COMS BC 3997 - F22: Problem Set 4

Introduction: This PDF comprises the written component of the this problem set. In addition to solving the problems found below, you will also need to complete the coding part of the assignment, found in the Github repo. Finally, we'd like to remind you that all work should be yours and yours alone. This being said, in addition to being able to ask questions at office hours, you are allowed to discuss questions with fellow classmates, provided 1) you note the people with whom you collaborated, and 2) you **DO NOT** copy any answers. Please write up the solutions to all problems independently.

Collaborators:

Problem 1: (Optimal) Control and Planning Concepts (5 points)

Are the following statements true or false? Make sure to briefly explain why in 1-3 sentences.

(a) A PID controller computed about the unstable equilibrium for a pendulum (swung up to the vertical position) will be able to control the pendulum back to that point under any perturbation. Aka if someone knocks the pendulum, the controller can always swing it back up within a reasonable amount of time.

(b) Consider the same setup as part a except now you have an LQR controller. Will that always be able to control it back to the upright point within a reasonable amount of time?

(c) The Line Search Parameter in DDP makes it work better and converge faster but is not necessary in most cases.

(d) LQR-RRT will always find the goal faster than standard euclidean RRT.

(e) DDP always finds a globally optimal trajectory.

Solution 1:

(a)

(b)

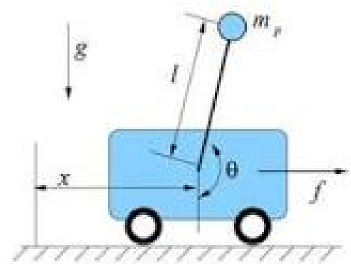
(c)

(d)

(e)

Problem 2: Optimal Control (3 points)

Suppose we are again trying to control a Cart-Pole to hold the pole upright. Note that the the cart can move left or right along the track and is powered, while the pendulum has no motor so the control action $a \in \mathcal{R}$ and the state $s \in \mathcal{R}^4 = [x, \theta, \dot{x}, \dot{\theta}]^T$. Assume that we start at $s_0 = [0, \frac{3\pi}{4}, 0, 0]^T$ and have a goal as mentioned earlier of $s_g = [0, \pi, 0, 0]^T$. We solve the LQR problem using a cost function of the form $J(s, a) = (s - s_g)^T Q(s - s_g) + a^T Ra$. Finally, assume that the default control input is 0.



(a) What would the optimal feedback control, a^*, be at x_0 if we are are using LQR-Control at the goal with $K = [1, 4, 10, -10]$.

(b) Assume that sometime later we have now moved to the state $[0.1, \frac{7\pi}{8}, -1, 1]^T$. Now what would the optimal feedback control be?

(c) Assume the controller from part (b) agressively overshot the stable upright point and used a large amount of control input. How might you adjust the controller to mitigate that error?

Solution 2:

(a)

(b)

(c)

Problem 3: Filtering (7 Points + 2 Bonus)

For this problem, you will be playing a typing simulation. Let random variable E represent the observed key press, and X represent the hidden (intended) key press. We have a language with 4 letters (A, B, C, D), and a keyboard arranged as a circle.

A	B
C	D

At any time, the probability of hitting the intended key is 50%, and the probability of hitting the neighboring keys is 25%. For example, $P(E|X = \mathrm{B})$:

0.25	0.5
0	0.25

In the rest of this problem we will construct and solve for the probabilities of different things occurring using a filtering model over the belief state. Consider the following transition model for $P(X'|X)$:

	A'	B'	C'	D'
Begin	1	0	0	0
A	0.5	0.5	0	0
B	0	0.5	0.5	0
C	0.5	0	0	0.5
D	0.25	0.25	0.25	0.25

(a) What is the probability of the sequence of letters "A B B C D"?

(b) What is the probability of the sequence of letters "A A B A D"?

(c) What is $P(X_3 = x|X_1 = \mathrm{A}, X_2 = \mathrm{B})$ for all x?

(d) Finally we consider the full filtering problem in which we compute $P(X_n|E_1, \ldots, E_n)$. Let "A B B C D" be the sequence of observed key strokes. What is the current belief state of the model? We will break this problem down into a few steps.

  (i) First lets assume that for time $n = 1$, $B(X_1 = A) = 1$, as our model begins in state A. Compute $P(X_2 = x|E_1, \ldots, E_n)$ where E is defined as above (aka $E_1 = A$, $E_2 = B \ldots$).

