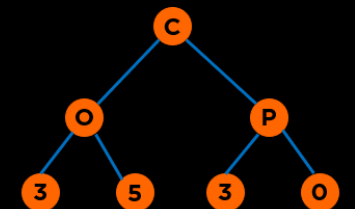


Final Exam Review



Categories of Data Structures

Linear Ordered

Lists

Stacks

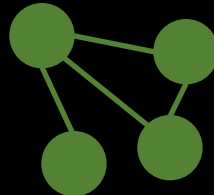
Queues



Non-linear Ordered

Trees

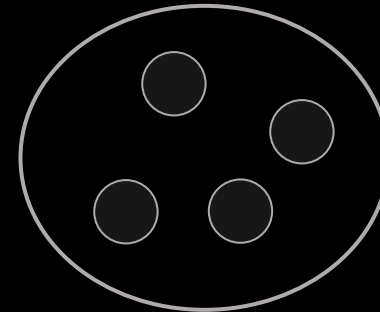
Graphs



Not Ordered

Sets

Tables/Maps



Announcements

- If you are a student in campus or hybrid sections (including Section OVER), you must take the exam between 2 pm - 10 pm EST on April 12 (this Wednesday). This means you must start by 8 pm EST or else you will lose time.
- The exam will be on Honorlock.
- It will cover Modules 1-3, 5-8 (everything till Kruskal's Algorithm).
- Topics and expectations guide on Canvas.
- You are allowed to use one page of crib sheet with handwritten notes + 4 sheets of blank scratch paper.

Mini Review – Linked Lists

Consider a class `List` that implements an `ordered` list backed by a singly linked list with a head pointer. The invariant “`ordered`” is maintained always. Given that representation, what is the worst-case time complexity of the following operations? Assume the list is sorted in ascending order.

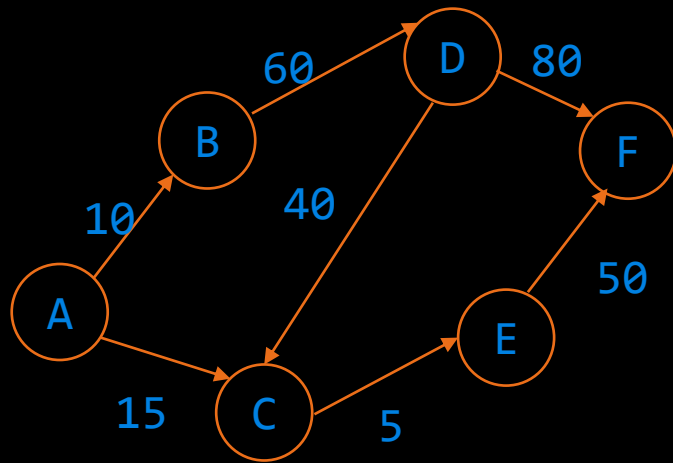
- A. Insert an item
- B. Finding the minimum element
- C. Delete the largest element from list
- D. Finding the largest element
- E. Finding a random element, `n`
- F. Deleting the minimum element in the list

Mini Review – Linked Lists

Consider a class List that implements an **ordered** list backed by a singly linked list with a head pointer. The invariant “**ordered**” is maintained always. Given that representation, what is the worst-case time complexity of the following operations? Assume the list is sorted in ascending order.

- A. Insert an item : $O(n)$
- B. Finding the minimum element : $O(1)$
- C. Delete the largest element from list : $O(n)$
- D. Finding the largest element : $O(n)$
- E. Finding a random element, n : $O(n)$
- F. Deleting the minimum element in the list : $O(1)$

