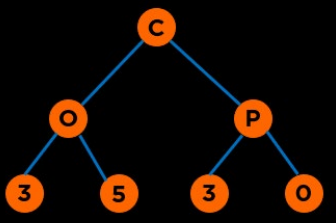
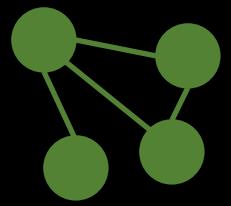
Sets, Maps and Hash Tables

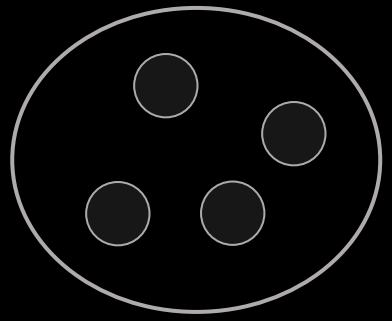


Data Structures & Algorithm

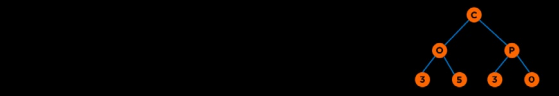
Categories of Data Structures

Linear Ordered	Non-linear Ordered	Not Ordered
Lists	Trees	Sets
Stacks	Graphs	Tables/Maps
Queues		





Sets

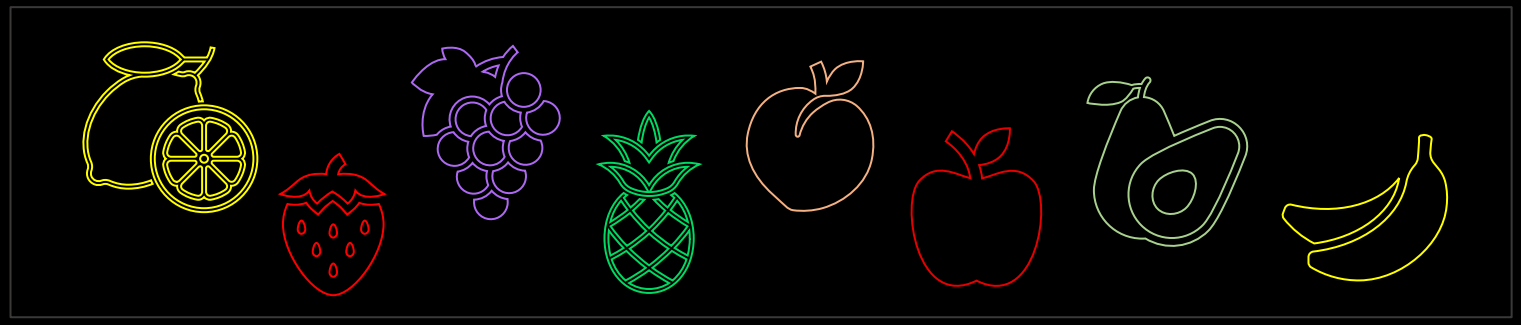


Sets

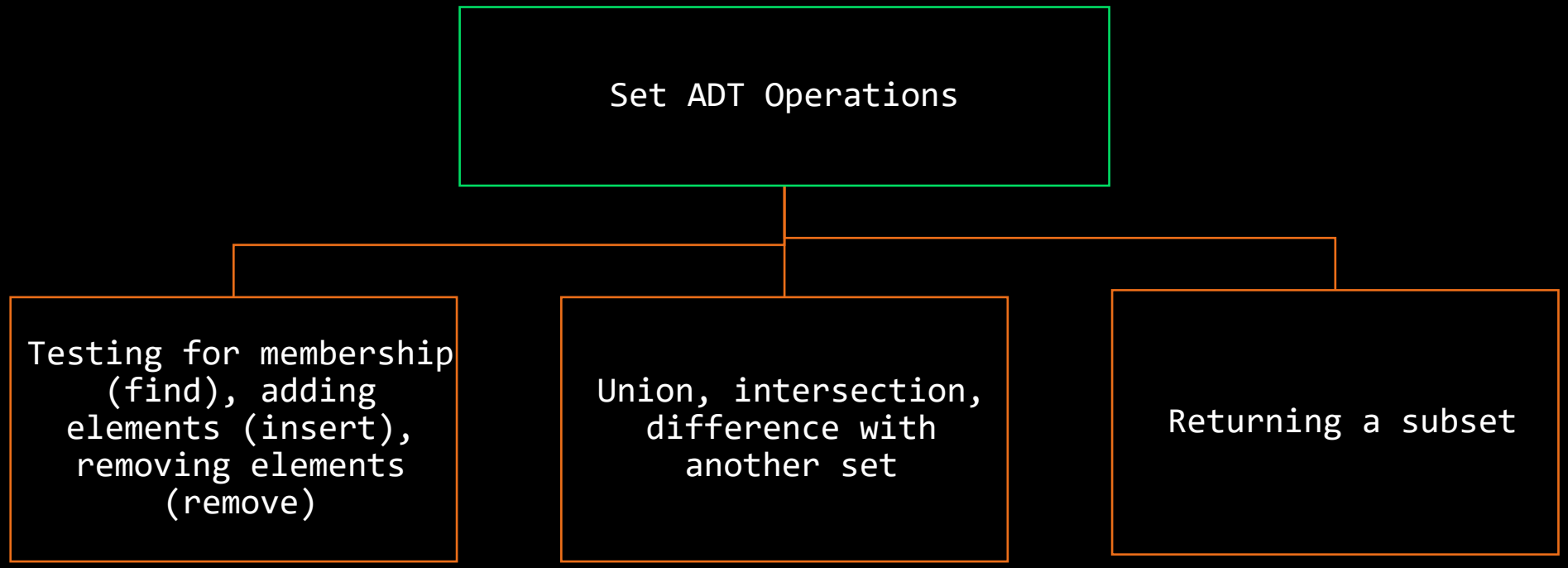
A set is a collection that contains no duplicate elements

Set objects

- are not indexed
- do not reveal the order of insertion of items
- do enable efficient search and retrieval of information
- do allow removal of elements without moving other elements around

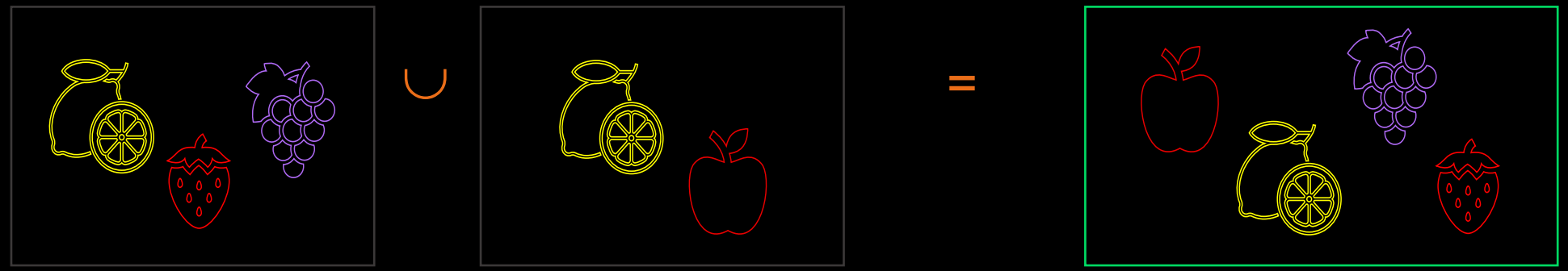


Sets

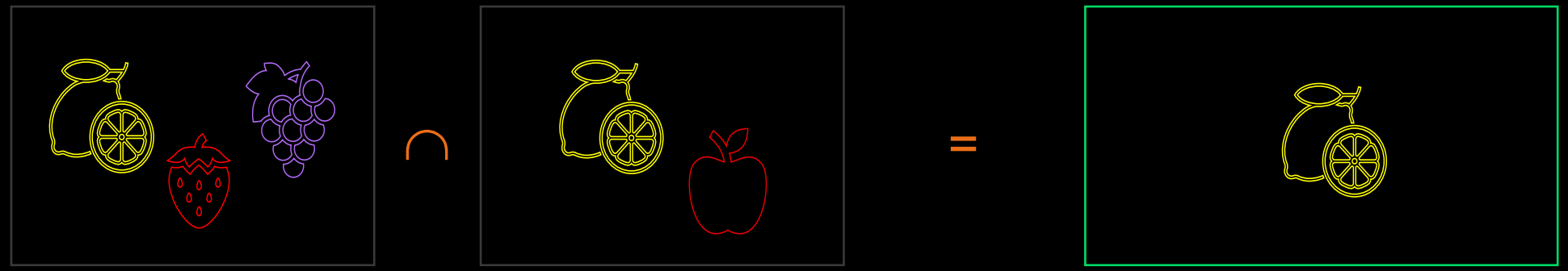

Set ADT Operations
Testing for membership (find), adding elements (insert), removing elements (remove)
Union, intersection, difference with another set
Returning a subset

Sets

Union of two sets, A ∪ B is a set whose elements belong either to A or B or to both A and B.

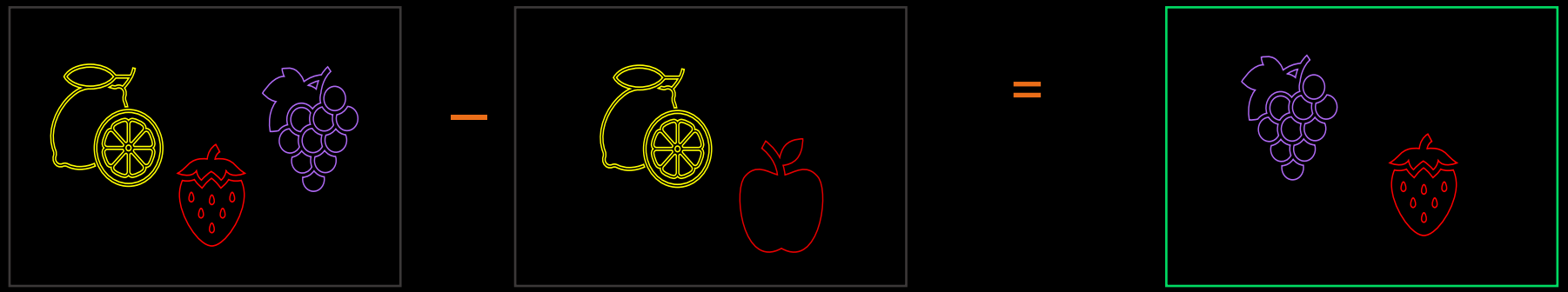


Intersection of sets A ∩ B is the set whose elements belong to both A and B.

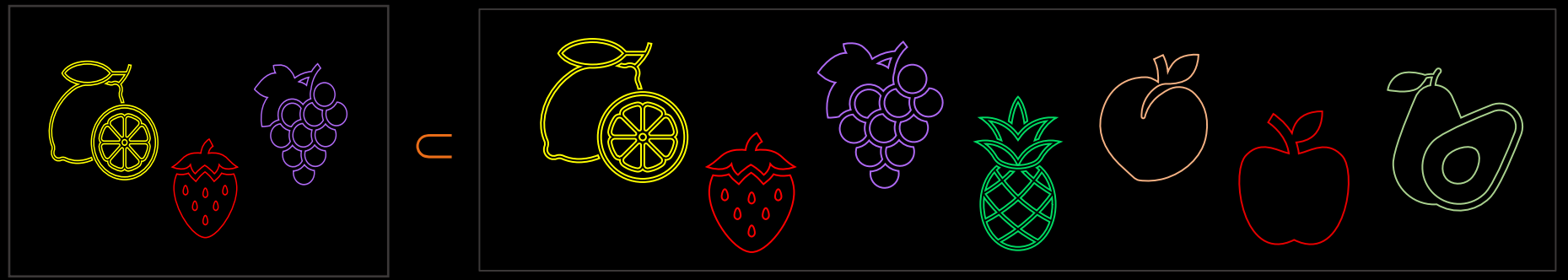


Sets

Difference of sets A - B is the set whose elements belong to A but not to B.

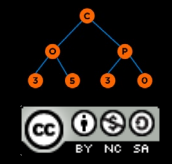


Set A is a subset of set B, A ⊂ B if every element of set A is also an element of set B.



Lists vs Sets

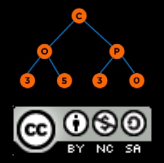
	Lists	Sets
Order and Access through Element Index	Yes	No
Duplicates	Yes	No
Implementations	Array Based, Linked Lists	Array Based, Tree Based



Sets in C++

	std::set	std::unordered_set
Order in Elements	Yes	No
Initialization	std::set<type> s;	std::unordered_set<type> s;
Common Methods	insert, erase, find, count, size, empty	insert, erase, find, count, size, empty, bucket_size, load_factor
Implementations	Binary Search Tree (TreeSet)	Hash Table (Hash Set)
Time Complexity of Common Operations	O(log n) for a Self-Balancing BST, e.g. Red Black Tree	O(1) + O(k) for hash

http://www.cplusplus.com/reference/set/set/
http://www.cplusplus.com/reference/unordered_set/unordered_set/



Sets in C++ Example

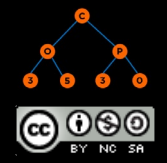
```
01  // Ordered tree-based set
02  set <int> s1;
03
04  // insert elements in random order
05  s1.insert(5);
06  s1.insert(2);
07  s1.insert(4);
08  s1.insert(11);
09  s1.insert(2); // only one 2 will be added to the set
10
11  // printing set
12  set <int> :: iterator itr1;
13  cout << "The set s1 is : ";
14  for (itr1 = s1.begin(); itr1 != s1.end(); ++itr1)
15            cout << "   " << *itr1;
```

```
The set s1 is :    2   4   5   11
```

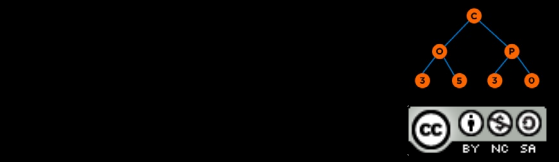
```
01  //Unordered Set – Hash-based
02  unordered_set <int> s2;
03
04  // insert elements in random order
05  s2.insert(5);
06  s2.insert(2);
07  s2.insert(4);
08  s2.insert(11);
09  s2.insert(2); // only one 2 will be added to the set
10
11  // printing set
12  unordered_set <int> :: iterator itr2;
13  cout << "The set s2 is:";
14  for (itr2 = s2.begin(); itr2 != s2.end(); ++itr2)
15            cout << "   " << *itr2;
16  cout << endl;
17  cout << "Bucket count: " << s2.bucket_count();
18  cout << "\nLoad Factor: " << s2.load_factor();
19  cout << "\nMax Load Factor:" << s2.max_load_factor();
```

```
The set s2 is: 11   4   5   2
Bucket count: 7
Load Factor: 0.571429
Max Load Factor: 1
```

https://onlinegdb.com/SyMCuHOlD



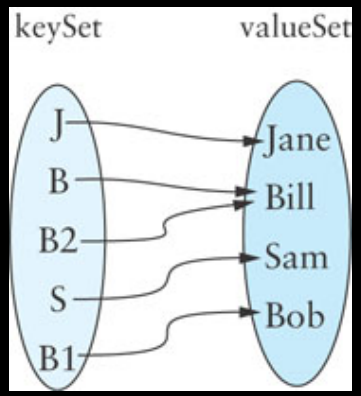
Maps



Maps

A map is a collection of key-value pairs that do not contain duplicate keys.

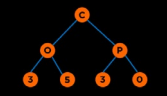
- Maps are sort of an abstraction over Sets
- The Keys in a map are a Set.
- Values can be non-unique [Many-to-One Relationship, Onto Mapping]
- If you store values along with keys in a Set data structure, you get a Map



{(J, Jane), (B, Bill),
(S, Sam), (B1, Bob),
(B2, Bill)}

Maps

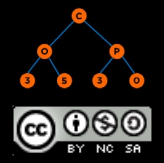
Type of item	Key	Value
University student	Student ID number	Student name, address, major, grade point average
Online store customer	E-mail address	Customer name, address, credit card information, shopping cart
Inventory item	Part ID	Description, quantity, manufacturer, cost, price



Maps in C++

	std::map	std::unordered_map
Order in Elements	Yes	No
Initialization (Internally stored as pairs)	std::map<type, type> m;	std::unordered_map <type, type> m;
Common Methods	insert, [], erase, find, count, size, empty	insert, [], erase, find, count, size, empty, bucket_size, load_factor
Implementations	Binary Search Tree (TreeMap)	Hash Table (Hash Map)

http://www.cplusplus.com/reference/map/map/
http://www.cplusplus.com/reference/unordered_map/unordered_map/



Maps in C++ Example

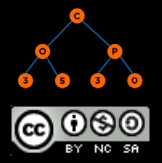
```
01 // Ordered tree-based map
02 map<char,int> table;
03
04 // insert elements in random order
05 table['b']=30;
06 table['a']=10;
07 table['c']=50;
08 table['a']=40;
09
10 // printing map
11 for(auto member: table)
12       cout << member.first << " " << member.second <<"\n";
```

```
a 40
b 30
c 50
```

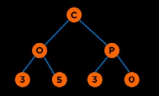
```
01 //Unordered Map – Hash-based
02 unordered_map<char,int> table_unordered;
03
04 // insert elements in random order
05 table_unordered['b']=30;
06 table_unordered['a']=10;
07 table_unordered['c']=50;
08 table_unordered['a']=40;
09
10 // printing set
11 for(auto member: table_unordered)
12       cout << member.first << " " << member.second <<"\n";
13
14 cout << "Load Factor: " << table_unordered.load_factor();
```

```
c 50
b 30
a 40
Load Factor: 0.428571
```

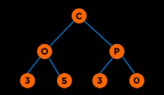
https://onlinegdb.com/SkHykUnlP



Questions

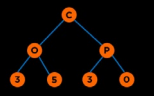


Hash Tables



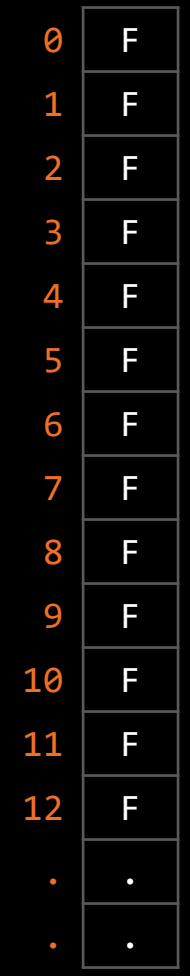
Problems with Tree Based Maps and Sets

- If the datatypes are comparable such as integers or characters, tree-based maps and sets makes sense. What if the data itself is incomparable?
- Common operations such as insert( ) or search( ) are O(log n). Can we do better than this?



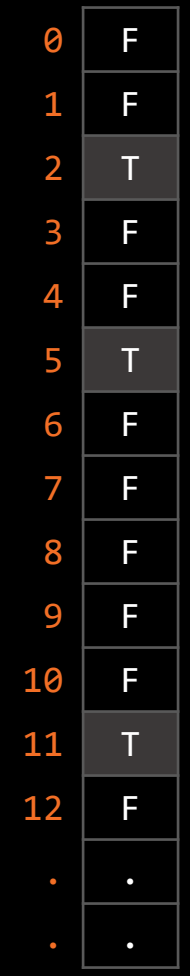
What if we use an Array: Exploiting constant access?

- Let's say we want to insert 11, 2 and 5 into a set
- Initially all values are false



What if we use an Array: Exploiting constant access?

- Let's say we want to insert 11, 2 and 5 into a set
- Initially all values are false
- When we insert an item, we set the value at index to true



What if we use an Array: Exploiting constant access?

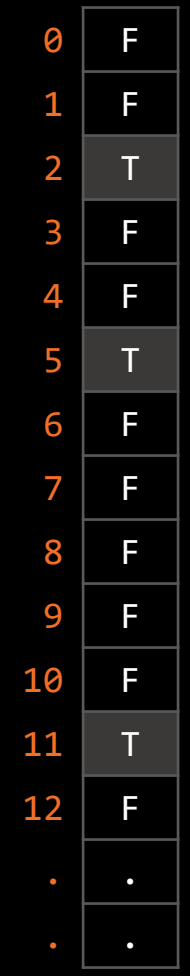
```
01  class ArraySet
02  {
03      private:
04          bool set[100] = {0};
05      public:
06          void insert(int value);
07          bool search(int value);
08  };
09
10  void ArraySet::insert(int value)
11  {
12      set[value] = 1;
13  }
14
15  bool ArraySet::search(int value)
16  {
17      return set[value];
18  }
```

```
19  int main()
20  {
21      ArraySet testSet;
22      testSet.insert(5);
23      std::cout << std::boolalpha << testSet.search(15) <<"\n";
24      std::cout << std::boolalpha << testSet.search(5);
25      return 0;
26  }
```

https://onlinegdb.com/TFfwCnEEl

What if we use an Array: Exploiting constant access?

- Let's say we want to insert 11, 2 and 5 into a set
- Initially all values are false
- When we insert an item, we set the value at index to true
- Common operations
  - Insert:
  - Find:



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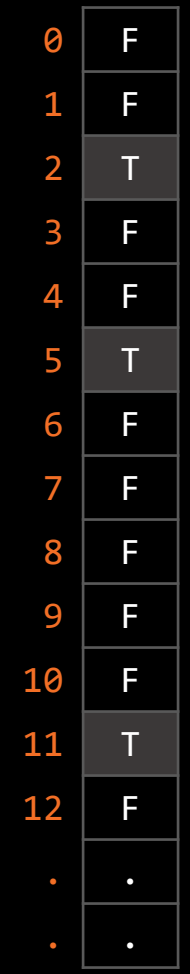
  - Insert: O(1)
  - Find: O(1)



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- Initially all values are false
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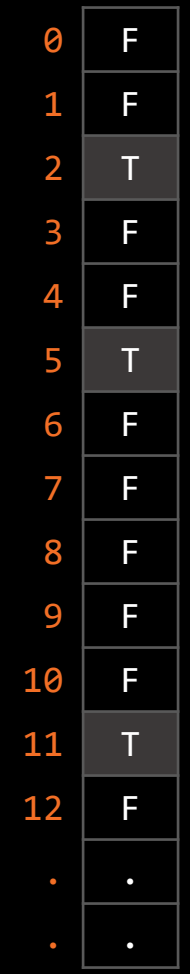
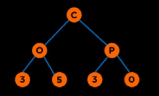
- Problems with this: wastes memory and other datatypes?



How to deal with Strings?

- Problems with this: wastes memory and other datatypes?
- What if we want to store: “cat” or “dog”?



How to deal with Strings?

- Problems with this: wastes memory and other datatypes?
- What if we want to store: “cat” or “dog”?
  - Idea: Convert “cat” or “dog” into a number
  - Approach: Use the first letter – ‘c’ = 3, ‘d’ = 4

How to deal with Strings?

- Problems with this: wastes memory and other datatypes?
- What if we want to store: “cat” or “dog”?
  - Idea: Convert “cat” or “dog” into a number
  - Approach: Use the first letter – ‘c’ = 3, ‘d’ = 4
  - Problem: What happens with “cap”?

How to deal with Strings?

- What if we want to store: “cat” or “dog”?
  - Idea: Convert “cat” or “dog” into a number
  - Approach: Use the first letter – ‘c’ = 3, ‘d’ = 4
  - Problem: What happens with “cap”? – “Collision”
- To fix this use all digits by multiplying each by a power of 27
  - Index of “cat” is $(3 \times 27^2) + (1 \times 27^1) + (20 \times 27^0) = 2234$.

How to deal with Strings?

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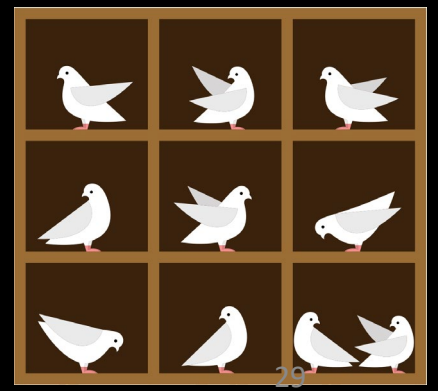
  - Index of “cat” is $(3 \times 27^2) + (1 \times 27^1) + (20 \times 27^0) = 2234$.
- As long as base >=26, we will get a unique number and no collisions. If it is less than 26, we are guaranteed for collisions due to pigeonhole principle



How to deal with Strings?

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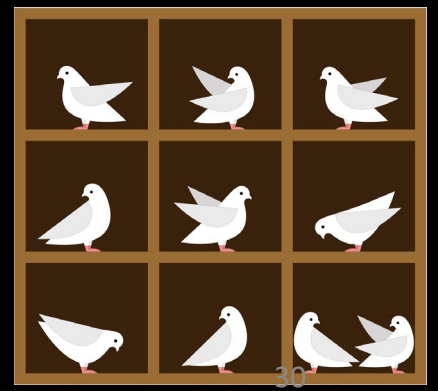
  - Index of “cat” is $(3 \times 27^2) + (1 \times 27^1) + (20 \times 27^0) = 2234$.
- As long as base >=26, we will get a unique number and no collisions. If it is less than 26, we are guaranteed for collisions due to pigeonhole principle
  - If base = 2, index of “ac” is $(1 \times 2^1) + (3 \times 2^0) = 5$
  - If base = 2, index of “e” is $(5 \times 2^0) = 5$
  - If base = 27, index of “ac” is $(1 \times 27^1) + (3 \times 27^0) = 30$
  - If base = 27, index of “e” is $(5 \times 27^0) = 5$



How to deal with Strings – ASCII and Unicode?

- Increase the base for other characters as 26 characters is too restrictive
  - ASCII: 128 characters
  - Unicode: 143,859 characters

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- Problem: ?

How to deal with Strings – ASCII and Unicode?

- Increase the base for other characters as 26 characters is too restrictive
  - ASCII: 128 characters
  - Unicode: 143,859 characters
- Fixed the problem of storing other datatypes
- Problem: How do we store large values? Overflows, lead to collisions again. And we are now wasting even more space.

Crux of the Problem

Approach

- Data -> Hash Function -> Hash Code

Hash code values for different data map to same index in array even after increasing a lot of space in table:

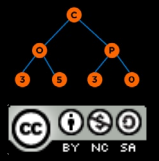
- “cat” -> transform2(“cat”) -> 34
- “cat” -> transform127(“cat”) -> 48534
- “cat” -> transform143859(“cat”) -> 62,086,379,522 -> 1956837378

1. poor hash functions
2. limitations of language

Collisions are Inevitable due to overflows!

Crux of the Problem

- Problem
  - Wastes memory if we have hash tables that are large
  - Has collisions – based on language limitations or poor hash functions
- Solution
  - allow collisions
  - use collision resolution strategies
  - use small table sizes initially and increase it as per need when performance is affected



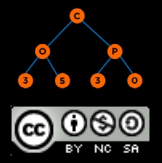
Hash Tables

Approach

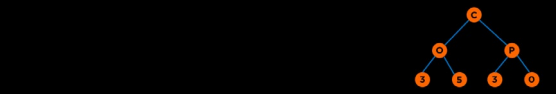
- Data -> Hash Function -> Hash Code -> Reduce -> Index
- Insert the data (D) at the index in the table and if there is some other data at the index which is not D, then there is a collision and use a collision resolution mechanism

Hash Function

- A function that converts a data object to a hash code.
- Properties
  - Input: Object x
  - Output: An integer representation of x
  - If x is equal to y, H(x) = H(y)
  - If x is not equal to y, it would be great if H(x) is not equal to H(y)

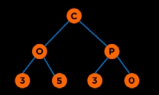


Hash Function



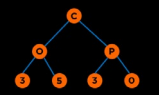
Hash Function Examples

- A function, H( ) that converts a data object, x to a hash code.
  - H(x): { return 0; }
  - H(x): { return Sum of all ASCII values; }
  - H(x): { return Powers of 31 with ASCII; }
  - H(x): { return Random Number; }
  - H(x): { return Current Time; }



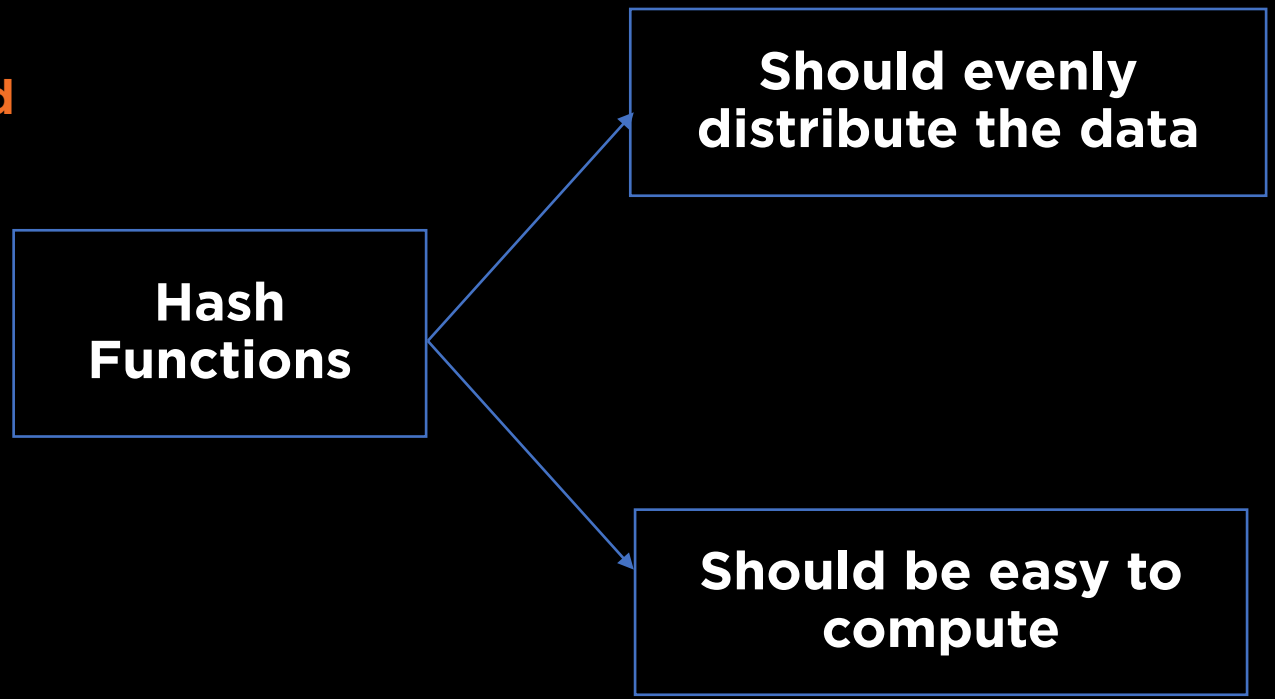
Hash Function Examples

- A function, H( ) that converts a data object, x to a hash code.
  - Poor - H(x): { return 0; }
  - Ok - H(x): { return Sum of all ASCII values; }
  - Good - H(x): { return Powers of 31 with ASCII; }
  - Invalid - ~~H(x): { return Random Number; }~~
  - Invalid - ~~H(x): { return Current Time; }~~



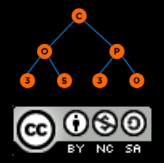
Hash Function Examples

- A function, H( ) that converts a data object, x to a hash code.
  - H(x): { return Powers of 31 with ASCII; }
  - Primes are usually used over composites
  - Smaller primes are preferred for faster calculations



Collision Resolution

- Buckets and Load Factor
- Separate Chaining
  - Fixed
  - Resizable
- Open Addressing
  - Linear Probing
  - Quadratic Probing



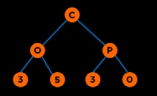
Hash Tables

Approach

- Data -> Hash Function -> Hash Code -> Reduce -> Index

- Powers of 31 method (Good)
- System to add all characters ASCII values in a String (Better)
- Constant (Bad)

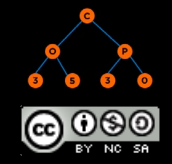
- Hashcode%10
- Hashcode/10
- Midsquare



Hash Tables and Map vs Set

```
01  class Set
02  {
03      private:
04          string arraySet[100];
05      public:
06          void insert(int value);
07          bool search(int value);
08  };
09
10  void ArraySet::insert(int value)
11  {
12      //find the hash of the value
13      //reduce the hash to get an index
14      //check if value is not at index
15              //insert value at index
16      //otherwise, use collision resolution strategy
17  }
18
19  bool ArraySet::search(int value)
20  {
21      //find the hash of the value
22      //reduce the hash to get an index
23      //check if value is not at index
24              //return false
25      //otherwise, search based on collision resolution strategy
26  }
```

Remember C++ Unordered Maps and Sets are backed by Hash Tables



Sets and Maps in C++ Example

```
01  //Unordered Set – Hash-based
02  unordered_set <int> s2;
03
04  // insert elements in random order
05  s2.insert(5);
06  s2.insert(2);
07  s2.insert(4);
08  s2.insert(11);
09  s2.insert(2); // only one 2 will be added to the set
10
11  // printing set
12  unordered_set <int> :: iterator itr2;
13  cout << "The set s2 is:";
14  for (itr2 = s2.begin(); itr2 != s2.end(); ++itr2)
15            cout << "   " << *itr2;
16  cout << endl;
17  cout << "Bucket count: " << s2.bucket_count();
18  cout << "\nLoad Factor: " << s2.load_factor();
19  cout << "\nMax Load Factor:" << s2.max_load_factor();
```

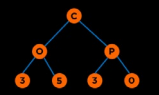
```
The set s2 is: 11   4   5   2
Bucket count: 7
Load Factor: 0.571429
Max Load Factor: 1
```

```
01  //Unordered Map – Hash-based
02  unordered_map<char,int> table_unordered;
03
04  // insert elements in random order
05  table_unordered['b']=30;
06  table_unordered['a']=10;
07  table_unordered['c']=50;
08  table_unordered['a']=40;
09
10  // printing set
11  for(auto member: table_unordered)
12        cout << member.first << " " << member.second <<"\n";
13
14  cout << "Load Factor: " << table_unordered.load_factor();
```

```
c 50
b 30
a 40
Load Factor: 0.428571
```

https://onlinegdb.com/SyMCuHOlD https://onlinegdb.com/SkHykUnlP

Questions



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10.1.2 Two Sum Problem

N-sum is a common problem where you are given an array and asked to see if there are ***N*** numbers that add up to a target. For this stepik module, you'll be asked to complete Two-Sum. This means you'll be given an array of integers and you have to determine if there are 2 values that sum to a desired target. The method signature is pair<int, int> two_sum(vector<int> arr, int target), which returns a pair of the indices whose values sum to the desired target. If no such 2 value exists, return the pair {-1,-1}. Make sure that the smaller index is first.

Example:
arr = [3, 5, 11, 12, 15]
target = 17
Output = {1,3}

https://stepik.org/lesson/390638/step/2

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Example:
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target = 17
Output = {1,3}

```
01  pair<int, int> two_sum(vector<int>& arr, int target)
02  {
03      unordered_map<int, int> map;
04      pair<int, int> result(-1, -1);
05      for (int i = 0; i < arr.size(); i++)
06      {
07          int diff = target - arr[i];
08          if(map.count(diff))          //check if complement is present in the set
09          {
10              result.first = map[diff];
11              result.second = i;
12              break;
13          }
14          map[arr[i]] = i;             //add the element to the set otherwise
15      }
16      return result;
17  }
```

https://stepik.org/lesson/390638/step/2