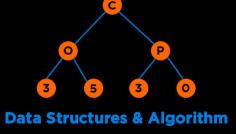
Sorting



Categories of Data Structures

Linear Ordered

Non-linear Ordered

Not Ordered

Lists

Trees

Sets

Stacks

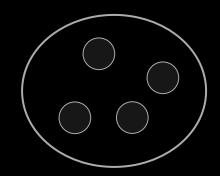
Graphs

Tables/Maps

Queues









Problem (Sort)

Input: Unordered Collection of size n, C_i [0....n-1]

Output: Ordered Collection of size n, C_o [0.....n-1]

Example: C_o for ascending sort

 $C_o[0] <= C_o[1] <= ... <= C_o[n-1]$









Premise:

- Find the smallest/largest element, e₁ in a Collection, C_i
- Move this element, e₁ to its correct position
- Find the next smallest/largest element, e₂ in C_i
- Move this element, e₂ to its correct position

:

Repeat this entire process C_i·size() - 1 times



Example:

Initial array





Example:

Initial array



1st pass



Example: Initial array 1st pass 2nd pass

Example: Initial array 1st pass 2nd pass 3rd pass



Example: Initial array 1st pass 2nd pass 3rd pass 4th pass



Example: Initial array 1st pass 2nd pass 3rd pass 4th pass



Selection Sort Code

```
void selectionSort(int array[], int size)
02
03
        for (int i = 0; i < size - 1; i++)
04
05
             int min_index = i;
             for (int j = i + 1; j < size; j++)
06
07
                  if (array[j] < array[min_index])</pre>
08
                       min_index = j;
09
10
             // put min at the correct position
11
             swap(&array[min_index], &array[i]);
12
13
14
```

Time Complexity

For very large n we can ignore all but the significant term in the expression, so the number of

- comparisons is $O(n^2)$
- exchanges is O(n)

An $O(n^2)$ sort is called a *quadratic sort*

Selection Sort Complexity

	Selection Sort	Bubble Sort	Insertion Sort
Worst Case	0(n²)		
Average Case	O(n²)		
Best Case	O(n²)		
Space	0(1)		



Premise:

- Swap adjacent elements, e_i and e_{i+1} in a Collection, C_i if they are out of order
- Repeat swapping till you reach the end of the Collection to bubble up the largest element after each iteration

Repeat this entire process C_i.size() - 1 times stopping at C_i.size() - i after ith iteration

Example:

Initial array





Initial array	29	10	14	37	13
1st pass	10	14	29	13	37

Initial array	29	10	14	37	13
1st pass	10	14	29	13	37
2 nd pass	10	14	13	29	37

Initial array	29	10	14	37	13
1st pass	10	14	29	13	37
2 nd pass	10	14	13	29	37
3 rd pass	10	13	14	29	37

nitial array	29	10	14	37	13
1st pass	10	14	29	13	37
2 nd pass	10	14	13	29	37
3 rd pass	10	13	14	29	37
4 th pass	10	13	14	29	37



Initial array	29	10	14	37	13
1st pass	10	14	29	13	37
2 nd pass	10	14	13	29	37
3 rd pass	10	13	14	29	37
4 th pass	10	13	14	29	37
Final array	10	13	14	29	37



Bubble Sort Code

```
void bubbleSort(int array[], int size)
02
      for (int i = 0; i < size - 1; i++)
03
04
        int swapped = 0;
05
        for (int j = 0; j < size - i - 1; ++j)
06
07
          if (array[j] > array[j + 1])
08
10
            int temp = array[j];
            array[j] = array[j + 1];
11
12
            array[j + 1] = temp;
13
            swapped = 1;
14
15
16
        // If there is no swapping in the last swap, then the array is already sorted.
        if (swapped == 0)
17
18
          break;
19
20
```

Time Complexity

In the worst case,

- comparisons is $O(n^2)$
- exchanges is $O(n^2)$

Compared to selection sort with its $O(n^2)$ comparisons and O(n) exchanges, bubble sort usually performs worse

Bubble Sort Complexity

	Selection Sort	Bubble Sort	Insertion Sort
Worst Case	O(n²)	O(n²)	
Average Case	O(n ²)	O(n²)	
Best Case	O(n ²)	0(n)	
Space	0(1)	0(1)	



Premise:

- Keeps a track of two regions: Sorted and Unsorted
- Initially, the sorted region has one element
- Insert the first element in the unsorted region in the correct place in the sorted region

Repeat this entire process till there are no more elements in unsorted region

Example:

Initial array





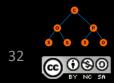


Initial array	29	10	14	37	13
1st pass	29	10	14	37	13
2 nd pass	10	29	14	37	13

Initial array	29	10	14	37	13
1st pass	29	10	14	37	13
2 nd pass	10	29	14	37	13
3 rd pass	10	14	29	37	13











Insertion Sort Code

```
void insertionSort(int array[], int size)
02
03
      for (int i = 1; i < size; i++)
04
05
       int key = array[i];
06
       int j = i-1;
07
       // Compare key with each element in sorted till smaller value is found
98
       while (key < array[j] && j >= 0)
09
10
11
         array[j+1] = array[j];
12
         j--;
13
       array[j+1] = key;
14
15
16
```

Insertion Sort Pseudocode

Time Complexity

In the worst case,

- comparisons is $O(n^2)$
- exchanges is $O(n^2)$

Insertion Sort Complexity

	Selection Sort	Bubble Sort	Insertion Sort
Worst Case	O(n²)	O(n²)	O(n²)
Average Case	O(n²)	O(n²)	O(n²)
Best Case	O(n ²)	0(n)	0(n)
Space	0(1)	0(1)	0(1)

Resources

- https://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html
- https://www.programiz.com/dsa
- https://www.youtube.com/user/AlgoRythmics/videos
- https://www.toptal.com/developers/sorting-algorithms

Questions



Premise:

- Merge sort splits the array in half, sorts the two smaller halves, then merges the two sorted halves together.
- Divide the array to be sorted into smaller subarrays till you reach a size of 1
- In the Conquer step, sort the two subarrays
- In the Combine step, combine two sorted arrays

i

Repeat this till you merge all elements in one array

Example:

Initial array

6 5 22 10 9 1

Example:

Initial array 6 5 22 10 9 1 6 5 22 10 9 1

Divide

Example:

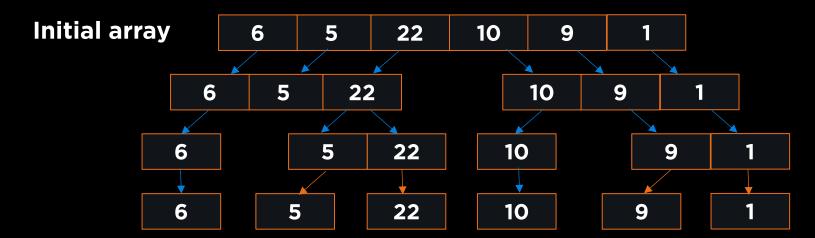
Initial array 6 5 22 10 9 1 6 5 22 10 9 1 6 5 22 10 9 1

Divide





Example:



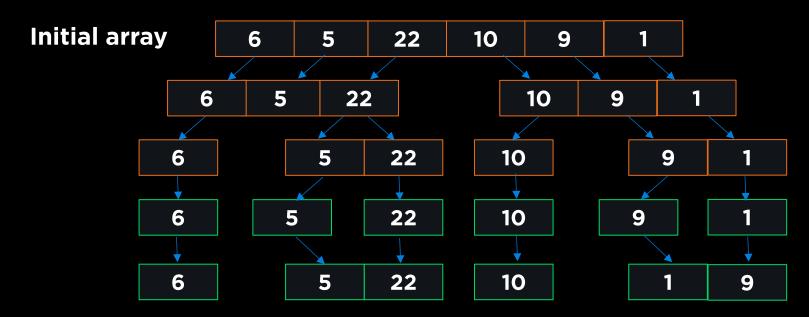
Divide

Example:

Initial array

Conquer

Example:

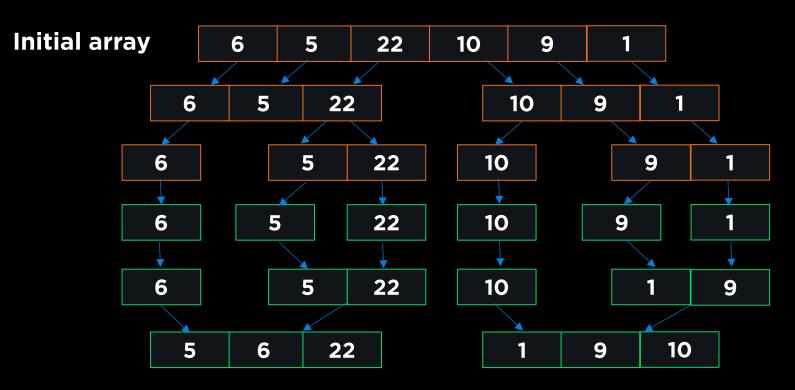


Combine





Example:



Combine

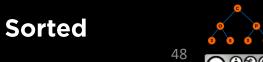




Example:

Initial array

Combine



Example:





Merge Sort Pseudocode

```
mergeSort(int [] numbers, int start, int end)
{
    if (start<end) //base case is start = end and sorting an array of 1
    {
        middle = (start+end)/2;
        mergeSort(numbers, start, middle);
        mergeSort(numbers, middle+1, end);
        merge(numbers, start, middle, end);
    }
}</pre>
```

mergeSort(A, 0, length(A)-1)

Merge Algorithm

- 1. Access the first item from both sequences.
- 2. while not finished with either sequence
- Compare the current items from the two sequences, copy the smaller current item to the output sequence and access the next item from the input sequence whose item was copied.
- 4. Copy any remaining items from the first sequence to the output sequence.
- 5. Copy any remaining items from the second sequence to the output sequence.



```
void mergeSort(int arr[], int left, int right)
1.
2.
       if (left < right)</pre>
4.
         // m is the point where the array is divided into two subarrays
         int mid = left + (right - left) / 2;
6.
          mergeSort(arr, left, mid);
8.
          mergeSort(arr, mid + 1, right);
9.
10.
         // Merge the sorted subarrays
          merge(arr, left, mid, right);
11.
12.
13.
```

mergeSort(A, 0, length(A)-1)



```
14. // Merge two subarrays from arr
15. void merge(int arr[], int left, int mid, int right)
17.
      // Create X ← arr[left..mid] & Y ← arr[mid+1..right]
18.
      int n1 = mid - left + 1;
      int n2 = right - mid;
19.
20.
      int X[n1], Y[n2];
21.
22.
      for (int i = 0; i < n1; i++)
23.
        X[i] = arr[left + i];
      for (int j = 0; j < n2; j++)
25.
        Y[j] = arr[mid + 1 + j];
27.
    // Merge the arrays X and Y into arr
28.
      int i, j, k;
      i = 0;
29.
30.
      i = 0;
      k = left;
```

```
32. while (i < n1 && j < n2)
33. {
34.    if (X[i] <= Y[j])
35.    {
36.        arr[k] = X[i];
37.        i++;
38.    }
39.    else
40.    {
41.        arr[k] = Y[j];
42.        j++;
43.    }
44.    k++;
45. }</pre>
```

```
46. // When we run out of elements
   in either X or Y append the rema
  ining elements
      while (i < n1)
49.
        arr[k] = X[i];
        i++;
51.
        k++;
52.
      while (j < n2)
        arr[k] = Y[j];
        j++;
        k++;
58.
59.
```

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```

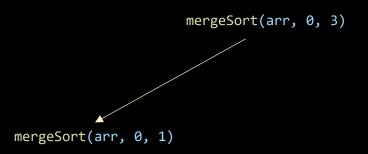
arr

15	2	7	0
0	1	2	3

mergeSort(arr, 0, 3)

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```





```
15 2 7 0
```

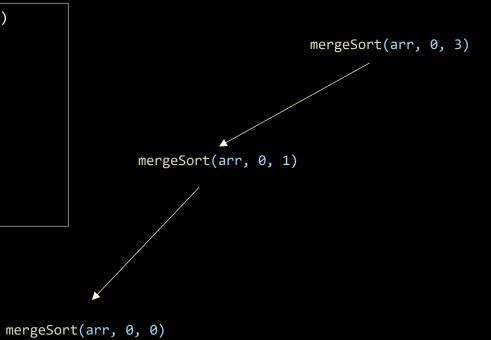
2

3

arr

1

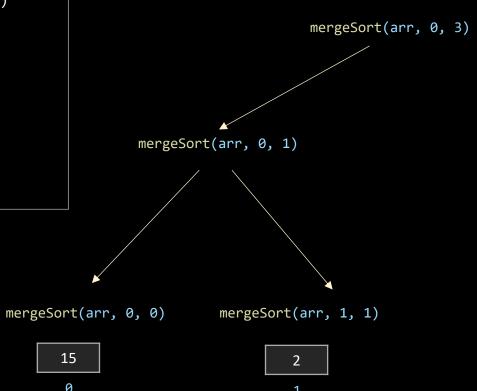
```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```



0

15	2	7	0
0	1	2	3

```
void mergeSort(int arr[], int left, int right)
         if (left < right)</pre>
           int mid = left + (right - left) / 2;
           mergeSort(arr, left, mid);
           mergeSort(arr, mid + 1, right);
           // Merge the sorted subarrays
           merge(arr, left, mid, right);
11.
12.
```



15 2 7 0

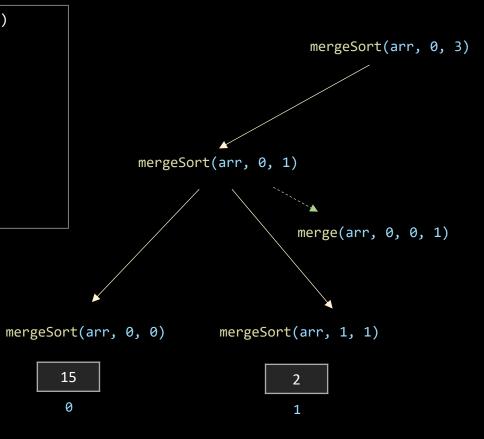
2

3

1

arr

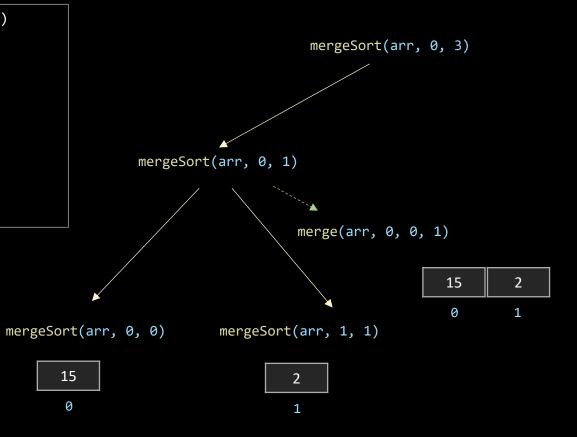
```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.       // Merge the sorted subarrays
10.       merge(arr, left, mid, right);
11.    }
12. }</pre>
```



15	2	7	0
0	1	2	3

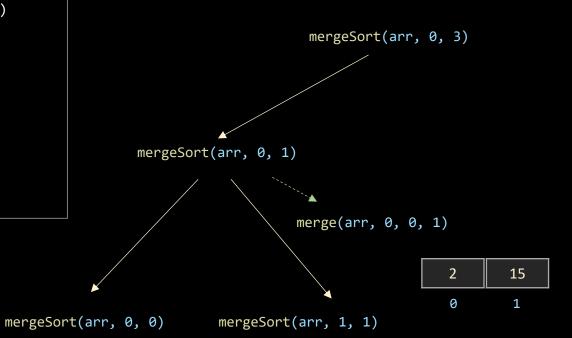
```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.

9.       // Merge the sorted subarrays
10.       merge(arr, left, mid, right);
11.    }
12. }</pre>
```



15	2	7	0
0	1	2	3

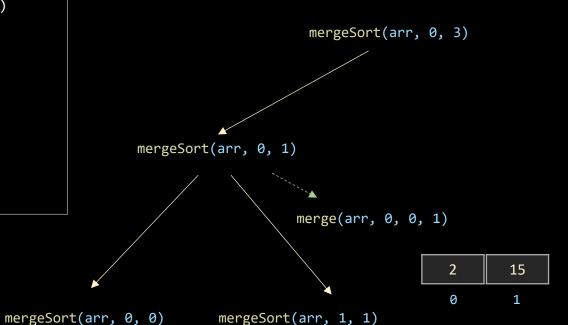
```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```





arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```



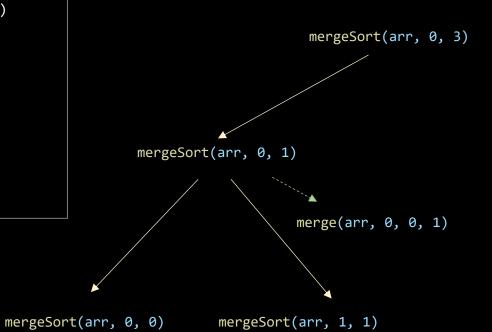
2	15	7	0
0	1	2	3





arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.    // Merge the sorted subarrays
10.      merge(arr, left, mid, right);
11.    }
12. }</pre>
```



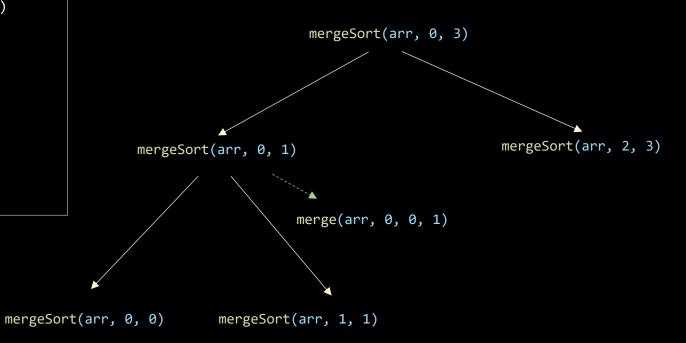
2	15	7	0
0	1	2	3





arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```

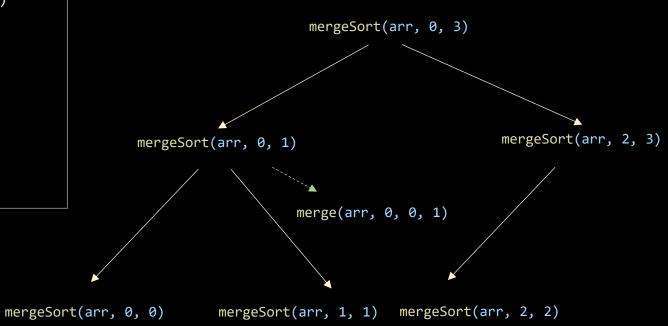


2	15	7	0
0	1	2	3



arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```



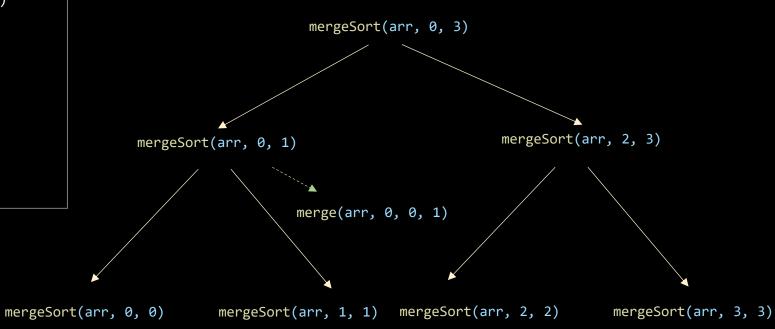
arr

2	15	7	0
0	1	2	3



arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```



arr

2	15	7	0
0	1	2	3

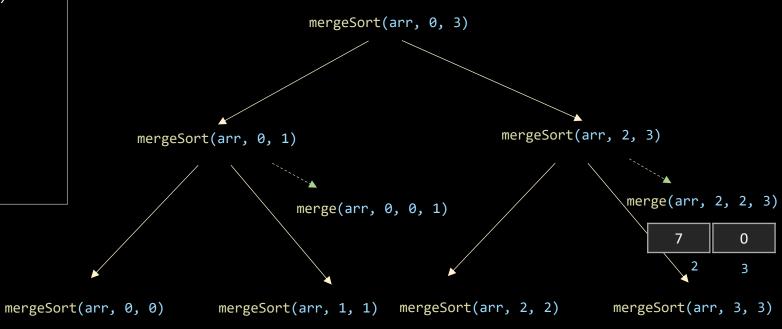
7





arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```



arr

2	15	7	0
0	1	2	3

7

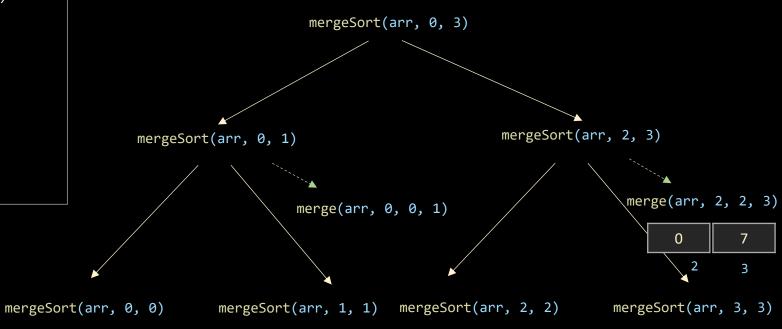


0



arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```



arr

2	15	0	7
0	1	2	3

7

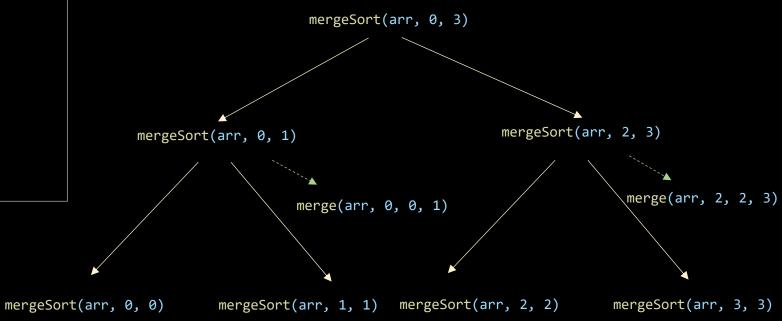


0



arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.       // Merge the sorted subarrays
10.       merge(arr, left, mid, right);
11.    }
12. }</pre>
```



arr

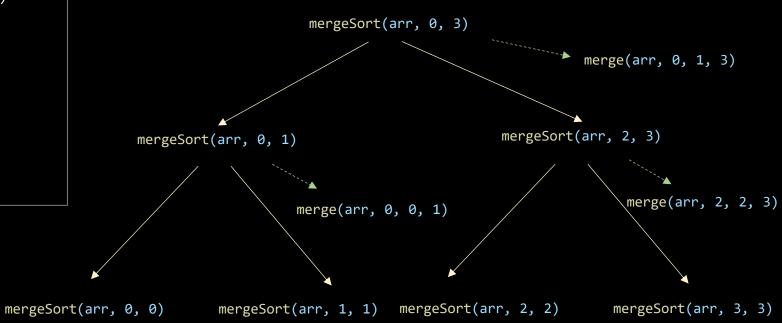
2	15	0	7
0	1	2	3

7



arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.       // Merge the sorted subarrays
10.       merge(arr, left, mid, right);
11.    }
12. }</pre>
```



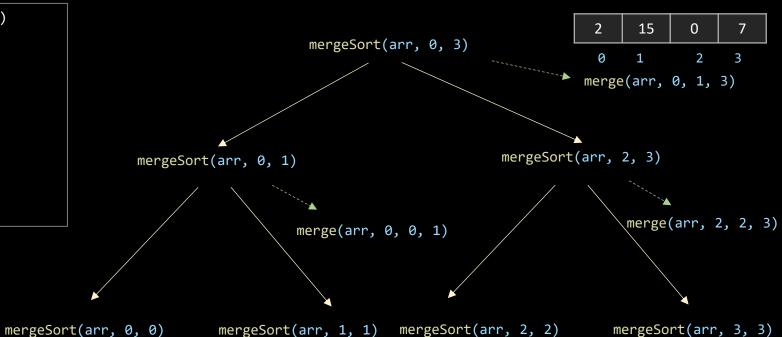
2	15	0	7
0	1	2	3





arr

```
1. void mergeSort(int arr[], int left, int right)
2. {
3.    if (left < right)
4.    {
5.       int mid = left + (right - left) / 2;
6.       mergeSort(arr, left, mid);
7.       mergeSort(arr, mid + 1, right);
8.
9.    // Merge the sorted subarrays
10.    merge(arr, left, mid, right);
11.    }
12. }</pre>
```



arr

2	15	0	7
0	1	2	3

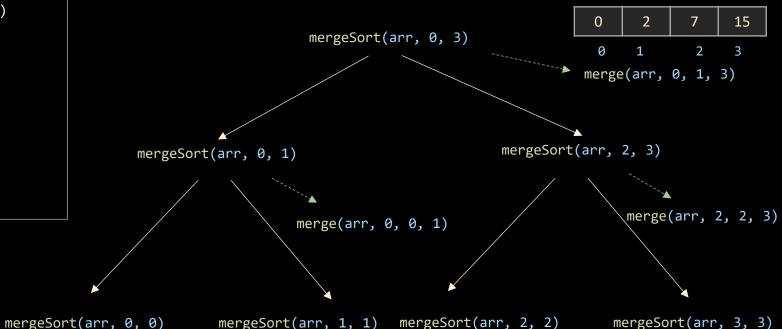
mer gesor e(arr, s, s,





arr

```
void mergeSort(int arr[], int left, int right)
         if (left < right)</pre>
           int mid = left + (right - left) / 2;
           mergeSort(arr, left, mid);
           mergeSort(arr, mid + 1, right);
           // Merge the sorted subarrays
           merge(arr, left, mid, right);
11.
12.
```



arr

0	2	7	15
0	1	2	3

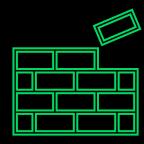
mergeSort(arr, 1, 1) mergeSort(arr, 2, 2) mergeSort(arr, 3, 3)



Merge Sort Time Complexity

	Merge Sort	
Worst Case	O(n*log n)	
Average Case	O(n*log n)	
Best Case	O(n*log n)	
Space	0(n)	

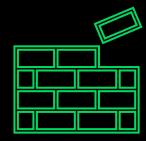
Quick Sort



Premise:

- Quicksort rearranges the array into two parts called partitioning
- A pivot is selected, and the following is executed:
 - All the elements in the left subarray are less than or equal to the pivot
 - All the elements in the right subarray are larger than the pivot

The process is repeated until the array is sorted



Algorithm for Quicksort

```
if first < last then
```

- Partition the elements in the subarray first . . . last so that the pivot value is in its correct place (subscript pivIndex)
- Recursively apply quicksort to the subarray first . . . pivIndex 1
- Recursively apply quicksort to the subarray pivIndex + 1 . . . last

7	4	9	3	2	8	6	5



Example:

Initial array

7	4	9	3	2	8	6	5
---	---	---	---	---	---	---	---

- Define the pivot value as the contents of table[first].
- Initialize up to first and down to last.
- do
- Increment up until up selects the first element greater than the pivot value or up has reached last.
- Decrement down until down selects the first element less than or equal to the pivot value or down has reached first.
- if up < down then
- Exchange table[up] and table[down].
- while up is to the left of down
- Exchange table[first] and table[down].
- Return the value of down to pivIndex.



Example:

Initial array

	Up						Down
7	4	9	3	2	8	6	5

Pivot

- Define the pivot value as the contents of table[first].
- Initialize up to first and down to last.
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Example:

Initial array

7	4	5	3	2	8	6	9

Down

Up

Pivot

- Define the pivot value as the contents of table[first].
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- dc
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Example:

Initial array

6	4	5	3	2	7	8	9
_	_	_		_	_		_

Down

Up

Pivot

- Define the pivot value as the contents of table[first].
- Initialize up to first and down to last.
- dc
- Increment up until up selects the first element greater than the pivot value or up has reached last.
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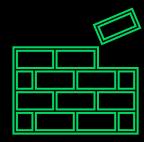
Quick Sort Code

```
1. void quickSort(int array[], int low, int high)
2. {
3.    if (low < high)
4.    {
5.       int pivot = partition(array, low, high);
6.       quickSort(array, low, pivot - 1);
7.       quickSort(array, pivot + 1, high);
8.    }
9. }</pre>
```

```
36. int main()
37. {
38.    int data[] = {15, 0, 5, 6};
39.    int n = sizeof(data) / sizeof(data[0]);
40.    quickSort(data, 0, n - 1);
41.    return 0;
42. }
```

```
int partition(int array[], int low, int high)
11.
12.
         // Select the pivot element
13.
         int pivot = array[low];
14.
         int up = low, down = high;
15.
         while(up < down)</pre>
17.
18.
             for (int j = up; j < high; j++)
19.
               if(array[up] > pivot)
21.
                    break;
22.
                up++;
23.
             for (int j = high; j > low; j--)
25.
               if(array[down] < pivot)</pre>
27.
                    break;
                down--:
29.
             if(up < down)</pre>
31.
                swap(&array[up], &array[down]);
32.
         swap(&array[low], &array[down]);
34.
         return down;
```





Time Complexity:

- If the pivot value is a random value selected from the current subarray,
 - then statistically half of the items in the subarray will be less than the pivot and half will be greater
 - thus there will be log n levels of recursion
- Partitioning requires n moves
- Total time: O(n log n) on average
- A quicksort will give very poor behavior if, each time the array is partitioned, a subarray is empty. In that case, the sort will be O(n²). Under these circumstances, the overhead of recursive calls and the extra run-time stack storage required by these calls makes this version of quicksort a poor performer relative to the quadratic sorts



Quick Sort Time Complexity

	Merge Sort	Quick Sort
Worst Case	O(n*log n)	O(n²)
Average Case	O(n*log n)	O(n*log n)
Best Case	O(n*log n)	O(n*log n)
Space	0(n)	0(log n)*

^{*} Recursion Stack

Other Sorts

- Sleep sort
- Counting sort
- Tim Sort
- Radix Sort
- Bucket Sort

Resources

- https://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html
- https://www.programiz.com/dsa
- https://www.youtube.com/user/AlgoRythmics/videos
- https://www.toptal.com/developers/sorting-algorithms

Questions



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