COMP 3721

Tutorial 2

- Countability

- DFA

Countability

1) Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

Hints:
It is equinumerous with $\mathbb{N}$.
Find a way to enumerate every element in it within finite steps.
Using conclusions from the lecture.

Countability

- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

- Proof 1(a little bit complicated):
  - Find a bijection between $\mathbb{N} \times \mathbb{N}$ and $\mathbb{N}$.

  - The following function is such a bijection:
  - f(a, b) = (a + b - 1)*(a + b - 2)/2 + a

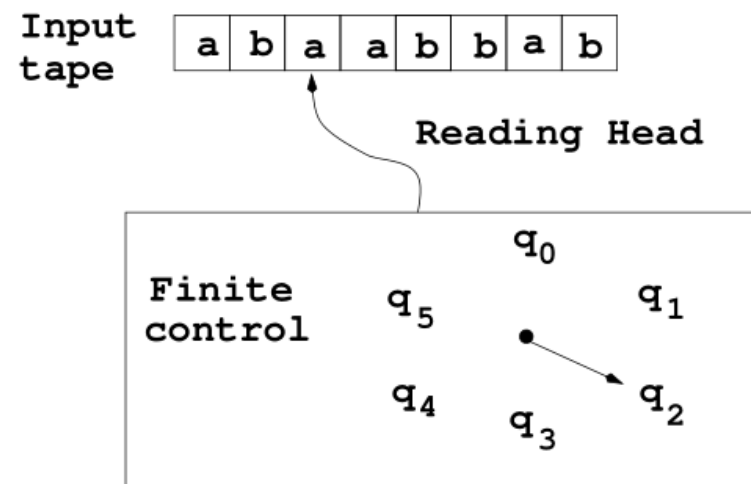 - Prove that it is a bijection….

Countability

- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

- Proof 2:
  - A way to enumerate all elements:

    - enumerate tuple (a, b) where:
    - a + b = 0
    - a + b = 1
    - a + b = 2
    - …
  - Every (a, b) will be reached in finite step..

(0,0)	(0,1)	(0,2)	(0,3)	…
(1,0)	(1,1)	(1,2)	(1,3)	
(2,0)	(2,1)	(2,2)	(2,3)	
(3,0)	(3,1)	(3,2)	(3,3)	
…				

Countability

- Prove that $\mathbb{N} \times \mathbb{N}$ is countable.

- Proof 3:
  - Use that:
 - the Cartesian product of two countable sets is countable…

DFA

DFA

1) Construct a DFA for accepting the language with regular expression $b^*a(a \cup b)^*$.

2) Construct a DFA for accepting each of the following languages:
   a) $\{w \in \{a, b\}^* : w$ contains the string *abbab*$\}$.
   b) $\{w \in \{a, b\}^*: w$ don't have *abb* as a substring$\}$.
   c) $\{w \in \{a, b\}^*: w$ has a number of *a*'s divisible by 3$\}$.
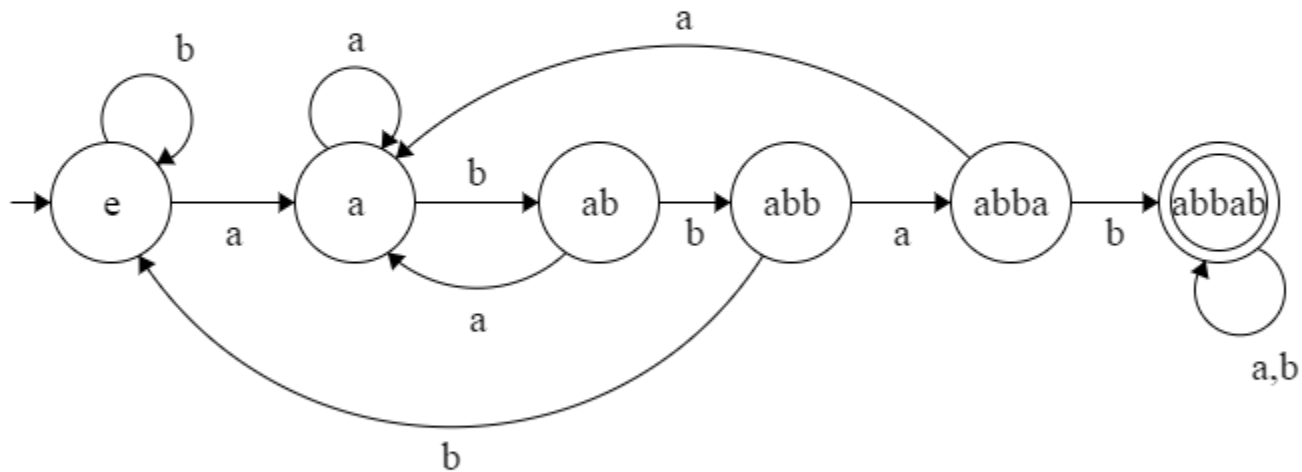   d) $\{w \in \{a, b\}^*: w$ has an odd number of *a*'s and an even number of *b*'s$\}$.

