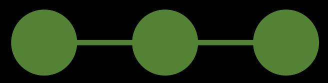
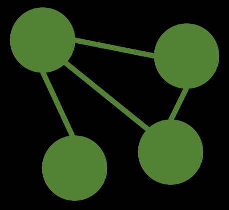
Final Exam Review

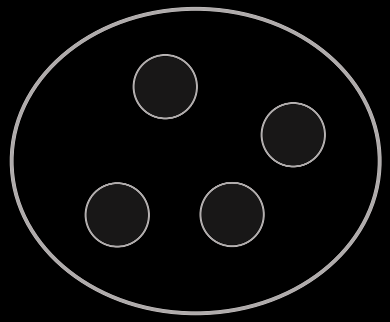
Data Structures & Algorithm

Categories of Data Structures

Linear Ordered	Non-linear Ordered	Not Ordered
Lists	Trees	Sets
Stacks	Graphs	Tables/Maps
Queues		



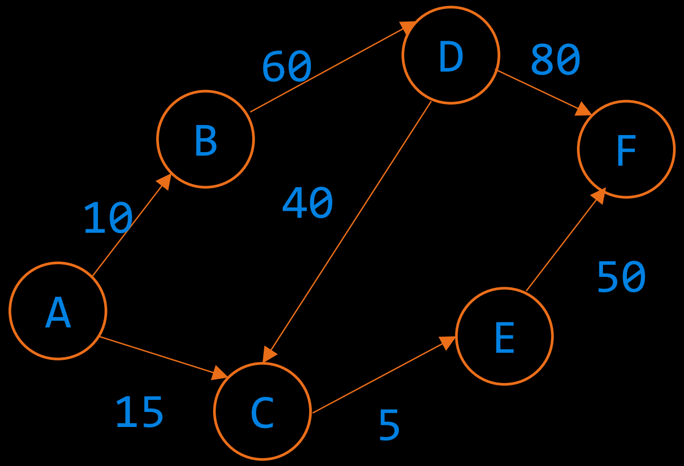




Announcements

- Exam Timing
  - Campus students: you can take the exam on July 25 between 6 to 10 pm EST. This means you must start by 8 pm EST or else you will lose time.
  - UFOL/UDER students: can take the exam between July 25, 6 pm to July 27, midnight.
- The exam will be over Honorlock and you are allowed one double sided handwritten sheet of notes.
- The exam duration is 2 hours.
- Exam 2 Topics and Expectations Guide: Link

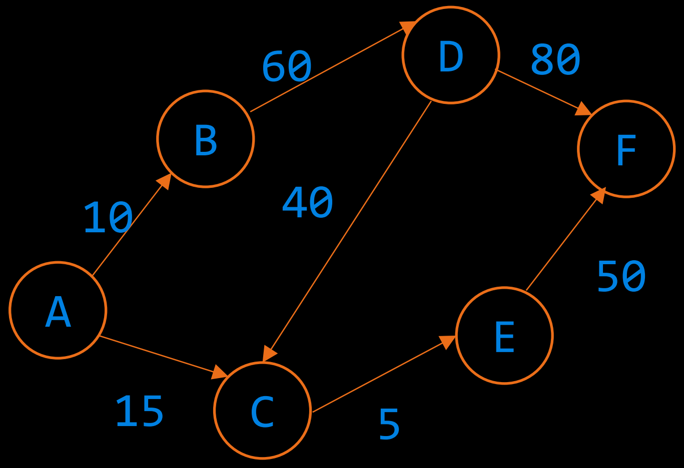
Common Representations



G

- Edge List
- Adjacency Matrix
- Adjacency List

Edge List

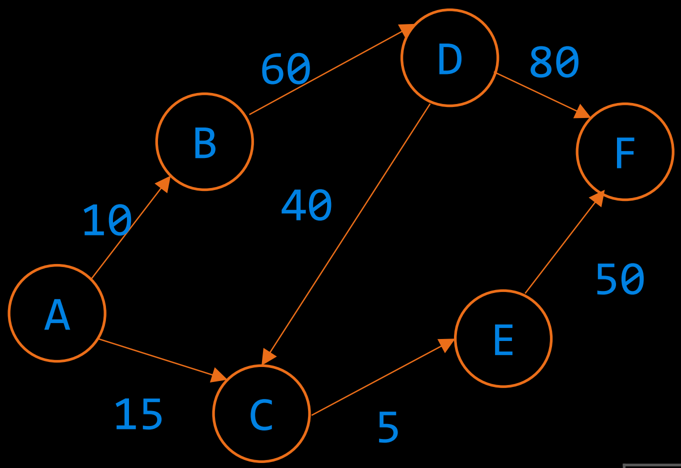


G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

G = {(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)}

Edge List



G = {(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)}

G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

Common Operations:

1. Connectedness

   Is A connected to B?

   ~ O(E)

2. Adjacency

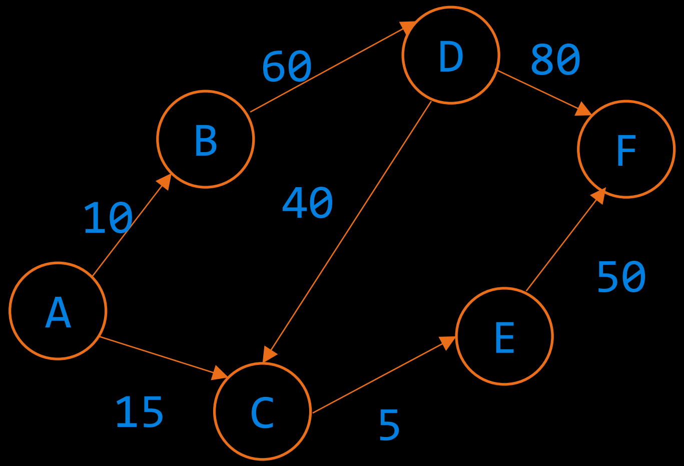
   What are A’s adjacent nodes?

   ~ O(E)

O(|E|) ~ O(|V| * |V|)

Space: O(E)

Adjacency Matrix



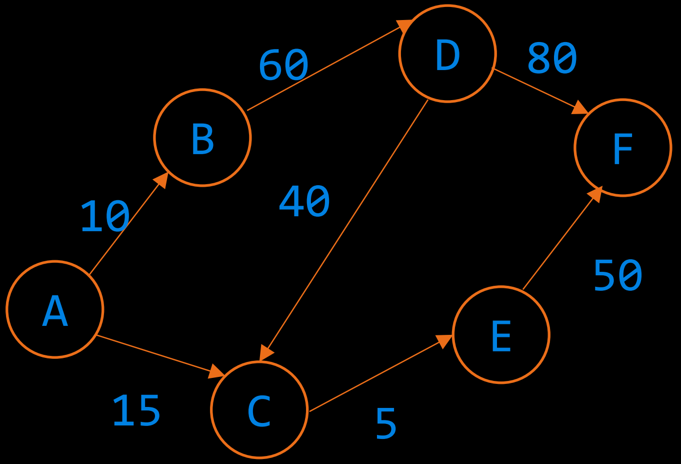
G

	A	B	C	D	E	F
A	0	10	15	0	0	0
B	0	0	0	60	0	0
C	0	0	0	0	5	0
D	0	0	40	0	0	80
E	0	0	0	0	0	50
F	0	0	0	0	0	0

Insertion:

```
G[from][to] = weight; (if there is an edge, “from” -> “to”)
G[from][to] = 0;            (otherwise)
```

Adjacency Matrix Implementation



Insertion:

G[from][to] = weight; (if there is an edge, “from” -> “to”)

G[from][to] = 0; (otherwise)

```
#include <iostream>
#include<map>
#define VERTICES 6
using namespace std;
int main()
{
        int no_lines, wt, j=0;
        string from, to;
        int graph [VERTICES][VERTICES] = {0};
        map<string, int> mapper;
        cin >> no_lines;
        for(int i = 0; i < no_lines; i++)
        {
                cin >> from >> to >> wt;
                if (mapper.find(from) == mapper.end())
                        mapper[from] = j++;
                if (mapper.find(to) == mapper.end())
                        mapper[to] = j++;
                graph[mapper[from]][mapper[to]] = wt;
        }
        return 0;
}
```

Input

7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50

G

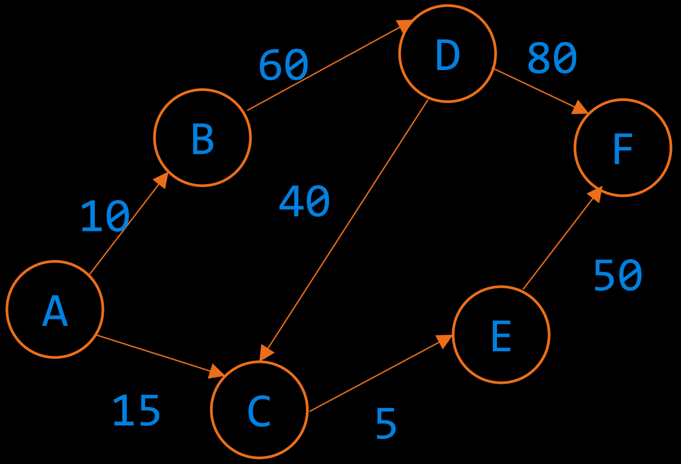
Map

A 0
B 1
C 2
D 3
E 4
F 5

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

https://www.onlinegdb.com/Hy8M0CnsS

Adjacency Matrix



G

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Map
A 0
B 1
C 2
D 3
E 4
F 5

Common Operations:

1. Connectedness

Is A connected to B?

