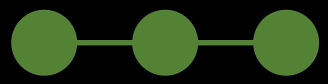
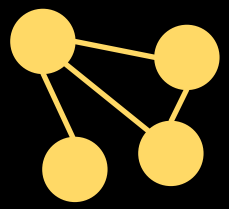
Graphs

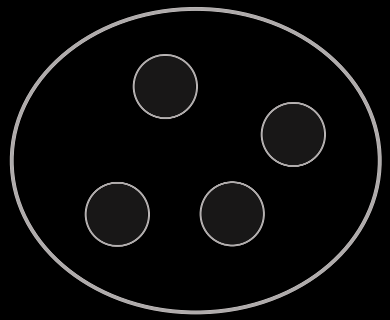
Data Structures & Algorithm

Categories of Data Structures

Linear Ordered	Non-linear Ordered	Not Ordered
Lists	Trees	Sets
Stacks	Graphs	Tables/Maps
Queues		



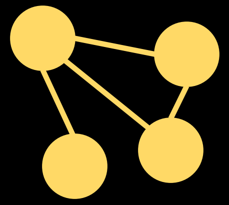




Recap

Non-linear Ordered

Graphs



- **Graphs**
  - Terminology
  - Types
- **Graph Implementations**
  - Edge List
  - Adjacency Matrix
  - Adjacency List

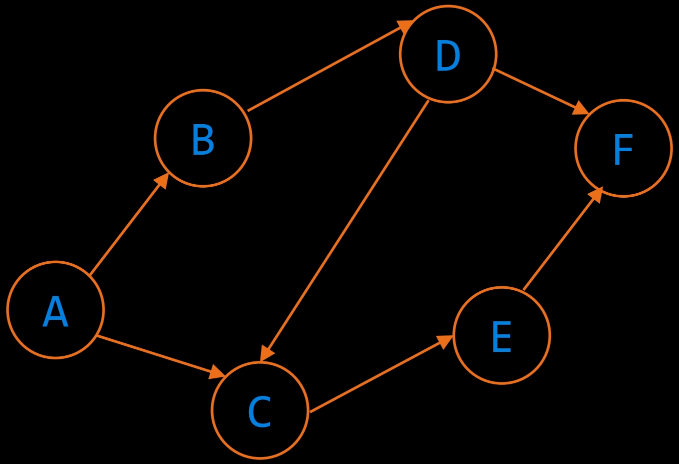
One Graph API

```
class Graph
{
    private:
      //Graph Data Structure

   public:
      Graph();
      Graph(int V); //Creates graph with v vertices
      int V();      //Returns number of vertices
      int E();      //Returns number of edges

      void insertEdge(int from, int to, int weight);
      bool isEdge(int from, int to);
      int getWeight(int from, int to);
      vector<int> getAdjacent(int vertex);
      void printGraph();
};
```

https://stepik.org/lesson/351255/step/1?unit=335132

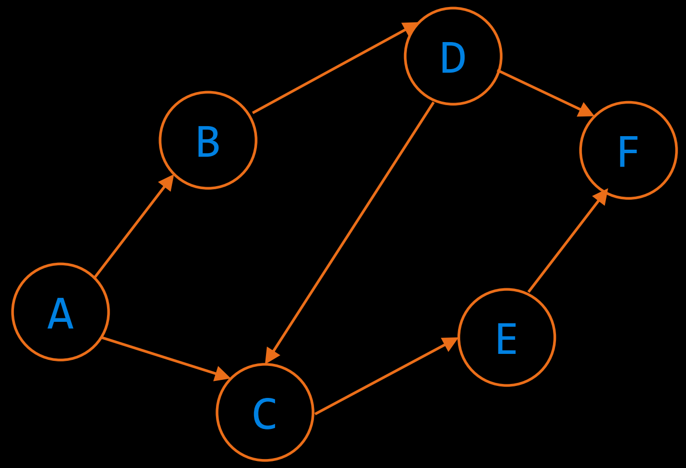
Graph Traversal

Breadth First Search



Valid BFS:

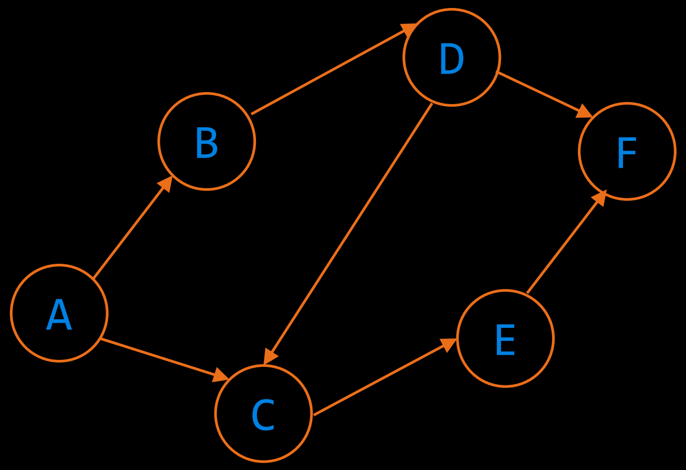
Breadth First Search



Valid BFS: A, B, C, D, E, F

Breadth First Search

```
Algorithm for Breadth-First Search

1.    Take an arbitrary start vertex, mark it identified,
     and place it in a queue.
2.    while the queue is not empty
3.        Take a vertex, u, out of the queue and visit u.
4.        for all vertices, v, adjacent to this vertex, u
5.              if v has not been identified or visited
6.                    Mark it identified
7.                    Insert vertex v into the queue.
8.          We are now finished visiting u.
```

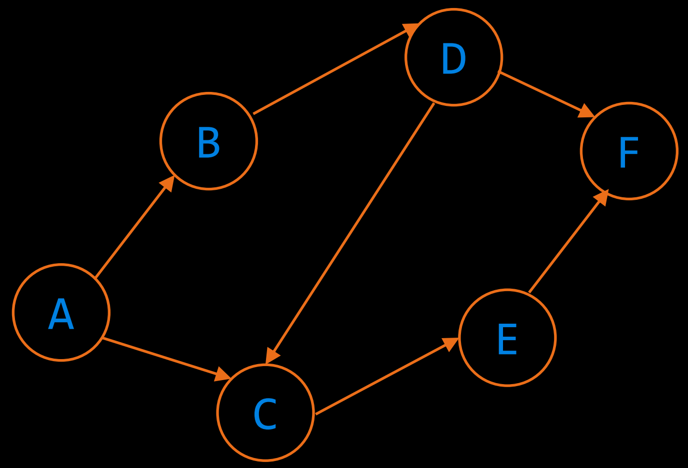


https://onlinegdb.com/B12M0og1P

Breadth First Search

Algorithm for Breadth-First Search

1. Take an arbitrary start vertex, mark it identified, and place it in a queue.
2. while the queue is not empty
3. Take a vertex, *u*, out of the queue and visit *u*.
4. **for** all vertices, *v*, adjacent to this vertex, *u*
5. **if** *v* has not been identified or visited
6. Mark it identified
7. Insert vertex *v* into the queue.
8. We are now finished visiting *u*.




```
string source = "A";
std::set<string> visited;
std::queue<string> q;

visited.insert(source);
q.push(source);
cout<<"BFS: ";

while(!q.empty())
{
        string u = q.front();
        cout << u;
        q.pop();
        vector<string> neighbors = graph[u];
        std::sort(neighbors.begin(), neighbors.begin() + neighbors.size());
        for(string v: neighbors)
        {
                if(visited.count(v) == 0)
                {
                        visited.insert(v);
                        q.push(v);
                }
        }
}
```

https://onlinegdb.com/B12M0og1P

Breadth First Search: Alternate way (7.2.2)

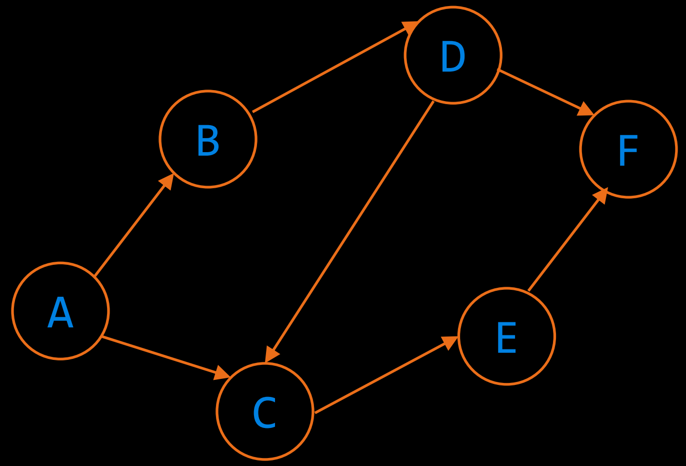
```
Algorithm for Breadth-First Search

1.     Take an arbitrary start vertex, mark it identified,
      and place it in a queue.
2.     while the queue is not empty
3.          Take a vertex, u, out of the queue and visit u.
4.          for all vertices, v, adjacent to this vertex, u
5.                 if v has not been identified or visited
6.                        Mark it identified
7.                        Insert vertex v into the queue.
8.         We are now finished visiting u.
```

```
// Visited Vertices Alternate
set<string> visited;
visited.insert(source);
if(visited.count(v)==0)
      visited.insert(v);
```

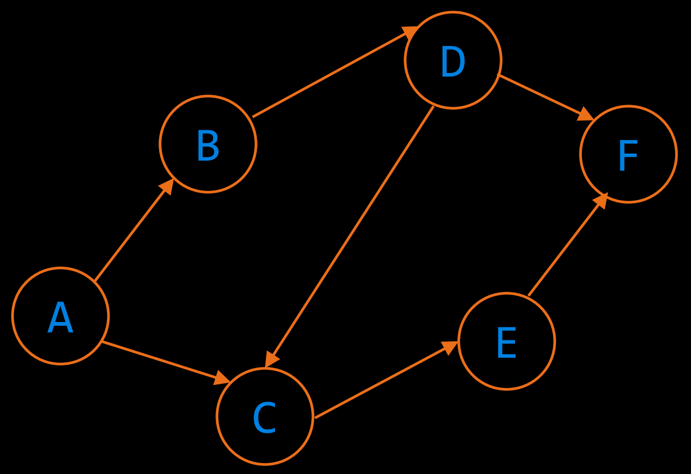
```
void bfs(const Graph& graph, int src)
{
    vector<bool> visited(graph.numVertices);
    queue<int> q;

    visited[src] = true;
    q.push(src);

    while (!q.empty())
    {
        int u = q.front();
        cout << u << " ";
        q.pop();

        for (int v : graph.adjList[u])
        {
            if (!visited[v])
            {
                visited[v] = true;
                q.push(v);
            }
        }
    }
}
```

https://stepik.org/lesson/390631/step/2?unit=379731

Depth First Search



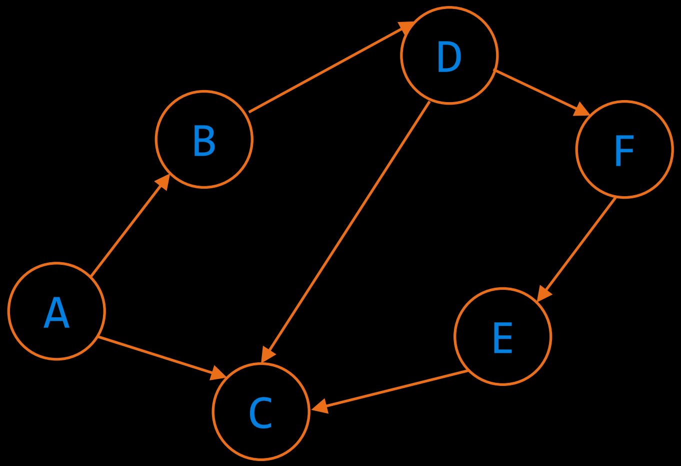
Depth First Search



Valid DFS: A, B, D, C, E, F

Depth First Search – Modified BFS

```
Algorithm for Depth-First Search

1.     Take an arbitrary start vertex, mark it identified,
      and place it in a stack.
2.     while the stack is not empty
3.          Take a vertex, u, out of the stack and visit u.
4.          for all vertices, v, adjacent to this vertex, u
5.                if v has not been identified or visited
6.                      Mark it identified
7.                      Insert vertex v into the stack.
8.          We are now finished visiting u.
```

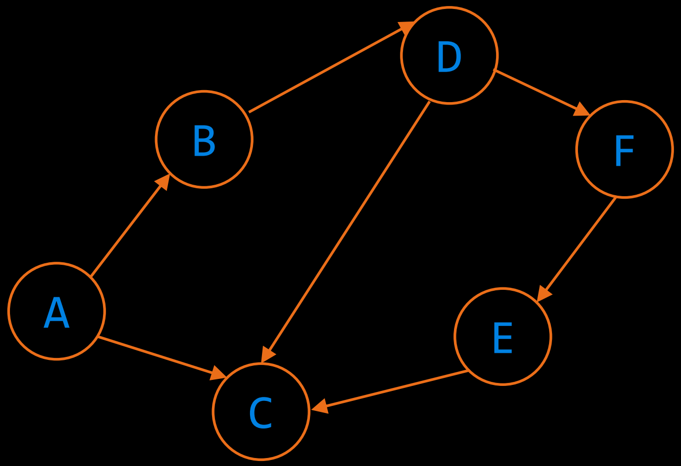


https://onlinegdb.com/SkNolhlkP

Depth First Search – Modified BFS

Algorithm for Depth-First Search

1. Take an arbitrary start vertex, mark it identified, and place it in a stack.
2. while the stack is not empty
3. Take a vertex, *u*, out of the stack and visit *u*.
4. for all vertices, *v*, adjacent to this vertex, *u*
5. if *v* has not been identified or visited
6. Mark it identified
7. Insert vertex *v* into the stack.
8. We are now finished visiting *u*.




```
string source = "A";
std::set<string> visited;
std::stack<string> s;

visited.insert(source);
s.push(source);
cout<<“DFS: ";

while(!s.empty())
{
      string u = s.top();
      cout<<u;
      s.pop();
      vector<string> neighbors = graph[u];
      for(string v: neighbors)
      {
            if(visited.count(v)==0)
            {
                  visited.insert(v);
                  s.push(v);
            }
      }
}
```

https://onlinegdb.com/SkNolhlkP

BFS vs DFS

```
string source = "A";
std::set<string> visited;
std::queue<string> q;

visited.insert(source);
q.push(source);
cout<<"BFS: ";

while(!q.empty())
{
    string u = q.front();
    cout<<u;
    q.pop();
    vector<string> neighbors = graph[u];
    for(string v: neighbors)
    {
        if(visited.count(v)==0)
        {
            visited.insert(v);
            q.push(v);
        }
    }
}
```

```
string source = "A";
std::set<string> visited;
std::stack<string> s;

visited.insert(source);
s.push(source);
cout<<“DFS: ";

while(!s.empty())
{
    string u = s.top();
    cout<<u;
    s.pop();
    vector<string> neighbors = graph[u];
    for(string v: neighbors)
    {
        if(visited.count(v)==0)
        {
            visited.insert(v);
            s.push(v);
        }
    }
}
```

Theoretical Complexity: O(V+E)

One Graph API

```
class Graph
{
    private:
      //Graph Data Structure

   public:
      Graph();
      Graph(int V); //Creates graph with v vertices
      int V();      //Returns number of vertices
      int E();      //Returns number of edges

      void insertEdge(int from, int to, int weight);
      bool isEdge(int from, int to);
      int getWeight(int from, int to);
      vector<int> getAdjacent(int vertex);
      void printGraph();
};
```

```
class Path
{
   public:
      //find all paths from g
      Path(Graph g, int s);

      //is there a path from s to v
      bool hasPathTo(int s);

      //path from s to v
      vector<int> pathTo(int s);
}
```

Questions

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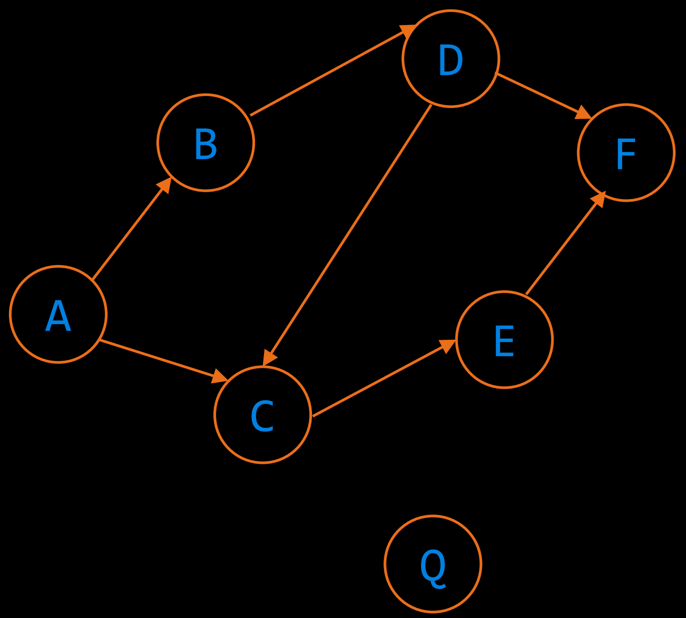
Questions

Shortest Path

s-t Path

Is there a path between vertices s and t?

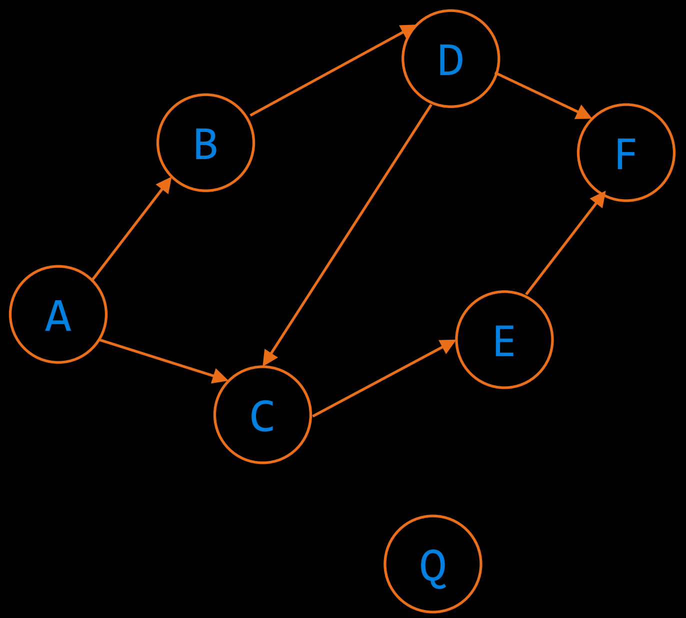


Is there a path between vertices A and C? - Yes

Is there a path between vertices A and Q? - No

s-t Path

Is there a path between vertices s and t?



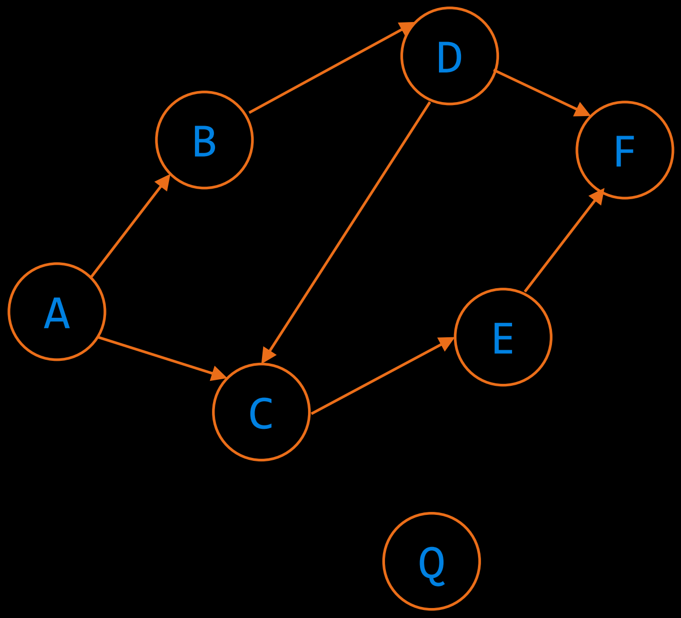
Is there a path between vertices A and C? - Yes

Is there a path between vertices A and Q? - No

Solution

Perform DFS or BFS with source “s” and if we encounter “t” in the path/traversal, then return True otherwise False

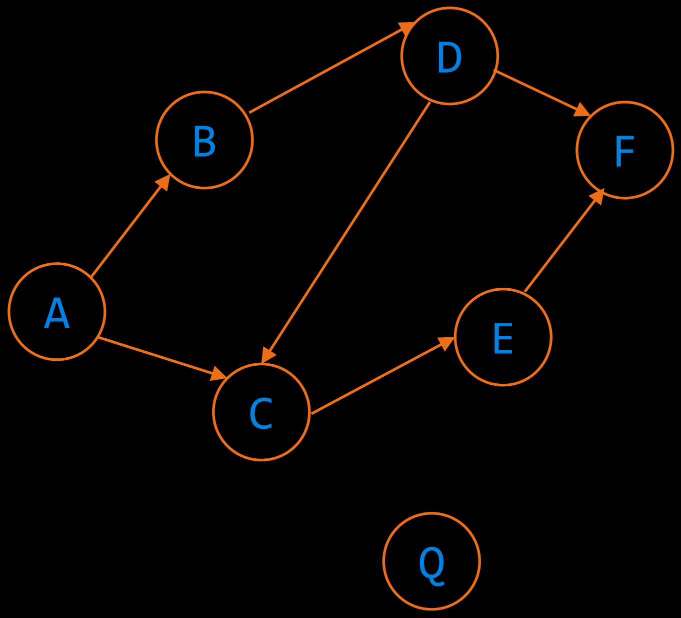
7.2.1 DFS to Find Whether a Given Vertex is Reachable (Iterative)



s-t Path

```
bool dfs(const Graph& graph, int src, int dest)
{
    set<int> visited;
    stack<int> s;
    visited.insert(src);
    s.push(src);
    while(!s.empty())
    {
        int u = s.top();
        s.pop();
        for(auto v: graph.adjList[u])
        {
            if(v == dest)
                return true;
            if ((visited.find(v) == visited.end()))
            {
                visited.insert(v);
                s.push(v);
            }
        }
    }
    return false;
}
```

7.2.1 DFS to Find Whether a Given Vertex is Reachable (Recursive)



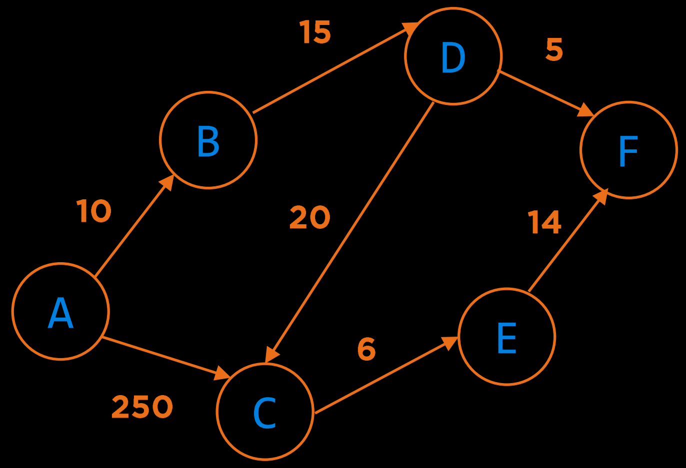
s-t Path: Recursive

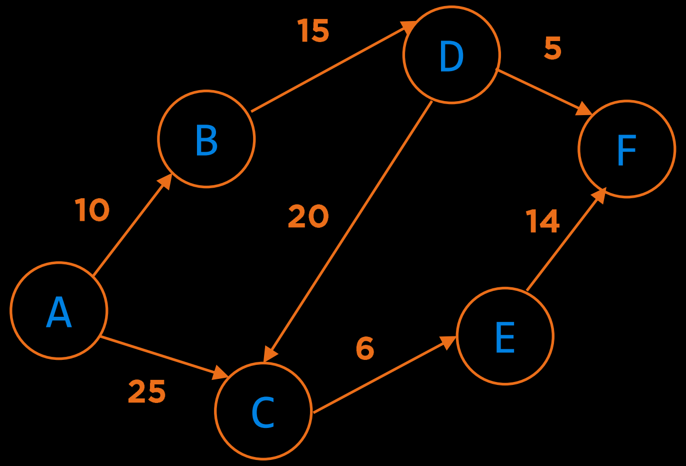
```
bool dfs_helper(const Graph& graph, int src, int dest, vector<bool>& visited)
{
    visited[src] = true;

    if (src == dest)
        return true;

    for (int neighbor : graph.adjList[src]) {
        if (!visited[neighbor]) {
            if (dfs_helper(graph, neighbor, dest, visited))
                return true;
        }
    }
    return false;
}

bool dfs(const Graph& graph, int src, int dest)
{
    vector<bool> visited(graph.numVertices);
    return dfs_helper(graph, src, dest, visited);
}
```

Problem with s-t Path

What if the edges are weighted?

