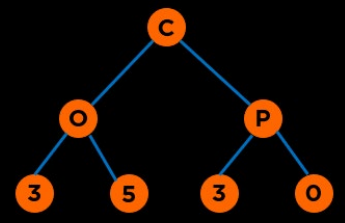
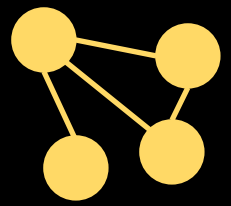
Graphs

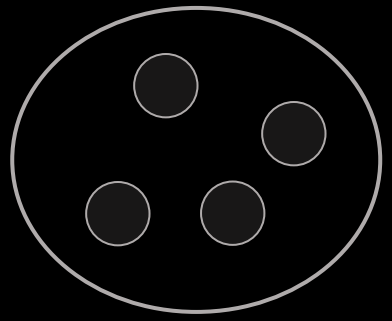


Data Structures & Algorithm

Categories of Data Structures

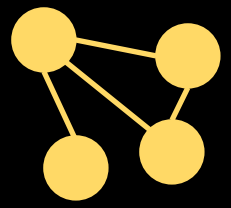
Linear Ordered	Non-linear Ordered	Not Ordered
Lists	Trees	Sets
Stacks	Graphs	Tables/Maps
Queues		

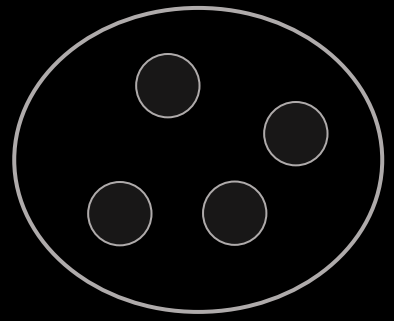




Categories of Data Structures

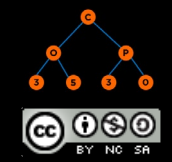
Linear Ordered	Non-linear Ordered	Not Ordered
Lists	Trees	Sets
Stacks	Graphs	Tables/Maps
Queues		





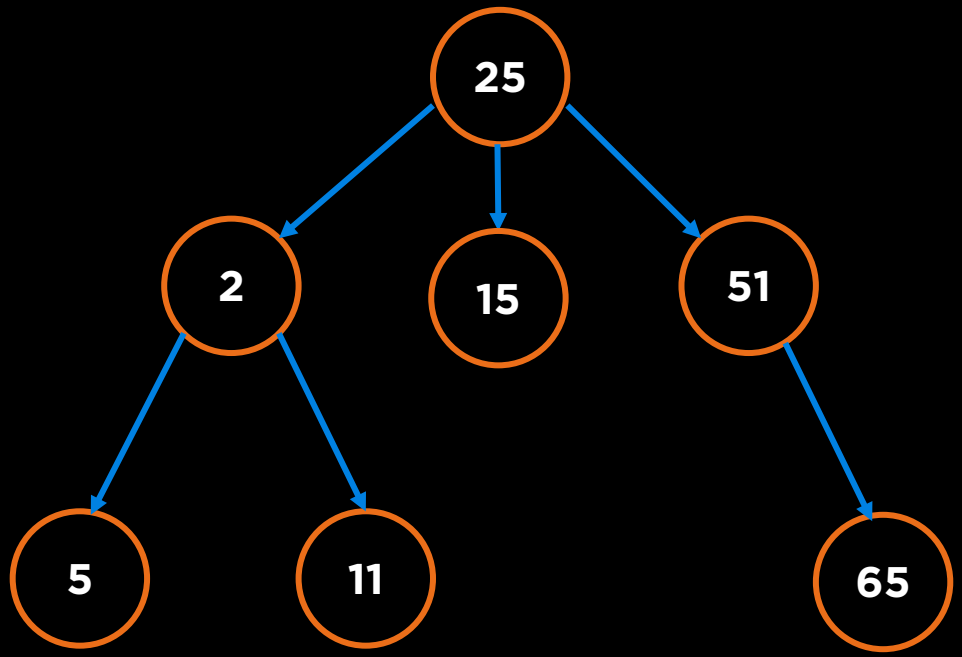
Agenda

- Graphs
  - Terminology
  - Types
  - Use cases
- Graph Implementations
  - Edge List
  - Adjacency Matrix
  - Adjacency List



Trees

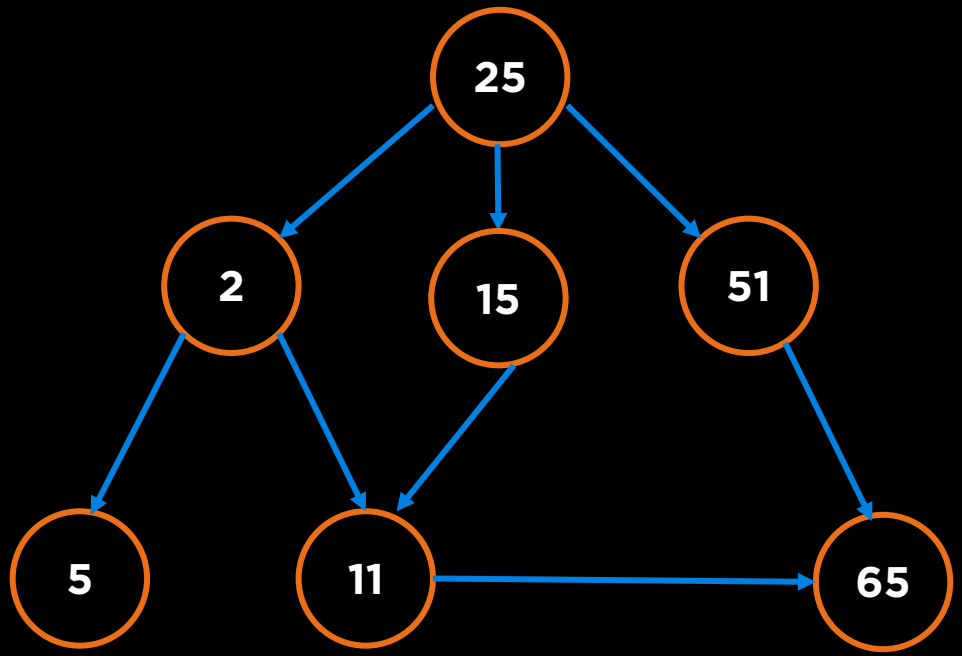
Hierarchical, Acyclic, and Exactly one path between two nodes



Graphs

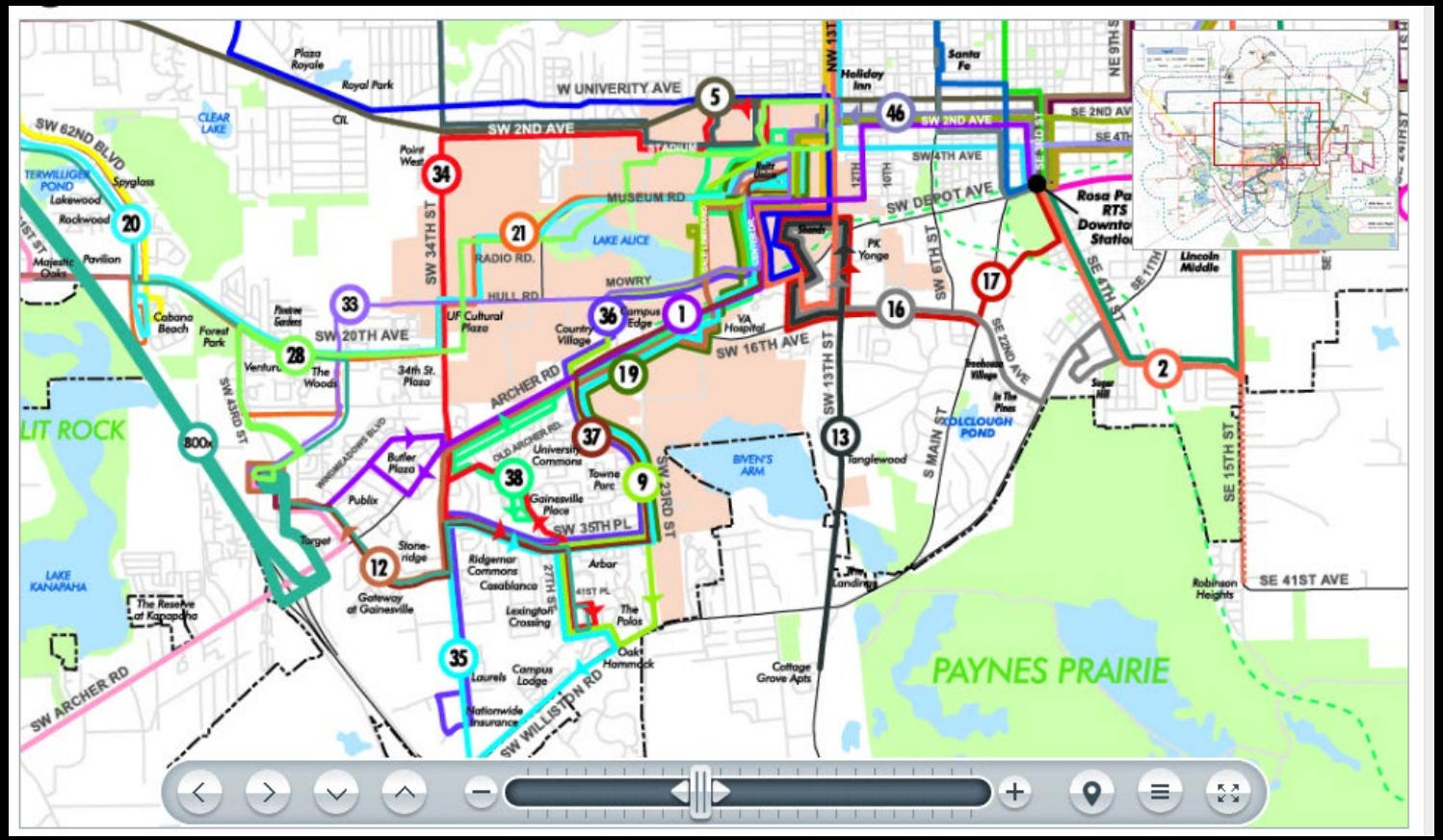
An ordered pair of a set of nodes and a set of edges.

