COMS BC 3997 - F22: Problem Set 1

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Introduction: Welcome to the first "real" problem set of the semester! As you are hopefully already aware, this PDF comprises the written component of the first problem set. In addition to solving the problems found below, you will also need to complete the coding part of the assignment, found in the Github repo. Finally, we'd like to remind you that all work should be yours and yours alone. This being said, in addition to being able to ask questions at office hours, you are allowed to discuss questions with fellow classmates, provided 1) you note the people with whom you collaborated, and 2) you **DO NOT** copy any answers. Please write up the solutions to all problems independently.

Collaborators:

Problem 1 (4 points): Which of the following are true and which are false? Explain your answers.

(a) Depth-first search always expands at least as many nodes as A* search with an admissible heuristic.

(b) h(n) = 0 is an admissible heuristic for the 15-Puzzle.

(c) Breadth-first search is complete even if zero step costs are allowed.

(d) Assume that a Queen can move on a chessboard any number of squares in a straight line, vertically or horizontally, in one move, but cannot jump over other pieces. Manhattan distance is an admissible heuristic for the problem of moving the Queen from square A to square B in the smallest number of moves.

Solution 1:

(a)

(b)

(c)

(d)

Problem 2: (2 points) The iterative lengthening search algorithm is to uniform cost search what iterative deepening is to depth first search. The idea is to use increasing limits on path cost. At each iteration, UCS is run up to a path cost of λ. Then for each new iteration the cost is increased to the lowest path cost of any node in the previous iteration whose cost was greater than λ.

(a) Show that this algorithm is optimal when all path costs are positive.

(b) Consider a uniform tree with branching factor b, solution depth d, and unit step costs. How many **iterations** will iterative lengthening require to find the solution?

Solution 2:

(a)

(b)

Problem 3: (2 points) Describe a scenario in which iterative deepening search performs much worse than depth-first search (for example, $O(d^2)$ vs. $O(d)$).

Solution 3:

Problem 4: (3 points) Show that each of the following statements are true, or give a counterexample (note: this is not a CS theory class and so you do not need to provide a full proof - but you should make sure to provide a complete explanation):

(a) Breadth-first search is a special case of uniform-cost search.

(b) Uniform-cost search is a special case of A* search.

Solution 4:

(a)

(b)

Problem 5: (2 points) Construct a heuristic that is admissible but NOT consistent (hint: you likely want to construct an artificial example with very few nodes).

Solution 5:

Problem 6: (3 Points) Consider the AdaptiveStep-RRT (AS-RRT) algorithm (the blue lines indicate new steps taken in AS-RRT compared to standard RRT):

Algorithm 1 AS-RRT (start and goal states s_0, s_G, initial tree $T = s_0$, max and min extend distance d_{max}, d_{min}, step size change δ) $\rightarrow$ (final tree T)

for $i = 1 : N$ **do**
  Sample states $s \in \mathcal{S}$ until s is collision-free
  Find closest state $s_c \in T$
  Set $d = d_{max}$
  while $d > d_{min}$ **do**
  Extend s_c toward s with distance d resulting in state s'
  if isCollisionFreePath(s_c, s') **then**
    Add s' to T
    Break
  else
    $d = d - \delta$
  Return T

(a) Assume that we are using a data structure for our tree where insert and the nearest searches take $O(log|T|)$ and that sample, extend, and isCollisionFreePath are $O(1)$ operations, what is the worst case time coplexity for ONE iteration through the main loop of AdaptiveStep-RRT?

(b) Do you think that the assumption that isCollisionFreePath is always an $O(1)$ operation is reasonable? Why or why not?

Solution 6:

(a)

(b)