DFA

1) Construct a DFA for accepting the language with regular expression $b^*a(a \cup b)^*$.
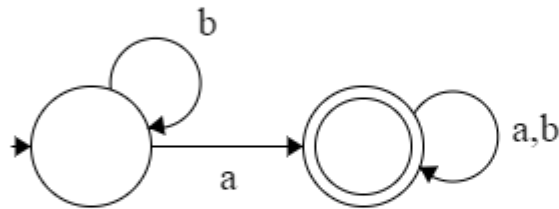
DFA

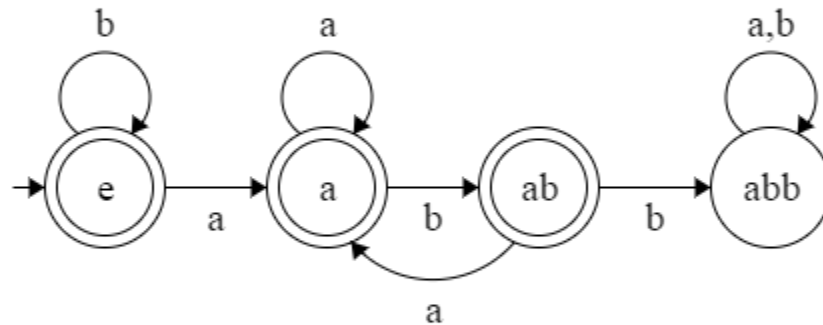2) Construct a DFA for accepting each of the following languages:
   a) {w ∈ {a, b}* : w contains the string *abbab*}.

DFA

2) Construct a DFA for accepting each of the following languages:

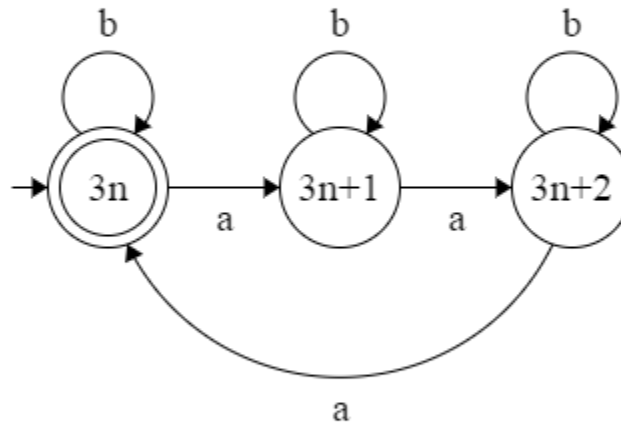   a) {w ∈ {a, b}* : w contains the string *abbab*}.

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DFA

2) Construct a DFA for accepting each of the following languages:
   a) {$w \in \{a, b\}^*$: w contains the string *abbab*}.
   b) {$w \in \{a, b\}^*$: w don't have *abb* as a substring}.
   c) {$w \in \{a, b\}^*$: w has a number of a's divisible by 3}.

DFA

2) Construct a DFA for accepting each of the following languages:

a) {w ∈ {a, b}* : w contains the string *abbab*}.

b) {w ∈ {a, b}*: w don't have *abb* as a substring}.

c) {w ∈ {a, b}*: w has a number of *a*'s divisible by 3}.

d) {w ∈ {a, b}*: w has an odd number of *a*'s and an even number of *b*'s}.