COMS BC3159: Problem Set 3

Due Wednesday, October 23, 2024 11:59 PM

Please show your work for all solutions to receive partial/full credit. Always turn in *only* your own, independent work to **Gradescope** (assign each question to a page in your submission). For our late policy, refer to the syllabus. All other questions can be posted on Slack.

Collaborators:

AI Tool Disclosure:

The following exercises will continue to explore trajectory optimization.

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1 Solving Linear Systems (6 Points)

For the following questions, briefly explain your answer in 1-3 sentences.

(a) Why might you want to use a factorization method over a standard matrix inverse when you are solving linear system(s) of the form $Ax = b$?

(b) Why might you prefer to use a factorization based method over an iterative method when you are solving linear system(s) of the form $Ax = b$?

(c) Why might you prefer to use an iterative method over a factorization based method when you are solving linear system(s) of the form $Ax = b$?

2 True or False? (8 Points)

For the following statements, please indicate if the statement is true or false and then briefly explain your answer in 1-3 sentences.

(a) Constrained optimization problems can always find a local minima via gradient descent.

(b) Unconstrained optimization problems can always find a local minima via gradient descent.

(c) Iterative methods for linear system(s) of the form $Ax = b$ where $A \in \mathcal{R}^N$ will always converge in N iterations.

(d) Adding additional constraints into direct transcription methods does not change the overall structure or algorithmic flow of the problem.

3 Pendulum (8 points)

Assume you are solving the trajectory optimization problem for a pendulum swinging up from the downward, stable, equilibrium of $x_s = (0, 0)$ to the upward, unstable, equilibrium of $x_g = (\pi, 0)$ under the following cost function where $Q = I,\ R = 0.1I,\ \text{and}\ Q_D = 100I$:

$$J = (x_N - x_g)^T Q_N (x_N - x_g) + \sum_{k=0}^{N-1} (x_k - x_g)^T Q (x_k - x_g) + u_k^T R u_k$$

(a) Assuming that $N = 10$ knot points along the trajectory, how many decision variables are there when solving this using a direct transcription method?

(b) Assuming that $N = 10$ knot points along the trajectory, how many decision variables are there when solving this using a differential dynamic programming method?

(c) Assuming that $N = 10$ knot points along the trajectory, what is the minimum number of constraints we need to solve this problem using a direct transcription method?

(d) Assuming that $N = 10$ knot points along the trajectory, what is the minimum number of constraints we need to solve this problem using a differential dynamic programming method?

4 More optimization (4 Points)

(a) True or False? Quadratic penalty methods transform constrained problems into unconstrained problems that can be quickly solved to very high precision in very few iterations. Please explain your answer in 1-3 sentences.

(b) Imagine you are using an augmented Lagrangian method to solve a 1-dimensional optimization problem with a constraint $g(x) = 3x - 7$ starting with $\mu = 10$. After the first outer iteration the current value of $x = 4$. What would you update λ be? Assume that we initialized $\lambda = 0$.

5 Loss (6 Points)

Assume x is a vector in $\mathcal{R}^N$ and that we are optimizing the loss:

$$\mathcal{L}(x) = \frac{1}{2}x^T Q x + 1 \quad \text{where} \quad Q = 3I$$

(a) What is the dimension of Q?

(b) What is the gradient, ∇, of $\mathcal{L}(x)$

(c) What is the hessian, ∇^2, of $\mathcal{L}(x)$