The algorithms do not consider the weights.

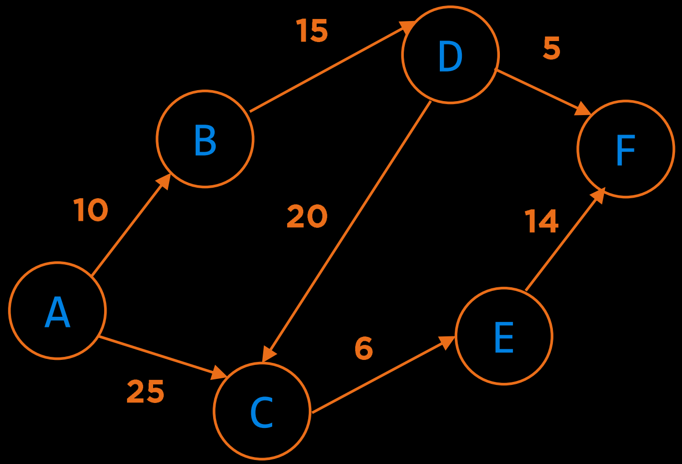




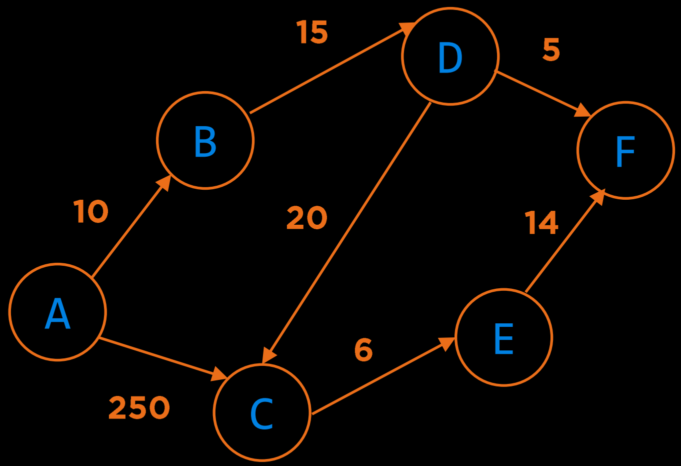
Problem with s-t Path

What if the edges are weighted?

The algorithms do not consider the weights.



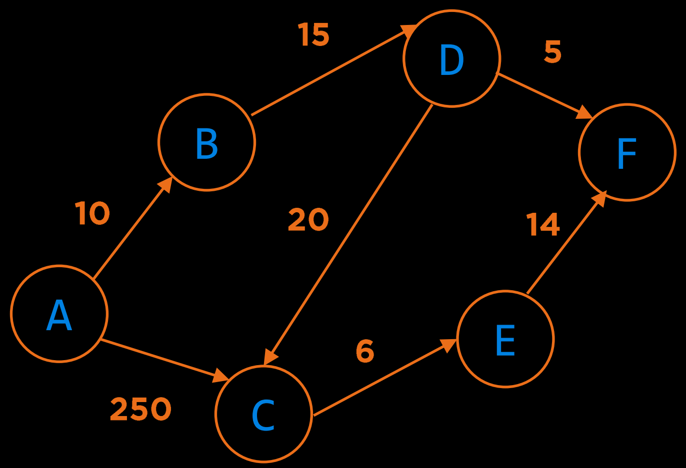
Example 1: Path for A to C will be A-B-D-C for a DFS traversal which will have a total cost of 45 against 25 for the path directly from A-C.



Example 2: Path for A to C will be A-C for a BFS traversal which might have a total cost of 250 against 45 for the path directly from A-B-D-C.

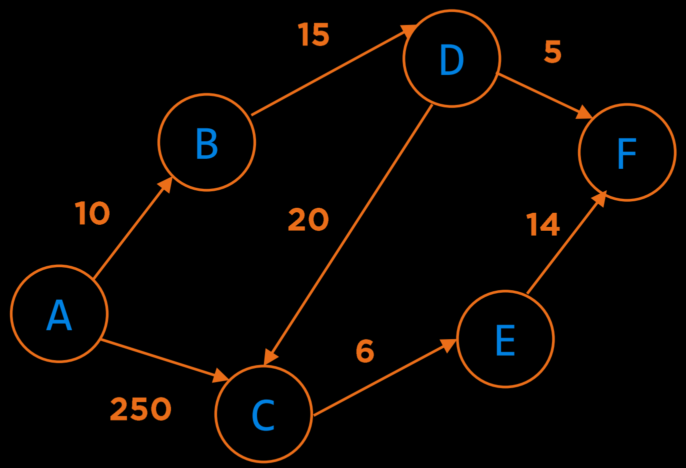
Shortest Weighted s-t Path

What is the shortest weighted path between vertices s and t?



Shortest Weighted s-t Path

What is the shortest weighted path between vertices s and t?

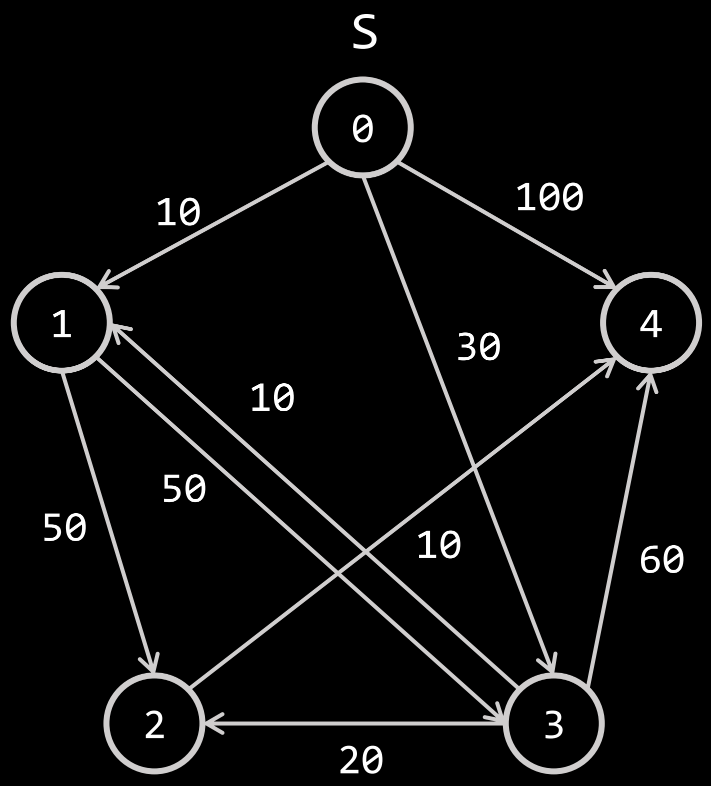


- Dijkstra's Algorithm
  - Single Source: Path to all vertices
  - Directed Graphs
  - No negative weights allowed
  - No negative weight cycles allowed
- Bellman Ford
  - Single Source: Path to all vertices
  - Negative Weights allowed
  - No negative weight cycles allowed
- Floyd-Warshall
  - All pair shortest paths
- A* Search

Dijkstra's Shortest Path Algorithm

Example

- Specify a source vertex, S
- Initialize two arrays and two sets
    - Set S will contain the vertices for which we have computed the shortest distance
        - Initially S will be empty
    - Set V-S will contain the vertices we still need to process
        - Initialize V-S by placing all vertices into it
    - d[v] will contain shortest distance from s to v
        - Initially all d[v]'s will be set to infinity except for source which will be 0
    - p[v] will predecessor of v in the path from s to v
        - Initially all p[v]'s will be set to -1



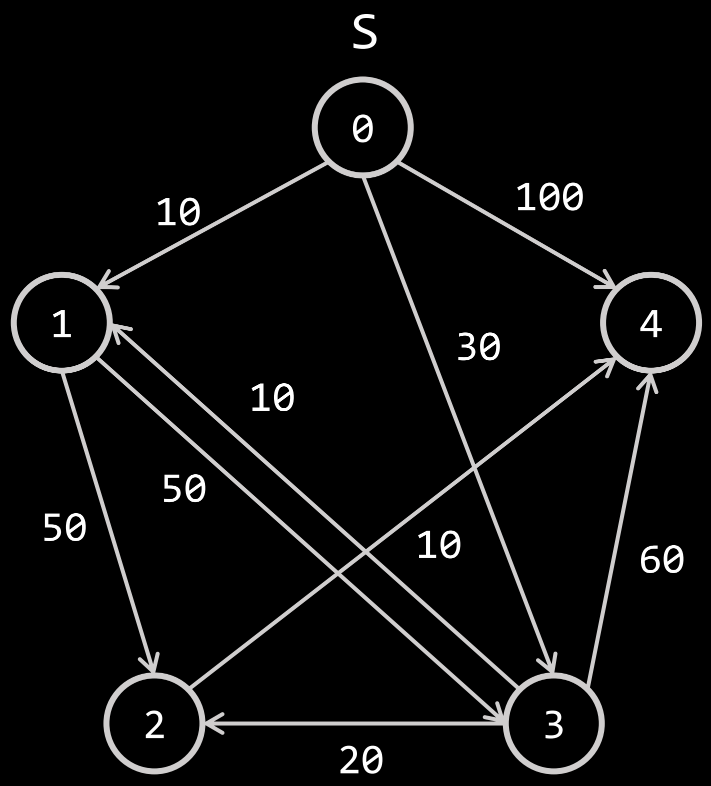
Dijkstra's Shortest Path Algorithm

Example

Computed, S = {}

Needs processing, V-S = {0, 1, 2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	∞	-1
2	∞	-1
3	∞	-1
4	∞	-1



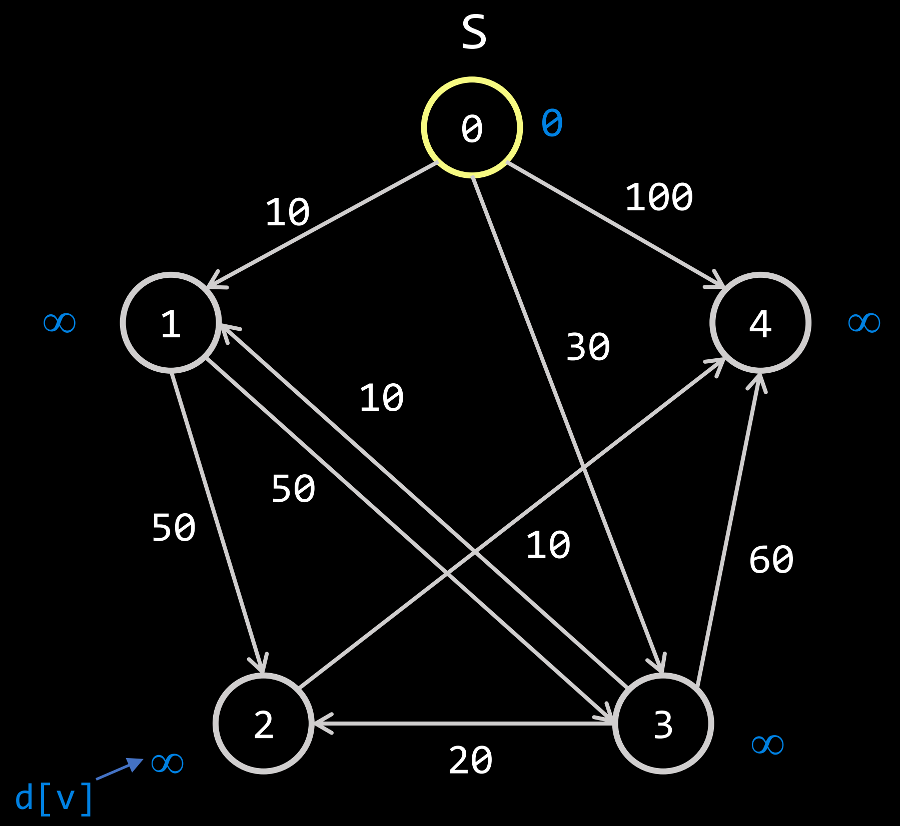
Dijkstra's Shortest Path Algorithm

Example: Start with vertex that has minimum distance in d[v], i.e. 0 and add to Computed

Computed, S = {0}

Needs processing, V-S = {1, 2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	∞	-1
2	∞	-1
3	∞	-1
4	∞	-1



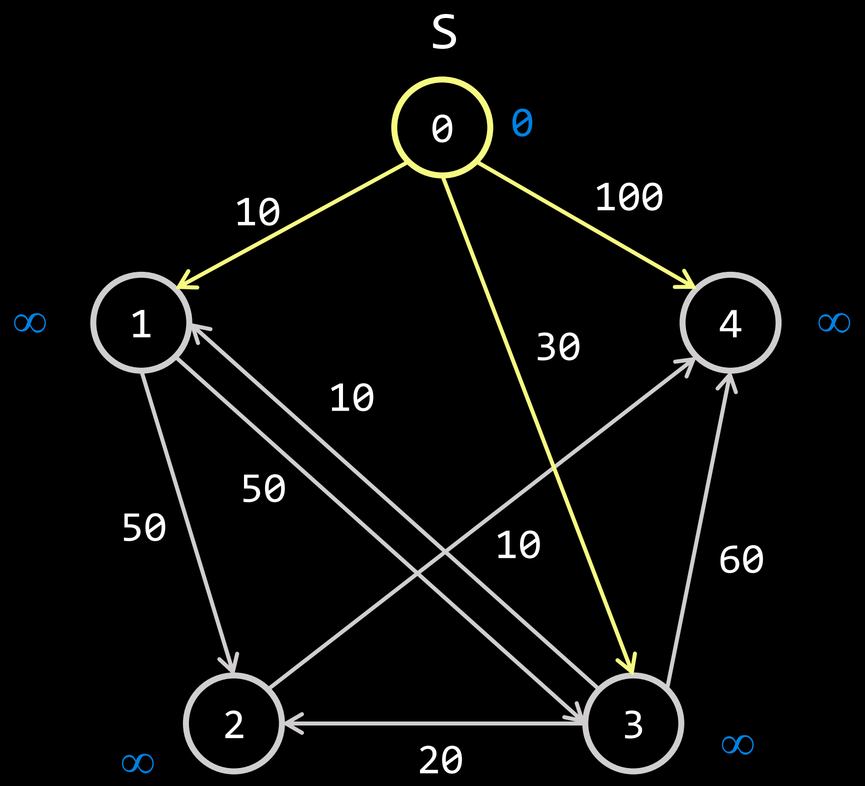
Dijkstra’s Shortest Path Algorithm

Example: Process edges adjacent to the vertex 0 and update distances based on relaxation*

Computed, S = {0}

Needs processing, V-S = {1, 2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	∞	-1
2	∞	-1
3	∞	-1
4	∞	-1



* Relaxation

```
if (dist[v] > dist[u] + w)
        dist[v] = dist[u] + w;
```

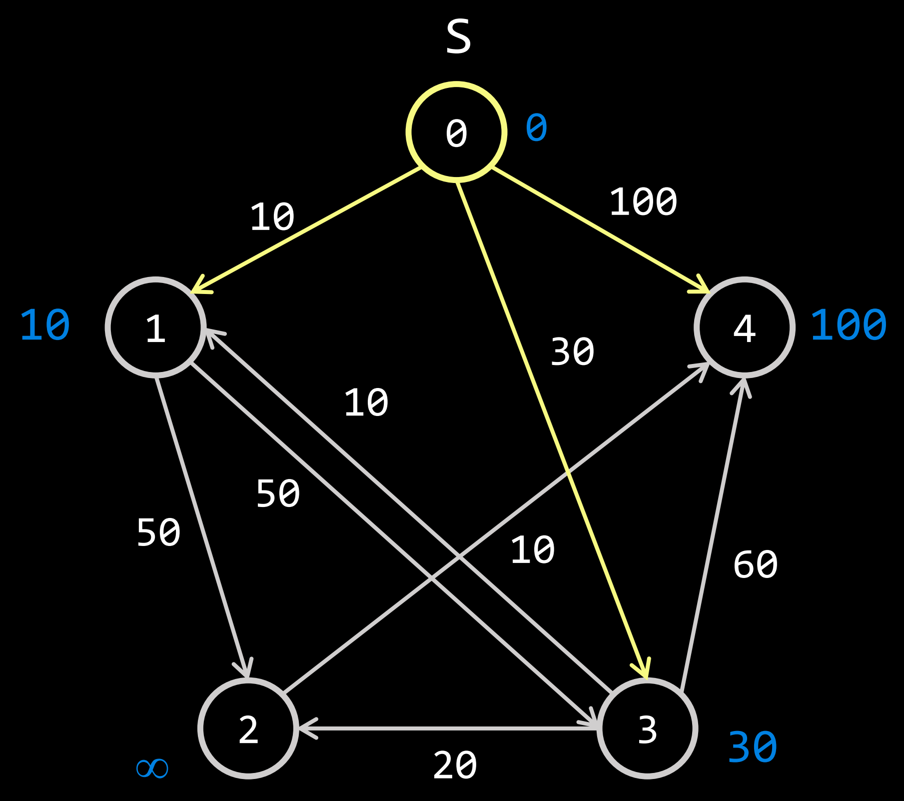
Dijkstra's Shortest Path Algorithm

Example: Process edges adjacent to the vertex 0 and update distances based on relaxation*

Computed, S = {0}

Needs processing, V-S = {1, 2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	∞	-1
3	30	0
4	100	0




```
* Relaxation
  if (dist[v] > dist[u] + w)
        dist[v] = dist[u] + w;
```

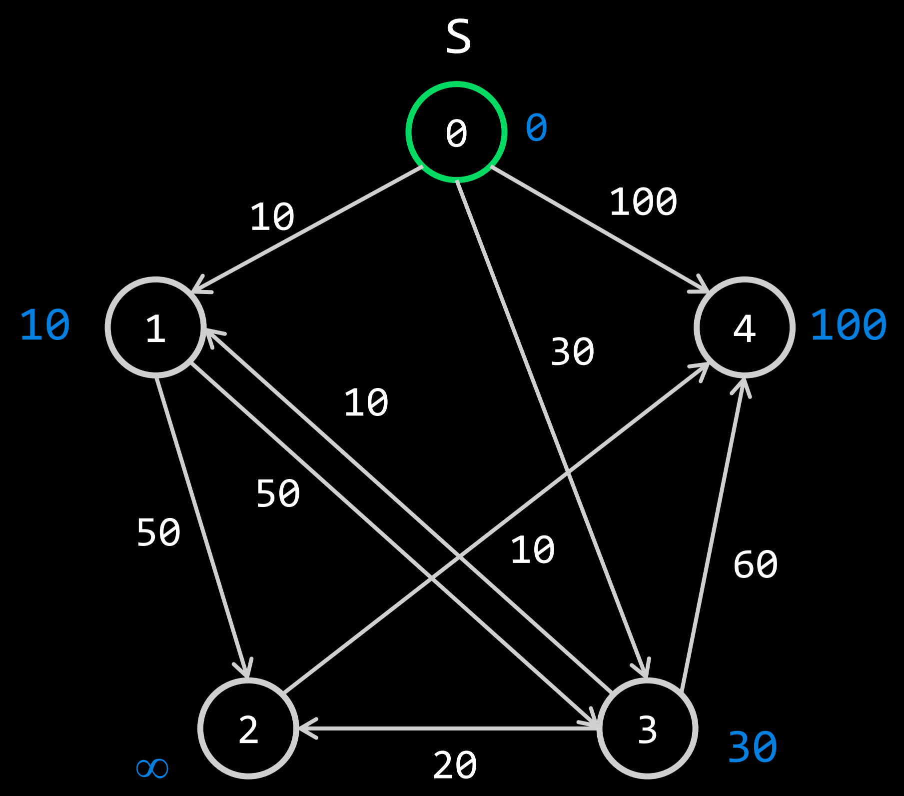
Dijkstra's Shortest Path Algorithm

Example: 0 is now done. Next, repeat the process picking the minimum element in d[v] that has not been computed