G = (V, E)

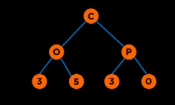


Graphs

Example



Graph Terminology

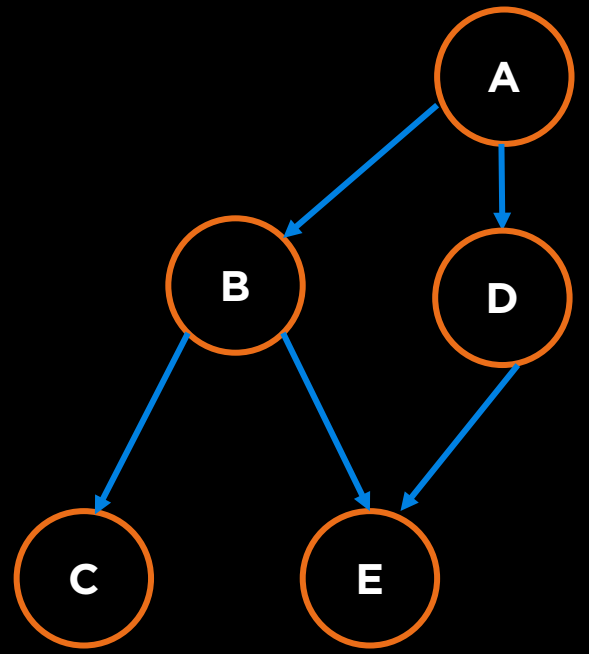


Graph: Terminology

Vertex

Each node in a Graph is called a Vertex

```
V = {A, B, C, D, E}

|V| is the number of vertices in the graph

|V| = 5
```

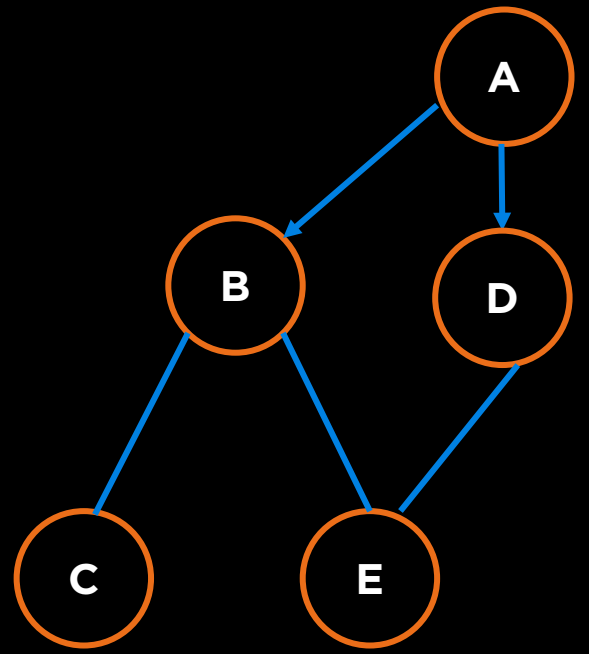


Graph: Terminology

Edge

The connections between two nodes is called an edge.

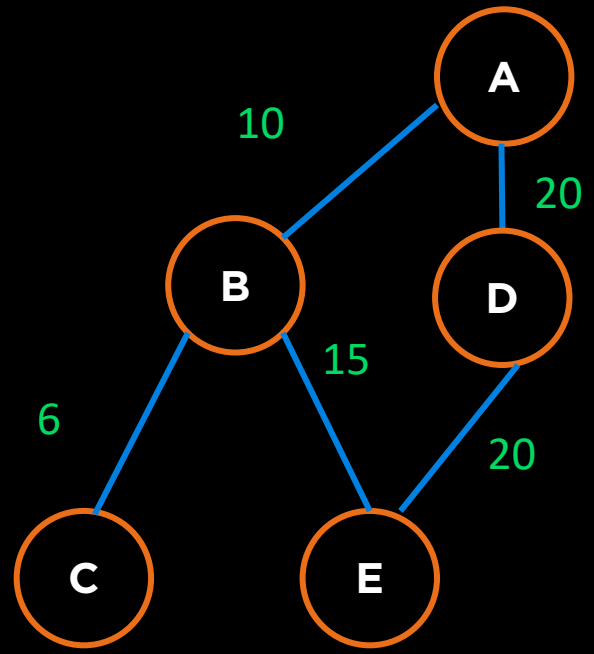
```
E = {(A,B), (A,D), {B,C}, {B,E}, {D,E}}

|E| is the number of edges in the graph

|E| = 5
```



Graph: Terminology

Weight

The edges in a graph may have associated values known as their weights. A weight is like a cost to travel from one vertex to the other over the edge.

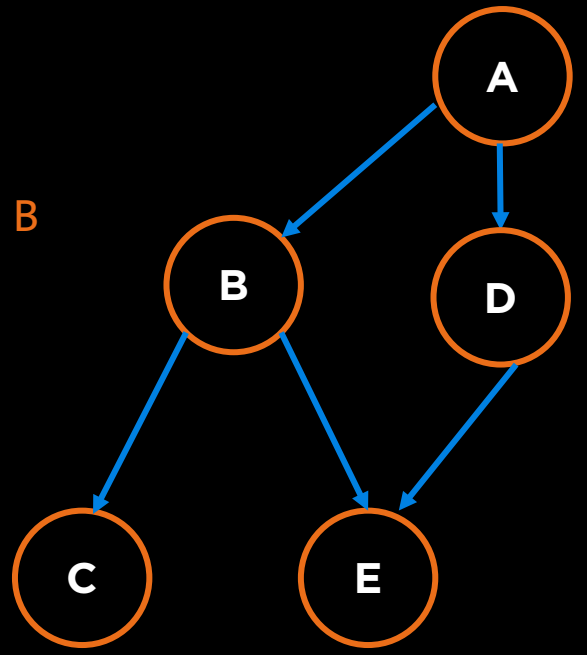


Graph: Terminology

Adjacent Vertices

A vertex is adjacent to another vertex if there is an edge to it from that other vertex.

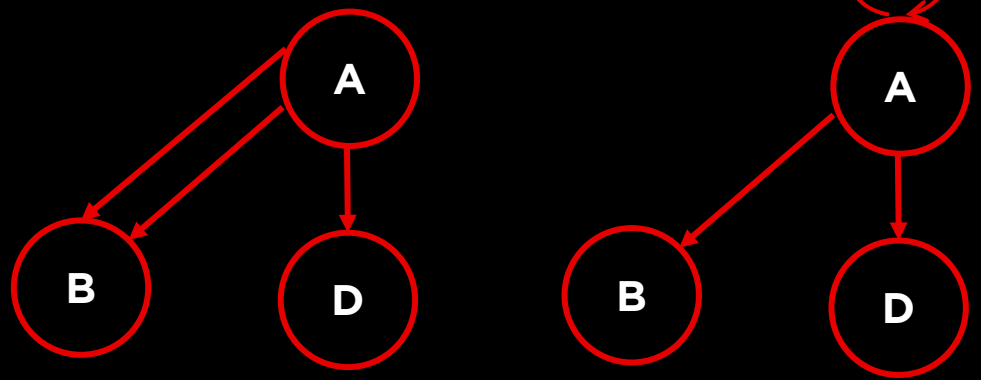
`B is adjacent to A but A is not adjacent to B`

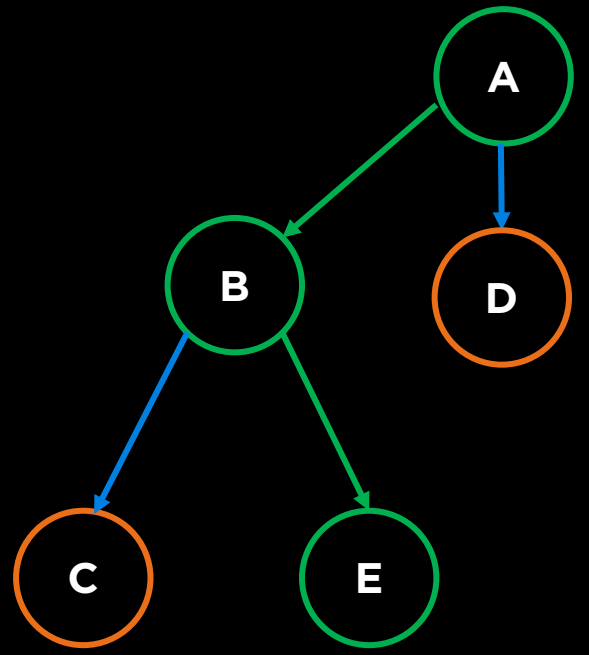


Graph: Terminology

Simple Graph

A simple graph is a graph with no edges that connect a vertex to itself, i.e. no “loops” and no two edges that connect the same vertices, i.e. no “parallel edges”.



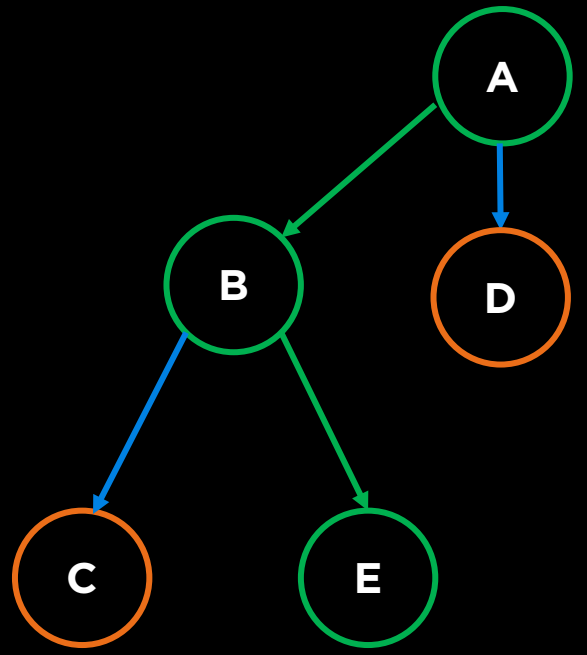


Graph: Terminology

Path

A path is a sequence of vertices in which each successive vertex is adjacent to its predecessor.

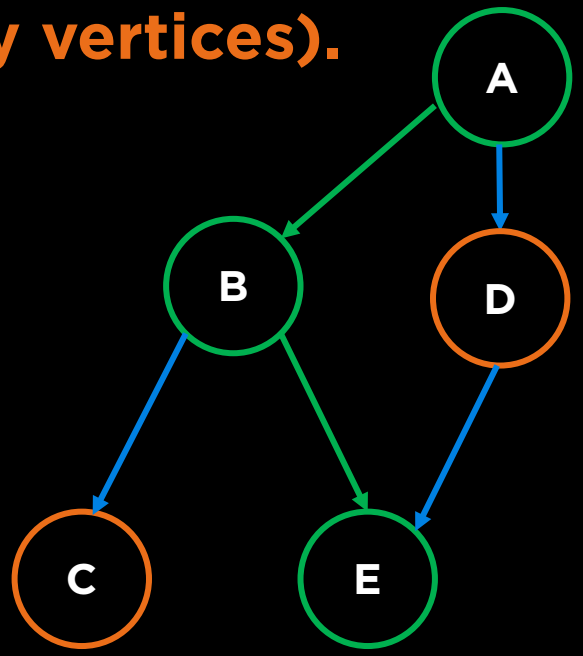
```
Path from A to E: A, B, E
```



Graph: Terminology

Simple Path

In a simple path, the vertices and edges are distinct except that the first and last vertex may be the same (no repeated intermediatory vertices).

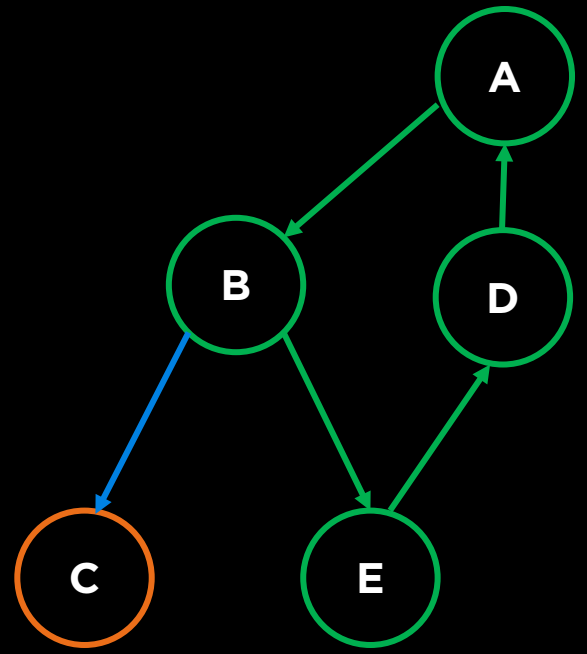


Graph: Terminology

Cycle

A cycle is a simple path in which only the first and final vertices are the same.

