COMP3721 Tutorial 10

CSE, HKUST

R.E. Languages

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Union: If L_1 and L_2 are recursively enumerable languages then there exists two Turing machines M_1 and M_2 that semi-decide L_1 and L_2, respectively. Let M be the 2-tape Turing machine that operates as follows:

(i) Copy the input string w from the first tape to the second tape.

(ii) Simulate M_1 on the first tape and M_2 on the second tape alternatively (i.e. do one step of M_1 on first tape, then one step of M_2 on second tape, and so on). If M_1 or M_2 halts, then M halts.

We claim M semi-decides $L_1 \cup L_2$.

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Intersection: If L_1 and L_2 are recursively enumerable languages then there exists two Turing machines M_1 and M_2 that semi-decide L_1 and L_2, respectively. Let M be the Turing machine that operates as follows:

(i) Simulate M_1 on the string w.

(ii) If M_1 halts, then simulate M_2 on the string w.

(iii) If M_2 halts, then M halts.

We claim M semi-decides $L_1 \cap L_2$.

Preparing the Midterm

Q2. Write regular expression for
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Solution: L can be written as $L = L'\{a\}L'$ where L' is the language that contains at least one b. It is easy to obtain that $R(L') = (a \cup b)^* b (a \cup b)^*$. Then we have $R(L) = (a \cup b)^* b (a \cup b)^* a (a \cup b)^* b (a \cup b)^*$

Preparing the Midterm

Q3. Let L_1 be a regular language on Σ, and let L_2 be an arbitrary language on Σ. We define

$$\frac{L_1}{L_2} = \{w \in \Sigma^* : wv \in L_1 \text{ for some } v \in L_2\}.$$

Show that $\frac{L_1}{L_2}$ is regular.

Preparing the Midterm

Q3. Let L_1 be a regular language on Σ, and let L_2 be an arbitrary language on Σ. We define

$$\frac{L_1}{L_2} = \{w \in \Sigma^* : wv \in L_1 \text{ for some } v \in L_2\}.$$

Show that $\frac{L_1}{L_2}$ is regular.

Solution: Since L_1 is regular, there is some DFA $M = (K, \Sigma, \delta, s, F)$ that accepts L_1. Now we construct a DFA $M' = (K, \Sigma, \delta, s, F')$ by defining

$$F' = \{q \in K : (q, y) \vdash_M^* (f, e) \text{ for some string } y \in L_2 \text{ and } f \in F\}$$

We claim that M' accepts $\frac{L_1}{L_2}$ and hence $\frac{L_1}{L_2}$ is regular. To see this, consider any string w. $w \in \frac{L_1}{L_2}$, if and only if $wv \in L_1$ for some $v \in L_2$, if and only if $(s, wv) \vdash_M^* (q, v) \vdash_M^* (f, e)$ for some $v \in L_2$ and some $f \in F$, if and only if $(s, w) \vdash_{M'}^* (q, e)$ for some $q \in F'$, if and only if w is accepted by M'.

Preparing the Midterm

Q4. Write a context-free grammar that generates the following language.

$$L = \{a^n b^m : m = 2n \geq 0 \text{ or } m = n \geq 0\}$$

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Solution: Note that L can be written as

$$L = \{a^n b^n : n \geq 0\} \cup \{a^n b^{2n} : n \geq 0\}$$

We define the grammar $G = (V, \sigma, R, S)$ that generates L as follows.

(i) $\Sigma = \{a, b\}$.

(ii) $V = \Sigma \cup \{S, A, B\}$ where nonterminal S stands for strings in L, A stands for strings of the form $a^n b^n$, and B stands for strings of the form $a^n b^{2n}$.

(iii) $R = \{S \to A, S \to B, A \to aAb, A \to e, B \to aBbb, B \to e\}$.