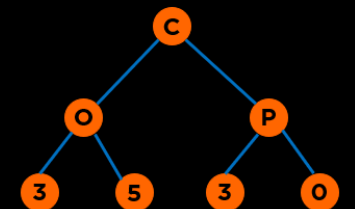


Final Exam Review



Categories of Data Structures

Linear Ordered

Lists

Stacks

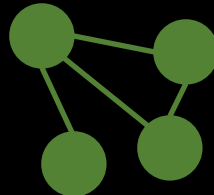
Queues



Non-linear Ordered

Trees

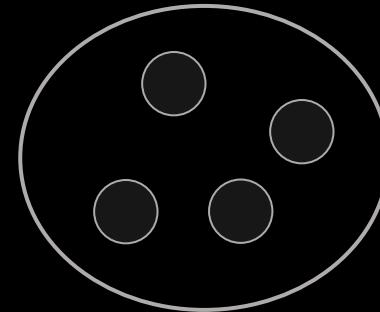
Graphs



Not Ordered

Sets

Tables/Maps



Announcements

- You must take the exam between 1pm - 6pm EST on November 30 at this link: [Final Exam: Exam 2 - Honorlock](#)
- The exam will be over Honorlock and you are allowed one double sided handwritten sheet of notes.
- The exam duration is 2 hours. This means you must start by 4 pm EST or else you will lose time.
- Exam 2 Topics and Expectations Guide: [Link](#)
- Exam reviews: [Exam 2 Resources](#)

Mentimeter

```
1  #include <iostream>
2  #include <vector>
3
4  int sequence(int n)
5  {
6      std::vector<int> dp1(n + 1, 0);
7      std::vector<int> dp2(n + 2, 0);
8      dp1[1] = 1;
9      dp2[1] = 1;
10
11     for(int i = 2; i <= n; i++)
12     {
13         dp1[i] = dp1[i - 1] + dp1[i - 2];
14         dp2[i] = dp1[i] * dp1[i];
15     }
16
17     return dp2[n];
18 }
19
20 int main()
21 {
22     std::cout << sequence(6);
23     return 0;
24 }
25
```

Mentimeter

	0	1	2	3	4	5	6	7
{}	0	0	0	0	0	0	0	0
{1}	0							
{1, 2}	0							
{1, 2, 3}	0							
{1, 2, 3, 4}	0							

	V_i	W_i
1	3	1
2	6	3
3	28	5
4	30	7

$$OPT(i, W) = \begin{cases} 0 & \text{if } i=0 \\ OPT(i-1, w) & \text{if } w_i > W \\ \max\{OPT(i-1, w), v_i + OPT(i-1, W-w_i)\} & \text{otherwise} \end{cases}$$

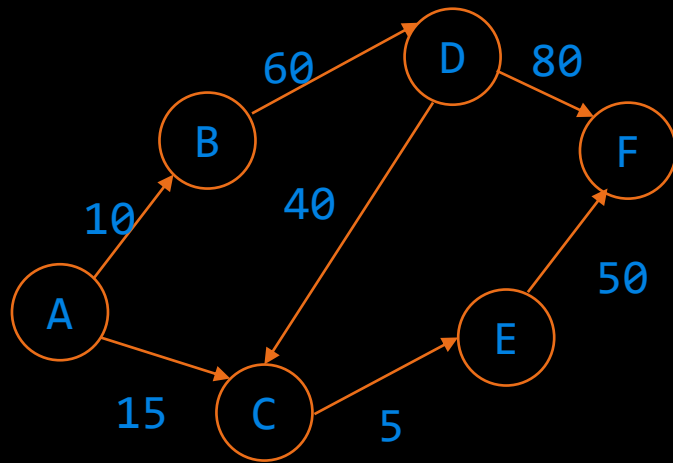
Mentimeter

	0	1	2	3	4	5	6	7
{}	0	0	0	0	0	0	0	0
{1}	0	3	3	3	3	3	3	3
{1, 2}	0	3	3	6	9	9	9	9
{1, 2, 3}	0	3	3	6	9	28	31	31
{1, 2, 3, 4}	0	3	3	6	9	28	31	31

	V_i	W_i
1	3	1
2	6	3
3	28	5
4	30	7

$$OPT(i, W) = \begin{cases} 0 & \text{if } i=0 \\ OPT(i-1, w) & \text{if } w_i > W \\ \max\{OPT(i-1, w), v_i + OPT(i-1, W-w_i)\} & \text{otherwise} \end{cases}$$

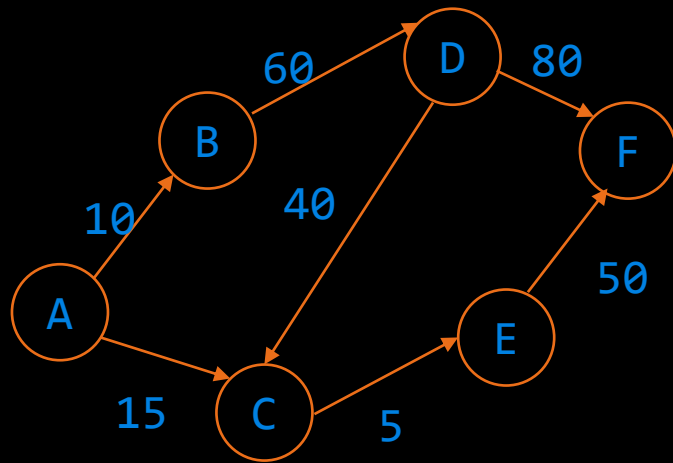
Common Representations



G

- Edge List
- Adjacency Matrix
- Adjacency List

Edge List

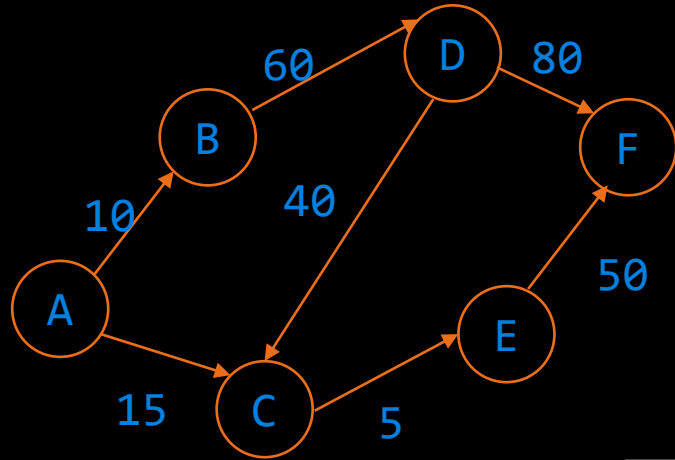


G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

$G = \{(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)\}$

Edge List



$G = \{(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)\}$

G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

Common Operations:

1. Connectedness

Is A connected to B?

$\sim O(E)$

2. Adjacency

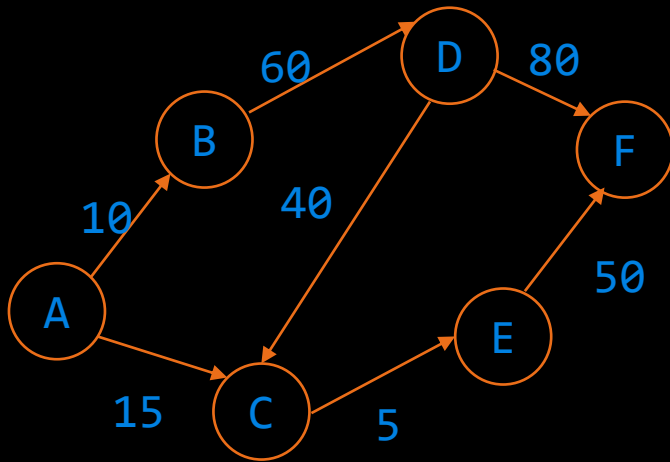
What are A's adjacent nodes?