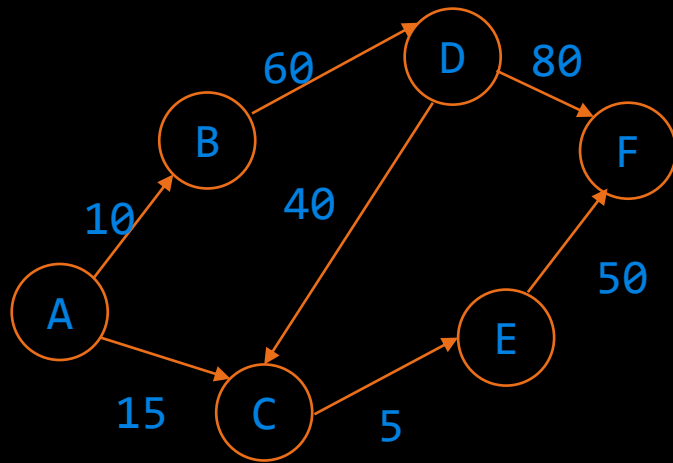
Common Representations



G

- Edge List
- Adjacency Matrix
- Adjacency List

Edge List

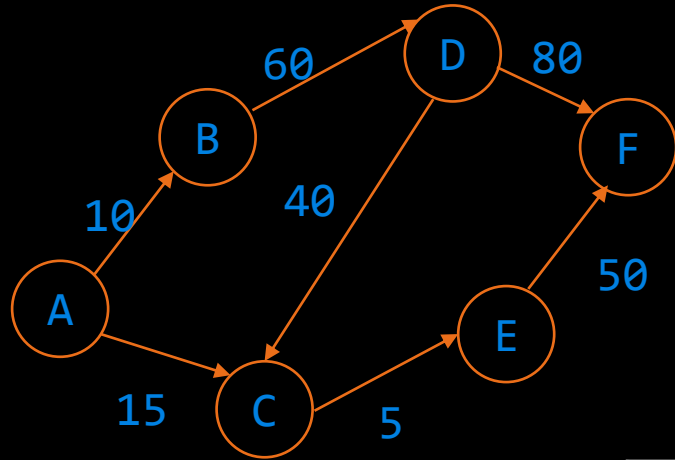


G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

$G = \{(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)\}$

Edge List



$G = \{(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)\}$

G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

Common Operations:

1. Connectedness

Is A connected to B?

$\sim O(E)$

2. Adjacency

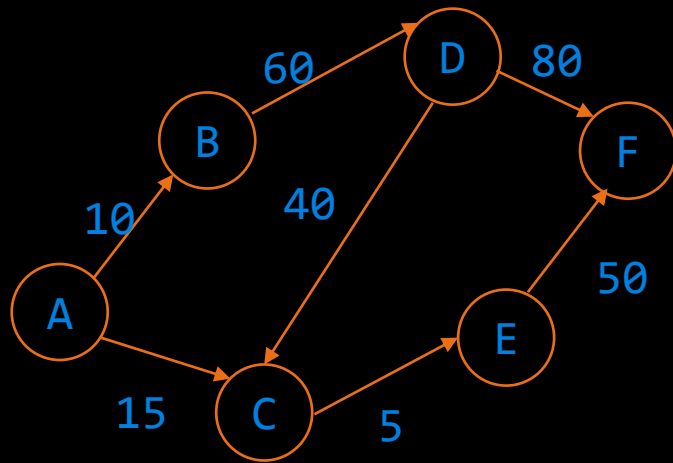
What are A's adjacent nodes?

$\sim O(E)$

$O(|E|) \sim O(|V| * |V|)$

Space: $O(E)$

Adjacency Matrix



G

A

B

C

D

E

F

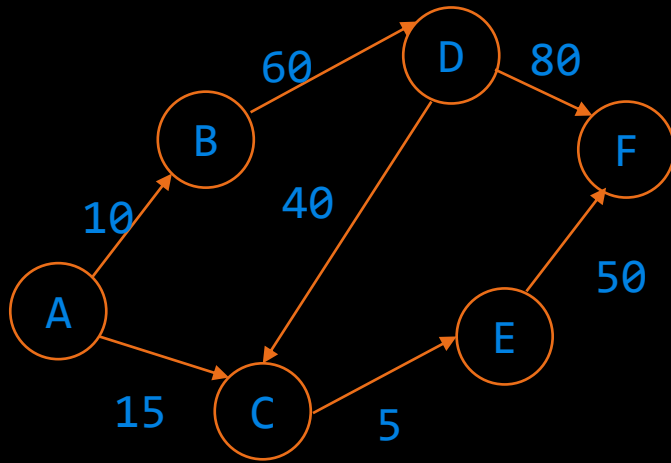
	A	B	C	D	E	F
A	0	10	15	0	0	0
B	0	0	0	60	0	0
C	0	0	0	0	5	0
D	0	0	40	0	0	80
E	0	0	0	0	0	50
F	0	0	0	0	0	0

Insertion:

$G[\text{from}][\text{to}] = \text{weight};$ (if there is an edge, “from” \rightarrow “to”)

$G[\text{from}][\text{to}] = 0;$ (otherwise)

