## Problem 4: Due March 7 at 10:00 AM

We are a small oil extracting firm. We own an oil reservoir that contains $\bar{s} = 500$ thousand barrels of oil and $s_t$ denotes how much oil we have in our reservoir at time $t = 0, 1, 2, ....$ The cost of extracting oil is given by the function,

$$C(e_t, s_t) = e_t^2,$$

where $e_t$ is the amount extracted at time $t$ in thousands. The oil remaining in our reservoir next period is,

$$s_{t+1} = s_t - e_t.$$

We do not have any on-land oil storage so it must be immediately sold. The world price of oil is \$30 per barrel (\$30,000 per thousand barrels). We discount the future with a discount factor of $\beta = 0.95$.

1. Write out our extraction problem in Bellman form
2. Solve for our infinite-horizon value function using value function iteration with a convergence criterion of $||V_{old} - V_{new}||_\infty < 10^{-4}$ where $||\cdot||_\infty$ is the max norm. Use units of thousands of barrels (e.g. initial condition for simulation is reservoir = 500) and thousands of dollars.
   - Solve using a 4th, 7th, and 10th degree approximations with Chebyshev polynomials
3. Simulate 50 periods beginning at time $t = 0$ from the initial condition above and plot the extraction trajectory and the oil reservoir stock trajectory for both sets of basis functions. How much do the simulated trajectories differ between degrees of approximation and across the types of basis functions? If you find a difference in trajectories between basis functions using the same degree approximation, explain why you think this is? Hint: think about where in the state space you move through during the simulation and the grid point locations for each basis function.

Now suppose that oil becomes more costly to extract as we deplete the reservoir:

$$C(e_t, s_t) = \frac{\bar{s}}{s_t} e_t^2.$$

1. Write out our extraction problem in Bellman form

2. Solve for our infinite-horizon value function using value function iteration with a convergence criterion of $||V_{old} - V_{new}||_{\infty} < 10^{-4}$ where $||\cdot||_{\infty}$ is the max norm. Use units of thousands of barrels (e.g. initial condition for simulation is reservoir = 500)

   - Solve using a 10th degree approximation with Chebyshev polynomials

3. Simulate 50 periods beginning at time $t = 0$ from the initial condition above and plot the extraction trajectory and the oil reservoir stock trajectory for both sets of basis functions

4. How does an increasing cost of extraction affect the optimal extraction trajectory?