Computed, S = {0}

Needs processing, V-S = {1, 2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	∞	-1
3	30	0
4	100	0



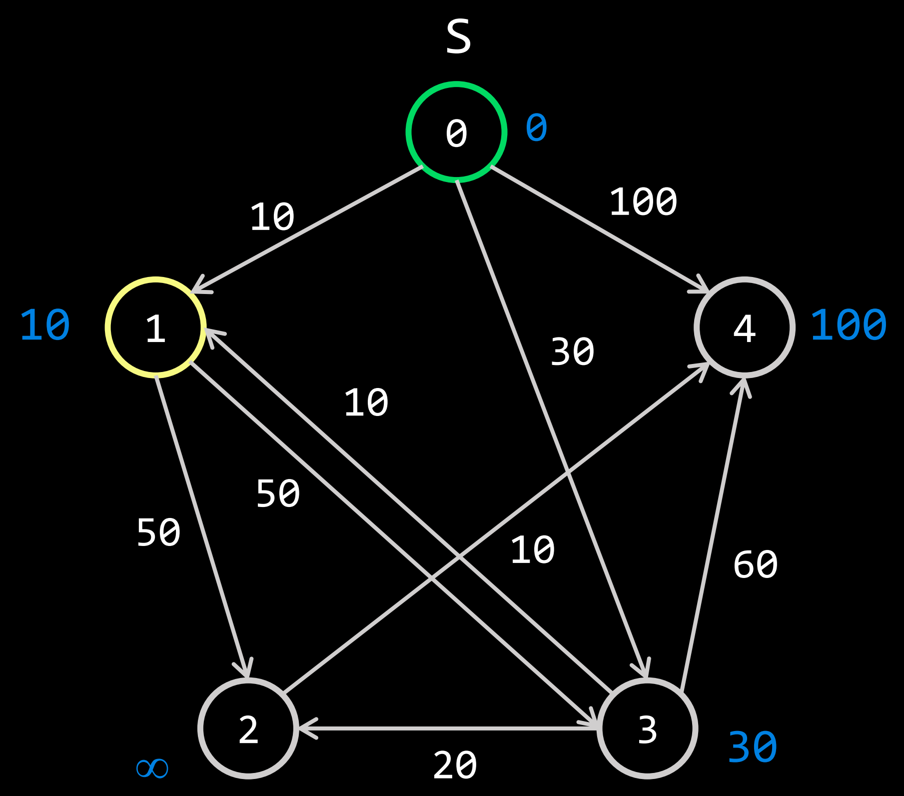
Dijkstra's Shortest Path Algorithm

Example: Pick 1

Computed, S = {0}

Needs processing, V-S = {1, 2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	∞	-1
3	30	0
4	100	0



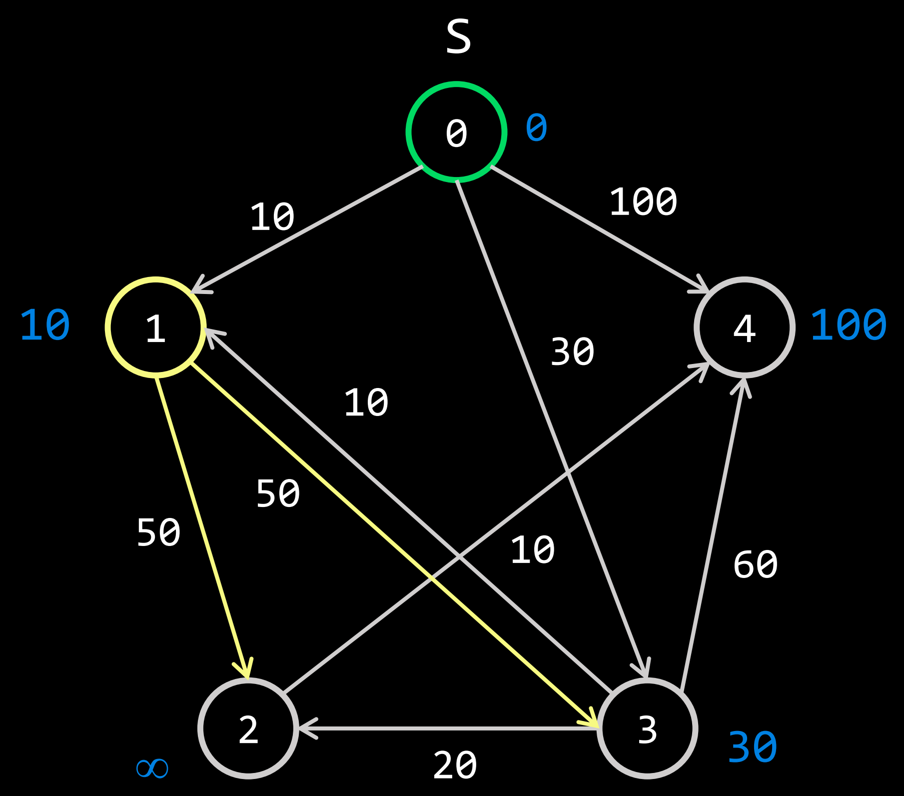
Dijkstra’s Shortest Path Algorithm

Example: Process edges adjacent to the vertex 1 and update distances based on relaxation

Computed, S = {0}

Needs processing, V-S = {1, 2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	∞	-1
3	30	0
4	100	0



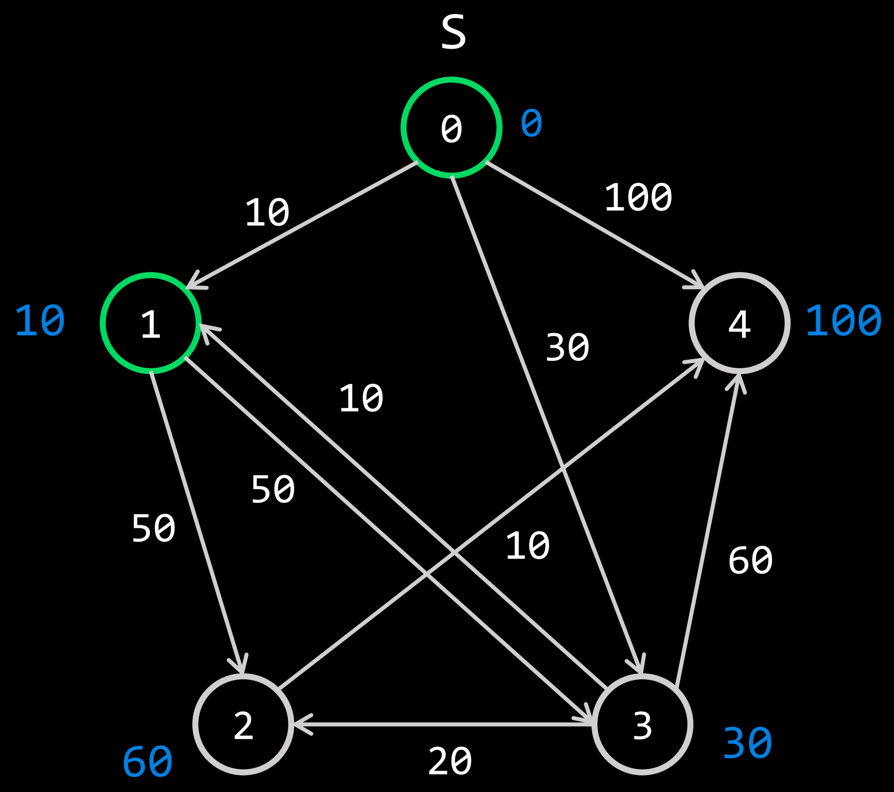
Dijkstra’s Shortest Path Algorithm

Example: 1 is now done. Next, repeat the process picking the minimum element in d[v] that has not been computed

Computed, S = {0, 1}

Needs processing, V-S = {2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	60	1
3	30	0
4	100	0



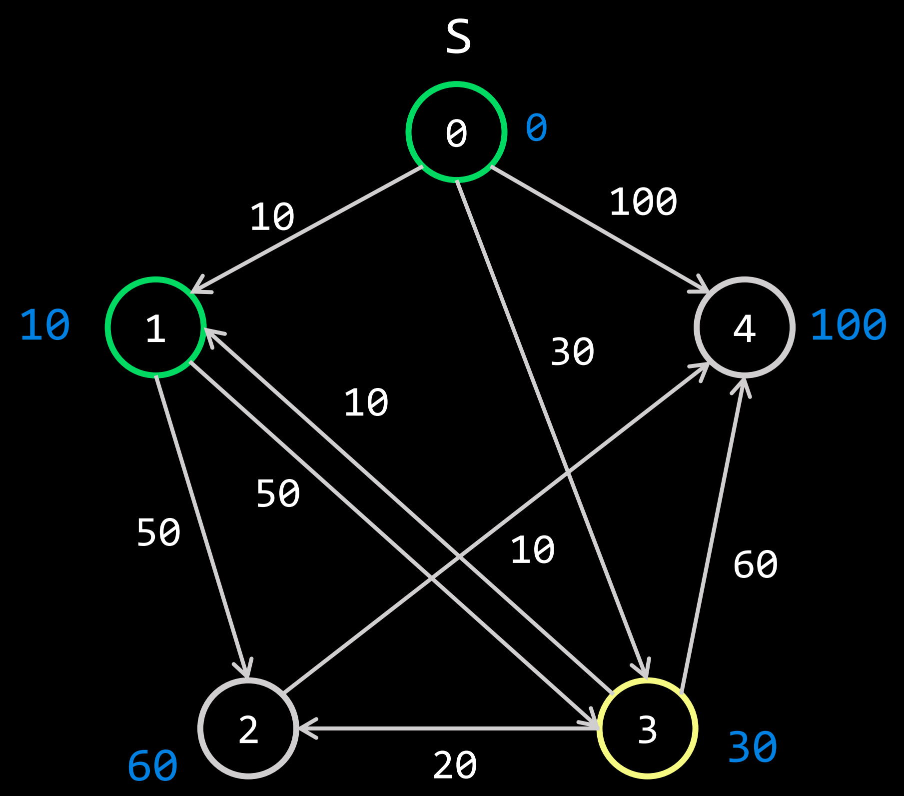
Dijkstra's Shortest Path Algorithm

Example: Pick 3

Computed, S = {0, 1}

Needs processing, V-S = {2, 3, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	60	1
3	30	0
4	100	0



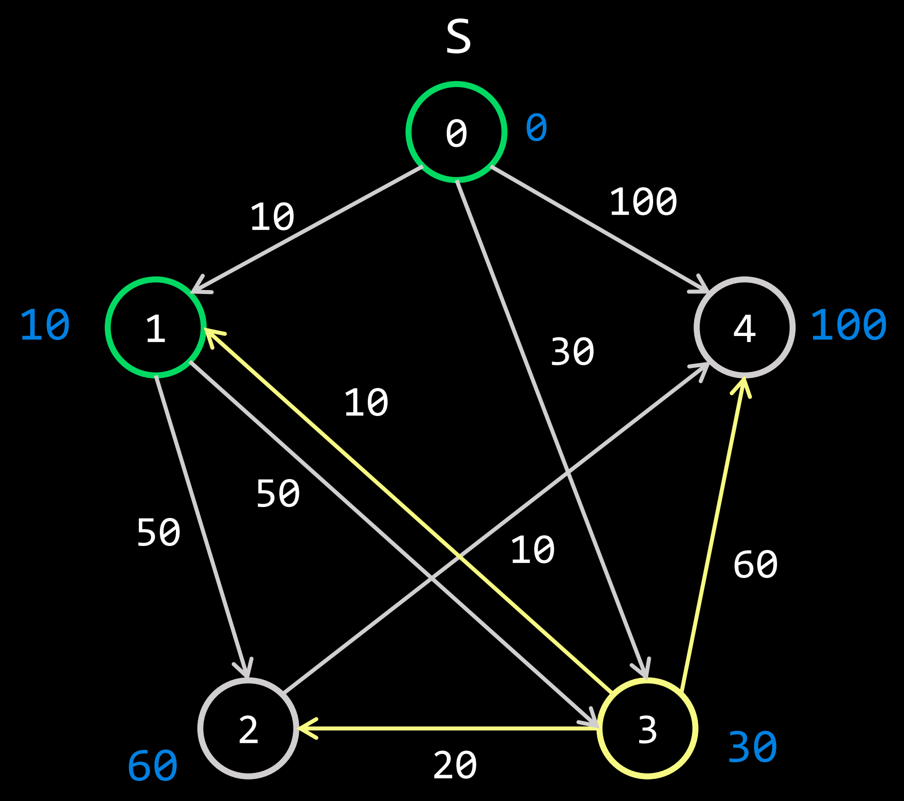
Dijkstra's Shortest Path Algorithm

Example: Process edges adjacent to the vertex 3 and update distances based on relaxation

Computed, S = {0, 1, 3}

Needs processing, V-S = {2, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	60	1
3	30	0
4	100	0



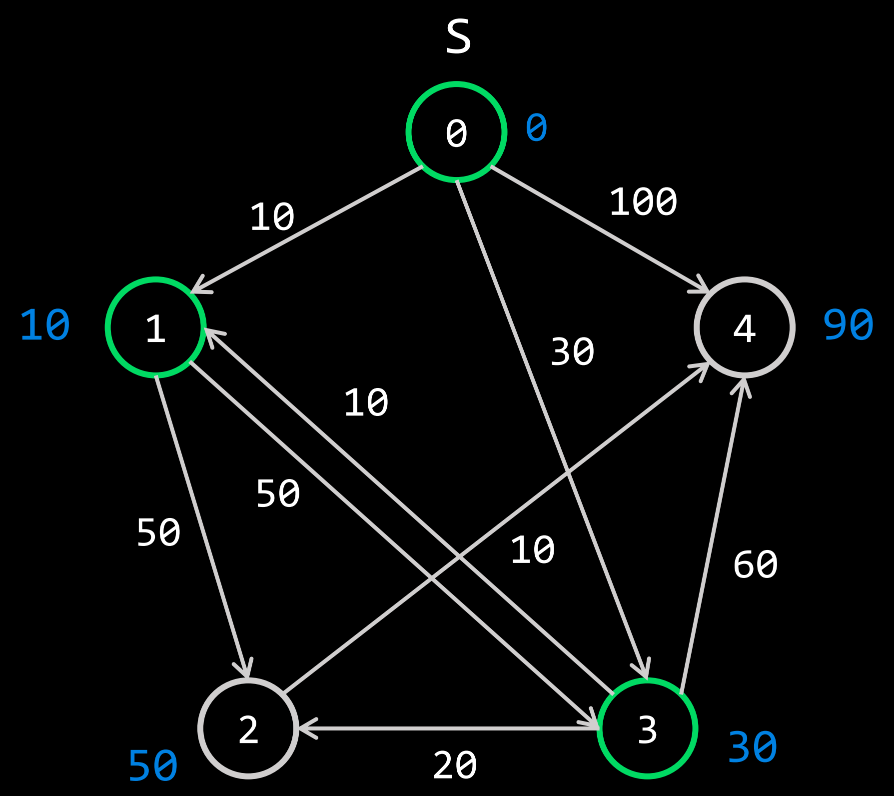
Dijkstra's Shortest Path Algorithm

Example: 3 is now done

Computed, S = {0, 1, 3}

Needs processing, V-S = {2, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	90	3



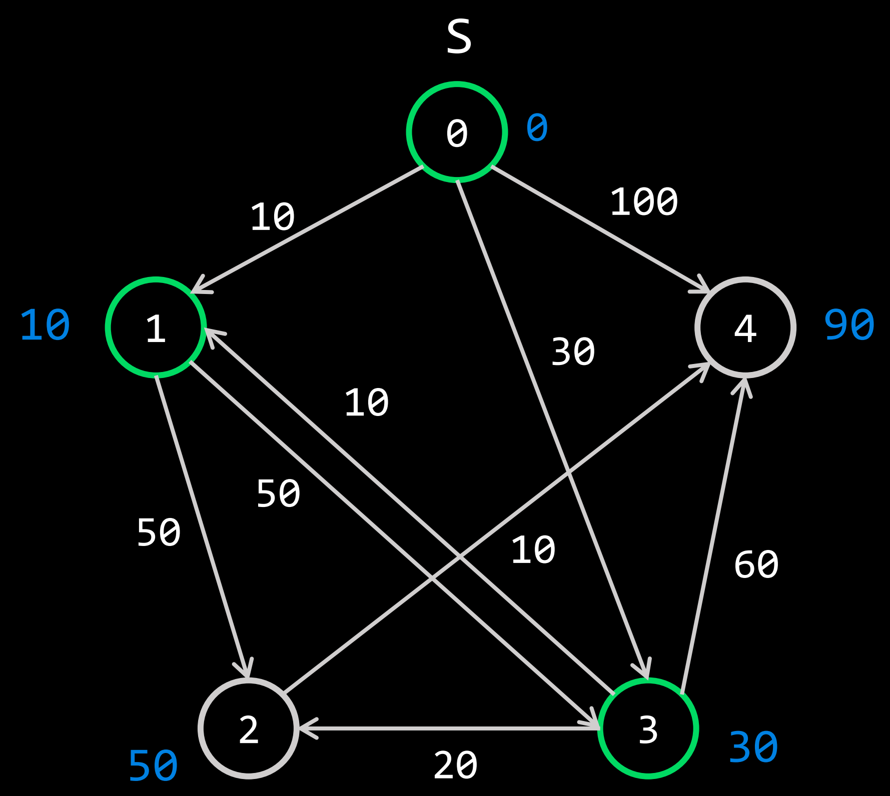
Dijkstra's Shortest Path Algorithm

Example: Next, repeat the process picking the minimum element in d[v] that has not been computed

Computed, S = {0, 1, 3}

Needs processing, V-S = {2, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	90	3



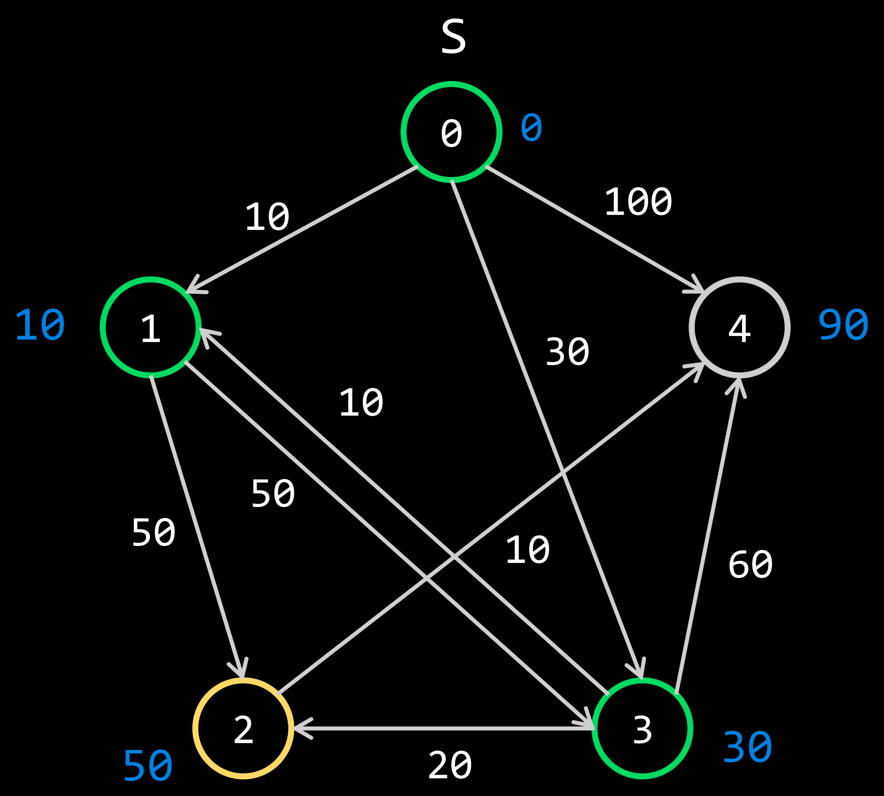
Dijkstra’s Shortest Path Algorithm

Example: Pick 2

Computed, S = {0, 1, 3}

Needs processing, V-S = {2, 4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	90	3



