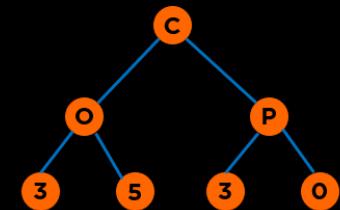


# Algorithm Paradigms



# Categories of Data Structures

Linear Ordered

Non-linear Ordered

Not Ordered

Lists

Trees

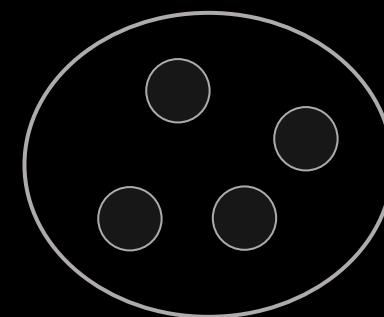
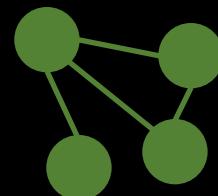
Sets

Stacks

Graphs

Tables/Maps

Queues



# Categories of Algorithms

Brute Force

Divide & Conquer

Greedy

Dynamic Programming

Selection Sort

Binary Search

Minimum Spanning Tree

Knapsack

Bubble Sort

Merge Sort

Shortest Paths

Fibonacci

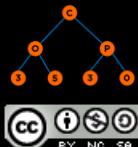
Insertion Sort

Quick Sort

NP Complete Problems

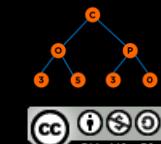


# Algorithmic Paradigms



# Algorithmic Paradigms

	Properties	Examples
Brute Force	<ul style="list-style-type: none"> <li>Generate and Test an Exhaustive Set of all possible combinations</li> <li>Can be computationally very expensive</li> <li>Guarantees correct solution</li> </ul>	<ul style="list-style-type: none"> <li>Finding divisors of a number, <math>n</math> by checking if all numbers from <math>1..n</math> divides <math>n</math> without remainder</li> <li>Finding duplicates using all combinations</li> <li>Bubble/Selection Sort</li> </ul>
Divide and Conquer	<ul style="list-style-type: none"> <li>Break the problem into subcomponents typically using recursion</li> <li>Solve the basic component</li> <li>Combine the solutions to sub-problems</li> </ul>	<ul style="list-style-type: none"> <li>Quick Sort</li> <li>Merge Sort</li> <li>Binary Search</li> <li>Peak Finding</li> </ul>
Dynamic Programming	<ul style="list-style-type: none"> <li><b>Optimal substructure:</b> solution to a large problem can be obtained by solution to a smaller optimal problems e.g., Shortest path in a graph (Longest path does not follow optimal substructure)</li> <li><b>Overlapping sub-problems:</b> space of sub-problems must be small, that is, any recursive algorithm solving the problem should solve the same sub-problems over and over, rather than generating new sub-problems.</li> <li>Guarantees optimal solution in terms of correctness and time</li> </ul>	<ul style="list-style-type: none"> <li>Fibonacci Sequence</li> <li>Assembly Scheduling</li> <li>Knapsack</li> <li>Bin packing</li> </ul>
Greedy Algorithms	<ul style="list-style-type: none"> <li>Local optimal solutions at each stage</li> <li>Does not guarantee optimal solution</li> </ul>	<ul style="list-style-type: none"> <li>Prim's Algorithm</li> <li>Dijkstra's Algorithm</li> <li>Kruskal's Algorithm</li> <li>Bin packing</li> </ul>



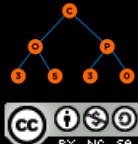
# Optimization problems

- **Optimization Problem:** the problem of finding the best solution from all feasible solutions.
- **Constraints to a Problem:** Minimize or Maximize an Objective function (e.g., go from A->B in 10 hours, Objective function: minimum time). Objective functions define the objective of the optimization and is a single scalar value that is formulated from a set of design responses.
- **Feasible Solution:** A feasible solution is a set of values for the decision variables that satisfies all of the constraints in an optimization problem.
- **Optimal Solution**
  - An *optimal solution* is a feasible solution where the objective function reaches its maximum (or minimum) value.
  - A *globally optimal solution* is one where there are no other feasible solutions with better objective function values.
  - A *locally optimal solution* is one where there are no other feasible solutions “in the vicinity” with better objective function values – you can picture this as a point at the top of a “peak” or at the bottom of a “valley” which may be formed by the objective function and/or the constraints.

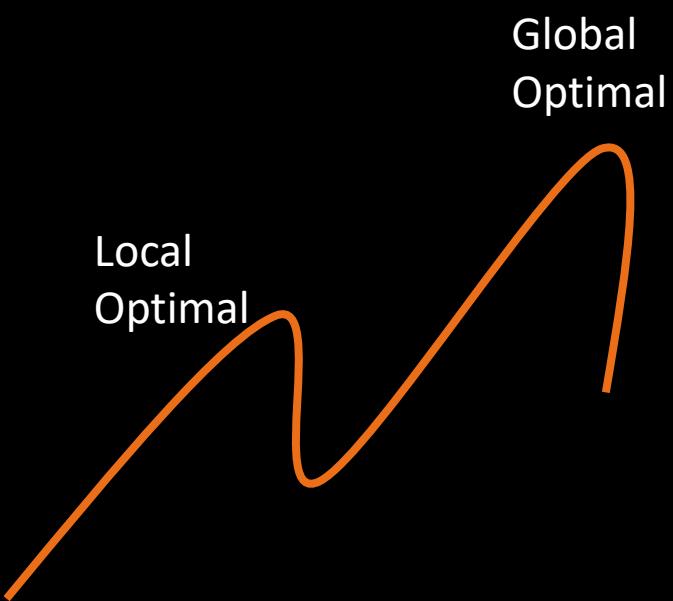
# Optimization problems

- Optimization Problem: find code that runs fast in the course where fast means that it takes less than 1 second to execute and is passing all tests.
- Constraints to a Problem: Minimize an Objective function - minimize time. Constraints: must execute in less than 60 sec.
- Feasible Solution: Lot of student code that passes all tests and executes in less than 1 second.
- Optimal Solution
  - A *globally optimal solution* is the fastest running code.
  - A *Locally optimal solution* is one where you pick the submissions which were submitted earlier than later based on a heuristic.

# Greedy Algorithms

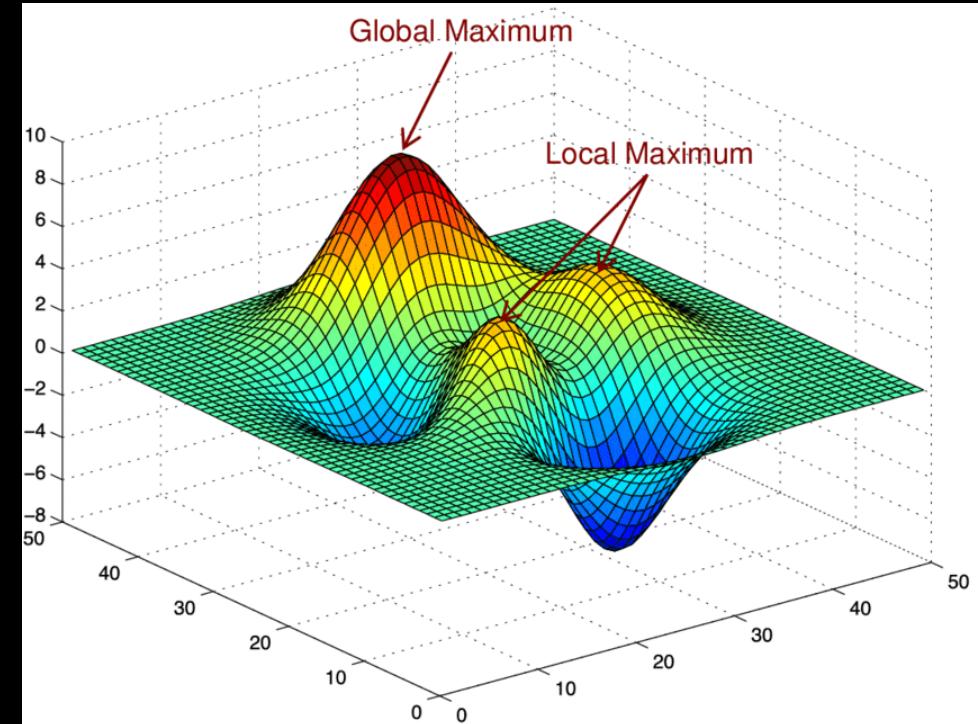
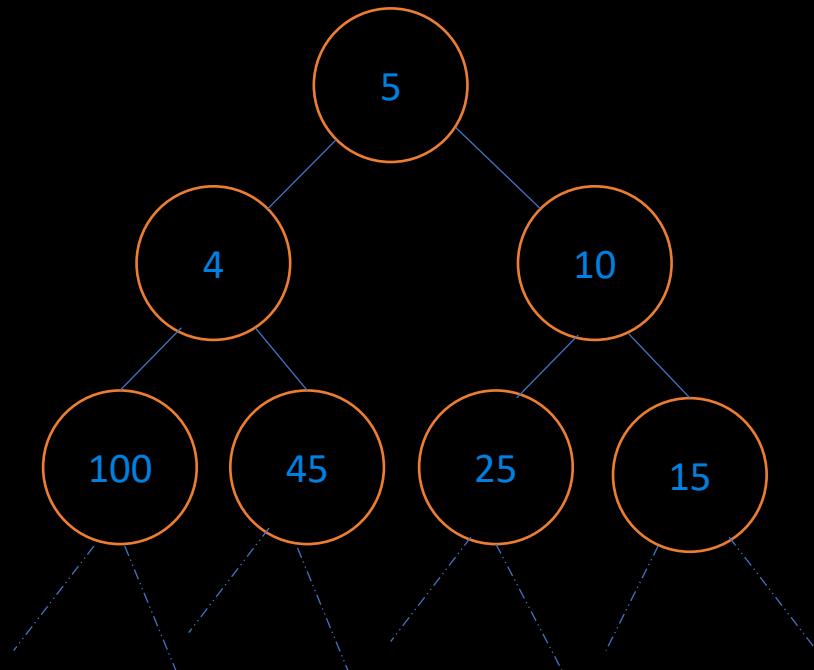


# Greedy Algorithms



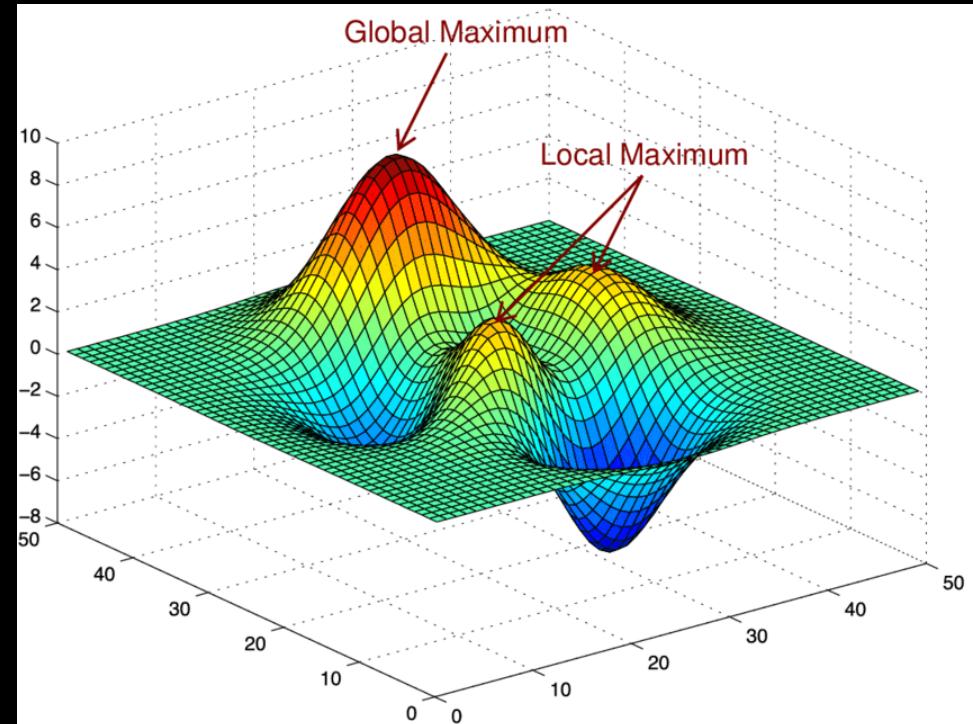
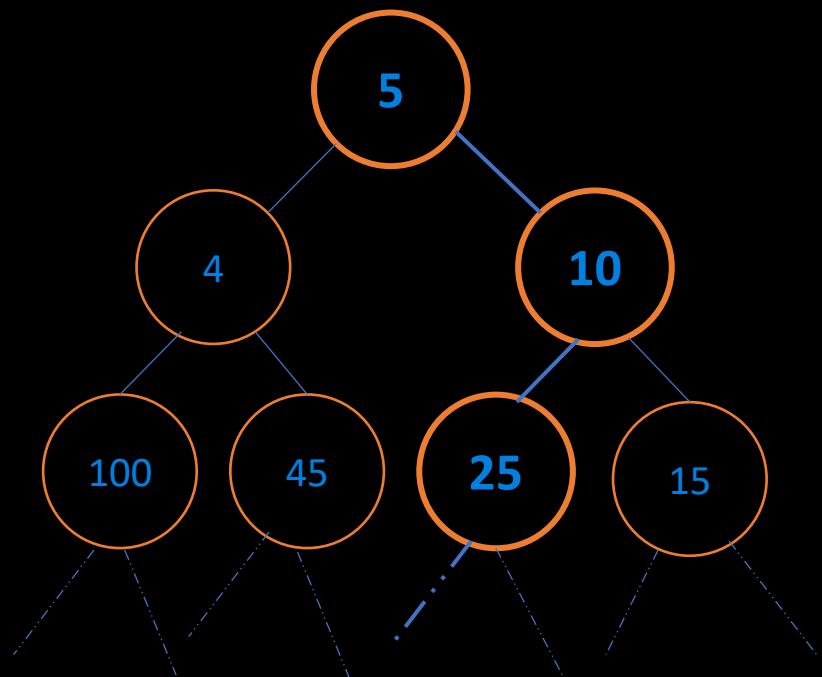
- Greedy algorithms work in phases.
  - In each phase a decision is made that appears to be good.
  - Generally, this means that a local optimum is chosen.
- At the end, we hope the local optimum found is the global optimum.
- Prim's algorithm and Dijkstra's algorithm are greedy algorithms.

# Greedy Algorithms

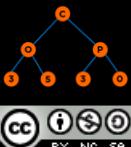


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# Greedy Algorithms

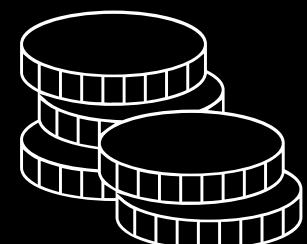


<https://images.app.goo.gl/nehZPYDjn3LxjJey8>



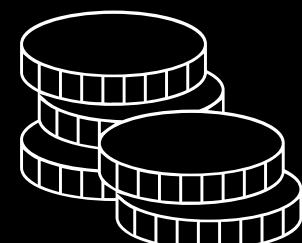
# Coin Change

- Make change with the smallest number of coins.
- Example: 32 cents
  - Quarters (0.25)
  - Dimes (0.10)
  - Nickels (0.05)
  - Pennies (0.01)



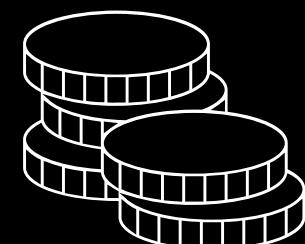
# Coin Change

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- Start with the largest denomination and dispense as many of those fit.
- Example: 32 cents
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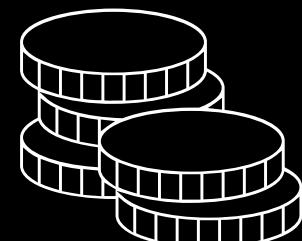
# Coin Change

- Make change with the smallest number of coins.
- Start with the largest denomination and dispense as many of those fit.
- Example: 32 cents
  - Quarters (0.25) - 1
  - Dimes (0.10) - 0
  - Nickels (0.05) - 1
  - Pennies (0.01) - 2
- = 4 coins, the minimum possible



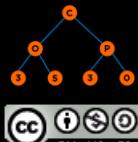
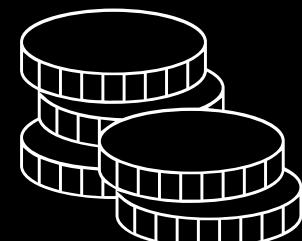
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- Example, alternate universe: 32 cents
  - Gryffindor (0.14)
  - Slytherin (0.08)
  - Hufflepuffs (0.01)



# Coin Change

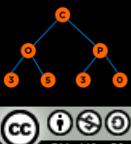
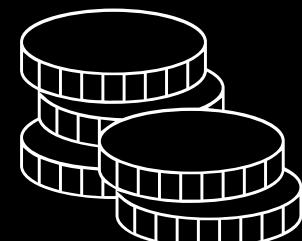
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  - Slytherin (0.08) - 0
  - Hufflepuffs (0.01) - 4
  - = 6 coins, not the minimum possible as 4 Slytherins is global optimal



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- = 6 coins, not the minimum possible as 4 Slytherins is global optimal

A coin system is called "canonical" if the greedy algorithm always solves change-making optimally. It is possible to test whether a coin system is canonical in polynomial time.



# Bin Packing

If we have boxes that each require 14 units, 16 units, 4 units and 6 units of space, how many minimum bins are required to store all the four boxes if each bin can take at most 20 units of space using the following Greedy strategies

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- Best Fit: scan the bins and place the new item in the bin that finds the spot that creates the smallest empty space

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# Bin Packing

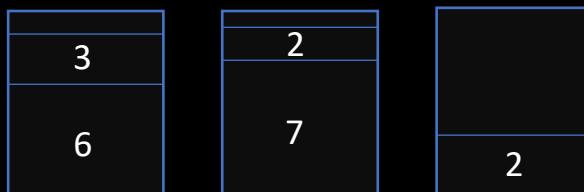
If we have boxes that each require 6 units, 3 units, 7 units, 2 units, and 2 units of space, how many minimum bins are required to store all the five boxes if each bin can take at most 10 units of space using the following Greedy strategies

- First Fit: scan the bins and place the new item in the first bin that is large enough.
- Best Fit: scan the bins and place the new item in the bin that finds the spot that creates the smallest empty space

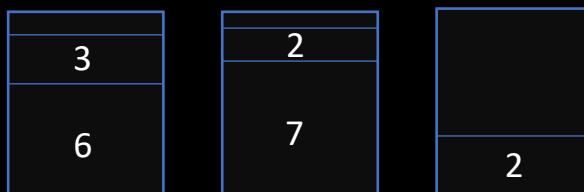
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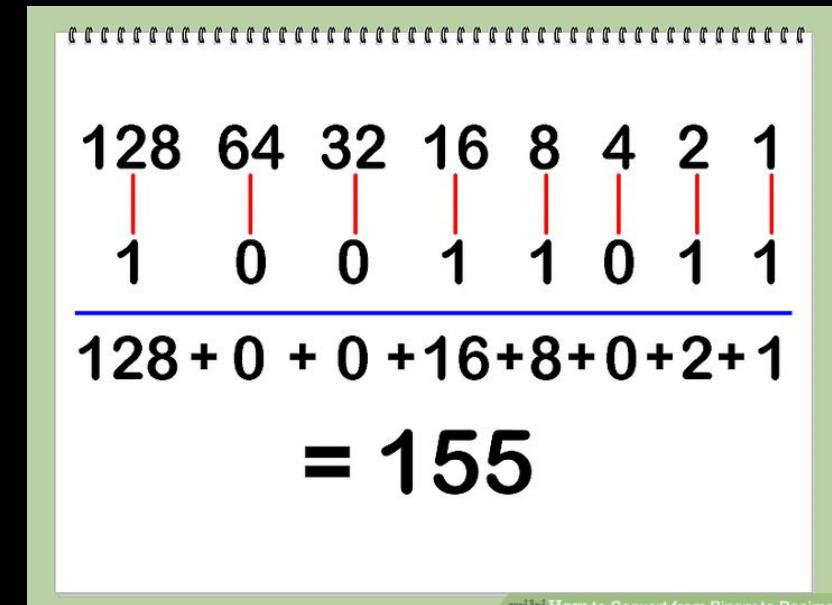
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# Greedy Algorithm for Converting Decimal to Binary

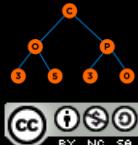
- Convert decimal to binary.
- Start with the largest power of two that will fit.
- Assign 1 to that place. Subtract that value.
- Repeat.