$\sim O(E)$

$O(|E|) \sim O(|V| * |V|)$

Space: $O(E)$

Adjacency Matrix



G

A

B

C

D

E

F

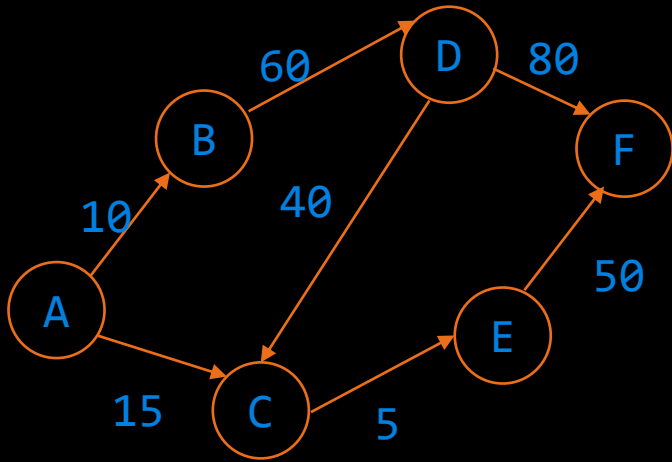
	A	B	C	D	E	F
A	0	10	15	0	0	0
B	0	0	0	60	0	0
C	0	0	0	0	5	0
D	0	0	40	0	0	80
E	0	0	0	0	0	50
F	0	0	0	0	0	0

Insertion:

$G[\text{from}][\text{to}] = \text{weight};$ (if there is an edge, “from” -> “to”)

$G[\text{from}][\text{to}] = 0;$ (otherwise)

Adjacency Matrix Implementation



Input

```
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50
```

G

Map

```
A 0
B 1
C 2
D 3
E 4
F 5
```

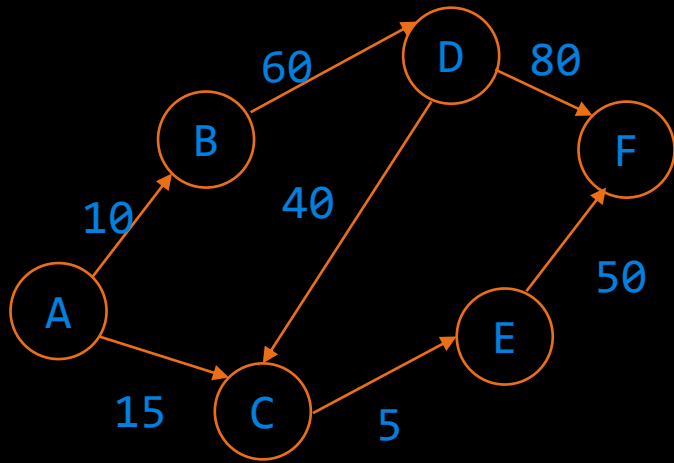
	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Insertion:

```
G[from][to] = weight; (if there is an edge, "from" -> "to")
G[from][to] = 0;      (otherwise)
```

```
01 #include <iostream>
02 #include<map>
03 #define VERTICES 6
04 using namespace std;
05 int main()
06 {
07     int no_lines, wt, j=0;
08     string from, to;
09     int graph [VERTICES][VERTICES] = {0};
10     map<string, int> mapper;
11     cin >> no_lines;
12     for(int i = 0; i < no_lines; i++)
13     {
14         cin >> from >> to >> wt;
15         if (mapper.find(from) == mapper.end())
16             mapper[from] = j++;
17         if (mapper.find(to) == mapper.end())
18             mapper[to] = j++;
19         graph[mapper[from]][mapper[to]] = wt;
20     }
21     return 0;
22 }
```

Adjacency Matrix



G

Map

A 0
B 1
C 2
D 3
E 4
F 5

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Common Operations:

1. Connectedness

Is A connected to B?

$G["A"]["B"] \sim O(1)$

2. Adjacency

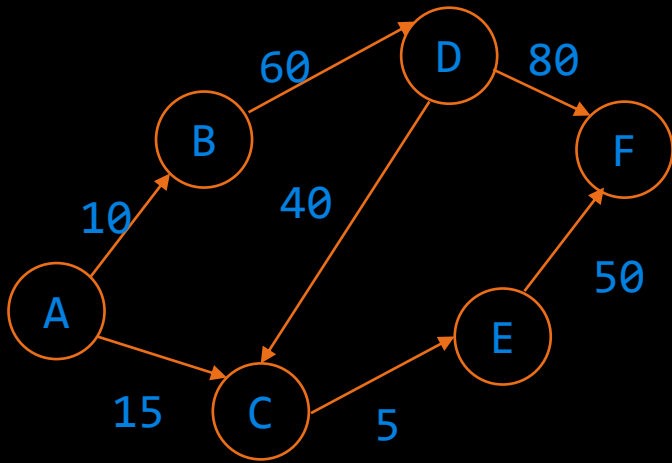
What are A's adjacent nodes?

for each element x in $G["A"]$
if $x \neq 0$

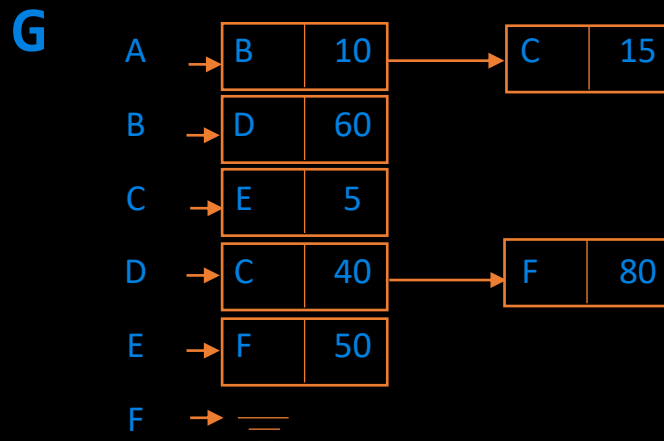
$\sim O(|V|)$

Space: $O(|V| * |V|)$

Adjacency List



Sparse Graph:
Edges \sim Vertices



Common Operations:

1. Connectedness

Is A connected to B?
for each element x in G["A"]
if x != 'B'
~ $O(\text{outdegree}|V|)$

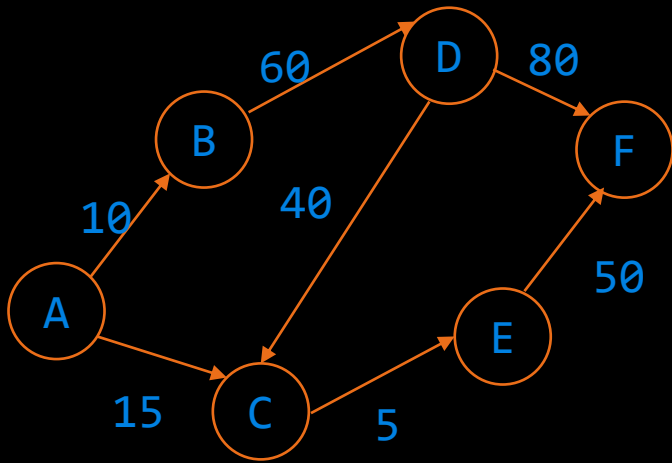
2. Adjacency

What are A's adjacent nodes?

$G["A"] \sim O(\text{outdegree}|V|)$

Space: $O(|V| + |E|)$

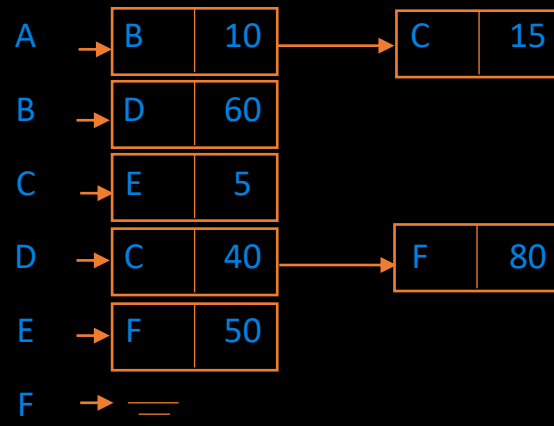
Adjacency List Implementation



Input

```
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50
```

G



Insertion:

If to or from vertex not present add vertex

Otherwise add edge at the end of the list

```
01 #include <iostream>
02 #include<map>
03 #include<vector>
04 #include<iterator>
05 using namespace std;
06
07 int main()
08 {
09     int no_lines;
10     string from, to, wt;
11     map<string, vector<pair<string,int>>> graph;
12     cin >> no_lines;
13     for(int i = 0; i < no_lines; i++)
14     {
15         cin >> from >> to >> wt;
16         graph[from].push_back(make_pair(to, stoi(wt)));
17         if (graph.find(to)==graph.end())
18             graph[to] = {};
19     }
20 }
```

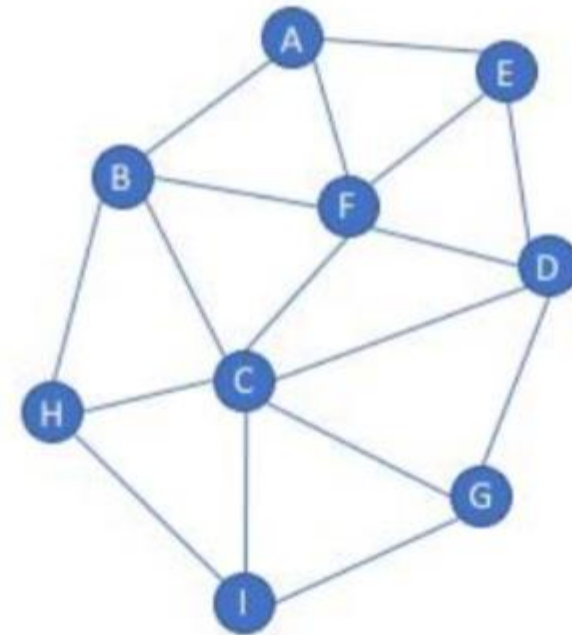
Graph Implementation

	Edge List	Adjacency Matrix	Adjacency List
Time Complexity: Connectedness	$O(E)$	$O(1)$	$O(\text{outdegree}(V))$
Time Complexity: Adjacency	$O(E)$	$O(V)$	$O(\text{outdegree}(V))$
Space Complexity	$O(E)$	$O(V*V)$	$O(V+E)$

Graph - BFS

- Which of the following are valid breadth first search traversals for this graph?

- a) AFBEDCHGI
- b) ICHGBFDAE
- c) DCFEGHIBA
- d) EAFDBHCIG
- e) FAEDCBGHIH



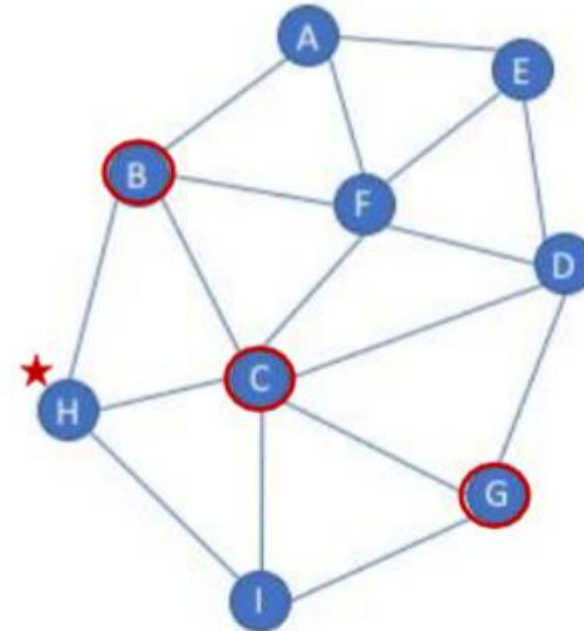
Graph - BFS

- Which of the following are valid breadth first search traversals for this graph?

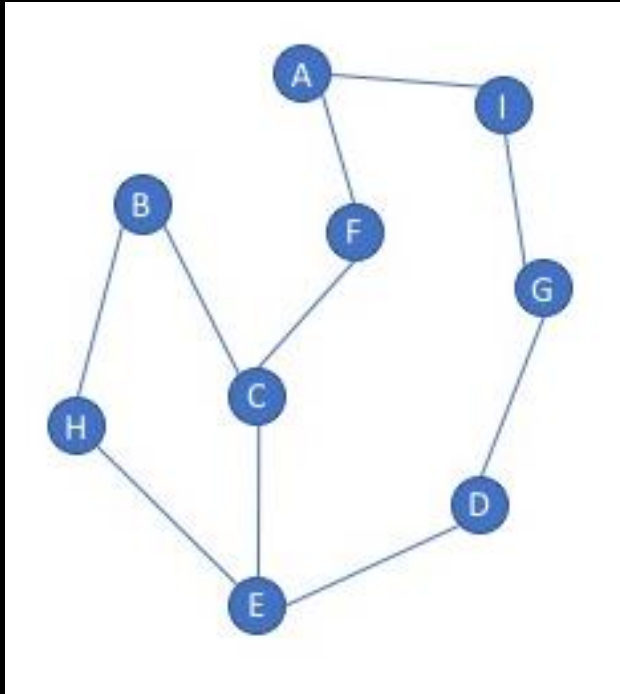
- a) AFBEDCHGI
- b) ICHGBFDAE
- c) DCFEGHIBA
- d) EAFDBHCIG
- e) FAEDCBGHI

All the options except for d
Why not d?

** H is visited before C and G

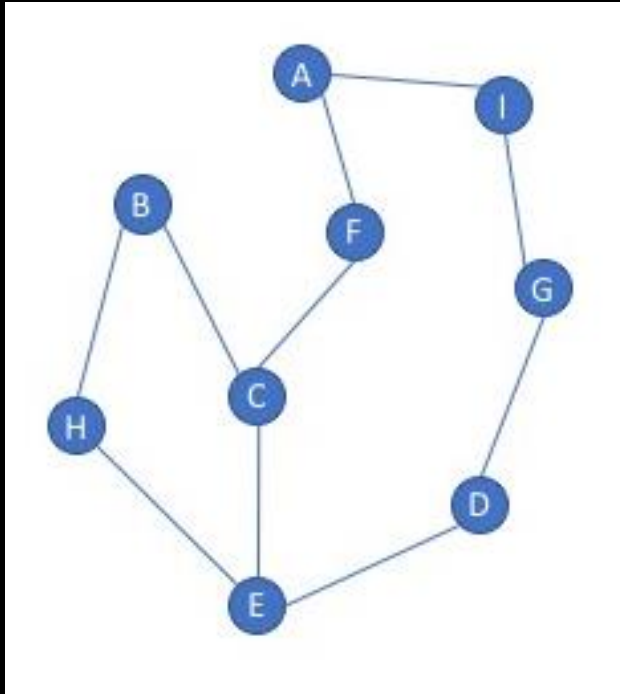


Valid DFS: Which DFS are valid?