A – B – E – D - A is a cycle.

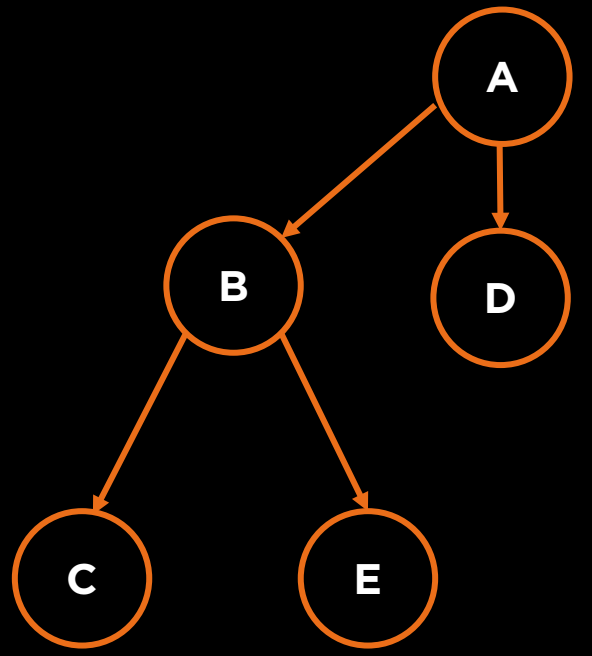


Graph: Terminology

Connected Vertex

Two vertices are connected if there is a path between them.

```
A and C are connected
D and C are not connected
```

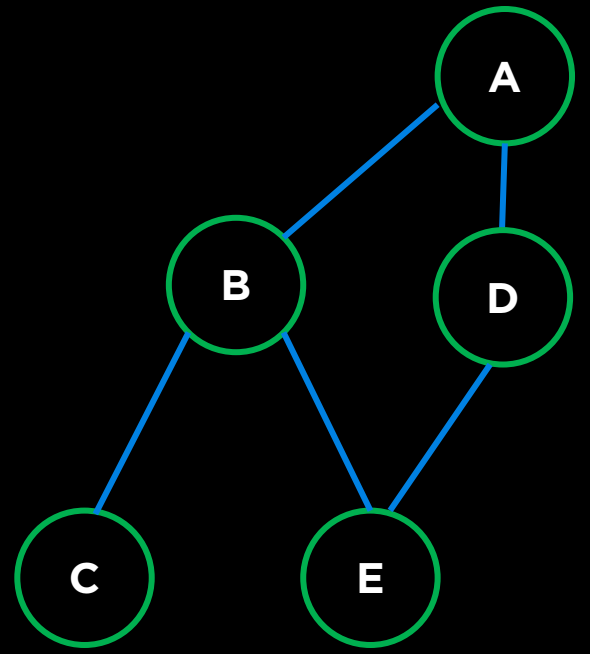


Graph: Terminology

Connected Graph

An undirected graph is called a connected graph if there is a path from every vertex to every other vertex.

```
This is a connected graph
```



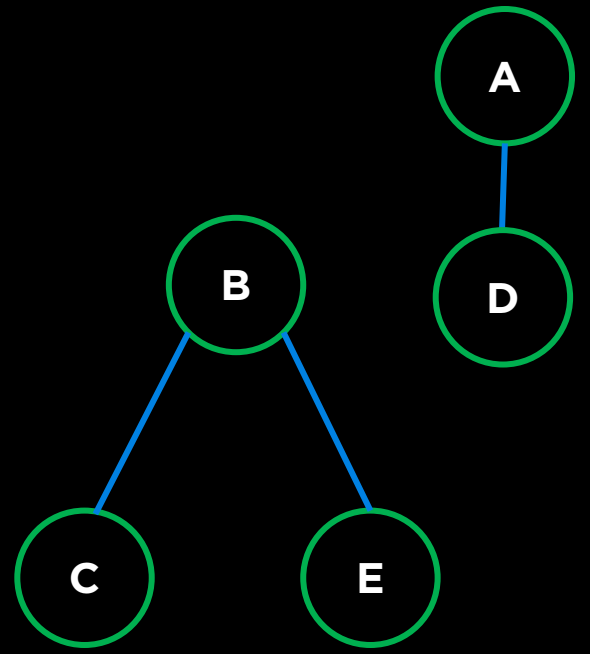
Graph: Terminology

Connected Graph

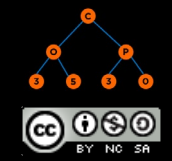
An undirected graph is called a connected graph if there is a path from every vertex to every other vertex.

This is not a connected graph.