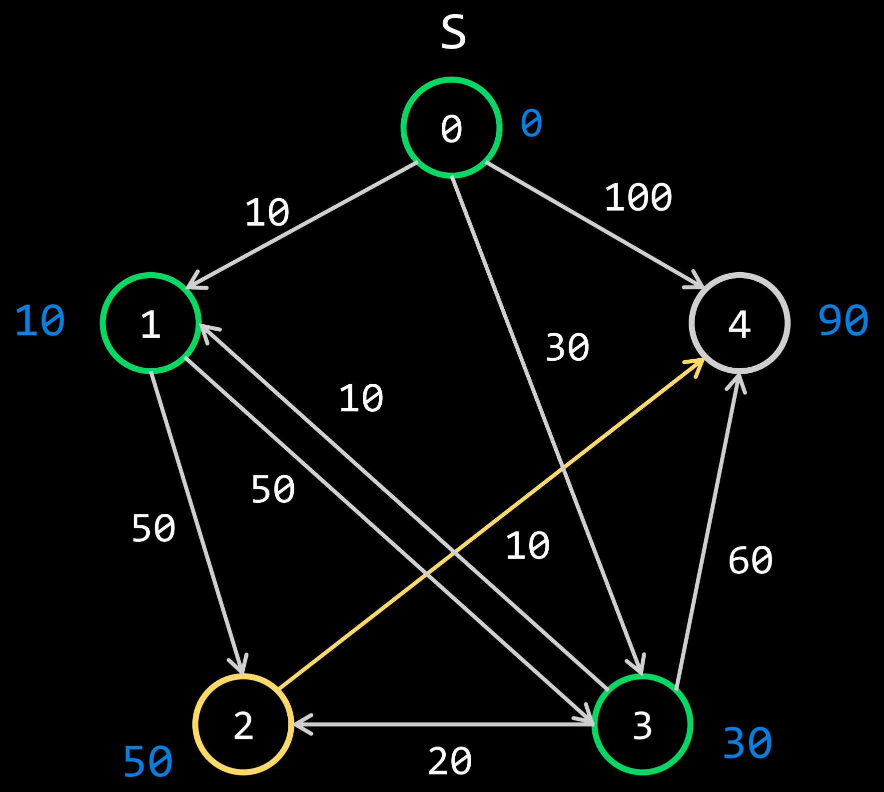
Dijkstra's Shortest Path Algorithm

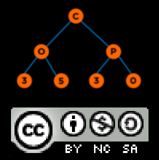
Example: Process edges adjacent to the vertex 2 and update distances based on relaxation

Computed, S = {0, 1, 2, 3}

Needs processing, V-S = {4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	90	3





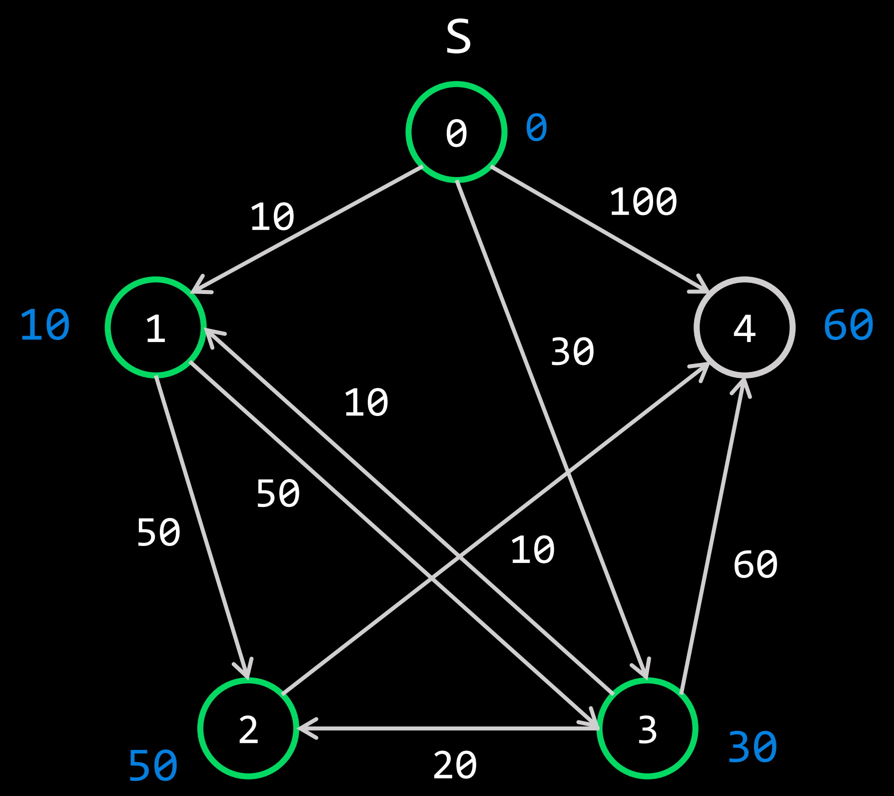
Dijkstra's Shortest Path Algorithm

Example: 2 is now done

Computed, S = {0, 1, 2, 3}

Needs processing, V-S = {4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	60	2



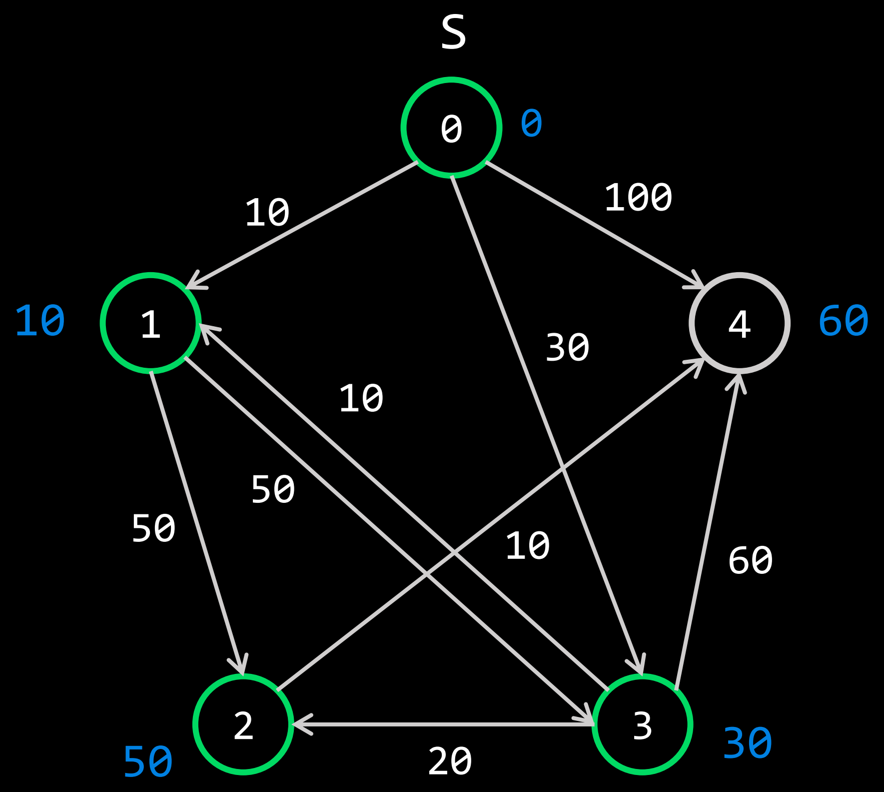
Dijkstra's Shortest Path Algorithm

Example: Next, repeat the process picking the minimum element in d[v] that has not been computed

Computed, S = {0, 1, 2, 3}

Needs processing, V-S = {4}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	60	2



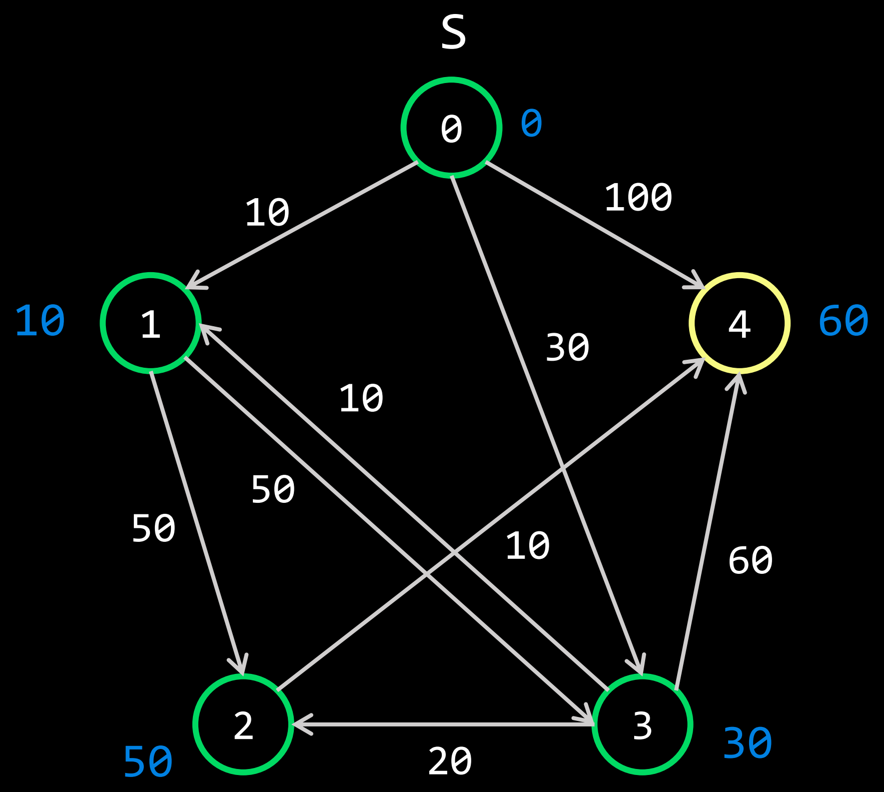
Dijkstra's Shortest Path Algorithm

Example: Pick 4. Process edges adjacent to the vertex 4 and update distances based on relaxation

Computed, S = {0, 1, 2, 3, 4}

Needs processing, V-S = {}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	60	2



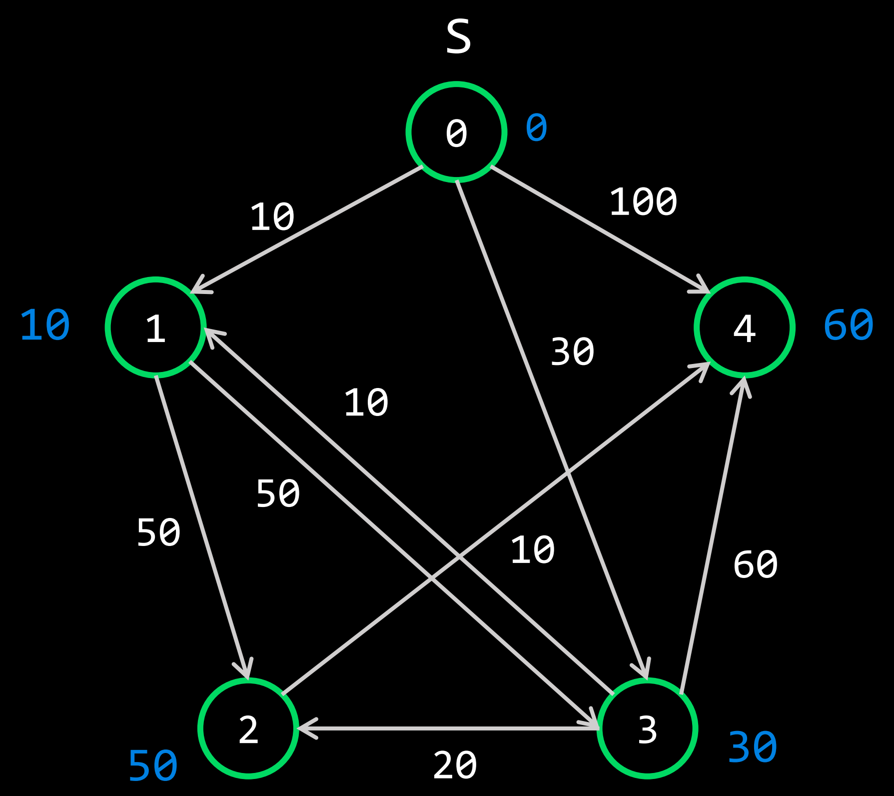
Dijkstra's Shortest Path Algorithm

Example: 4 is now done and V-S is empty. Stop.

Computed, S = {0, 1, 2, 3, 4}

Needs processing, V-S = {}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	60	2



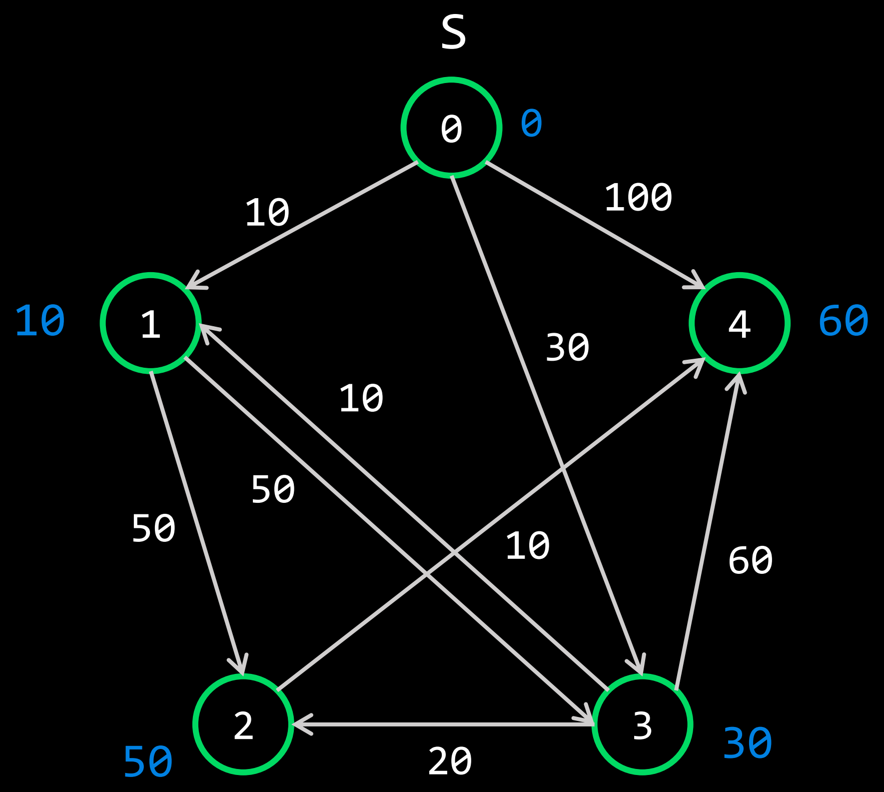
Dijkstra's Shortest Path Algorithm

Example: 4 is now done and V-S is empty. Stop.

Computed, S = {0, 1, 2, 3, 4}

Needs processing, V-S = {}

v	d[v]	p[v]
0	0	-1
1	10	0
2	50	3
3	30	0
4	60	2



Path from 0 to 4: 0 3 2 4 (Cost: 60)

Dijkstra's Shortest Path Algorithm

Dijkstra's Algorithm

```
1.  Initialize S with the start vertex, s, and V–S with the remaining vertices.
2.  for all v in V–S
3.          Set p[v] to s.
4.          if there is an edge (s, v)
5.                  Set d[v] to w(s, v).
            else
6.                  Set d[v] to ∞.
7.  while V–S is not empty
8.          for all u in V–S, find the smallest d[u].
9.          Remove u from V–S and add u to S.
10.         for all v adjacent to u in V–S
11.                 if d[u] + w(u, v) is less than d[v].
12.                         Set d[v] to d[u] + w(u, v).
13.                         Set p[v] to u.
```

Dijkstra's Shortest Path Algorithm

```
Dijkstra's:
    PQ.add(source, 0)
    For other vertices v, PQ.add(v, infinity)
    While PQ is not empty:
        p = PQ.removeSmallest()
        Relax all edges from p

Relaxing an edge u → v with weight w:
    If d[u] + w < d[v]:
        d[v] = d[u] + w
        p[v] = u
        PQ.changePriority(v, d[v])
```

Dijkstra's Shortest Path Algorithm

```
Dijkstra's:
    PQ.add(source, 0)                                  O(V*log V)
    For other vertices v, PQ.add(v, infinity)
    While PQ is not empty:
        p = PQ.removeSmallest()                        O(V*log V)
        Relax all edges from p

Relaxing an edge u → v with weight w:
    If d[u] + w < d[v]:
        d[v] = d[u] + w
        p[v] = u
        PQ.changePriority(v, d[v])                     O(E*log V)
```

Dijkstra's Properties

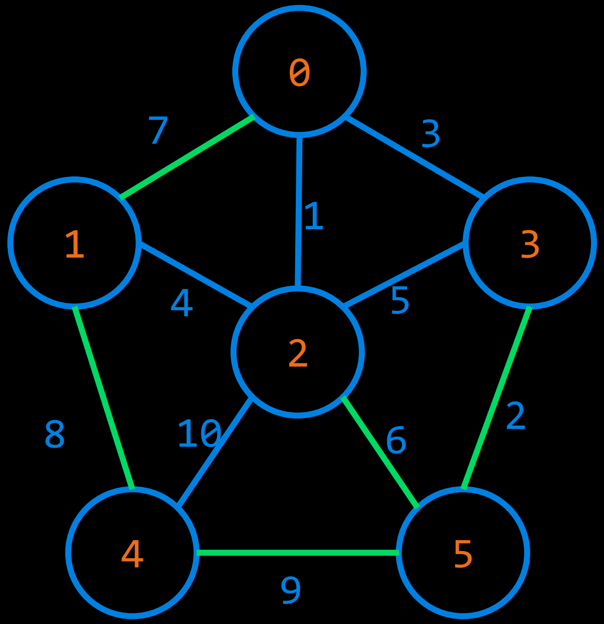
- Greedy Algorithm: Visit vertices in order of best-known distance from source. On visit, relax every edge from the visited vertex
- Dijkstra's is guaranteed to return a correct result if all edges are non-negative.
- Dijkstra's is guaranteed to be optimal so long as there are no negative edges.
- Overall runtime: O(V*log(V) + V*log(V) + E*logV).
  - Assuming E > V, this is just O(E log V) for a connected graph.

Questions

Minimum Spanning Tree

Spanning Tree

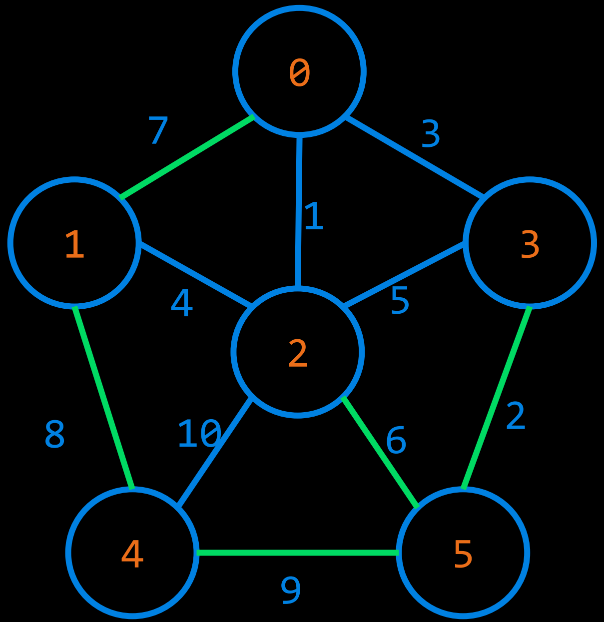
- A spanning tree is a subset of the edges of a graph such that there is only one edge between each vertex, and all of the vertices are connected. The tree is connected and acyclic.
- The cost of a spanning tree is the sum of the weights of the edges.
- Minimum spanning tree is the spanning tree with the smallest cost.
- Spanning tree with N vertices will have N-1 edges.
- Used in networks, laying wires for electricity/telephones, routing for internet connections, etc.



Minimum Spanning Tree – Prim's

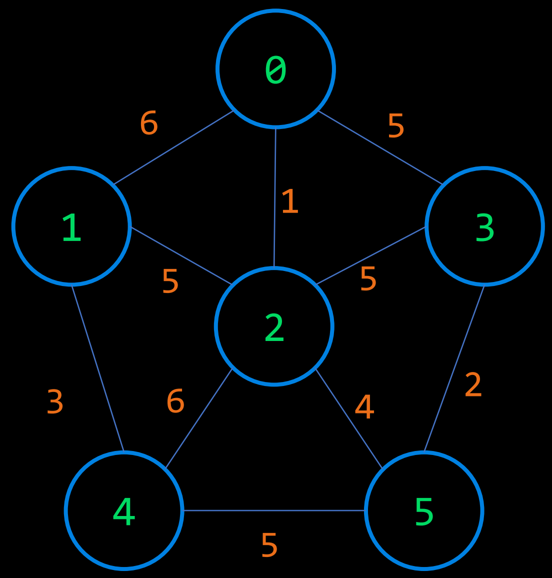
Prim’s Algorithm

- Prims algorithm analyzes all the connections between vertices and finds the set with minimum total weight that makes the graph connected.
- The vertices are divided into two sets:
  - S, the set of vertices in the spanning tree
  - And V-S, the remaining vertices
- Next, we choose the edge with the smallest weight that connects a vertex in S to a vertex in V-S and add it to the minimum spanning tree.



