COMP3721 Question Bank #3

Turing Machines

Question 1

Let M be the Turing machine $(K, \Sigma, \delta, s, \{h\})$, where

$$K = \{q_0, q_1, q_2, h\},$$
$$\Sigma = \{a, \sqcup, \triangleright\},$$
$$s = q_0,$$

and δ is given by the following table.
Let $n \geq 0$. Describe carefully what M does when started in the configuration $(q_0, \triangleright \sqcup \underbrace{a...a}_{n} \underline{a})$.

q,	σ	$\delta(q, \sigma)$
q_0	a	$(q_1, \leftarrow)$
q_0	$\sqcup$	$(h, \sqcup)$
q_0	$\triangleright$	$(q_0, \rightarrow)$
q_1	a	$(q_2, \sqcup)$
q_1	$\sqcup$	$(h, \sqcup)$
q_1	$\triangleright$	$(q_1, \rightarrow)$
q_2	a	(q_2, a)
q_2	$\sqcup$	$(q_0, \leftarrow)$
q_2	$\triangleright$	$(q_2, \rightarrow)$

Question 2

Design and write out in full a Turing machine that scans to the right until it finds two consecutive a's and then halts. The alphabet of the Turing machine should be $\{a, b, \sqcup, \triangleright\}$.

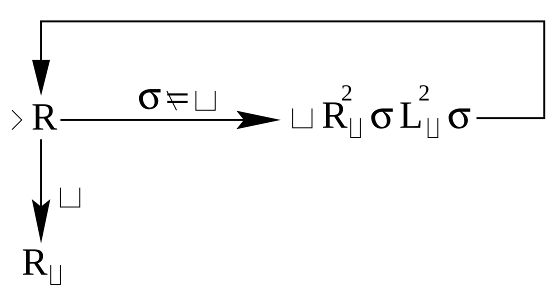
Question 3

Explain what this machine does on the input $\triangleright\underline{\sqcup}w\sqcup$.

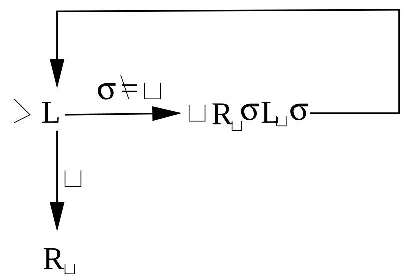
$$> R \overset{a\neq\sqcup}{\rightarrow} R \overset{b\neq\sqcup}{\rightarrow} R_\sqcup a R_\sqcup b$$

Question 4

(a) Trace the operation of the following Turing machine when started on $\triangleright\underline{\sqcup}aabb$.



(b) Trace the operation of the following Turing machine when started on $\triangleright\sqcup aabb\underline{\sqcup}$.



Recursive & R.E. Languages

Question 1

1. Give a Turing machine that decides the regular language a^*ba^*b.
2. Give a Turing machine that semidecides the regular language a^*ba^*b.

Question 2

Give an example of a Turing machine with two halting states, y and n, that does not decide a language.

Question 3

Describe in words an implementation of a Turing machine that *decides* the following language

$$L = \{\#w_1\#w_2\#...\#w_n : w_i \in \{0,1\}^*, n \geq 0, w_i \neq w_j \text{ for each } i \neq j\}$$

Question 4

a) Construct a Turing machine out of basic machines that transforms $\triangleright\underline{\sqcup}w\sqcup$ to $\triangleright\sqcup w\underline{\sqcup}w^R\sqcup$, where $w \in \{a,b\}^*$.

b) Describe in words how a double-tape Turing machine transforms $\triangleright\underline{\sqcup}w\sqcup$ to $\triangleright\sqcup w\underline{\sqcup}w^R\sqcup$, where $w \in \{a,b\}^*$.

Question 5

Prove that a language L is recursive if and only if L and $\overline{L}$ are both recursively enumerable.

Question 6

We know that the class of recursively enumerable languages is not closed under complementation. Show that it is closed under union and intersection.

Undecidable problems

Question 1

Say that Turing machine M **uses** k tape squares on input string w if and only if there is a configuration of M, $(q, u\underline{a}v)$, such that $(s, \rhd\underline{\sqcup}w) \vdash_M^* (q, u\underline{a}v)$ and $|uav| \geq k$.

(a) Show that the following problem is solvable: Given a Turing machine M, an input string w, and a number k, does M use k tape squares on input w ?

(b) Show that the following problem is undecidable: Given a Turing machine M and an input string w, does there exist a $k \geq 0$ such that M does not use k tape squares on input w ? (That is, does M use a finite amount of tape on input w ?)

Question 2

Which of the following problems about Turing machines are solvable, and which are undecidable? Explain your answers carefully.

(a) To determine, given a Turing machine M, a state q, and a string w, whether M ever reaches state q when started with input w from its initial state.

(b) To determine, given a Turing machine M and a symbol σ, whether M ever writes the symbol σ when started on the empty tape.

(c) To determine, given a Turing machine M, whether M ever writes any symbol (nonblank) when started on the empty tape.

(d) To determine, given a Turing machine M, whether the language semidacided by M is finite.