G[“A”][“B”] ~ O(1)

2. Adjacency

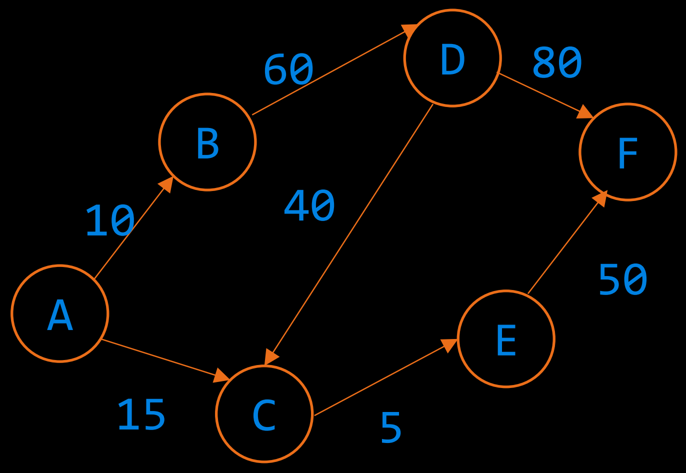
What are A’s adjacent nodes?

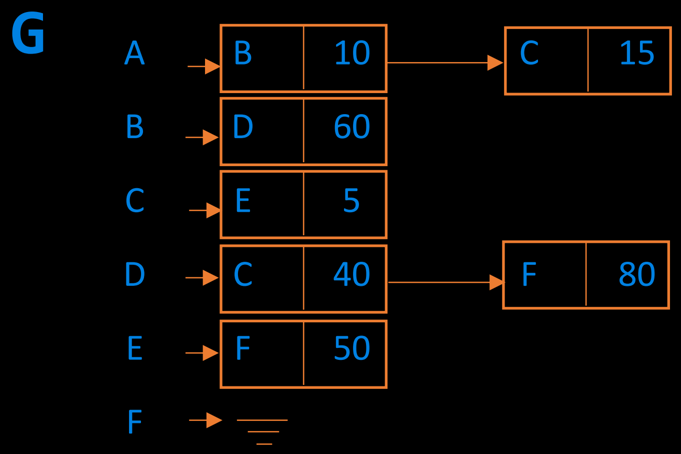
```
for each element x in G[“A”]
    if x ! = 0
```

~ O(|V|)

Space: O(|V| * |V|)

Adjacency List





Sparse Graph:
Edges ~ Vertices

Common Operations:

1. Connectedness

   Is A connected to B?

   for each element x in G["A"]
   if x ! = 'B'
   ~ O(outdegree|V|)

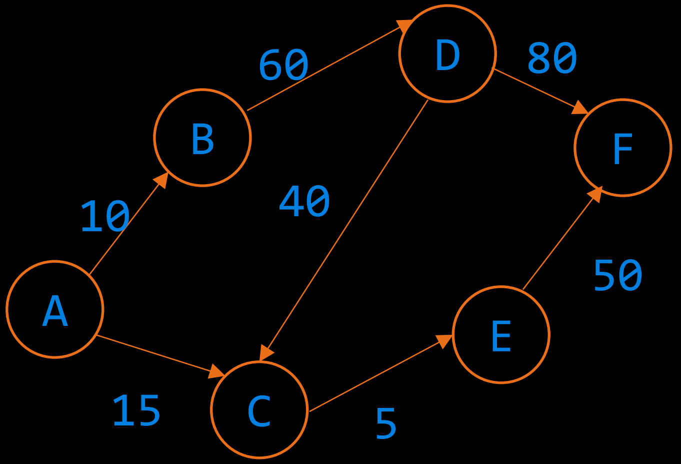
2. Adjacency

   What are A's adjacent nodes?

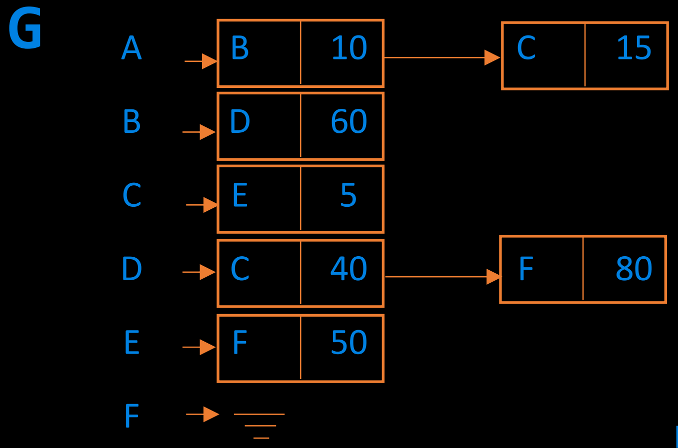
   G["A"] ~ O(outdegree|V|)

Space: **O(|V| + |E|)**

Adjacency List Implementation



Input
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50



Insertion:

If to or from vertex not present add vertex

Otherwise add edge at the end of the list

```
#include <iostream>
#include<map>
#include<vector>
#include<iterator>
using namespace std;

int main()
{
        int no_lines;
        string from, to, wt;
        map<string, vector<pair<string,int>>> graph;
        cin >> no_lines;
        for(int i = 0; i < no_lines; i++)
        {
                cin >> from >> to >> wt;
                graph[from].push_back(make_pair(to, stoi(wt)));
                if (graph.find(to)==graph.end())
                        graph[to] = {};
        }
}
```

https://onlinegdb.com/HkJq9iFaI

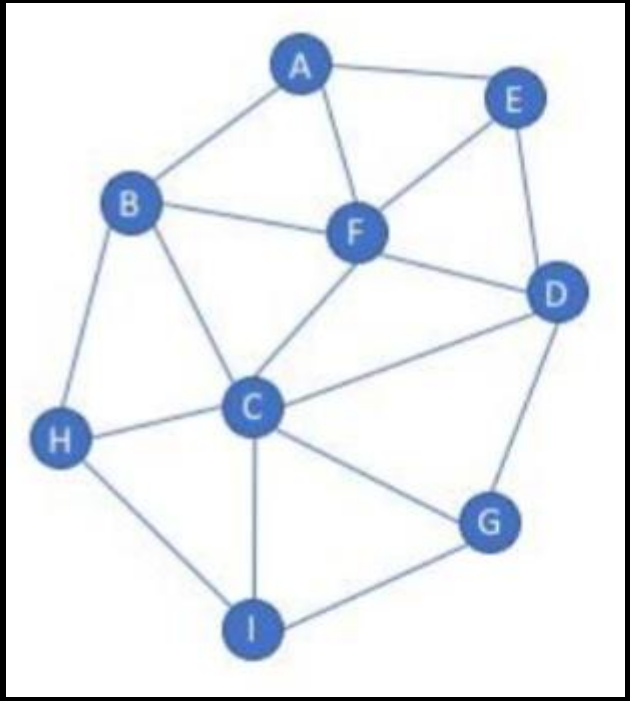
Graph Implementation

	Edge List	Adjacency Matrix	Adjacency List
Time Complexity: Connectedness	O(E)	O(1)	O(outdegree(V))
Time Complexity: Adjacency	O(E)	O(V)	O(outdegree(V))
Space Complexity	O(E)	O(V*V)	O(V+E)

Alt Text for the Graph on Next Slide

Vertex: Neighbors of Vertex (Edges pointing from a vertex to the neighbor)

A: B, E, F
B: A, C, F, H
C: B, D, F, G, H, I
D: C, E, F, G
E: A, D, F
F: A, B, C, D, E
G: C, D, I
H: B, C, I
I: C, G, H



Graph - BFS

- Which of the following are valid breadth first search traversals for this graph?