- Example: 280
  - $2^8 = 256$
  - ninth significant digit = 1
    - 100000000
    - $280 - 256 = 24$
  - $2^4 = 16$
  - fifth significant digit = 1
    - 100010000
    - $24 - 16 = 8$
  - $2^3 = 8$
  - fourth significant digit = 1
    - 100011000
    - $8 - 8 = 0$ , done



wikiHow to Convert from Binary to Decimal

# Huffman Trees



# Rationale

- **How is text transmitted?**
- **How many bits we need to encode a character?**

# Rationale

- How is text transmitted?
  - 1's and 0's
- How many bits we need to encode a character?
  - $\lceil \log_2 n \rceil$

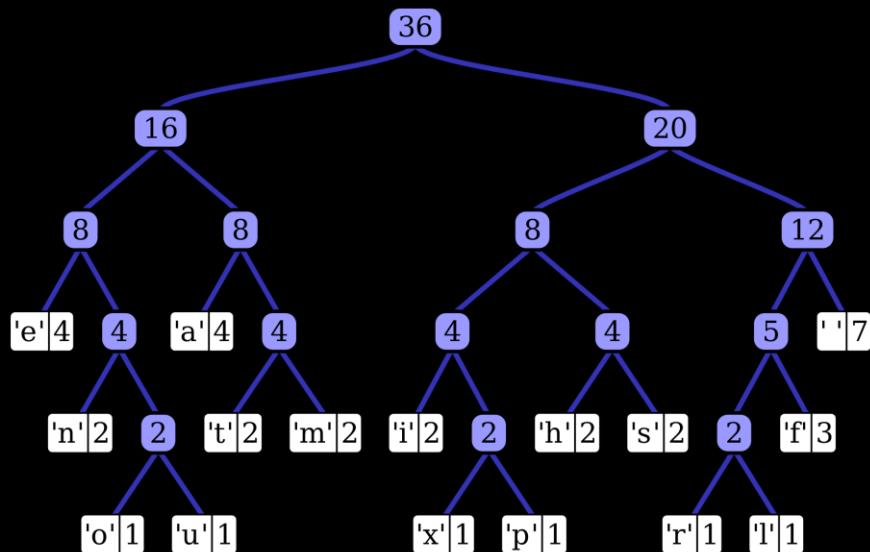
# Rationale

- How is text transmitted? : mississippi
- How many bits we need to encode a character?
  - $\lceil \log_2 4 \rceil = 2$  bits per character
  - $11 * 2 = 22$  bits to transmit “mississippi”

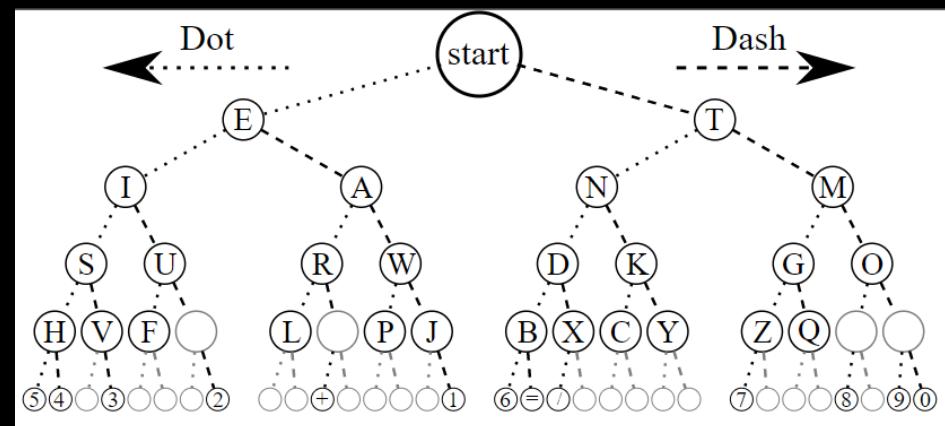
# Huffman Encoding

Huffman Encoding uses prefix-free codes (no codeword is a prefix of another). A prefix code is a uniquely decodable code; given a complete and accurate sequence, a receiver can identify each word without requiring a special marker between words.

Prefix free



Not Prefix free



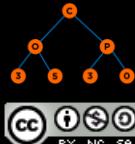
[https://en.wikipedia.org/wiki/Huffman\\_coding#/media/File:Huffman\\_tree\\_2.svg](https://en.wikipedia.org/wiki/Huffman_coding#/media/File:Huffman_tree_2.svg)  
<https://commons.wikimedia.org/wiki/File:Morse-code-tree.svg>

# Algorithm for Huffman Encoding

1. Create a table with symbols and their frequencies

mississippi

Character	Frequency
m	1
i	4
s	4
p	2

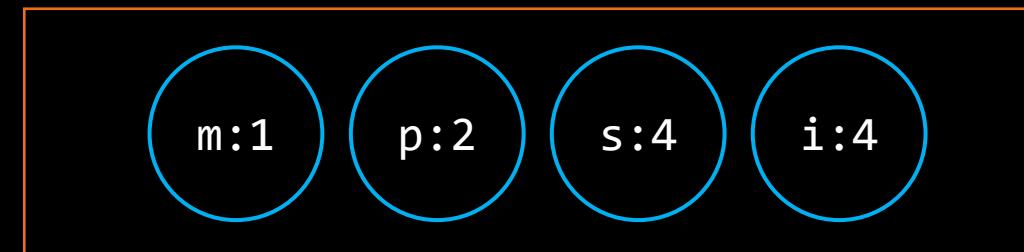


# Algorithm for Huffman Encoding

2. Construct a set of trees with root nodes that contain each of the individual symbols and their weight (frequency).
3. Place the set of trees into a priority queue.

mississippi

Character	Frequency
m	1
i	4
s	4
p	2



# Algorithm for Huffman Encoding

4. while the priority queue has more than one item

    Remove the two trees with the smallest weights.

    Combine them into a new binary tree in which the weight of the tree root is the sum of the weights of its children.

    Insert the newly created tree back into the priority queue.

mississippi

Character	Frequency
m	1
i	4
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p	2

m:1

p:2

s:4

i:4

# Algorithm for Huffman Encoding

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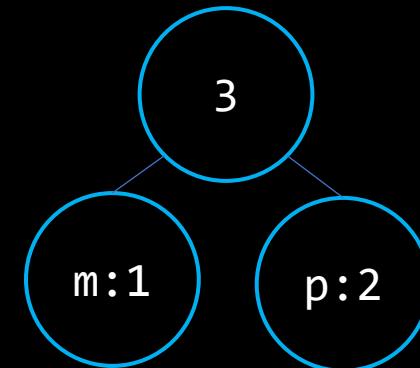
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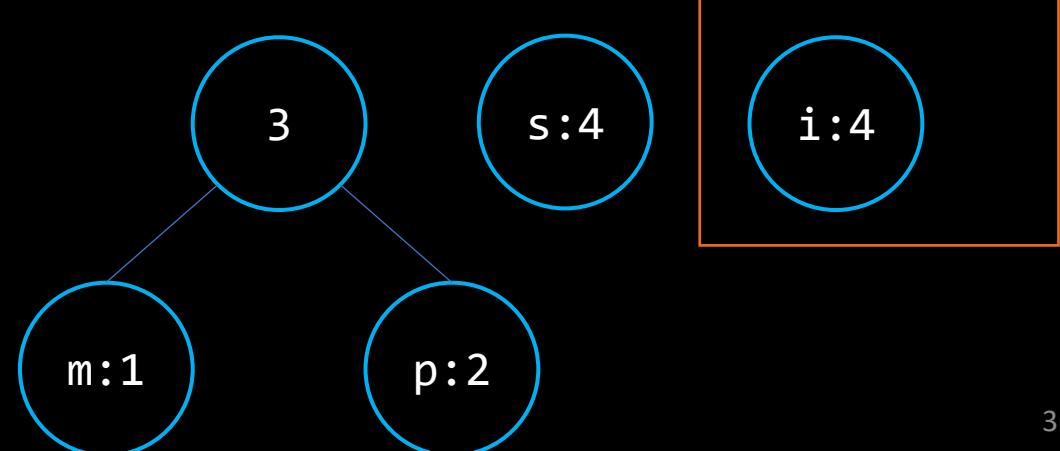


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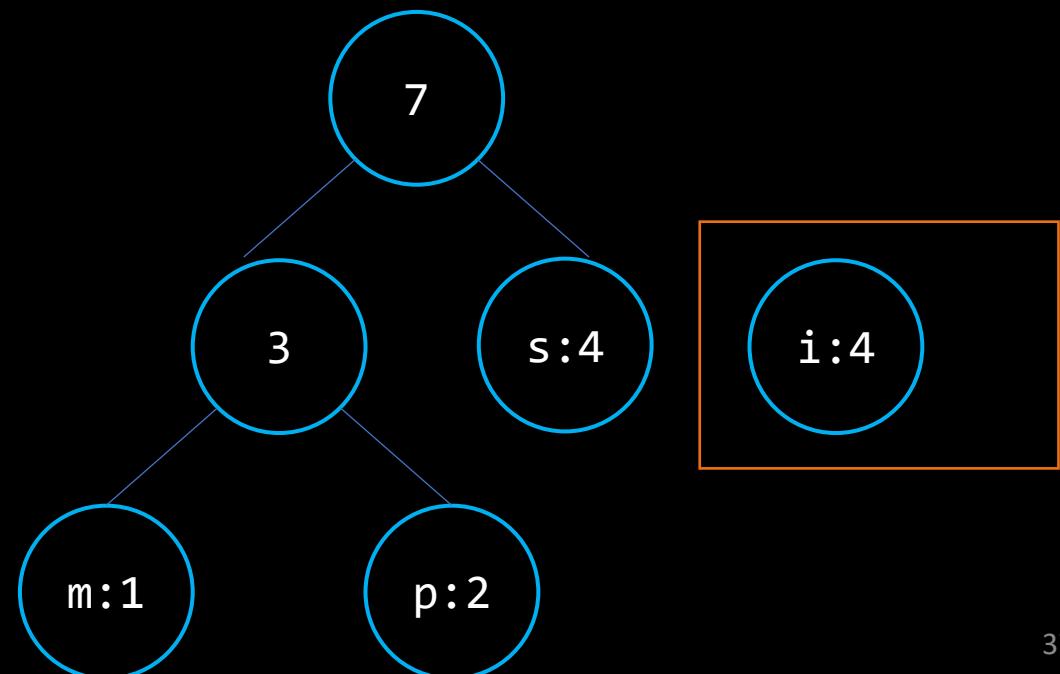


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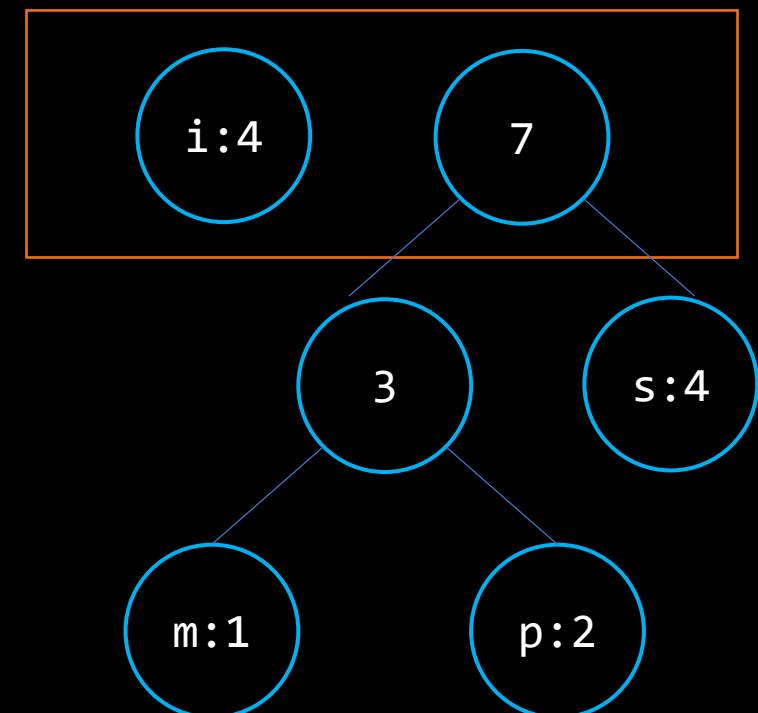
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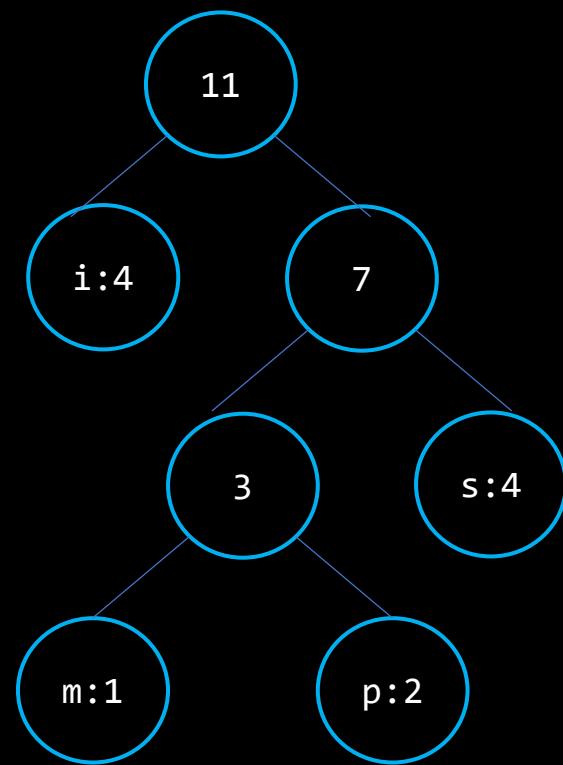
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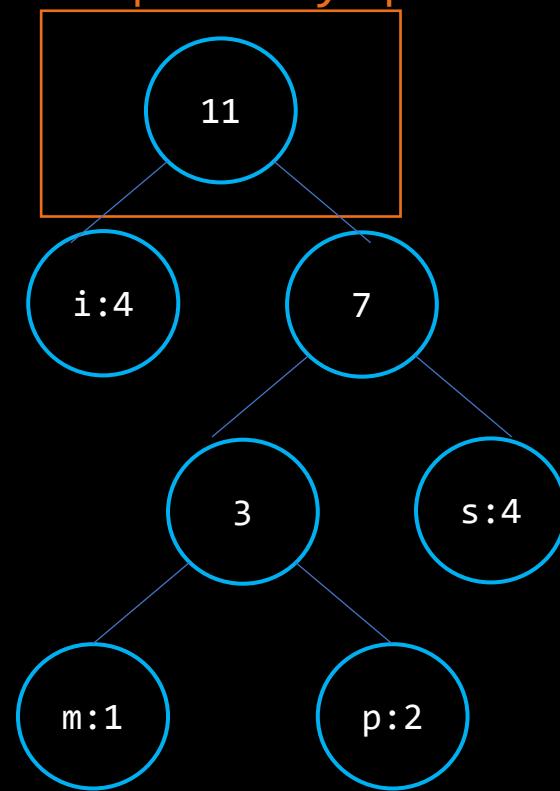
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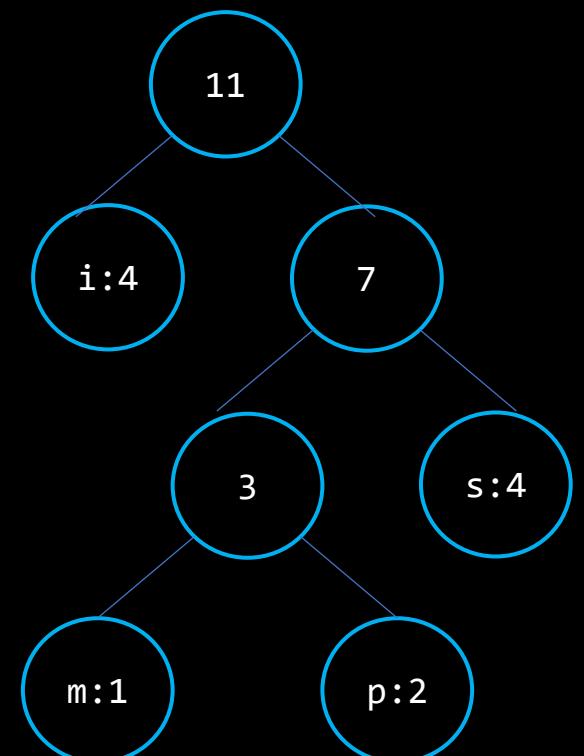


# Algorithm for Huffman Encoding

5. Traverse the resulting tree to obtain binary codes for characters: 0 towards left, 1 to towards right

mississippi

Character	Frequency	Code	Bits per Code
m	1	100	3
i	4	0	1
s	4	11	2
p	2	101	3



# Algorithm for Huffman Encoding

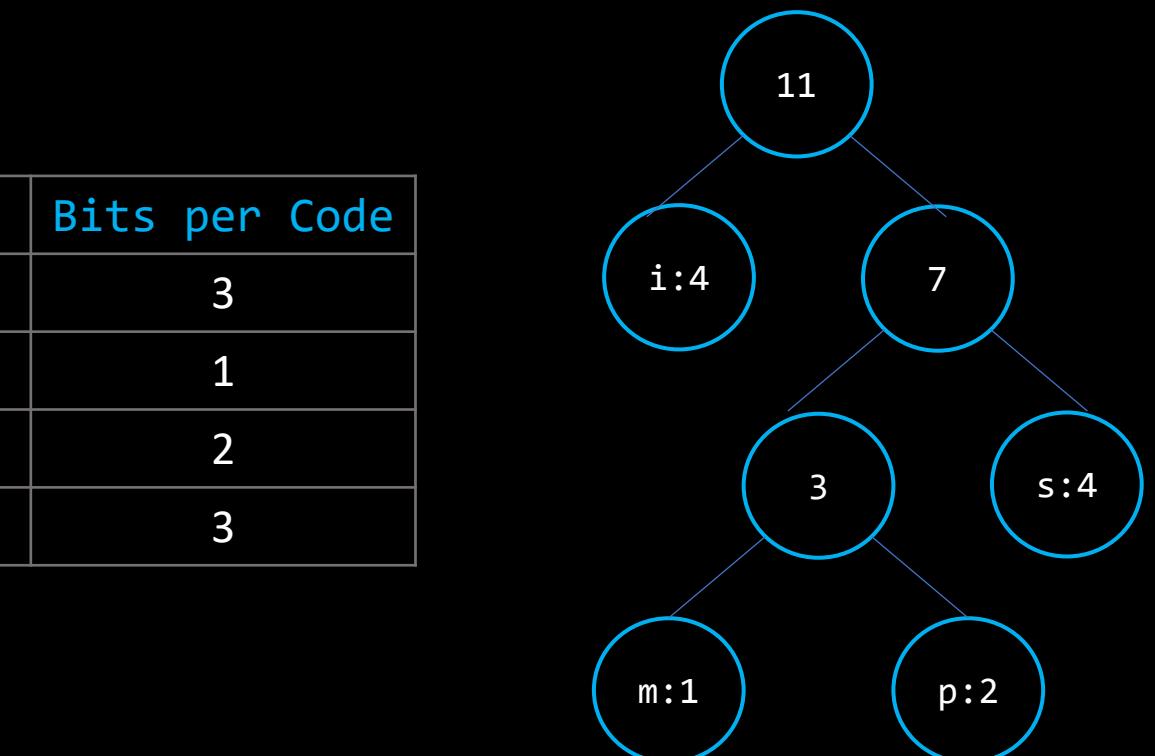
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mississippi

Character	Frequency	Code	Bits per Code
m	1	100	3
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p	2	101	3

Total bits for “mississippi” =

$$1*3 + 4*1 + 4*2 + 2*3 = 21 \text{ bits}$$



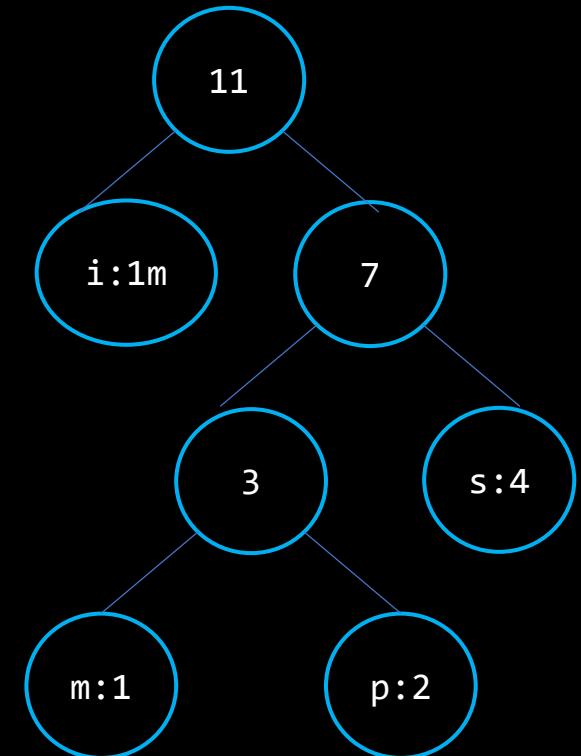
Compression:  $21/22 \sim 95.5\%$

# Algorithm for Huffman Encoding

What if there were 1 million succeeding ‘i’ in “mississippi”?

mississippi...