Connected components:
{A,D} and {B,C,E}

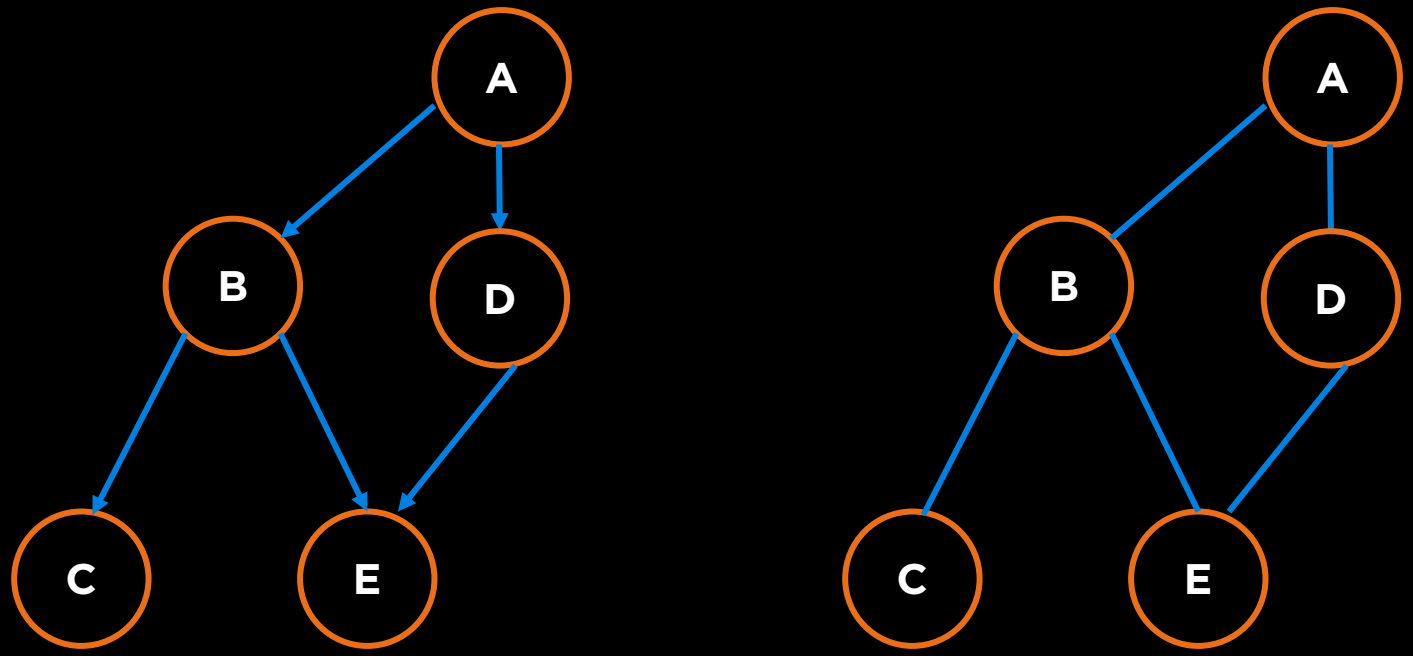


Graph Types



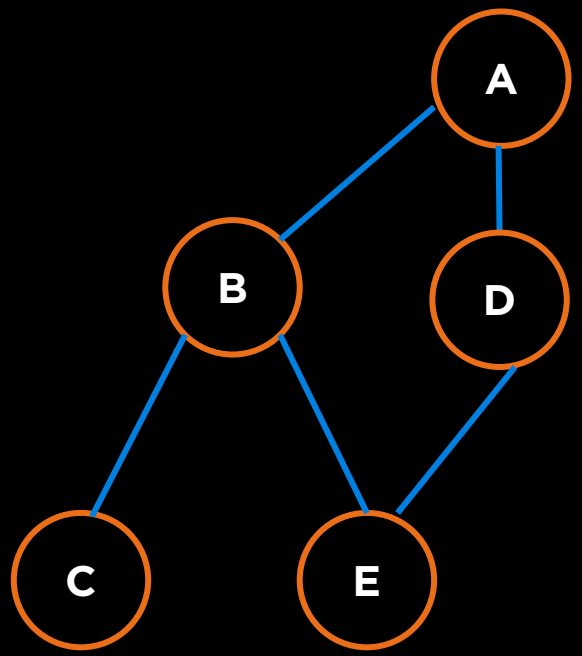
Graph Types

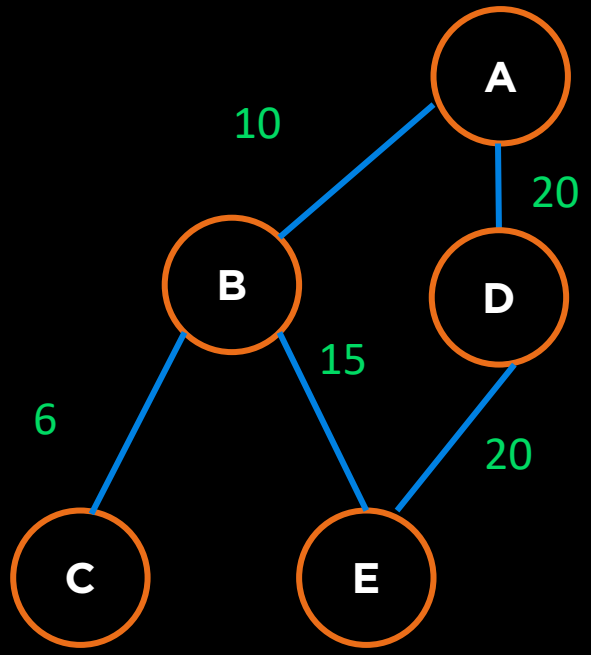
Directed (Digraph) vs Undirected



Graph Types

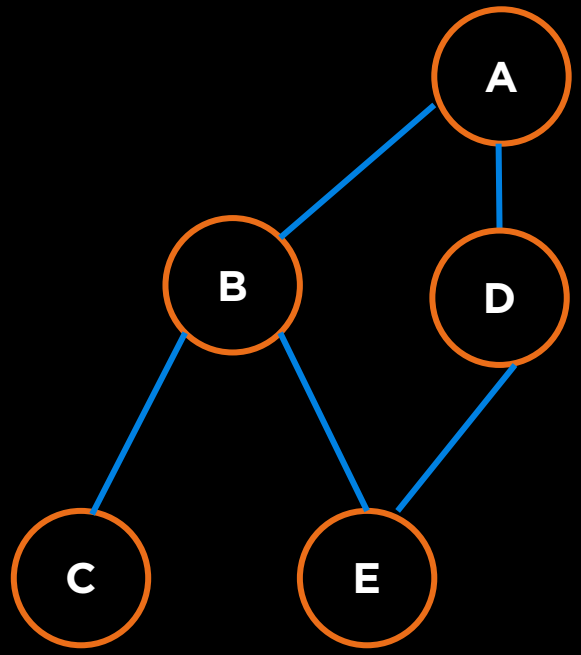
Weighted vs Unweighted

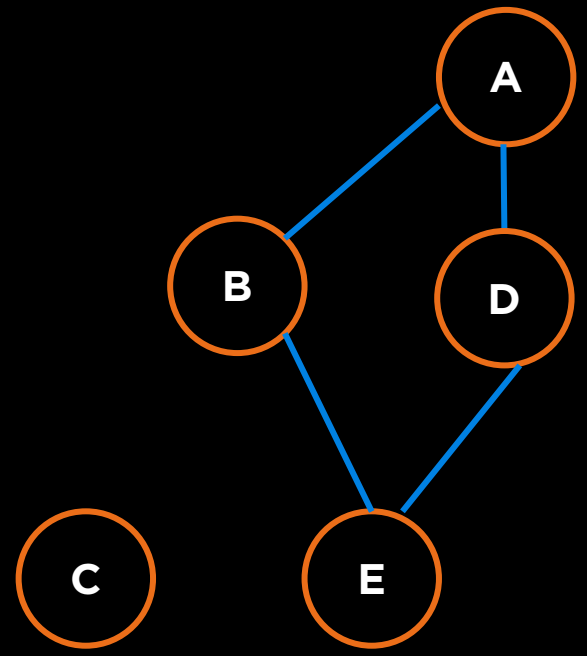




Graph Types

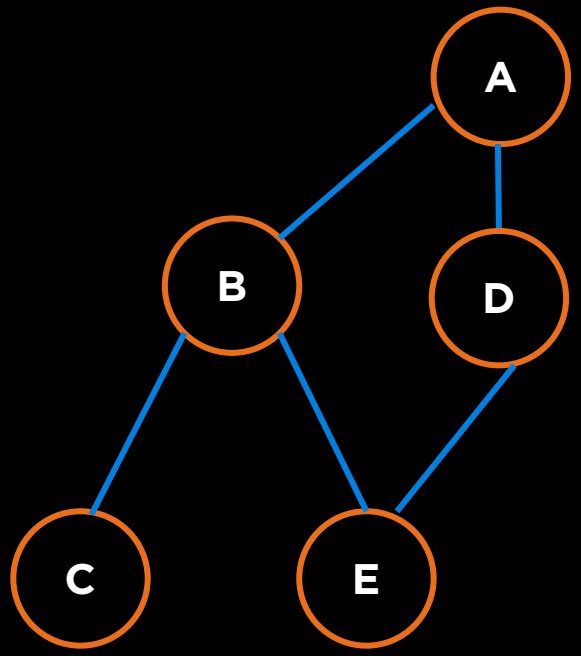
Connected vs Unconnected



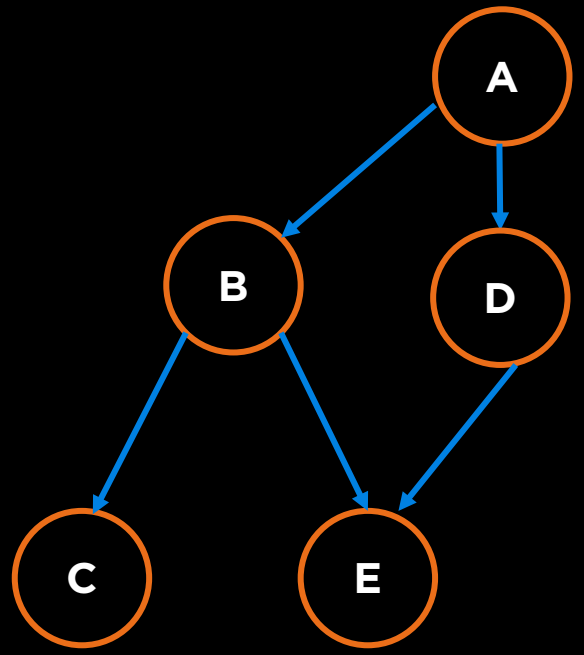


Graph Types

Cyclic vs Acyclic



Cyclic

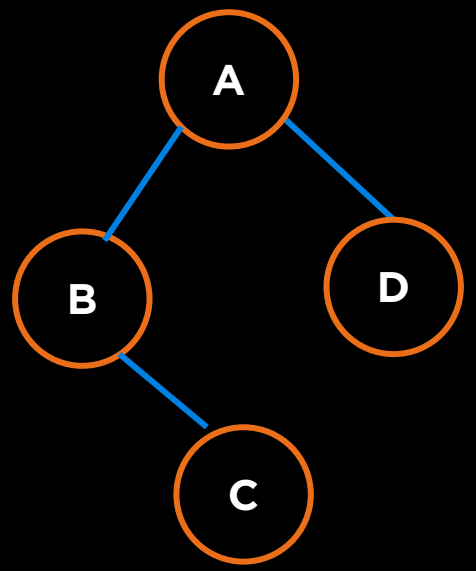


Acyclic

Graph Types

Dense vs Sparse

- The density of a graph is the ratio of $|E|$ to $|V|^2$
- We can assume that $|E|$ is
  - ~ $|V|^2$ for a dense graph [Density ~ 1]
  - ~ $|V|$ for a sparse graph [Density ~ 0]



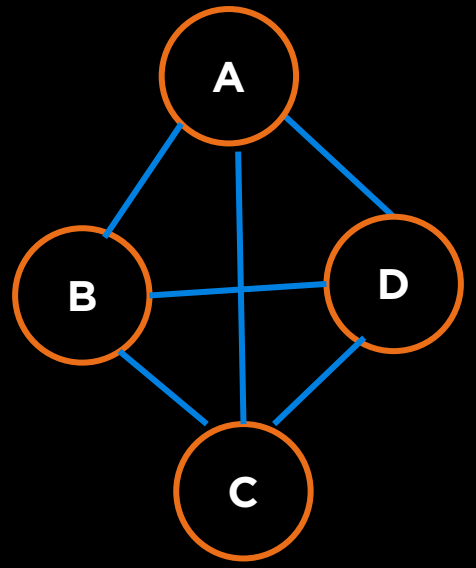
Sparse

Directed Graphs:

$0 <= |E| <= |V|(|V|-1)$

Undirected Graphs:

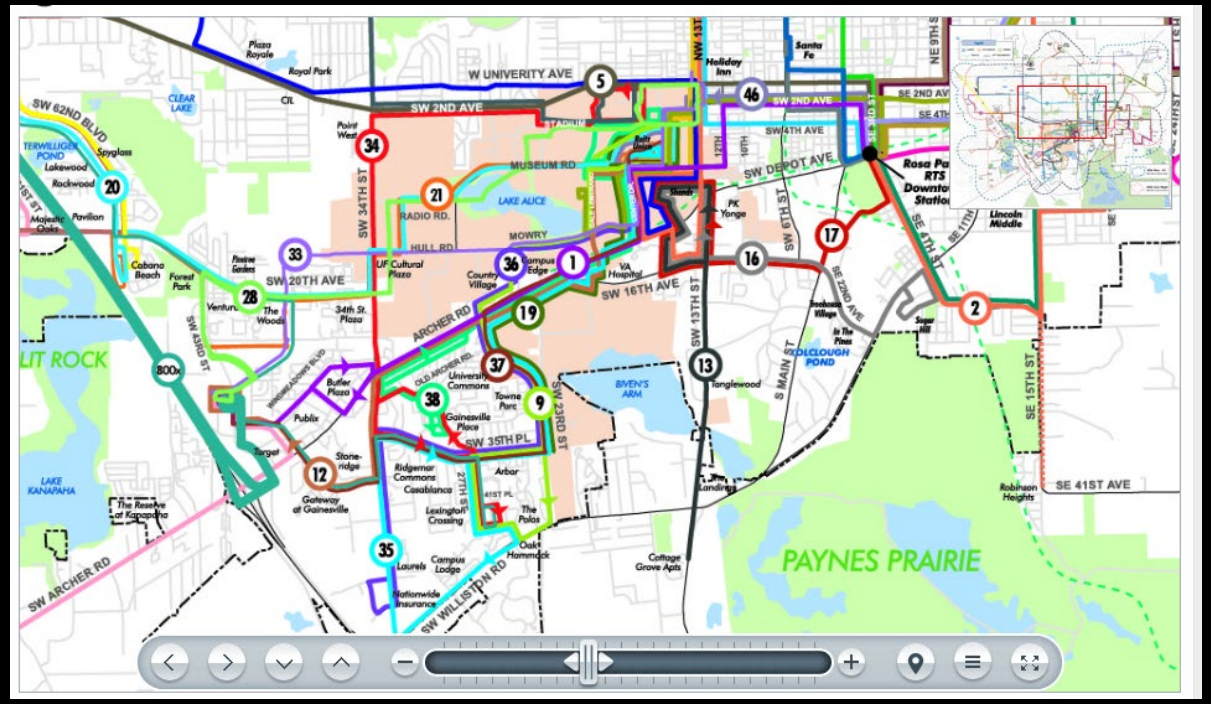
$0 <= |E| <= |V|(|V|-1)/2$



Dense

Graphs

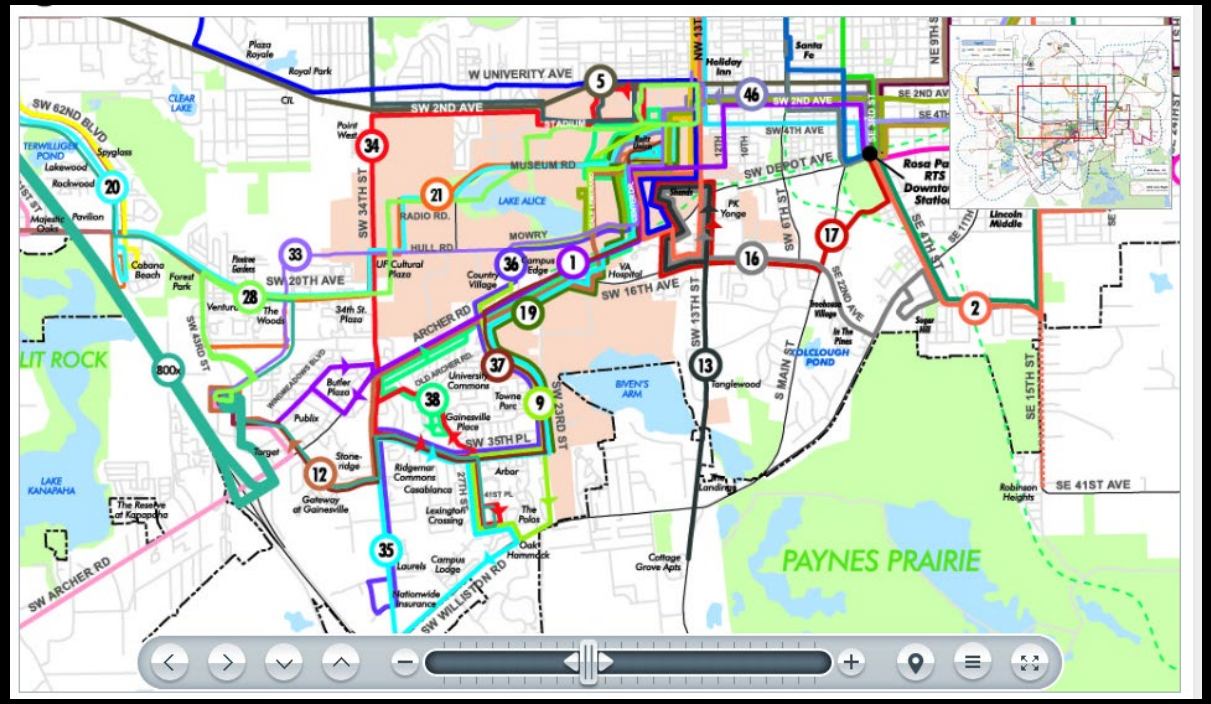
Example



Undirected
Directed
Cyclic
Connected

Graphs

Example



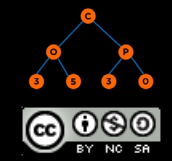
Undirected
Directed
Cyclic
Connected

Graph Problems

Common Examples:

- Social Networks
- World Wide Web
- Maps

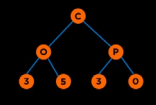
Weighted? Directed?