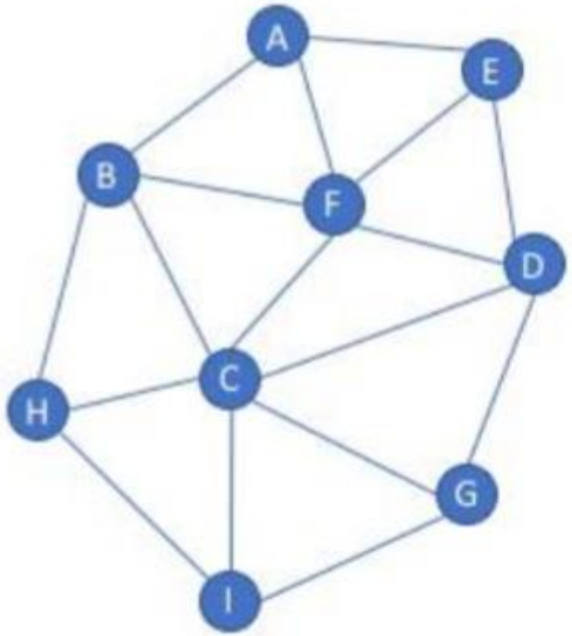
a) A F B E D C H G I

b) I C H G B F D A E

c) D C F E G H I B A

d) E A F D B H C I G

e) F A E D C B G I H



Graph - BFS

- Which of the following are valid breadth first search traversals for this graph?

a) A F B E D C H G I

b) I C H G B F D A E

c) D C F E G H I B A

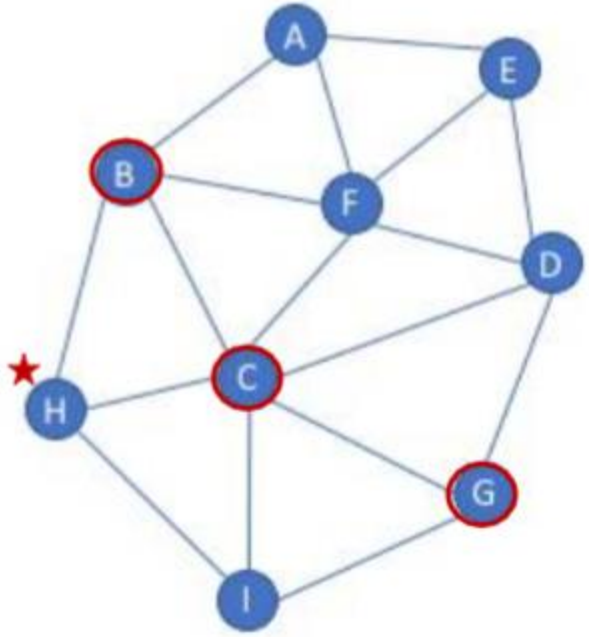
d) E A F D B H C I G

e) F A E D C B G I H

All the options except for d

Why not d?

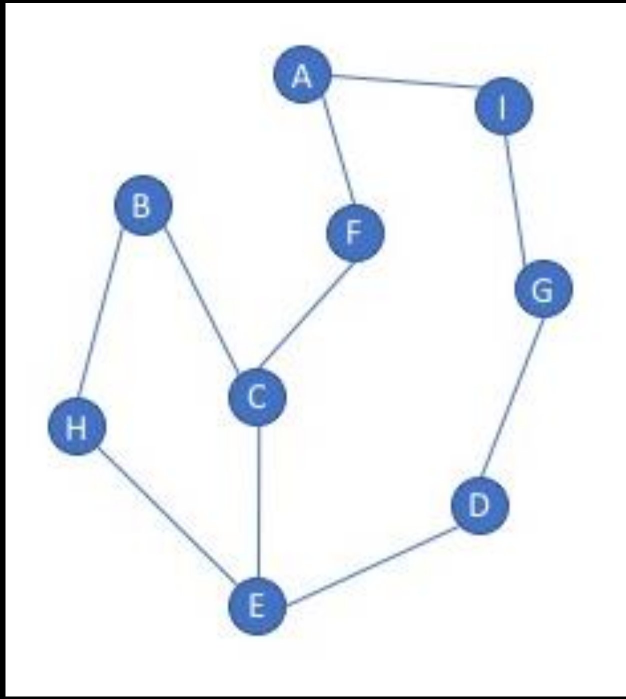
** H is visited before C and G



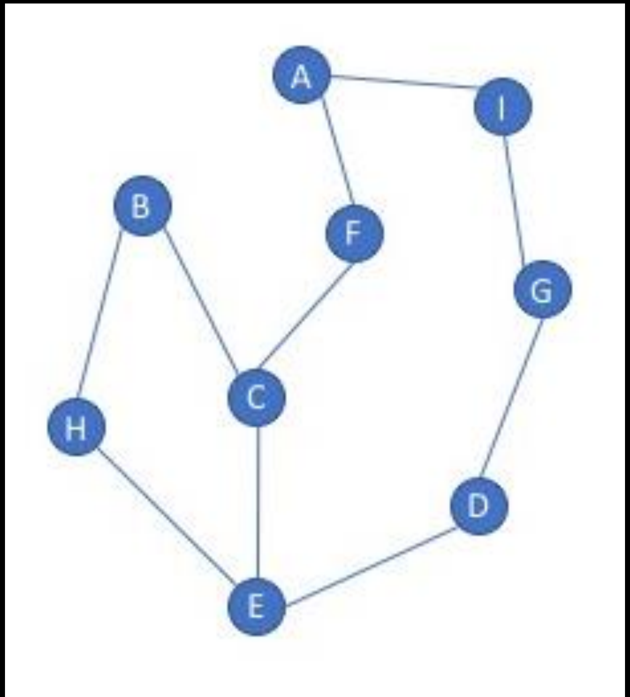
Alt Text for the Graph on Next Slide

Vertex: Neighbors of Vertex (Edges pointing from a vertex to the neighbor)

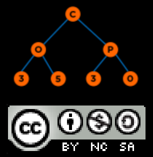
A: F, I
B: C, H
C: B, E, F
D: E, G
E: C, D, H
F: A, C
G: D, I
H: B, E
I: A, G



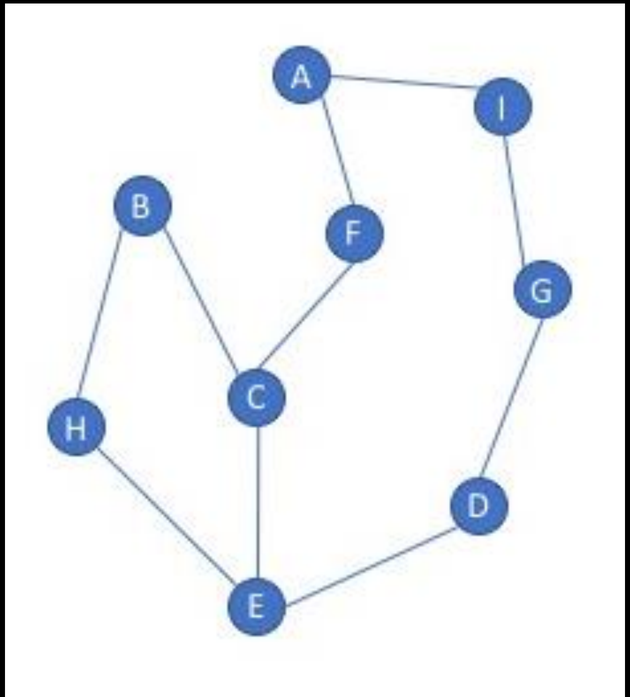
Valid DFS: Which DFS are valid?



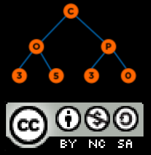
- H E C B D G I A F
- C E H B D G I A F
- A F C E H B I G D
- D E C B H F A I G



Valid DFS: Which DFS are valid?



- H E C B D G I A F
- C E H B D G I A F
- A F C E H B I G D
- D E C B H F A I G



Applied Traversal

A connected component of a graph is a collection of vertices in which any two vertices in a component have a path between them.Given an unweighted and undirected graph represented as an adjacency list, write a function using pseudocode or C++ code which will return the number of vertices in the largest component of the graph. You do not need to write main method for reading and creating a graph. Assume that the graph is passed into your function which has the following signature:

```
unsigned int largest_component(unordered_map<int, vector<int>>& adjListGraph){
        // code here
}
```

Example input:

```
0: 2, 3
1: 4
2: 0, 6
3: 0
4: 1, 5
5: 4
6: 2
7:
```

Example output: 4

Explanation: There are three connected components: (0,2,3,6); (1,4,5); and (7). Of these, the largest has 4 vertices.

Also, state the Big O complexity of the solution in the worst case.

Applied Traversal

```cpp
#include <unordered_map>
#include <vector>
#include <unordered_set>
using namespace std;

unsigned int dfs(int node, unordered_map<int, vector<int>>& adjListGraph, unordered_set<int>& visited) {
    visited.insert(node);
    unsigned int size = 1;  // Start with the current node

    for (int neighbor : adjListGraph[node]) {
        if (visited.find(neighbor) == visited.end()) {
            size += dfs(neighbor, adjListGraph, visited);
        }
    }
    return size;
}

unsigned int largest_component(unordered_map<int, vector<int>>& adjListGraph) {
    unordered_set<int> visited;
    unsigned int maxComponentSize = 0;

    for (const auto entry : adjListGraph) {
        int node = entry.first;
        if (visited.find(node) == visited.end()) {
            unsigned int componentSize = dfs(node, adjListGraph, visited);
            if (componentSize > maxComponentSize) {
                maxComponentSize = componentSize;
            }
        }
    }

    return maxComponentSize;
}
```

BFS vs DFS

BFS

```cpp
string source = "A";
std::set<string> visited;
std::queue<string> q;

visited.insert(source);
q.push(source);
cout<<"BFS: ";

while(!q.empty())
{
	string u = q.front();
	cout << u;
	q.pop();
	vector<string> neighbors = graph[u];
	for(string v: neighbors)
	{
		if(visited.count(v)==0)
		{
			visited.insert(v);
			q.push(v);
		}
	}
}
```

DFS

```cpp
string source = "A";
std::set<string> visited;
std::stack<string> s;

visited.insert(source);
s.push(source);
cout<<“DFS: ";

while(!q.empty())
{
	string u = s.top();
	cout << u;
	s.pop();
	vector<string> neighbors = graph[u];
	for(string v: neighbors)
	{
		if(visited.count(v)==0)
		{
			visited.insert(v);
			s.push(v);
		}
	}
}
```

Theoretical Complexity: O(V+E)

BFS Pseudocode

- Write pseudocode/code for implementing the **Breadth First Search Algorithm** of a graph, G that takes a source vertex S as input. (8).

