COMP 3721

Tutorial 5

October 6, 2016

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- $V = \{a, b, c, d, A, B, X, S\}$
- $\Sigma = \{a, b, c, d\}$
- $R = \{S \rightarrow aSd, S \rightarrow A, S \rightarrow B, A \rightarrow aAc, A \rightarrow X, B \rightarrow bBd, B \rightarrow X, X \rightarrow bXc, X \rightarrow e\}$

Context-Free Grammar

(2) Let $G = (V, \Sigma, R, S)$, where $V = \{a, b, S\}$, $\Sigma = \{a, b\}$, and $R = \{S \to aSb, S \to aSa, S \to bSa, S \to bSb, S \to e\}$. Show that $L(G) = \{w \in \{a, b\}^* : w \text{ has even length}\}$.

Proof

- It is easy to see that any string generated by G have even length since every rule of G generates even number of terminals.
- Now it suffices to show that any string w of even length can be generated by G. We prove this by induction on the length of w.
  (i) When $|w| = 0$, it is obvious that w can be generated by G.
  (ii) Suppose that when $|w| = 2i$, w can be generated by G.
  (iii) We show that when $w = 2(i+1)$, w can be generated by G. Suppose w starts with a and ends with a (the other three cases can be handled in a similar way). Then $w = aya$ for some y with $|y| = 2i$. By the inductive hypothesis, w can be generated as follows.

$$S \Rightarrow aSa \Rightarrow^* aya = w$$

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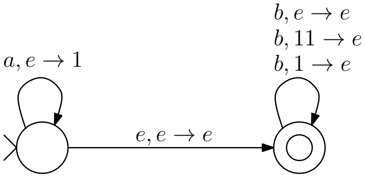

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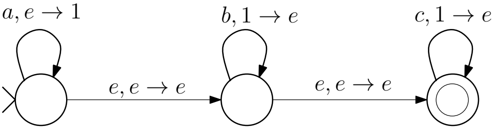

Pushdown Automata

(3) Construct a pushdown automaton that accepts the following language.

$$\{w \text{ in } \{a, b\}^* : w \text{ has twice as many } b\text{'s as } a\text{'s}\}$$

Solution