$.
b) Convert the NFA of (a) to DFA.

The start state is E(q0) = Q0.
The accepting states are those contains q0. {Q0, Q3, Q4}

NFA = DFA

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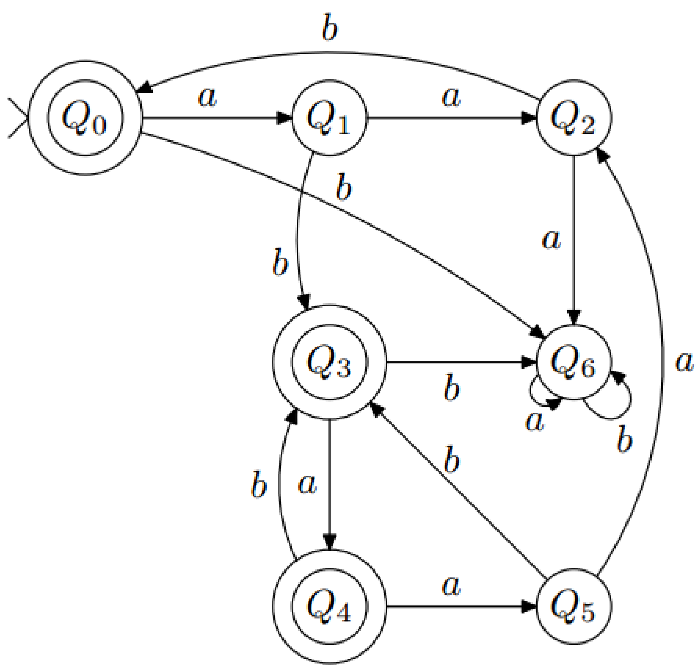

DFA = NFA = Regular Expression

3) Prove that if *L* is regular, then so is

$$Pref(L) = \{w : wu \in L \text{ for some string } u\}$$

Hints:

Construct NFA/DFA accepting *Pref(L)* from that accepting *L*

DFA = NFA = Regular Expression

3) Prove that if *L* is regular, then so is

$$Pref(L) = \{w : wu \in L \text{ for some string } u\}$$

Proof: Since L is regular, it is accepted by some NFA.

M = {K, Σ, Δ, s, F}. Next we will construct a NFA M′ that accepts Pref(L). We let M′ = (K, Σ, Δ, s, F′) where

F′ = {q ∈ K : in M, there is a path from q to f for some f ∈ F}.

We claim that M′ accepts Pref(L) and hence Pref(L) is regular.

DFA = NFA = Regular Expression

3) Prove that if *L* is regular, then so is

$$Pref(L) = \{w : wu \in L \text{ for some string } u\}$$

Proof(cont'd):

To see why M' accepts Pref(L) , consider any string w.

w ∈ Pref(L) iff

wy ∈ L for some y ∈ $\Sigma*$, iff

wy is accepted by M, iff

$(s, wy) \vdash_M^* (q, y)) \vdash_M^* (f, e)$ for some q ∈ K and f ∈ F, iff

$(s', w) \vdash_{M'} (q, e)$ for some q ∈ F' iff w is accepted by M'