- Also, state the Big O complexity of the traversal in the worst case (2).

Graph Algorithm Mix n Match

- Finds the shortest paths in a weighted graph
- Find the minimum cost connected network
- Scheduling algorithm, list steps in a process
- Finds the shortest path in an unweighted graph

Prim’s or Kruskals
BFS
DFS
Topological Sort
Dijkstra’s Algorithm

Graph Algorithm Mix n Match

- Finds the shortest paths in a weighted graph
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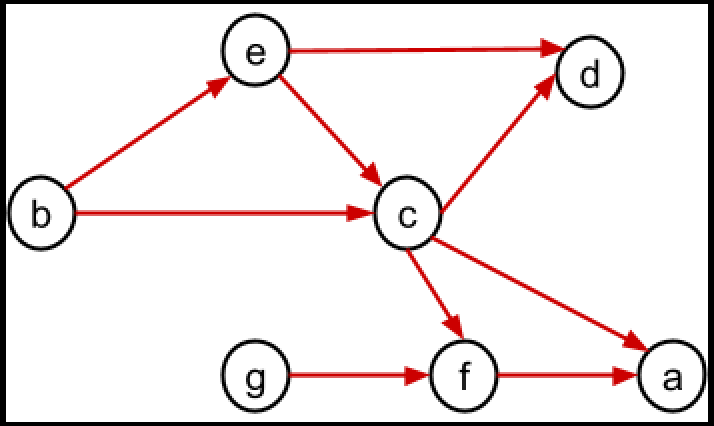
- Scheduling algorithm, list steps in a process
- Finds the shortest path in an unweighted graph

Prim's or Kruskals
BFS
DFS
Topological Sort
Dijkstra's Algorithm

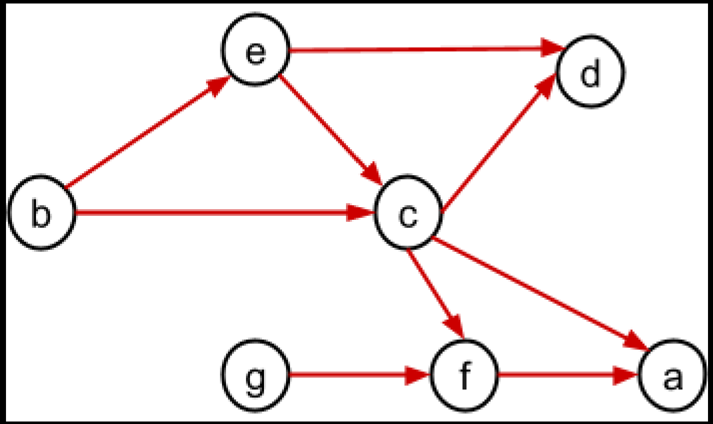
Alt Text for the Graph on Next Slide

Vertex: Neighbors of Vertex (Edges pointing from a vertex to the neighbor)

A: -
B: C, E
C: A, D, F
D: -
E: C, D
F: A
G: F

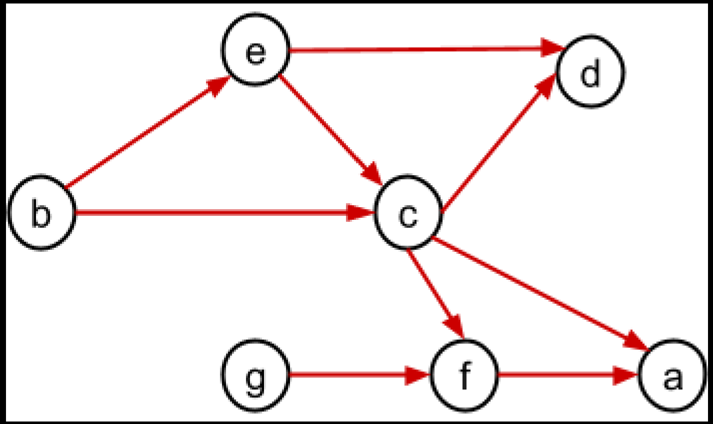


Which of the choices below represent a valid topological sort ordering of this graph?



- b, e, c, g, f, a, d
- b, a, c, g, f, e, d
- b, g, f, c, e, a, d
- b, e, c, g, a, f, d
- b, g, e, c, d, f, a
- b, f, c, g, a, e, d

Which of the choices below represent a valid topological sort ordering of this graph?



- b, e, c, g, f, a, d
- b, a, c, g, f, e, d
- b, g, f, c, e, a, d
- b, e, c, g, a, f, d
- b, g, e, c, d, f, a
- b, f, c, g, a, e, d

What does this code do?

```cpp
#include <set>
#include <stack>
using namespace std;

bool doSomething(const Graph& graph, int src, int dest)
{
    set<int> visited;
    stack<int> s;
    visited.insert(src);
    s.push(src);
    while(!s.empty())
    {
        int u = s.top();
        s.pop();
        for(auto v: graph.adjList[u])
        {
            if(v == dest)
                return true;
            if ((visited.find(v) == visited.end())) {
                visited.insert(v);
                s.push(v);
            }
        }
    }
    return false;
}
```

What does this code do?

```cpp
#include <set>
#include <stack>
using namespace std;

bool doSomething(const Graph& graph, int src, int dest)
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    visited.insert(src);
    s.push(src);
    while(!s.empty())
    {
        int u = s.top();
        s.pop();
        for(auto v: graph.adjList[u])
        {
            if(v == dest)
                return true;
            if ((visited.find(v) == visited.end())) {
                visited.insert(v);
                s.push(v);
            }
        }
    }
    return false;
}
```

Returns whether a given vertex is reachable from another vertex using DFS

Scenario

A county government maintains a network of roads. The county government has tabulated the cost of maintaining each road. They need to minimize the cost of road maintenance but ensure that all places in the county are accessible.

Which graph algorithm that we discussed in class could they use to solve this problem? What are the vertices, what are the edges, what are the edge values?

Scenario

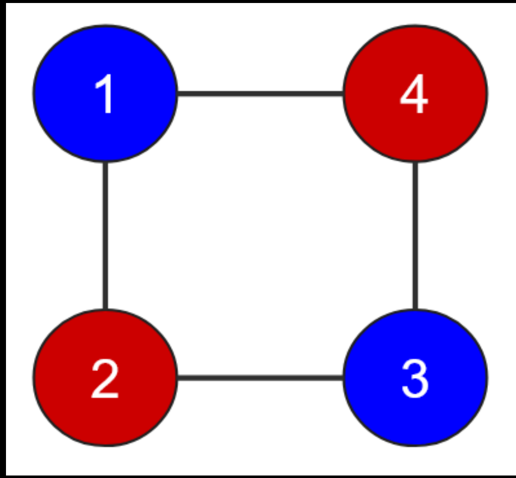
A county government maintains a network of roads. The county government has tabulated the cost of maintaining each road. They need to minimize the cost of road maintenance but ensure that all places in the county are accessible.

Which graph algorithm that we discussed in class could they use to solve this problem? What are the vertices, what are the edges, what are the edge values?

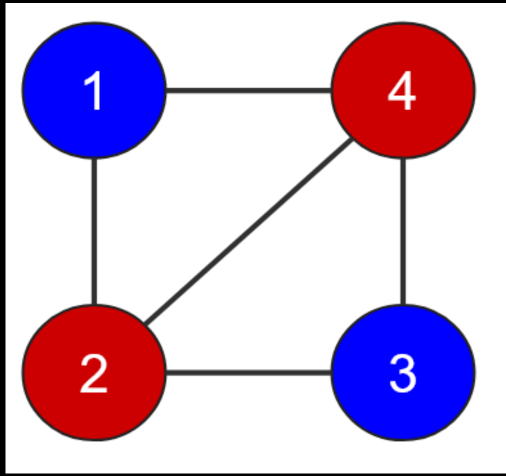
- Prim's or Kruskals algorithm for minimum spanning tree.
- Roads are edges.
- Ends of roads are vertices.
- Edge weights are cost for maintaining roads.

Scenario

A bipartite graph is known formally as an undirected graph whose nodes can be grouped into two sets such that no edge connects a node to another node in its own set (that is, every edge connects a node from one set to another). If the following graphs are colored using two colors denoting different sets, are these graphs bipartite?



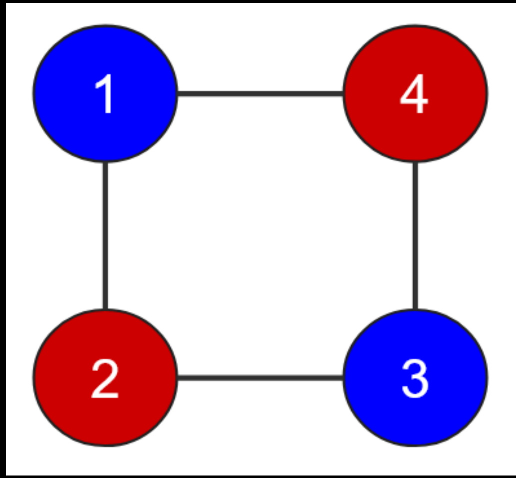
Graph 1



Graph 2

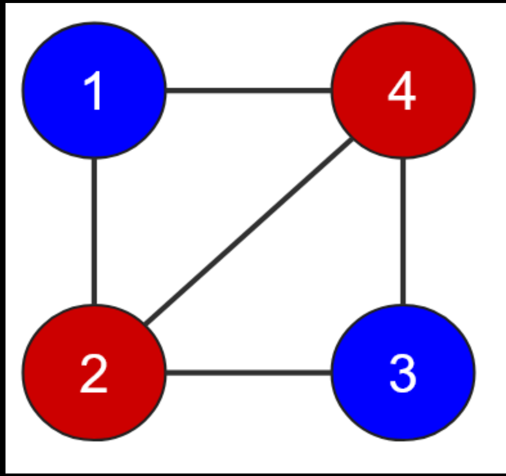
Scenario

A bipartite graph is known formally as an undirected graph whose nodes can be grouped into two sets such that no edge connects a node to another node in its own set (that is, every edge connects a node from one set to another). If the following graphs are colored using two colors denoting different sets, are these graphs bipartite?



Graph 1

Bipartite



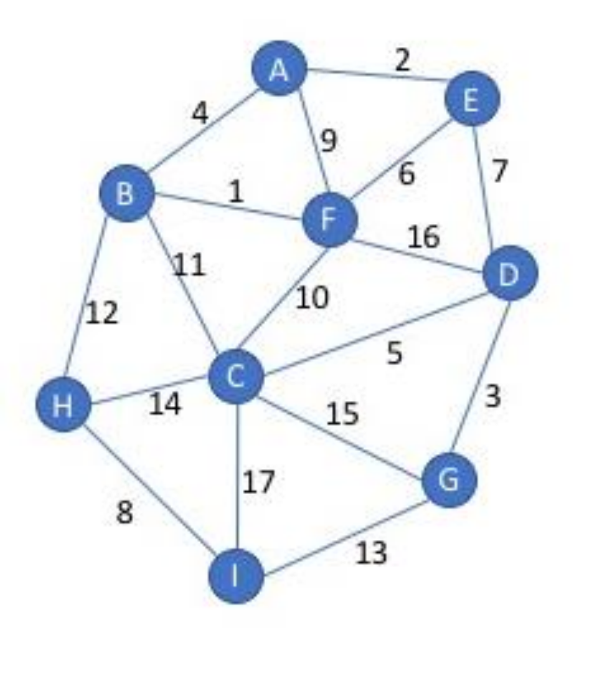
Graph 2

Non-Bipartite (Edge 2-4) in the same set

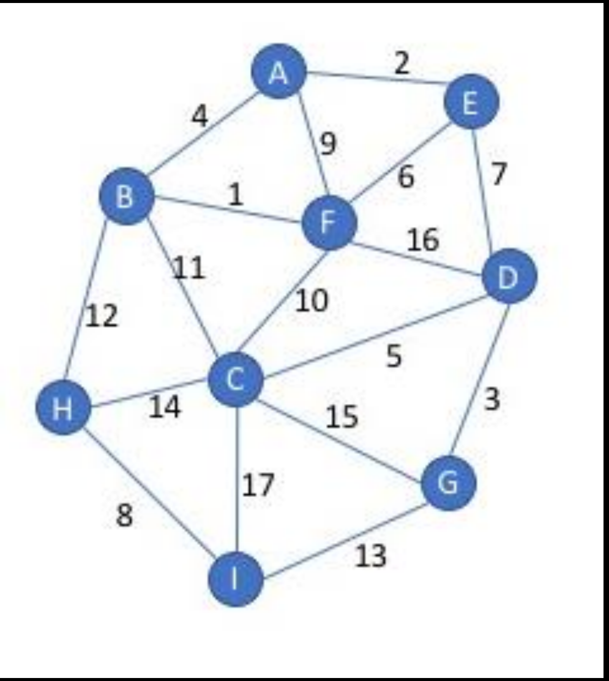
Alt Text for the Graph on Next Slide

Vertex: <Neighbors of Vertex (Edges pointing from a vertex to the neighbor), edge weight>

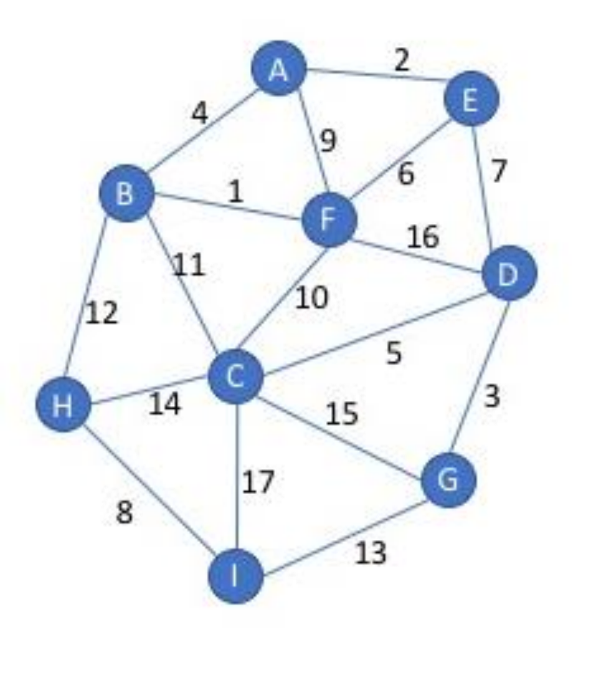
A: <B, 4>, <E, 2>, <F, 9>
B: <A, 4>, <C, 11>, <F, 1>, <H, 12>
C: <B, 11>, <D, 7>, <F, 10>, <G, 15>, <H, 14>, <I, 17>
D: <C, 5>, <E, 7>, <F, 16>, <G, 3>
E: <A, 2>, <D, 7>, <F, 6>
F: <A, 9>, <B, 1>, <C, 10>, <D, 16>, <E, 6>
G: <C, 15>, <D, 3>, <I, 13>
H: <B, 12>, <C, 14>, <I, 8>
I: <C, 17>, <G, 13>, <H, 8>



MST using Prims starting from “I”



MST using Prims starting from “I”

