# COMP 3721

Tutorial 4

- NFA -> Regular Expression

- NFA = DFA = Regular Expressions

- Pumping Theorem

# NFA -> Regular Expression

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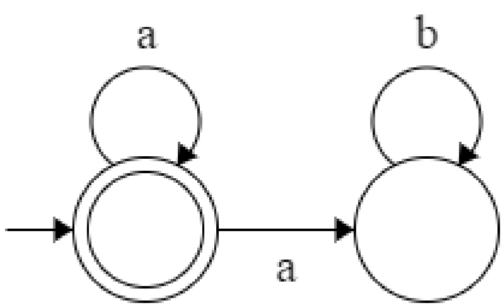


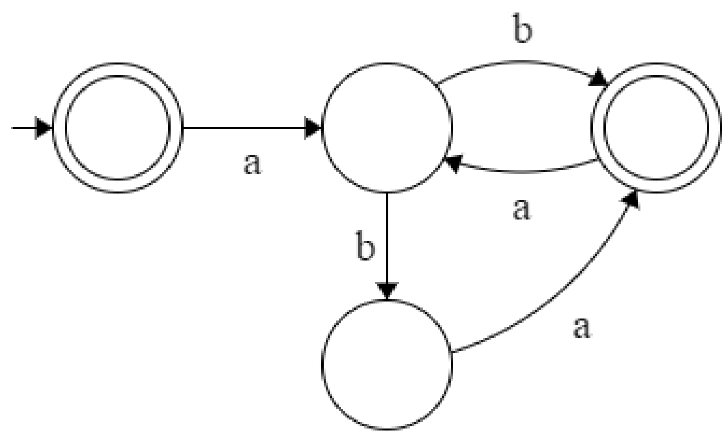

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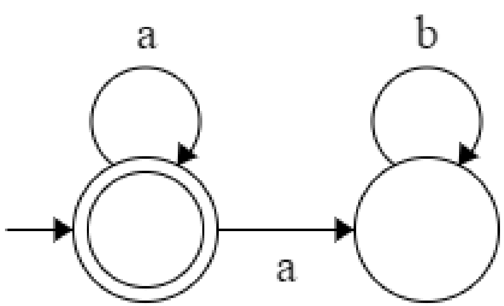


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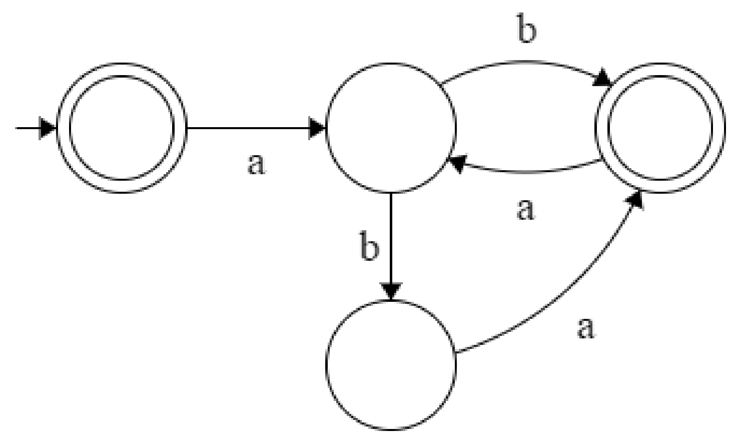

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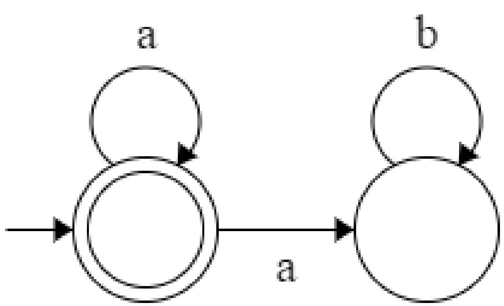