Character	Frequency	Code	Bits per Code
m	1	100	3
i	1,000,004	0	1
s	4	11	2
p	2	101	3



Total bits for “mississippi...” =  $1*3 + 1000004*1 + 4*2 + 2*3 = 1,000,021$  bits [Huffman codes]

Total bits for “mississippi...” through regular transmission =  $1000011*2 = 2,000,022$  bits

Compression:  $1000021/2000022 \sim 50\%$

# Algorithm for Huffman Encoding: Interface

```
class huffman_tree
{
private:
//add your data structures here

public:

/*
Preconditions: input is a string of characters with ascii values 0-127
Postconditions: reads the characters of input and constructs a
huffman tree based on the character frequencies of the file contents
*/
huffman_tree(const string& input) {}

/*
Preconditions: input is a character with ascii value between 0-127
Postconditions: returns the Huffman code for character if character
is in the tree and an empty string otherwise.
*/
string get_character_code(char character) const { return ""; }
```

```
/*
```

Preconditions: *input* is a string of characters with ascii values 0-127

Postconditions: returns the Huffman encoding for the contents of  
*file\_name* if file name exists and an empty string  
otherwise. If the file contains letters not present in the  
*huffman\_tree*, return an empty string

```
*/
```

```
string encode(const string& input) const { return ""; }
```

```
/*
```

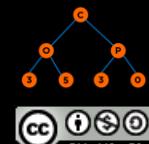
Preconditions: *string\_to\_decode* is a string containing Huffman-encoded text

Postconditions: returns the plaintext represented by the string if the string  
is a valid Huffman encoding and an empty string otherwise

```
*/
```

```
string decode(const string& string_to_decode) const { return ""; }
```

```
};
```



# Questions

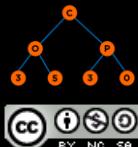
# Mentimeter

Code

6498 872



# Dynamic Programming



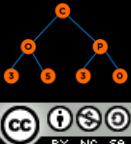
# Dynamic Programming

- Top-down DP: Memoization
- Bottom-up DP: Tabulation

# Fibonacci Sequence

```
int standardFibonacci(int n)
{
    if (n <= 1)
        return n;
    return standardFibonacci(n-1) + standardFibonacci(n-2);
}
```

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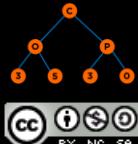


# Fibonacci Sequence

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int standardFibonacci(int n)
{
    if (n <= 1)
        return n;
    return standardFibonacci(n-1) + standardFibonacci(n-2);
}
```

Time Complexity:  
Space Complexity:

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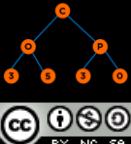


# Fibonacci Sequence

```
int standardFibonacci(int n)
{
    if (n <= 1)
        return n;
    return standardFibonacci(n-1) + standardFibonacci(n-2);
}
```

Time Complexity:  $O(2^n)$   
Space Complexity:  $O(n)$

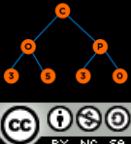
<https://onlinedb.com/S1VSAMjUu>



# Fibonacci Sequence: Tabulation

```
int bottomUpDPFibonacci(int n)
{
    vector<int> dp(n+1);
    dp[0] = 0;
    dp[1] = 1;
    for (int i = 2; i <= n; i++)
        dp[i] = dp[i - 1] + dp[i - 2];
    return dp[n];
}
```

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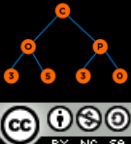


# Fibonacci Sequence: Tabulation

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    vector<int> dp(n+1);
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}
```

Time Complexity:  
Space Complexity:

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# Fibonacci Sequence: Tabulation

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int bottomUpDPFibonacci(int n)
{
    vector<int> dp(n+1);
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    for (int i = 2; i <= n; i++)
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    return dp[n];
}
```

Time Complexity:  $O(n)$   
Space Complexity:  $O(n)$

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# Fibonacci Sequence: Memoization

```
int fib(vector<int> &dp, int n)
{
    if (n <= 1)
        return n;
    if (dp.at(n) != 0)
        return dp.at(n);
    dp.at(n) = fib(dp, n - 1) + fib(dp, n - 2);
    return dp.at(n);
}
```

```
int topDownDPFibonacci(int n)
{
    vector<int> dp(n + 1, 0);
    return fib(dp, n);
}
```

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# Fibonacci Sequence: Memoization

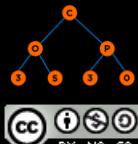
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}
```

```
int mFibonacci(int n)
{
    vector<int> dp(n + 1, 0);
    return fib(dp, n);
}
```

```
int main()
{
    mFibonacci(5);
    return 0;
}
```

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0	0	0	0	0	0
0	1	2	3	4	5



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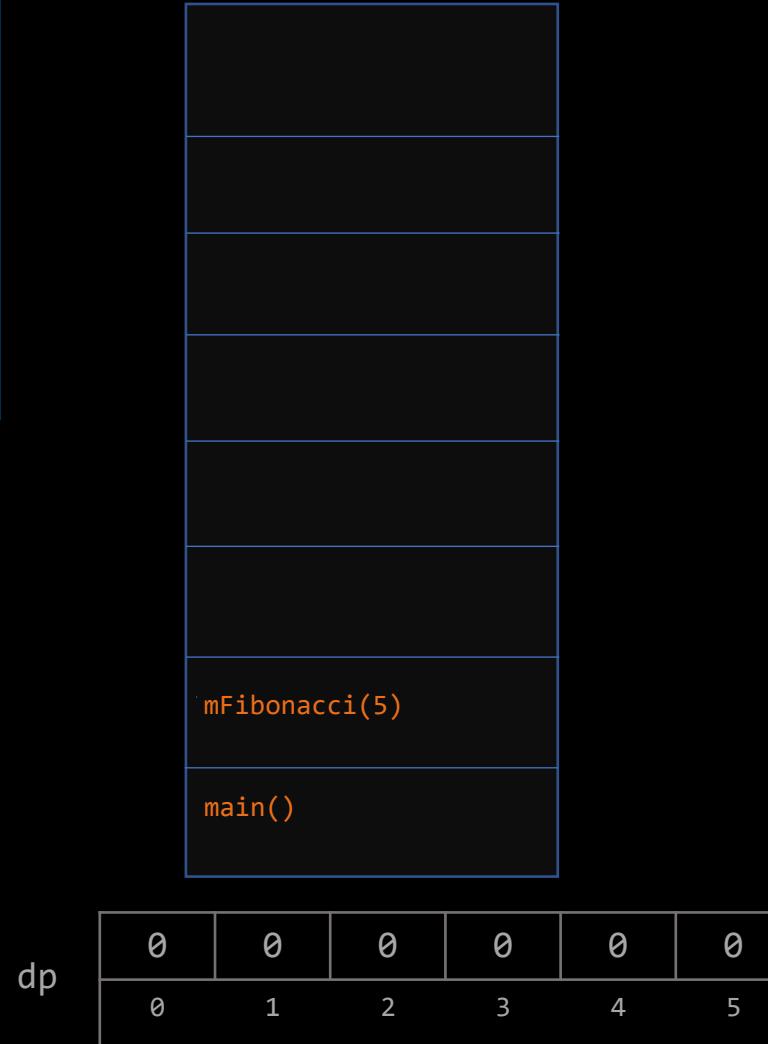
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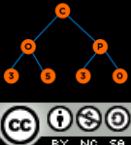
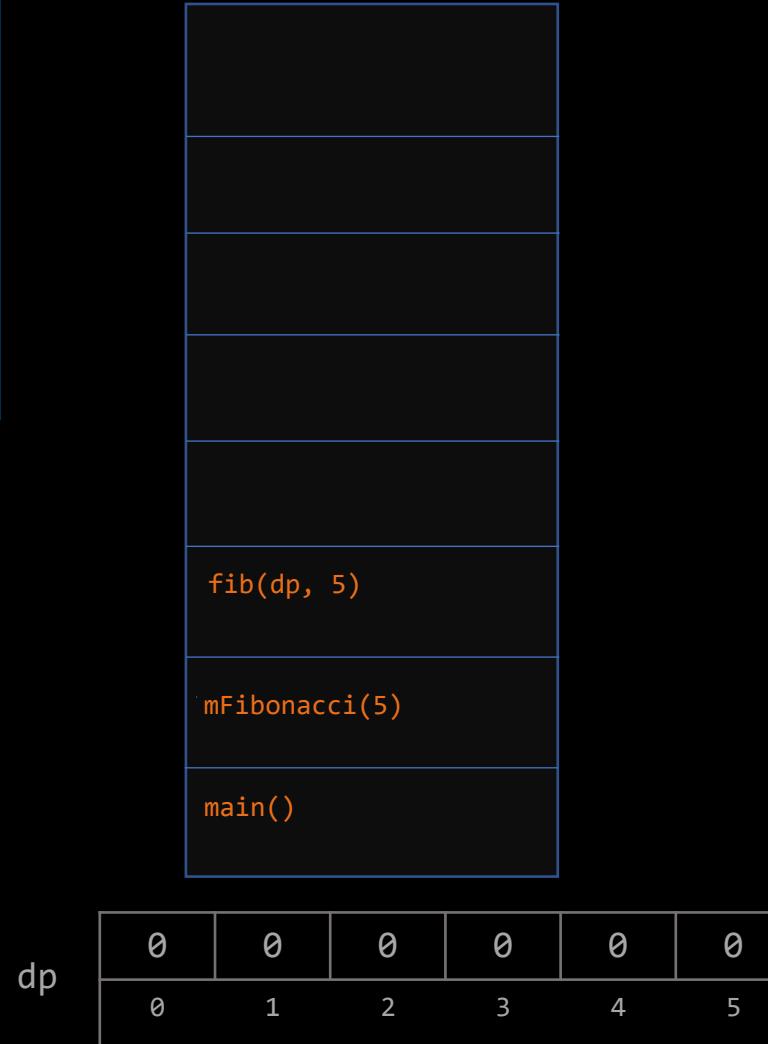
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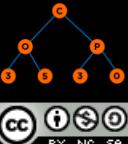
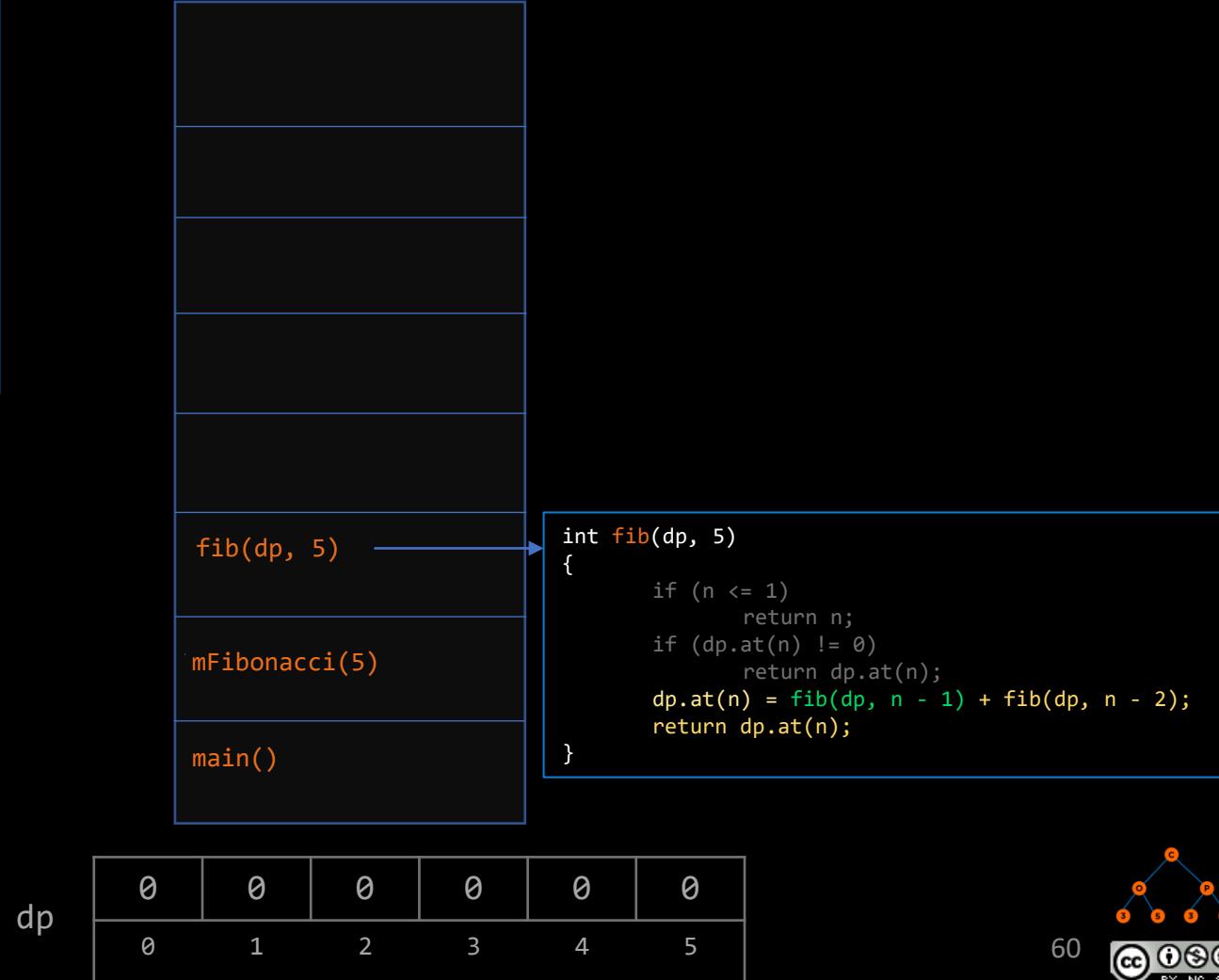
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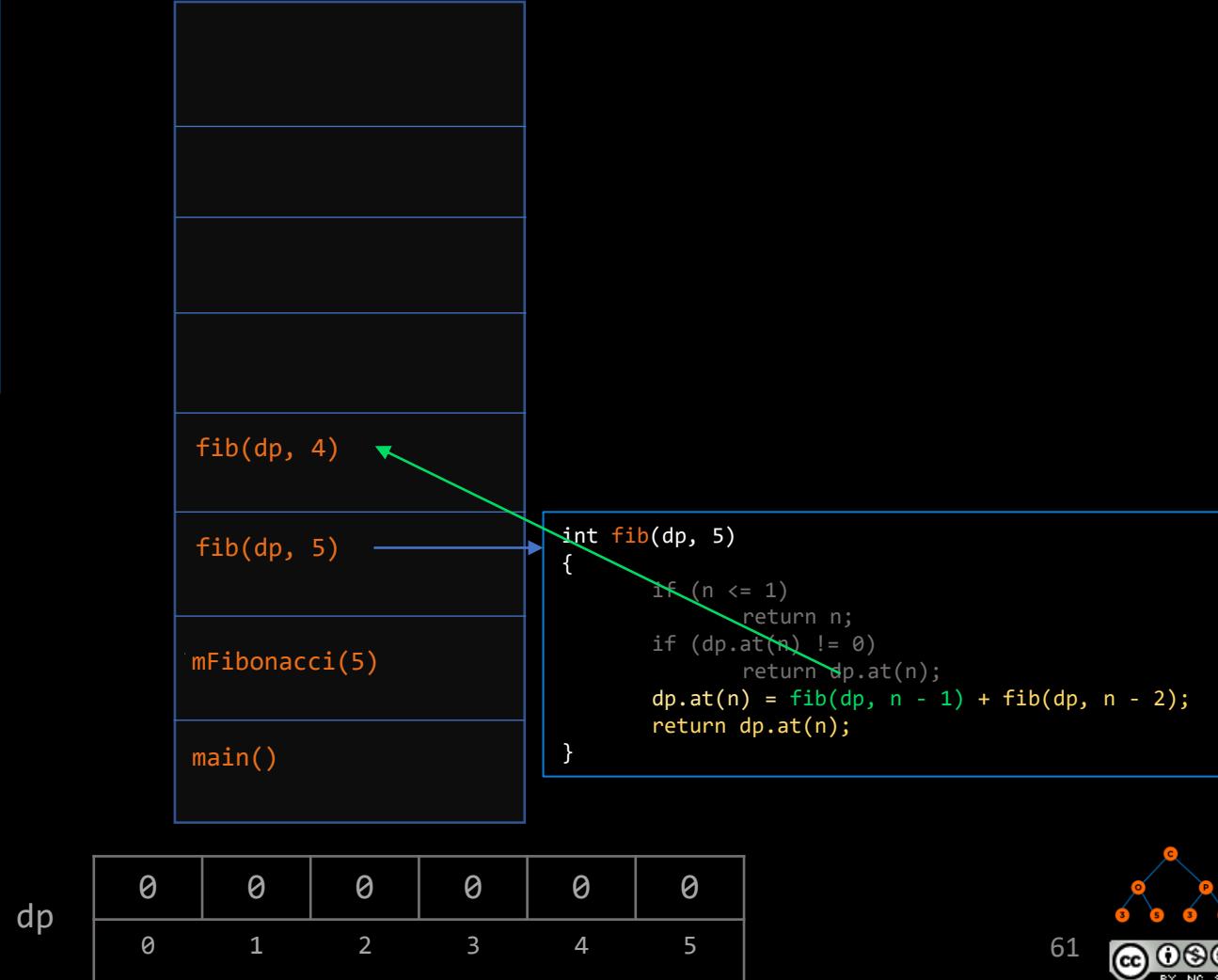


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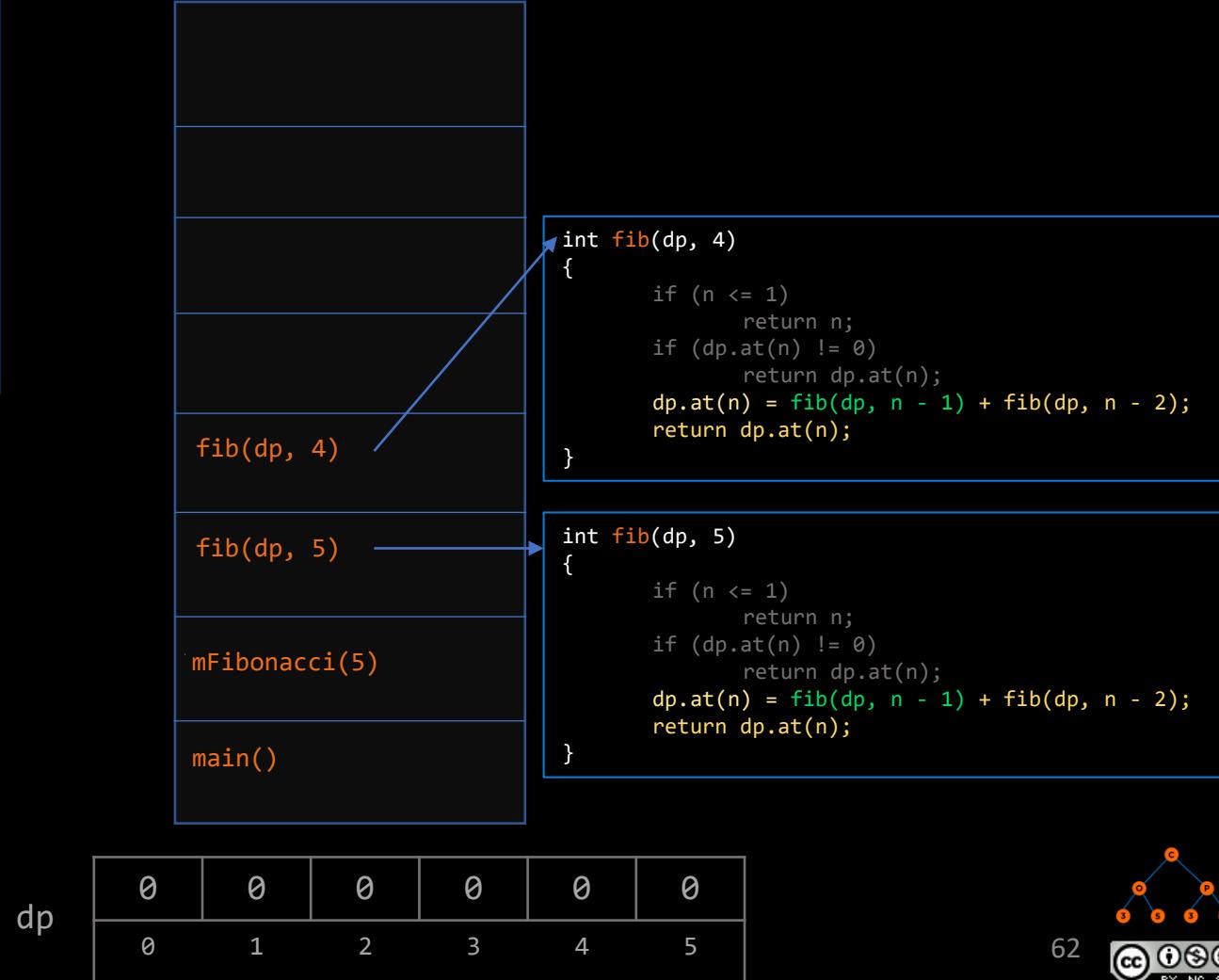
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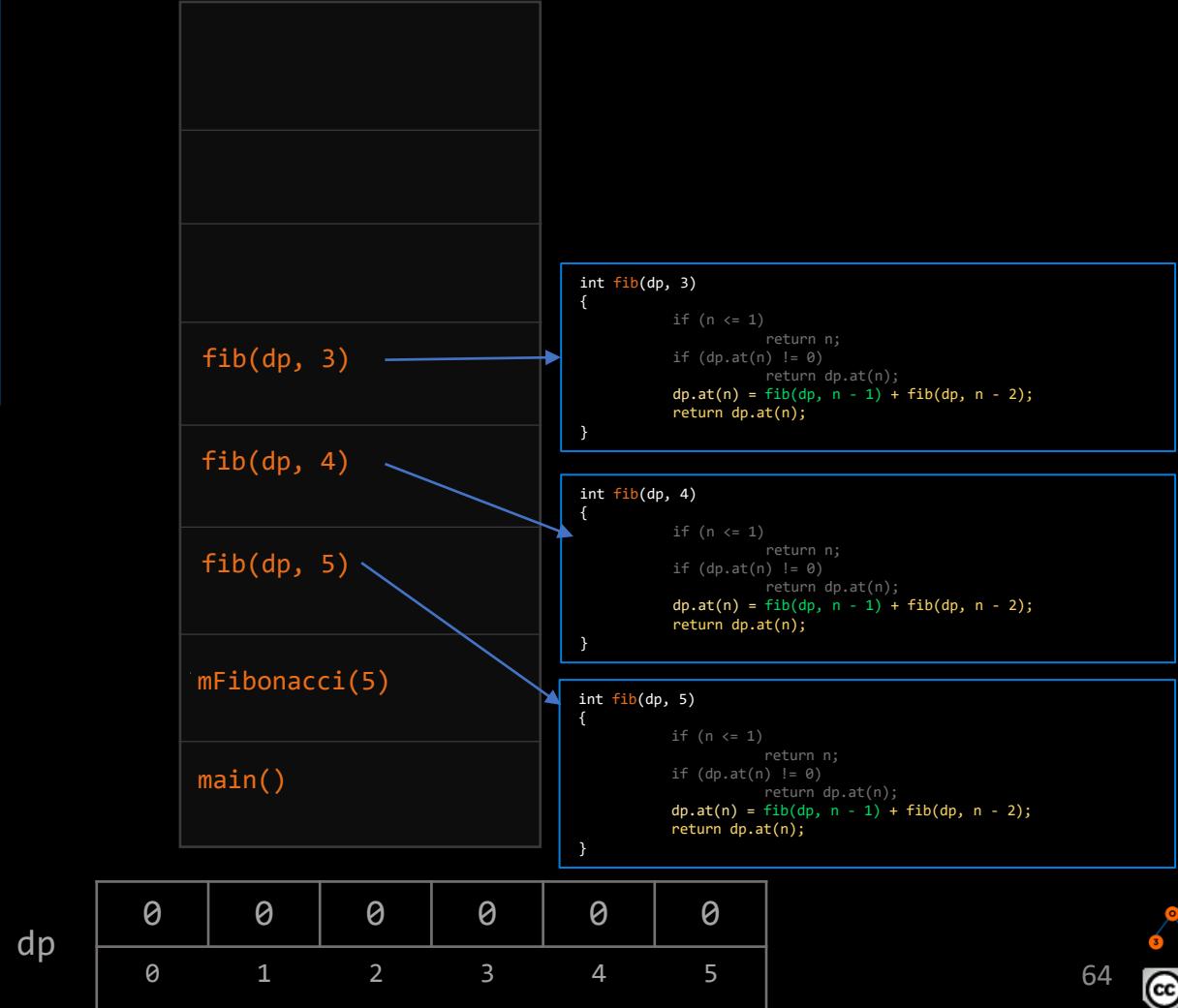
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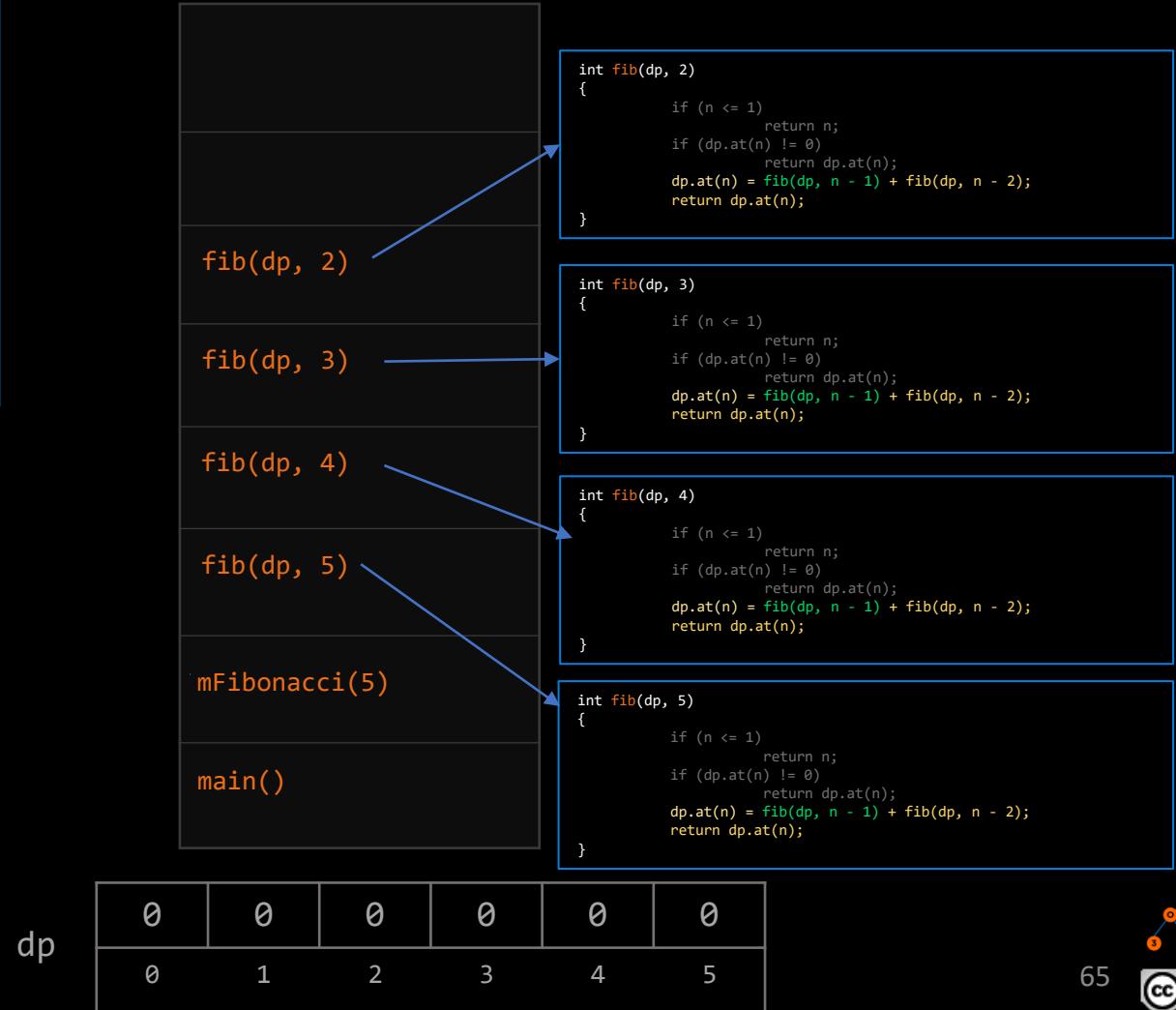
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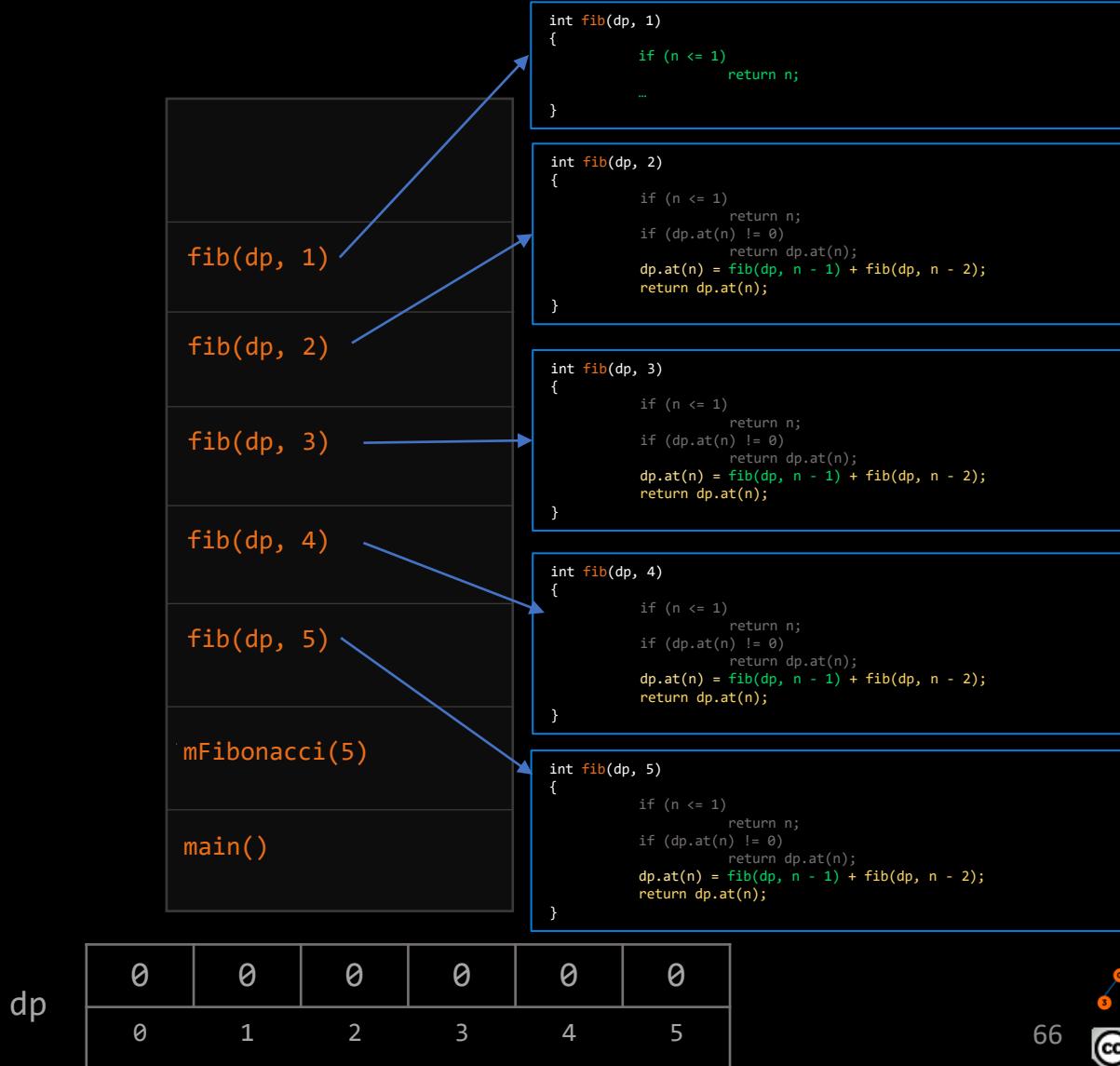
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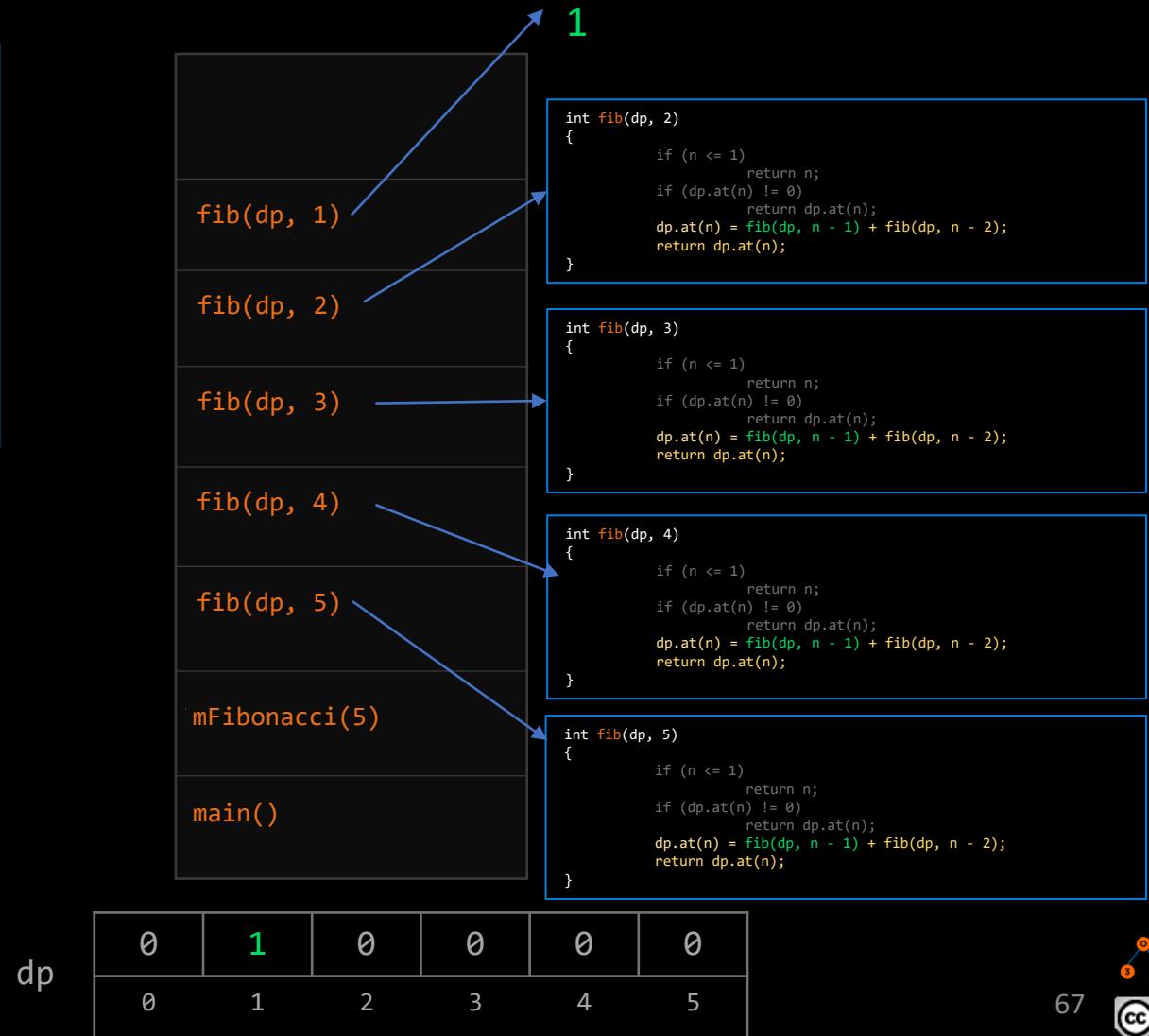
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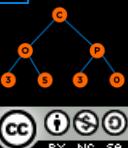
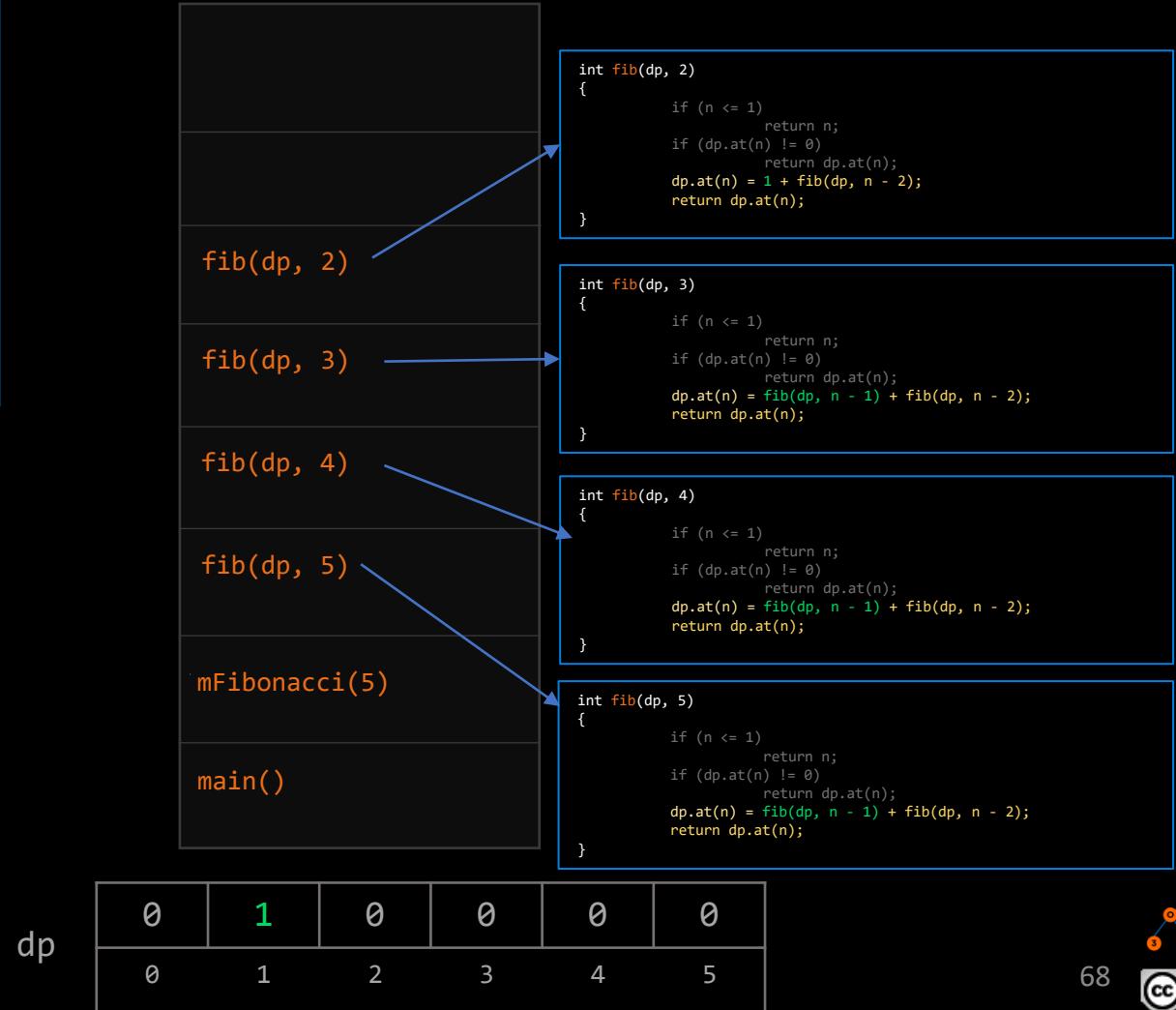
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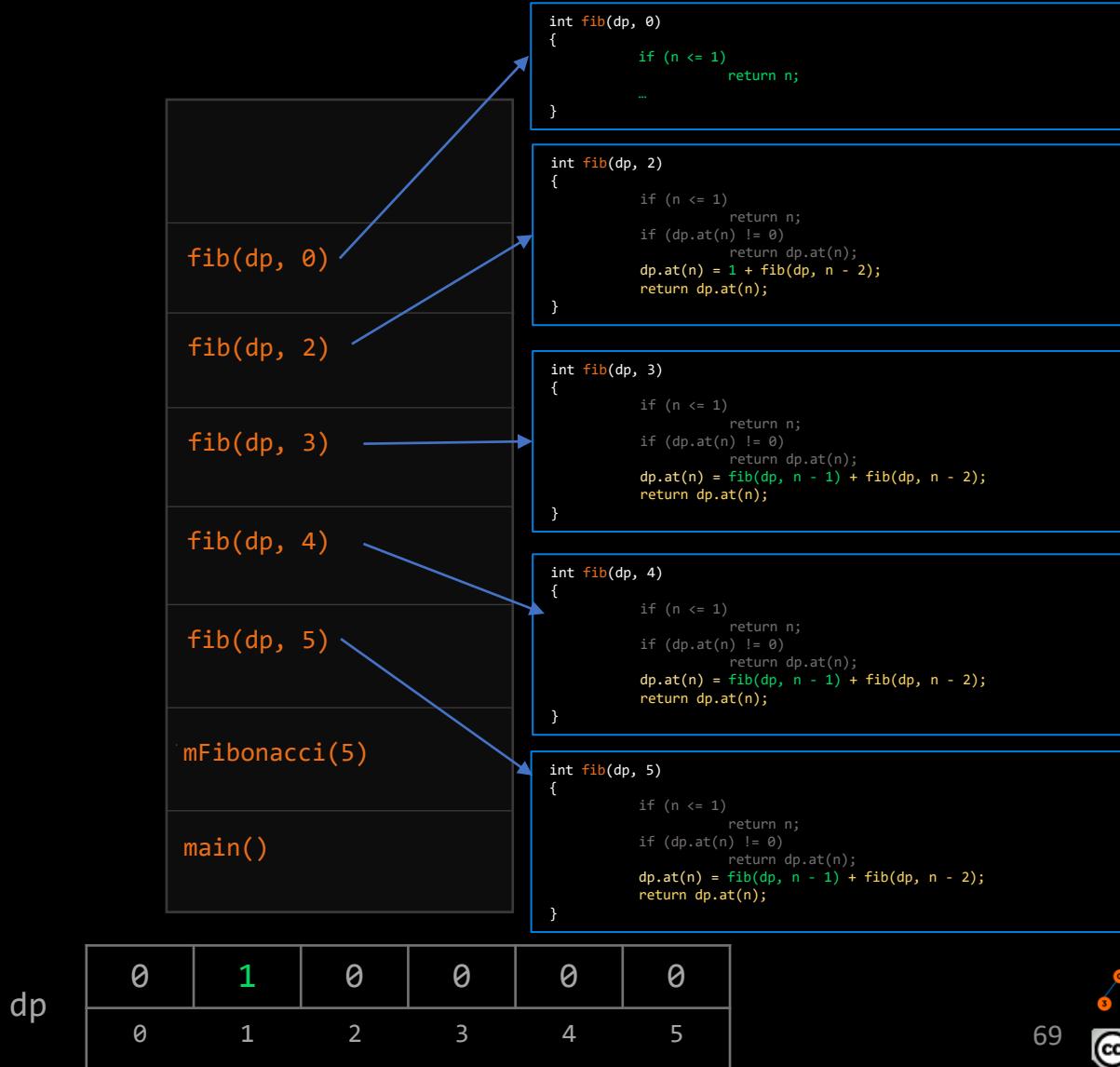
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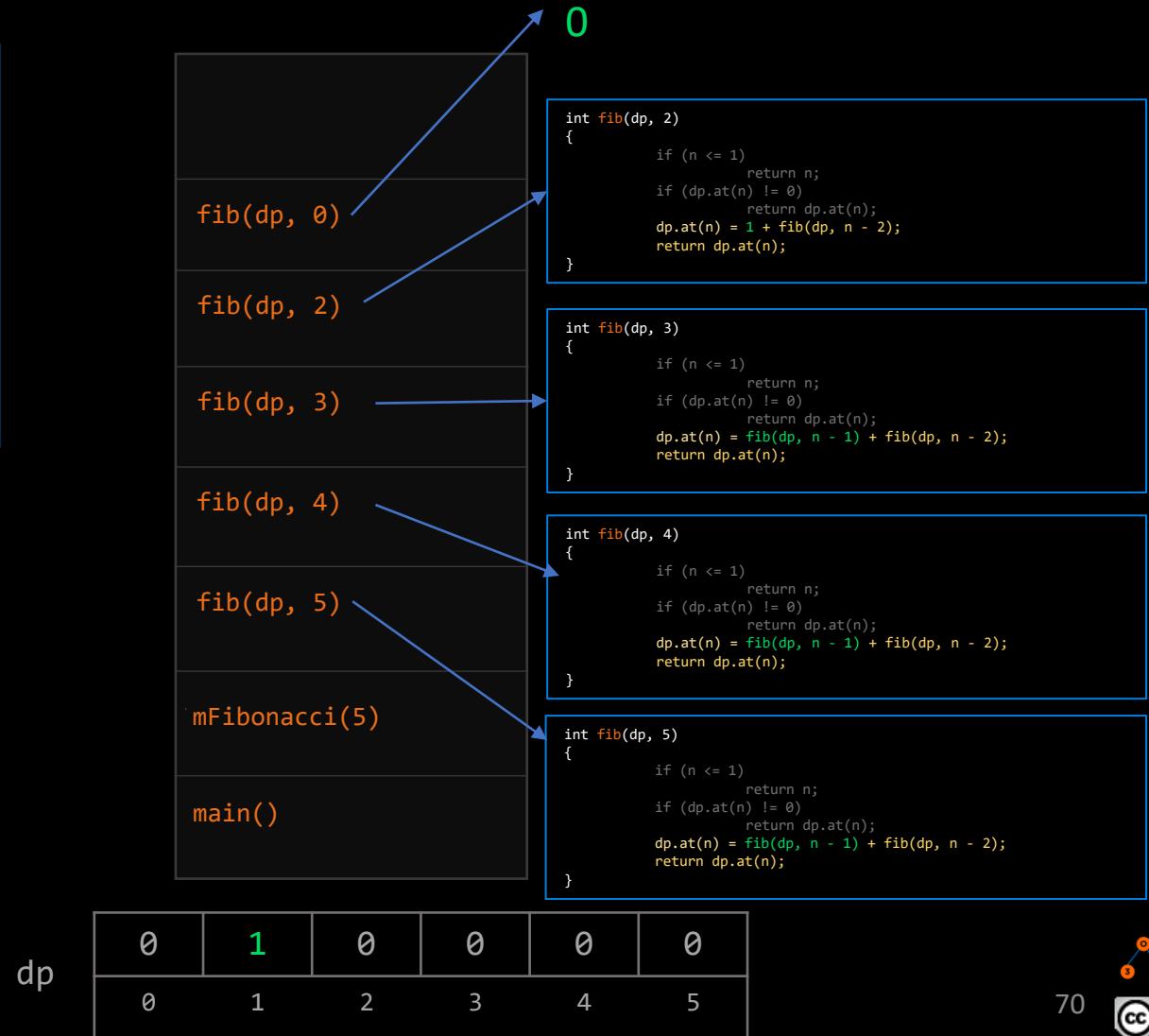