- HECBDGIAF
- CEHBDGIAF
- AFCEHBIGD
- DECBHFAIG

Valid DFS: Which DFS are valid?



- HECBDGIAF
- **CEHBDGIAF**
- AFCEHBIGD
- **DECBHFAIG**

BFS Pseudocode

- Write pseudocode/code for implementing the **Breadth First Search Algorithm** of a graph, G that takes a source vertex S as input. (8).
- Also, state the Big O complexity of the traversal in the worst case (2).

BFS

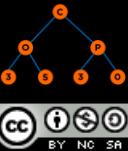
vs

DFS

```
01 string source = "A";
02 std::set<string> visited;
03 std::queue<string> q;
04
05 visited.insert(source);
06 q.push(source);
07 cout<<"BFS: ";
08
09 while(!q.empty())
10 {
11     string u = q.front();
12     cout << u;
13     q.pop();
14     vector<string> neighbors = graph[u];
15     for(string v: neighbors)
16     {
17         if(visited.count(v)==0)
18         {
19             visited.insert(v);
20             q.push(v);
21         }
22     }
23 }
```

```
01 string source = "A";
02 std::set<string> visited;
03 std::stack<string> s;
04
05 visited.insert(source);
06 s.push(source);
07 cout<<"DFS: ";
08
09 while(!s.empty())
10 {
11     string u = s.top();
12     cout << u;
13     s.pop();
14     vector<string> neighbors = graph[u];
15     for(string v: neighbors)
16     {
17         if(visited.count(v)==0)
18         {
19             visited.insert(v);
20             s.push(v);
21         }
22     }
23 }
```

Theoretical Complexity: $O(V+E)$



Graph Algorithm Mix n Match

- Finds the shortest paths in a weighted graph
- Find the minimum cost connected network
- Scheduling algorithm, list steps in a process
- Finds the shortest path in an unweighted graph

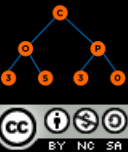
Prim's or Kruskals

BFS

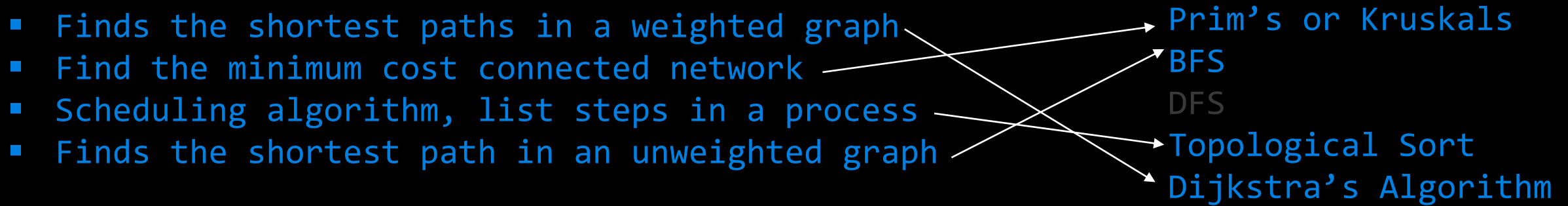
DFS

Topological Sort

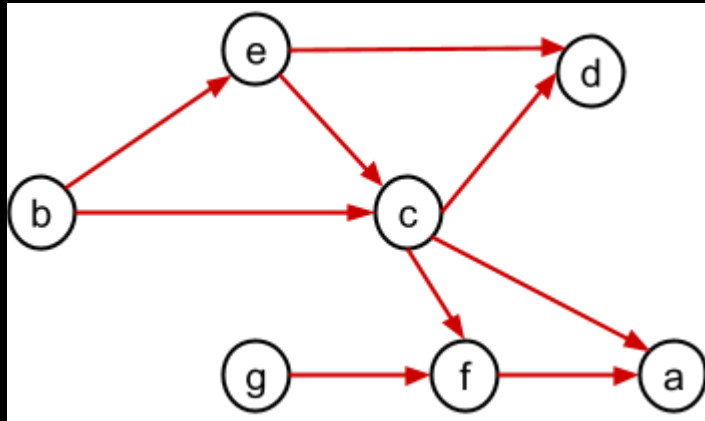
Dijkstra's Algorithm



Graph Algorithm Mix n Match

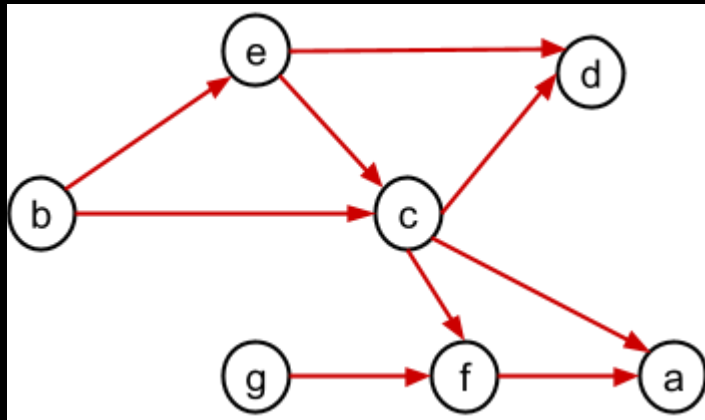
- Finds the shortest paths in a weighted graph
 - Find the minimum cost connected network
 - Scheduling algorithm, list steps in a process
 - Finds the shortest path in an unweighted graph
- Prim's or Kruskals
BFS
DFS
Topological Sort
Dijkstra's Algorithm
- 

Which of the choices below represent a valid topological sort ordering of this graph?



- b, e, c, g, f, a, d
- b, a, c, g, f, e, d
- b, g, f, c, e, a, d
- b, e, c, g, a, f, d
- b, g, e, c, d, f, a
- b, f, c, g, a, e, d

Which of the choices below represent a valid topological sort ordering of this graph?

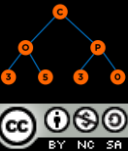


- b, e, c, g, f, a, d
- b, a, c, g, f, e, d
- b, g, f, c, e, a, d
- b, e, c, g, a, f, d
- b, g, e, c, d, f, a
- b, f, c, g, a, e, d

What does this code do?

```
#include <set>
#include <stack>
using namespace std;

bool doSomething(const Graph& graph, int src, int dest)
{
    set<int> visited;
    stack<int> s;
    visited.insert(src);
    s.push(src);
    while(!s.empty())
    {
        int u = s.top();
        s.pop();
        for(auto v: graph.adjList[u])
        {
            if(v == dest)
                return true;
            if ((visited.find(v) == visited.end())) {
                visited.insert(v);
                s.push(v);
            }
        }
    }
    return false;
}
```

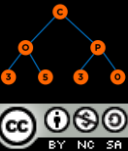


What does this code do?

```
#include <set>
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{
    set<int> visited;
    stack<int> s;
    visited.insert(src);
    s.push(src);
    while(!s.empty())
    {
        int u = s.top();
        s.pop();
        for(auto v: graph.adjList[u])
        {
            if(v == dest)
                return true;
            if ((visited.find(v) == visited.end())) {
                visited.insert(v);
                s.push(v);
            }
        }
    }
    return false;
}
```

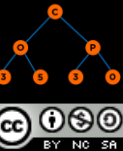
Returns whether a given vertex is reachable from another vertex using DFS



Scenario

A county government maintains a network of roads. The county government has tabulated the cost of maintaining each road. They need to minimize the cost of road maintenance but ensure that all places in the county are accessible.

Which graph algorithm that we discussed in class could they use to solve this problem? What are the vertices, what are the edges, what are the edge values?



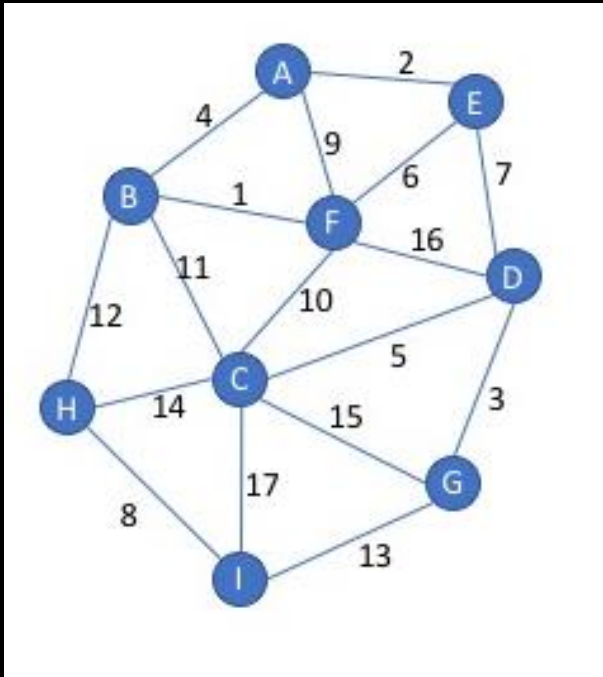
Scenario

A county government maintains a network of roads. The county government has tabulated the cost of maintaining each road. They need to minimize the cost of road maintenance but ensure that all places in the county are accessible.

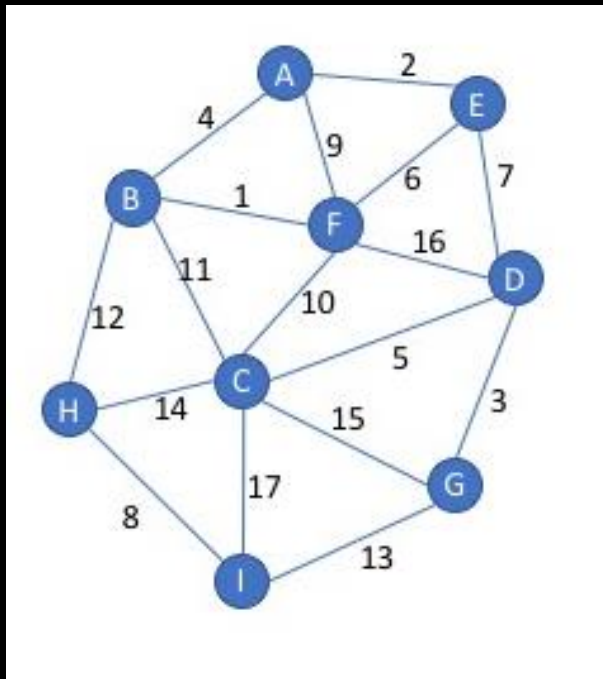
Which graph algorithm that we discussed in class could they use to solve this problem? What are the vertices, what are the edges, what are the edge values?

- Prim's or Kruskal's algorithm for minimum spanning tree.
- Roads are edges.
- Ends of roads are vertices.
- Edge weights are cost for maintaining roads.

MST using Prim's starting from "I"



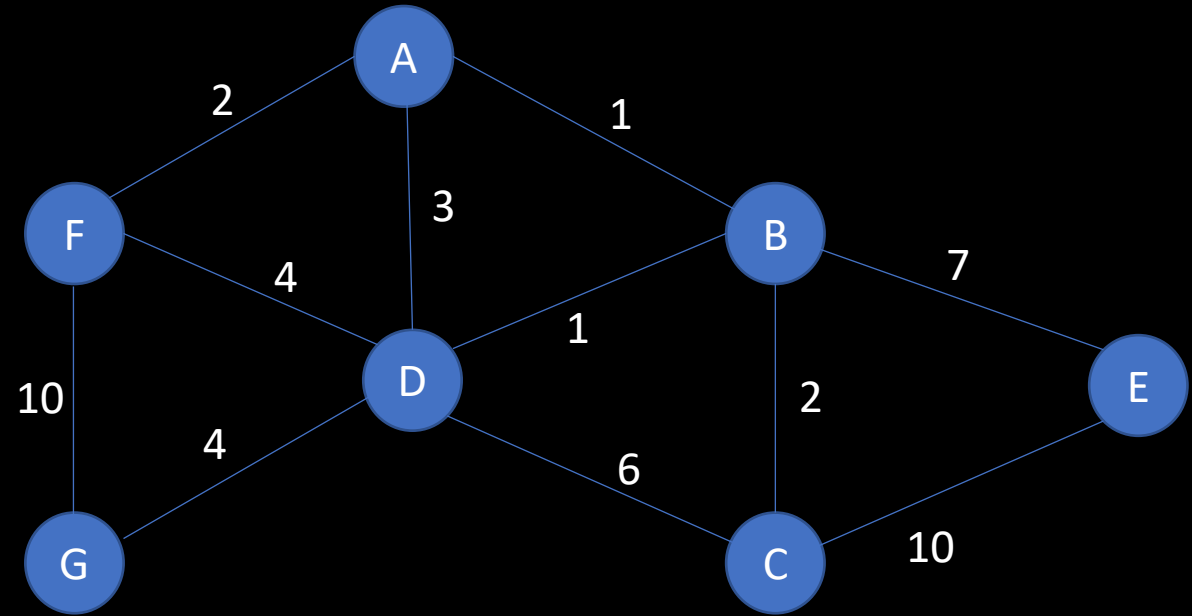
MST using Prim's starting from "I"



I H B F A E D G C

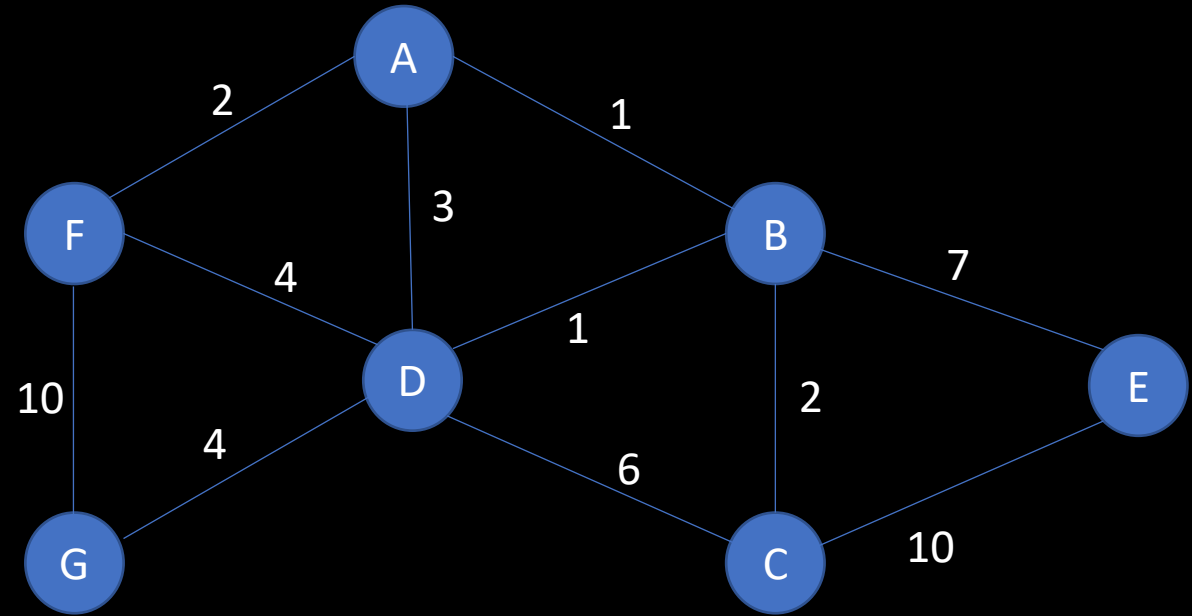
Dijkstra with A as source

v	D(v)	P(v)
A		
B		
C		
D		
E		
F		
G		

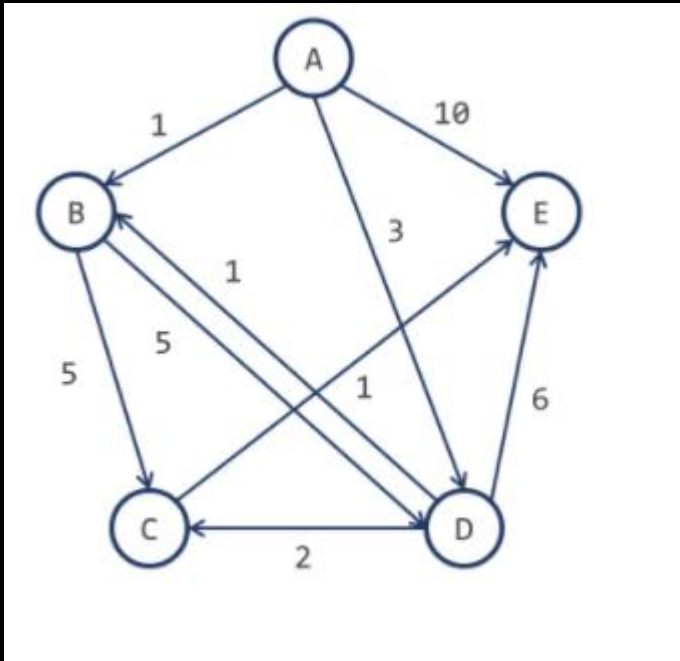


Dijkstra with A as source

v	D(v)	P(v)
A	0	NA
B	1	A
C	3	B
D	2	B
E	8	B
F	2	A
G	6	D



Dijkstra with A as source

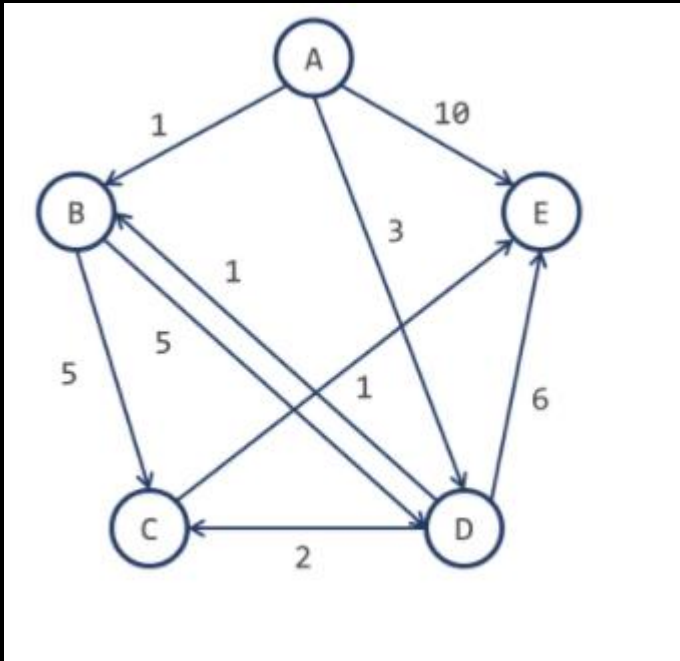


V
B
C
D
E

$d(V)$

$p(V)$

Dijkstra with A as source



V

B

C

D

E

$d(V)$

1

5

3

6

$p(V)$

A

D

A

C

Algorithmic Paradigms

Algorithmic Paradigms

	Properties	Examples
Brute Force	<ul style="list-style-type: none">▪ Generate and Test an Exhaustive Set of all possible combinations▪ Can be computationally very expensive▪ Guarantees optimal solution	<ul style="list-style-type: none">▪ Finding divisors of a number, n by checking if all numbers from $1..n$ divides n without remainder▪ Finding duplicates using all combinations▪ Bubble/Selection Sort
Divide and Conquer	<ul style="list-style-type: none">▪ Break the problem into subcomponents typically using recursion▪ Solve the basic component▪ Combine the solutions to sub-problems	<ul style="list-style-type: none">▪ Quick Sort▪ Merge Sort▪ Binary Search▪ Peak Finding
Dynamic Programming	<ul style="list-style-type: none">▪ Optimal substructure: solution to a large problem can be obtained by solution to a smaller optimal problems▪ Overlapping sub-problems: space of sub-problems must be small, that is, any recursive algorithm solving the problem should solve the same sub-problems over and over, rather than generating new sub-problems.▪ Guarantees optimal solution	<ul style="list-style-type: none">▪ Fibonacci Sequence▪ Assembly Scheduling▪ Knapsack
Greedy Algorithms	<ul style="list-style-type: none">▪ Local optimal solutions at each stage▪ Does not guarantee optimal solution	<ul style="list-style-type: none">▪ Prim's Algorithm▪ Dijkstra's Algorithm▪ Kruskal's Algorithm

Bin Packing

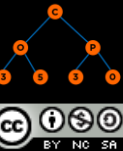
If we have packets that each require 7 units, 8 units, 2 units and 3 units of space, how many minimum bins are required to store all the four packets if each bin can take at most 10 units of space using the following Greedy strategies

- **First Fit:** scan the bins and place the new item in the first bin that is large enough.
- **Best Fit:** scan the bins and place the new item in the bin that finds the spot that creates the smallest empty space

Algorithm for Huffman Encoding

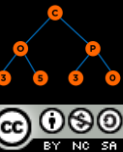
Given this file, generate a Huffman Tree and identify the codes of each character.

care racecar era



Algorithm for Huffman Encoding

1. Create a table with symbols and their frequencies
2. Construct a set of trees with root nodes that contain each of the individual symbols and their weight (frequency).
3. Place the set of trees into a min priority queue.
4. while the priority queue has more than one item
 Remove the two trees with the smallest weights.
 Combine them into a new binary tree in which the weight of the tree root is the sum of the weights of its children.
 Insert the newly created tree back into the priority queue.
5. Traverse the resulting tree to obtain binary codes for characters



Algorithm for Huffman Encoding

1. Create a table with symbols and their frequencies

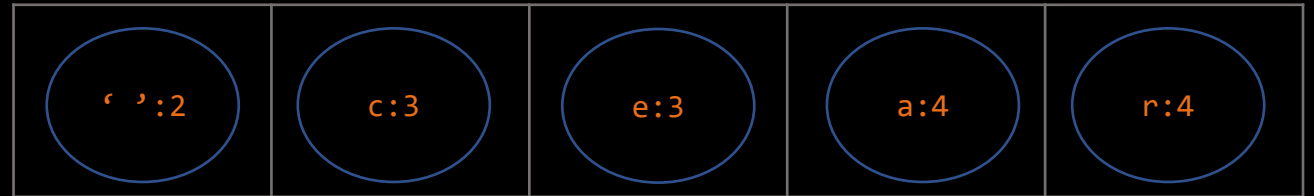
care racecar era

Character	Frequency
a	4
r	4
c	3
e	3
' ,	2

Algorithm for Huffman Encoding

- Construct a set of trees with root nodes that contain each of the individual symbols and their weight (frequency).
- Place the set of trees into a min priority queue.

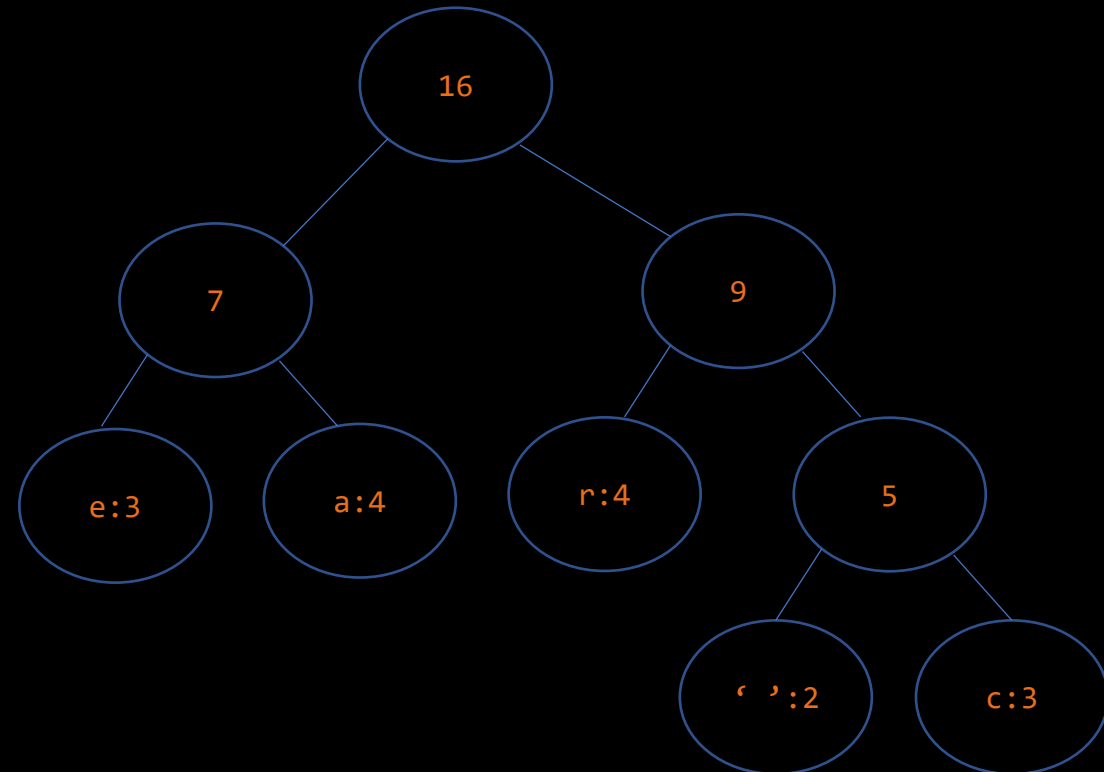
Character	Frequency
a	4
r	4
c	3
e	3
' '	2



Algorithm for Huffman Encoding

4. while the priority queue has more than one item
Remove the two trees with the smallest weights.
Combine them into a new binary tree in which the weight of the tree root is the sum of the weights of its children.
Insert the newly created tree back into the priority queue.

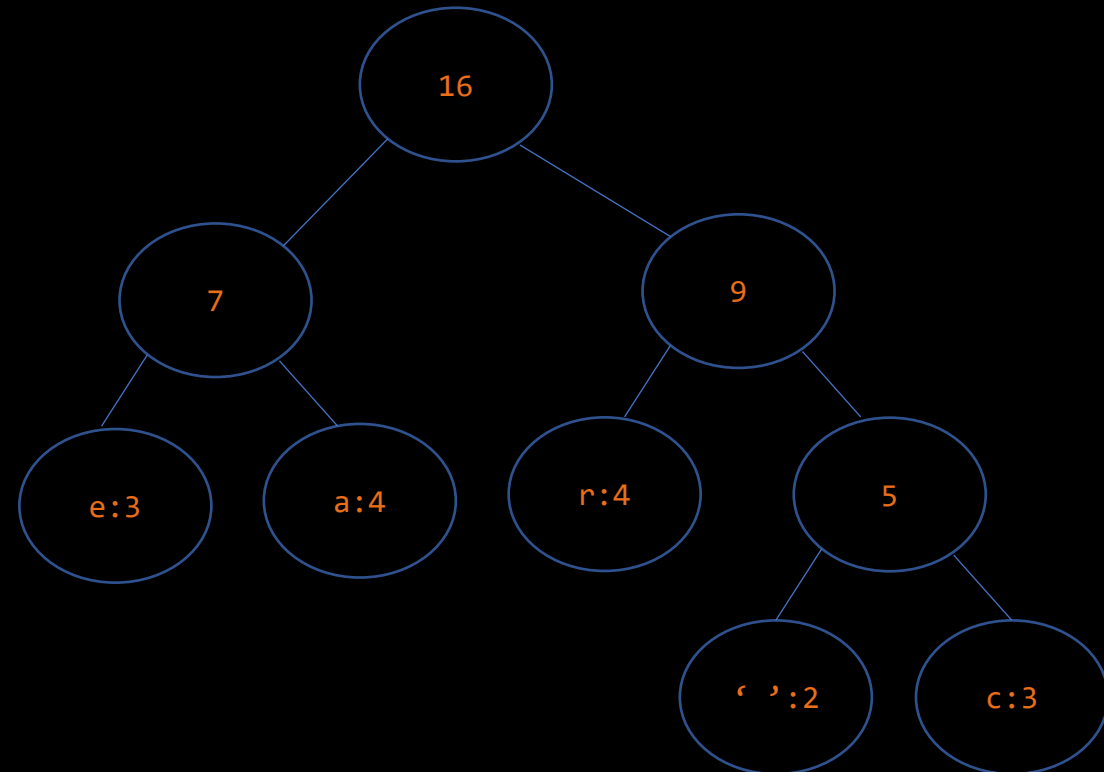
Character	Frequency
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Algorithm for Huffman Encoding

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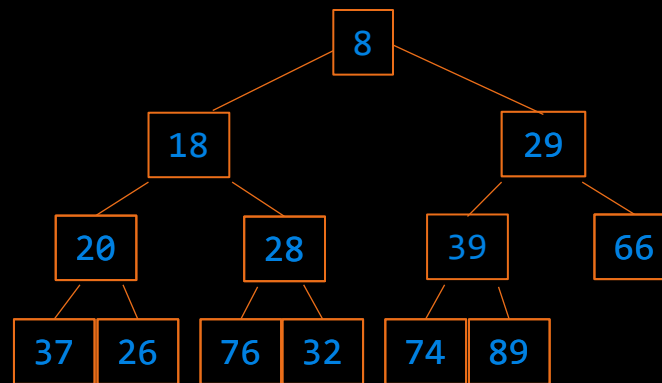
Character	Frequency	Huffman Code
a	4	01
r	4	10
c	3	111
e	3	00
' '	2	110



