Problem 4: Due March 7 at 10:00 AM

We are a small oil extracting firm. We have a lease to an oil reservoir that contains $\bar{s} = 500$ thousand barrels of oil and s_t denotes how much oil we have in our reservoir at time t. The cost of extracting oil is given by the function,

$$C(e_t, s_t) = e_t^2,$$

where e_t is the amount extracted at time t in thousands. We do not have any on-land oil storage so it must be immediately sold. The world price of oil is \$30 per barrel (\$30,000 per thousand barrels). We discount the future with a discount factor of $\beta = 0.95$.

1. Write out our extraction problem in Bellman form
2. Solve for our value function using value function iteration with a convergence criterion of $||V_{old} - V_{new}||_\infty < 10^{-4}$ where $||\cdot||_\infty$ is the max norm. Use units of thousands of barrels (e.g. initial condition for simulation is reservoir = 500)
   - Solve using a 10th degree approximation with Chebyshev polynomials and cubic splines
3. Simulate 50 periods beginning from the initial condition above and plot the extraction trajectory and the oil reservoir stock trajectory for both sets of basis functions

Now suppose that oil becomes more costly to extract as we deplete the reservoir:

$$C(e_t, s_t) = \frac{\bar{s}}{s_t} e_t^2.$$

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3. Simulate 50 periods beginning from the initial condition above and plot the extraction trajectory and the oil reservoir stock trajectory for both sets of basis functions
4. How does an increasing cost of extraction affect the optimal extraction trajectory?