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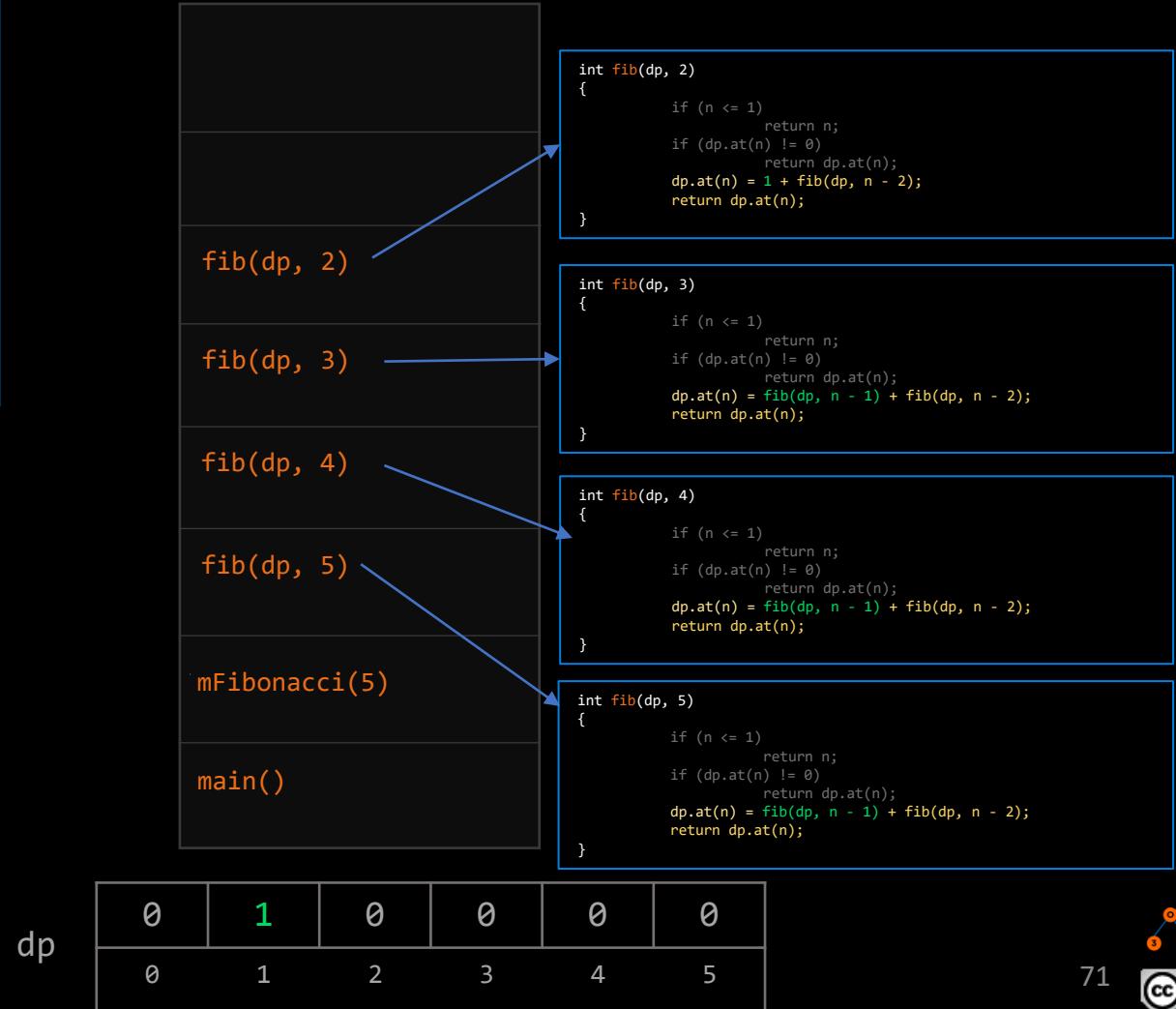
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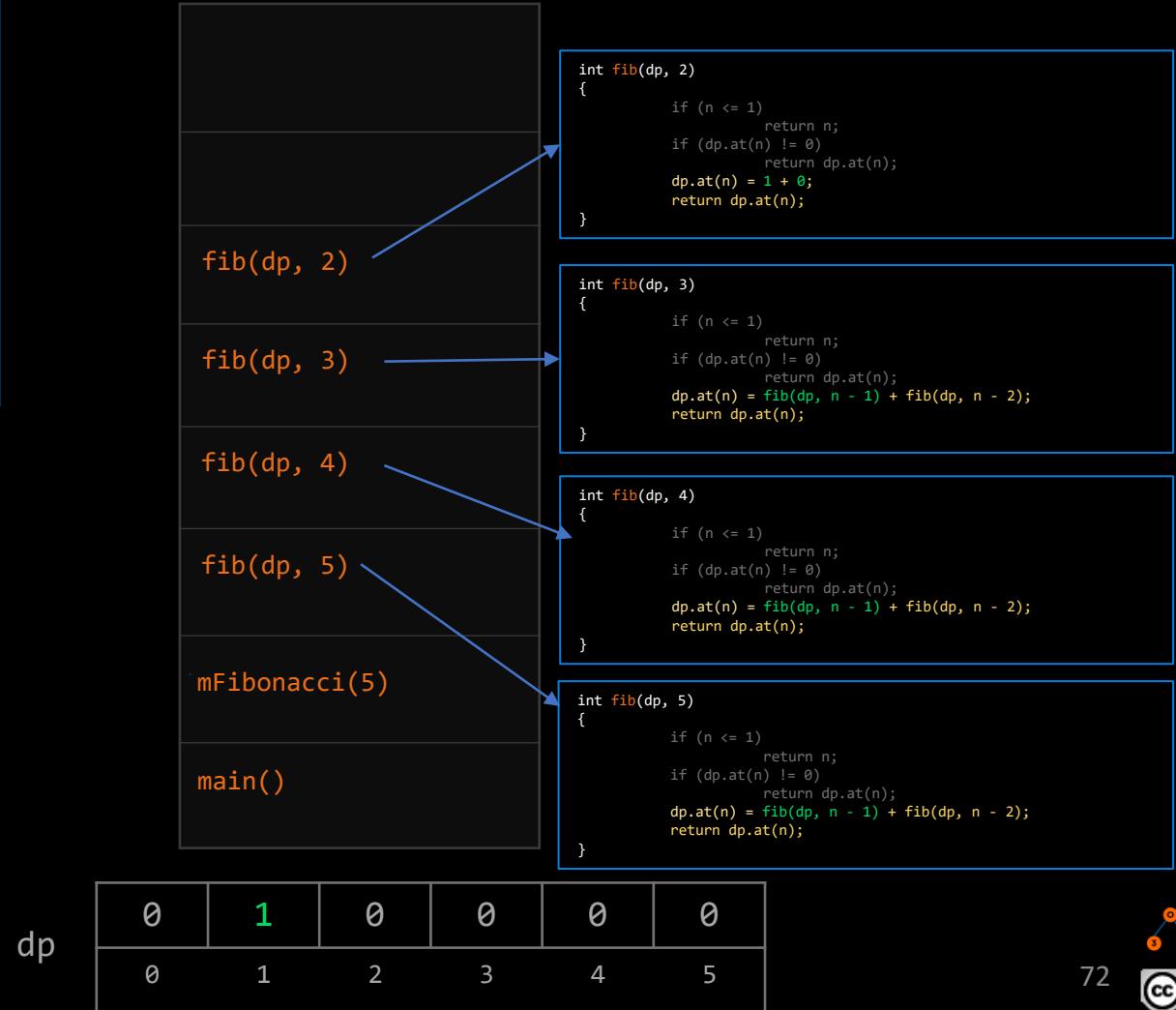
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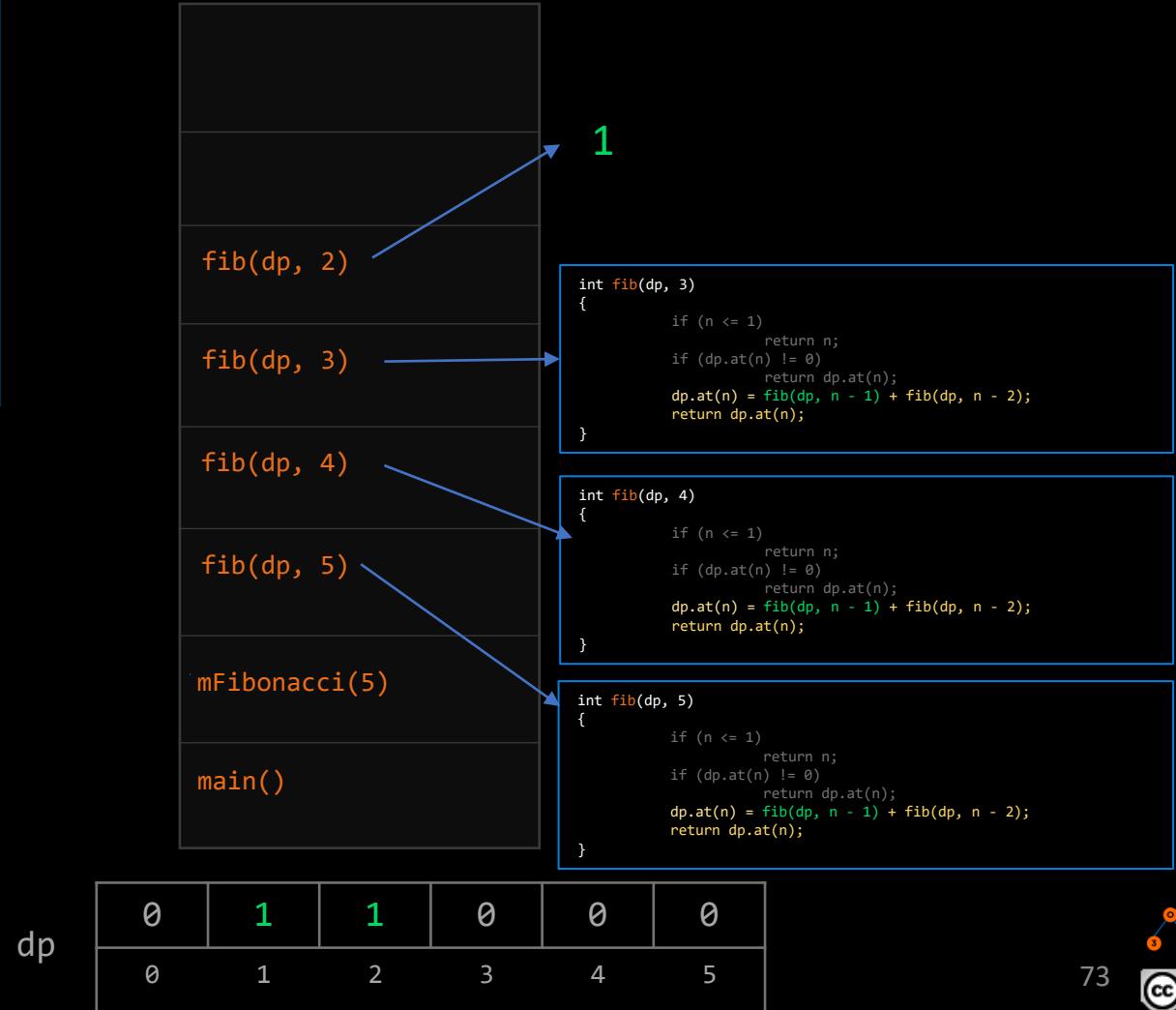
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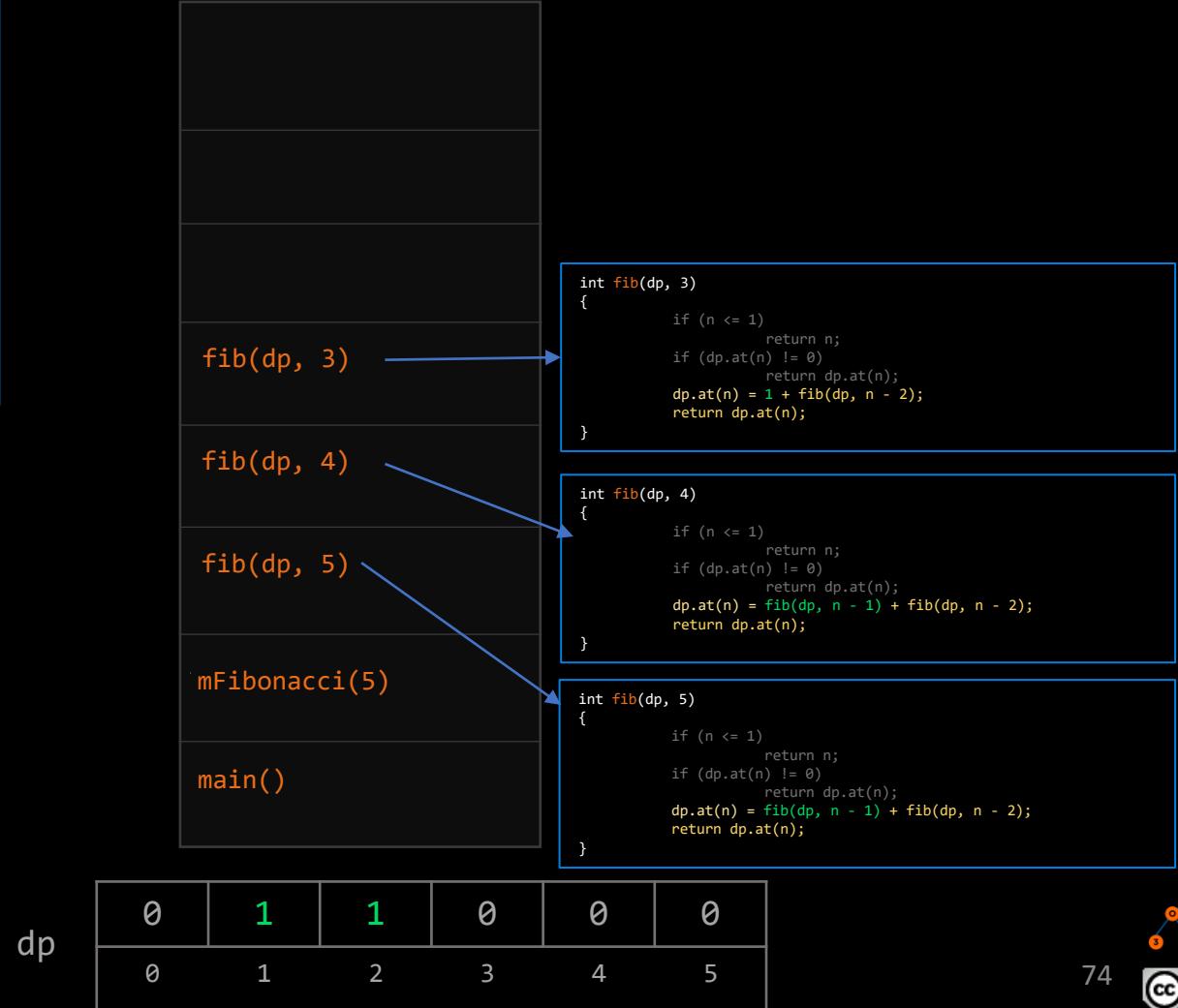
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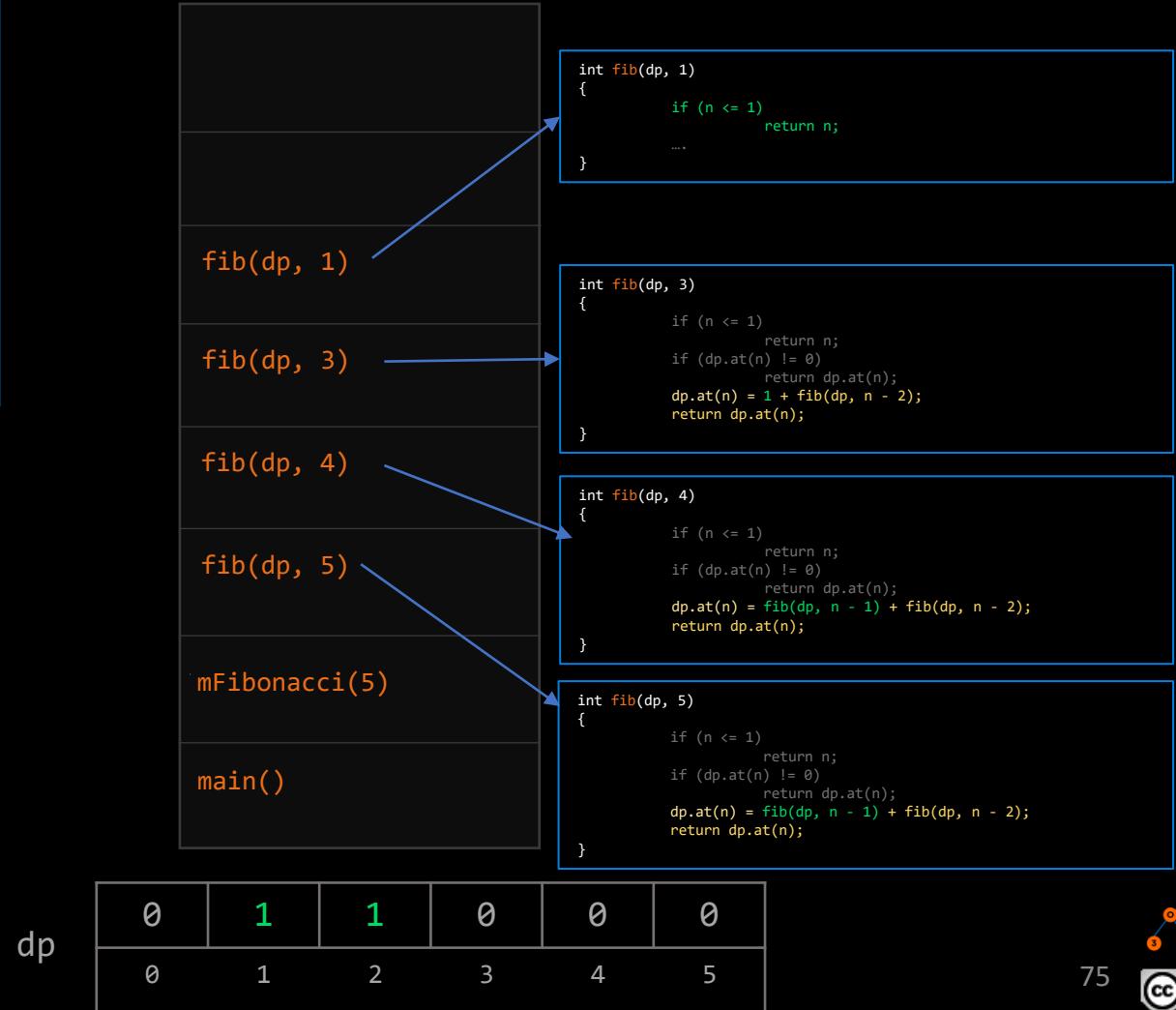
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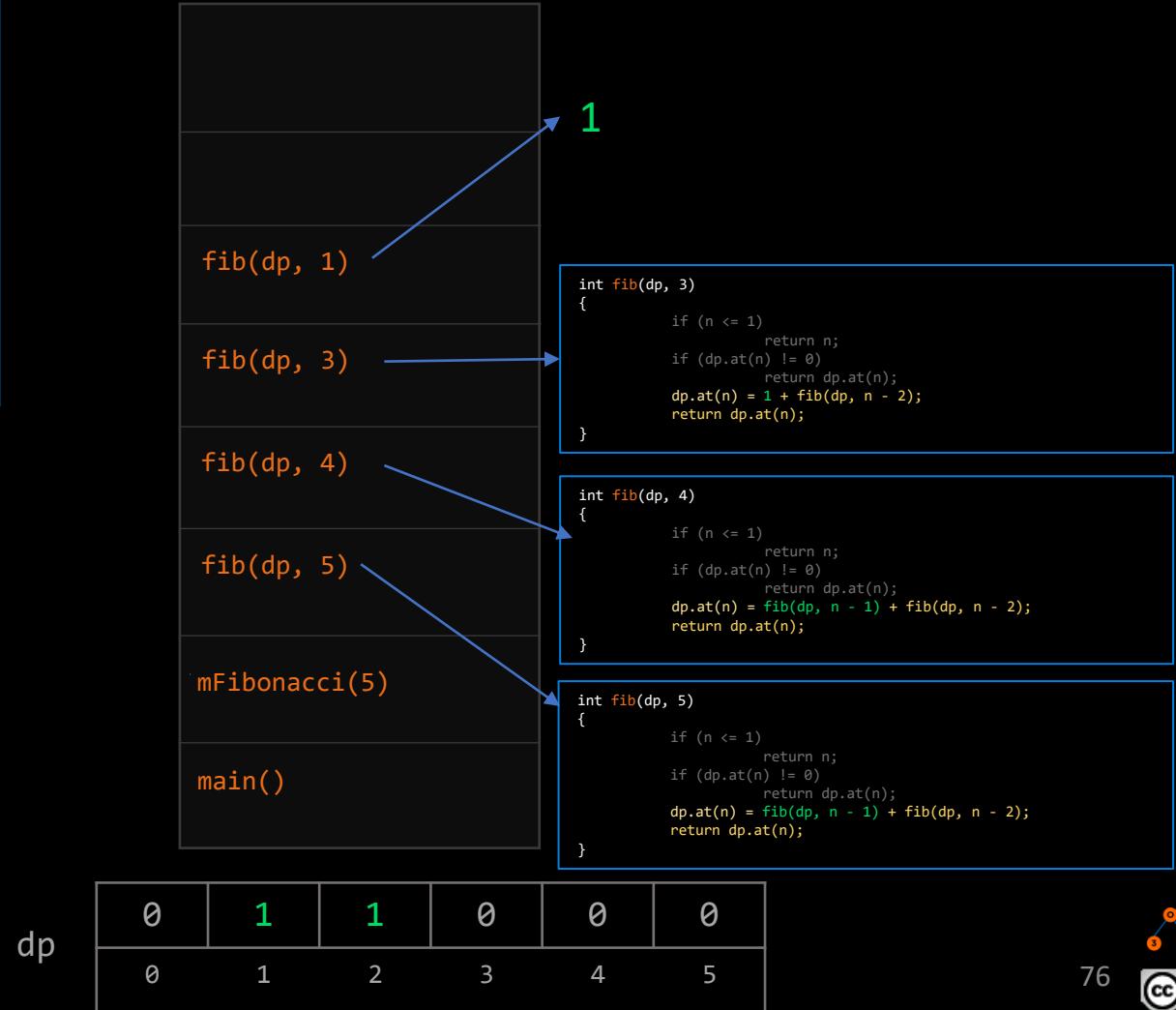
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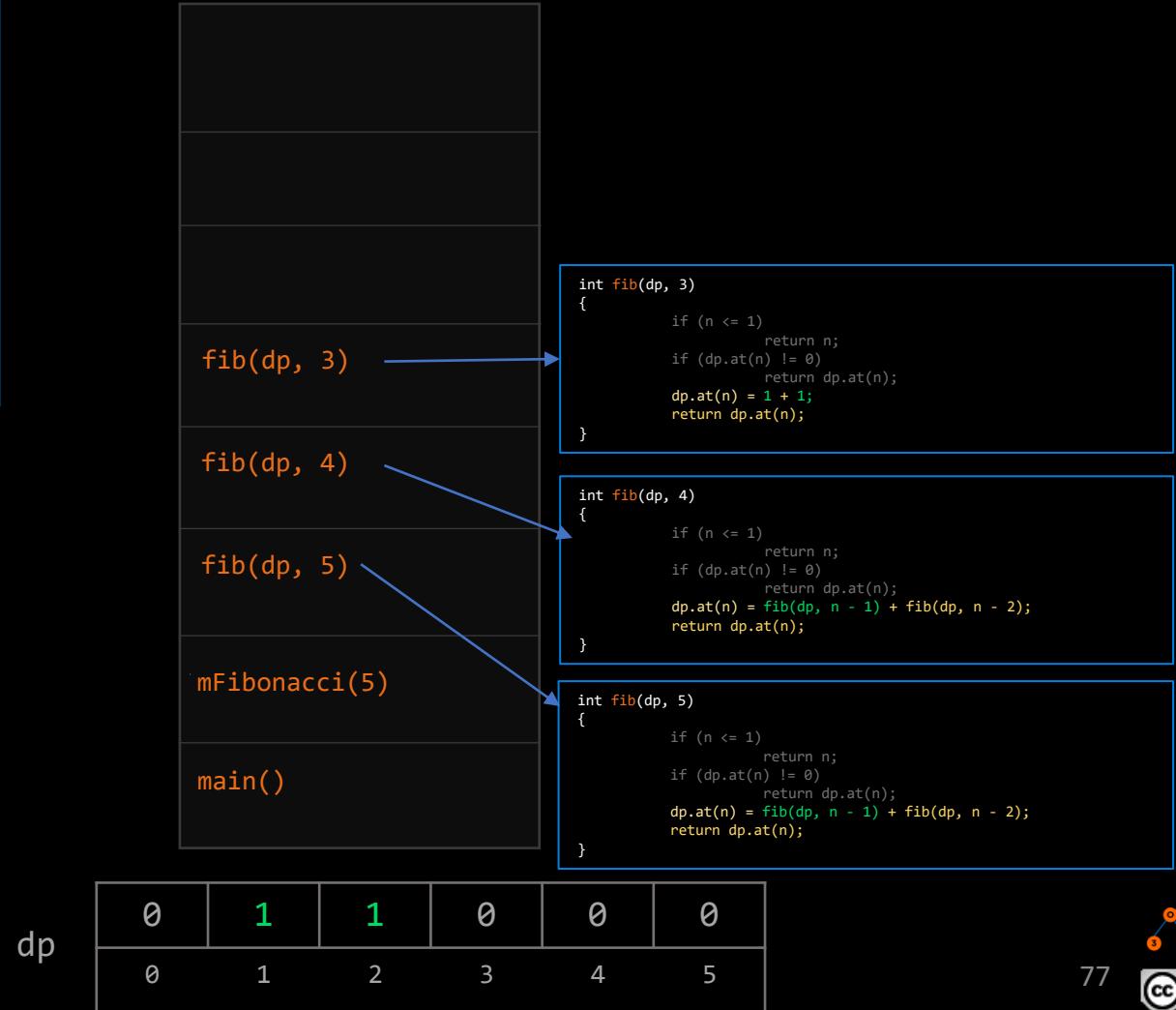
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}
```

