COMS BC 3159 - S24: Problem Set 3

Introduction: The following exercises will explore trajectory optimization.

Finally, we'd like to remind you that all work should be yours and yours alone. This being said, in addition to being able to ask questions at office hours, you are allowed to discuss questions with fellow classmates, provided 1) you note the people with whom you collaborated, and 2) you **DO NOT** copy any answers. Please write up the solutions to all problems independently.

Without further ado, let's jump right in!

Collaborators:
AI Tool Disclosure:

Problem 1 (6 Points):

Please answer the following questions in 1-3 sentences.

(a) Why might you want to use a factorization method over a standard matrix inverse when you are solving linear system(s) of the form $Ax = b$?

(b) Why might you prefer to use an iterative method over a factorization based method when you are solving linear system(s) of the form $Ax = b$?

(c) Why might you want to use a preconditioner with an iterative method?

Solution 1:

(a)

(b)

(c)

Problem 2 (8 Points):

For the following questions please indicate if the statement is true or false and then explain your answer in 1-3 sentences.

(a) Constrained optimization problems can always find a local minima via gradient descent.

(b) Iterative methods for linear system(s) of the form $Ax = b$ where $A \in \mathcal{R}^N$ will always converge in N iterations.

(c) Adding additional constraints into direct transcription methods does not change the overall structure or algorithmic flow of the problem.

(d) Adding additional constraints into differential dynamic programming methods does not change the overall structure or algorithmic flow of the problem.

Solution 2:

(a) ☐ True ☐ False

(b) ☐ True ☐ False

(c) ☐ True ☐ False

(d) ☐ True ☐ False

Problem 3 (8 Points):

Assume you are solving the trajectory optimization problem for a pendulum swinging up from the downward, stable, equilibrium of $x_s = (0, 0)$ to the upward, unstable, equilibrium of $x_g = (\pi, 0)$ under the following cost function where $Q = I, R = 0.1I, Q_D = 100I$:
$J = (x_N - x_g)^T Q_N (x_N - x_g) + \sum_{k=0}^{N-1} (x_k - x_g)^T Q (x_k - x_g) + u_k^T R u_k$.

(a) Assuming that $N = 10$ knot points along the trajectory, how many decision variables are there when solving this using a direct transcription method?

(b) Assuming that $N = 10$ knot points along the trajectory, how many decision variables are there when solving this using a differential dynamic programming method?

(c) Assuming that $N = 10$ knot points along the trajectory, how many constraints are there when solving this using a direct transcription method?

(d) Assuming that $N = 10$ knot points along the trajectory, how many constraints are there when solving this using a differential dynamic programming method?

Note: Please add 1 sentence describing why you wrote down your answer so that we can provide partial credit if you are incorrect.

Solution 3:

(a)

(b)

(c)

(d)

Problem 4 (4 Points)

(a) True or False. Quadratic penalty methods transform constrained problems into unconstrained problems that can be quickly solved to very high precision in very few iterations. Please explain your answer in 1-3 sentences.

(b) Imagine you are using an augmented Lagrangian method to solve a 1-dimensional optimization problem with a constraint $g(x) = 3x - 7$ starting with $\mu = 10$. After the first outer iteration the current value of $x = 4$. What would you update λ be? Assume that we initialized $\lambda = 0$.

Solution 4:

(a) ☐ True ☐ False

(b)

Problem 5 (6 Points)

Assume x is a vector in $\mathcal{R}^N$ and that we are optimizing the loss:

$$\mathcal{L}(x) = \frac{1}{2}x^T Qx + 1 \quad \text{where} \quad Q = 3I$$

(a) What is the dimension of Q?

(b) What is the gradient, ∇, of $\mathcal{L}(x)$

(c) What is the hessian, ∇^2, of $\mathcal{L}(x)$

Solution 5:

(a)

(b)

(c)