Graph Problems

Common Examples:

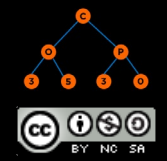
- Social Networks (Unweighted, Undirected)
- World Wide Web (Unweighted, Directed)
- Maps (Weighted, Undirected)



Graph Problems

There are lots of interesting questions we can ask about a graph:

- What is the shortest route from S to T? What is the longest without cycles?
- Are there cycles?
- Is there a tour you can take that only uses each node (station) exactly once?
- Is there a tour that uses each edge exactly once?

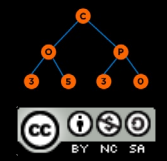


Graph Problems

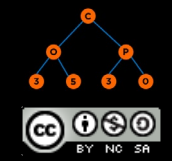
Some well-known graph problems and their common names:

- s-t Path. Is there a path between vertices s and t?
- Connectivity. Is the graph connected, i.e. is there a path between all vertices?
- Biconnectivity. Is there a vertex whose removal disconnects the graph?
- Shortest s-t Path. What is the shortest path between vertices s and t?
- Cycle Detection. Does the graph contain any cycles?
- Euler Tour. Is there a cycle that uses every edge exactly once?
- Hamilton Tour. Is there a cycle that uses every vertex exactly once?
- Planarity. Can you draw the graph on paper with no crossing edges?
- Isomorphism. Are two graphs isomorphic (the same graph in disguise)?

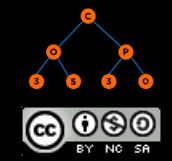
Often can’t tell how difficult a graph problem is without very deep consideration.



Questions

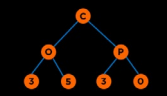


Graph Implementations



Graph API

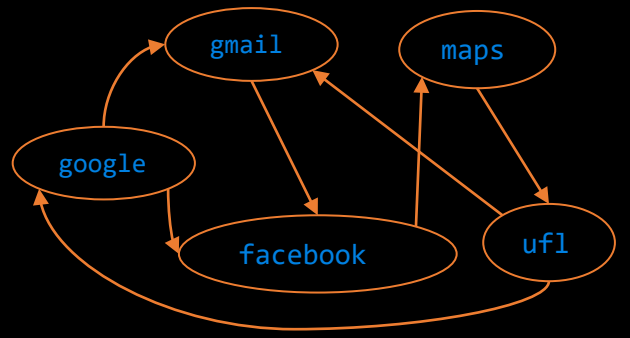
- No common ADT for Graphs
- Graphs were present before Object Oriented Programming
- API must include Graph methods, including their signatures and behaviors
- Defines how Graph client programmers must think.
- An underlying data structure to represent our graphs.
- Our choices can have profound implications on:
  - Runtime
  - Memory usage
  - Difficulty of implementing various graph algorithms

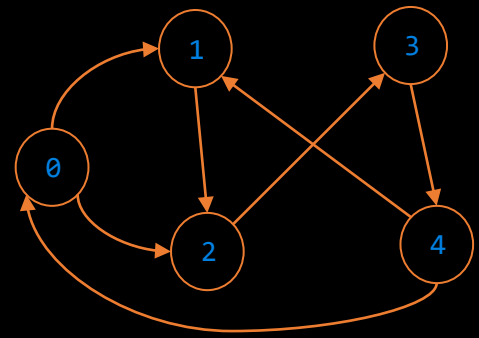


Common Convention

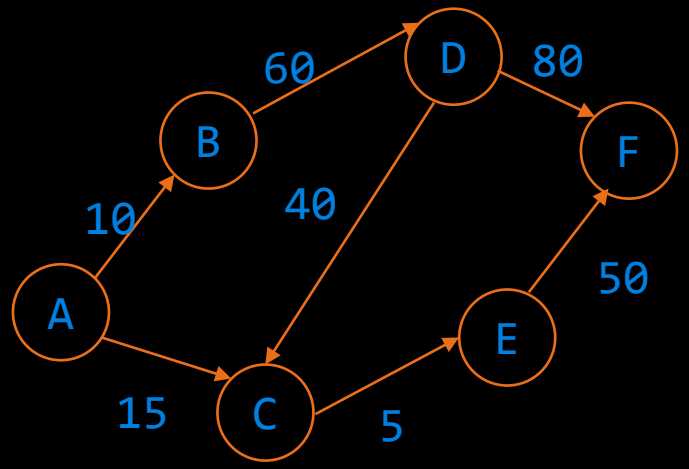
- Map labels to numbers, e.g. If node is called “google.com”, assign it a number, say 0.
- Use a map data structure to achieve this: map<string, int>
- To find a vertex by label, you’d need to use find the value of the label which is then passed into the operation you are trying to perform.

Label	Graph_Index
google.com	0
gmail.com	1
facebook.com	2
maps.com	3
ufl.edu	4





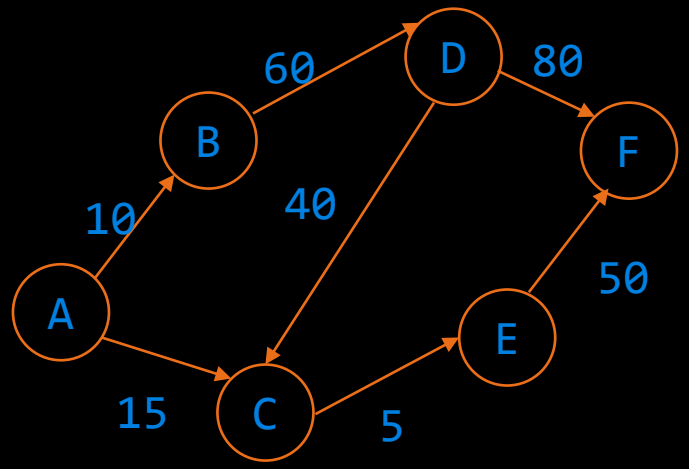
Common Operations



G

- Connectedness
- Neighborhood or Adjacency

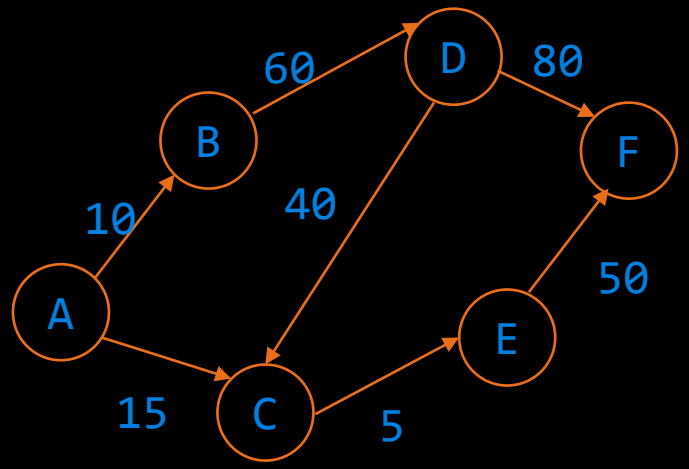
Common Representations



G

- Edge List
- Adjacency Matrix
- Adjacency List

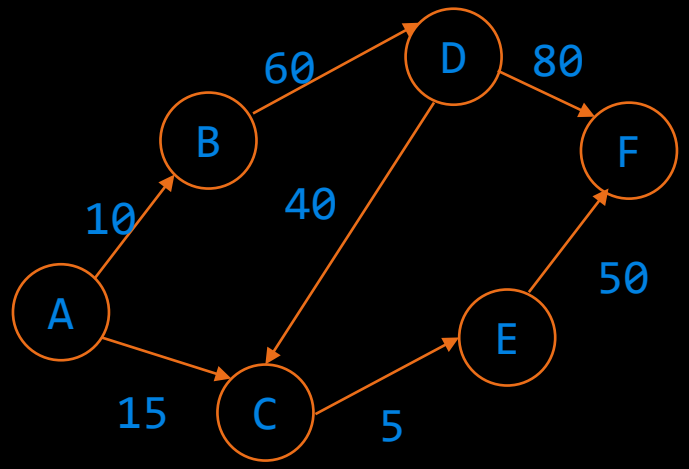
Edge List



G

G = {(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)}

Edge List

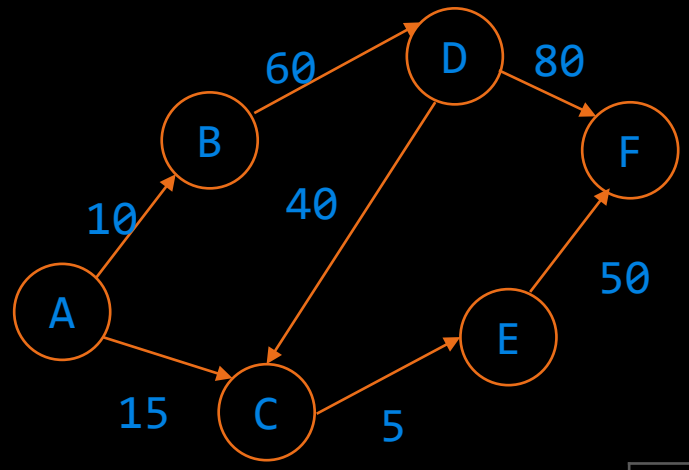


G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

G = {(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)}

Edge List



G = {(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)}

G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

Common Operations:

1. Connectedness

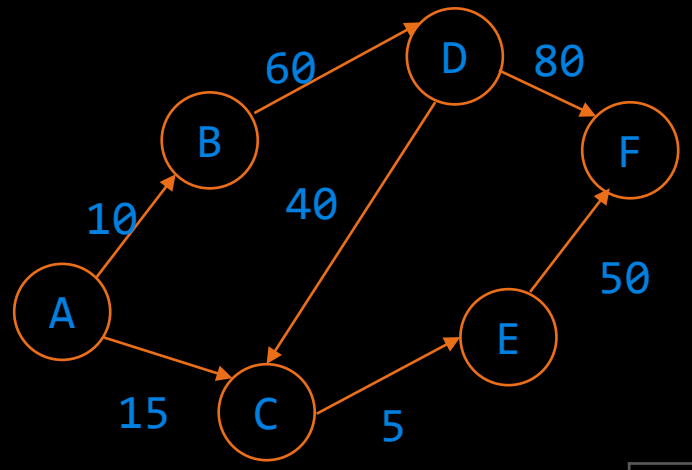
   Is A connected to B?

2. Adjacency

   What are A's adjacent nodes?

Space: ?

Edge List



G = {(A,B), (A,C), (B,D), (D,C), (D,F), (E,F), (C,E)}

G

A	B	10
A	C	15
B	D	60
D	C	40
D	F	80
E	F	50
C	E	5

Common Operations:

1. Connectedness

Is A connected to B?