Adjacency Matrix Implementation



Input

```
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50
```

G

Map

```
A 0
B 1
C 2
D 3
E 4
F 5
```

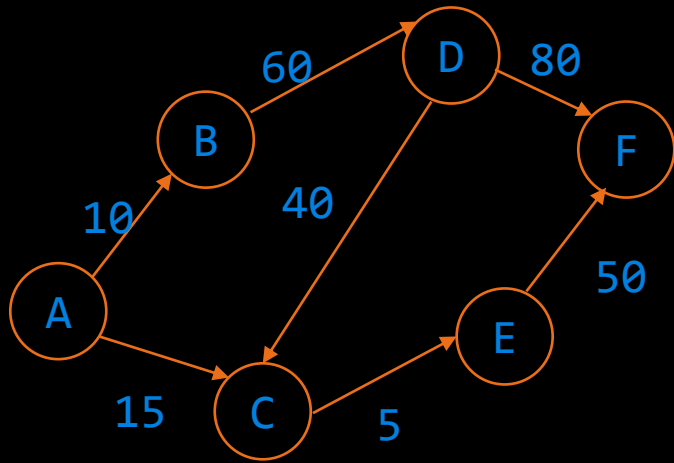
	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Insertion:

```
G[from][to] = weight; (if there is an edge, "from" -> "to")
G[from][to] = 0;      (otherwise)
```

```
01 #include <iostream>
02 #include<map>
03 #define VERTICES 6
04 using namespace std;
05 int main()
06 {
07     int no_lines, wt, j=0;
08     string from, to;
09     int graph [VERTICES][VERTICES] = {0};
10     map<string, int> mapper;
11     cin >> no_lines;
12     for(int i = 0; i < no_lines; i++)
13     {
14         cin >> from >> to >> wt;
15         if (mapper.find(from) == mapper.end())
16             mapper[from] = j++;
17         if (mapper.find(to) == mapper.end())
18             mapper[to] = j++;
19         graph[mapper[from]][mapper[to]] = wt;
20     }
21     return 0;
22 }
```

Adjacency Matrix



G

Map

A 0
B 1
C 2
D 3
E 4
F 5

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Common Operations:

1. Connectedness

Is A connected to B?

$G["A"]["B"] \sim O(1)$

2. Adjacency

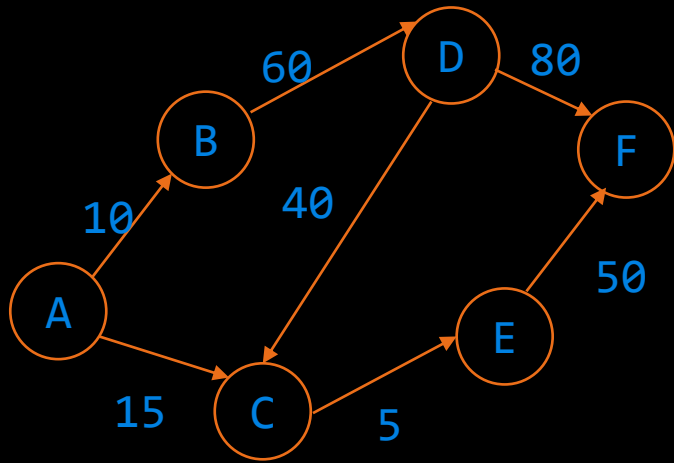
What are A's adjacent nodes?

for each element x in $G["A"]$
if $x \neq 0$

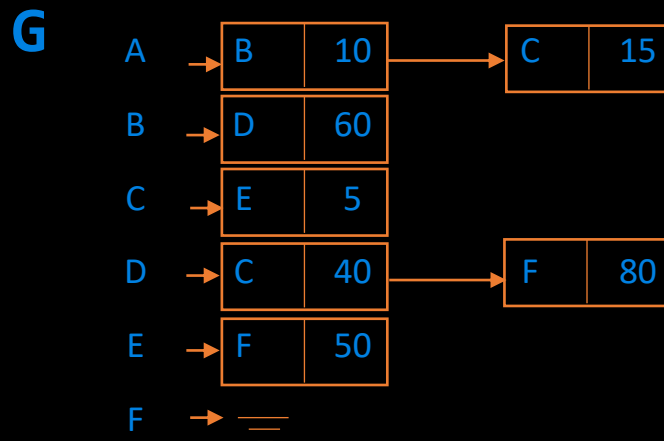
$\sim O(|V|)$

Space: $O(|V| * |V|)$

Adjacency List



Sparse Graph:
Edges \sim Vertices



Common Operations:

1. Connectedness

Is A connected to B?
for each element x in G["A"]
if x != 'B'
~ $O(\text{outdegree}|V|)$