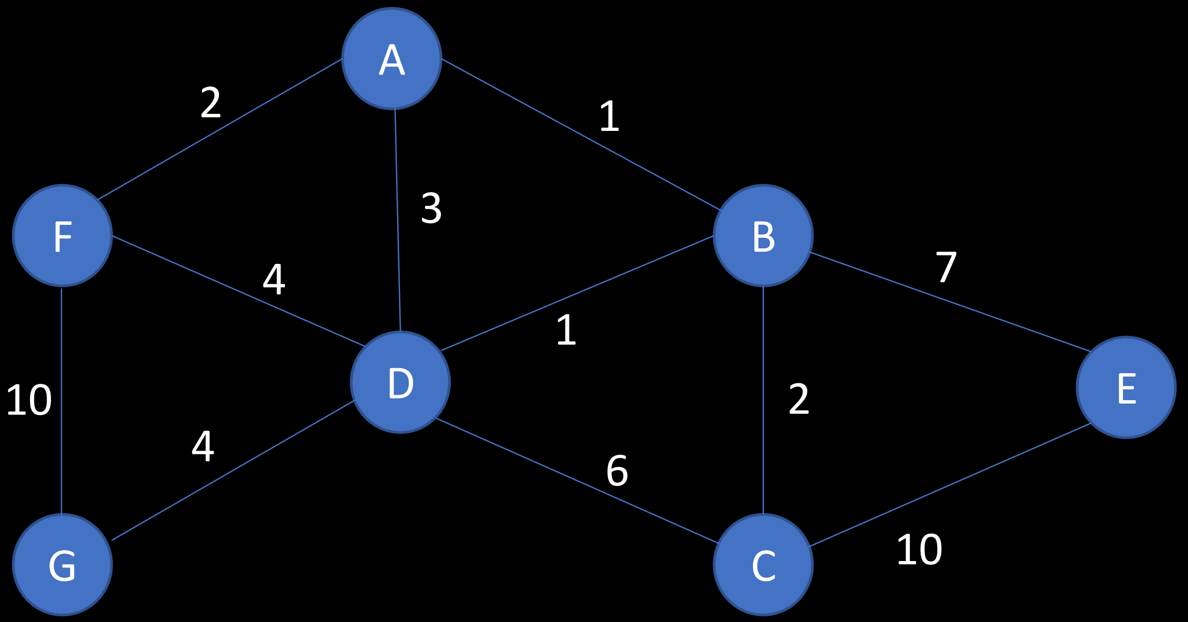


I H B F A E D G C

Alt Text for the Graph on Next Slide

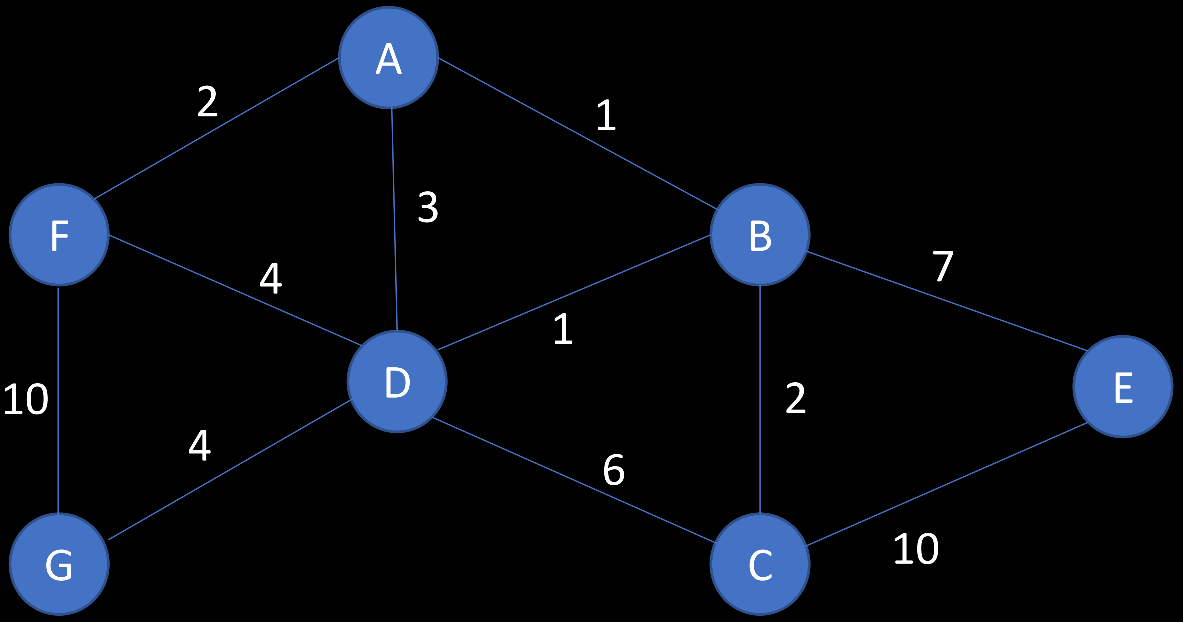
Vertex: <Neighbors of Vertex (Edges pointing from a vertex to the neighbor), edge weight>

A: <B, 1>, <D, 3>, <F, 2>
B: <A, 1>, <C, 2>, <D, 1>, <E, 7>
C: <B, 2>, <D, 6>, <E, 10>
D: <A, 3>, <B, 1>, <C, 6>, <F, 4>, <G, 4>
E: <B, 7>, <C, 10>
F: <A, 2>, <D, 4>, <G, 10>
G: <D, 4>, <F, 10>



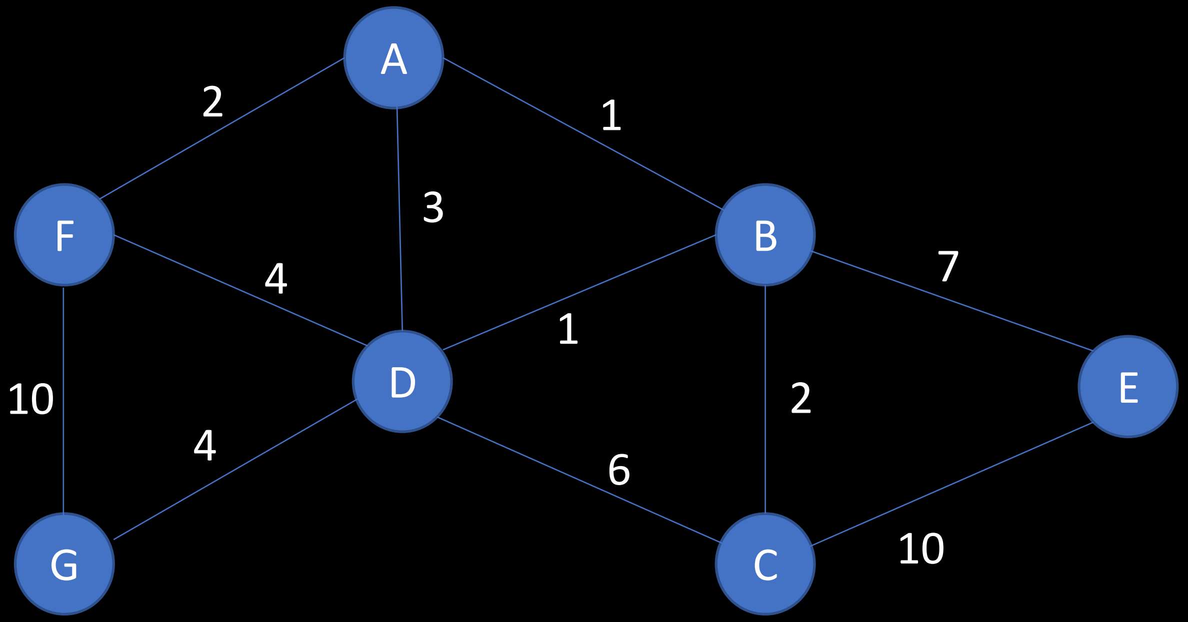
Dijkstra with A as source

v	D(v)	P(v)
A		
B		
C		
D		
E		
F		
G		



Dijkstra with A as source

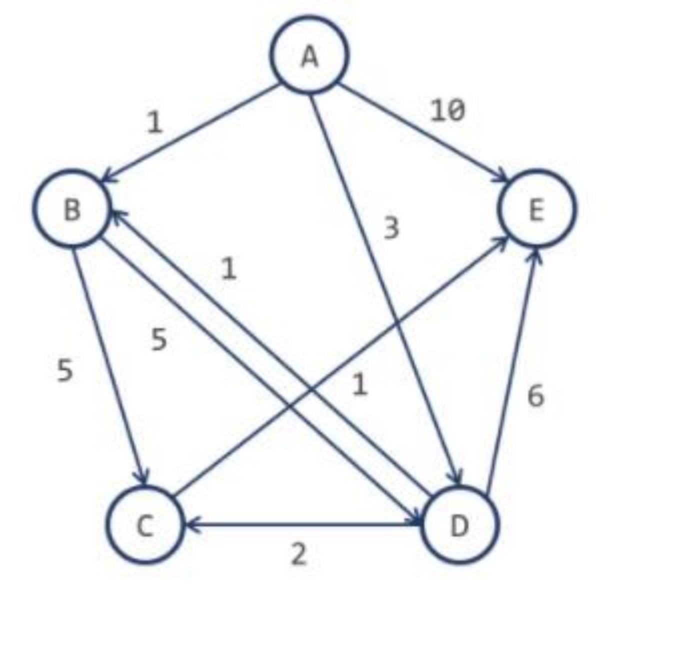
v	D(v)	P(v)
A	0	NA
B	1	A
C	3	B
D	2	B
E	8	B
F	2	A
G	6	D



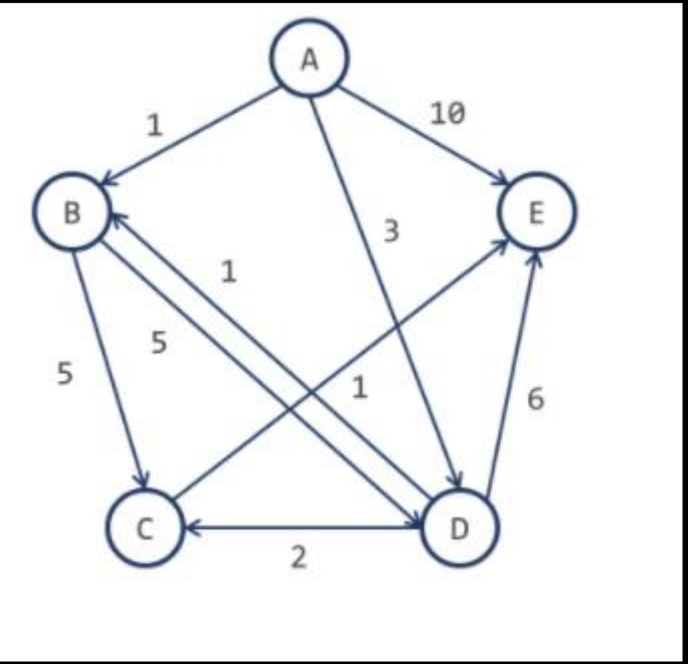
Alt Text for the Graph on Next Slide

Vertex: <Neighbors of Vertex (Edges pointing from a vertex to the neighbor), edge weight>