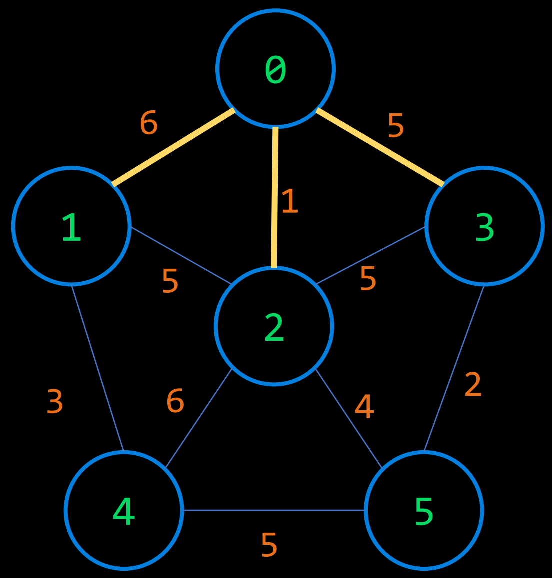
Prim’s Algorithm

V	V-S
	0 1 2 3 4 5



Prim's Algorithm

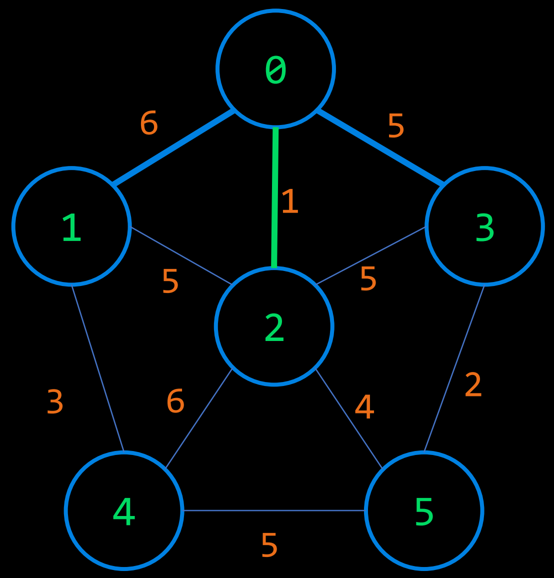
V	V-S
0	1 2 3 4 5



Prim’s Algorithm

V
0 2

V-S
1 3 4 5



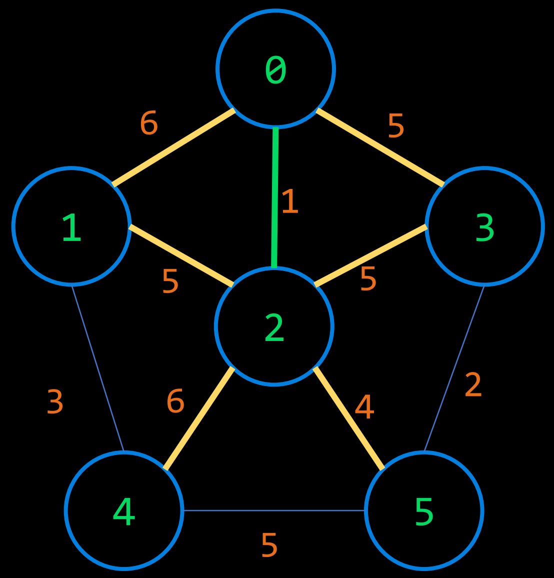
Prim’s Algorithm

V

0 2

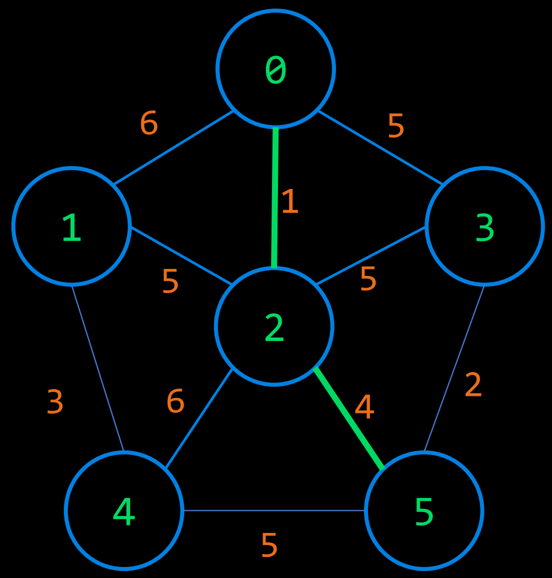
V-S

1 3 4 5



Prim’s Algorithm

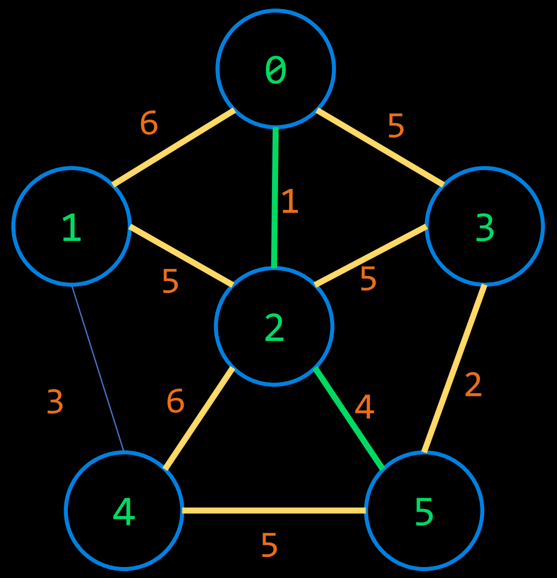
V	V-S
0 2 5	1 3 4



Prim's Algorithm

V
0 2 5

V-S
1 3 4



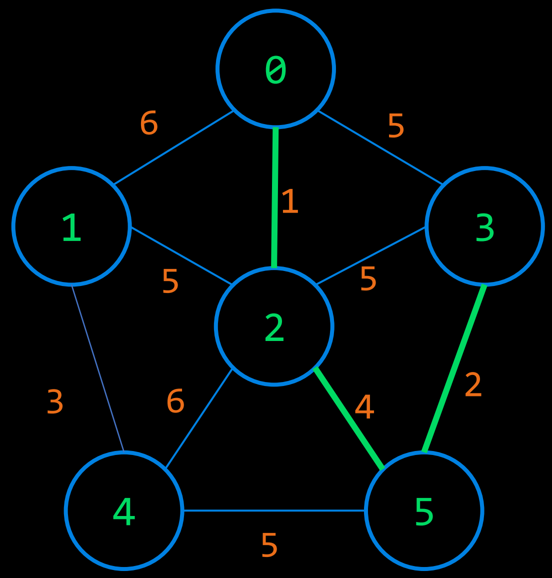
Prim’s Algorithm

V

0 2 3 5

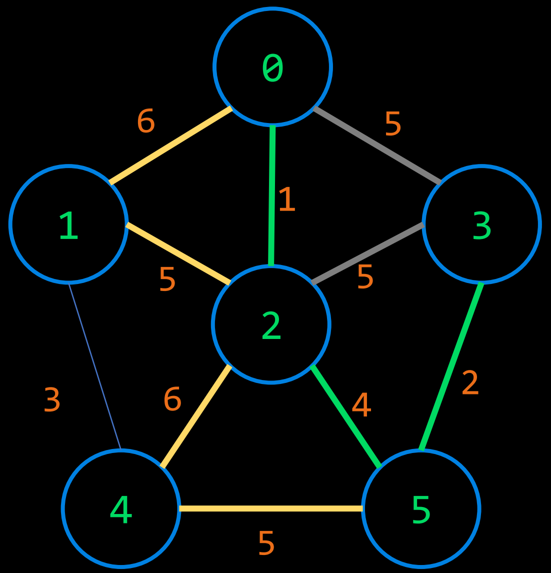
V-S

1 4



Prim’s Algorithm

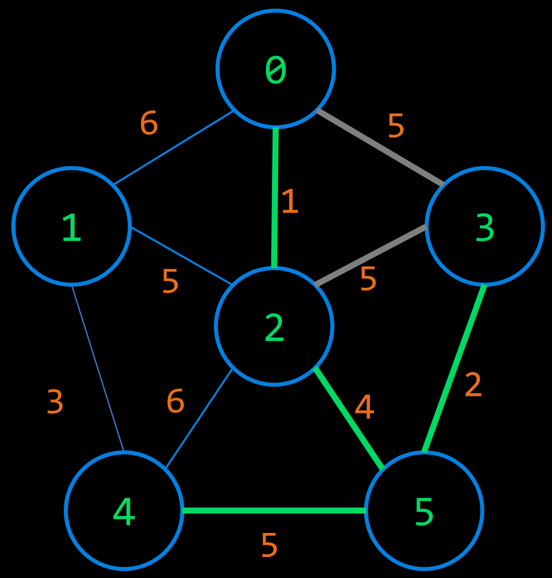
V	V-S
0 2 3 5	1 4



We choose the edge with the smallest weight that connects a vertex in S to a vertex in V-S and add it to the minimum spanning tree. Option to pick 1- or 4-5. Pick any.

Prim’s Algorithm

V	V-S
0 2 3 4 5	1

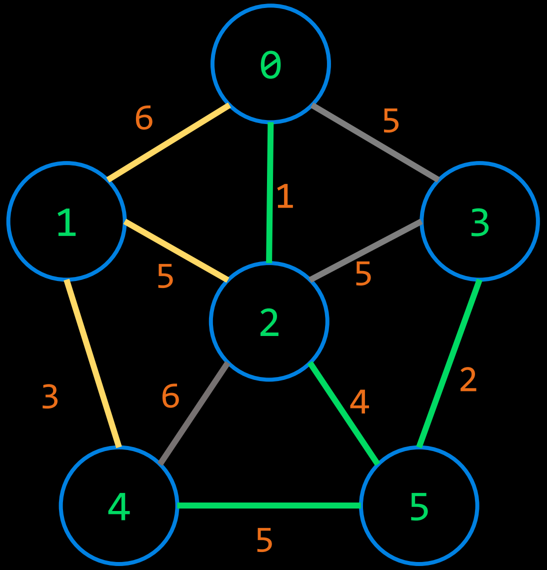


Pick 4-5.

Prim's Algorithm

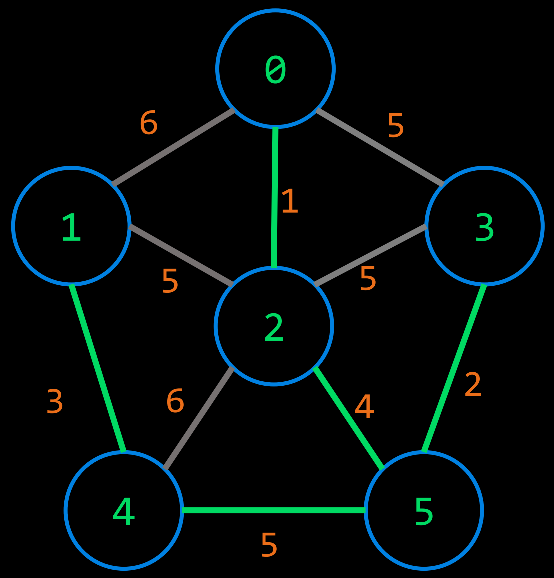
V
0 2 3 4 5

V-S
1



Prim's Algorithm

V	V-S
0 1 2 3 4 5	



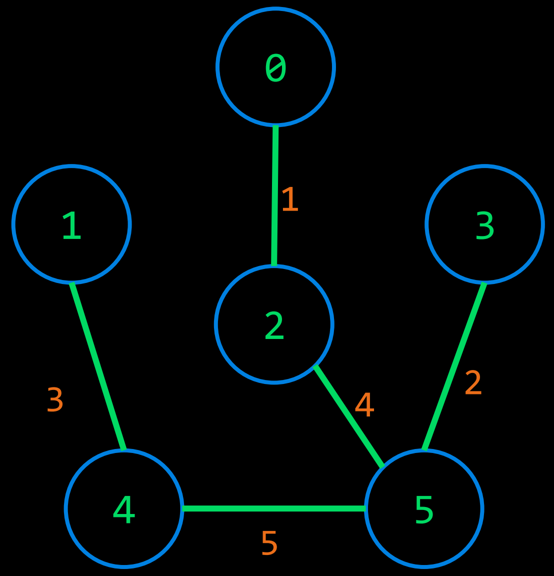
Pick 1-4.

Prim's Algorithm

V

0 1 2 3 4 5

V-S



Sum of MST = 15.

Prim’s Algorithm

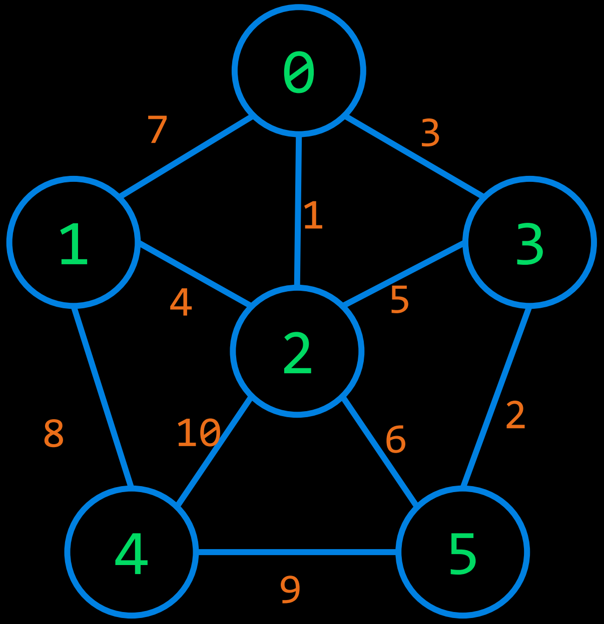
```
Input: An undirected, connected, weighted graph G.
Output: T, a minimum spanning tree for G.

T := ∅.
Pick any vertex in G and add it to T.

For j = 1 to n-1
      Let C be the set of edges with one endpoint inside T and
one endpoint outside T.
      Let e be a minimum weight edge in C.
      Add e to T.
      Add the endpoint of e not already in T to T.
End-for

Complexity: O(EV) or O(E log V) – using priority queues
```

Minimum Spanning Tree – Kruskal’s

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
2-3	5
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10

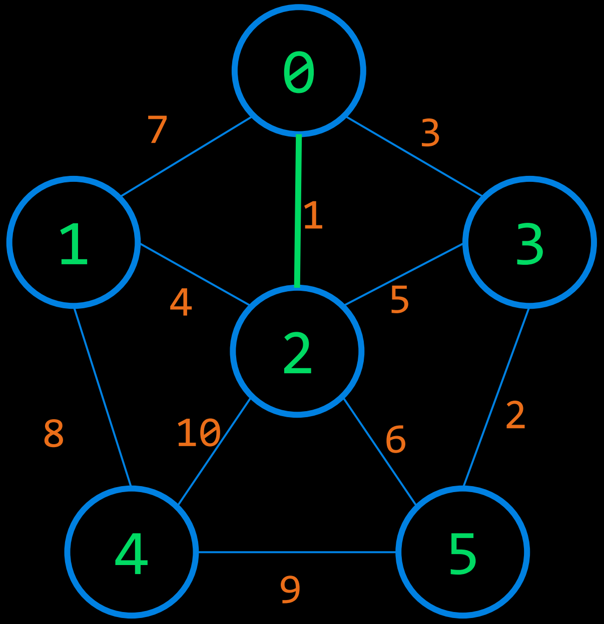


Add edges in order as long as they don’t create a cycle

Kruskal's Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
2-3	5
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10



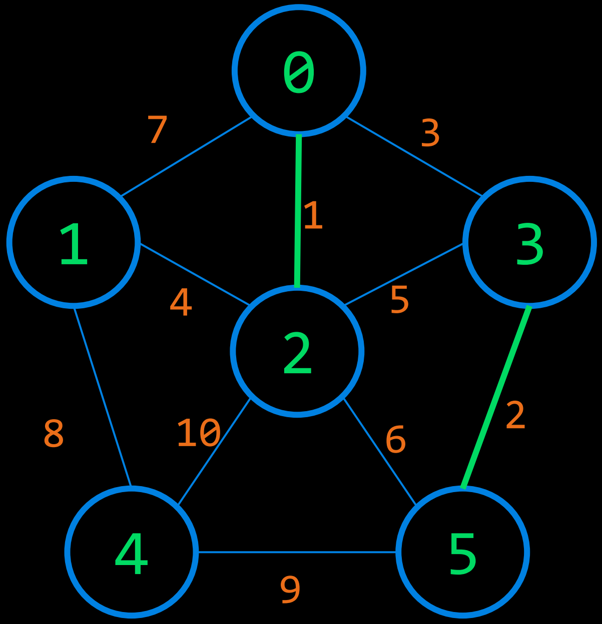
Add edges in order as long as they don't create a cycle

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
2-3	5
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10

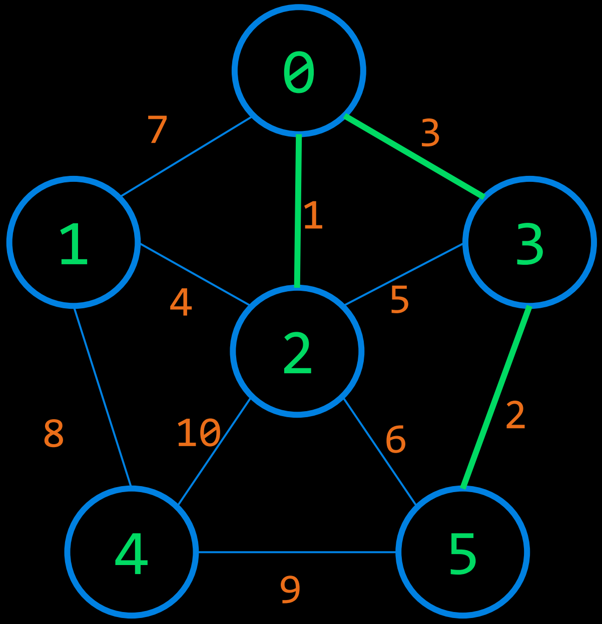
Add edges in order as long as they don’t create a cycle



Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
2-3	5
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10

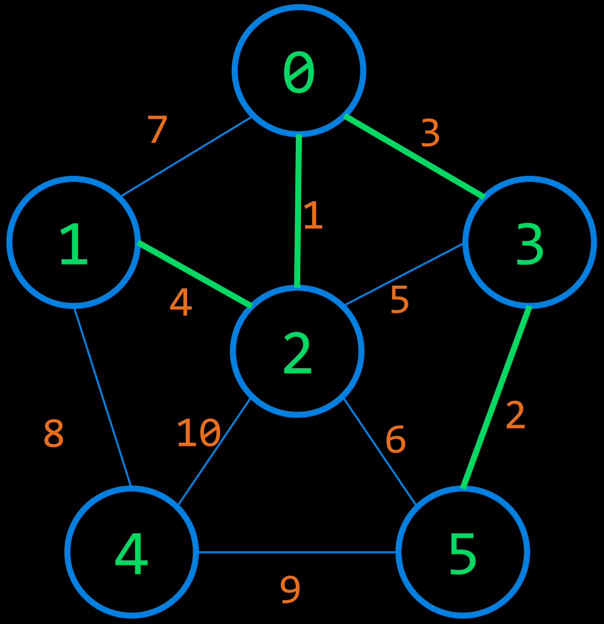


Add edges in order as long as they don’t create a cycle

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
2-3	5
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10

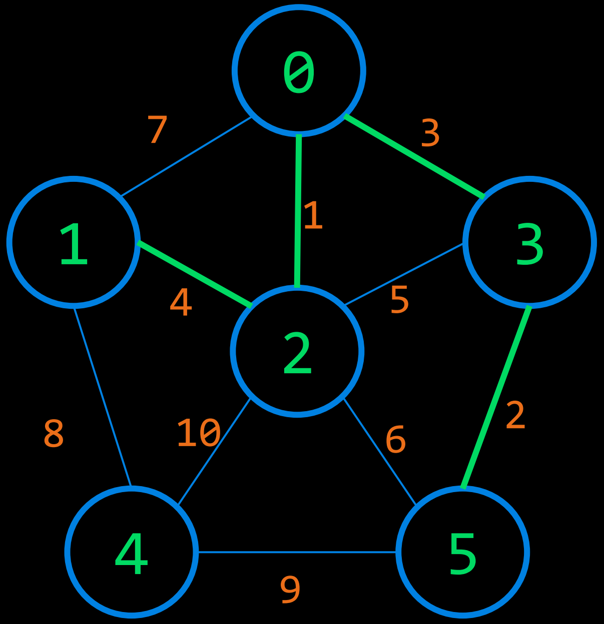


Add edges in order as long as they don’t create a cycle

Kruskal's Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
~~2-3~~	~~5~~
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10

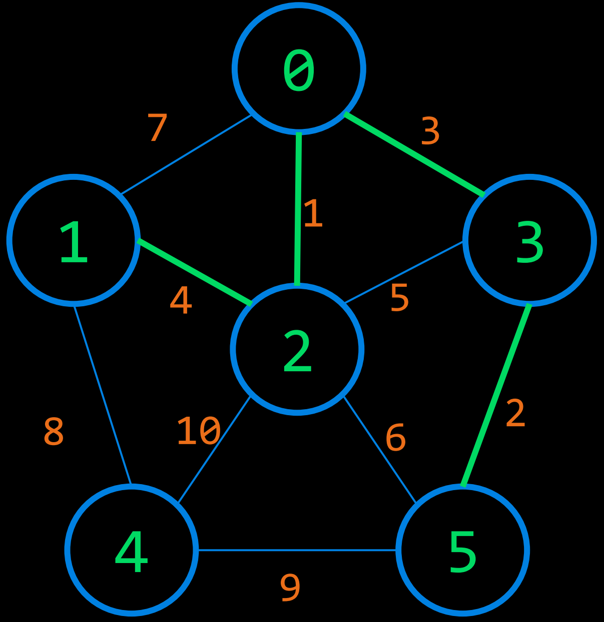


Add edges in order as long as they don't create a cycle

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
~~2-3~~	~~5~~
~~2-5~~	~~6~~
0-1	7
1-4	8
4-5	9
2-4	10

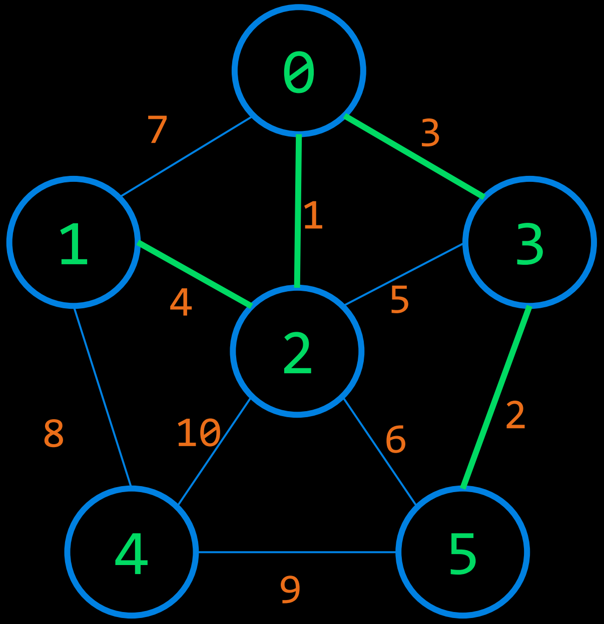


Add edges in order as long as they don’t create a cycle

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
~~2-3~~	~~5~~
~~2-5~~	~~6~~
~~0-1~~	~~7~~
1-4	8
4-5	9
2-4	10



