COMP 3721

Tutorial 1

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- No fixed Office Hour

- Tutorials:
  - Selected Questions from Question Bank
  - Solutions will be posted on course webpage after each tutorial.

- Sets
- Languages and Regular Expressions

Sets

1) Determine whether each of the following is true or false.
   a) $\emptyset \in \emptyset$
   b) $\emptyset \subseteq \emptyset$
   c) $\{a, b\} \subseteq \{a, b, \{a, b\}\}$
   d) $\{a, b\} \in \{a, b, \{a, b\}\}$

Sets

1) Determine whether each of the following is true or false.
   a) $\emptyset \in \emptyset$ False. Empty set contains no elements.
   b) $\emptyset \subseteq \emptyset$
   c) $\{a, b\} \subseteq \{a, b, \{a, b\}\}$
   d) $\{a, b\} \in \{a, b, \{a, b\}\}$

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Languages and Regular Expressions

- Definition:

Regular expressions

Regular expressions are a *finite* representation of languages.

Inductive definition of *regular expressions* for languages over an alphabet Σ. A regular expression is a string over alphabet $\Sigma_1 = \Sigma \cup \{(,), \emptyset, \cup, ^*\}$.

1. $\emptyset$ and each $\sigma \in \Sigma$ are regular expressions.
2. If α and β are regular expressions, then

$$(\alpha\beta), (\alpha \cup \beta), \alpha^*$$

   are regular expressions.
3. Nothing else is a regular expression.

Languages and Regular Expressions

2) Show that if *a* and *b* are distinct symbols, then

$$\{a, b\}^* = \{a\}^*(\{b\}\{a\}^*)^*$$

Hint:

- Two sets A and B are *equal* $(A = B)$ if $A \subseteq B$ and $B \subseteq A$.

Languages and Regular Expressions

2) Show that if *a* and *b* are distinct symbols, then

$$\{a, b\}^* = \{a\}^*(\{b\}\{a\}^*)^*$$

Proof:

It is clear that $\{a\}^*(\{b\}\{a\}^*)^* \subseteq \{a, b\}^*$.

Need to show that $\{a, b\}^* \subseteq \{a\}^*(\{b\}\{a\}^*)^*$.

Languages and Regular Expressions

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For any string $w \in \{a, b\}^*$. If we mark all the occurrence of *b*'s in *w*, then *w* can be rewritten as:

$$w \in a^*ba^*b \cdots a^*ba^*ba^*$$

Languages and Regular Expressions

3) True or False. Justification.
   a) $abcd \in (a(cd)^*b)^*$
   b) $\{a^n b^n : n \geq 0\}\{b^n c^n : n \geq 0\} = \{a^n b^{2n} c^n : n \geq 0\}$

Languages and Regular Expressions

3) True or False. Justification.

a) $abcd \in (a(cd)^*b)^*$

False.

Because any nonempty string in $(a(cd)^*b)^*$ must ends with *b*.

Languages and Regular Expressions

3) True or False. Justification.

b) $\{a^n b^n : n \geq 0\}\{b^n c^n : n \geq 0\} = \{a^n b^{2n} c^n : n \geq 0\}$

False.

Counter example: abbbcc

Languages and Regular Expressions

4) Let $\Sigma = \{a, b\}$. Write regular expressions for the following sets.
   a) All strings in Σ^* with no more than 3 *a*'s.
   b) All strings in Σ^* with a number of *a*'s divisible by 3.
   c) All strings in Σ^* that does not have *aab* as a substring.

Languages and Regular Expressions

4) Let $\Sigma = \{a, b\}$. Write regular expressions for the following sets.
   a) All strings in Σ^* with no more than 3 *a*'s.
   $$b^*(a \cup \emptyset)b^*(a \cup \emptyset)b^*(a \cup \emptyset)b^*$$
   b) All strings in Σ^* with a number of *a*'s divisible by 3.

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   c) All strings in Σ^* that does not have *aab* as a substring.

      $$(ab \cup b)^*a^*$$

Languages and Regular Expressions

5) Prove that if *L* is regular, then so is

$$\text{Pref}(L) = \{w : wu \in L \text{ for some string u}\}.$$

Hint: Follow the definition of regular expression.

Languages and Regular Expressions

- Proof(sketch):
  i. We first prove that if for any $L(a), a \in \Sigma \cup \emptyset$, then *Pref(L(a))* is regular.

  ii. Then we prove that if L1 and L2 are regular and have their *Pref* also regular, so do their *union*, *concatenation* and *Kleene Star*.