

# ECON 509: Numerical Methods in Economics

Fall 2017: Tuesday 5:10-8:00

Room: Heady 0272

<http://github.com/IASTATEECON509>

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## Course Overview

**Course Objective:** The objective of this course is to familiarize you with the fundamentals of computation, and a set of state-of-the-art methods, primarily for dynamic modeling. In this course you should learn why we need computational methods for certain types of problems, the theory behind the methods, and most importantly, how to use them in practice. The course begins with the basics of computing, then moves into optimization and methods for dynamic economic models. We will conclude with Monte Carlo methods, methods for empirical research, and how to use the high performance computing resources on campus. The beginning of the course is heavy on theory to understand what is happening inside your machine when solving numerical models. As we begin studying techniques to solve economic problems you will also apply them in practice. Keep in mind this is not a programming course. Beyond the first week or two we will be solely focusing on methodology although we will work through some applications. In this class we will be using Julia. Julia was selected because it's open source, many packages exist to employ the methods we will learn and practice, and it's fast and intuitive. Please set up a JuliaBox account before class starts at [juliabox.com](http://juliabox.com). JuliaBox allows you to code using cloud (and multi-core) resources for free. Much of what we do can be ported to R, Python, MATLAB, and C.\*

**Prerequisites:** ECON 500 and ECON 501, or ECON 600 and ECON 601.

**Readings:** Miranda and Fackler (2002) will be referred to frequently and is recommended reading. Nocedal and Wright (2006) is highly useful as a detailed reference for optimization and Judd (1998) takes a more detailed look at the fundamental numerical methods in economics. Miranda and Fackler (2002) and Nocedal and Wright (2006) are available as eBooks in the ISU library. Please read *Learn Julia the Hard Way* for a brief introduction to coding in Julia. The remainder of the required readings will be from journal articles or excerpts from texts which will be accessible online.

**Grading:** 5% of your grade is seminar attendance, 10% of your grade is class participation, 10% is your presentation of a paper, 25% of your grade is the final project, and 50% of your grade is the problem sets.

**Office Hours:** Wednesday 1:30-3:00 in 479 Heady.

**Contacting Me:** Please put ECON 509 in the subject line of all e-mails and I will respond within 24 hours.

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\*See <http://quant-econ.net> for guides and packages for economics research in Julia.

# Assignments

1. You are required to attend at least two economics seminars with a numerical component. More information on the specific seminars will come later since the set of papers being presented is still being finalized. A list of the known numerical seminars are below with titles, and you can most likely plan to attend the ones without titles:
  - August 30: Ivan Rudik - *Calibrating Informational Dynamics: Learning the Sensitivity of Climate to Emissions*
  - October 16: Derek Lemoine - *Innovation-Led Transitions in Energy Supply*
  - October 18: Benjamin Moll
  - October 23: Aubhik Khan
2. Problem sets will be due periodically. You must submit your code on Github (url above) to your group's repository as well as a separate pdf of your results. We will learn how to use Git during class. You may work in a group of **three or fewer people**. Groups should turn in one assignment with all members' names on the front page of your submitted pdf. **You must use Julia and you must clearly comment your code.**
3. There will be a final project for the course, due at the end of the semester, where each student will submit the beginning of a numerically-driven research paper. During dead week, each student will present their completed work which should have a first-take at a numerical/computational model and preliminary results. The paper should be at least 7 pages including tables or graphs and should:
  - Have an introduction that clearly states the economic question you are answering, frames your research in the context of the existing literature, and tells the reader what you are doing to advance economic knowledge.
  - Analytically develop the model, provide proofs for theoretical results if there are any.
  - Describe how you solve the model.
  - Have preliminary results.
4. Starting near the middle of the course, one student a week will present either a paper that applies methods we have learned in the previous week, or a paper that extends methodology for computing dynamic models. More information will come later in the course.

## Tentative Course Schedule

### 1 Introduction to Computation

#### Theory

Miranda and Fackler (2002, Chapters 1, 2, and 5); Judd (1998, Chapters 2, 3 and 7)

## 2 Git and Julia Tutorial

**Application:** SourceTree, Types, Control Flow

Learn Julia the Hard Way

## 3 Rootfinding and Optimization

**Theory:** (Quasi-)Newton, Nelder-Mead, Line Search, Trust Region

Miranda and Fackler (2002, Chapters 3 and 4); Judd (1998, Chapter 4 and 5); Nocedal and Wright (2006, Chapters 2-6)

## 4 Dynamic Programming

**Theory:** Markov Chains, Bellman's Principle of Optimality, Projection

Ljungqvist and Sargent (2004); Adda and Cooper (2003); Cai and Judd (2014); Fernández-Villaverde et al. (2016)

**Application:** Monetary Policy, Climate Change, Bioeconomic Modeling, Agriculture

Cogley et al. (2007); Springborn and Sanchirico (2013); Lemoine and Traeger (2014); Traeger (2014); Cai et al. (2015); Livingston et al. (2015); Rudik (2017)

## 5 Continuous Time Optimal Control

**Theory:** Hamiltonians, Backwards Shooting/Integration, Relaxation Algorithm

Judd (1998, Chapter 10); Brunner and Strulik (2002); Caputo (2005); Trimborn et al. (2008)

**Application:** Climate Change, Resource Depletion, Shallow Lakes, Antibiotics

Goulder and Mathai (2000); Laxminarayan and Brown (2001); Maler et al. (2003); Venables (2014); Lemoine and Rudik (2017)

## 6 Methods for Accelerating and Improving Approximation in Dynamic Programming

**Theory:** Modified Policy Iteration, Sparse Grids, Endogenous Grids, Adaptive Grids, Stochastic Simulation, Precomputation, Envelope Condition Methods, Multicollinearity, Error Analysis

Smolyak (1963); Puterman and Shin (1978); Santos (2000); Carroll (2006); Winschel and Kratzig (2010); Malin et al. (2011); Peralta-alva and Santos (2014); Maliar and Maliar (2014); Judd et al. (2014, 2015); Maliar and Maliar (2015); Judd et al. (2016); Brumm and Scheidegger (2017); Judd et al. (2017)

## 7 Markov Chain Monte Carlo

**Theory:** Metropolis-Hastings, Gibbs Sampler, Importance Sampling

Chib and Greenberg (1995, 1996)

**Application:** Non-Conjugate Learning, Uncertainty Shocks

Lieli and Springborn (2012); Orlik and Veldkamp (2014)

## 8 Maximum Likelihood

**Application:** Simulating Data Sets, Computing Standard Errors, Dynamic Discrete Choice

Rust (1987); Scott (2013)

## 9 High Performance Computing: Computing Clusters and GPUs

Aldrich et al. (2011); Aldrich (2014); Cai et al. (2014)

## 10 Final Project Presentations

## References

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