A: <B, 1>, <D, 3>, <E, 10>
B: <C, 5>, <D, 5>
C: <E, 1>
D: <B, 1>, <C, 2>, <E, 6>
E: -

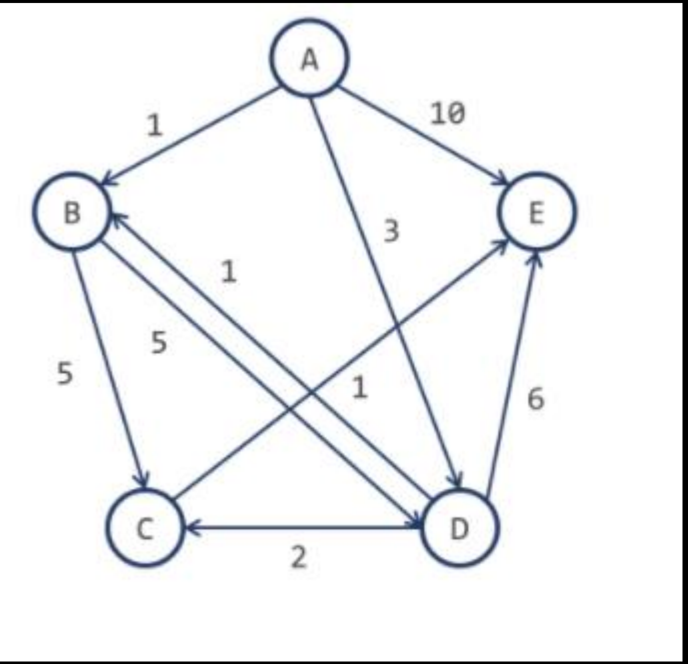


Dijkstra with A as source



V	d(V)	p(V)
B		
C		
D		
E		

Dijkstra with A as source



V	d(V)	p(V)
B	1	A
C	5	D
D	3	A
E	6	C

Algorithmic Paradigms

Algorithmic Paradigms

	Properties	Examples
Brute Force	▪ Generate and Test an Exhaustive Set of all possible combinations ▪ Can be computationally very expensive ▪ Guarantees optimal solution	▪ Finding divisors of a number, n by checking if all numbers from 1..n divides n without remainder ▪ Finding duplicates using all combinations ▪ Bubble/Selection Sort
Divide and Conquer	▪ Break the problem into subcomponents typically using recursion ▪ Solve the basic component ▪ Combine the solutions to sub-problems	▪ Quick Sort ▪ Merge Sort ▪ Binary Search ▪ Peak Finding
Dynamic Programming	▪ Optimal substructure: solution to a large problem can be obtained by solution to a smaller optimal problems ▪ Overlapping sub-problems: space of sub-problems must be small, that is, any recursive algorithm solving the problem should solve the same sub-problems over and over, rather than generating new sub-problems. ▪ Guarantees optimal solution	▪ Fibonacci Sequence ▪ Assembly Scheduling ▪ Knapsack
Greedy Algorithms	▪ Local optimal solutions at each stage ▪ Does not guarantee optimal solution	▪ Prim’s Algorithm ▪ Dijkstra’s Algorithm ▪ Kruskal’s Algorithm

Bin Packing

If we have packets that each require 7 units, 8 units, 2 units and 3 units of space, how many minimum bins are required to store all the four packets if each bin can take at most 10 units of space using the following Greedy strategies

- First Fit: scan the bins and place the new item in the first bin that is large enough.

- Best Fit: scan the bins and place the new item in the bin that finds the spot that creates the smallest empty space

Algorithm for Huffman Encoding

Given this file, generate a Huffman Tree and identify the codes of each character.

```
care racecar era
```

Use the following constraints when building the tree:

- the node with a lower frequency is attached to the left in case two nodes are merged after extraction from the priority queue;
- if two nodes have the same priority when merging, then the merged nodes are resolved as follows:
  - if both nodes have letters (a.k.a. left nodes) then the letter with lower ascii value will be the left node when combining two nodes into a tree.
  - if one or both nodes have cumulative frequencies, then the node with more number of nodes in the tree will be attached to the right, in other words, the smaller tree will be towards the left.
- traversing left from a node appends '0' to the Huffman code and traversing right appends '1'.

Algorithm for Huffman Encoding

1. Create a table with symbols and their frequencies

2. Construct a set of trees with root nodes that contain each of the individual symbols and their weight (frequency).

3. Place the set of trees into a min priority queue.

4. while the priority queue has more than one item
   Remove the two trees with the smallest weights.
   Combine them into a new binary tree in which the weight of the tree root is the sum of the weights of its children.
   Insert the newly created tree back into the priority queue.

5. Traverse the resulting tree to obtain binary codes for characters

Algorithm for Huffman Encoding

1. Create a table with symbols and their frequencies

```
care racecar era
```

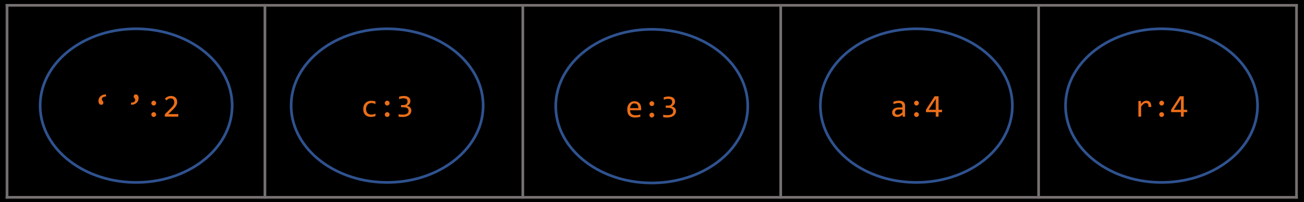
Character	Frequency
a	4
r	4
c	3
e	3
‘ ’	2