```
int main()
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<https://onlinegdb.com/S1VSAMjUu>



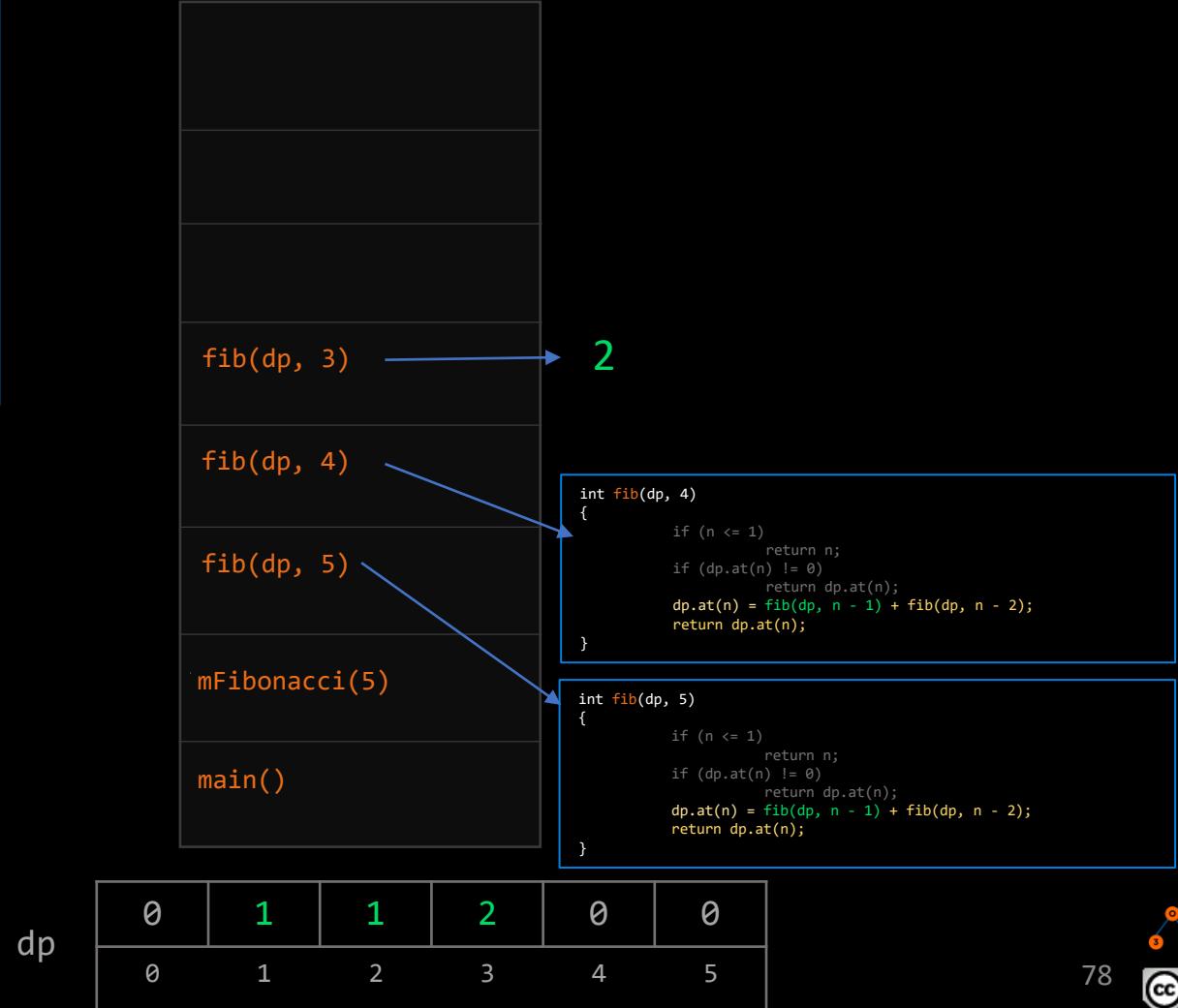
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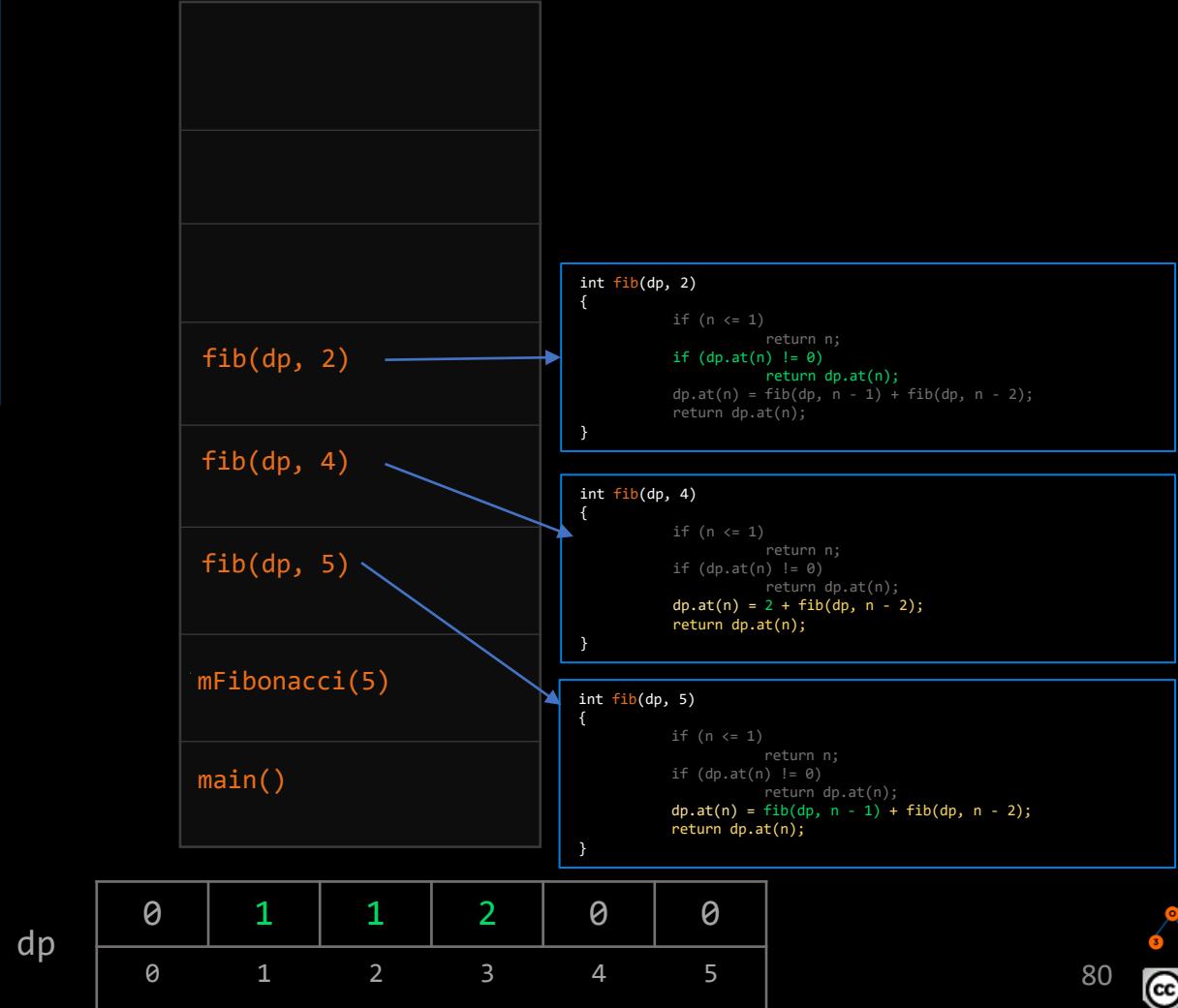
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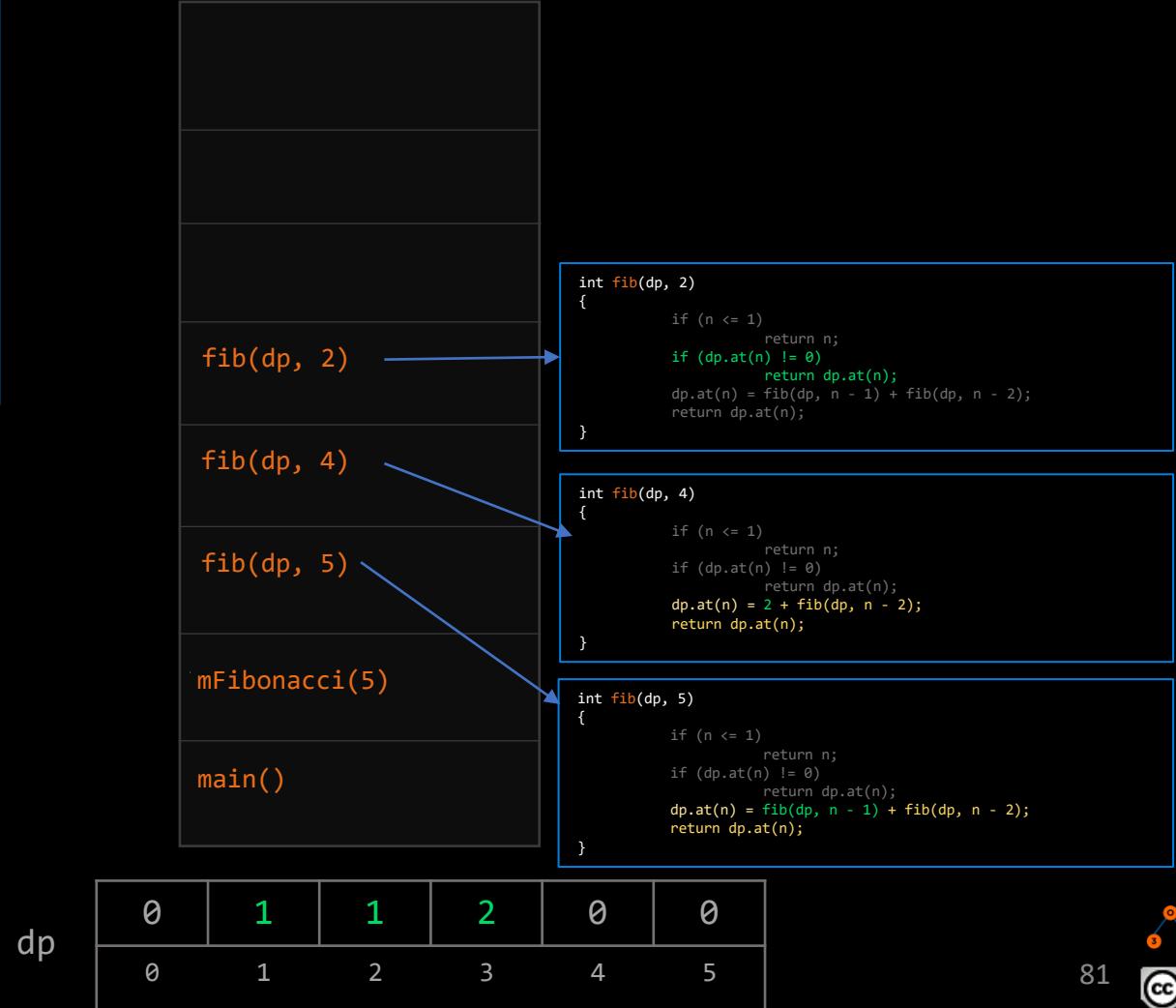
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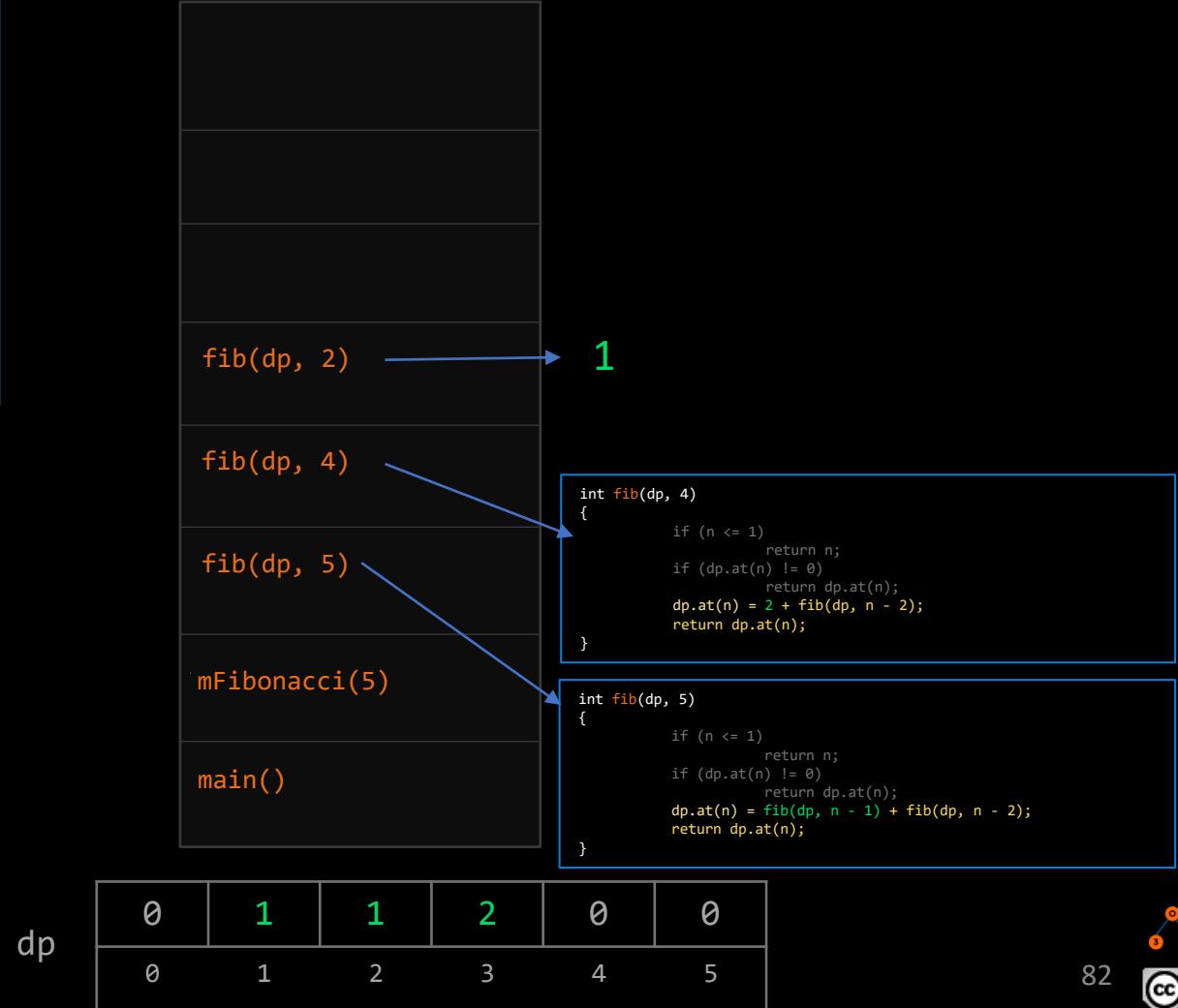
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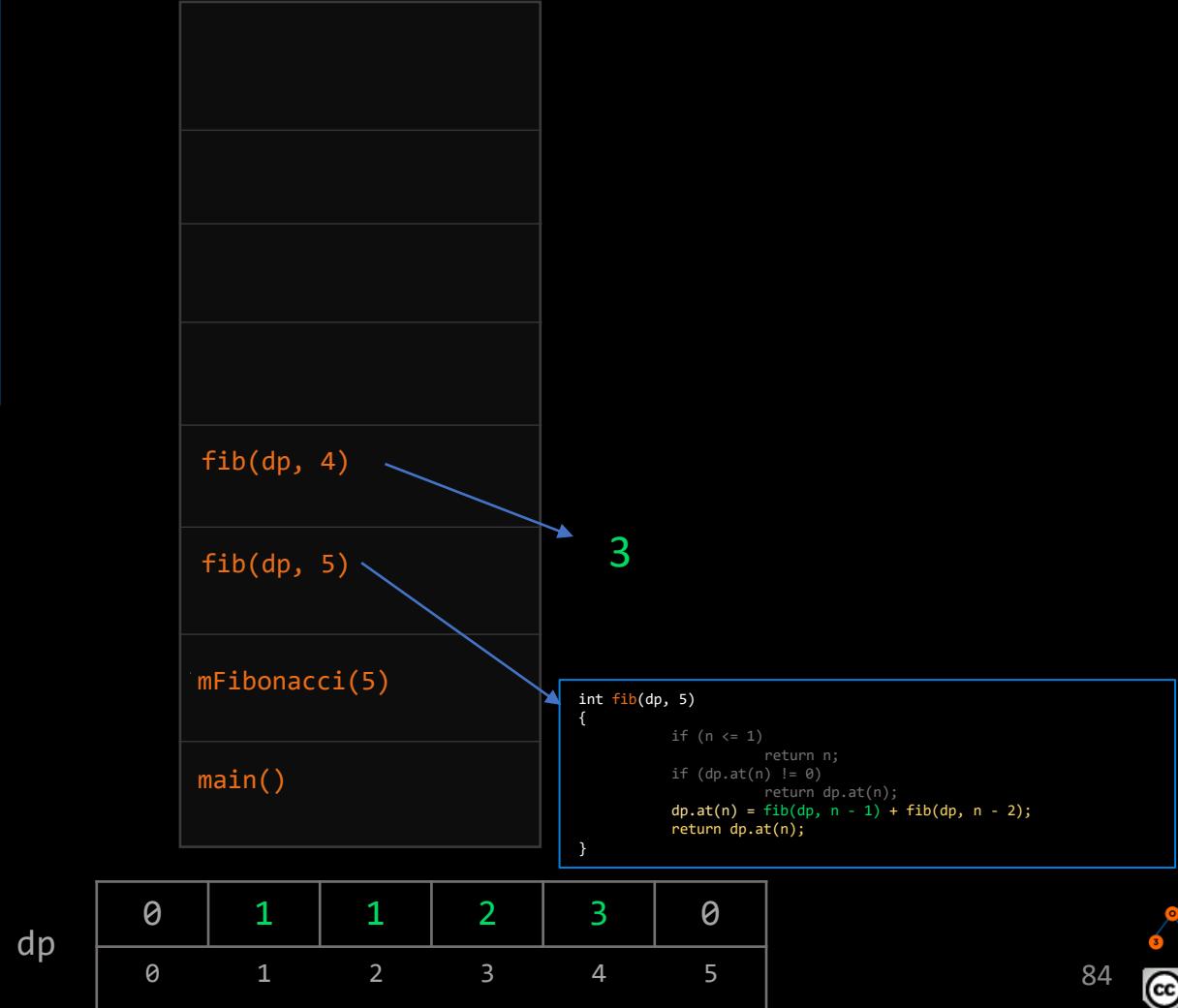
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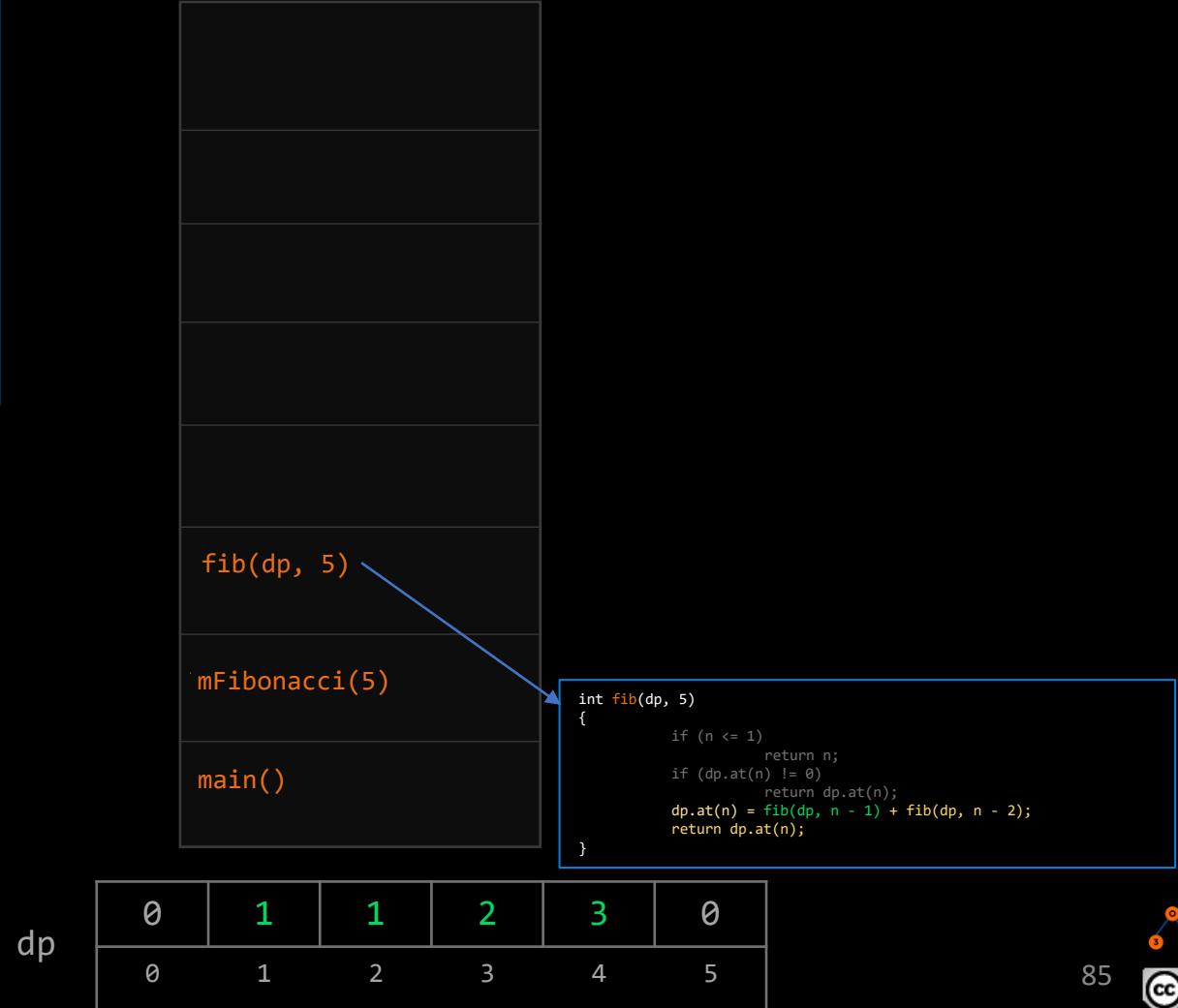
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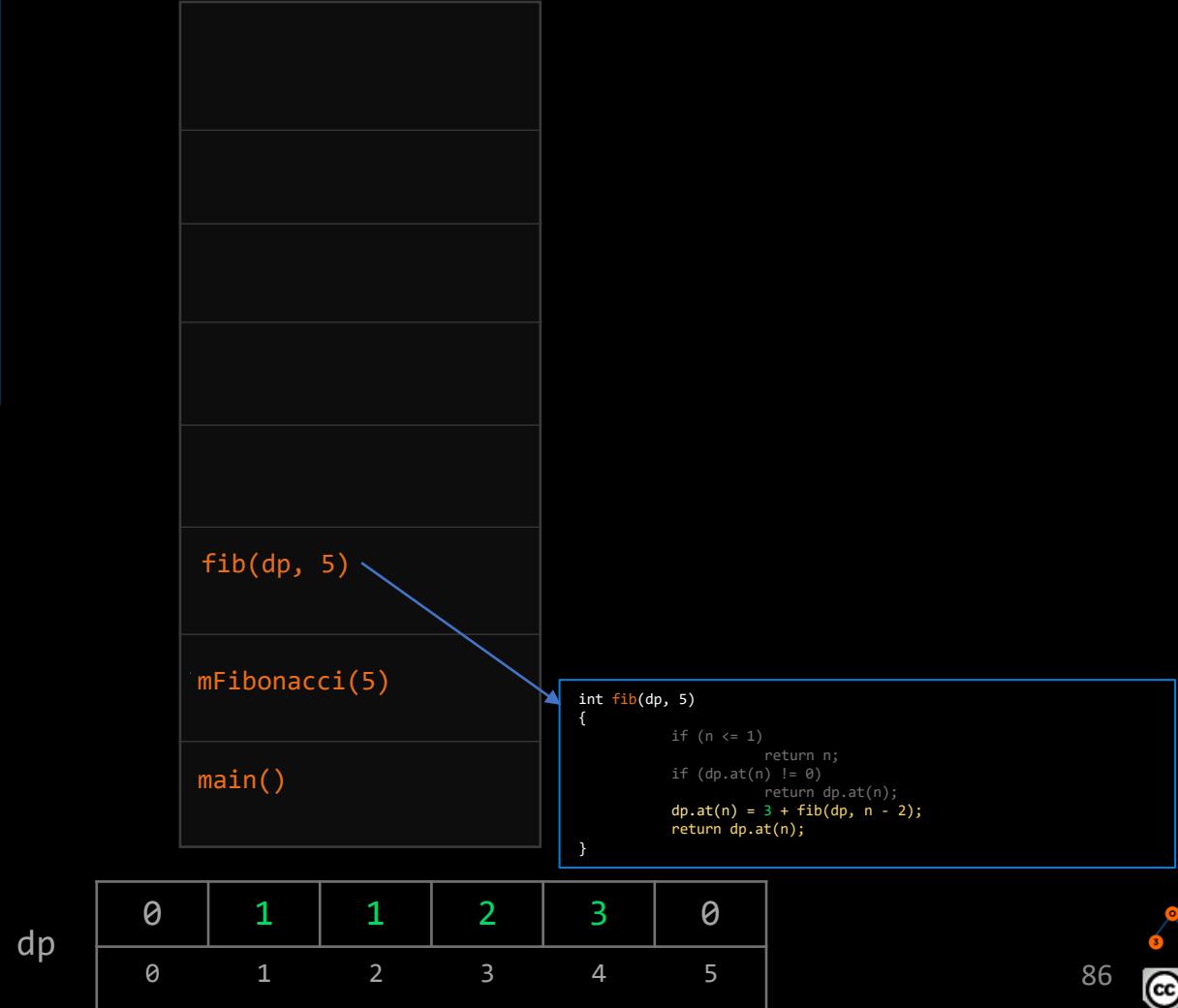
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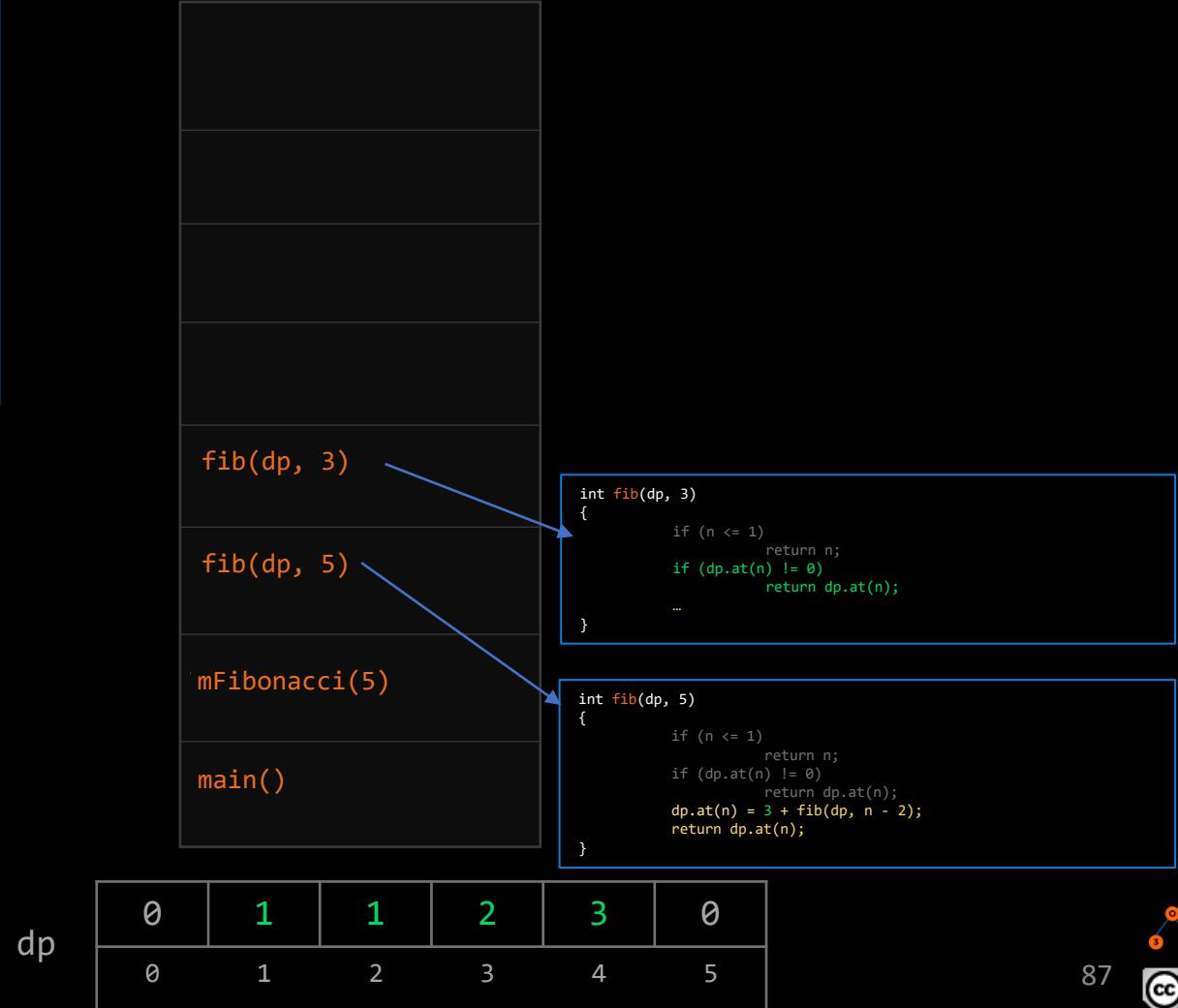
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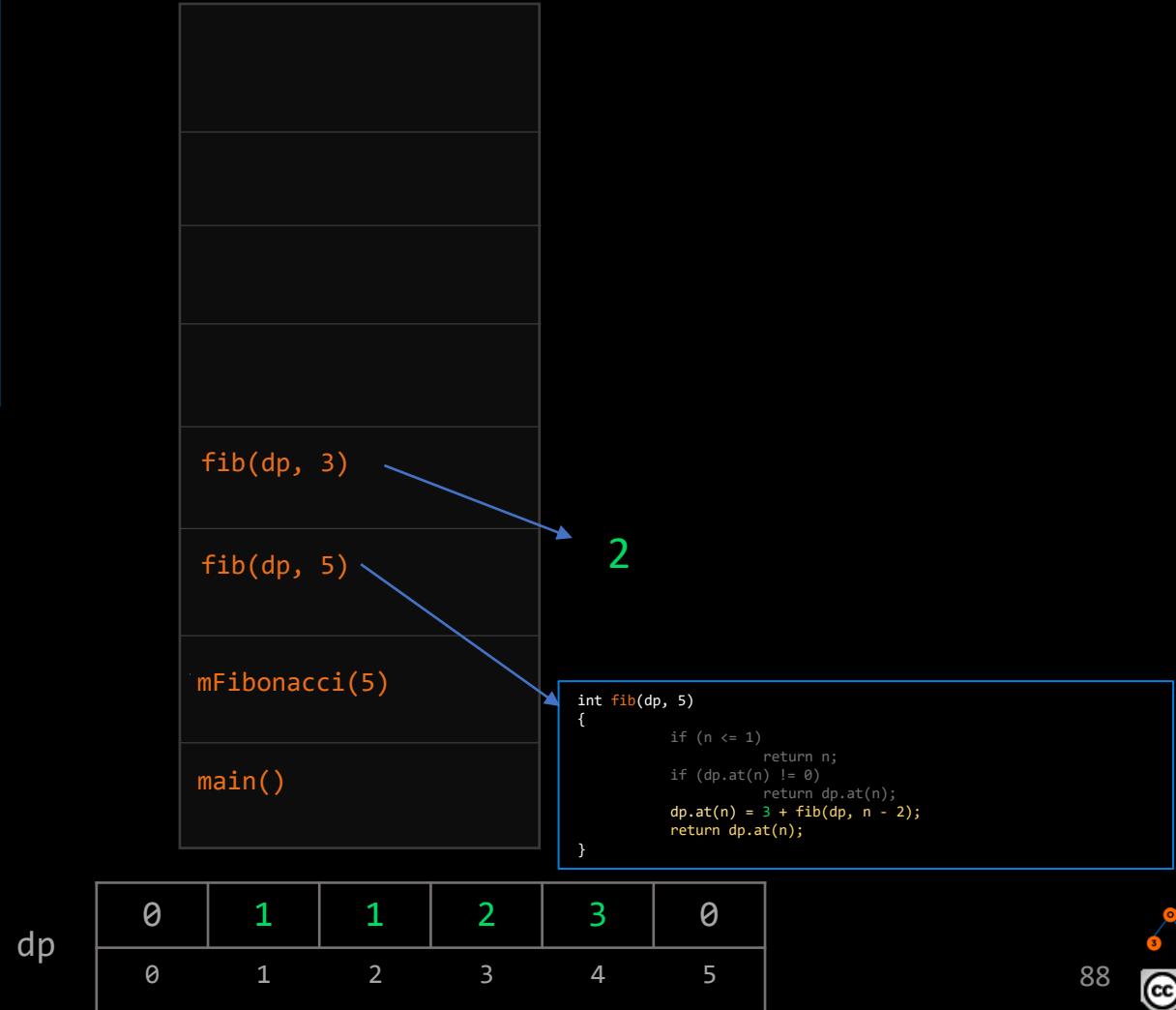
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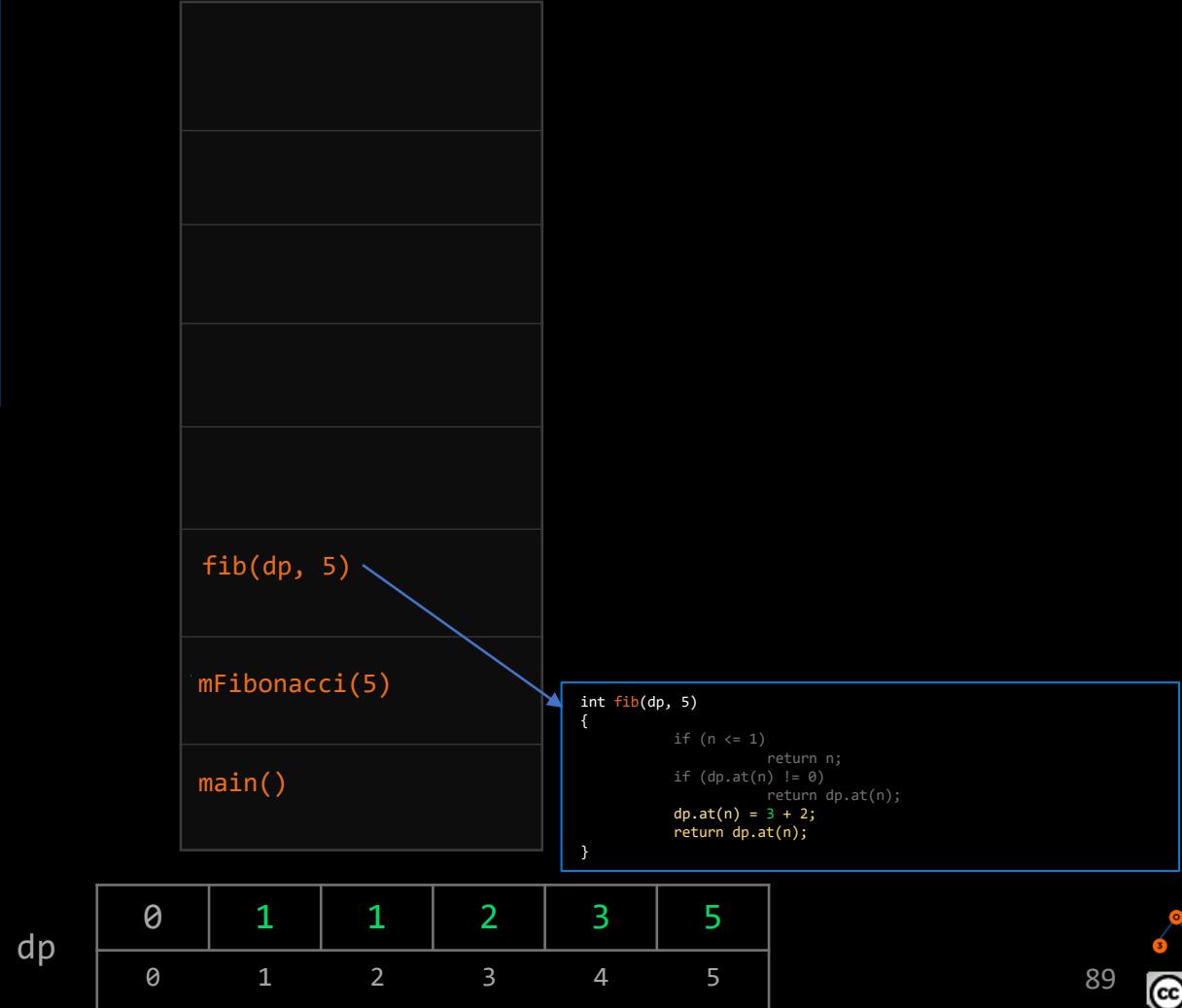
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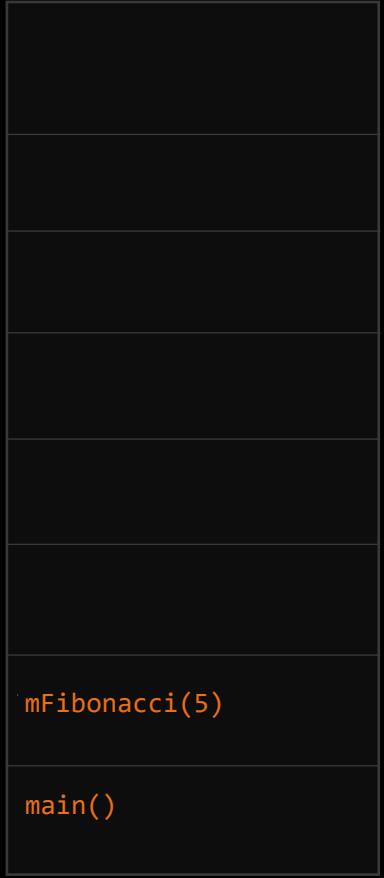
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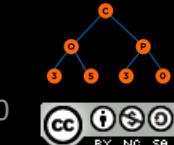
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dp	0	1	1	2	3	5
	0	1	2	3	4	5



