COMS BC 3997 - F22: Problem Set 3

Introduction: This PDF comprises the written component of the this problem set. In addition to solving the problems found below, you will also need to complete the coding part of the assignment, found in the Github repo. Finally, we'd like to remind you that all work should be yours and yours alone. This being said, in addition to being able to ask questions at office hours, you are allowed to discuss questions with fellow classmates, provided 1) you note the people with whom you collaborated, and 2) you **DO NOT** copy any answers. Please write up the solutions to all problems independently.

Collaborators:

Problem 1: Configuration Space (2 points)

Consider the following 2-link, 2-joint robot, tethered to the center of the square. There is one obstacle in the space (the blue circle). Assume only the "joint" (aka the black circle) can collide with the obstacle (while this is not a great assumption in this case, in practice things like this are done all the time).

Draw out the configuration space for this setup in the box on the right. Make sure to label the axis on the right – aka we are moving from X,Y space to what?

Hint: What about the robot can move? And what kind of move might cause a collision to occur?
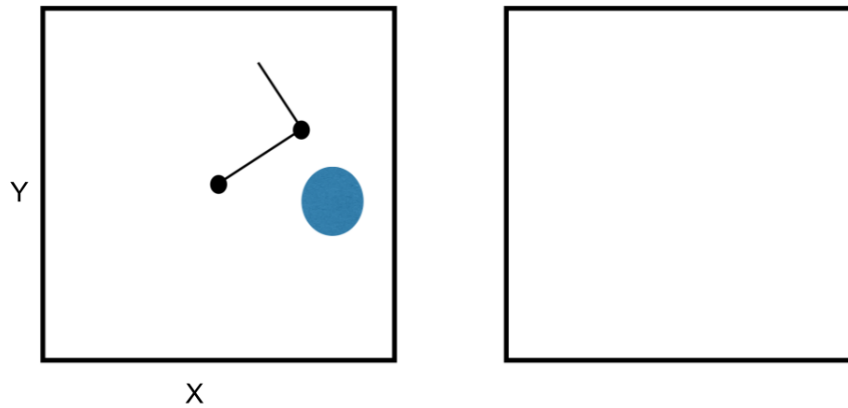
Problem 2: (Optimal) Control (5 points)

Suppose we are trying to control a Cart-Pole to hold the pole upright. Note that the the cart can move left or right along the track and is powered, while the pendulum has no motor ($u \in \mathcal{R}$ and $x \in \mathcal{R}^4 = [x, \theta, \dot{x}, \dot{\theta}]$). Assume that we start at $x_0 = [0, \frac{3\pi}{4}, 0, 0]$ and have a goal as mentioned earlier of $x_g = [0, \pi, 0, 0]$. Finally, assume that the default control input is 0.
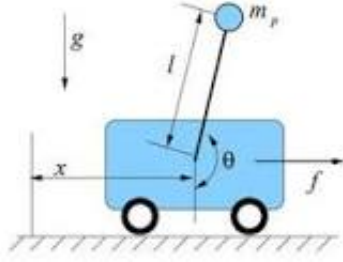
(a) What would the feedback control, u, be at x_0 if we are are using P-Control at the goal with $K_P = [1, 4]$.

(b) What would the feedback control, u, be at x_0 if we are are using PD-Control at the goal with $K_P = [1, 4]$ and $K_D = [0.1, 0.1]$.

(c) Assume that sometime later we have now moved to the state $[0.1, \frac{7\pi}{8}, -1, 1]$. Now what would the feedback control be in we are using PD-Control at the goal with $K_P = [1, \frac{8}{\pi}]$ and $K_D = [0.1, 0.1]$.

(d) Assume the controller from part (c) overshot the stable upright point. How might you adjust the controller to mitigate that error?

(e) Assume the controller from part (d) had a steady state error. How might you adjust the controller to mitigate that error?

Solution 2:

(a)

(b)

(c)

(d)

(e)

Problem 3: Machine Learning - Linear Models (5 points)
You are a turnip farmer, and after a few years of tumultuous harvests, you decide to take a quantitative approach to agriculture. You decide to model your annual turnip yield (y) as a linear function of a single variable: the amount of rainfall in the previous year (x) with the following data points:

turnip yield (y)	rainfall (x)
24	10
33	14
34	12

Assume your are looking for a parameter θ, and your model is $f_\theta(x) = \theta x$.

(a) Suppose you are using a quadratic loss function, $L(X, Y) = \sum [f_\theta(x_i) - y_i]^2$, and you can choose θ to be equal to either 1, 2, or 3. Which is the best option for your data? Why?

(b) Now suppose you are using an L_1 loss function, $L(X, Y) = \sum |f_\theta(x_i) - y_i|$, and you can choose θ to be equal to either 1, 2, or 3. Which is the best option for your data? Why?

(c) Why might the L_1 loss function be hard to use in practice? Hint: how do you optimize it and what might go wrong?

Solution 3:

(a)

(b)

(c)

Problem 4: Machine Learning - Nonlinear Models (4 points)
You are working for a hospital, and are given a dataset of patient information
with hundreds of continuous features (height, weight, blood pressure, etc) along
with whether the patient has diabetes – that is a vector of features $f_i(x)$ for
a patient x and the output of whether or not they have the disease y. Your
goal is to predict whether incoming patients will have diabetes. You decide that
logistic regression would be a good way to solve the problem.

(a) Assume that you have learned a vector of weights w. What equation would
you compute to predict if the patient has diabetes. Hint: your answer will
be in terms of some subset of $f_i(x), x, y, w$.

(b) Assume you have 200 features and data from only 12 patients. After you
have optimized the model (explain each in 1-3 sentences):

   (i) How accurate do you think the optimized model will be for those 12
   patients?

   (ii) How accurate do you think the optimized model will be for other
   patients not in the dataset?

(c) Now we are going to use a very large Neural Network instead of logistic
regression. Again assume you have 200 features per patient and data from
only 12 patients. After you have optimized the model how will the NN
model compare to the logistic regression model?

Solution 4:

(a)

(b)   (i)
      (ii)

(c)

Problem 5: Convex Optimization (7 Points) Answer the following questions about the function:

$$y(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{17}{2}x^2 + 15x + 210$$

(a) Compute the derivative $y'(x)$.

(b) What are the critical points of the function?

(c) Show whether each point is a local minima or a local maxima. Hint: you want to compute $y''(x)$.

(d) Compute a quadratic approximation $\hat{y}(x)$ of $y(x)$ at $x = 7$.

(e) Starting from that approximation take two gradient *descent* steps using a learning rate of $\alpha = 0.01$. Where do you end up? Hint: you may need to compute a second approximation!

(f) If you were going to do that gradient descent step again starting at $x = 7$ would you increase or decrease the learning rate? Why? Explain in 1-3 sentences.

Solution 5:

(a)

(b)

(c)

(d)

(e)

(f)