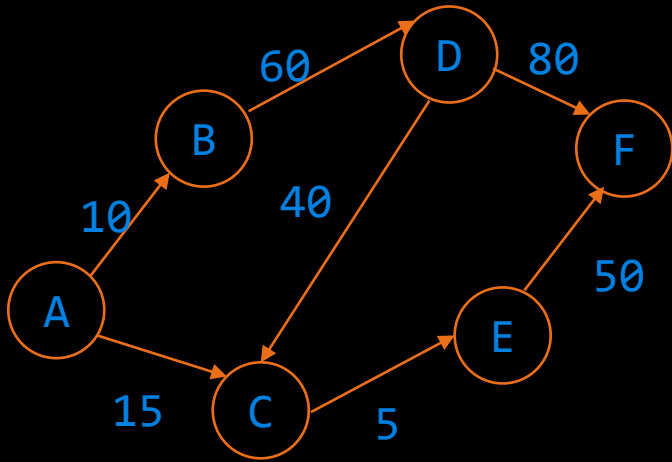
2. Adjacency

What are A's adjacent nodes?

$G["A"] \sim O(\text{outdegree}|V|)$

Space: $O(|V| + |E|)$

Adjacency List Implementation



Input

7

A B 10

A C 15

B D 60

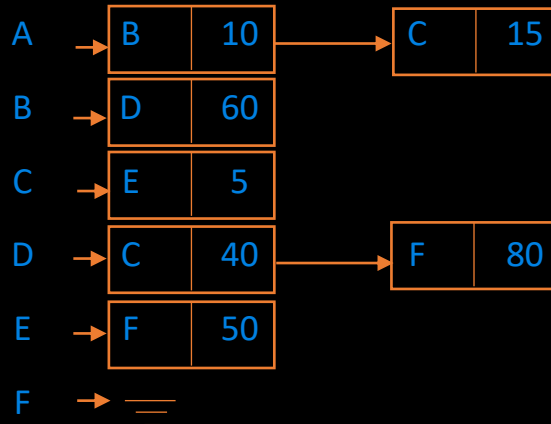
D C 40

C E 5

D F 80

E F 50

G



Insertion:

If to or from vertex not present add vertex

Otherwise add edge at the end of the list

```
01 #include <iostream>
02 #include<map>
03 #include<vector>
04 #include<iterator>
05 using namespace std;
06
07 int main()
08 {
09     int no_lines;
10     string from, to, wt;
11     map<string, vector<pair<string,int>>> graph;
12     cin >> no_lines;
13     for(int i = 0; i < no_lines; i++)
14     {
15         cin >> from >> to >> wt;
16         graph[from].push_back(make_pair(to, stoi(wt)));
17         if (graph.find(to)==graph.end())
18             graph[to] = {};
19     }
20 }
```

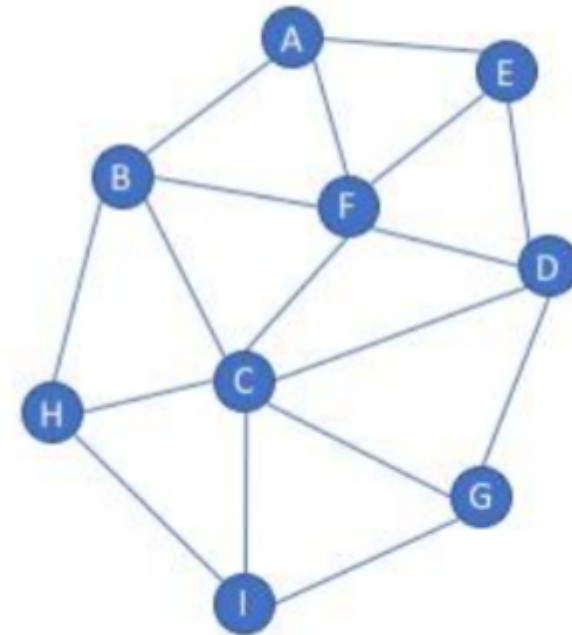
Graph Implementation

	Edge List	Adjacency Matrix	Adjacency List
Time Complexity: Connectedness	$O(E)$	$O(1)$	$O(\text{outdegree}(V))$
Time Complexity: Adjacency	$O(E)$	$O(V)$	$O(\text{outdegree}(V))$
Space Complexity	$O(E)$	$O(V*V)$	$O(V+E)$

Graph - BFS

- Which of the following are valid breadth first search traversals for this graph?