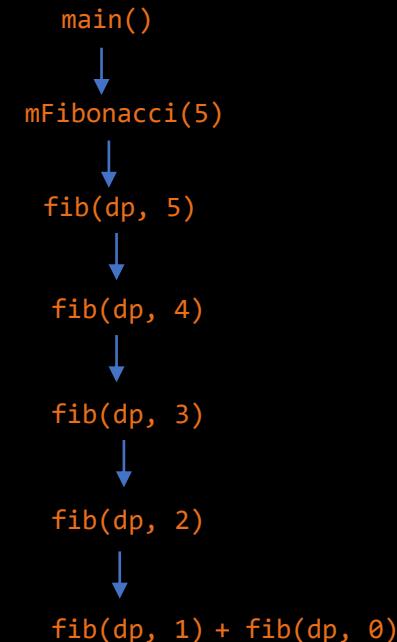
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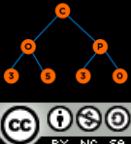
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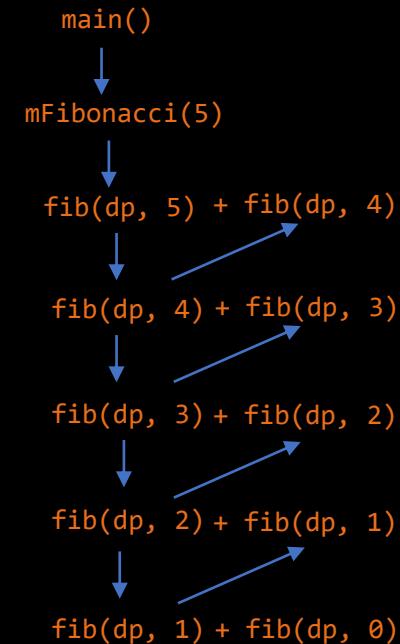
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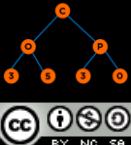
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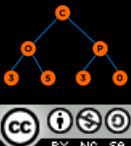
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    return dp.at(n);
}
```