Add edges in order as long as they don’t create a cycle

Kruskal’s Algorithm

Arrange edges in ascending order

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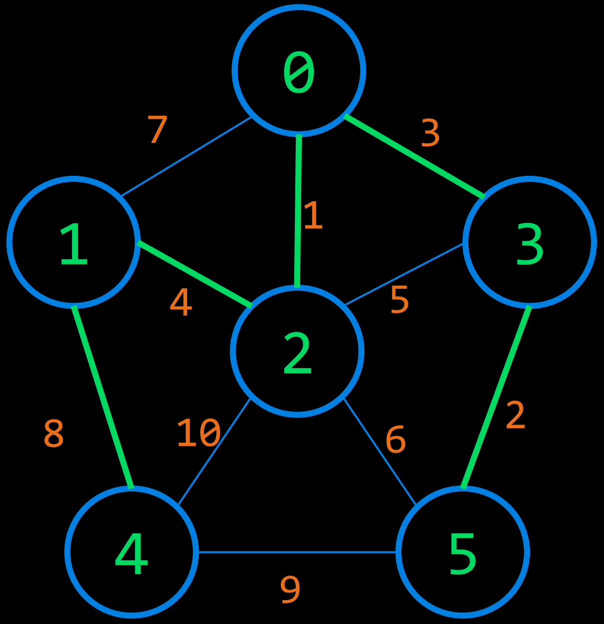
~~2-3~~	~~5~~
~~2-5~~	~~6~~
~~0-1~~	~~7~~
1-4	8
4-5	9
2-4	10

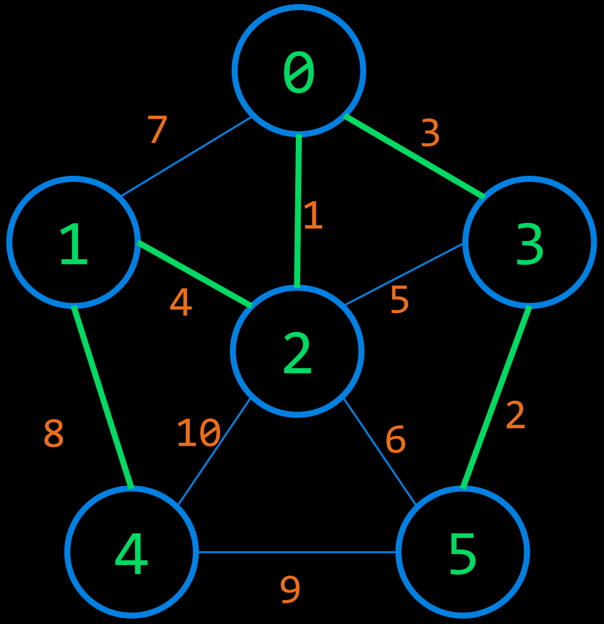


Add edges in order as long as they don’t create a cycle

Kruskal's Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
~~2-3~~	~~5~~
~~2-5~~	~~6~~
~~0-1~~	~~7~~
1-4	8
~~4-5~~	~~9~~
2-4	10

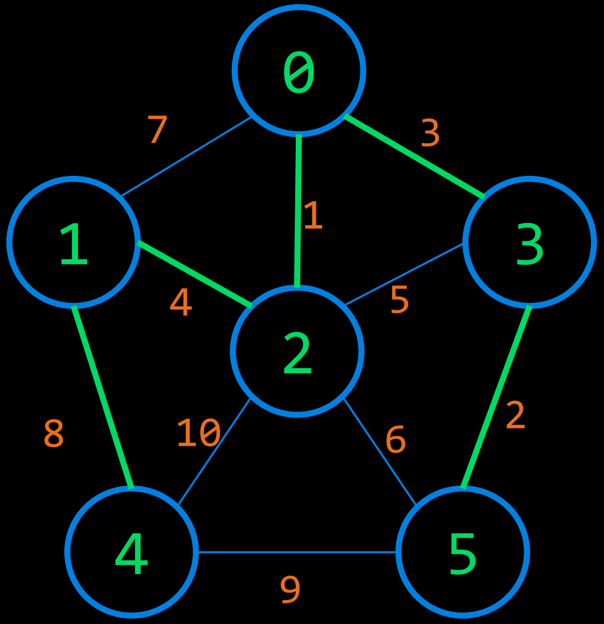


Add edges in order as long as they don't create a cycle

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
~~2-3~~	~~5~~
~~2-5~~	~~6~~
~~0-1~~	~~7~~
1-4	8
~~4-5~~	~~9~~
~~2-4~~	~~10~~

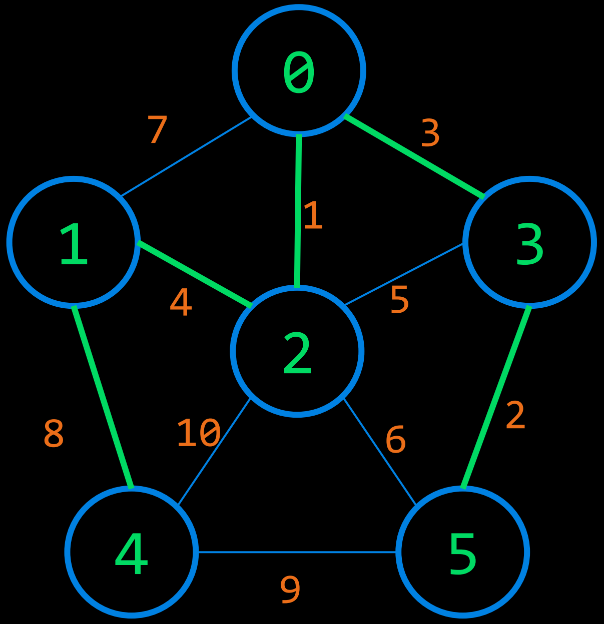


Add edges in order as long as they don’t create a cycle

Kruskal's Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
~~2-3~~	~~5~~
~~2-5~~	~~6~~
~~0-1~~	~~7~~
1-4	8
~~4-5~~	~~9~~
~~2-4~~	~~10~~



Minimum Spanning Tree Sum = 18

Questions

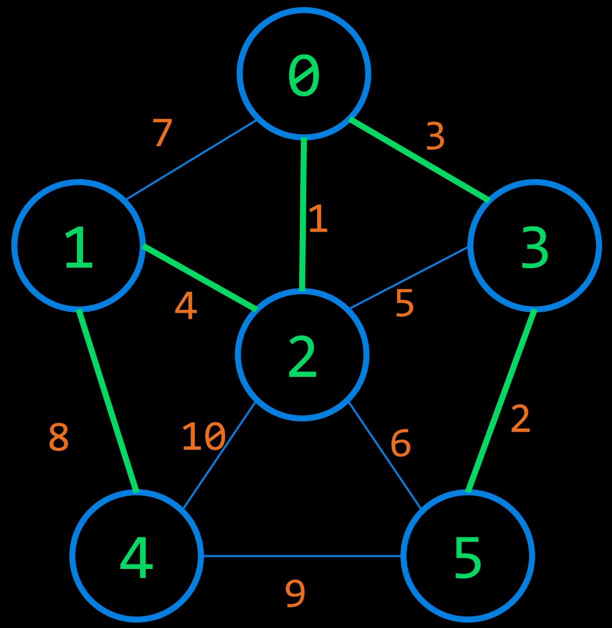
Kruskal's Algorithm

How can we detect a cycle when adding an edge?

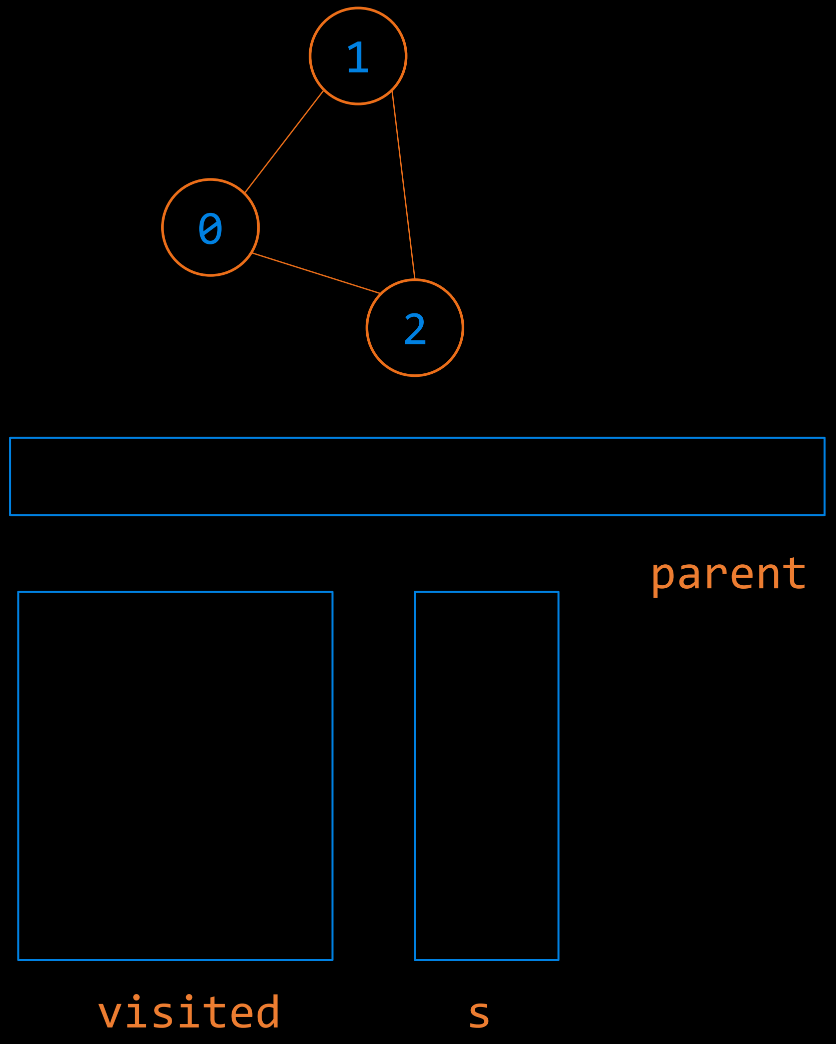
Method 1:

Cycle Detection using DFS. Find back edges.

Back Edge: An edge that connects an ancestor during DFS traversal.

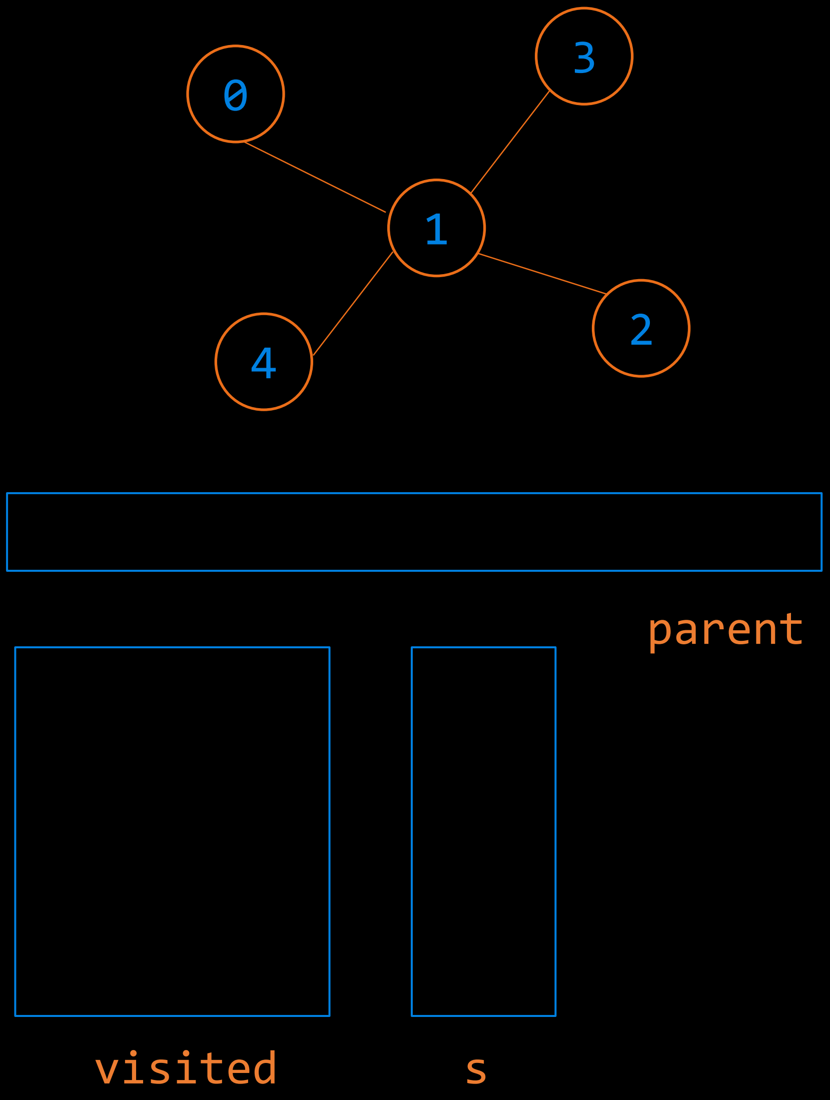


7.3.1 Detect whether there is a Cycle in an Undirected Graph



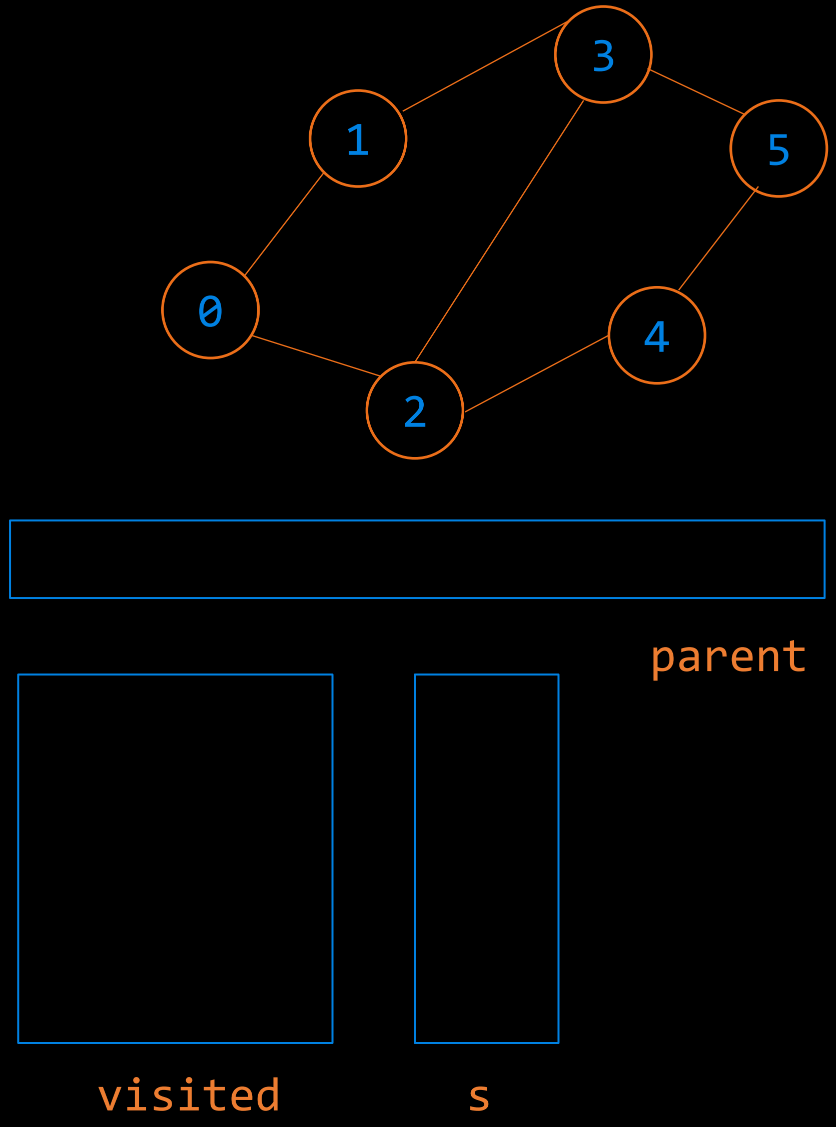

```
bool anyCycle(const Graph& graph)
{
    set<int> visited;
    vector<int> parent(graph.numVertices, -1);
    stack<int> s;
    visited.insert(0);
    s.push(0);
    while(!s.empty())
    {
            int u = s.top();
            s.pop();
            for(auto v: graph.adjList[u])
            {
                if ((visited.find(v)==visited.end()))
                {
                    visited.insert(v);
                    s.push(v);
                    parent[v] = u;
                }
                else if (parent[u] != v)
                    return true;
            }
     }
     return false;
}
```

7.3.1 Detect whether there is a Cycle in an Undirected Graph



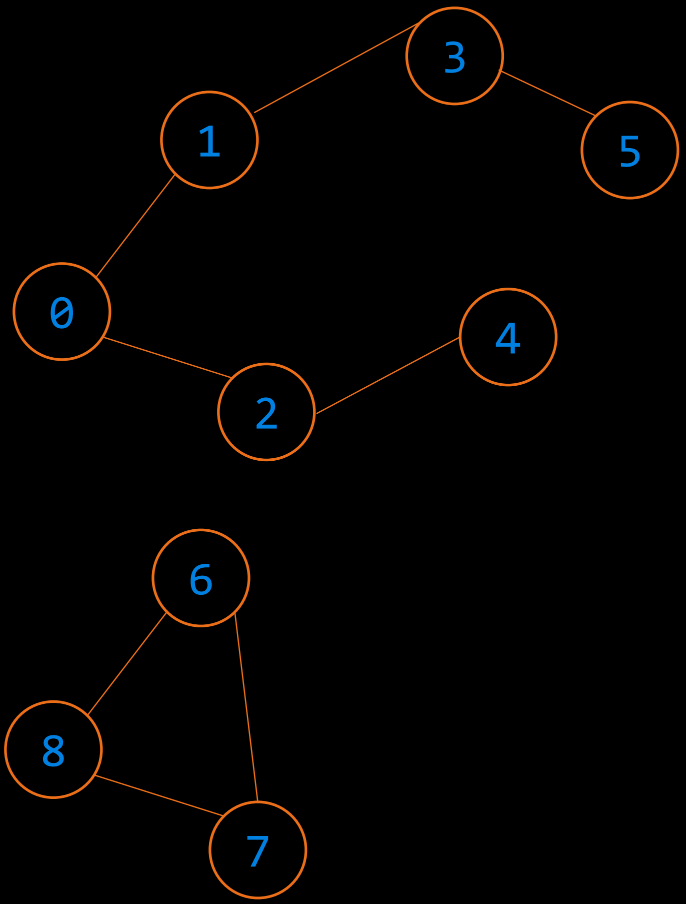

```
bool anyCycle(const Graph& graph)
{
    set<int> visited;
    vector<int> parent(graph.numVertices, -1);
    stack<int> s;
    visited.insert(0);
    s.push(0);
    while(!s.empty())
    {
            int u = s.top();
            s.pop();
            for(auto v: graph.adjList[u])
            {
                if ((visited.find(v)==visited.end()))
                {
                    visited.insert(v);
                    s.push(v);
                    parent[v] = u;
                }
                else if (parent[u] != v)
                    return true;
            }
    }
    return false;
}
```

7.3.1 Detect whether there is a Cycle in an Undirected Graph

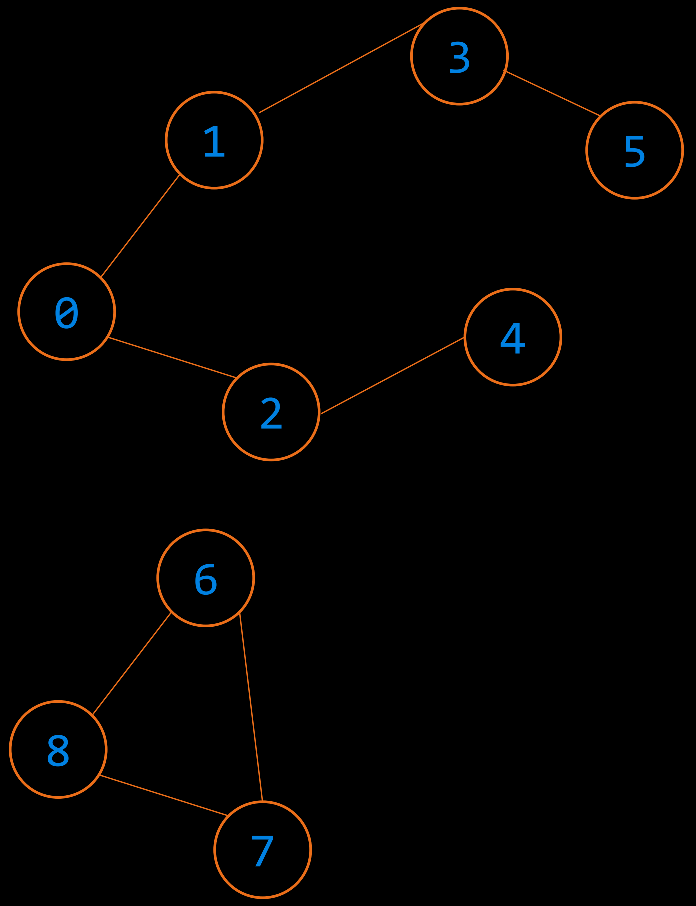



```cpp
bool anyCycle(const Graph& graph)
{
    set<int> visited;
    vector<int> parent(graph.numVertices, -1);
    stack<int> s;
    visited.insert(0);
    s.push(0);
    while(!s.empty())
    {
            int u = s.top();
            s.pop();
            for(auto v: graph.adjList[u])
            {
                if ((visited.find(v)==visited.end()))
                {
                    visited.insert(v);
                    s.push(v);
                    parent[v] = u;
                }
                else if (parent[u] != v)
                    return true;
            }
    }
    return false;
}
```

7.3.1 Detect whether there is a Cycle in an Undirected Graph



7.3.1 Detect whether there is a Cycle in an Undirected Graph




```cpp
bool anyCycle(const Graph& graph)
{
    set<int> visited;
    vector<int> parent(graph.numVertices, -1);
    stack<int> s;
    for(int i=0; i<graph.numVertices; i++)
    {
        if ((visited.find(i)==visited.end()))
        {
            visited.insert(i);
            s.push(i);
            while(!s.empty())
            {
                int u = s.top();
                s.pop();
                for(auto v: graph.adjList[u])
                {
                    if ((visited.find(v)==visited.end()))
                    {
                        visited.insert(v);
                        s.push(v);
                        parent[v] = u;
                    }
                    else if (parent[u] != v)
                        return true;
                }
            }
        }
    }
    return false;
}
```

Kruskal’s Algorithm

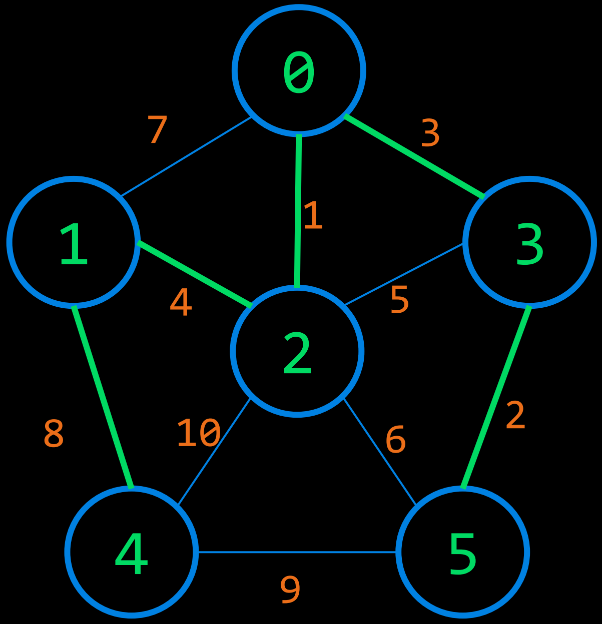
How can we detect a cycle when adding an edge?