~ O(E)

2. Adjacency

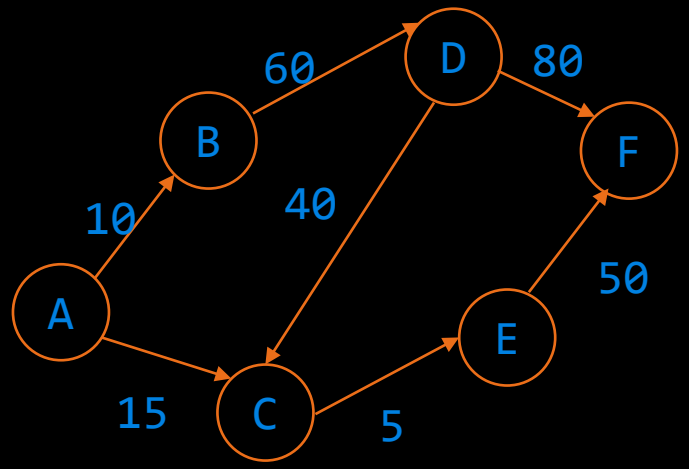
What are A's adjacent nodes?

~ O(E)

O(|E|) ~ O(|V| * |V|)

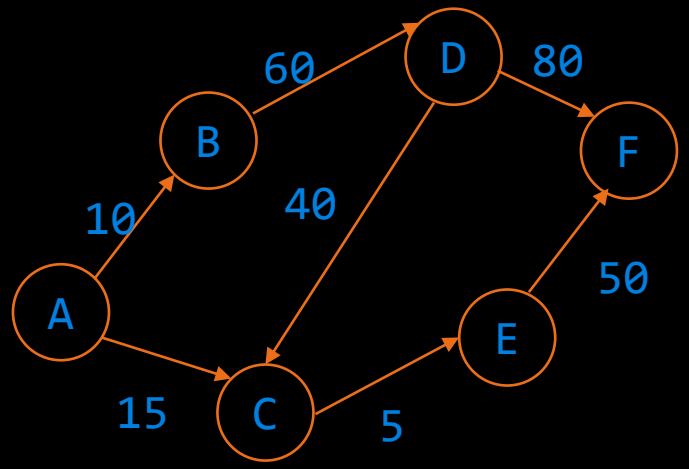
Space: **O(E)**

Adjacency Matrix



G

Adjacency Matrix



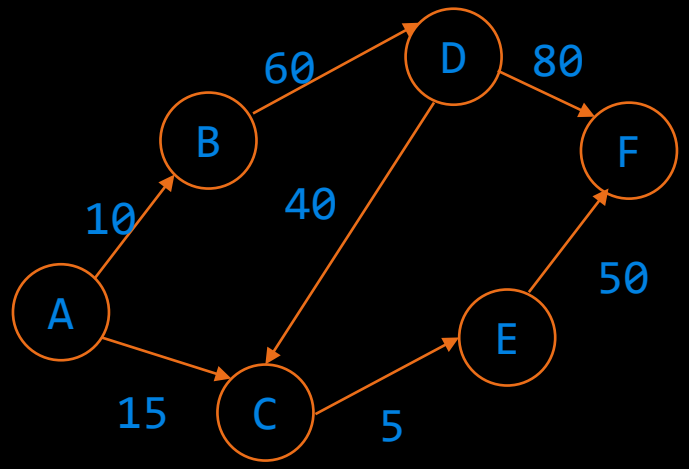
G

Insertion:

G[from][to] = weight; (if there is an edge, “from” -> “to”)

G[from][to] = 0; (otherwise)

Adjacency Matrix



G

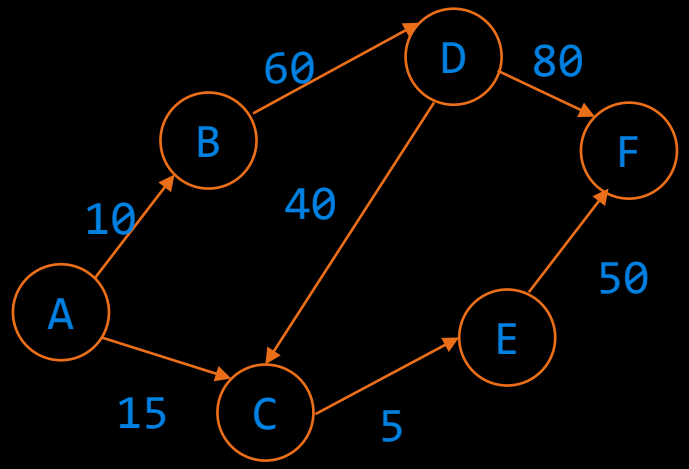
	A	B	C	D	E	F
A	0	10	15	0	0	0
B	0	0	0	60	0	0
C	0	0	0	0	5	0
D	0	0	40	0	0	80
E	0	0	0	0	0	50
F	0	0	0	0	0	0

Insertion:

G[from][to] = weight; (if there is an edge, “from” -> “to”)

G[from][to] = 0; (otherwise)

Adjacency Matrix



G

	A	B	C	D	E	F
A	0	10	15	0	0	0
B	0	0	0	60	0	0
C	0	0	0	0	5	0
D	0	0	40	0	0	80
E	0	0	0	0	0	50
F	0	0	0	0	0	0

Insertion:

G[from][to] = weight; (if there is an edge, “from” -> “to”)

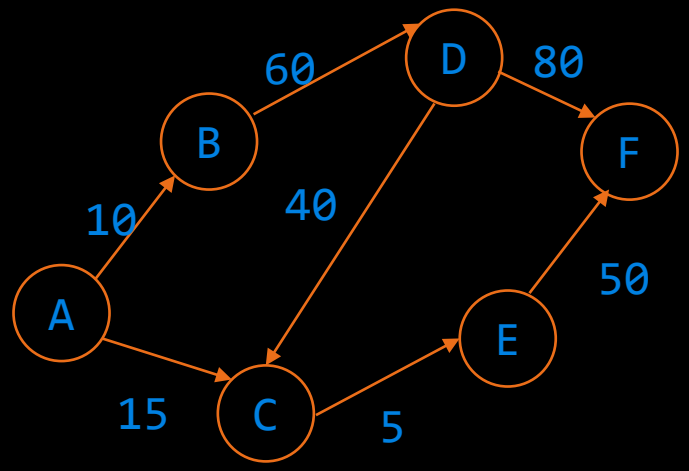
G[from][to] = 0; (otherwise)

Adjacency Matrix Implementation

Insertion:

G[from][to] = weight; (if there is an edge, “from” -> “to”)

G[from][to] = 0; (otherwise)



Input

```
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50
```

G

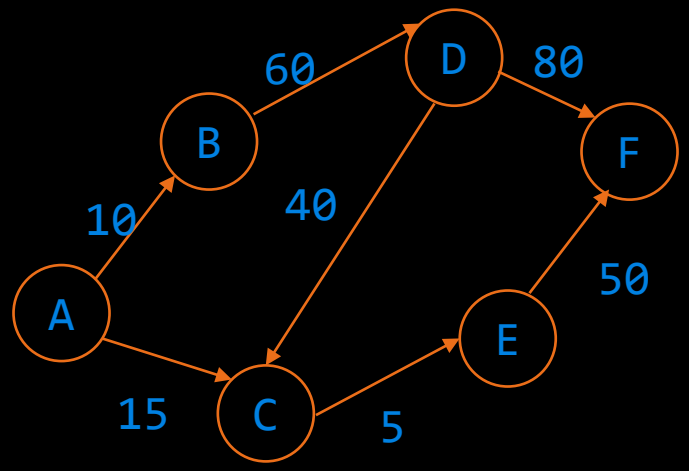
Map

```
A 0
B 1
C 2
D 3
E 4
F 5
```

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

https://www.onlinegdb.com/Hy8M0CnsS

Adjacency Matrix Implementation



Input

```
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50
```

G

Map

```
A 0
B 1
C 2
D 3
E 4
F 5
```

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Insertion:

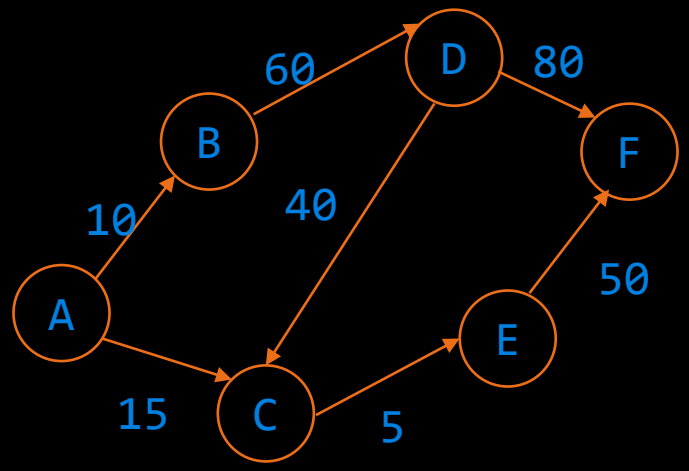
G[from][to] = weight; (if there is an edge, “from” -> “to”)

G[from][to] = 0; (otherwise)

```
01  #include <iostream>
02  #include<map>
03  #define VERTICES 6
04  using namespace std;
05  int main()
06  {
07          int no_lines, wt, j=0;
08          string from, to;
09          int graph [VERTICES][VERTICES] = {0};
10          map<string, int> mapper;
11          cin >> no_lines;
12
13
14
15
16
17
18
19
20
21          return 0;
22  }
```

https://www.onlinegdb.com/Hy8M0CnsS

Adjacency Matrix Implementation



Input

```
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50
```

G

Map

```
A 0
B 1
C 2
D 3
E 4
F 5
```

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Insertion:

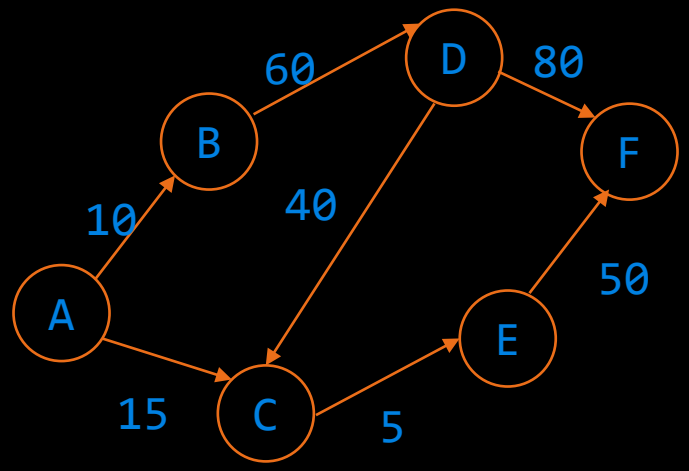
G[from][to] = weight; (if there is an edge, “from” -> “to”)

G[from][to] = 0; (otherwise)

```
01  #include <iostream>
02  #include<map>
03  #define VERTICES 6
04  using namespace std;
05  int main()
06  {
07          int no_lines, wt, j=0;
08          string from, to;
09          int graph [VERTICES][VERTICES] = {0};
10          map<string, int> mapper;
11          cin >> no_lines;
12          for(int i = 0; i < no_lines; i++)
13          {
14                  cin >> from >> to >> wt;
15                  if (mapper.find(from) == mapper.end())
16                          mapper[from] = j++;
17                  if (mapper.find(to) == mapper.end())
18                          mapper[to] = j++;
19                  graph[mapper[from]][mapper[to]] = wt;
20          }
21          return 0;
22  }
```

https://www.onlinegdb.com/Hy8M0CnsS

Adjacency Matrix



Map
A 0
B 1
C 2
D 3
E 4
F 5

G

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Common Operations:

1. Connectedness

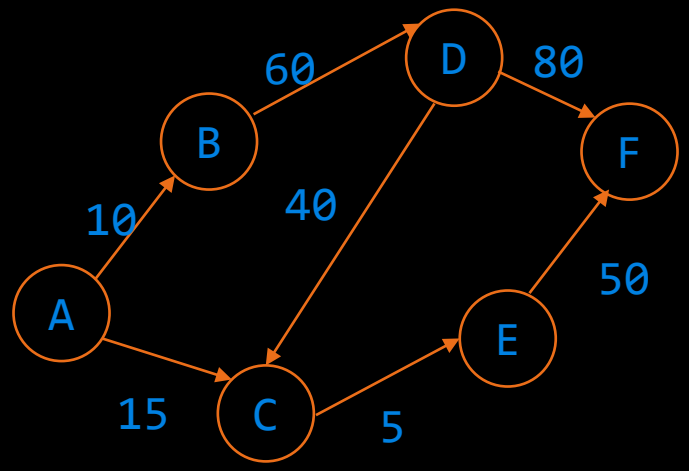
   Is A connected to B?