```
int topDownDPFibonacci(int n)
{
    vector<int> dp(n + 1, 0);
    return fib(dp, n);
}
```

Time Complexity:  
Space Complexity:

<https://onlinegdb.com/S1VSAMjUu>



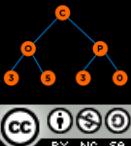
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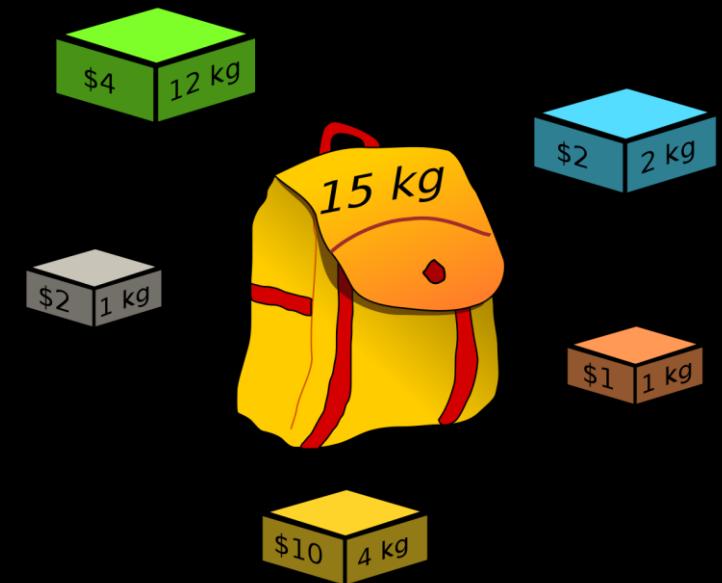
Time Complexity:  $O(n)$   
Space Complexity:  $O(n)$

<https://onlinegdb.com/S1VSAMjUu>



# Knapsack Problem

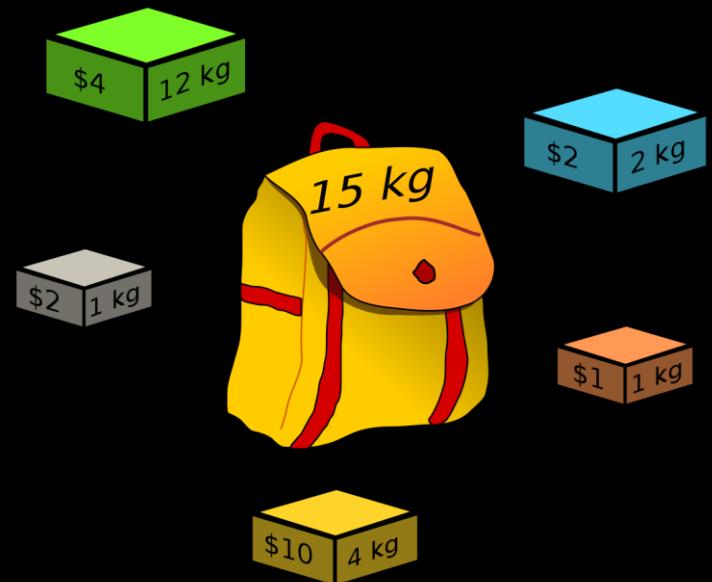
- One or more Constraints
- Bounded / Unbounded / Fractional Items



<https://images.app.goo.gl/vQejKPdsUynyp3As6>

# 0-1 Knapsack Problem

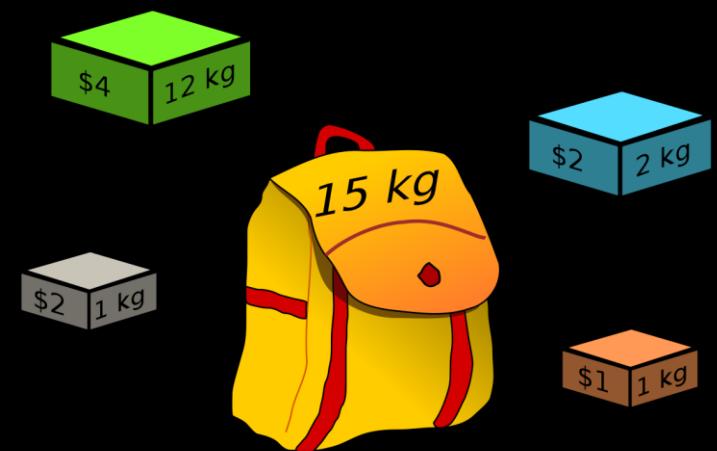
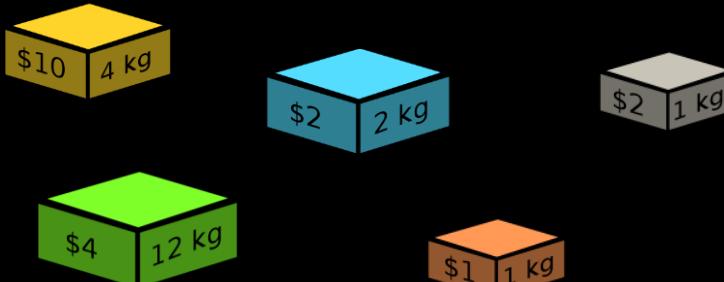
- Items are bounded, non-fractional and only one knapsack allowed.
- Maximize profit/value



<https://images.app.goo.gl/vQejKPdsUynyp3As6>

# 0-1 Knapsack Problem

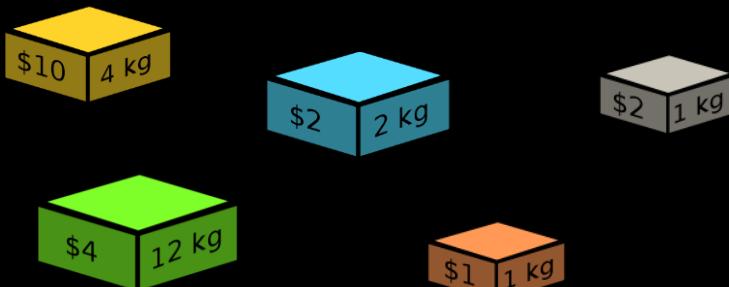
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- Maximize profit/value
- Brute Force:  $2^N$  combinations of sets for N-items in a Set



<https://images.app.goo.gl/vQejKPdsUynyp3As6>

# 0-1 Knapsack Problem

- Items are bounded, non-fractional and only one knapsack allowed.
- Maximize profit/value
- Brute Force:  $2^N$  combinations of sets for N-items in a Set



Brute Force:  
For every item, we have a choice -  
Should we add this item or not?

# 0-1 Knapsack Problem

- Items are bounded, non-fractional and only one knapsack allowed.
- Maximize profit/value
- Brute Force:  $2^N$  combinations of sets for N-items in a Set
- For example, if we had three items

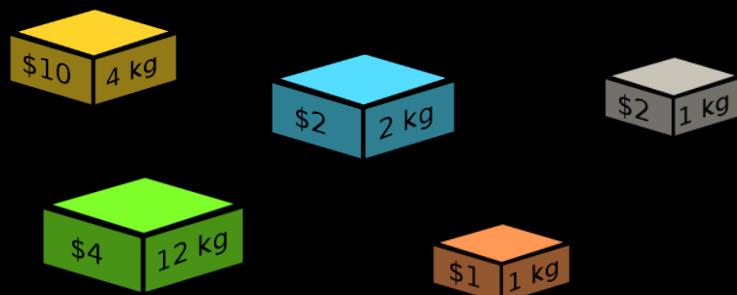


Binary Rep.	Items	Weight of items	Value
000	$\emptyset$	0	0
001	\$1 1 kg	1	1
010	\$2 1 kg	1	2
100	\$2 2 kg	2	2
011	\$1 1 kg, \$2 1 kg	3	3
110	\$2 2 kg, \$2 1 kg	3	4
101	\$1 1 kg, \$2 2 kg	3	3
111	\$1 1 kg, \$2 1 kg, \$2 2 kg	5	5

# 0-1 Knapsack Problem

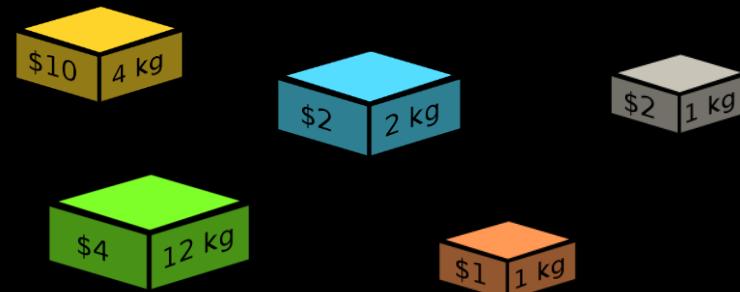
- Items are bounded, non-fractional and only one knapsack allowed.
- Maximize profit/value
- Brute Force:  $2^N$  combinations of sets for N-items in a Set
- For example, if we had five items

Number of  
combinations?



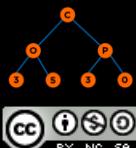
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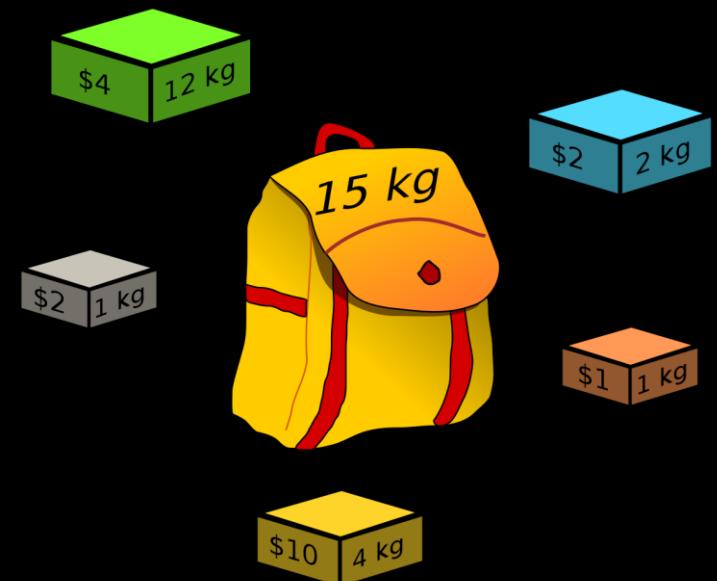
Binary Rep.	Items	Weight of items	Value
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01000	\$10 4 kg	4	10
10000	\$4 12 kg	12	4
⋮	⋮	⋮	⋮
11111	\$2 2 kg \$2 1 kg \$1 1 kg \$10 4 kg \$4 12 kg	20	19

101



# 0-1 Knapsack Problem

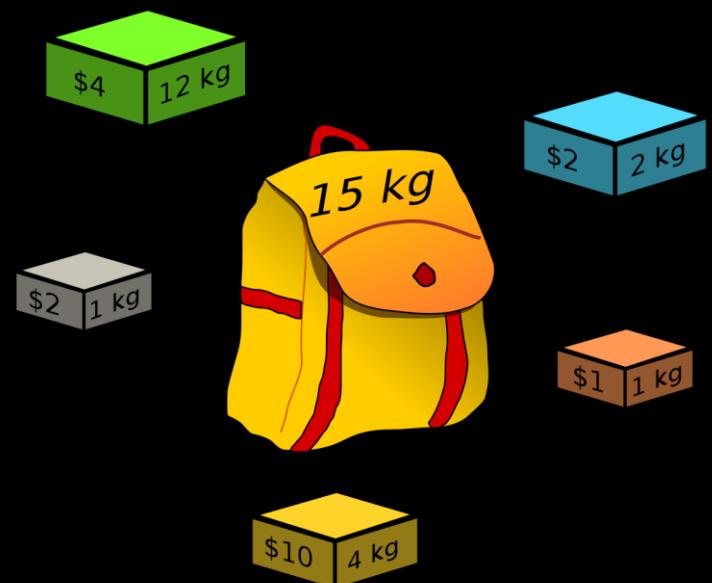
- Items are bounded, non-fractional and only one knapsack allowed.
- Maximize profit/value
- Greedy Algorithm?



<https://images.app.goo.gl/vQejKPdsUynyp3As6>

# Knapsack Problem

- $\text{OPT}(i, W)$  = optimal value of max weight subset that uses items 1, ..., i with weight limit  $W$
- Greedy:
  - Repeatedly add item with maximum  $v_i/w_i$  ratio ....
  - Capacity  $M=7$ , Number of objects  $n = 3$
  - $w = [5, 4, 3]$
  - $v = [10, 7, 5]$  (ordered by  $v_i/w_i$  ratio)



<https://images.app.goo.gl/vQejKPdsUynyp3As6>

# Knapsack Problem

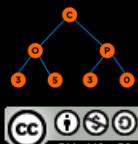
- Dynamic Programming
  - Think about the following:
    1. How can we break down the problem into smaller chunks?
    2. What will happen when we have no items to choose from?
    3. What will happen when we have one item to choose from?
    4. What will happen when we have one item to choose from, but the weight of the item is more than the capacity of the knapsack?

# Knapsack Problem

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*Mathematical representation of the problem*

$OPT(i, w)$  = optimal value of max weight subset that uses items 1, ..., i with weight limit  $w$  (If we have  $i$  items and a Knapsack of capacity  $w$ , what is the optimal value?)



# Knapsack Problem

- Dynamic Programming
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*Mathematical representation of the problem*

$\text{OPT}(i, w)$ :

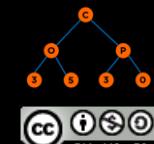
- 0 if  $i = 0$
- $\text{OPT}(i - 1, w)$  if  $w_i > w_k$
- $\max(\text{OPT}(i - 1, w), v_i + \text{OPT}(i - 1, w - w_i))$  otherwise

# Knapsack Problem

- $OPT(i, W)$  = optimal value of max weight subset that uses items  $1, \dots, i$  with weight limit  $W$   
(If we have  $i$  items and a Knapsack of capacity  $W$ , what is the optimal value?)
- Dynamic Programming
  - Case 1: item  $i$  is not included:
    - Take best of  $\{1, 2, \dots, i-1\}$  using weight limit  $W$ :  $OPT(i-1, W)$
  - Case 2: item  $i$  with weight  $w_i$  and value  $v_i$  is included:
    - only possible if  $W \geq w_i$
    - new weight limit =  $W - w_i$
    - Take best of  $\{1, 2, \dots, i-1\}$  using weight limit  $W - w_i$  and add  $v_i$ :
      - $OPT(i-1, W-w_i) + v_i$

$OPT(i, W)$ :

```
- 0                                     if i = 0  
-  $OPT(i - 1, W)$                       if  $w_i > W$   
-  $\max(OPT(i - 1, W), v_i + OPT(i - 1, W - w_i))$  otherwise
```



# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

$\text{OPT}(i, W)$ :

- $0$  if  $i = 0$
- $\text{OPT}(i - 1, W)$  if  $w_i > W$
- $\max(\text{OPT}(i - 1, W), v_i + \text{OPT}(i - 1, W - w_i))$  otherwise

$\text{OPT}(i, W)$ : optimal value of max weight subset that uses items 1, ..., i with weight limit W

# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

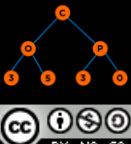
# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}			
{0, 1}			
{0, 1, 2}			
{0, 1, 2, 3}			

$\text{OPT}(i, w)$ :

- 0 if  $i = 0$
- $\text{OPT}(i - 1, w)$  if  $w_i > w$
- $\max(\text{OPT}(i - 1, w), v_i + \text{OPT}(i - 1, w - w_i))$  otherwise

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# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

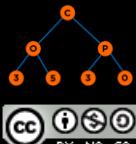
# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}	0	0	0
{0, 1}	0		
{0, 1, 2}	0		
{0, 1, 2, 3}	0		

$\text{OPT}(i, w)$ :

- 0 if  $i = 0$
- $\text{OPT}(i - 1, w)$  if  $w_i > w$
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# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

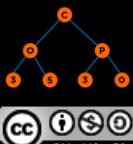
# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}	0	0	0
{0, 1}	0	10	10
{0, 1, 2}	0		
{0, 1, 2, 3}	0		

$\text{OPT}(i, w)$ :

- 0 if  $i = 0$
- $\text{OPT}(i - 1, w)$  if  $w_i > w$
- $\max(\text{OPT}(i - 1, w), v_i + \text{OPT}(i - 1, w - w_i))$  otherwise

$\text{OPT}(i, w)$ : optimal value of max weight subset that uses items 1, ..., i with weight limit w



# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

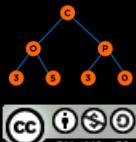
# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}	0	0	0
{0, 1}	0	10	10
{0, 1, 2}	0	20	30
{0, 1, 2, 3}	0		

$\text{OPT}(i, w)$ :

- 0 if  $i = 0$
- $\text{OPT}(i - 1, w)$  if  $w_i > w$
- $\max(\text{OPT}(i - 1, w), v_i + \text{OPT}(i - 1, w - w_i))$  otherwise

$\text{OPT}(i, w)$ : optimal value of max weight subset that uses items 1, ..., i with weight limit w



# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

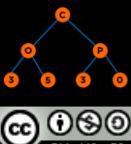
# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}	0	0	0
{0, 1}	0	10	10
{0, 1, 2}	0	20	30
{0, 1, 2, 3}	0	30	50

$\text{OPT}(i, w)$ :

- 0 if  $i = 0$
- $\text{OPT}(i - 1, w)$  if  $w_i > w$
- $\max(\text{OPT}(i - 1, w), v_i + \text{OPT}(i - 1, w - w_i))$  otherwise

$\text{OPT}(i, w)$ : optimal value of max weight subset that uses items 1, ..., i with weight limit w



# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}	0	0	0
{0, 1}	0	10	10
{0, 1, 2}	0	20	30
{0, 1, 2, 3}	0	30	50

Which items we selected?

# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}	0	0	0
{0, 1}	0	10	10
{0, 1, 2}	0	20	30
{0, 1, 2, 3}	0	30	50

Which items we selected?

Item 3 is in the bag as value of bag with item 3 in it is different from the value with Item {1, 2}.

# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}	0	0	0
{0, 1}	0	10	10
{0, 1, 2}	0	20	30
{0, 1, 2, 3}	0	30	50

Which items we selected?

Item 2 is in the bag as value of bag with item 2 in it is different from the value with Item {1}.

# Knapsack Problem

Weight of Knapsack,  $W = 2$

Value ( $v_i$ )	Weight ( $w_i$ )
10	1
20	1
30	1

Grid Size

# rows: No. of items + 1  
# columns:  $W + 1$

	0	1	2
{0}	0	0	0
{0, 1}	0	10	10
{0, 1, 2}	0	20	30
{0, 1, 2, 3}	0	30	50

Which items we selected?

Item 2 and 3

# Knapsack Problem - 11.2 Stepik

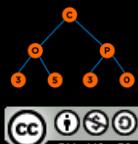
```
int backpack(int W, vector<int> weights, vector<int> values)
{
    int m = weights.size();    // Number of items
    int dp[m + 1][W + 1];      // dp[i][j] is the max value for the first i items with knapsack of capacity j

    for (int i = 0; i <= m; i++)
    {
        for (int j = 0; j <= W; j++)
        {
            if (i == 0 || j == 0) //If there are no items (i = 0) or the capacity of knapsack is zero (j = 0)
                dp[i][j] = 0;
            else if (weights[i - 1] > j) //If weight is bigger than the capacity
                dp[i][j] = dp[i - 1][j];
            else
                dp[i][j] = max(dp[i - 1][j], values[i - 1] + dp[i - 1][j - weights[i - 1]]);
        }
    }

    return dp[m][W];
}
```

# Use cases of Data Structures and Algorithms

- Priority Queues / heaps
  - Prim's algorithm – choosing next edge to add
  - Dijkstra's algorithm – choosing next vertex to set distance
  - Huffman compression algorithm
  - Operating systems - load balancing and interrupt handling
- Binary search trees / hash tables
  - Anything where data is stored with a key / any database
  - Customer list with email address as a key
  - Students with ID number as key
- Minimum Spanning Tree / Graphs
  - Network design
  - Cluster analysis
- Shortest paths / Graphs
  - Delivery planning/scheduling
- Bin packing
  - scheduling problems - scheduling conferences
- Knapsack
  - choosing investment portfolio
  - resource allocation
  - [https://www.vice.com/en\\_us/article/4x385b/the-world-is-knapsack-problem-and-were-just-living-in-it](https://www.vice.com/en_us/article/4x385b/the-world-is-knapsack-problem-and-were-just-living-in-it)



# Questions