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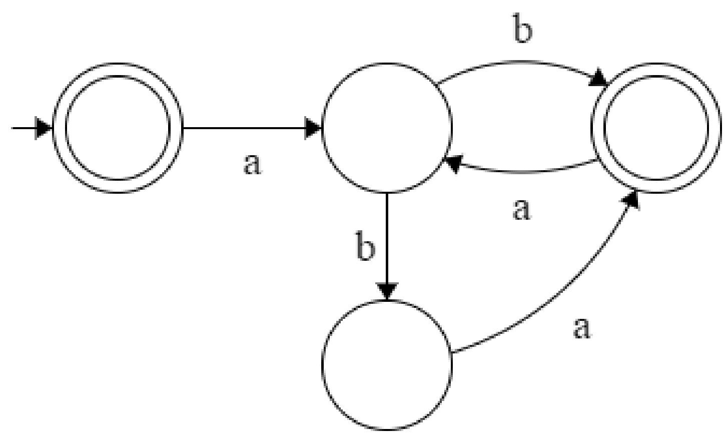


$(a(ba \cup b))^*$

# NFA = DFA = Regular Expressions

2) Prove that if L is regular, then the following languages are also regular

   a) $Subseq(L) = \{w_1 \ldots w_k : x = x_0 w_1 \ldots w_k x_k \in L \text{ for some } x_0, \ldots, x_k\}.$

   b) $L^R = \{w : w^R \in L\}.$

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Proof:

Since L is regular, it is accepted by some NFA *M = {K, Σ, Δ, s, F}.*

Next we will construct a NFA *M'* that accepts *Subseq(L)*.

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Next we will construct a NFA *M′* that accepts *Subseq(L)*. We look at any pair *(p, q)* of states in *M*, if there is a path from *p* to *q*, we draw an arrow from *p* to *q* labeled with *e*. More precisely, *M′ = (K, Σ, Δ′, s, F)* where

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We claim that *M′* accepts *Subseq(L)* and hence *Subseq(L)* is regular. Details are omitted

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   b) $L^R = \{w : w^R \in L\}$.

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   Since L is regular, it is accepted by some NFA *M = {K, Σ, Δ, s, f}*. Without loss of generality, we assume that *M* has single final state.

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Next we will construct a NFA $M'$ that accepts $L^R$ by reversing all the transitions in $M$.

More precisely, we let $M' = (K, \Sigma, \Delta', s', f')$ where

$s' = f, f' = s,$ and $\Delta' = \{(p, a, q) : (q, a, p) \in \Delta\}$.

We claim that $M'$ accepts $L^R$ and hence $L^R$ is regular. Details are omitted

# Pumping Theorem

We use the pumping theorem to prove that a language is NOT regular. To prove that $L$ is not regular, we show the following, which contradicts the P.T.:

$$\begin{aligned}
&\forall n \geq 1, \\
&\exists w \in L, |w| \geq n, \\
&\forall x, y, z \text{ such that } w = xyz \text{ and } y \neq e,\ |xy| \leq n, \\
&\text{but } \exists i \geq 0,\ xy^i z \notin L.
\end{aligned}$$

# Pumping Theorem

- Assume $L$ is regular.
- Then, by Pumping Theorem, there exists an integer $n$ such that *all* strings of length $\geq n$ in $L$ can be pumped.
- Find *one* string $w$ in $L$ that has length $\geq n$ and is not pumpable no matter how you split it into 3 parts, i.e.,:
  - Choose *one* string $w \in L$ with $|w| \geq n$.
  - Consider *all* valid ways of splitting $w$ into $x, y, z$.
  - For each way of splitting, demonstrate *one* value $i$ such that $xy^iz \notin L$
- The existence of one string $w$ that is not pumpable contradicts the Pumping Theorem.
- Hence $L$ is not regular.

# Pumping Theorem

3) Are the following languages on *{a, b}* regular or not? Prove your answer.
   a) $L = \{a^i b^j \, : \, i > j \geq 1\}$
   b) $L = \{ww \, : \, w \in \{a, b\}^*\}$
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Then $x = a^i$, $y = a^j$, $z = a^k b^n$ for $i, k \geq 0$ and $j > 0$.

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   Proof.

   In fact,

   $L = (babbabbab)^+ = (babbabbab)(babbabbab)^*$

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   Counter example:

   $\{a^n b^n : n \geq 0\}$ is a subset of $(a \cup b)^*$, but is not regular.

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   True.

   Regular language is closed under *Concatenation* and *Complementation.*

   $L' = L\overline{L}$

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   False.

   Can be proved by pumping theorem.

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