- a) AFBEDCHGI
- b) ICHGBFDAE
- c) DCFEGHIBA
- d) EAFDBHCIG
- e) FAEDCBGHIH



Graph - BFS

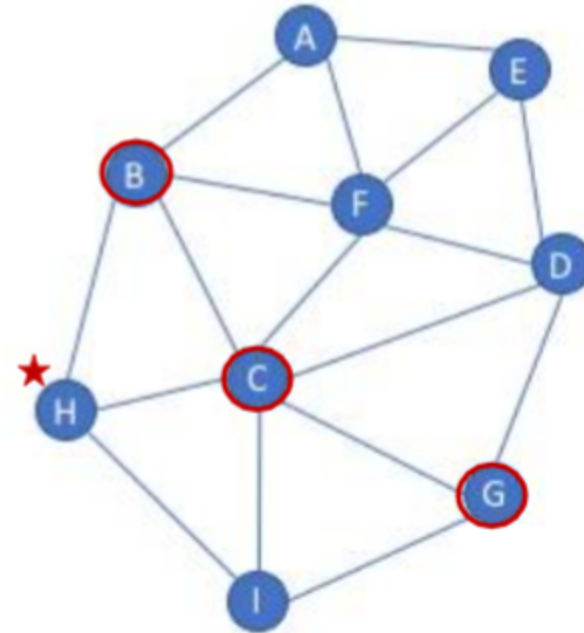
- Which of the following are valid breadth first search traversals for this graph?

- a) AFBEDCHGI
- b) ICHGBFDAE
- c) DCFEGHIBA
- d) EAFDBHCIG
- e) FAEDCBGHI

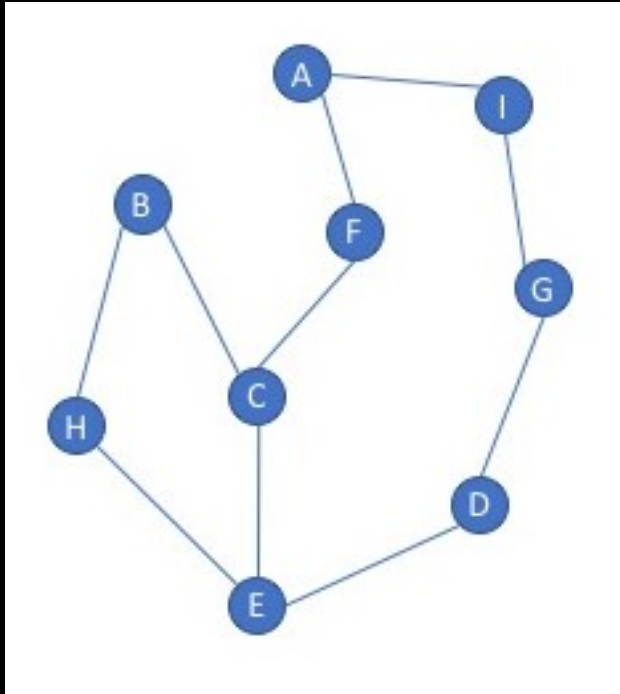
All the options except for d

Why not d?

** H is visited before C and G

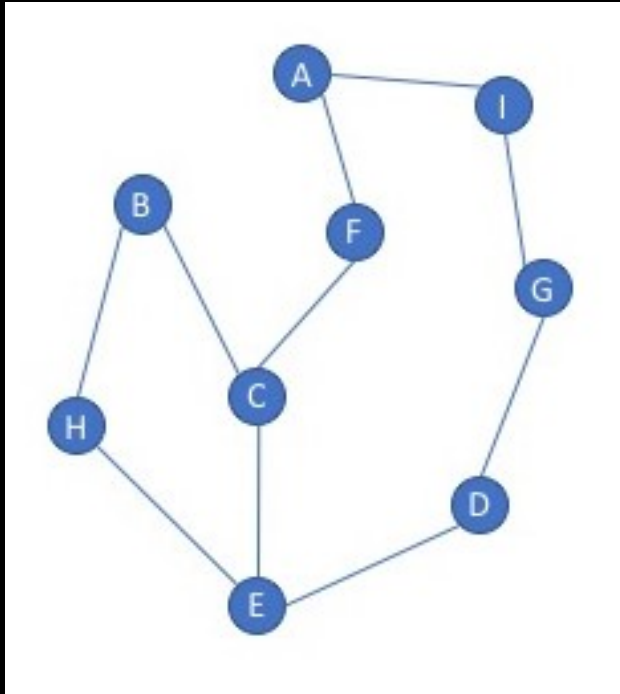


Valid DFS: Which DFS are valid?



- HECBDGIAF
- CEHBDGIAF
- AFCEHBIGD
- DECBHFAIG

Valid DFS: Which DFS are valid?