Questions

Binary Heap

Heap Representation



```
int Heap[];
```

For a node at position p ,

L. child position: $2p + 1$

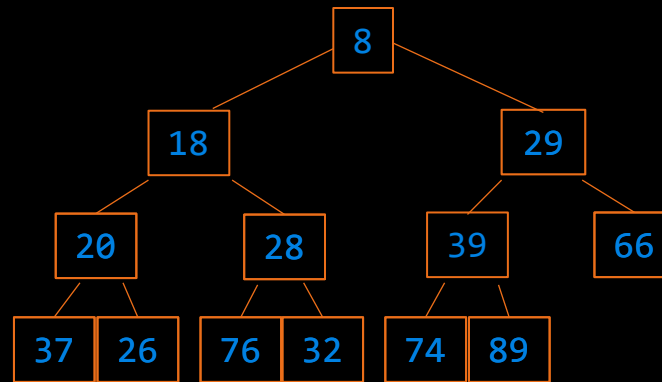
R. child position: $2p + 2$

A node at position c can find its parent at $\text{floor}((c - 1)/2)$

0	1	2	3	4	5	6	7	8	9	10	11	12	13
8	18	29	20	28	39	66	37	26	76	32	74	89	

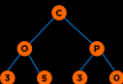
Binary Heap Insertion

Heap Insertion



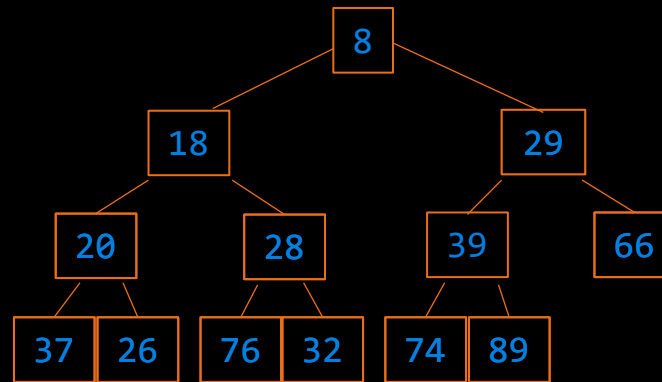
Algorithm for Inserting in a Heap

1. Insert the new item in the next position at the bottom of the heap.
2. **while** new item is not at the root and new item is smaller than its parent
3. Swap the new item with its parent, moving the new item up the heap.



Binary Heap Insertion

Heap Insertion



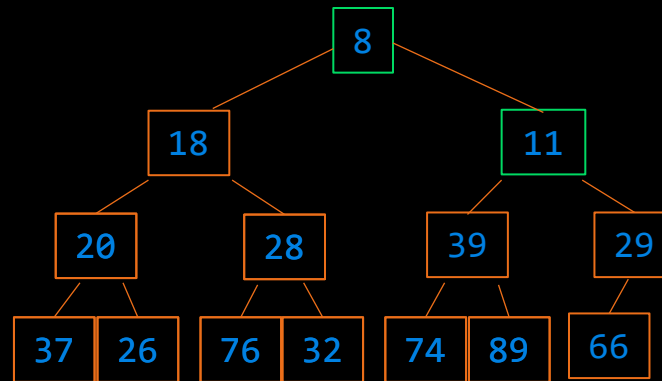
1. Insert the new element at the end of the array and set child to `arr.size() - 1`
2. Set parent to $(\text{child} - 1) / 2$
3. while ($\text{parent} \geq 0$ and $\text{arr}[\text{parent}] > \text{arr}[\text{child}]$)
 - Swap $\text{arr}[\text{parent}]$ and $\text{arr}[\text{child}]$
 - Set child equal to parent
 - Set parent equal to $(\text{child}-1)/2$

insert 11

0	1	2	3	4	5	6	7	8	9	10	11	12	13
8	18	29	20	28	39	66	37	26	76	32	74	89	

Binary Heap Insertion

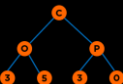
Heap Insertion



1. Insert the new element at the end of the array and set child to `arr.size() - 1`
2. Set parent to $(\text{child} - 1) / 2$
3. while (`parent >= 0` and `arr[parent] > arr[child]`)
 - Swap `arr[parent]` and `arr[child]`
 - Set child equal to parent
 - Set parent equal to $(\text{child}-1)/2$

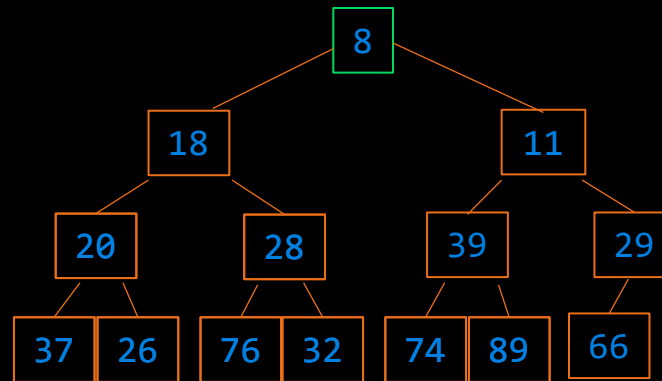
child = 13 | 6 | 2
parent = 6 | 2 | 0

$O(\log n)$ time to insert!



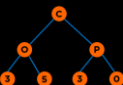
Binary MinHeap Deletion

Heap Deletion (ExtractMin)



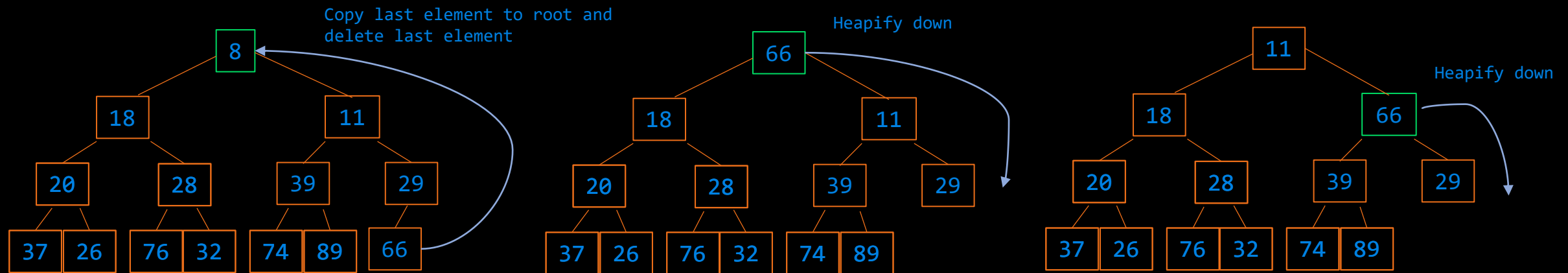
Algorithm for Removal from a Heap

1. Remove the item in the root node by replacing it with the last item in the heap (LIH).
2. **while** item LIH has children and item LIH is larger than either of its children
3. Swap item LIH with its smaller child, moving LIH down the heap.



Binary MinHeap Deletion

Heap Deletion (ExtractMin)

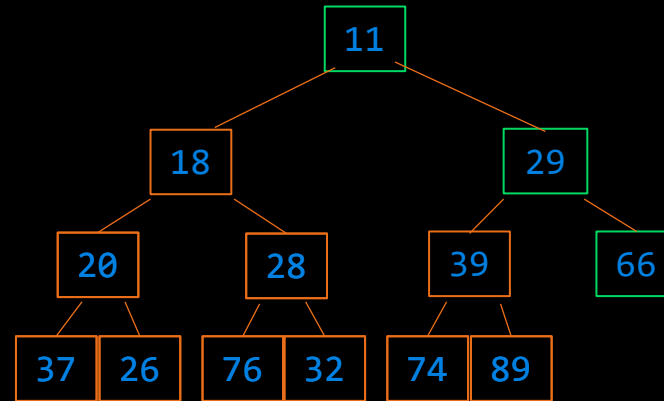


Algorithm for Removal from a Heap

1. Remove the item in the root node by replacing it with the last item in the heap (LIH).
2. **while** item LIH has children and item LIH is larger than either of its children
3. Swap item LIH with its smaller child, moving LIH down the heap.

Binary MinHeap Deletion

Heap Deletion (ExtractMin)



$O(\log n)$ time to ExtractMin!