Method 1:

Cycle Detection using DFS.

Works correctly but is computationally more expensive.

Complexity: O(E (V+E))

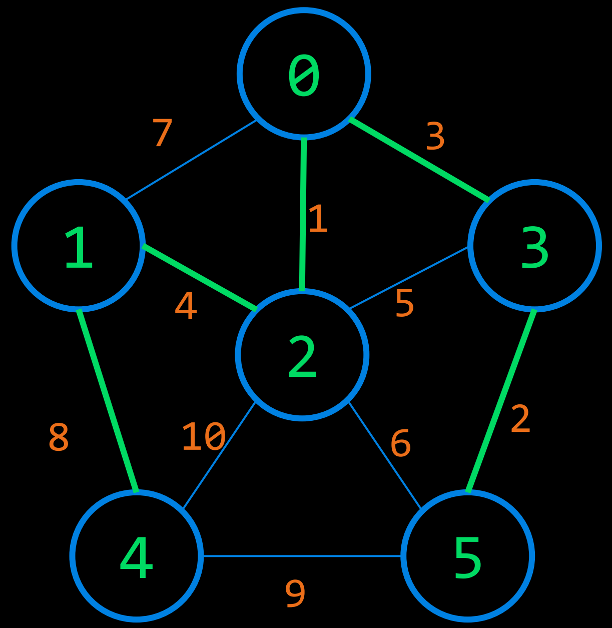


Kruskal's Algorithm

How can we detect a cycle when adding an edge?

Method 2a:

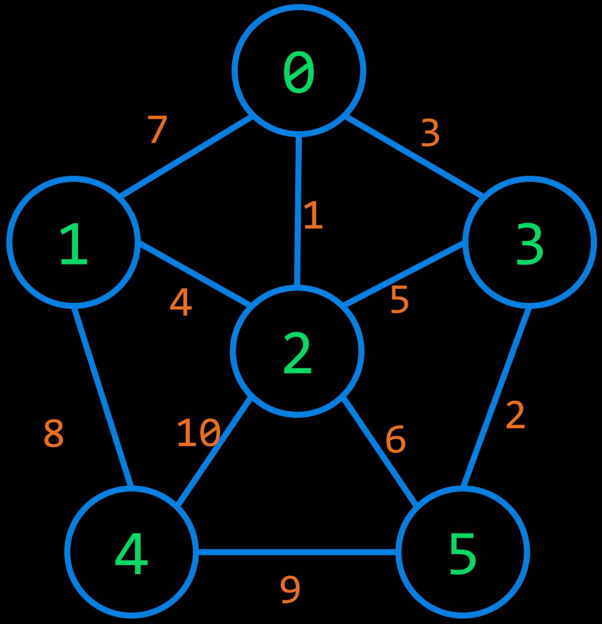
- Create an empty set, S.
- For each edge, E:
    - If either of the vertices connecting E is not a part of the set, add the vertices of E to S
    - If, both the vertices are part of the set S, **ignore the edge as it forms a cycle.**



Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
2-3	5
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10



Add edges in order as long as they don’t create a cycle

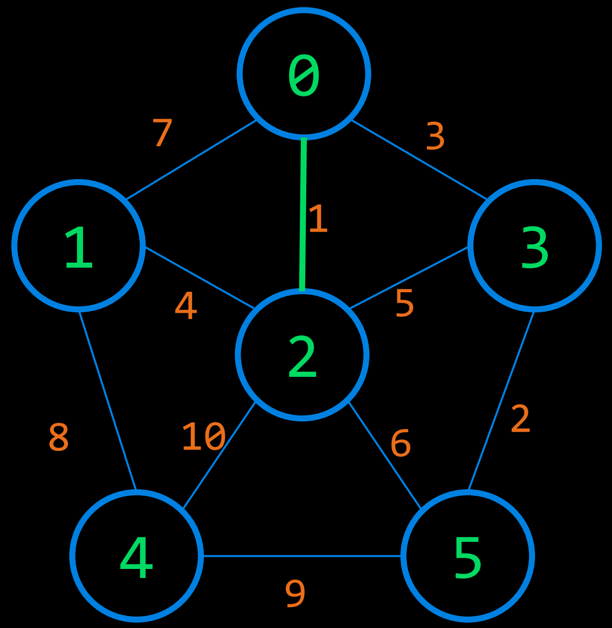
Kruskal's Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
2-3	5
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10

Set

0 2



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Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
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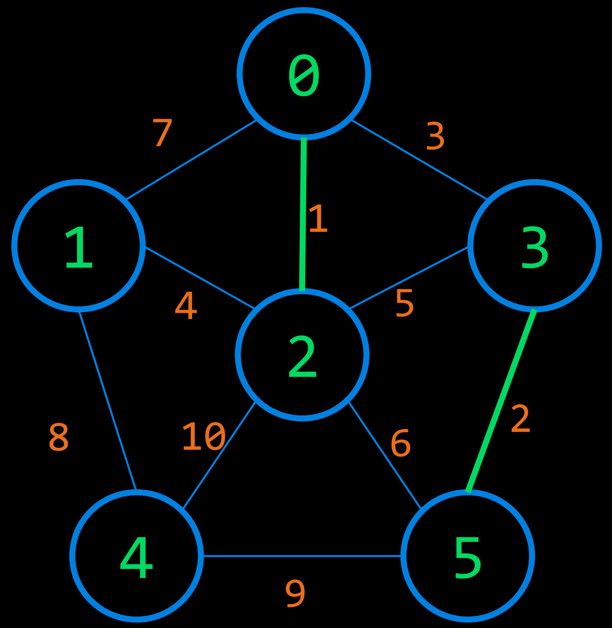
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10

Set

0 2 3 5



Add edges in order as long as they don’t create a cycle

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
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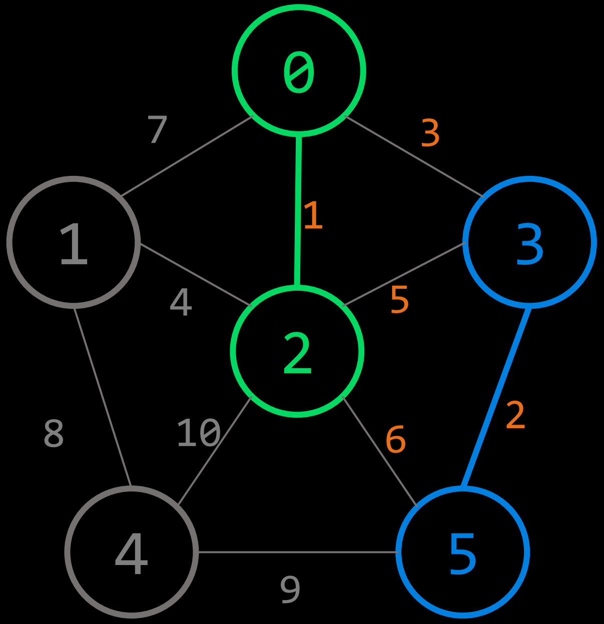
Set

0 2 3 5



Disconnected Components

Add edges in order as long as they don’t create a cycle

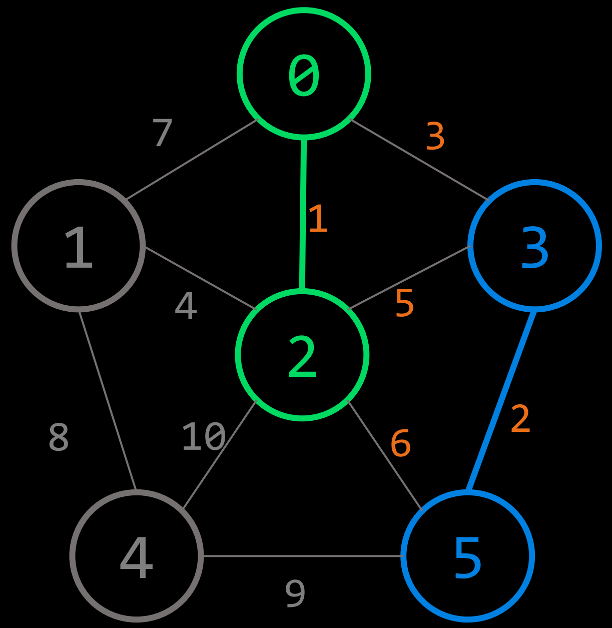
Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
2-3	5
2-5	6
0-1	7
1-4	8
4-5	9
2-4	10

Set

0 2 3 5



Disconnected Components
0-3, 2-3, 2-5 will never be picked in MST

Add edges in order as long as they don’t create a cycle

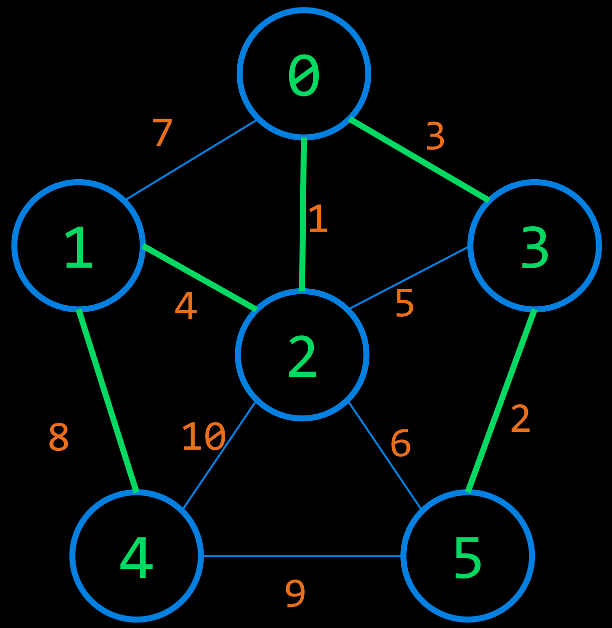
Kruskal's Algorithm

How can we detect a cycle when adding an edge?

Method 2a:

- Create an empty set, S.
- For each edge, E:
  - If either of the vertices connecting E is not a part of the set, add the vertices of E to S
  - If, both the vertices are part of the set S, ignore the edge as it forms a cycle.

This will not work whenever we pick edges in an order such that we have two disconnected components. Adding edges leads to connected components!



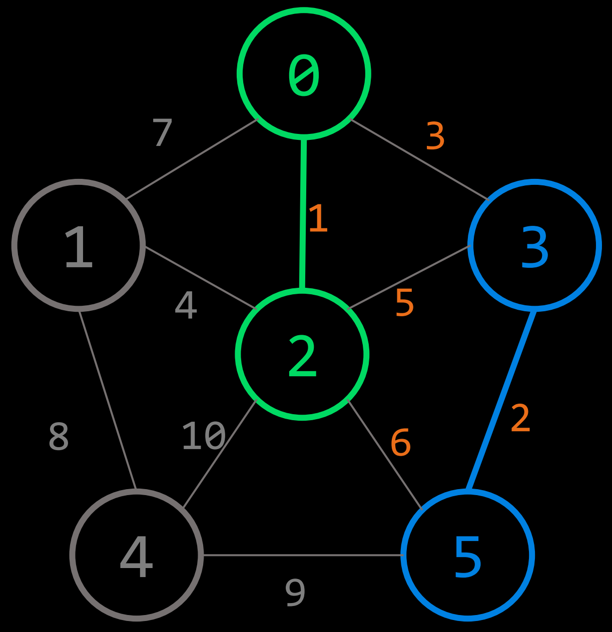
Kruskal’s Algorithm

How can we detect a cycle when adding an edge?

Method 2b:

Disjoint Sets – Weighted Union

- A group of sets. There is no item in common in any of the sets.
- Operations:
  - find(i) identify the set that contains i
  - union(i, j) merge the set that contains i and the set that contains j
- Disjoint sets represent connected components.
- A cycle is created by adding an edge for which both vertices are in the same connected component.
- Complexity: O(E log V)



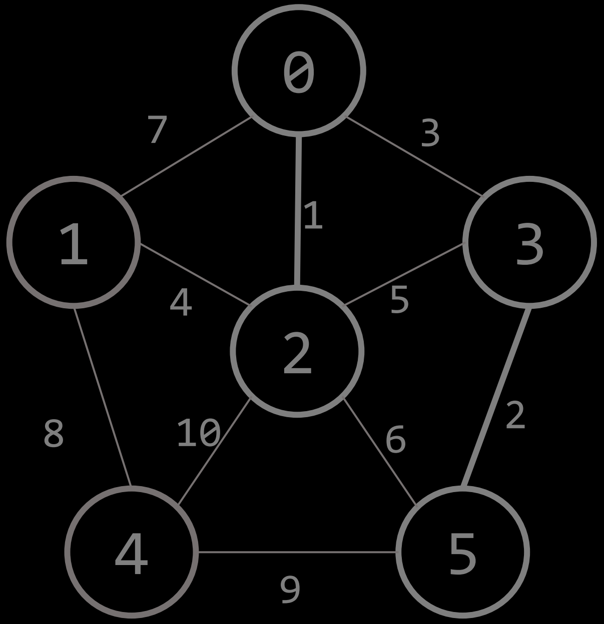
Disconnected Components
Two connected components:
{0,2} and {3,5}

Disjoint Sets

Disjoint Sets – Union/Find

- Optimally represented as an array where each index stores the parent of the “index” vertex. An entire set is represented as a tree.

- Operations:
  - union(i, j) merge the set that contains i and the set that contains j
  - find(i) identify the set that contains i



Disjoint Sets

Disjoint Sets – Union/Find

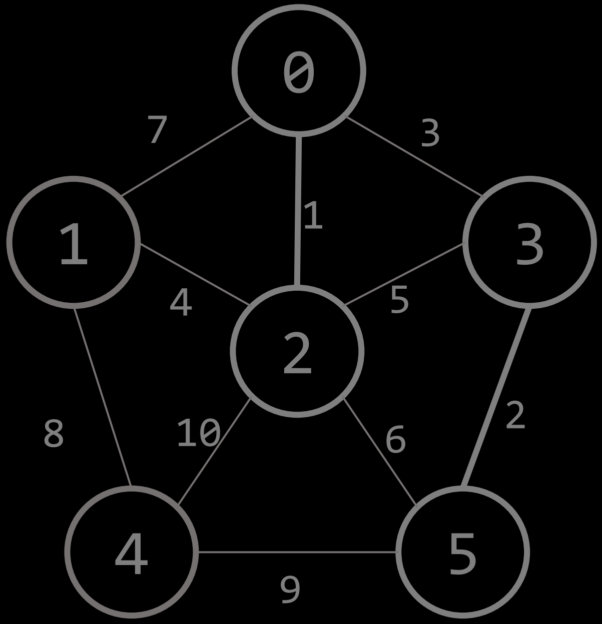
- Optimally represented as an array where each index stores the parent of the “index” vertex. An entire set is represented as a tree.

- Operations:
  - union(i, j) merge the set that contains i and the set that contains j

```
pi = find(i)
pj = find(j)
arr [pi] = pj
```

  - find(i) identify the set that contains i

```
if(arr[i]) == -1)
        return i
else
        return find(arr[i])
```



Disjoint Sets

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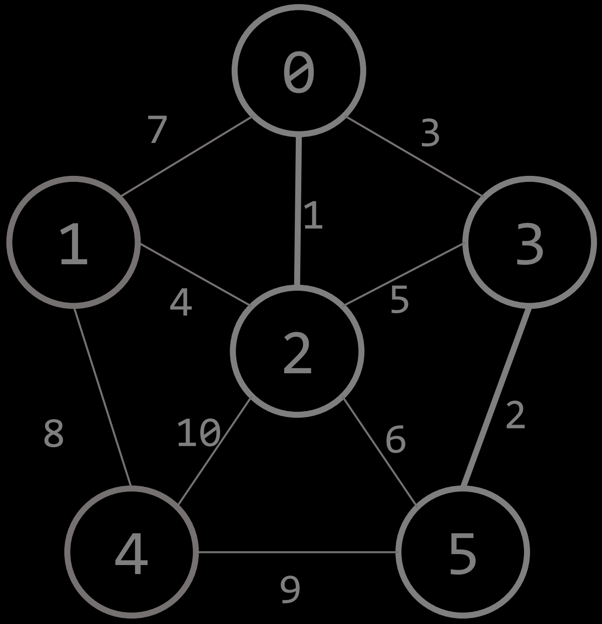

Kruskal's Algorithm

Arrange edges in ascending order

0-2	1
3-5	2
0-3	3
1-2	4
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2-5	6
0-1	7
1-4	8
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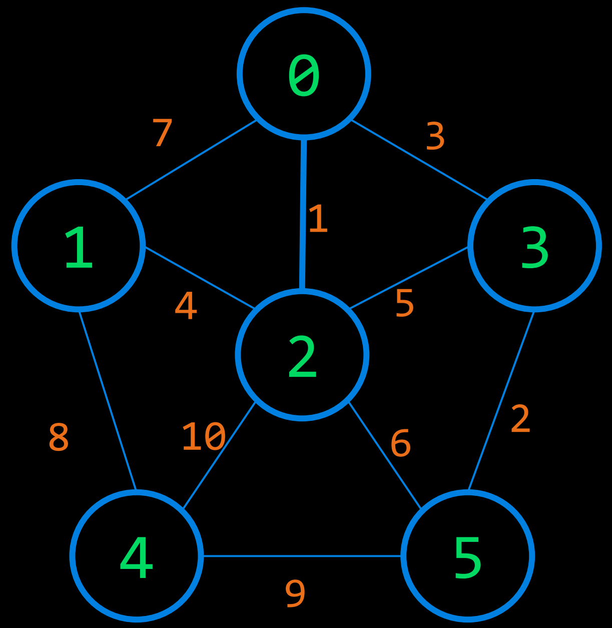
2-4	10




```
Kruskal's algorithm – choose an edge i-j
                     if find(i) != find(j)    //not in same disjoint set
                        union(i, j)   //add the edge, which joins the components
```

Disjoint sets: {0}, {1}, {2}, {3}, {4}, {5}

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
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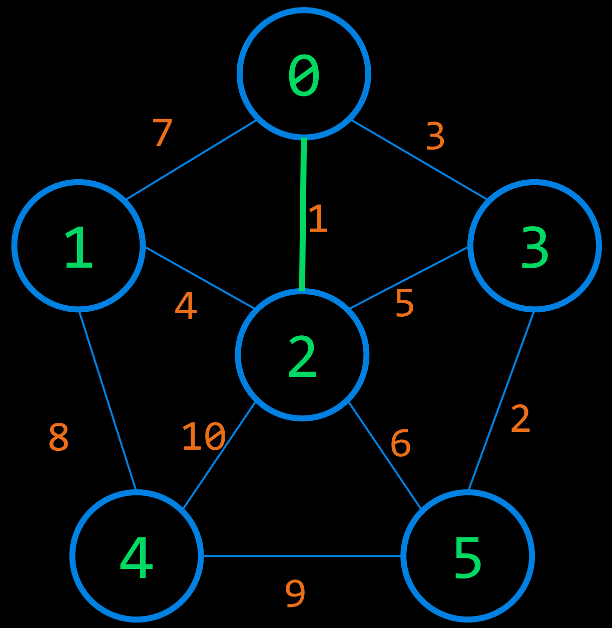


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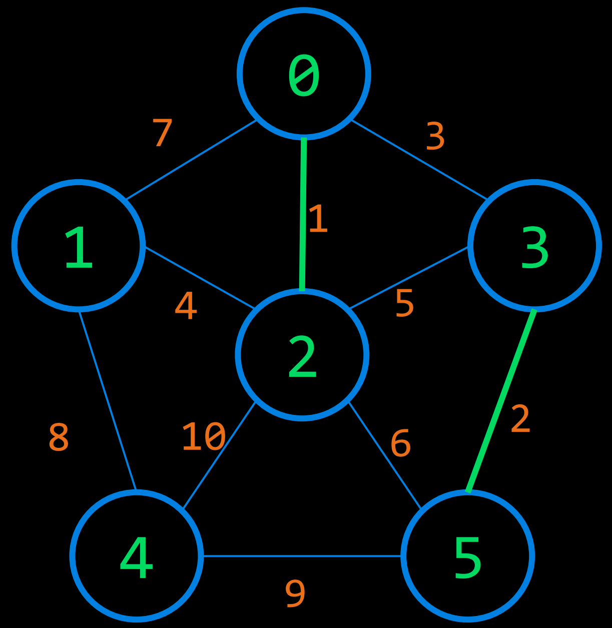