Algorithm for Huffman Encoding

2. Construct a set of trees with root nodes that contain each of the individual symbols and their weight (frequency).

3. Place the set of trees into a min priority queue.

Character	Frequency
a	4
r	4
c	3
e	3
‘ ’	2

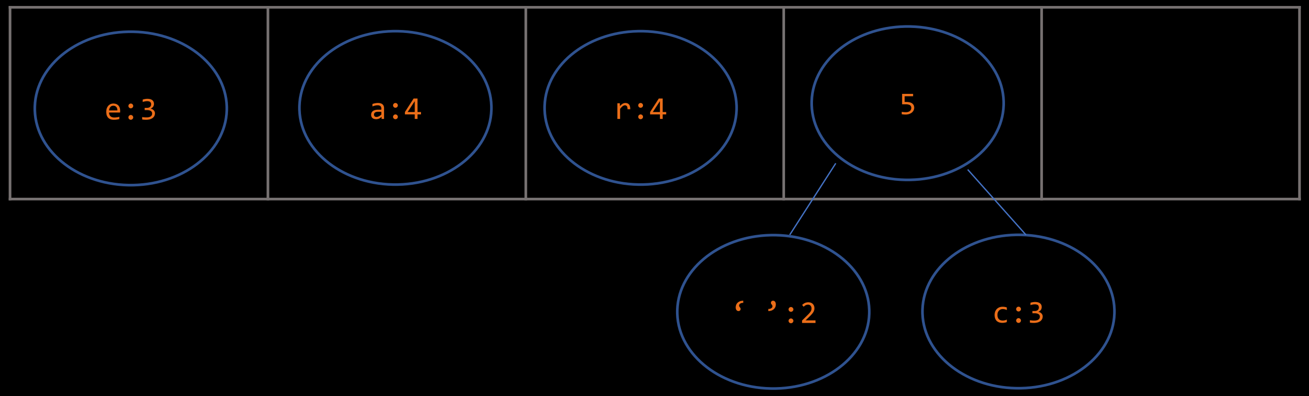


Algorithm for Huffman Encoding

2. Construct a set of trees with root nodes that contain each of the individual symbols and their weight (frequency).

3. Place the set of trees into a min priority queue.

Character	Frequency
a	4
r	4
c	3
e	3
‘ ’	2

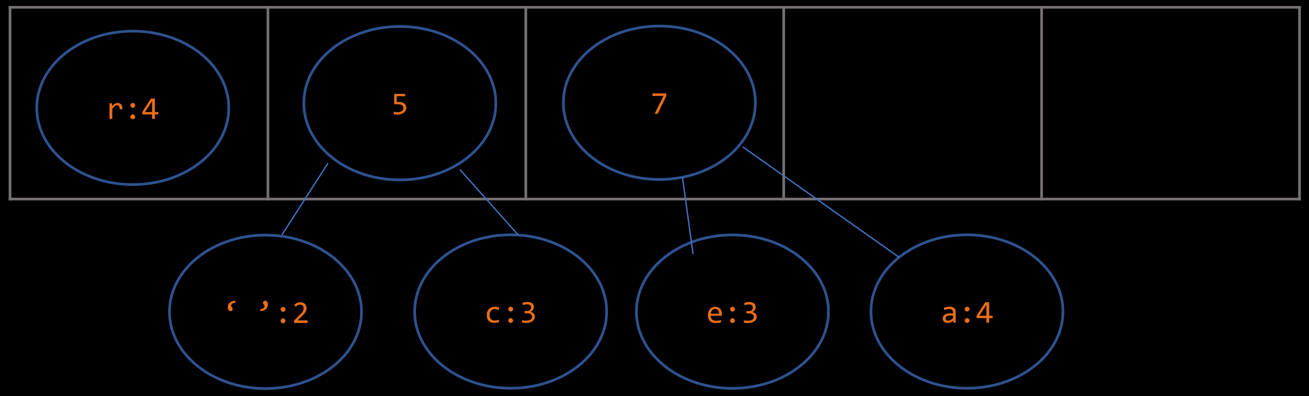


Algorithm for Huffman Encoding

2. Construct a set of trees with root nodes that contain each of the individual symbols and their weight (frequency).

3. Place the set of trees into a min priority queue.

Character	Frequency
a	4
r	4
c	3
e	3
‘ ’	2

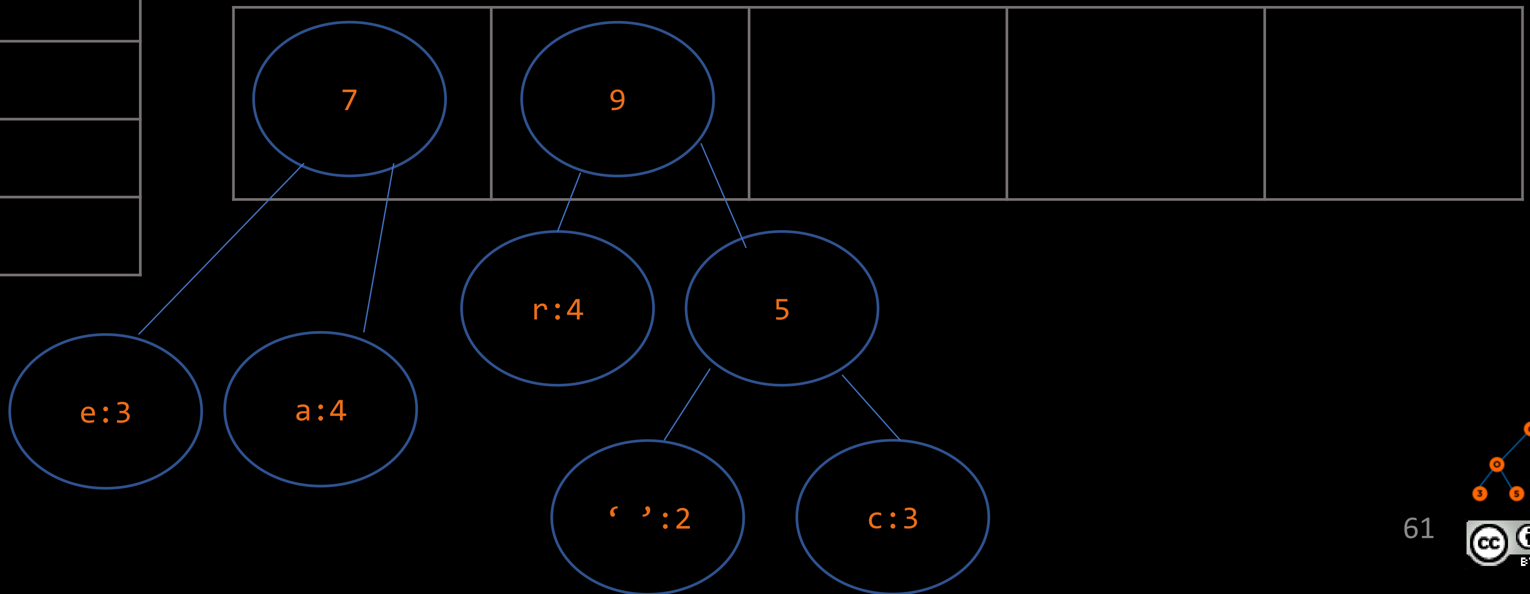


Algorithm for Huffman Encoding

2. Construct a set of trees with root nodes that contain each of the individual symbols and their weight (frequency).

3. Place the set of trees into a min priority queue.

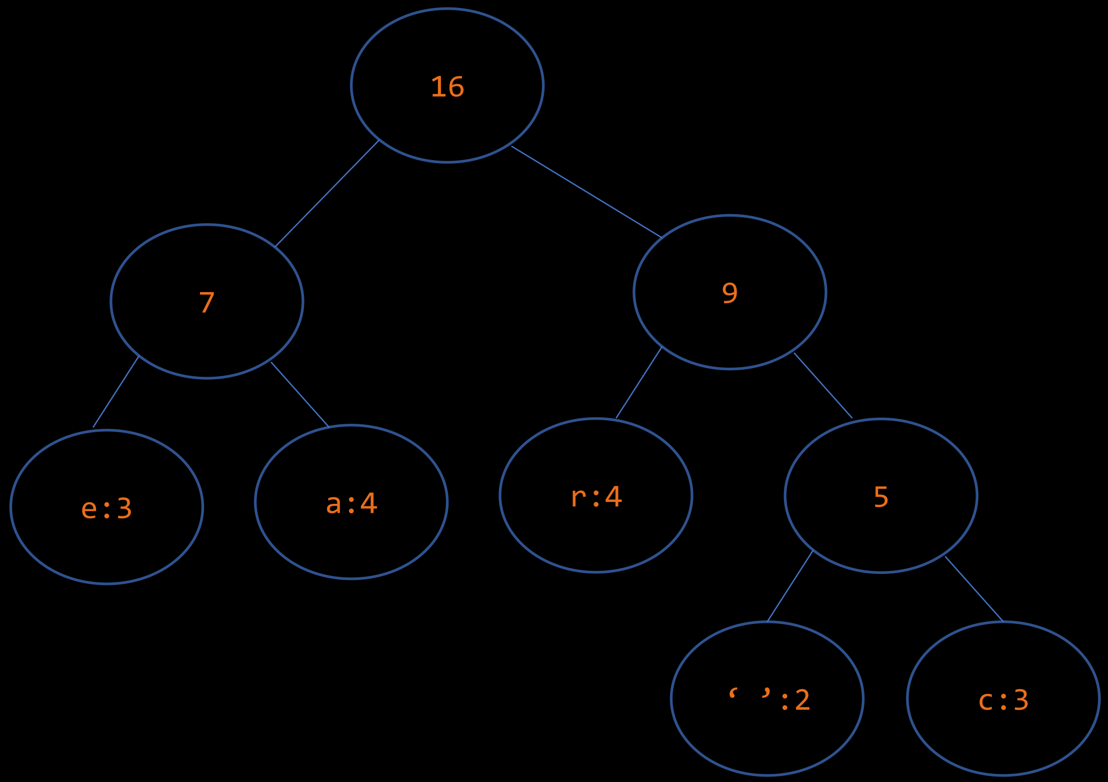
Character	Frequency
a	4
r	4
c	3
e	3
‘ ’	2



Algorithm for Huffman Encoding

4. while the priority queue has more than one item
    Remove the two trees with the smallest weights.
    Combine them into a new binary tree in which the weight of the tree root is the sum of the weights of its children.
    Insert the newly created tree back into the priority queue.

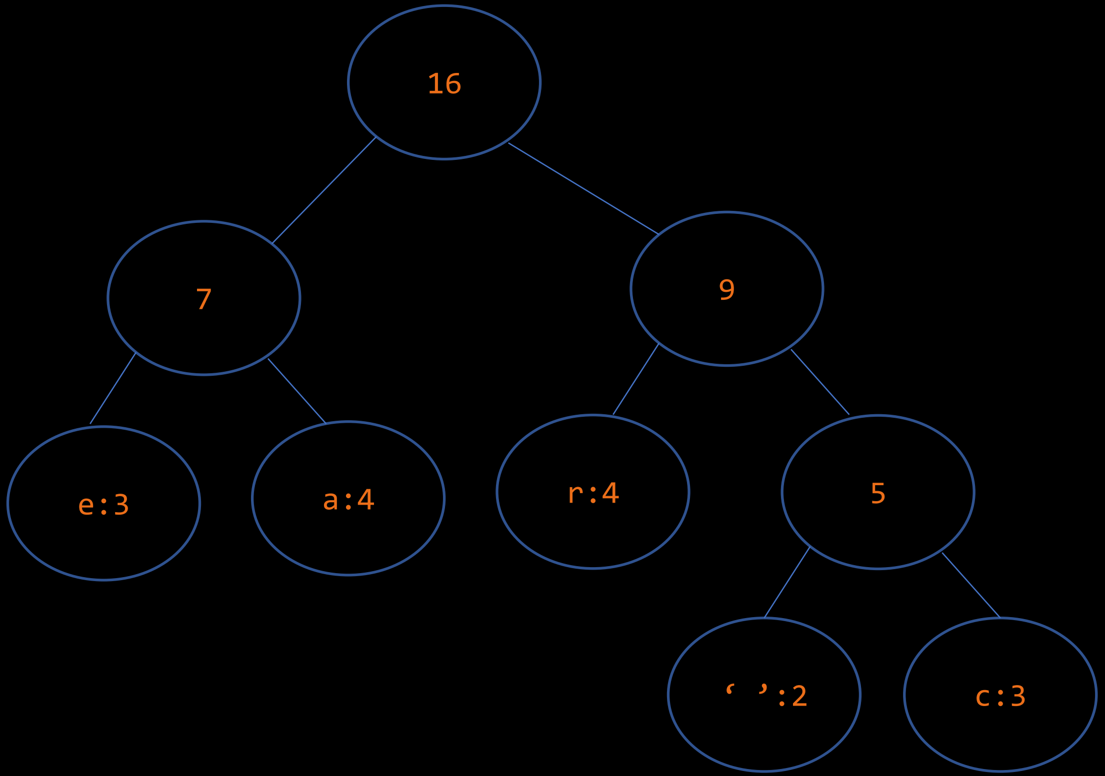
Character	Frequency
a	4
r	4
c	3
e	3
‘ ’	2



Algorithm for Huffman Encoding

4. while the priority queue has more than one item
   Remove the two trees with the smallest weights.
   Combine them into a new binary tree in which the weight of the tree root is the sum of the weights of its children.
   Insert the newly created tree back into the priority queue.

Character	Frequency	Huffman Code
a	4	01
r	4	10
c	3	111
e	3	00
‘ ’	2	110



Questions