- HECBDGIAF
- **CEHBDGIAF**
- AFCEHBIGD
- **DECBHFAIG**

Graph Algorithm Mix n Match

- Finds the shortest paths in a weighted graph
- Find the minimum cost connected network
- Scheduling algorithm, list steps in a process
- Finds the shortest path in an unweighted graph

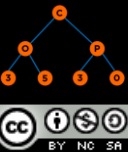
Prim's or Kruskals

BFS

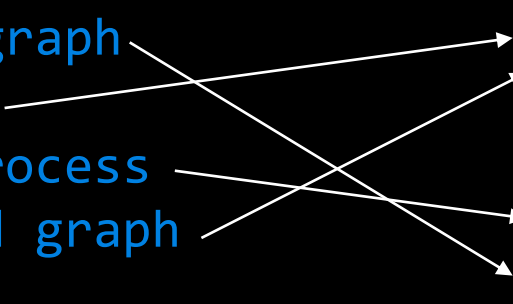
DFS

Topological Sort

Dijkstra's Algorithm



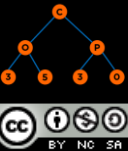
Graph Algorithm Mix n Match

- Finds the shortest paths in a weighted graph
 - Find the minimum cost connected network
 - Scheduling algorithm, list steps in a process
 - Finds the shortest path in an unweighted graph
- Prim's or Kruskals
BFS
DFS
Topological Sort
Dijkstra's Algorithm
- 

What does this code do?

```
#include <set>
#include <stack>
using namespace std;

bool doSomething(const Graph& graph, int src, int dest)
{
    set<int> visited;
    stack<int> s;
    visited.insert(src);
    s.push(src);
    while(!s.empty())
    {
        int u = s.top();
        s.pop();
        for(auto v: graph.adjList[u])
        {
            if(v == dest)
                return true;
            if ((visited.find(v) == visited.end())) {
                visited.insert(v);
                s.push(v);
            }
        }
    }
    return false;
}
```