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Disjoint sets: {0, 2}, {1}, {3, 5}, {4}

Kruskal’s Algorithm

Arrange edges in ascending order

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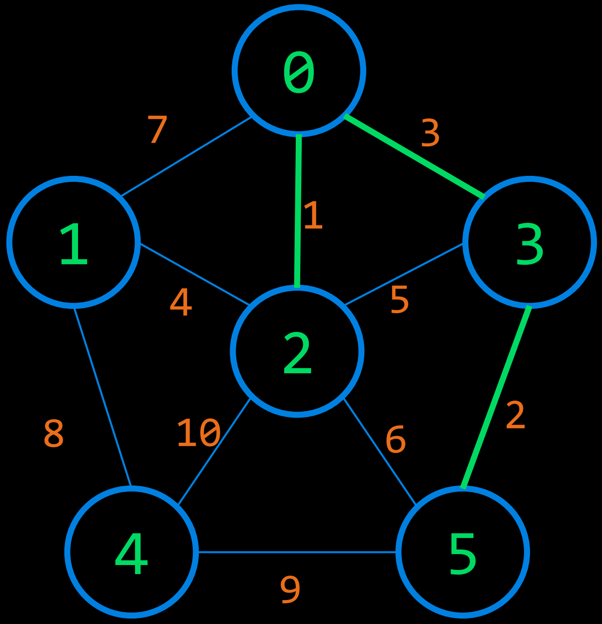


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```

Disjoint sets: {0, 2, 3, 5}, {1}, {4}

Kruskal's Algorithm

Arrange edges in ascending order

0-2	1
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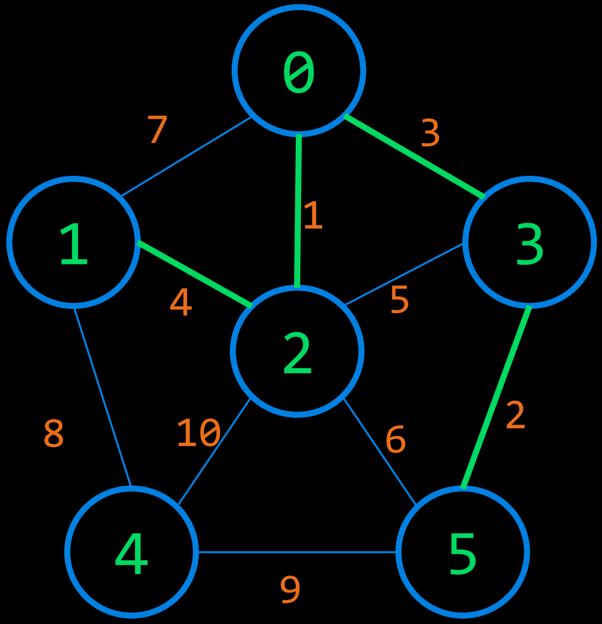


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Disjoint sets: {0, 1, 2, 3, 5}, {4}

Kruskal’s Algorithm

Arrange edges in ascending order

0-2	1
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1-2	4
~~2-3~~	~~5~~
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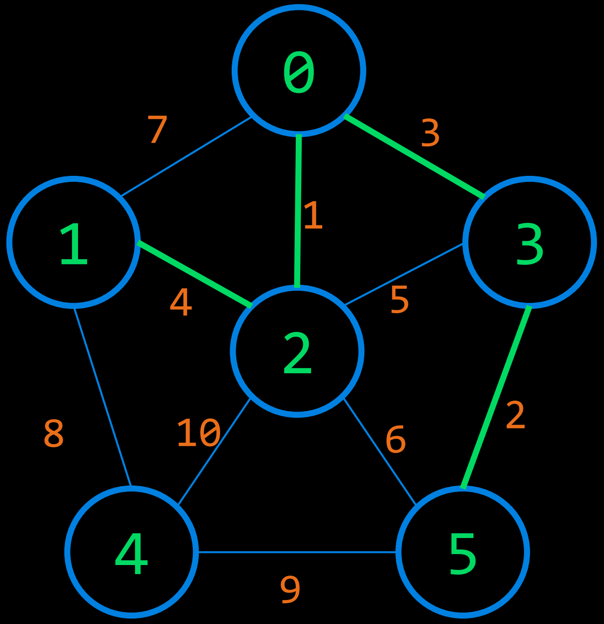


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Disjoint sets: {0, 1, 2, 3, 5}, {4}
```

Kruskal's Algorithm

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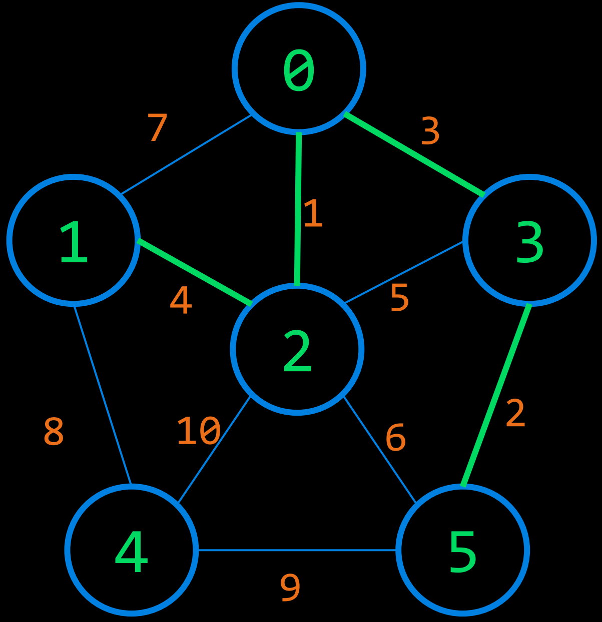


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Disjoint sets: {0, 1, 2, 3, 5}, {4}

Kruskal's Algorithm

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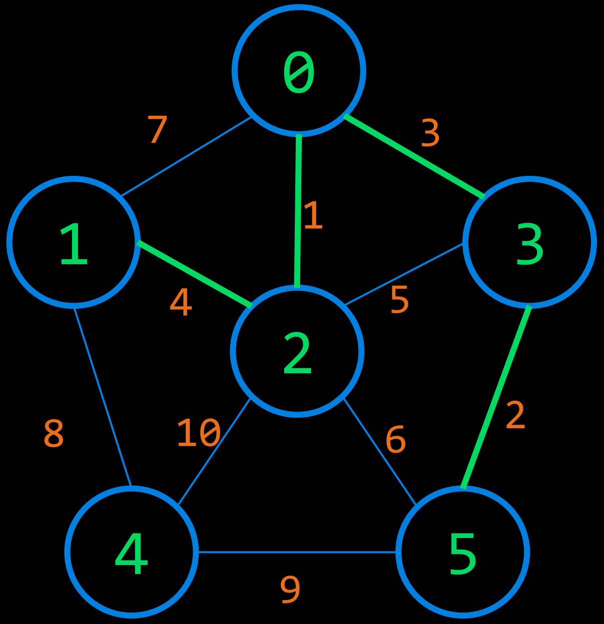
~~0-1 7~~
1-4 8
4-5 9
2-4 10

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Disjoint sets: {0, 1, 2, 3, 5}, {4}

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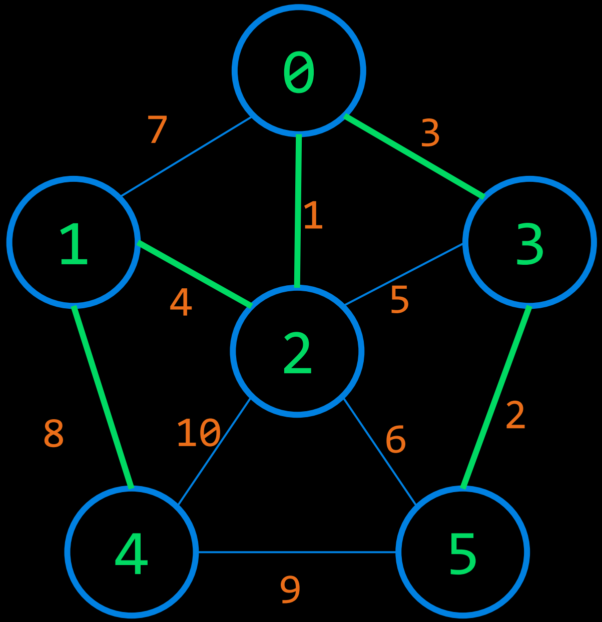
2-4	10

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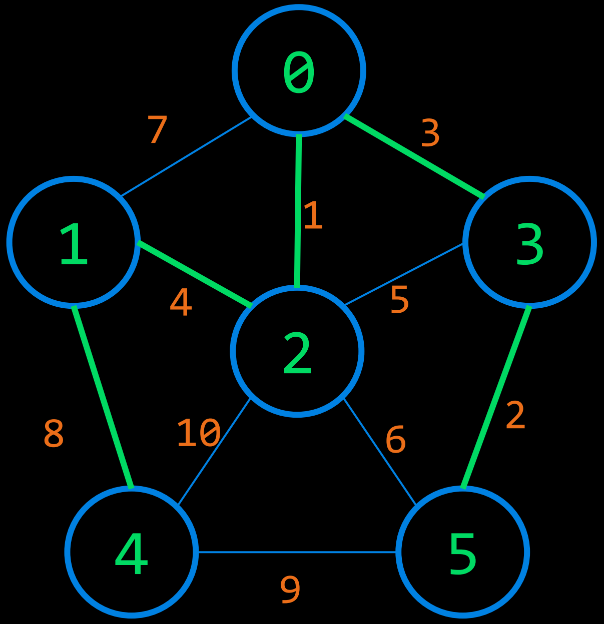
~~0-1~~	~~7~~
1-4	8
~~4-5~~	~~9~~
~~2-4~~	~~10~~

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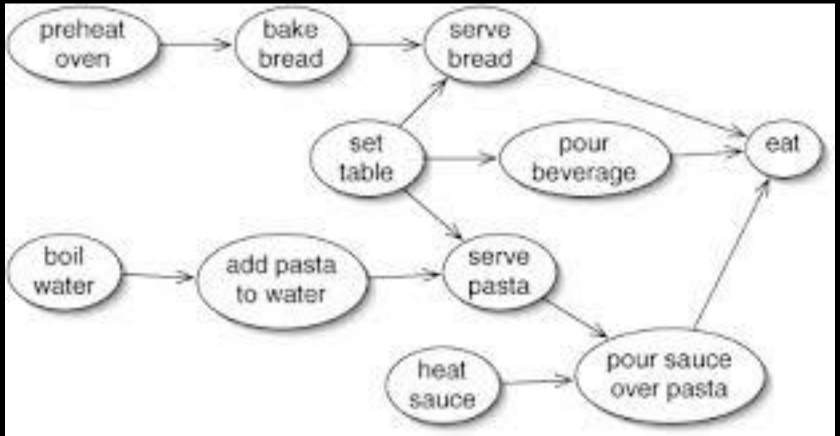
                      if find(i) != find(j)    //not in same disjoint set
                         union(i, j)   //add the edge, which joins the components
```

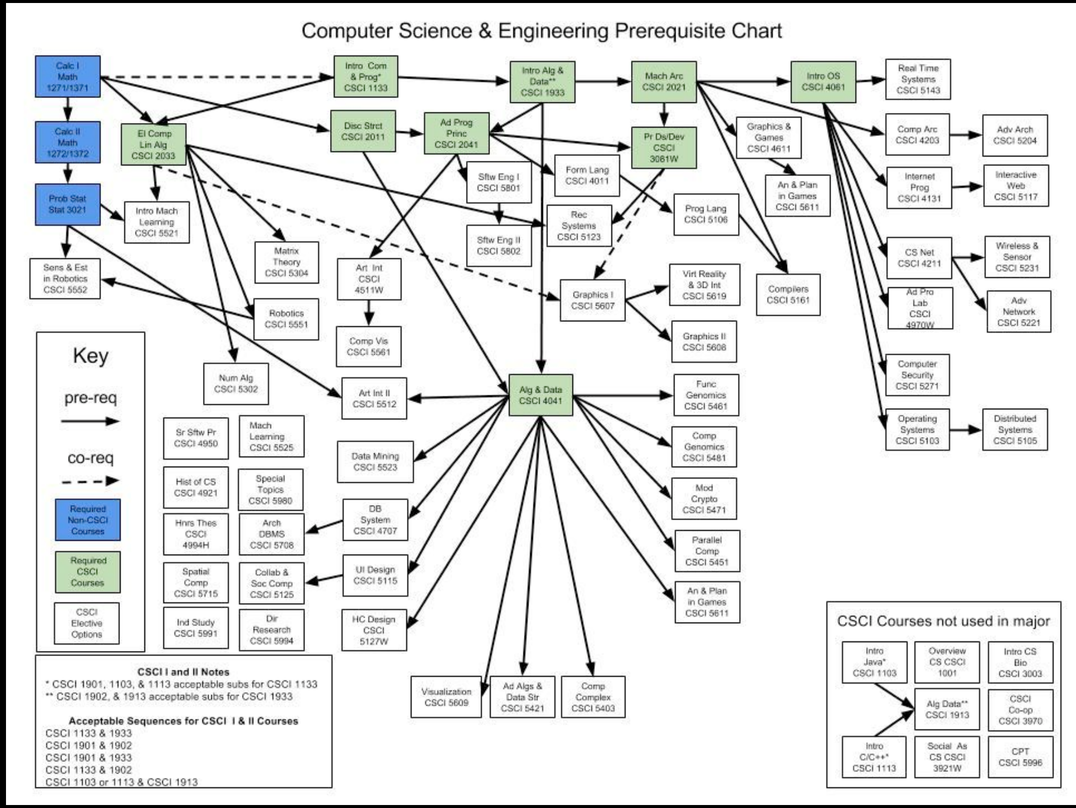
Disjoint sets: {0, 1, 2, 3, 4, 5}

Topological Sort

Topological Sort

A topological sort is an ordering of vertices such that if there is an edge from v_i to v_j, then v_j comes after v_i



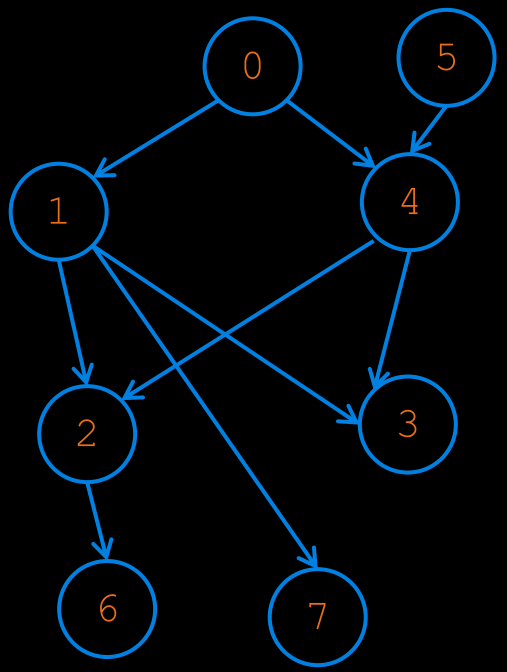


Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.



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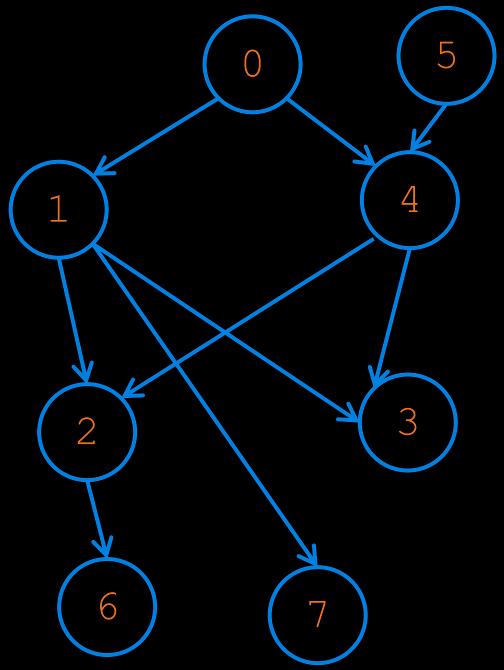


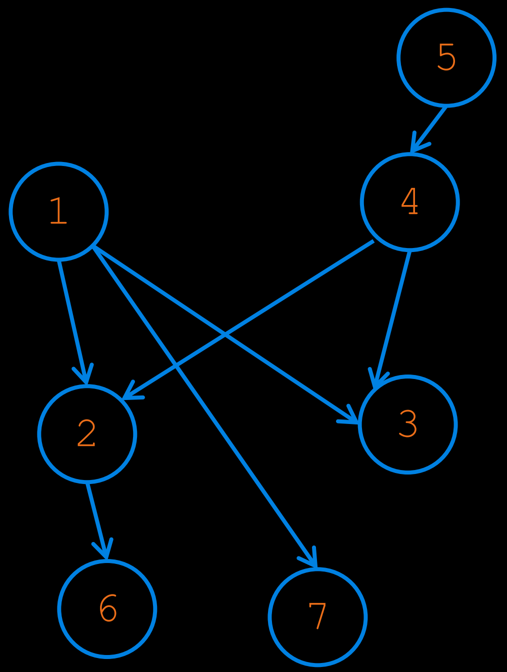
V0 = {0, 5}
Sort Order = {}

Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.



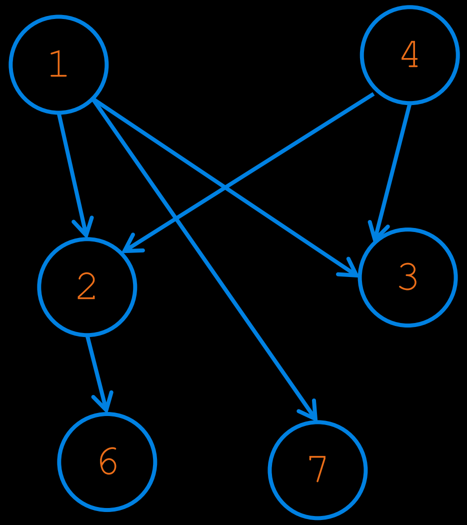
V0 = {5, 1}
Sort Order = {0}

Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.



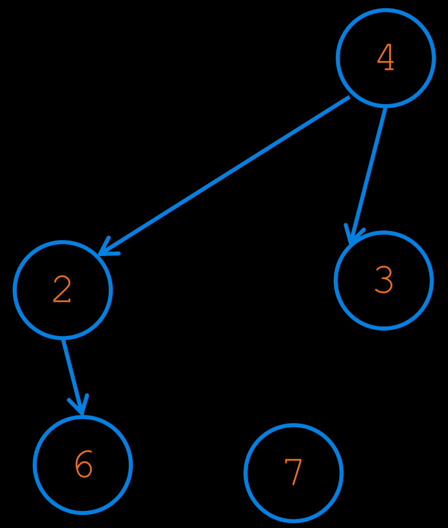

```
V0 = {1, 4}
Sort Order = {0, 5}
```

Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.



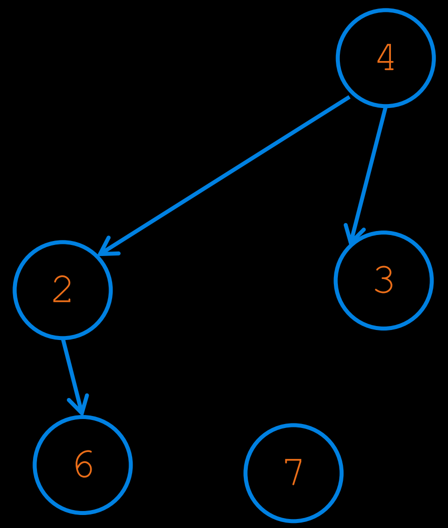
V0 = {4, 7}
Sort Order = {0, 5, 1}

Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.



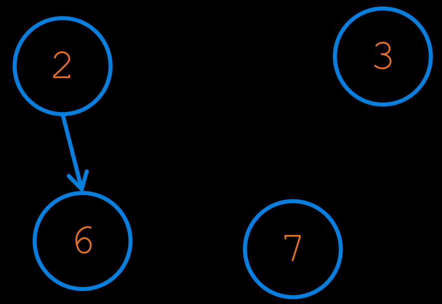
V0 = {4, 7}
Sort Order = {0, 5, 1}

Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.



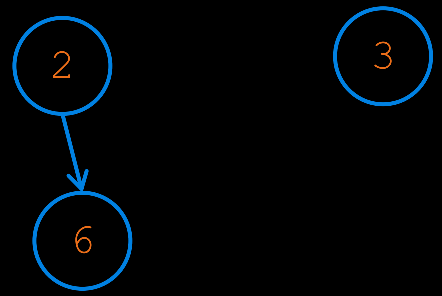
V0 = {7, 2, 3}
Sort Order = {0, 5, 1, 4}

Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.



V0 = {2, 3}
Sort Order = {0, 5, 1, 4, 7}

Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.


```
V0 = {3, 6}
Sort Order = {0, 5, 1, 4, 7, 2 }
```

Topological Sort

A simple algorithm to find a topological ordering is first to find any vertex with no incoming edges.

We can then print this vertex, and remove it, along with its edges, from the graph.

Then we apply this same strategy to the rest of the graph.

V0 = {6}
Sort Order = {0, 5, 1, 4, 7, 2, 3 }

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```
V0 = {}
Sort Order = {0, 5, 1, 4, 7, 2, 3, 6}
```

Topological Sort Pseudocode

```
void Graph::topsort( )
{
        Queue<Vertex> q;
        int counter = 0;
        q.makeEmpty( );
        for each Vertex v
             if( v.indegree == 0 )
                  q.enqueue( v );
        while( !q.isEmpty( ) )
        {
                Vertex v = q.dequeue( );
                v.topNum = ++counter;  // Assign next number
                for each Vertex w adjacent to v
                       if( --w.indegree == 0 )
                              q.enqueue( w );
        }
        if( counter != NUM_VERTICES )
               throw CycleFoundException{ };
}
```

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Questions

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