  Hint: Use both a motion and sensor model to propagate the belief state through time and then update it for the evidence observed! $P(X_n|E_1, ..., E_n) \propto P(E_n|X_n) \sum_{x_{n-1}} P(X_n|x_{n-1})B(x_{n-1})$

(ii) Continue filtering and compute $P(X_3 = x|E_1, \ldots, E_n)$

(iii) For 2 Bonus Points continue filtering and compute $P(X_4 = x|E_1, \ldots, E_n)$

Note: if you would like to check your answer you can keep going and compute $P(X_5 = x|E_1, \ldots, E_n)$ which should be [0, 0.04, 0, 0.96].

Solution 3:

(a)

(b)

(c)

(d) (i)

(ii)

(iii)

Problem 4: Computer Vision (10 Points)

(a) Assume you have a grayscale input image of a cat with the following pixel values:

22	84	1	13
87	7	144	89
222	11	12	144
0	95	45	230

Also assume you have the following filter:

-1	1	1
-2	0	1
3	-1	0

(i) What is the size of the output feature map assuming no use of padding or overlap over the edges of the image?

(ii) What would the resulting feature map values be?

(b) Would you classify this image as a dog assume the feature map is passed into an MLP layer using sigmoid as the activation function, assuming the feature map is processed in row major order, neuron weights of [-0.1, -0.2, 0.1, 0.2], a neuron bias of [3.0], and a threshold of $p = 0.5$?

(i) What is the output of the MLP?

(ii) Would that output be classified as a dog?

(c) True or False and explain in 1-3 sentences:

(i) The Gaussian filter helps smooth out images and is NOT helpful for finding features such as edges.

(ii) The derivative filter helps smooth out images and is NOT helpful for finding features such as edges.

(iii) The RANSAC algorithm is always able to find a model that fits all of the datapoints.

(iv) Convolutional Neural Networks can only fit data to predict one class at a time.

Solution 4:

(a) (i)

(ii)

(b) (i)

(ii)

(c) (i)

(ii)

(iii)

(iv)

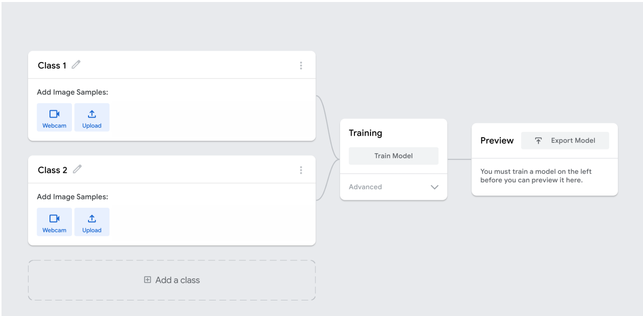
Problem 5: Computer Vision Experiment (3 Points)

A software based question in the written?!? What!?! Well it turns out this is more of an experiment with an online interface than a true coding questions so its best if you upload a screenshot to show that you did it!

Go to https://teachablemachine.withgoogle.com/.

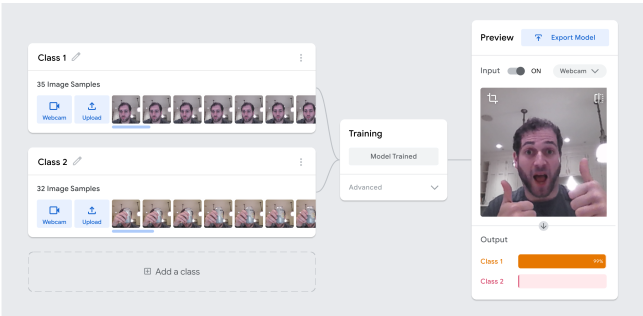
There you will find a whole host of documentation and videos on how to get started using their free platform for training simple neural network models. It turns out that while training a large, sophisticated model can take a long time, training a simple, small model is a breeze!

Click on `Get Started` − > `Image Project` − > `Standard Image Model`. You should arrive at a screen that looks something like the below:



You'll want to pick between 2-5 objects that you want to train a model to differentiate between (you can add more classes by clicking `Add a class` at the bottom). You can name the classes and the use your webcam or upload pictures of the classes. Try to pick objects that look distinctively different for at least a few of the classes (and then feel free to throw in one hard one if you'd like). Make sure you upload / take 30-50 images of each class.

Then click `Train Model` which will likely take a surprisingly short amount of time! Then you can preview your results. Please submit a screenshot of the final trained model running on one of your classes like the image below of me!



Solution 5: