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Optimal CO_2 Abatement in the Presence of Induced Technological Change

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This paper explores the significance of policy-induced technological change for the design of carbon-abatement policies. We derive analytical expressions characterizing optimal CO_2 abatement and carbon tax profiles under different specifications for the channels through which technological progress occurs. We consider both R & D-based and learning-by-doing-based knowledge accumulation, and we examine each specification under both a cost-effectiveness and a benefit–cost policy criterion.

We show analytically in a cost-effectiveness setting that the presence of induced technological change (ITC) always implies a lower time profile of optimal carbon taxes. The same is true in a benefit–cost setting as long as damages are convex in the atmospheric CO_2 concentration. The impact of ITC on the optimal abatement path varies. When knowledge is gained through R & D investments, the presence of ITC justifies shifting some abatement from the present to the future. However, when knowledge is accumulated via learning-by-doing the impact on the timing of abatement is analytically ambiguous.

Illustrative numerical simulations indicate that the impact of ITC upon overall costs and optimal carbon taxes can be quite large in a cost-effectiveness setting but typically is much smaller under a benefit–cost policy criterion. The impact of ITC on the timing of abatement is very slight, but the effect (applicable in the benefit–cost case) on cumulative abatement over time can be large, especially when knowledge is generated through learning-by doing.

Key Words: climate policy; carbon tax; technological change.

1. INTRODUCTION

Over the past decade considerable efforts have been directed toward evaluating alternative policies to reduce the atmospheric accumulation of greenhouse gases, particularly carbon dioxide (CO_2). Initial assessments tended to disregard interconnections between technological change and CO_2-abatement policies, treating the rate of technological progress as autonomous—that is, unrelated to policy changes or associated changes in relative prices. Recently, however, several researchers have emphasized that CO_2 policies and the rate of technological change are connected: to the extent that public policies affect the prices of carbon-based fuels, they affect incentives to invest in research and development (R & D) aimed at

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bringing alternative fuels on line earlier or at lower cost. Such policies may also prompt R & D oriented toward the discovery of new production methods that require less of *any* kind of fuel. Moreover, climate policies can affect the growth of knowledge through impacts on learning by doing (LBD): to the extent that these policies affect producers' experience with alternative energy fuels or energy-conserving processes, they can influence the rate of advancement of knowledge.

Thus, through impacts on patterns of both R & D spending and learning by doing, climate policy can alter the path of knowledge acquisition. What does this connection imply for the design of CO_2-abatement policy? In particular, how do the optimal timing and extent of carbon emissions abatement, as well as the optimal time path of carbon taxes, change when we recognize the possibility of induced technological change (ITC)?

Policymakers and researchers are divided on these questions. Wigley *et al.* [31] have argued that the prospect of technological change justifies relatively little current abatement of CO_2 emissions: better to wait until scientific advances make such abatement less costly. In contrast, Ha-Duong *et al.* [8] have maintained that the potential for ITC justifies relatively more abatement in the near term, in light of the ability of current abatement activities to contribute to learning by doing. Yet another possibility is that ITC makes it optimal to increase abatement in all periods and thus achieve more ambitious overall targets for atmospheric CO_2 concentrations.

In addition to these disagreements on the optimal profile for abatement, there are differing viewpoints concerning the optimal carbon tax profile. One frequently heard claim is that induced technological change justifies a higher carbon tax trajectory than would be optimal in the absence of ITC. The argument is that in the presence of ITC, carbon taxes not only confer the usual environmental benefit by forcing agents to internalize the previously external costs from CO_2 emissions, but also yield the benefit of faster innovation, particularly in the supply of alternative energy technologies.[2] Another possibility, however, is that with technological progress, a lower carbon tax profile is all that is needed to achieve desired levels of abatement.

This paper aims to clarify the issues underlying these controversies. We derive analytical expressions characterizing the optimal paths of emissions abatement and carbon taxes under different specifications for the channels through which knowledge is accumulated, considering both *R & D-based* and *learning-by-doing-based* knowledge accumulation. We examine each of these specifications under two different optimization criteria: the *cost-effectiveness* criterion of obtaining by a specified date and thereafter maintaining, at minimum cost, a given target for the atmospheric CO_2 concentration; and the *benefit–cost* criterion under which we also choose the optimal concentration target, thus obtaining the path of carbon abatement that maximizes the benefits from avoided climate damages net of abatement

[2]Some have suggested that the innovation-related benefits from a carbon tax might be as large as the direct abatement costs associated with such a tax. If this were the case, then the overall cost (ignoring environmental benefits) of a carbon tax would be zero. Porter and van der Linde [25] advance a general argument consistent with this view, maintaining that environmental regulation often stimulates substantial technological progress and leads to significant long-run cost savings that make the overall costs of regulation trivial or even negative.

costs.[3] To gain a sense of plausible magnitudes, we also perform illustrative numerical simulations.

Our analysis is in the spirit of two studies by Nordhaus [16, 17]—the first to obtain analytical expressions for the optimal carbon tax trajectory—as well as more recent work by Farzin and Tahvonnen [4], Farzin [3], Peck and Wan [23], Sinclair [28], and Ulph and Ulph [29]. Our paper also complements work by Nordhaus [18], Nordhaus and Yang [20], and Peck and Teisberg [21, 22], in which numerical methods are used to obtain the optimal carbon abatement and carbon tax profiles under different exogenous technological specifications.[4] Another related paper is by Kolstad [12], who solves numerically for optimal emissions trajectories in the presence of endogenous learning. Kolstad's paper differs from ours, however, in that it focuses on learning that reduces uncertainty about CO_2-related damages, rather than on learning that improves abatement technologies and thus reduces abatement costs. Finally, our paper is closely related to the previously mentioned studies by Wigley *et al.* [31] and Ha-Duong *et al.* [8], as well as to working papers by Grubb [7], Goulder and Schneider [6], and Nordhaus [19] that analyze the implications of induced technological change for optimal climate policy.

The present investigation differs from each of these other studies in three ways. First, it derives *analytical* results revealing the impact of ITC on optimal time profiles for carbon taxes and carbon abatement. Second, it considers, in a unified framework, two channels for knowledge accumulation (R & D and learning by doing) and two policy criteria (cost-effectiveness and benefit–cost). In the model, policymakers (or the social planner) choose optimal paths of carbon abatement and carbon taxes, taking into account the impact of these taxes on technological progress and future abatement costs. Finally, it employs both analytical and numerical methods in an integrated, complementary way.

The analytical model reveals (contrary to what some analysts have suggested) that the presence of ITC generally implies a lower time profile of optimal carbon taxes.[5] The impact of ITC on the optimal abatement path varies. When knowledge is gained through R & D investments, the presence of ITC justifies shifting some abatement from the present to the future. However, when knowledge reflects learning by doing, the impact on the timing of abatement is analytically ambiguous.

When the government employs the benefit–cost policy criterion, the presence of ITC justifies greater overall (cumulative) abatement than would be warranted in its absence. This does not imply, however, that abatement rises in every period: when knowledge accumulation results from R & D expenditure, the presence of ITC implies a reduction of near-term abatement efforts, despite the overall increase in the scale of abatement over time.

Our numerical simulations reinforce the qualitative predictions of the analytical model. The quantitative impact on overall costs and optimal carbon taxes can be quite large in a cost-effectiveness setting but typically is much smaller under a benefit–cost policy criterion. The weak effect on the tax rate in the benefit–cost case reflects the relatively trivial impact of ITC on optimal CO_2 concentrations,

[3] This is equivalent to minimizing the sum of abatement costs and CO_2-related damages to the environment.

[4] The present paper also complements that of Manne and Richels [14], who employ a multiregion computable general equlibrium model to solve for Pareto-efficient paths of carbon abatement and taxes.

[5] However, in a benefit–cost setting, the opposite could be true if damages were concave in the atmospheric CO_2 concentration.

associated marginal damages, and (hence) the optimal tax rate. As for the optimal abatement path, the impact of ITC on the timing of abatement is very weak, but the effect on overall abatement (which applies in the benefit–cost case) can be large, especially when knowledge is accumulated via learning-by-doing.

The rest of the paper is organized as follows. Section 2 lays out the analytical model and applies it to the case in which the policy criterion is cost-effectiveness. Section 3 applies the model to the situation in which policymakers employ the broader benefit–cost criterion. Section 4 presents and interprets results from numerical simulations and includes a sensitivity analysis. The final section offers conclusions and indicates directions for future research.

2. OPTIMAL POLICY UNDER THE COST-EFFECTIVENESS CRITERION

In this section we consider optimal abatement when the policy criterion is cost-effectiveness (CE). We assume that producers are competitive and minimize costs. Let $C(A_t, H_t)$ be the economy's (aggregate) abatement-cost function, where A_t is abatement at time t and H_t is the stock of knowledge—or alternatively, the level of technology—at time t. We assume $C_A(\cdot) > 0$, $C_{AA}(\cdot) > 0$, $C_H(\cdot) < 0$, and $C_{AH}(\cdot) < 0$. The last two properties imply that increased knowledge reduces, respectively, total and marginal costs of abatement. Later on, we consider the implications of alternative assumptions. We also allow for the possibility that costs may depend on the relative amount of abatement (A_t/E_t^0) rather than the absolute level (A_t). In this case baseline emissions become an argument of the cost function. For expositional simplicity, however, we usually suppress E_t^0 from the cost function in the main text.

2.1. *Technological Change via R & D*

2.1.1. *The Problem and Basic Characteristics of the Solution*

Within our cost-effectiveness analysis, we consider two modes of knowledge accumulation. The first specification assumes that to accumulate knowledge, the economy must devote resources to research and development. We refer to this as the CE_R specification (where "R" indicates that the channel for knowledge accumulation is R & D). The planner's problem is to choose the time-paths of abatement and R & D investment that minimize the costs of achieving the concentration target.[6] Formally, the optimization problem is

$$\min_{A_t, I_t} \int_0^\infty (C(A_t, H_t) + p(I_t)I_t)e^{-rt}\,dt \tag{1}$$

$$\text{s.t.} \quad \dot{S}_t = -\delta S_t + E_t^0 - A_t \tag{2}$$

[6]Our analysis focuses on the social planner's problem. We disregard the market failure associated with knowledge spillovers, that is, with the inability of firms to appropriate the full social returns on their investments in knowledge. Our model implicitly assumes that any market failures associated with this appropriability problem have already been addressed through public policies.

$$\dot{H}_t = \alpha_t H_t + k\Psi(I_t, H_t) \tag{3}$$

$$S_0, H_0 \text{ given}$$

$$\text{and} \qquad S_t \leq \bar{S} \qquad \forall t \geq T \tag{4}$$

where A_t is abatement, I_t is investment in knowledge (i.e., R & D expenditure), S_t is the CO_2 concentration, H_t is the knowledge stock, $p(\cdot)$ is the real price of investment resources, r is the interest rate, δ is the natural rate of "removal" of atmospheric CO_2, E_t^0 is baseline emissions, α_t is the rate of *autonomous* technological progress, and k is a parameter that, as discussed below, indicates whether *induced* technological progress is present as well.

Expression (1) indicates that the objective is to minimize the discounted sum of abatement costs and expenditure on R & D into the infinite future. Expression (2) states that the change in the CO_2 concentration is equal to the contribution from current emissions $(E_t^0 - A_t)$ net of natural removal (δS_t).[7]

Expression (3) describes the evolution of the knowledge stock (H_t), that is, the process of technological change. In the case where $k > 0$, the planner will choose an optimal profile for investments in R & D consistent with meeting the concentration target at minimum cost. These R & D investments (I_t) serve to increase the stock of knowledge (H_t) through the knowledge-accumulation function $(\Psi(\cdot))$. This profile of investment can be interpreted as the *additional* R & D investment that the optimal carbon tax would induce on the part of competitive firms. Thus, the $k > 0$ case is the induced technological change, or "ITC" case. We also consider the situation where $k = 0$ and there is no possibility of induced technological change because the connection between additional R & D investments and the stock of knowledge is severed. We call this the no-ITC or "NITC" case. In much of this paper, we will compare optimal abatement and carbon tax paths between the ITC and NITC cases. In addition to induced technological change, we also allow for *autonomous* technological change at the rate α_t: even if there were no climate policies in place, it seems reasonable to assume that some technological progress would still occur. There may be nonclimate reasons for such progress, such as a desire on the part of firms to economize on costly fuel inputs.

Expression (4) shows that the target CO_2 concentration, $\bar{S}$, must be met by time T and maintained after that point in time. We assume $p(\cdot)$ is nondecreasing in I_t; that is, the average cost of R & D investment increases with the level of R & D. This captures in reduced form the idea that there is an increasing opportunity cost (to other sectors of the economy) of employing scientists and engineers to devise new abatement technologies.[8] We also assume that the knowledge-accumulation function $\Psi(\cdot)$ has the following properties: $\Psi(\cdot) > 0$, $\Psi_I(\cdot) > 0$, and $\Psi_{II}(\cdot) < 0$.

The current-value Hamiltonian associated with the optimization problem for $t < T$ is[9]

$$\mathscr{H}_t = -(C(A_t, H_t) + p(I_t)I_t) - \tau_t(-\delta S_t + E_t^0 - A_t) + \mu_t(\alpha_t + k\Psi(I_t H_t))$$

[7]For analytical convenience, we postulate a simple stock-flow relationship here. A more complicated equation of motion, such as the one introduced in the numerical simulations, would not alter the qualitative analytical results obtained here.

[8]This issue is discussed in greater detail by Goulder and Schneider [6].

[9]This Hamiltonian actually corresponds to the problem of maximizing the negative of costs.

where $-\tau_t$ and μ_t are the shadow values of S_t and H_t, respectively. For $t \geq T$, however, we must form the following Lagrangian:

$$\mathscr{L}_t = \mathscr{H}_t + \eta_t(\bar{S} - S_t).$$

From the maximum principle, we obtain a set of first-order conditions, assuming an interior solution, as well as costate equations, state equations, and transversality conditions. Two key equations are

$$C_A(\cdot) = \tau_t \tag{5}$$

and

$$\dot{\tau}_t = \begin{cases} (r+\delta)\tau_t & \text{for } t < T \\ (r+\delta)\tau_t - \eta_t & \text{for } t \geq T. \end{cases} \tag{6}$$

In this problem, $-\tau_t$ is the shadow value of a small additional amount of CO_2 at time t. This shadow value is negative, since CO_2 is a "bad" from the policymaker's perspective. Thus τ_t represents the (positive) shadow *cost* of CO_2 or, equivalently, the benefit from an incremental amount of abatement (a small reduction in the CO_2 concentration). In a decentralized competitive economy in which all other market failures have been corrected, the optimal carbon tax is τ_t, the shadow cost of CO_2. By Eq. (5), this is equal to the marginal abatement cost at the optimal level of abatement. Equation (5) states that abatement should be pursued to the point at which marginal cost equals marginal benefit, while Eq. (6) states that the optimal carbon tax grows at the rate $(r + \delta)$ (at least for points in time up until T).[10] The two equations together imply that in an optimal program, the discounted marginal costs of abatement must be equal at all points in time (up to T), where the appropriate discount rate is $(r + \delta)$.[11] In the Appendix we demonstrate that this corresponds to an optimal abatement profile that slopes upward over time (whether or not there is induced technological change) so long as baseline emissions are not declining "too rapidly."

2.1.2. *Implications of ITC*

We now examine the effect of ITC on abatement costs and on the optimal carbon tax and abatement profiles. We do this by considering the significance of a change in the parameter k. As mentioned above, the case of $k = 0$ corresponds to a scenario with no induced technological change (the NITC scenario), while positive values of k imply the presence of induced technological change (the ITC

[10] After T, matters are complicated by the η_t term in Eq. (6).

[11] The appropriate discount rate is not simply r. Consider an arbitrary path of emissions leading to a given concentration S_T at the time T. Since CO_2 is removed naturally, altering this path by increasing emissions slightly at time t and reducing emissions slightly at a later time t' leads to greater overall removal and thus leads to a CO_2 concentration at time T that is less than S_T. Equivalently (as seen in the sensitivity analysis in Section 3), S_T can be achieved with less cumulative emissions abatement if the path of abatement is oriented more toward the future. Hence there is a value to postponing abatement beyond that implied by interest rate, r; this additional value is captured in the appearance of δ in the discount rate.

scenario). Our analysis will focus on incremental increases in k from the point $k = 0$.[12]

If (as is assumed) $C_H(\cdot) < 0$, then additional knowledge is clearly valuable (i.e., the multiplier μ is positive). When $k = 0$, all of the growth in knowledge is due to the autonomous term, and knowledge grows at the rate α_t. In contrast, for strictly positive values of k, the planner will find it optimal for society to accumulate at least some additional knowledge, assuming an interior solution.[13] This additional knowledge causes a decrease[14] in optimized costs to a degree dictated by μ_t. Thus, as would be expected, the introduction of the ITC option lowers the costs of achieving the given concentration target.

Next we examine the impact of introducing ITC on the optimal time profiles of abatement and carbon taxes. Differentiating Eq. (5) with respect to k and rearranging, we obtain

$$\frac{dA_t}{dk} = \frac{d\tau_t/dk - C_{AH}(\cdot)\, dH_t/dk}{C_{AA}(\cdot)}. \tag{7}$$

For the moment, assume that the first term in the numerator is zero, i.e., that ITC has no impact on the shadow cost of CO_2. Under this assumption, we are left only with what we shall refer to as the *knowledge-growth effect*: to the extent that knowledge has increased as a result of ITC ($dH_t/dk > 0$) and has thus reduced marginal abatement costs ($-C_{AH}(\cdot) > 0$), abatement tends to rise.[15]

The knowledge-growth effect is represented in Fig. 1 by the upward pivot of the abatement profile from the initial path 1 to path 2. At time 0 path 2 coincides with the initial path because knowledge is initially fixed at H_0: there can be no knowledge-growth effect at time 0.[16] The distance between paths 1 and 2 grows over time, representing the fact that the knowledge-growth effect becomes larger over time. This follows from the fact that there is no depreciation of knowledge in our model: whatever additional knowledge was induced by ITC at time t remains at time $t' > t$, and there might have been a further increment to knowledge at this later time.

Note that path 2 involves more abatement in every period than does the first path. Given that the same $\bar{S}$ constraint holds and that the initial path satisfied this constraint, path 2 clearly cannot be optimal. Path 2 was obtained under the assumption that the introduction of ITC had no impact on the shadow cost of CO_2. In fact, however (as shown in the Appendix), under the maintained assumption that

[12] The focus here on differential changes does not limit the generality of the analysis. Our analytical results are independent of the initial value of k. Given the smooth nature of our problem, results that hold for small changes in k around any initial value will carry over qualitatively for large changes around the point 0. This is confirmed in the numerical simulations.

[13] A corner solution arises if even the first increment of knowledge has marginal returns smaller than marginal costs. In this case, the social planner does not invest in additional knowledge; even here, though, we know that knowledge at least will not decrease from the baseline path.

[14] Throughout, when we use the words "increase" and "decrease" we will mean nonstrict increases and decreases, thus including the possibility that the variable stays constant.

[15] Note that the denominator of Eq. (7) is positive by assumption.

[16] We are stating that $dH_0/dk = 0$. This simply expresses the notion that the initial value for H_t (i.e., H_0) is not affected by different values for k. It remains true, however, that dH_t/dt is positive at all points in time. Even at time 0, the time-derivative of H_t is positive as a result of autonomous knowledge growth and induced investment in knowledge.

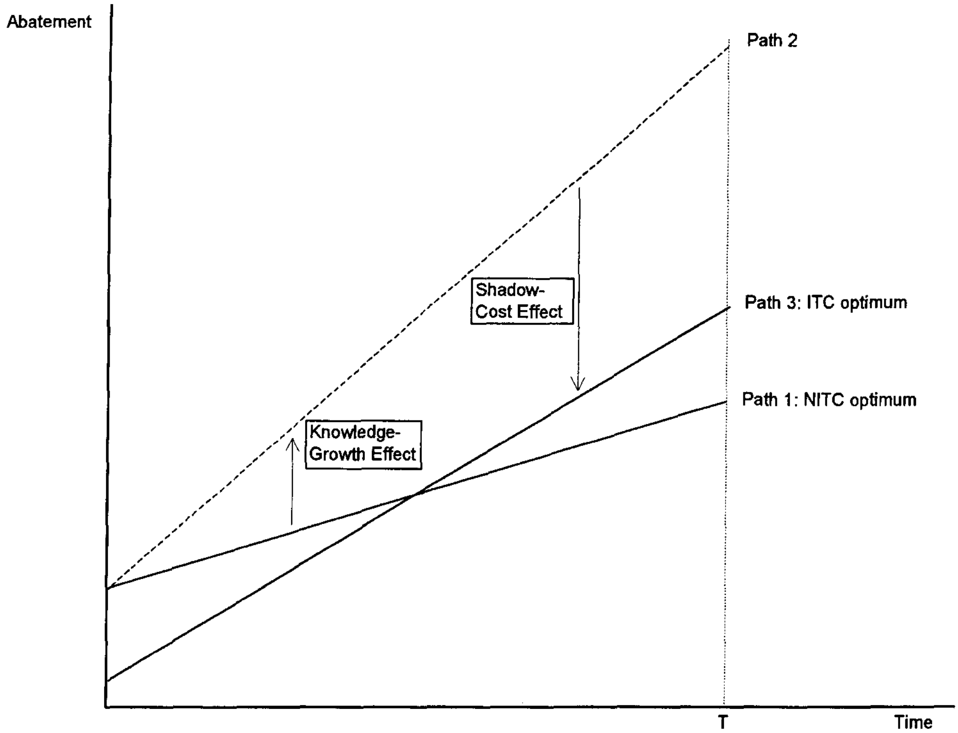

FIG. 1. Knowledge-growth and shadow-cost effects (drawn for CE_R model).

$C_{AH}(\cdot) < 0$, the shadow cost of CO_2 at all points in time decreases in magnitude in the presence of ITC: $d\tau_t/dk \leq 0 \; \forall t$. The basic explanation for this *shadow-cost effect* is as follows. If we are armed with the potential to develop new technologies rapidly through ITC, the prospect of being given an additional amount of CO_2 at time t and still being expected to meet the $\bar{S}$ constraint by time T is less worrisome than it would be if we had only autonomously advancing abatement technologies at our disposal.[17] Note that since the optimal carbon tax is the shadow cost of CO_2, it follows that the presence of ITC lowers carbon taxes.

This result contradicts the notion that the induced-innovation benefit from carbon taxes justifies a higher carbon tax rate. Figure 2 demonstrates our result heuristically by offering a static representation of this dynamic problem.[18] Cost-effective abatement (depicted in the upper panel) is achieved by a carbon tax set equal to the marginal abatement cost (MC) at the desired level of abatement. Technological progress causes the MC curve to pivot down, thus implying a lower optimal tax: it now takes a lower tax to yield the same amount of abatement. Note that this result depends on the assumption that *marginal* abatement costs are lowered by technological progress; i.e., $C_{AH} < 0$. It is possible to conceive of new

[17]The decline in the shadow cost reflects the maximum potential of the ITC option over the entire time horizon; i.e., the fall in the shadow cost corresponds to optimal R & D, as is sensible in this model of an optimizing planner.

[18]The figure is not meant to represent a single year in the program, but rather an independent one-period analogue to our abatement problem.

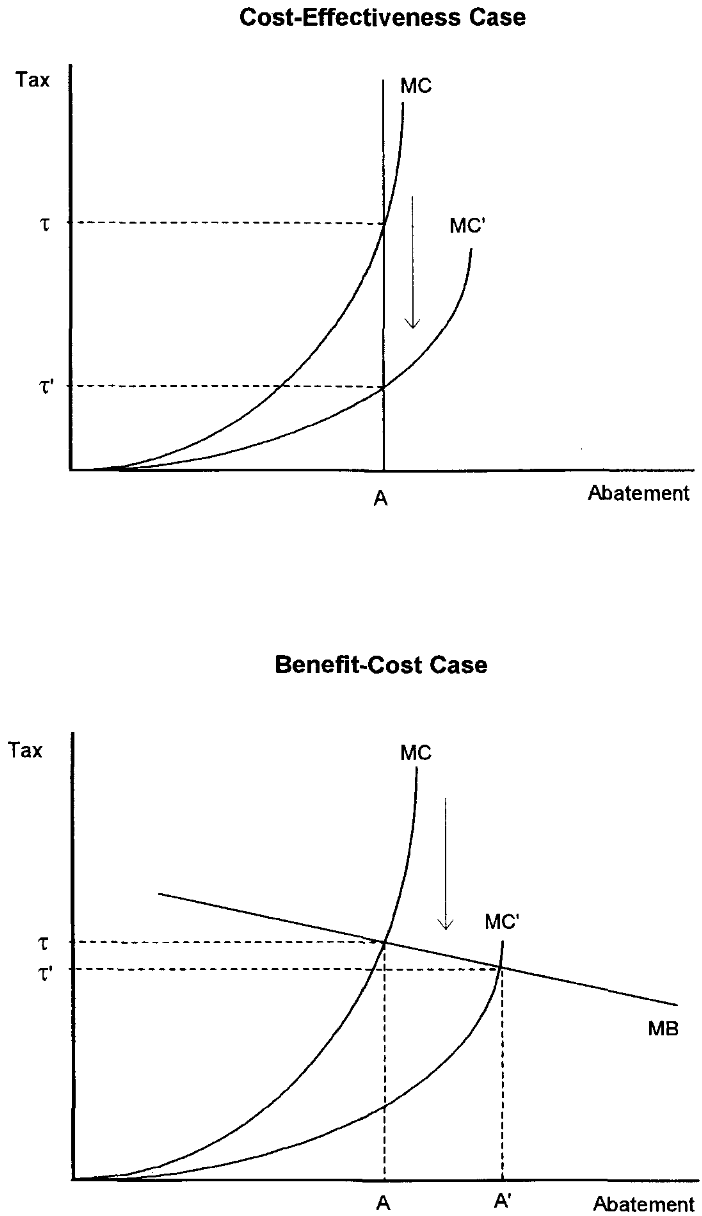

FIG. 2. Optimal climate policy in a static setting.

technologies that involve higher marginal abatement costs but that are nonetheless attractive because of lower fixed (and overall) abatement costs; however, this seems to be an unusual case.

Now we return to our analysis of the impact of ITC on abatement. The shadow-cost effect, reflected in the first term of the numerator in Eq. (7), shows up in Fig. 1 as the downward shift from path 2 to path 3. The shift is not parallel: as shown in the Appendix, tax rates at later points in time fall by greater absolute amounts than do early taxes, in such a way as to preserve the carbon tax growth rate at $(r + \delta)$. The downward shift is of a magnitude such that path 3 lies neither completely above nor completely below path 1: if it did, it would imply either overshooting or undershooting the constraint $\bar{S}$, which is likely to be suboptimal.[19] Together, the knowledge-growth and shadow-cost effects imply a new optimal abatement path that is steeper than the initial one: abatement is postponed from the present into the future.[20] Intuitively, ITC reduces the cost of future abatement relative to current abatement and thus makes postponing (some) current abatement more attractive. Thus, in a cost-effectiveness setting with R & D-based technological change, our analysis supports the claim of Wigley *et al.* [31] that future technological developments justify a more gradual approach to abatement.

At any given time t, we cannot be sure whether abatement rises or falls—this depends on whether the knowledge-growth effect or the shadow-cost effect dominates at that particular moment. But we can say something definite about abatement at time 0. Because knowledge is initially fixed at H_0, only the shadow-cost effect comes into play at time 0:

$$\frac{dA_0}{dk} = \frac{d\tau_0/dk}{C_{AA}(\cdot)} \le 0. \tag{8}$$

Thus, *initial* abatement weakly declines as a result of ITC.

These results depend on our assumption that $C_{AH}(\cdot) < 0$—that knowledge lowers marginal abatement costs. However, the possibility that $C_{AH}(\cdot) > 0$ cannot be ruled out. In this case (which we find somewhat implausible), ITC raises marginal costs, but presumably lowers total costs through greatly reduced sunk costs. Under these circumstances, the shadow-cost effect is positive and the presence of ITC raises the optimal carbon tax. The net effect of an increase in k on abatement at any arbitrary time t is (again) ambiguous, but initial abatement unambiguously rises.

2.1.3. *Summary*

Our results to this point are as follows. First, the solution to the cost-minimization problem (for any value of k) involves carbon taxes that rise over time at the rate $(r + \delta)$ for $t < T$, and that grow more slowly, and perhaps even decline, afterward. Second, the optimal abatement profile is upward sloping for $t < T$, as long as baseline emissions are not too steeply declining. Finally, assuming that ITC reduces marginal (and total) abatement costs, opening the ITC option causes optimized costs to fall, makes the entire carbon tax path fall (and by an equal proportion at all t), and causes initial abatement to fall and later abatement to rise. Table I summarizes the results regarding the implications of ITC.

[19] If baseline emissions were to rise sharply after T, then given the convexity of the abatement cost function, it might be optimal to more than meet the $\bar{S}$ requirement at time T to reduce the amount of abatement required afterward. However, it is the case nevertheless that one curve cannot lie above or below another over the entire infinite horizon: perpetual over- or undershooting of the constraint cannot be optimal.

[20] In characterizing the path as "steeper" we do not mean that the slope of the new path is everywhere greater than that of the old path. In fact, in the numerical simulations we will see that this is often not the case. We simply mean that, loosely speaking, less abatement is undertaken early on, and more later on.

TABLE I
Summary of Analytical Results

Policy criterion	Channel for technological change	Impacts of induced technological change on optimal solution: Tax path[a]	Abatement path
Cost-effectiveness	R & D	Falls by an equal proportion at all t	A_0 falls and later A rises; "steepening" of path
	LBD	Falls by an equal proportion at all t	Ambiguous effect on A_0 and on slope of path
Benefit–cost	R & D	Falls	A_0 falls; cumulative abatement rises; "steepening" of path
	LBD	Falls	Ambiguous effect on A_0; cumulative abatement increases

[a]Assuming damages are a convex function of the CO_2 concentration.

2.2. *Technological Change via Learning by Doing*

2.2.1. *The Problem and Basic Characteristics of the Solution*

Here we analyze a variant of the model presented above; now abatement itself yields improvements in technology. This is the "CE_L" model, where the "L" refers to learning by doing. The optimization problem is now

$$\min_{A_t} \int_0^\infty C(A_t, H_t) e^{-rt}\, dt$$

$$\begin{aligned} \text{s.t.} \quad & \dot{S}_t = -\delta S_t + E_t^0 - A_t \\ & \dot{H}_t = \alpha_t H_t + k\Psi(A_t, H_t) \\ & S_0, H_0 \text{ given} \\ & \text{and} \quad S_t \le \bar{S} \quad \forall t \ge T. \end{aligned}$$

This problem is virtually the same as the CE_R model of the previous section, except for a change in the $\Psi(\cdot)$ function: now induced knowledge growth is a function of the current level of abatement rather than R & D investment. Equivalently, current knowledge depends on cumulative abatement, which is regarded as a measure of experience. The first-order condition for abatement is now given by

$$C_A(\cdot) - \mu_t k \Psi_A(\cdot) = \tau_t. \tag{9}$$

Equation (9) states that the marginal benefit of abatement (τ_t, the value of the implied reduction in the CO_2 concentration) should equal the gross marginal cost of abatement ($C_A(\cdot)$) adjusted for the cost-reduction associated with the learning by doing stemming from that abatement ($\mu_t k \Psi_A(\cdot)$).

As in the CE_R model, the optimal carbon tax here is equal to τ_t.[21] Since the costate equation for τ_t is unchanged from before, we can refer to earlier results and conclude that the carbon tax grows at the rate $(r + \delta)$ for $t < T$, and that it grows more slowly, and perhaps declines, thereafter.

Although the CE_R and CE_L models are similar as regards the carbon tax path, they differ with respect to the characteristics of the optimal abatement path. In particular, it is no longer unambiguously true that the abatement path is positively sloped for $t < T$, even in the case in which baseline emissions are growing over time. This is demonstrated in the Appendix; the basic reason is that the cost-reduction due to learning by doing does not necessarily grow with time.[22]

2.2.2. *Implications of ITC*

Now consider what happens to the optimal tax and abatement paths when we introduce ITC, i.e., increase k from the point $k = 0$. As before, our assumption that $C_H(\cdot) < 0$ directly implies that the presence of ITC causes optimized costs to fall. Perhaps more substantively, under the assumption that $C_{AH}(\cdot) < 0$, we again find that the presence of ITC causes the shadow cost of the CO_2 concentration, and thus the optimal carbon tax, to decline (and increasingly so for higher t).

To analyze the impact of ITC on the abatement path, we differentiate Eq. (9) with respect to k. Evaluating this at $k = 0$ yields

$$\frac{dA_t}{dk} = \frac{d\tau_t/dk + \mu_t \Psi_A(\cdot) - C_{AH}(\cdot)\, dH_t/dk}{C_{AA}(\cdot)}. \tag{10}$$

As in the CE_R model, we observe the negative shadow-cost effect $(d\tau_t/dk)$ and the positive knowledge-growth effect $(-C_{AH}(\cdot)\, dH_t/dk)$. In our LBD specification, however, the presence of ITC has an additional, positive effect on abatement which we term the *learning-by-doing effect* $(\mu_t \Psi_A(\cdot))$. This effect reflects the fact that in the learning-by-doing specification, there is an additional marginal benefit (the learning) from abatement. Other things being equal, this further marginal benefit justifies additional abatement. Thus, under this specification the presence of ITC has three effects on abatement, one negative (the shadow-cost effect), and two positive (the knowledge-growth and learning-by-doing effects).[23] The net effect is ambiguous. Even at time 0, when the knowledge-growth effect does not come into play, we are still left with the opposing shadow-cost and learning-by-doing effects:

$$\frac{dA_o}{dk} = \frac{d\tau_o/dk + \mu_0 \Psi_A(\cdot)}{C_{AA}(\cdot)}.$$

[21] We have assumed no spillovers in the model (or at least none that have not been fully addressed by other government policies); the cost-reduction from learning by doing is fully appropriated by agents.

[22] In an NITC scenario, the abatement path will unambiguously slope upward for $t < T$, given that baseline emissions do not decline too rapidly. See the Appendix for details.

[23] Evaluating at an arbitrary nonzero initial value of k adds extra terms which are difficult to sign. Unlike in the R & D-specification, here we cannot be fully confident that our differential analysis around the point $k = 0$ carries over to the case of large increases in k from 0. However, the numerical simulations below indicate that the qualitative results obtained here carry through even for large changes in k.

Thus, in contract to the CE_R model, the presence of ITC no longer implies unambiguously that initial abatement will fall. If the learning-by-doing effect is strong enough, initial abatement rises. (This in fact happens in most of the numerical simulations presented in Section 4.)[24] These results offer partial support for Ha-Duong *et al.*'s [8] claim that because of learning by doing, ITC justifies higher initial abatement. Higher initial abatement may be justified, but this is not always the case.

2.2.3. *Summary*

We can summarize our results for the CE_L case as follows. The optimal carbon tax grows at the rate $(r + \delta)$ for $t < T$, but will grow more slowly, and perhaps even decline, after that. The slope of the optimal abatement path is of ambiguous sign throughout (unless we are in an NITC scenario, in which case abatement unambiguously rises over time, at least for $t < T$, if baseline emissions do not decline too rapidly). The presence of ITC lowers optimized costs and makes the entire carbon tax path fall by an equal proportion at all $t < T$. The impact on initial abatement is analytically ambiguous. These effects of ITC are noted in Table I.

3. OPTIMAL POLICY UNDER THE BENEFIT–COST CRITERION

We now analyze optimal tax and abatement profiles in a benefit–cost (BC) framework. No longer is there an exogenously given concentration target; rather the object is to minimize the sum of abatement costs, investment costs (in the R & D model), *and damages from CO_2* over an infinite horizon.

3.1. *Technological Change via R & D*

3.1.1. *The Problem and Basic Characteristics of the Solution*

In the R & D-based specification (hereafter referred to as the BC_R model), the problem is

$$\min_{A_t, I_t} \int_0^{\infty} (C(A_t, H_t) + p(I_t) I_t + D(S_t)) e^{-rt}\, dt$$

$$\text{s.t.} \qquad \dot{S}_t = -\delta S_t + E_t^0 - A_t$$

$$\dot{H}_t = \alpha_t H_t + k\Psi(I_t, H_t)$$

$$\text{and} \qquad S_0, H_0 \text{ given,}$$

where $D(S_t)$ is the damage function, assumed to have the following properties: $D'(\cdot) > 0$ and $D''(\cdot) > 0$. This is not completely uncontroversial. Although most would accept that damages are a convex function of climate change, it is also widely felt—see, e.g., Dickinson and Cicerone [1]—that climate change forcing is a

[24] The learning-by-doing effect can be quite large, as abatement, by increasing the stock of knowledge, lowers the cost of future abatement over the entire remaining time horizon.

concave function of changes in the atmospheric CO_2 concentration. Thus our $D(\cdot)$ function—relating damages to concentrations—could be concave. The shape of the damage function is critical in predicting the impacts of ITC.

The current-value Hamiltonian associated with the optimization problem is

$$\mathscr{H}_t = -(C(A_t, H_t) + p(I_t)I_t + D(S_t)) - \tau_t\big(-\delta\dot{S}_t + E_t^0 - A_t\big) + \mu_t(\alpha_t + k\Psi(I_t, H_t)).$$

From the maximum principle, assuming an interior solution, we obtain a set of necessary conditions, of which the most important to us are

$$C_A(\cdot) = \tau_t \tag{11}$$

and

$$\dot{\tau}_t = (r+\delta)\tau_t - D'(\cdot). \tag{12}$$

As before, $-\tau_t$ is the negative shadow value of a small additional amount of CO_2. Hence τ_t again represents the marginal benefit of abatement. Equation (11) thus states that abatement should be pursued up to the point at which marginal cost equals marginal benefit. Equation (12) can be integrated, using the relevant transversality condition as a boundary condition, to obtain

$$\tau_t = \int_t^{\infty} D'(S_s)e^{-(r+\delta)(s-t)}\,ds. \tag{13}$$

Equation (13) states that the shadow cost of an increment to the CO_2 concentration equals the discounted sum of marginal damages that this increment would inflict over all future time. Alternatively, the marginal benefit from incremental CO_2 abatement equals the discounted sum of the avoided damages attributable to such abatement.

As in the CE_R model, the optimal carbon tax is equal to τ_t, and thus, by Eq. (11), to the marginal abatement cost at the optimum. Using Eq. (13), we demonstrate in the Appendix that in the BC_R model, the optimal carbon tax may either rise or fall over time. This contrasts with the results from the cost-effectiveness models, in which the optimal carbon tax rose at the rate $(r+\delta)$(at least for $t < T$). The reason for the ambiguity is that although there is a tendency for the BC_R shadow cost to grow at the rate $(r+\delta)$, there is also a tendency for it to decline over time because an extra amount of CO_2 later on would inflict marginal damages over a shorter time horizon. The Appendix shows that given the convex damage function which we think reasonable, a sufficient condition ensuring that the tax path slopes upward is that the optimized path of CO_2 also slopes upward.

Given rising taxes and a baseline emissions path that rises (or at least does not fall too rapidly), we can also demonstrate that optimal abatement rises; otherwise, the slope of the abatement path is ambiguous. (See the Appendix for details.)

3.1.2. *Implications of ITC*

As before, the presence of ITC leads to lower optimized total costs (where these now include CO_2-related damages as well as abatement and investment costs). Just as before (and as proven in the Appendix), if we assume that knowledge reduces the marginal costs of abatement, the shadow-cost of CO_2 declines in the presence of ITC: $d\tau_t/dk \leq 0$. The intuition is similar to what it was in both the CE_R and CE_L models. Technological progress makes marginal abatement cheaper. Thus, when R & D investments are capable of yielding advanced technologies ($k > 0$), the prospect of being given an additional amount of CO_2 is less worrisome than it would be if we knew only more primitive abatement technologies would be available ($k = 0$). Since the optimal carbon tax is the shadow cost of CO_2, the presence of ITC lowers carbon taxes (the shadow-cost effect).[25] In this benefit–cost setting, we can also appeal to another piece of intuition. When ITC gives us the prospect of having more advanced technologies at our disposal, it makes sense that we would aim for more ambitious CO_2 concentration targets. Given a convex damage function, this would imply that marginal damages would be lower in the ITC world, and thus, by Eq. (13), optimal carbon taxes would be lower as well.

The result that ITC lowers optimal carbon taxes is perhaps surprising. Earlier, in a cost-effectiveness setting, we dismissed the claim that the presence of ITC should increase optimal taxes by appealing to a simple static graph; this graph showed that with ITC, it took a lower tax to achieve the same required level of abatement. But one might still have expected that in the broader, benefit–cost setting, if technology progressed sufficiently, it would make sense to *increase* the amount of abatement, and thus the optimal tax would increase.

The lower panel of Fig. 2 heuristically indicates that this notion is incorrect, at least under the assumption that the damage function is convex in the CO_2 concentration. The optimal amount of abatement and the optimal carbon tax are given by the intersection of the upward sloping MC curve and the downward sloping marginal abatement benefit (MB) curve.[26] If the MC curve were to pivot downward as a result of technological progress, the optimal amount of abatement would increase, but the optimal carbon tax would fall because we move to a lower point on the marginal benefit (marginal damage) curve.

If the damage function were linear, implying a flat marginal damage schedule, then the MC pivot would increase the optimal amount of abatement while leaving the optimal carbon tax unchanged. On the other hand, if damages were concave in the CO_2 concentration, then the MB curve would be upward sloping, and it is possible to envision a scenario in which a technology-driven fall in the MC schedule could actually increase the optimal carbon tax.[27]

Next we examine the implications of increasing k. Using the same approach as in the CE_R model, we obtain

$$\frac{dA_t}{dk} = \frac{d\tau_t/dk - C_{AH}(\cdot)\, dH_t/dk}{C_{AA}(\cdot)}. \tag{14}$$

[25] Unlike in the cost-effectiveness models, however, it is not necessarily true that taxes later on fall by greater amounts than do early taxes. See the Appendix.

[26] This MB curve conveys the same information as the schedule of marginal damages from additions to the stock of CO_2.

[27] See Repetto [26] for a discussion of nonconvex damages. Also note that, as before, if technological progress were to raise the MC schedule, then even with convex damages, the optimal carbon tax would rise (and the optimal scale of abatement would fall). This is confirmed in the Appendix.

Once again, the impact of ITC on abatement at time t is ambiguous because the shadow-cost effect and the knowledge-growth effect oppose one another. At time 0, however, the stock of knowledge is fixed at H_0, and thus only the shadow-cost effect comes into play:

$$\frac{dA_0}{dk} = \frac{d\tau_0/dk}{C_{AA}(\cdot)} \leq 0. \tag{15}$$

Thus initial abatement declines as a result of ITC (although this result is reversed if $C_{AH}(\cdot) < 0$).

In the cost-effectiveness analyses, where we had a fixed terminal constraint, $\bar{S}$, we knew that over the entire time horizon, cumulative abatement would be approximately the same under both ITC and NITC scenarios.[28] This implied that the shadow-cost and knowledge-growth effects would approximately balance one another out over the entire horizon; in terms of Fig. 1, the area under path 1 would roughly approximate the area under path 3.

In the benefit–cost framework, however, this is not the case. As demonstrated in the Appendix, the overall scale of abatement over the entire infinite horizon increases; that is to say, the knowledge-growth effect dominates the shadow-cost effect on average. Since CO_2 inflicts environmental damages, it seems reasonable that in the presence of ITC, which makes emissions abatement cheaper, the optimal balance of benefits and costs of emissions abatement would be struck at a higher level of abatement (on average) than would be optimal in the NITC scenario. This result is perhaps not very surprising.[29] Perhaps a more unexpected result is that *initial* abatement still falls, no matter how "large" or powerful the ITC option. Equation (14) indicates that this occurs because there is no separate analytical term representing an upward shift of abatement at all points in time. Rather, the increased scale of abatement is reflected completely in the steepening of the abatement path resulting from the interaction between the knowledge-growth and shadow-cost effects.

3.1.3. *Summary*

We have obtained the following main results for the BC_R case. First, the optimal carbon tax may either rise or fall over time, but if concentrations of CO_2 are increasing through time, then (given a convex damage function) the optimal carbon tax rises as well. Optimal abatement may either rise or fall over time, but, as long as baseline emissions are not falling too rapidly over time, it will rise if the carbon tax is rising. Second, as summarized in Table I, introducing the ITC option lowers optimized net costs and causes the entire carbon tax path to fall. Initial

[28] We say "approximately" because natural removal implies that two abatement paths leading to $\bar{S}$ need not involve exactly the same cumulative abatement. In fact, as will be seen in the sensitivity analysis in Section 4, paths which concentrate relatively more abatement in the future need less cumulative abatement to reach the same $\bar{S}$ constraint because they take better advantage of natural removal than do more heavily "front-loaded" abatement paths.

[29] What may be surprising, however, is that the result depends on the convexity of the damage function. With concave damages and a marginal damage schedule steeper than marginal cost (unlikely, given that we have a stock pollutant), a downward pivot in the marginal cost schedule could lead to less abatement.

abatement also falls, but cumulative abatement over the entire horizon rises; hence ITC implies a "steeper" abatement path.

3.2. *Technological Change via Learning by Doing*

Finally, we examine an LBD specification in a benefit–cost framework (the BC_L model).

3.2.1. *The Problem and Basic Characteristics of the Solution*

The optimization problem is now

$$\min_{A_t} \int_0^{\infty} (C(A_t, H_t) + D(S_t))e^{-rt}\, dt$$

$$\text{s.t.} \quad \dot{S}_t = -\delta S_t + E_t^0 - A_t$$

$$\dot{H}_t = \alpha_t H_t + k\Psi(A_t, H_t)$$

$$\text{and} \quad S_0, H_0 \text{ given.}$$

Thus, CO_2-related damages are part of the minimand, and abatement effort contributes to the change in the knowledge stock. The optimality conditions are the same as in the BC_R model, with one major change: the first-order condition for abatement is now

$$C_A(\cdot) - \mu_t k\Psi_A(\cdot) = \tau_t, \tag{16}$$

which is just as it was in the CE_L model (Eq. (9)).

As in the BC_R model, the slope of the carbon tax path is ambiguous (though it will be positive if the optimized CO_2 concentration rises over time, given convex damages). Thus the slope of the abatement path is ambiguous as well.

3.2.2. *Implications of ITC*

As always, the presence of ITC lowers overall optimized costs as well as the profile of optimal carbon taxes (assuming $C_H(\cdot) < 0$ and $C_{AH}(\cdot) < 0$). The impact of ITC on abatement is given by[30]

$$\frac{dA_t}{dk} = \frac{d\tau_t/dk + \mu_t\Psi_A(\cdot) - C_{AH}(\cdot)\, dH_t/dk}{C_{AA}(\cdot)}.$$

As in the CE_L model, ITC has three effects on abatement: the negative shadow-cost effect ($d\tau_t/dk$), the positive learning-by-doing effect ($\mu_t\Psi_A(\cdot)$), and the positive knowledge-growth effect ($-C_{AH}(\cdot)\, dH_t/dk$). The net effect on abatement at an arbitrary point in time t (including $t = 0$) is clearly ambiguous. At

[30]As in the CE_L analysis, we restrict our attention to the neighborhood around $k = 0$.

$t = 0$, in particular, the knowledge-growth effect drops out, leaving the negative shadow-cost and positive learning-by-doing effects

$$\frac{dA_0}{dk} = \frac{d\tau_0/dk + \mu_0\Psi_A(\cdot)}{C_{AA}(\cdot)}$$

and we cannot even claim that *initial* abatement declines unambiguously.

Although the components of the analysis here are the same as in the corresponding cost-effectiveness case, their overall impact is different. In the CE_L model, since the overall scale of abatement was approximately the same in both ITC and NITC scenarios, all three effects roughly balanced out over the entire horizon. In contrast, in this benefit–cost case, cumulative abatement increases.[31] Thus, on average the learning-by-doing and knowledge-growth effects dominate the shadow-cost effect.

3.2.3. *Summary*

The key results are as follows. The slope of the optimal carbon tax path is ambiguous. However, if the optimized CO_2 concentration rises (given a convex damage function), the tax rises as well. These results are similar to those in the CE_L model. Moreover, the slope of the optimal abatement path is of ambiguous sign throughout (unless we are in an NITC world with rising taxes and baseline emissions that are not declining too rapidly). As noted in Table I, although introducing the ITC option makes overall costs and the entire carbon tax path fall, it could lead to an increase in initial abatement. Furthermore, cumulative abatement over the entire time horizon increases.

4. NUMERICAL SIMULATIONS

Here we perform numerical simulations to gauge the quantitative significance of our results. We postulate functional forms and parameter values and solve for optimal paths. We then conduct sensitivity analysis to assess the robustness of our results. The numerical simulations reinforce our analytical findings and also point up several striking empirical regularities, as discussed below. We begin this section by describing the choice of functional forms and the methods used to calibrate the various parameters of the model. We then present and discuss the numerical results.

4.1. *Functional Forms and Parameter Values*

The numerical model is solved at 10-year intervals, with the year 2000 as the initial year. Although the planner's time horizon is infinite, we actually simulate over 41 periods (400 years) and impose steady-state conditions in the last simulated

[31] See the Appendix for details.

period. This enables us to project forward the values of this last period and thereby determine benefits and costs into the infinite future.[32]

The CO_2 concentration in 2000 is taken to be 360 parts per million by volume (ppmv), following the projections of the Intergovernmental Panel on Climate Change (IPCC) [9]. Baseline emissions for the period 2000 to 2100 roughly follow the IPCC's IS92(a) central scenario. After that time, we adopt a hump-shaped profile that peaks at 26 gigatons of carbon (GtC) in 2125 and flattens out to 18 GtC by 2200.[33]

In the analytical section, we assumed for expositional clarity that CO_2 in the atmosphere is naturally "removed" at a constant exponential rate. In the numerical simulations, we adopt Nordhaus' [18] slightly more complex and realistic model, which applies short-term and long-term removal rates to the flow and "stock" of emissions, respectively.[34]

$$\dot{S}_t = \beta(E_t^0 - A_t) - \delta(S_t - PIL)$$
$$\text{where} \quad \beta = 0.64$$
$$\text{and} \quad \delta = 0.008.$$

Thus, only 64% of current emissions actually contribute to the augmentation of atmospheric CO_2, and the portion of the current CO_2 concentration in excess of the preindustrial level (PIL = 278 ppmv) is removed naturally at a rate of 0.8% per annum.

For our benefit–cost simulations, we need to specify a CO_2 damage function. We assume this function to be quadratic and, following Nordhaus [18], who reviewed damage estimates from a number of studies, calibrate the remaining scale parameter so that a doubling of the atmospheric CO_2 concentration implies a loss of 1.33% of world output each year. Thus we have

$$D(S_t) = M_D S_t^{\alpha_D}$$
$$\text{where} \quad M_D = 0.0012$$
$$\text{and} \quad \alpha_D = 2.$$

[32] Specifically, we impose the requirement that the CO_2 concentration remain constant after the last period. For this to occur, abatement must also remain constant (given that baseline emissions are constant at that point in time). In the R & D simulations, we also impose the steady-state constraint that investment go to zero. Even when these constraints are imposed, our model does not yield a steady state with constant abatement costs because the stock of knowledge continues to grow; this continued growth is due both to autonomous technological change and (in the LBD simulations) to the increments to knowledge stemming from continued experience with abatement. In solving the numerical model, we assume that abatement costs beyond the last simulation period are constant, even though the analytical structure of the model implies that abatement costs would fall as knowledge continued to accumulate. Thus our approach overestimates the true future costs. We have verified that this inconsistency has no numerical significance by comparing numerical results under this approach with those from simulations that assume costs after year 2400 are zero and thus underestimate future costs. These two alternative specifications bound the truth about future costs. The numerical results under these two very different approaches are indistinguishable: discounting over a 400-year horizon makes the terminal conditions unimportant in practice.

[33] This profile is patterned after a scenario used by Manne and Richels [15].

[34] Some scholars endorse more sophisticated formulations, such as the five-box model of Maier-Reimer and Hasselmann [13].

The functional form assumed for the abatement-cost function is

$$C(A_t, H_t) = M_C \frac{A_t^{\alpha_{C1}}}{\left(E_t^0 - A_t\right)^{\alpha_{C2}}} \frac{1}{H_t}.$$

This form has the properties assumed in the analytical model, including the feature that knowledge lowers marginal abatement costs ($C_{AH}(\cdot) < 0$). It also has the property that marginal costs tend to infinity as abatement approaches 100% of baseline emissions.[35] We choose the parameters M_C, α_{C1}, and α_{C2} to meet the requirements that (1) a 25% emissions reduction in 2020 should cost between 0.5 and 4% of global GDP[36] and (2) the present value (at a 5% discount rate) of global abatement costs for reaching $S_t = 550$ ppmv by 2200 (in an NITC world) should be roughly \$600 billion (Manne and Richels [15]). The parameter values that best meet these requirements are $M_C = 83$, $\alpha_{C1} = 3$, and $\alpha_{C2} = 2$, but calibration of the cost function remains an area of considerable uncertainty, and sensitivity analyses in this respect are particularly important.

Following estimates common in the literature,[37] we take the rate of autonomous technological progress to be 0.5% per annum: $\alpha_t = 0.005$. The knowledge accumulation function exhibits the properties discussed in the analytical section and is given, in the R & D simulations, by

$$\begin{aligned} \Psi(I_t, H_t) &= M_\Psi I_t^\gamma H_t^\phi \\ \text{where} \quad M_\Psi &= 0.0022 \\ \gamma &= 0.5 \\ \text{and} \quad \phi &= 0.5. \end{aligned}$$

H_0, the initial knowledge stock, is normalized to unity. In the learning-by-doing simulations, the knowledge accumulation function is the same, with A_t replacing I_t. The function we use is fairly standard in the endogenous growth literature.[38] γ is chosen to be 0.5 to indicate diminishing returns to R & D investment,[39] while ϕ, which dictates the intertemporal knowledge spillover, is set to 0.5, a central value of the range typically seen in the literature. As ϕ is positive, it indicates that knowledge accumulation today makes future accumulation easier. This is the "standing on shoulders" case which has been used, for example, by Nordhaus [19]. It contrasts with the case where $\phi < 0$, which implies a limited pool of ideas which are slowly "fished out"—current knowledge accumulation makes future accumulation more difficult. M_Ψ is calibrated so that the cost-savings from ITC are approximately 30% in the CE_R model. This is consistent with Manne and Richels

[35] There is no backstop technology in the model.

[36] These calculations are based on results of a literature review in EPRI [2] and are extrapolated to the global economy.

[37] See Manne and Richels [14, 15].

[38] See, for example, Romer [27], Jones [10], or Jones and Williams [11]. We are grateful to William Nordhaus and Chad Jones for recommending this function and alerting us to its usefulness.

[39] Jones and Williams [11] dub this the "stepping on toes effect," for "an increase in R & D effort induces duplication that reduces the average productivity of R & D."

[14], who compare the costs of carbon abatement under different assumptions about technological progress.[40]

We assume that the price of investment funds is

$$p(I_t) = I_t.$$

Thus the average cost of R & D investment increases with scale; as mentioned earlier, this captures the idea that drawing scientists away from R & D in other sectors involves increasing costs. Following Manne and Richels [15], we take the discount rate to be 5%.[41] Finally, we model the NITC cases by setting $k = 0$ and the ITC cases by setting $k = 1$.

4.2. *Central Cases*

4.2.1. *CE_R Simulation*

In the cost-effectiveness cases (CE_R and CE_L), the concentration target ($\bar{S}$) is 550 ppmv, which must be reached by 2200. This scenario has received considerable attention in policy discussions. We first consider results for the CE_R case, both with and without ITC. The upper-left panels of Figs. 3 and 4 depict, respectively, the optimal abatement and carbon tax paths in this case.

Abatement. As predicted by the analytical model, the optimal abatement paths slope upwards for most of the horizon until 2200, the year in which the constraint is first imposed.[42] Figure 3 shows that the presence of ITC leads to a slightly "steeper" abatement profile, with less abatement during the first 125 years and more abatement after that. However, the effect of ITC on abatement is almost imperceptible. The minuteness of this "abatement-timing effect" is noteworthy, particularly in light of the fact that ITC lowers the discounted average costs of abatement by 30%. The sensitivity analysis below will show that the weakness of ITC's abatement-timing effect is robust to different parameter specifications.

Carbon Tax. The upper-left panel of Fig. 4 shows that the optimal carbon tax starts at a few dollars per ton and grows exponentially. Although not evident from the figure alone, the tax grows at the rate $(r + \delta)$, just as predicted by the analytical model. While ITC's impact on abatement was extremely small, its effect

[40] In the work by Manne and Richels [14, p. 64], GDP costs of abatement policy are approximately 90% lower in an optimistic technology scenario than in the central-case technology scenario. This difference in GDP costs does not account for the costs of developing the improved technologies that distinguish the optimistic scenario from the central-case scenario. We assume that R & D investments have a social rate of return of 50% (as in Nordhaus [19]) and then calculate the net cost savings from technological progress to be roughly 30%. (The R & D costs that generate 0.90 of abatement-cost savings amount to $(1/1.5)0.90$. Thus, the net cost savings from technological progress is given by $0.90 - (1/1.5)0.90 = 0.30$). We assume that this figure is relevant to the induced technological change which we study in our paper, and we then choose M_Ψ to generate this level of savings.

[41] The discount rate represents, in this context, the marginal product of capital, rather than the pure rate of time preference.

[42] A slight decline begins around 2170, as baseline emissions are declining rapidly at this point; this is fully consonant with the analytical model. In both the ITC and NITC cases, the level of abatement drops discontinously in the year 2200 and stays constant thereafter, maintaining the CO_2 concentration at the level $\bar{S}$. The constraint on the year-2200 concentration forces this discontinuity.

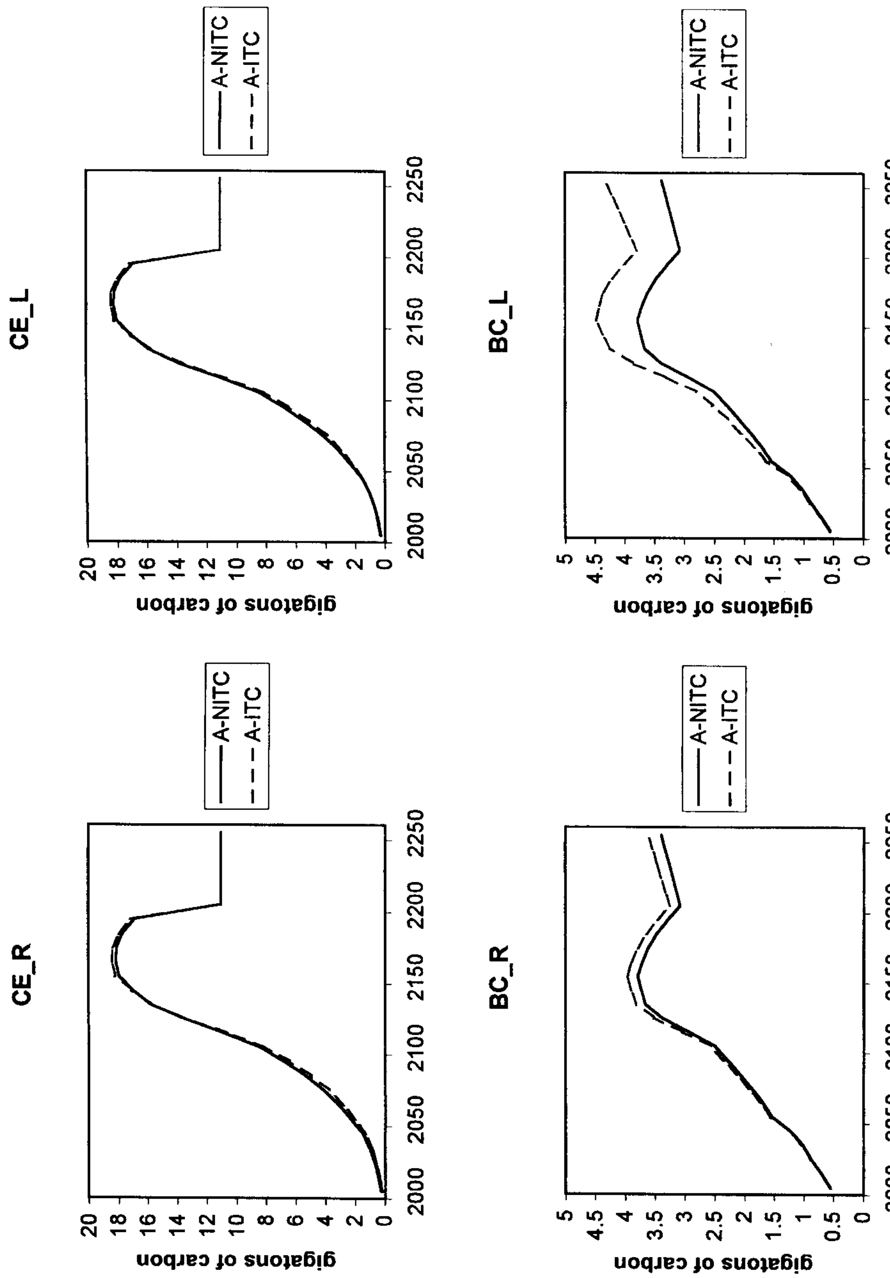

FIG. 3. Optimal abatement paths.

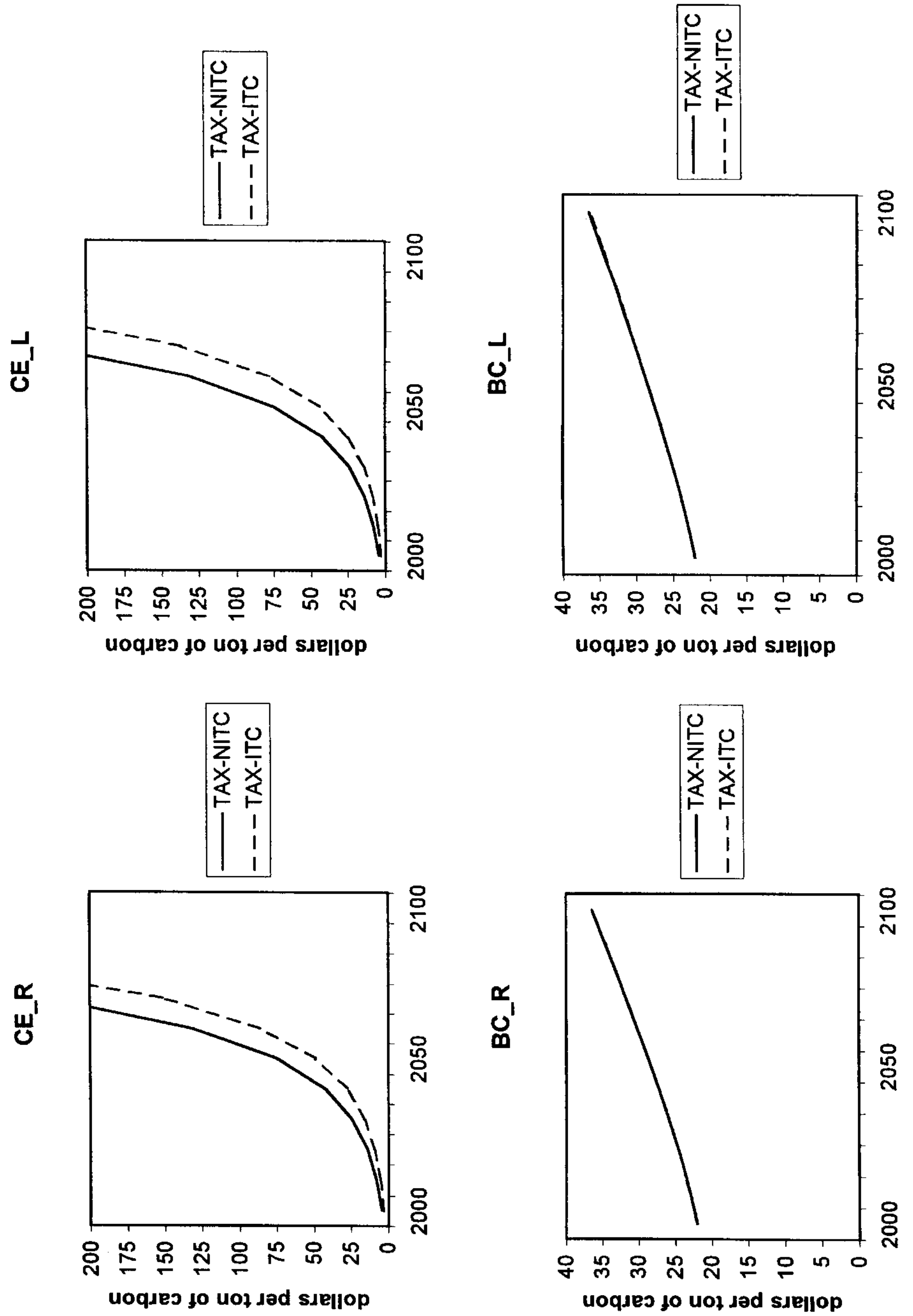

FIG. 4. Optimal carbon tax paths.

on the optimal tax is pronounced. The presence of ITC lowers the optimal carbon tax path at all points in time up to 2200 by about 35%, roughly in line with the 30% cost savings mentioned earlier.

4.2.2. *CE_L Simulation*

The upper-right panels of Figs. 3 and 4 depict the abatement and tax paths for the CE_L case. The results here are broadly similar to those in the CE_R case just discussed. Again the optimal abatement paths slope upward,[43] the optimal carbon tax rises at the rate $(r + \delta)$, and the presence of ITC causes a slight steepening of the abatement path and a sizable downward shift in the tax path. Here ITC implies a reduction in total costs of about 39% and a comparable (41%) lowering of the optimal carbon tax path.

Some differences between the CE_R and CE_L cases deserve mention. First, under learning by doing, the presence of ITC has an even smaller effect on optimal abatement timing than it does under R & D. This makes sense because the basic tendency toward postponing some abatement from the present to the future is offset in the CE_L case by the learning-by-doing effect, which prompts more abatement now in order to accumulate experience-based knowledge. (In fact, this learning-by-doing effect is large enough to cause initial abatement to rise in the CE_L simulation). A second difference is that ITC has a larger impact on taxes and costs in the CE_L case than it does in the CE_R case. This reflects the fact that under LBD-based ITC, technological progress comes about as a "free" by-product of abatement, rather than as a result of costly expenditures on R & D.

4.2.3. *BC_R Simulation*

We now turn to the benefit–cost cases. The lower-left panels of Figs. 3 and 4 depict the optimal abatement and tax paths in the BC_R model.

Abatement. The analytical model indicated that as long as taxes were rising and baseline emissions not declining "too rapidly," the abatement path would rise. In our simulations, abatement rises over the interval 2000–2150 and falls after that, largely matching the pattern of baseline emissions. As shown in Fig. 3, in the presence or absence of ITC, there is much less abatement here than in the CE cases (note the different scales used on the vertical axes). Correspondingly, the CO_2 concentration in 2200 that results from the optimal abatement path is above 800 ppmv, considerably higher than the 550 ppmv imposed in the CE simulations. These differences imply that the 550 ppmv target in the cost-effective analysis—a target given much attention in policy discussion—is too stringent from an efficiency point of view (given the parameters used here for the cost and damage functions). The presence of ITC implies a slight increase in the overall scale of abatement and steepening of the abatement path. Nonetheless, initial abatement falls (though only slightly). These outcomes all square with the predictions of the analytical model.

Carbon Tax. The lower left-panel of Fig. 4 shows that the optimal carbon tax profile is roughly linear in this simulation. This contrasts with the exponential shape in the CE simulations and conforms to the analysis of Section 3. Recall that

[43] Recall that the analytical model was unable to guarantee this result for the ITC scenario.

the shadow cost of the CO_2 concentration (i.e., the carbon tax) is given by the sum of marginal damages that a small additional amount of CO_2 would cause into the infinite future, discounted at the rate $(r + \delta)$. Although the shadow cost tends to rise at the rate $(r + \delta)$, this is offset by the fact that as time goes on, less time remains over which the incremental amount of CO_2 can inflict marginal damages. The combination of these two effects produces a linear carbon tax profile.

Again in striking contrast to the CE simulations, the impact of ITC on the optimal carbon tax path is virtually imperceptible in the BC_R central case. There are two reasons for the difference. First, as suggested by Fig. 2, the adjustment due to ITC is in both the quantity (that is, abatement) and price (tax) dimensions; in the CE cases, in contrast, adjustment can only occur in the price dimension because of the constraint on the terminal CO_2 concentration. In our central case, the marginal damage curve is very flat over the relevant range. As a result, nearly all of the adjustment to ITC in the BC cases comes via changes in the level of abatement.[44]

The second reason is more subtle and relates to the fact that the $\bar{S}$ constraint imposed in the cost-effectiveness scenarios is too stringent, as indicated by the fact that $\bar{S}$ is significantly lower than the optimal concentration for the year 2200 that emerges from the benefit–cost simulation. Consequently, levels of abatement are generally much higher in the cost-effectiveness cases, which implies that the potential gains from improved technology are also higher. Thus in the cost-effectiveness cases, it pays to accumulate more knowledge than it does in the benefit–cost settings. This implies that the downward pivot of the MC curve and associated impact on the carbon tax are larger in these cases.

Finally, the presence of ITC has an extremely small (2%) impact on average costs of abatement in the benefit–cost cases; as before, this contrasts with the result in the cost-effectiveness cases. Given that ITC in the benefit–cost cases has a small impact on the carbon tax, and in fact on the entire marginal abatement cost schedule, it should not be surprising that it has a correspondingly small impact on average costs. Similarly, the ITC-induced percentage increase in the net benefits of climate policy is very small.[45]

In a working paper circulated contemporaneously with early drafts of this paper, Nordhaus [19] independently obtained the result that, in a benefit–cost context, the presence of ITC has an imperceptible impact on the optimal carbon tax and on the net benefits from carbon abatement policy. Our BC results conform to Nordhaus', although, as discussed below, we obtain different results under alternative parameterizations.[46]

[44] In the central case, when the MC curve is relatively steep, the effect on abatement is not very large either.

[45] In other words, the net benefits of optimal abatement relative to a baseline of no abatement whatsoever, are scarcely bigger in an ITC scenario than they are in an NITC scenario.

[46] Nordhaus obtains his result in a dynamic optimization model in which technological change is driven by R & D expenditure. Some differences between the Nordhaus study and the present study are worth noting. His analysis explicitly models a production function, while ours represents production (or ease of substitution) in reduced form through the abatement-cost function. This enables us to obtain analytical results where Nordhaus relies solely on numerical simulations. Another difference is that the Nordhaus study considers only the BC_R case. In contrast, the present study considers both the benefit–cost and cost-effectiveness cases, and considers learning-by-doing- as well as R & D-based technological change. The present paper's attention to alternative policy specifications and knowledge-generation channels, along with the broad sensitivity analysis below, enable it to map out more broadly the conditions under which ITC has (or does not have) a significant impact on economic outcomes.

4.2.4. *BC_L Simulation*

Finally, we consider the BC_L case. The results for this case are displayed in the lower-right panels of Figs. 3 and 4. The effect of ITC on abatement (Fig. 3) is similar to the effect in the BC_R case, although the impact is somewhat more pronounced.[47] Initial abatement rises, indicating that the learning-by-doing effect outweighs the shadow-cost effect. Once again, the presence of ITC has a virtually imperceptible impact on the optimal carbon tax path, average costs per unit of abatement, and net benefits. The explanation is the same as that given in the BC_R case.

4.3. *Sensitivity Analysis*

Here we examine the sensitivity of the numerical results to changes in key parameters. For each of the four models, we examine six sets of variants of the central case. Table II presents summary statistics describing, in each variant, the percentage impact that ITC has on the abatement profile, the cumulative amount of abatement over the period 2000–2200, the terminal CO_2 concentration, the tax profile, and overall costs per unit of abatement. We report the impacts on abatement and taxes in the years 2000, 2050, and 2200 (or 2190).[48] For the benefit–cost cases, we also report the percentage impact of ITC on the net benefits of optimal climate policy relative to a zero-abatement baseline.

A higher discount rate (case 2a) reduces the importance of future benefits or costs relative to current ones. Since the costs of ITC are borne today, whereas the benefits are spread more uniformly through time, a higher discount rate tends to reduce the net benefits from ITC. This means that there will be less knowledge accumulation, which implies that the abatement-timing effect (the pivoting of the abatement path) is smaller. The reduced attractiveness of ITC implies, in the benefit–cost cases, that there will be a smaller impact on the overall scale of abatement as well. The opposite results hold under a lower discount rate (case 2b).

The next variant involves changes either in the constraint on year-2000 concentrations (in the cost-effectiveness cases) or in the parameters of the damage function (in the benefit–cost cases). In a cost-effectiveness setting, a tighter concentration constraint (case 3a) enforces greater overall abatement and therefore entails higher marginal costs of abatement. This confers higher value to ITC in terms of greater cost savings. The reverse applies when the constraint is more lax (case 3b).[49] In the benefit–cost simulations, case 3a imposes higher curvature on the damage function.[50] Thus the marginal damage function is steeper in the relevant range. As a result, ITC has a larger impact on the optimal carbon tax and there is less impact on quantity (abatement). Case 3b imposes a linear damage

[47]As when we compared the CE_R and CE_L models, this difference is due to the fact that ITC is "free" under an LBD specification but costly under R & D.

[48]In the cost-effectiveness simulations, we report these results for the year 2190; the year-2200 statistic is uninformative as both abatement and taxes in this first year of the constraint are identical across ITC and NITC runs.

[49]In Case 3b, the value 836.39 is chosen to match the optimized value of the year-2200 CO_2 concentration in the NITC scenario of the BC_R simulation.

[50]The multiplicative parameter in the damage function is recalibrated so that the cumulative amount of abatement in the NITC run remains constant.

TABLE II

Sensitivity Analysis (Percentage Changes, ITC Case Relative to NITC Case)

Policy Criterion: Cost-Effectiveness

	Abatement in year			Cumulative abatement	CO_2 concn. in 2200	Tax in year			Cost per unit of abatement
	2000	2050	2190			2000	2050	2190	
CE_R Model									
1. Central Case	−18.47	−12.39	1.46	−0.89	0.00	−34.83	−34.83	−34.83	−29.83
2a. $r = 0.075$	−14.92	−12.02	1.04	−0.60	0.00	−27.88	−27.88	−27.88	−24.14
2b. $r = 0.025$	−21.63	−11.18	2.09	−1.32	0.00	−44.49	−44.49	−44.49	−37.87
3a. $\bar{S} = 350$	−19.69	−4.16	0.52	−0.78	0.00	−57.14	−57.14	−57.14	−52.85
3b. $\bar{S} = 836.39$	−4.63	−3.98	0.97	−0.26	0.00	−9.06	−9.06	−9.06	−6.76
4a. $C = 12.14 * A^4/((E - A)^2 * H)$	−12.04	−8.35	1.52	−0.90	0.00	−34.74	−34.74	−34.74	−29.39
4b. $C = 105.7 * A^2/H$	−39.30	−31.89	0.00	−0.28	0.00	−39.30	−39.30	−39.30	−36.05
4c. $C = 438.4 * A^{1.5}/H$	0.00	−51.21	0.00	−0.06	0.00	−37.75	−37.75	−37.75	−36.19
5a. $M_\Psi = 0.0058$	−31.35	−21.16	2.31	−1.48	0.00	−54.43	−54.43	−54.43	−48.83
5b. $M_\Psi = 0.0015$	−9.74	−6.51	0.81	−0.47	0.00	−19.37	−19.37	−19.37	−15.97
6a. $\phi = 0.75$	−21.72	−15.27	2.00	−1.14	0.00	−40.14	−40.14	−40.14	−34.53
6a. $\phi = -0.5$	−10.96	−6.08	0.53	−0.39	0.00	−21.64	−21.64	−21.64	−18.20
7a. $\alpha_t = 0.01$	−12.71	−8.15	0.76	−0.50	0.00	−24.56	−24.56	−24.56	−20.43
7b. $\alpha_t = 0.00$	−25.60	−17.86	2.59	−1.47	0.00	−46.68	−46.68	−46.68	−41.28
CE_L Model									
1. Central Case	19.63	−6.01	1.32	−0.53	0.00	−41.15	−41.15	−41.15	−39.06
2a. $r = 0.075$	47.40	−2.00	0.86	−0.35	0.00	−36.36	−36.36	−36.36	−34.70
2b. $r = 0.025$	1.37	−7.25	2.06	−0.84	0.00	−46.25	−46.25	−46.25	−43.65
3a. $\bar{S} = 350$	−8.88	−2.48	0.31	−0.42	0.00	−42.44	−42.44	−42.44	−42.57
3b. $\bar{S} = 836.39$	79.70	15.92	1.77	−0.05	0.00	−24.88	−24.88	−24.88	−22.08
4a. $C = 12.14 * A^4/((E - A)^2 * H)$	0.34	−6.02	1.37	−0.65	0.00	−44.21	−44.21	−44.21	−42.40
4b. $C = 105.7 * A^2/H$	613.85	122.37	0.00	0.29	0.00	−27.10	−27.10	−27.10	−25.74
4c. $C = 438.4 * A^{1.5}/H$	15109.73	3003.29	0.00	0.18	0.00	−20.12	−20.12	−20.12	−18.73
5a. $M_\Psi = 0.0058$	18.04	−11.05	2.08	−0.95	0.00	−60.02	−60.02	−60.02	−57.79
5b. $M_\Psi = 0.0015$	16.30	−3.18	0.79	−0.29	0.00	−25.85	−25.85	−25.85	−24.26
6a. $\phi = 0.75$	18.24	−7.66	1.77	−0.71	0.00	−46.17	−46.17	−46.17	−43.95
6a. $\phi = -0.5$	24.25	−2.10	0.50	−0.16	0.00	−27.81	−27.81	−27.81	−26.05
7a. $\alpha_t = 0.01$	29.68	−3.07	0.83	−0.30	0.00	−33.94	−33.94	−33.94	−31.88
7b. $\alpha_t = 0.00$	9.11	−9.56	2.03	−0.88	0.00	−49.00	−49.00	−49.00	−46.96

TABLE II—(*Continued*)

Policy Criterion: Benefit–Cost
BC_R Model

	Abatement in year			Cumulative abatement	CO_2 concn. in 2200	Tax in year			Cost per unit of abatement	Net benefits
	2000	2050	2200			2000	2050	2200		
1. Central case	−0.00	1.59	5.77	3.73	−0.45	−0.01	−0.06	−0.52	−2.29	0.74
2a. $r = 0.075$	−0.00	1.04	4.08	2.57	−0.25	−0.00	−0.03	−0.27	−2.04	0.25
2b. $r = 0.025$	−0.03	3.01	9.63	6.41	−1.04	−0.07	−0.25	−1.24	−1.06	3.16
3a. $D = 1.14E^{-6} * S^3$	−0.01	1.42	5.43	3.56	−0.44	−0.02	−0.11	−1.02	−2.01	0.84
3b. $D = 1.64 * S$	0.00	1.80	6.14	3.92	−0.46	0.00	0.00	0.00	−2.55	0.67
3c. $D = 85.3 * S^{0.5}$	0.00	1.93	6.35	4.03	−0.47	0.00	0.04	0.26	−2.67	0.65
4a. $C = 47.62 * A^4/((E - A)^2 * H)$	−0.00	1.06	3.90	2.39	−0.28	−0.01	−0.05	−0.33	−1.43	0.41
4b. $C = 2.441 * A^2/H$	−0.02	4.29	17.58	11.00	−1.39	−0.02	−0.16	−1.89	−7.59	1.62
4c. $C = 5.608 * A^{1.5}/H$	−0.04	7.90	34.46	25.71	−3.52	−0.02	−0.17	−5.15	−15.35	4.59
5a. $M_\Psi = 0.0058$	−0.01	3.97	13.97	9.15	−1.10	−0.02	−0.15	−1.26	−5.32	1.83
5b. $M_\Psi = 0.0015$	−0.00	0.66	2.43	1.57	−0.19	−0.00	−0.03	−0.22	−0.99	0.31
6a. $\phi = 0.75$	−0.00	1.66	6.94	4.26	−0.52	−0.01	−0.07	−0.61	−2.71	0.78
6a. $\phi = -0.5$	−0.00	1.37	3.15	2.41	−0.28	−0.01	−0.05	−0.31	−1.25	0.61
7a. $\alpha_t = 0.01$	−0.00	1.42	3.75	2.83	−0.44	−0.01	−0.06	−0.51	−1.56	0.68
7b. $\alpha_t = 0.00$	−0.00	1.79	8.74	4.82	−0.45	−0.01	−0.06	−0.51	−3.19	0.79

BC_L Model

	Abatement 2000	Abatement 2050	Abatement 2200	Cumulative abatement	CO_2 concn. in 2200	Tax 2000	Tax 2050	Tax 2200	Cost per unit of abatement	Net benefits
1. Central case	0.47	5.61	23.64	14.45	−1.76	−0.03	−0.22	−2.06	−8.98	3.23
2a. $r = 0.075$	0.22	5.00	22.54	13.64	−1.34	−0.01	−0.13	−1.53	−10.21	1.50
2b. $r = 0.025$	1.69	6.91	24.67	15.57	−2.54	−0.19	−0.61	−3.12	−5.03	8.95
3a. $D = 1.114E^{-6} * S^3$	0.57	5.22	21.80	13.73	−1.69	−0.06	−0.41	−3.95	−7.95	3.78
3b. $D = 1.64 * S$	0.41	6.03	25.65	15.14	−1.81	0.00	0.00	−0.00	−9.91	2.80
3c. $D = 85.3 * S^{0.5}$	0.38	6.26	26.74	15.48	−1.84	0.02	0.13	1.07	−10.32	2.63
4a. $C = 47.62 * A/((E - A)^2 * H)$	0.23	4.38	17.41	10.35	−1.23	−0.03	−0.20	−1.44	−6.78	2.15
4b. $C = 2.441 * A^2/H$	1.55	15.47	74.51	43.18	−5.53	−0.10	−0.59	−7.75	−23.15	15.02
4c. $C = 5.608 * A^{1.5}/H$	3.86	23.16	141.78	93.45	−13.03	−0.07	−0.52	−18-99	−37.16	15.02
5a. $M_\Psi = 0.0058$	0.93	11.17	45.82	28.43	−3.45	−0.06	−0.45	−4.02	-15.74	6.43
5b. $M_\Psi = 0.0015$	0.25	2.91	12.39	7.52	−0.92	−0.02	−0.12	−1.07	−4.98	1.67
6a. $\phi = 0.75$	0.48	5.87	29.17	16.74	−2.06	−0.03	−0.24	−2.46	−10.55	3.42
6a. $\phi = -0.5$	0.45	4.76	12.63	9.16	−1.08	−0.03	−0.18	−1.21	−5.24	2.67
7a. $\alpha_t = 0.01$	0.48	5.08	16.66	11.66	−1.85	−0.03	−0.23	−2.16	−6.99	3.07
7b. $\alpha_t = 0.00$	0.46	6.18	32.65	17.44	−1.62	−0.03	−0.22	−1.88	−11.05	3.37

function, so that the marginal damage schedule is perfectly flat. In this case, there is no impact on the optimal carbon tax profile. All of the adjustment occurs in quantity (abatement). Even in this case, however, the effect on the abatement levels is quite small because the marginal cost curve is quite steep. Finally, in case 3c (applicable only in the BC simulations), we introduce a concave damage function, so that the marginal damage curve is upward sloping in abatement (but still flatter than the MC curve). As expected, taxes *rise* in this case.

In cases 4a, 4b, and 4c we alter the curvature of the cost function such that the *marginal* cost curve is, respectively, more convex than in the central case, strictly linear (less convex), and concave (much less convex). For the CE models, M_C in cases 4a, 4b, and 4c is calibrated such that the optimal tax path in the NITC world coincides with what it was in the central case.[51] For the BC models, M_C is calibrated such that the total amount of NITC abatement over the period 2000–2200 stays constant. Changes in the curvature of the cost function are most important to the results of the BC simulations. In case 4a, the marginal cost function is convex and steep in the relevant range. As a result, the downward-pivot of this function caused by ITC does not greatly alter the optimal levels of abatement. In contrast, when the marginal cost function is linear or concave (cases 4b and 4c) and much flatter in the relevant range, ITC has pronounced effects on optimal abatement. Indeed, in the concave case, ITC implies a 26% increase in cumulative abatement in the BC_R model and a 93% increase in the BC_L model! These larger impacts on abatement are associated with significant effects on average costs (costs per unit of abatement) and on the net benefits from optimal abatement. Thus, even if ITC's impact on the tax profile is small (a result attributable to the flatness of the marginal damage schedule), it may have a significant impact on abatement levels, abatement costs, and net benefits if the marginal cost function is concave and flat in the relevant range. Further research regarding the shape of the abatement cost schedule would seem necessary before one could confidently accept the Nordhaus [19] conclusion that ITC has only negligible effects.

In variants 5a and 5b we change the ease of accumulating knowledge when ITC is present by altering the multiplicative parameter M_Ψ in the $\Psi(\cdot)$ function. As expected, when the ITC option is made more powerful (case 5a), the effects of ITC are magnified. The reverse occurs when the ITC option is made weaker (case 5b).

Next, in variants 6a and 6b we consider alternative values for ϕ, which governs the intertemporal knowledge spillover. The central value is 0.5, indicating some degree of "standing on shoulders." Case 6a involves a value of 0.75 (a stronger positive intertemporal spillover); as expected, the effects of ITC are magnified, though only by a small amount. In case 6b, we set ϕ to -0.5 (which represents "fishing out"); here, the opposite holds, and the effects of ITC are (slightly) diminished.

Finally, in variants 7a and 7b we consider alternative rates of autonomous technological progress. The effects of ITC are muted, the higher the rate of autonomous technological change. This is highly sensible, given the idea of diminishing returns to R & D investments or LBD efforts.

[51]Thus we are assuring, for comparability across the cases, that the MC curve always intersects the vertical constraint (in the upper panel of Fig. 2) at the same point.

5. CONCLUSIONS

This paper has employed analytical and numerical models to examine the implications of induced technological change for the optimal design of CO_2-abatement policy. We obtain optimal time profiles for carbon taxes and CO_2 abatement under two channels for knowledge accumulation—R & D-based and LBD-based technological progress—and under both a cost-effectiveness and a benefit–cost policy criterion.

The analytical model reveals, in contrast with some recent claims, that the presence of ITC generally lowers the time profile of optimal carbon taxes. The impact of ITC on the optimal abatement path varies: when knowledge is gained through R & D investments, some abatement is shifted from the present to the future, but if the channel for knowledge-growth is learning by doing, the impact on the timing of abatement is analytically ambiguous.

When the government employs the benefit–cost policy criterion, the presence of ITC justifies greater overall (cumulative) abatement than would be warranted in its absence. However, ITC does not always promote greater abatement in all periods. When knowledge accumulation results from R & D expenditure, the presence of ITC implies a reduction in near-term abatement, despite the increase in overall abatement.

The numerical simulations reinforce the qualitative predictions of the analytical model. The quantitative impacts depend critically on whether the government is adopting the cost-effectiveness criterion or the benefit–cost criterion. ITC's effect on overall costs and optimal carbon taxes can be quite large in a cost-effectiveness setting: thus, policy-evaluation models that neglect ITC can seriously overstate both the costs of reaching stipulated concentration targets and the carbon taxes needed to elicit the desired abatement. On the other hand, the impact on costs and taxes is typically much smaller under a benefit–cost policy criterion. The weak effect on the tax rate in the benefit–cost case reflects the relatively trivial impact of ITC on CO_2 concentrations, associated marginal damages, and (hence) the optimal tax rate. As for the optimal abatement path, the impact of ITC on the timing of abatement is very weak, but the effect (present in the benefit–cost case) on total abatement over time can be large, especially when knowledge is accumulated via learning by doing.

Our work abstracts from some important issues. One is uncertainty. We have assumed both that knowledge accumulation is a deterministic process and that the cost of damage functions are perfectly known. In doing so, we have avoided difficult issues of abatement timing relating to irreversibilities and the associated need to trade off the "sunk costs and sunk benefits" of abatement policy.[52]

In addition, in this model the sole policy instrument available to the decision maker (social planner) is a tax on CO_2 emissions. It would be useful to extend the model to include two instruments: viz., a carbon tax and a subsidy to R & D. This would allow explorations of public policies that simultaneously consider two market failures—one attributable to the external costs from emissions of CO_2, and one attributable to knowledge spillovers, which force a wedge between the social and private returns to R & D. In this broader model, one could investigate optimal

[52] See Pindyck [24] and Ulph and Ulph [30].

combinations of carbon taxes and subsidies to R & D. It would also permit investigations of second-best policies: for example, optimal R & D subsidies in a situation in which the government is not able to levy a carbon tax. This approximates the situation implied by recent policy proposals of the Clinton administration.

APPENDIX A

A.1. *The Cost-Effectiveness Criterion*

A.1.1. *Technological Change via R & D*

We first demonstrate the basic characteristics of the slope of the optimal abatement path. We then go on to establish the implications of ITC. To determine how abatement changes over time, we differentiate the first-order condition governing abatement with respect to t. Note that the abatement-cost function is not necessarily time-stationary because costs may depend on baseline emissions, which usually vary through time. Differentiating Eq. (5) with respect to t yields

$$C_{AA}(\cdot)\dot{A}_t + C_{AH}(\cdot)\dot{H}_t + \frac{\delta C_A(\cdot)}{\delta t} = \dot{\tau}_t$$

$$\Leftrightarrow \quad \dot{A}_t = \frac{\dot{\tau}_t - C_{AH}(\cdot)\dot{H}_t - C_{AE}(\cdot)\dot{E}_t^0}{C_{AA}(\cdot)}.$$

We have established that for $t < T$, $\dot{\tau}_t > 0$ (see Eq. (6)), and we know that $\dot{H}_t \geq 0$. Previously we had assumed that $C_{AH}(\cdot) < 0$ and $C_{AA}(\cdot) > 0$. If costs do not depend on the level of emissions, then $C_{AE}(\cdot) = 0$ and Eq. (17) implies that abatement increases over time ($\dot{A}_t \geq 0$).

It is plausible that $C_{AE}(\cdot) < 0$, namely that the marginal cost of a fixed amount of abatement is greater, the lower the level of baseline emissions. This is consistent with the idea that abatement costs depend on relative, rather than absolute, levels of abatement. In this circumstance, $\dot{A}_t \geq 0$ so long as baseline emissions are not declining "too rapidly."

Next we move to the ITC/NITC comparison. Under the assumption that $C_{AH}(\cdot) < 0$, we prove the claim that $d\tau_0/dk \leq 0$. Suppose the opposite, i.e., suppose that

$$\frac{d\tau_0}{dk} > 0. \tag{17}$$

Equation (6) in the main text can be integrated, using the relevant transversality condition as a boundary condition, to obtain the following expression:

$$\tau_t = \int_{\max[t,T]}^{\infty} \eta_s e^{-(r+\delta)(s-t)}\, ds. \tag{18}$$

η_t, the multiplier on the $\bar{S}$ constraint, is zero if the constraint does not bind, and is typically positive, representing the shadow value of relaxing the constraint, if the constraint does bind. Thus, Eq. (18) states that the shadow cost of having a small additional amount of CO_2 at time t is dictated by how binding the $\bar{S}$ constraints are into the infinite future.[53] Combining Eq. (18) with our supposition (17) yields (assuming the proper regularity conditions hold)

$$\int_T^\infty \frac{d\eta_s}{dk} e^{-(r+\delta)s}\, ds > 0, \tag{19}$$

which states that overall, the $\bar{S}$ constraints from T onward become more binding, or costly. The supposition that τ_0 rises implies, from Eq. (8), that A_0 also rises. Noting from Eq. (6) that $\tau_t = \tau_0 e^{(r+\delta)t}$ for $t < T$, we see that our supposition implies that τ_t rises for all $t < T$. This in turn implies, from Eq. (7) (since $-C_{AH}(\cdot)dH_t/dk$ is clearly nonnegative), that A_t strictly rises for all $t < T$. In fact, as we shall now show, A_t strictly rises for all t, even beyond T. If abatement has strictly risen at every point in time up until T, then we know that S_T is now strictly less than it used to be in the NITC scenario, and thus certainly strictly less than $\bar{S}$. This itself is acceptable: it is easy to imagine situations in which, given a convex abatement-cost function and an emissions baseline that rises sharply after time T, an optimal program involves undershooting the constraint at the first point in time when it is imposed. However, the fact that S_T is now strictly less than $\bar{S}$ implies, by complementary slackness, that $\eta_T = 0$, and thus, since η_T is always nonnegative, that η_T is less than or equal to its value before the increase in k. In other words, $d\eta_T/dk \le 0$. But we know from Eq. (19) that the constraints from T onward are, on the whole, more binding, and thus we can now conclude, for sufficiently small ϵ' and all $\epsilon \in (0, \epsilon')$ as well, that

$$\int_{T+\epsilon}^\infty \frac{d\eta_s}{dk} e^{-(r+\delta)s}\, ds > 0 \quad \Leftrightarrow \quad \frac{d\tau_{T+\epsilon}}{dk} > 0 \quad \Leftrightarrow \quad \frac{dA_{T+\epsilon}}{dk} > 0.$$

Now we know that abatement has strictly risen for all $t < T + \epsilon$. The above argument can be repeated, in the style of a proof by induction, to show that $dS_{T+\epsilon}/dk < 0$, implying that $d\eta_{T+\epsilon}/dk \le 0$, and that, in turn, $dA_t/dk > 0\ \forall t$. Our supposition that $d\tau_0/dk > 0$ has led us to the conclusion that abatement rises at all points in time. Given that the initial program satisfied the constraints, a new program in which abatement is higher at every point clearly cannot be optimal. Thus we have a contradiction. We may conclude that $d\tau_0/dk \le 0$, and thus that $dA_0/dk \le 0$. Since the multiplier simply grows at the constant rate $(r + \delta)$ until time T, we have also shown that $d\tau_t/dk \le 0\ \forall t < T$, and in fact, that the absolute fall in the multiplier increases with t over this time range, but in such a way as to preserve the growth rate as $(r + \delta)$.

Note that if we had assumed $C_{AH}(\cdot) > 0$, then the above proof could be reversed to show that initial abatement and the entire tax path weakly rise. We would find that, in contrast to the normal case, ITC would cause a "flattening" rather than a "steepening" of the optimal abatement profile.

[53]As noted in the main text, this is in contrast to the benefit–cost cases, in which the shadow cost is given by the discounted sum of the marginal damages that a small additional amount of CO_2 would cause into the infinite future.

A.1.2. *Technological Change via Learning by Doing*

We start by establishing the slope of the optimal abatement path. It is necessary, however, first to examine the profile of μ_t, the shadow value of knowledge. The costate equation for μ_t, which is the same in both the CE_R and CE_L models, states that

$$\dot{\mu}_t = \mu_t(r - \alpha_t - k\Psi_H(\cdot)) + C_H(\cdot).$$

The shadow value grows at r because it is a *current-value* multiplier. The value of knowledge falls at α_t because new knowledge is being generated autonomously at that rate. Next, depending on the sign of $\Psi_H(\cdot)$—that is, depending on whether knowledge accumulation is characterized by "standing on shoulders" or "fishing out"—there is a third tendency for the shadow value either to fall or to rise over time. For example, when $\Psi_H(\cdot) < 0$, the "fishing out" case where further knowledge accumulation becomes more difficult the larger the current stock of knowledge, it is preferable to suffer this disadvantage over as short a time interval as possible. Thus in this case, the shadow value tends to rise over time. The opposite holds in the "standing on shoulders" case where $\Psi_H(\cdot) > 0$. Finally, since $C_H(\cdot) < 0$, there is a tendency for the shadow value of knowledge to fall over time because we have a shorter time range over which the knowledge will serve to reduce abatement costs. These four effects combine to make the slope of the μ_t path ambiguous in sign.

We now focus on the slope of the optimal abatement path in the CE_L model. Differentiating Eq. (9) and rearranging, we obtain

$$\dot{A}_t = \frac{\dot{\tau}_t + \dot{\mu}_t k\Psi_A(\cdot) + (\Psi_{AH}(\cdot)\mu_t k - C_{AH}(\cdot))\dot{H}_t - C_{AE}(\cdot)\dot{E}_t^0}{C_{AA}(\cdot) - \mu_t k\Psi_{AA}(\cdot)}.$$

The denominator is positive, but the numerator is of ambiguous sign because of the second and third terms. If we consider an NITC scenario in which $k = 0$, then we obtain

$$\dot{A}_t = \frac{\dot{\tau}_t - C_{AH}(\cdot)\dot{H}_t}{C_{AA}(\cdot)} - C_{AE}(\cdot)\dot{E}_t^0,$$

which, at least for $t < T$, is clearly positive, as discussed above, as long as $\dot{E}_t^0$ is not too negative. In the general LBD case with ITC, however, the optimal abatement path may very well slope downward (even if the emissions baseline is growing over time), in contrast to the R & D case.[54]

Now we examine the implications of ITC. The proof that $d\tau_0/dk \leq 0$ proceeds along the same lines as in the CE_R appendix. We suppose that τ_0 strictly rises, and this implies that abatement rises for all t, which cannot be optimal. (The extra learning-by-doing effect in Eq. (10) is positive and thus only strengthens the link

[54]Our numerical solutions confirm, however, that the abatement path typically does slope upward, even in the ITC learning-by-doing case.

between τ_t's rising and A_t's rising.[55] We conclude, as in the CE_R model, that the entire path of carbon taxes falls, and increasingly so for higher t (up to T). As noted in the text, however, this finding is not enough to assure us that initial abatement also falls.

A.2. *The Benefit–Cost Criterion*

A.2.1. *Technological Change via R & D*

First let us analyze the slope of the carbon tax path. We rearrange Eq. (12) to see that

$$\dot{\tau}_t = (r + \delta)\tau_t - D'(\cdot). \tag{20}$$

The first term on the right-hand side contributes to growth in τ_t, while the second contributes to its decline over time (an additional amount of CO_2 later on inflicts marginal damages over a shorter horizon). It is thus possible for the optimal carbon tax to decline over time.

Let us now consider the conditions under which the carbon tax will necessarily rise. Substituting Eq. (13) into Eq. (20) yields

$$\dot{\tau}_t = (r + \delta)e^{(r+\delta)t}\int_t^{\infty} D'(S_s)e^{-(r+\delta)s}\,ds - D'(S_t). \tag{21}$$

If we had a linear damage function, such that $D'(S_s)$ were constant and equal to $D'(S_t)$ for all $s > t$, then the first term in Eq. (21) would reduce to $D'(S_t)$, and we would conclude that $\dot{\tau}_t = 0$; i.e., the optimal tax path would be flat. If, however, $D'(S_s) > D'(S_t)\ \forall s > t$, the first term in Eq. (21) would be larger than $D'(S_t)$, and the tax path would be upward sloping. Given the convex damage function which we (and others, typically) assume, having an optimized S_t path that slopes upward ensures $D'(S_s) > D'(S_t)\ \forall S > t$, and is thus a sufficient condition for having an upward sloping tax path. Given that many other authors' simulations involve a steadily increasing optimized CO_2 concentration, it is easy to see why the literature frequently obtains optimal carbon taxes that forever rise.

What can we say about the slope of the abatement path in the BC_R model? Differentiating equation (11) with respect to t and rearranging, we obtain, as in the CE_R model,

$$\dot{A}_t = \frac{\dot{\tau}_t - C_{AH}(\cdot)\dot{H}_t - C_{AE}(\cdot)\dot{E}_t^0}{C_{AA}(\cdot)}.$$

The denominator and the second term in the numerator[56] are clearly positive, but the ambiguous slope of the optimal carbon tax path prevents us from concluding that optimized abatement must always rise over time. Once again, if the optimized S_t path is rising and the damage function is convex, then taxes rise, and thus so

[55] The learning-by-doing effect, however, prevents us from reversing the proof for the $C_{AH} > 0$ case. In that case, under a learning-by-doing specification, we cannot conclude anything about the impact of ITC on taxes or abatement.

[56] Assuming $C_{AH}(\cdot) < 0$.

does abatement, as long as the emission baseline is not declining too rapidly. As we see in our numerical simulations of the BC_R model, even though taxes are always rising, the optimal abatement profile actually slopes down during the time when baseline emissions are steeply decreasing.

Now we turn to the analysis of the implications of ITC; i.e., the effects of increasing k. We shall prove that $d\tau_0/dk \leq 0$, that $dA_0/dk \leq 0$, that the overall scale of abatement increases when we raise k, that the abatement path thus becomes steeper, and that $d\tau_t/dk \leq 0\ \forall t$. If we were to assume that knowledge *raises* marginal abatement costs ($C_{AH}(\cdot) \geq 0$), then the entire proof could be reversed to demonstrate that taxes rise, initial abatement rises, the overall scale of abatement falls, and the abatement path thus becomes flatter.

Suppose that τ_0 rises, and that thus, by Eq. (15), A_0 rises as well. We have, using Eq. (13) and (11),

$$\frac{d\tau_0}{dk} > 0 \quad \Leftrightarrow \quad \int_0^\infty \frac{dD'(S_s)}{dk} e^{-(r+\delta)s}\, ds > 0 \tag{22}$$

$$\Leftrightarrow \quad \int_0^\infty D''(S_s) \frac{dS_s}{dk} e^{-(r+\delta)s}\, ds > 0$$

$$\Leftrightarrow \quad \int_0^\infty D''(S_s) e^{-(r+\delta)s} \int_0^s \frac{dA_m}{dk} e^{-\delta(s-m)}\, dm\, ds < 0. \tag{23}$$

Given our convex damage function, this last equation (23) means that the overall scale of abatement becomes less ambitious when k rises.

For $t > 0$, we can use similar steps to obtain

$$\frac{d\tau_t}{dk} = -\int_t^\infty D''(S_s) e^{-(r+\delta)(s-t)} \int_0^s \frac{dA_m}{dk} e^{-\delta(s-m)}\, dm\, ds$$

$$\Leftrightarrow \quad \frac{d}{dt}\left(\frac{d\tau_t}{dk}\right) = -(r+\delta) \int_t^\infty D''(S_s) e^{-(r+\delta)(s-t)} \int_0^s \frac{dA_m}{dk} e^{-\delta(s-m)}\, dm\, ds$$

$$+ D''(S_t) \int_0^t \frac{dA_m}{dk} e^{-\delta(t-m)}\, dm. \tag{24}$$

Note that $d(d\tau_t/dk)/dt$ is clearly positive if each of the two terms on the right hand side of Eq. (24) is positive. Given Eq. (23), the first term is definitely nonnegative if

$$\int_0^t D''(S_s) e^{-(r+\delta)s} \int_0^s \frac{dA_m}{dk} e^{-\delta(s-m)}\, dm\, ds \geq 0.$$

The second term is clearly positive, assuming a convex damage function, if

$$\int_0^t \frac{dA_m}{dk} e^{-\delta(t-m)}\, dm \geq 0.$$

We thus are led to the following lemma. Given our assumption that the initial tax rises (Eq. (22)), and assuming a convex damage function, then

$$\int_0^t D''(S_s)e^{-(r+\delta)s}\int_0^s \frac{dA_m}{dk}e^{-\delta(s-m)}\,dm\,ds \geq 0 \tag{25}$$

and

$$\int_0^t \frac{dA_m}{dk}e^{-\delta(t-m)}\,dm \geq 0. \tag{26}$$

These together imply

$$\frac{d}{dt}\left(\frac{d\tau_t}{dk}\right) \geq 0.$$

Equations (22) and (15) together tell us that $dA_0/dk > 0$. This means (using inductive reasoning much like that used in the CE_R appendix),[57] that sufficiency conditions (25) and (26) hold for $t = \epsilon$ sufficiently close to 0, and also for $t = \epsilon'$ $\forall \epsilon' \in (0, \epsilon)$. Thus we conclude that

$$\frac{d}{dt}\left(\frac{d\tau_\epsilon}{dk}\right) \geq 0 \quad \Leftrightarrow \quad \frac{d\tau_\epsilon}{dk} \geq \frac{d\tau_0}{dk} > 0 \quad \Leftrightarrow \quad \frac{dA_\epsilon}{dk} > 0,$$

where this last implication is only strengthened by the $-C_{AH}(\cdot)\,dH_\epsilon/dk$ effect in Eq. (15). The whole chain of reasoning can be repeated inductively to imply that abatement strictly rises at every point in time. This, however, contradicts Eq. (23), which says that the overall scale of abatement is less ambitious. Thus, our supposition must be wrong. Thus, we conclude that τ_0 (weakly) falls, A_0 falls, a more ambitious overall scale of abatement is adopted, and the abatement path becomes "steeper," all as a result of the increase in k. That is to say,

$$\frac{d\tau_0}{dk} \leq 0$$

$$\frac{dA_0}{dk} \leq 0$$

and

$$\int_0^\infty D''(S_s)e^{-(r+\delta)s}\int_0^s \frac{dA_m}{dk}e^{-\delta(s-m)}\,dm\,ds \geq 0. \tag{27}$$

Using arguments similar to those used above, it is also possible to demonstrate that the entire path of carbon taxes must weakly fall: $d\tau_t/dk \leq 0$ $\forall t$. This does not mean, however, that *abatement* always weakly falls; the growth in H_t as a result of k counters the effect of the weakly falling carbon taxes, and we know, in fact, that overall we end up with a weakly more ambitious abatement path.

[57] Where we used a small ϵ and appealed to continuity to justify an inductive proof in a continuous-time problem.

A.2.2. *Technological Change via Learning by Doing*

Using methods virtually identical to those in the previous sections, we prove that (1) the carbon tax falls at all points in time, including time 0, (2) the impact on A_0 is ambiguous, and (3) the overall scale of abatement increases. Please refer to earlier sections of the Appendix corresponding to the CE_L and BC_R models; the proofs here are not substantively different.

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