2. Adjacency

   What are A's adjacent nodes?

Space: ?

Adjacency Matrix



Map
A 0
B 1
C 2
D 3
E 4
F 5

G

	0	1	2	3	4	5
0	0	10	15	0	0	0
1	0	0	0	60	0	0
2	0	0	0	0	5	0
3	0	0	40	0	0	80
4	0	0	0	0	0	50
5	0	0	0	0	0	0

Common Operations:

1. Connectedness

Is A connected to B?

`G["A"]["B"] ~ O(1)`

2. Adjacency

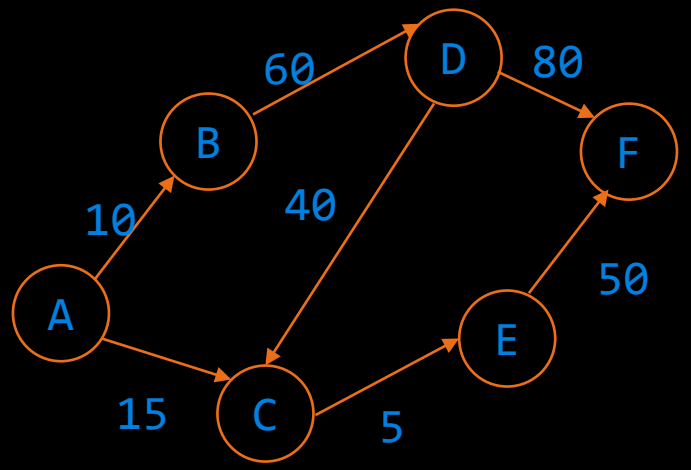
What are A's adjacent nodes?

```
for each element x in G["A"]
    if x ! = 0
```

`~ O(|V|)`

Space: `O(|V| * |V|)`

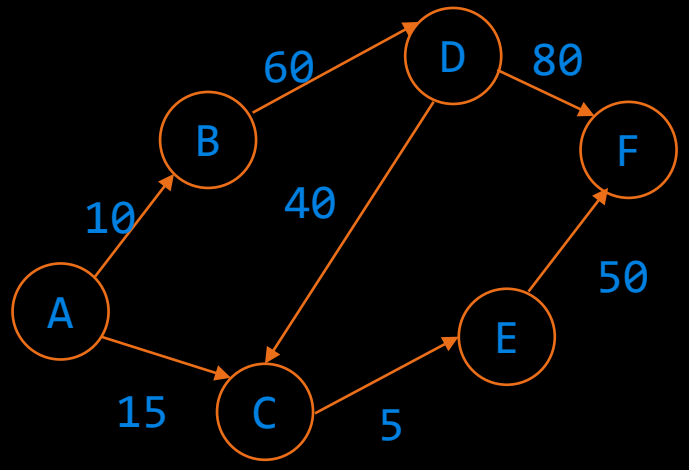
Adjacency Matrix Problem



G

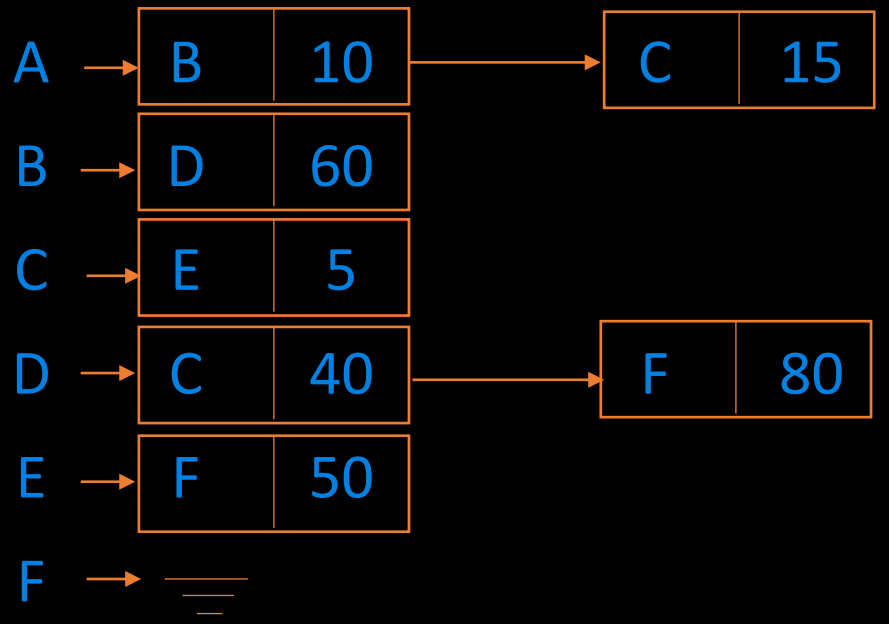
	A	B	C	D	E	F
A	0	10	15	0	0	0
B	0	0	0	60	0	0
C	0	0	0	0	5	0
D	0	0	40	0	0	80
E	0	0	0	0	0	50
F	0	0	0	0	0	0

Adjacency List

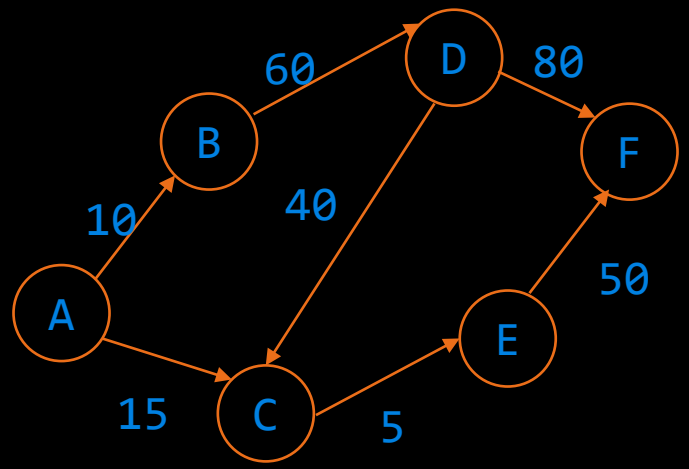


G

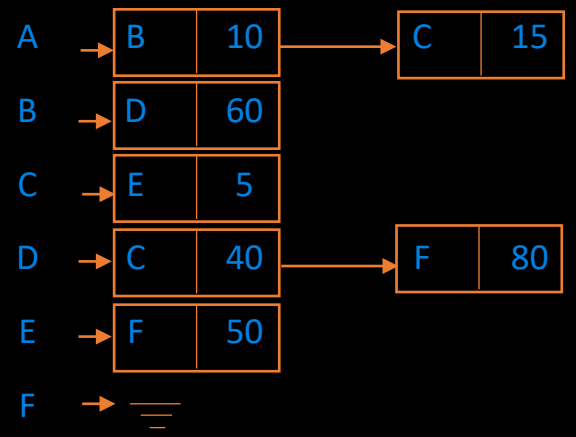
	A	B	C	D	E	F
A	0	10	15	0	0	0
B	0	0	0	60	0	0
C	0	0	0	0	5	0
D	0	0	40	0	0	80
E	0	0	0	0	0	50
F	0	0	0	0	0	0



Adjacency List



G



Common Operations:

1. Connectedness

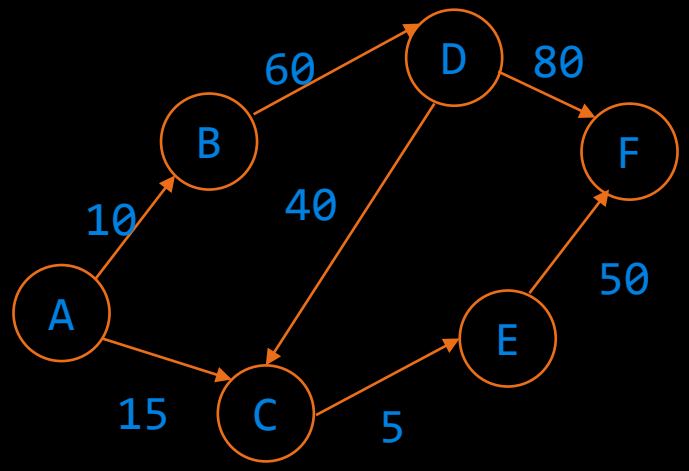
   Is A connected to B?

2. Adjacency

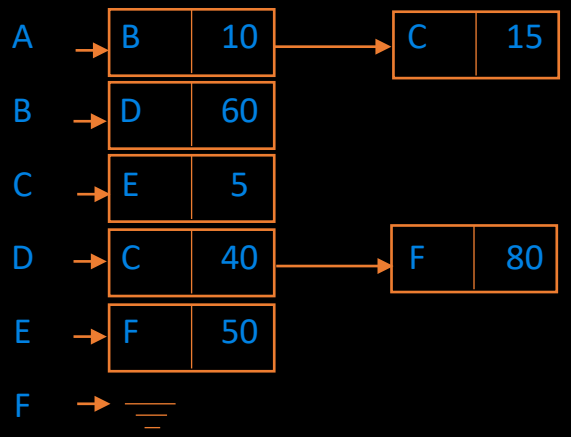
   What are A's adjacent nodes?

Space: ?

Adjacency List



G



Sparse Graph:
Edges ~ Vertices

Common Operations:

1. Connectedness

Is A connected to B?

```
for each element x in G["A"]
    if x ! = 'B'
```

~ O(outdegree|V|)

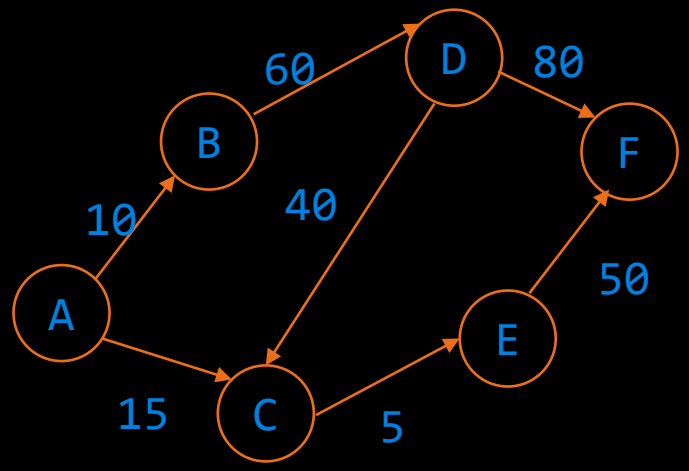
2. Adjacency

What are A's adjacent nodes?

G["A"] ~ O(outdegree|V|)

Space: O(|V| + |E|)

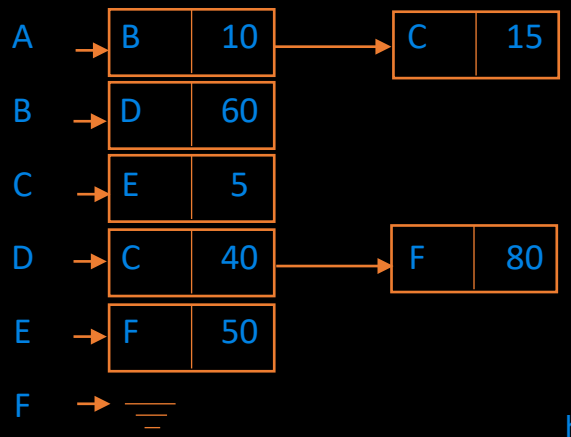
Adjacency List Implementation



Input

```
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50
```

G



Insertion:

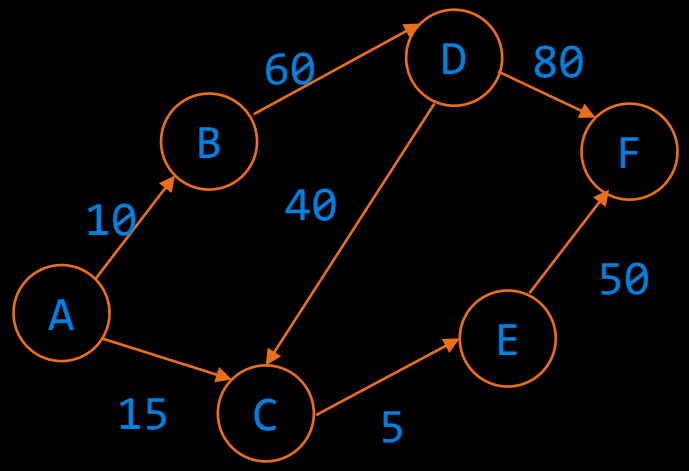
If to or from vertex not present add vertex

Otherwise add edge at the end of the list

```
01  #include <iostream>
02  #include<map>
03  #include<vector>
04  #include<iterator>
05  using namespace std;
06
07  int main()
08  {
09          int no_lines;
10          string from, to, wt;
11          map<string, vector<pair<string,int>>> graph;
12          cin >> no_lines;
13          for(int i = 0; i < no_lines; i++)
14          {
15
16
17
18
19          }
20  }
```

https://onlinegdb.com/HkJq9iFaI

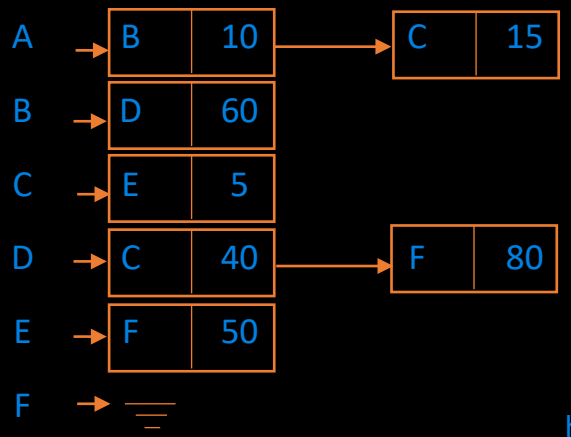
Adjacency List Implementation



Input

```
7
A B 10
A C 15
B D 60
D C 40
C E 5
D F 80
E F 50
```

G



Insertion:

If to or from vertex not present add vertex

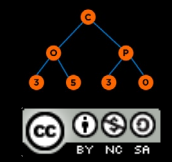
Otherwise add edge at the end of the list

```
01  #include <iostream>
02  #include<map>
03  #include<vector>
04  #include<iterator>
05  using namespace std;
06
07  int main()
08  {
09          int no_lines;
10          string from, to, wt;
11          map<string, vector<pair<string,int>>> graph;
12          cin >> no_lines;
13          for(int i = 0; i < no_lines; i++)
14          {
15                  cin >> from >> to >> wt;
16                  graph[from].push_back(make_pair(to, stoi(wt)));
17                  if (graph.find(to)==graph.end())
18                          graph[to] = {};
19          }
20  }
```

https://onlinegdb.com/HkJq9iFaI

Graph Implementation

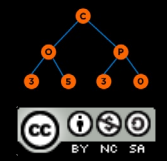
	Edge List	Adjacency Matrix	Adjacency List
Time Complexity: Connectedness	O(E)	O(1)	O(outdegree(V))
Time Complexity: Adjacency	O(E)	O(V)	O(outdegree(V))
Space Complexity	O(E)	O(V*V)	O(V+E)



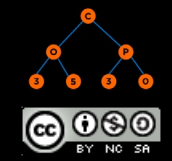
One Graph API

```
class Graph
{
    private:
      //Graph Data Structure

   public:
      Graph();
      Graph(int V); //Creates graph with v vertices
      int V();      //Returns number of vertices
      int E();      //Returns number of edges

      void insertEdge(int from, int to, int weight);
      bool isEdge(int from, int to);
      int getWeight(int from, int to);
      vector<int> getAdjacent(int vertex);
      void printGraph();
};
```

https://stepik.org/lesson/351255/step/1?unit=335132

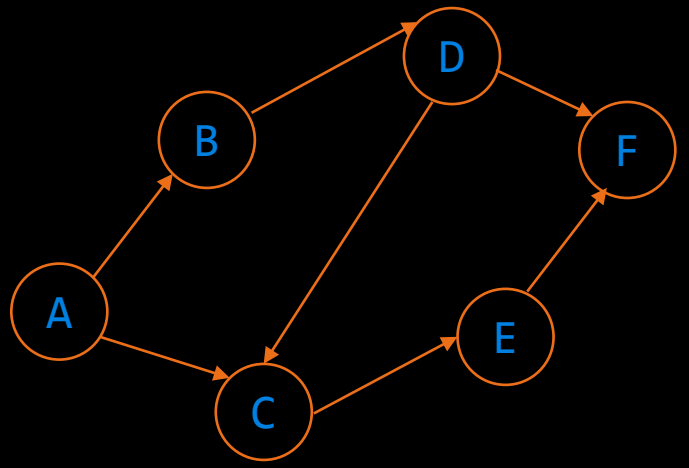


Graph Traversal



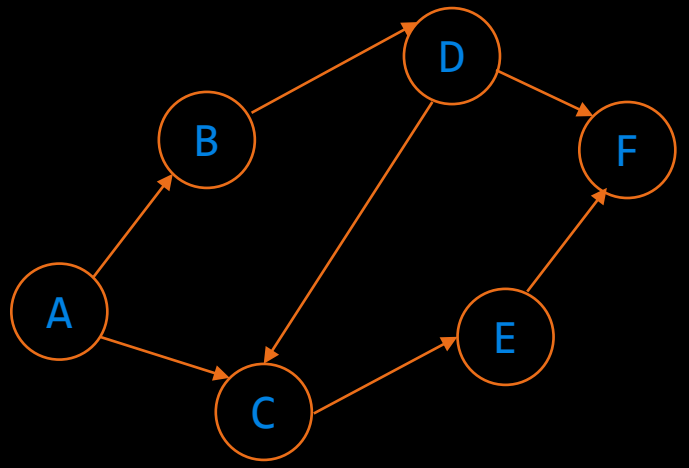
Breadth First Search

```
Algorithm for Breadth-First Search

1.     Take an arbitrary start vertex, mark it identified, and place it in a queue.
2.     while the queue is not empty
3.         Take a vertex, u, out of the queue and visit u.
4.         for all vertices, v, adjacent to this vertex, u
5.               if v has not been identified or visited
6.                     Mark it identified
7.                     Insert vertex v into the queue.
8.         We are now finished visiting u.
```



Breadth First Search

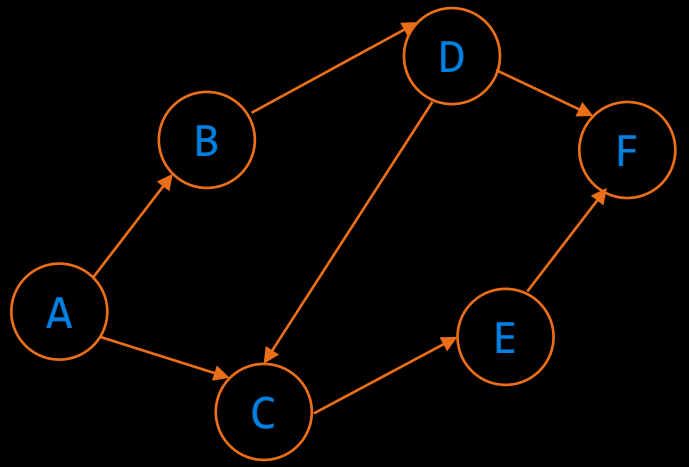
```
Algorithm for Breadth-First Search

1.     Take an arbitrary start vertex, mark it identified, and place it in a queue.
2.     while the queue is not empty
3.          Take a vertex, u, out of the queue and visit u.
4.          for all vertices, v, adjacent to this vertex, u
5.               if v has not been identified or visited
6.                    Mark it identified
7.                    Insert vertex v into the queue.
8.          We are now finished visiting u.
```



Valid BFS: A, B, C, D, E, F

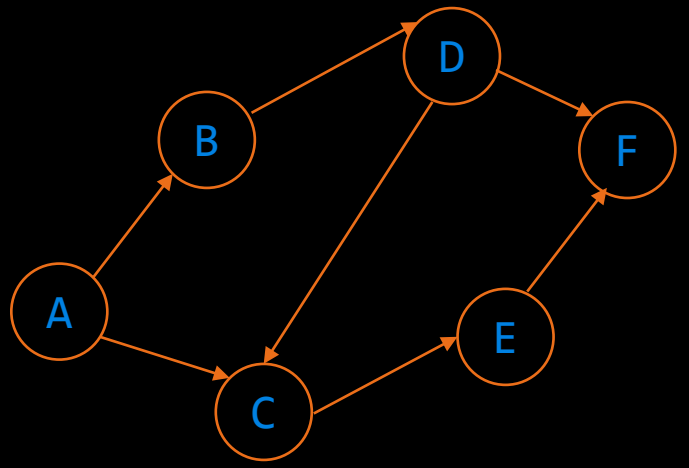
Depth First Search

```
Algorithm for Depth-First Search

1.    Take an arbitrary start vertex, mark it visited, and place it in a stack.
2.    while the stack is not empty
3.         the item on top of the stack is u
4.          if there is a vertex, v, adjacent to this vertex, u, that has not been visited
5.                Mark v visited
6.                 Push vertex v onto the top of the stack
7.         else
8.                pop stack
```



Depth First Search

```
Algorithm for Depth-First Search

1.    Take an arbitrary start vertex, mark it visited, and place it in a stack.
2.    while the stack is not empty
3.         the item on top of the stack is u
4.          if there is a vertex, v, adjacent to this vertex, u, that has not been visited
5.                Mark v visited
6.                 Push vertex v onto the top of the stack
7.         else
8.                pop stack
```

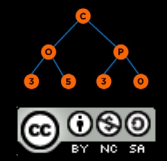


Valid DFS: A, B, D, C, E, F

One Graph API

```
class Graph
{
    private:
      //Graph Data Structure

   public:
      Graph();
      Graph(int V); //Creates graph with v vertices
      int V();      //Returns number of vertices
      int E();      //Returns number of edges

      void insertEdge(int from, int to, int weight);
      bool isEdge(int from, int to);
      int getWeight(int from, int to);
      vector<int> getAdjacent(int vertex);
      void printGraph();
};
```

```
class Path
{
   public:
      //find all paths from g
      Path(Graph g, int s);

      //is there a path from s to v
      bool hasPathTo(int s);

      //path from s to v
      vector<int> pathTo(int s);
}
```

Questions