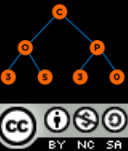


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    while(!s.empty())
    {
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        s.pop();
        for(auto v: graph.adjList[u])
        {
            if(v == dest)
                return true;
            if ((visited.find(v) == visited.end())) {
                visited.insert(v);
                s.push(v);
            }
        }
    }
    return false;
}
```

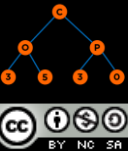
Returns whether a given vertex is reachable from another vertex using DFS



Scenario

A county government maintains a network of roads. The county government has tabulated the cost of maintaining each road. They need to minimize the cost of road maintenance but ensure that all places in the county are accessible.

Which graph algorithm that we discussed in class could they use to solve this problem? What are the vertices, what are the edges, what are the edge values?



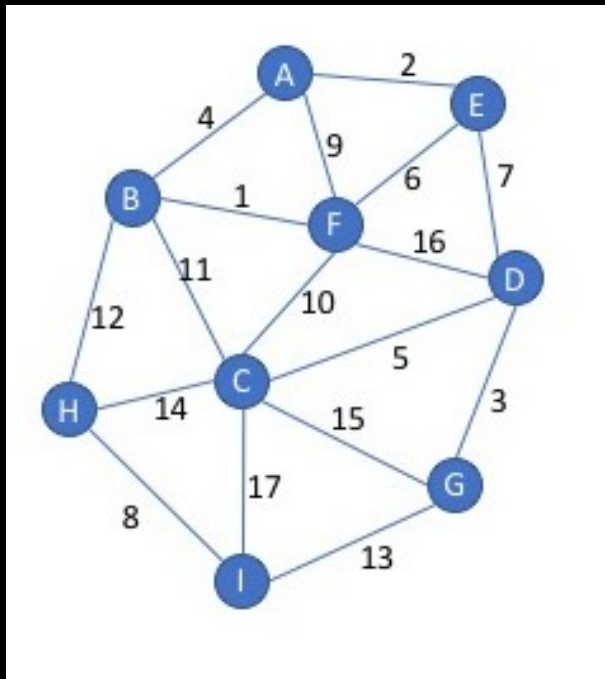
Scenario

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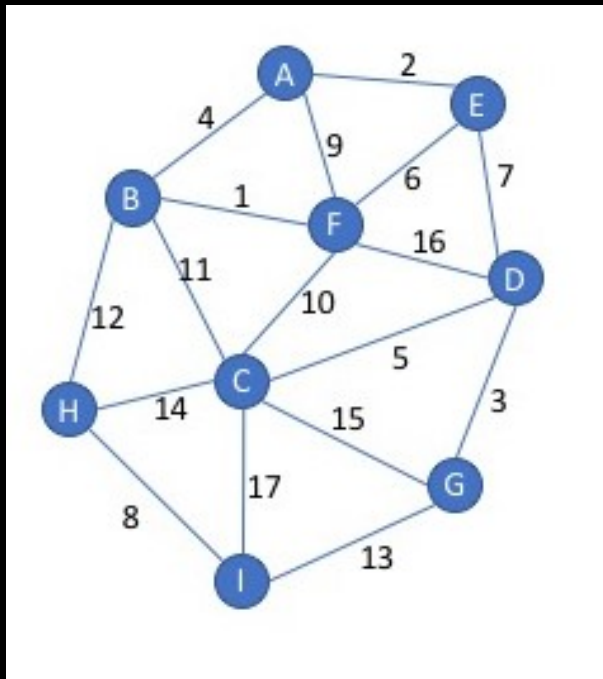
Which graph algorithm that we discussed in class could they use to solve this problem? What are the vertices, what are the edges, what are the edge values?

- Prim's or Kruskal's algorithm for minimum spanning tree.
- Roads are edges.
- Ends of roads are vertices.
- Edge weights are cost for maintaining roads.

MST using Prim's starting from "I"



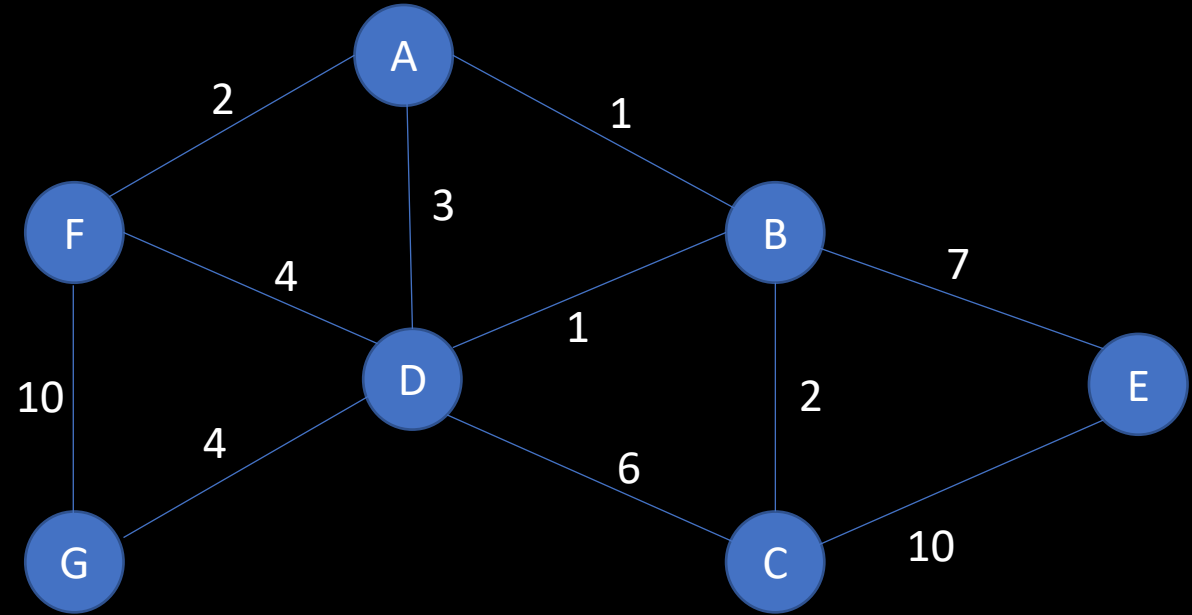
MST using Prim's starting from "I"



I H B F A E D G C

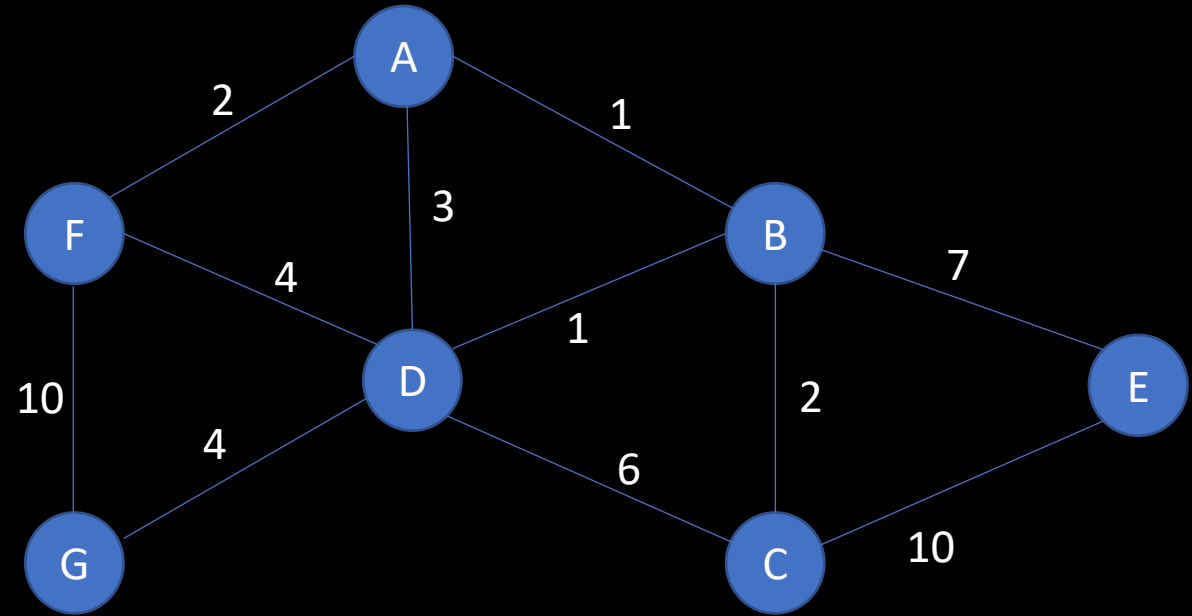
Dijkstra with A as source

v	D(v)	P(v)
A		
B		
C		
D		
E		
F		
G		

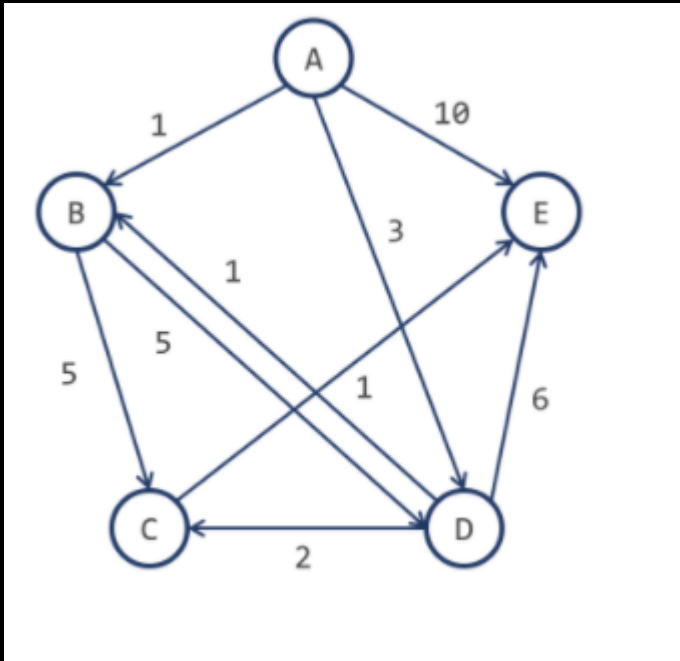


Dijkstra with A as source

v	D(v)	P(v)
A	0	NA
B	1	A
C	3	B
D	2	B
E	8	B
F	2	A
G	6	D



Dijkstra with A as source

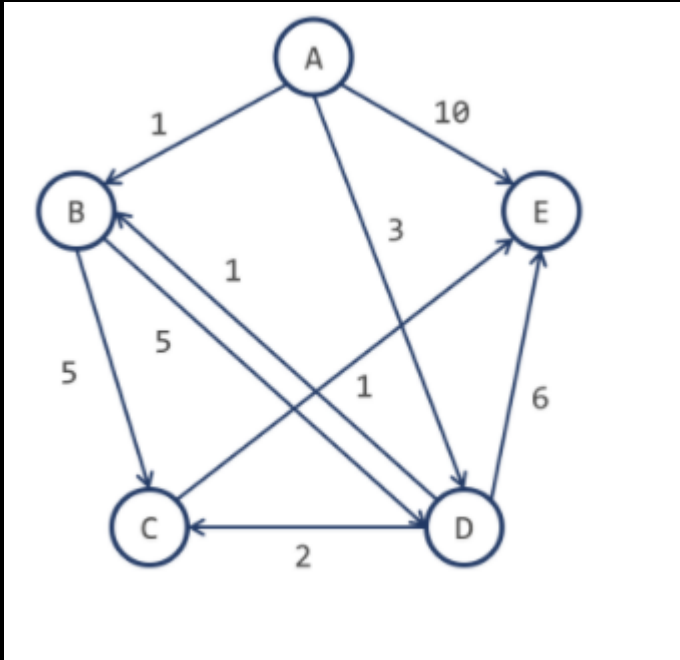


V
B
C
D
E

$d(V)$

$p(V)$

Dijkstra with A as source



V
B
C
D
E

$d(V)$
1
5
3
6

$p(V)$
A
D
A
C