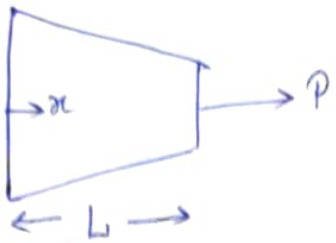
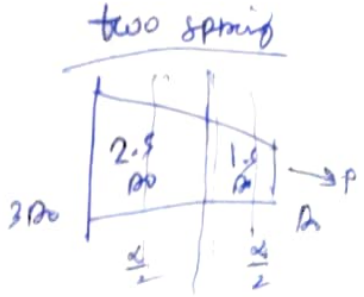
Dinayan Parbat
22M0032

1)





a) $K_0 = \frac{A_0 E}{L}$, $K_1 = \frac{5}{2} \times 2 \times \frac{A_0 E}{L}$ $= 5K_0$

$K_2 = \frac{3}{2} \times \frac{A_0 E}{L}$

$= 3K_0$

① First element: $\begin{Bmatrix} R_0' \\ R_1' \end{Bmatrix} = \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix} \begin{Bmatrix} u_0' \\ u_1' \end{Bmatrix}$

twice element: $\begin{Bmatrix} R_1^2 \\ R_2^2 \end{Bmatrix} = \begin{bmatrix} K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{Bmatrix} u_1^2 \\ u_2^2 \end{Bmatrix}$

Here, $u_0' = u_0$, $u_1' = u_1^2 = u_1$, $u_2^2 = u_2$

$R_0' = R_0$, $R_1' + R_1^2 = R_1$, $R_2^2 = R_2$

$R_0 = K_1 u_0 - K_1 u_1$ —— (i)

$R_1' + R_1^2 = R_1 = -K_1 u_0 + K_1 u_1 + K_2 u_1 - K_2 u_2$

$= -K_1 u_0 + (K_1 + K_2) u_1 - K_2 u_2$ —— (ii)

$R_2^2 = R_2 = -K_2 u_1 + K_2 u_2$ —— (iii)

In matrix form, $u_0 = 0$.

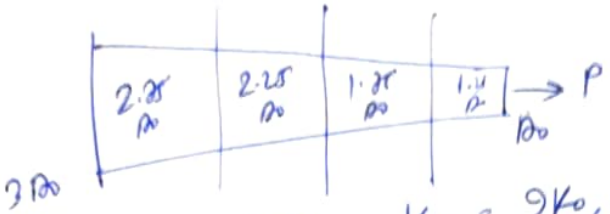
$$\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} 0 \\ P \end{Bmatrix} = \begin{bmatrix} 8K_0 & -3K_0 \\ -3K_0 & 3K_0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Solving, $u_1 = \frac{3}{8} u_2$ $\qquad P = (-3u_1 + 3u_2) K_0$

$\Rightarrow P = \frac{15}{8} u_2 K_0$

$\Rightarrow u_2 = 0.533 \frac{P}{K_0}$



$K_0 = \frac{AE}{L}$, $K_1 = 11K_0$, $K_2 = 9K_0$, $K_3 = 7K_0$, $K_4 = 5K_0$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{Bmatrix} = \begin{bmatrix} K_1 & -K_1 & 0 & 0 & 0 \\ -K_1 & K_1+K_2 & -K_2 & 0 & 0 \\ 0 & -K_2 & K_2+K_3 & -K_3 & 0 \\ 0 & 0 & -K_3 & K_3+K_4 & -K_4 \\ 0 & 0 & 0 & -K_4 & K_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}$$

$$\begin{Bmatrix} R_2 \\ R_3 \\ R_4 \\ R_5 \end{Bmatrix} = K_0 \begin{bmatrix} 20 & -9 & 0 & 0 \\ -9 & 16 & -7 & 0 \\ 0 & -7 & 12 & -5 \\ 0 & 0 & -5 & 5 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix}$$

$R_5 = P$, $R_2 = R_3 = R_4 = 0$.

$20u_2 = 9u_3$, $-9u_2 + 16u_3 - 7u_4 = 0$

$-7u_3 + 12u_4 - 5u_5 = 0$, $P = (-5u_4 + 5u_5) K_0$

Solving, $P = 1.8335\, u_5 K_0$

$\Rightarrow u_5 = 0.544 \frac{P}{K_0}$

Exact sol^n, $u = \frac{PL \ln(3)}{2EA_0} = 0.5493\ P/K_0$

% error $= \frac{0.5493 - 0.544}{0.5493} \times 100 = 0.928\%$ error (4 springs)

$= \frac{0.5493 - 0.5333}{0.5493} \times 100 = 2.912\%$ error (2 springs)