Team Control Number

20190444

Problem Chosen

A

2019

ShuWei Cup

Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Summary

Improve the level of hospital is a difficult question. The whole questions offer different aspects like disease, hospital, device and so on. For question 1, we use gray forecasting model to predict the future data of GPA, popularity and other factors and finally predict the proportion of aging in the next decade. We conclude that the percentage of aging will rise to 17% in 2023. For question 2, we take Neural Network to fit the past decade data of disease and predict the trend of different disease in the future. We conclude that influenza and malignant tumor are the most popular disease in the coming 5-year. For question 3, we introduce the multi-server system to describe the queuing system and compute the proper data of the port of different inspection sites. For question 4, we use Cooperation-Competition model to describe and analyze the relationship between public hospital and private hospital. We conclude that the private hospital should offer more and medical resource for public hospital while public hospital should enlarge their scale. For question 5, we offer some suggestion for the "14TH Five-Year plan" based on the total discussion and analysis.

Key word:* *Gray Forecasting, Neural Network, Data Fitting, Queue System, Cooperation-Competition model, discrete programming

Content

1. Introduction

1.1 Background

Due to the rapid development of economy and the continuous improvement of medical security level, the aging problem in China is becoming more and more serious. Until 2019, China has ranked 10th in the list of aging countries in the world, so faced with this situation, more attention should be paid to aging problem. And at the same time, the increasing degree of aging leads to the further enhancement of medical service requirements.

The rapid growth of medical demand urges us to optimize the current medical service system, so as to achieve a more efficient and diverse service for the audience. From the perspective of the hospital itself, in recent years, private hospitals and public hospitals have been booming, but due to their different characteristics and functions, how to make their symbiosis and coordinated development becomes an urgent problem which needs to be solved. In the confront of different patients with different examination needs, in order to further save time, we also need a practical queuing theory. From the perspective of the government, it is crucial to further promote the integration of urban and rural medical system and form a scientific and effective social security system to achieve the goal of the “14th Five-Year Plan”.

1.2 Work

We are required to establish a correct and effective prediction and analysis model to judge the current aging degree of our country and predict the situation in the next ten years. According to the analysis and prediction results, put forward suggestions for upgrading the medical system to meet the medical needs of residents.

In order to analysis the most common types of diseases, we take Beijing as an example, analyzes the most common types of diseases and puts forward suggestions for the overall development of public hospitals.

Owing to improve working efficiency of medical service system. We present a common queuing theory and its related optimal queuing methods.Besides, we also put forward a reasonable model of competition and cooperation for the operation of private hospitals and public hospitals

According to the analysis and judgment results of the first four questions, write a proposal for the medical management department.

2. Problem analysis

2.1 Data analysis

Data from Beijing Statistical Yearbook(2008-2016), Report on population health and health management in Beijing(2008-2016), National Population Health Science Data Center(http://www.phsciencedata.cn/Share/index.jsp)(2009-2018), National Bureau of Statistics(http://www.stats.gov.cn/tjsj/)(2009-2018).

2.2 Analysis of question one

In recent years, the aging of our country is increasing, so it is very important to focus on the aging trend of China and the medical needs of the residents. Through literature review, we selected the following important factors：total population at the end of the year(X_1), consumption level of residents(X_2), gross national income(X_3), gross domestic product(X_4), per capita GDP(X_5).

2.3 Analysis of question two

We select Beijing as our target, and discuss about infectious disease(hepatitis B, syphilis, influenza, Tuberculosis) and noninfectious disease(Malignant tumor, Heart disease, Cerebral vascular disease, Chronic respiratory disease) which include 8 various disease in total.

2.4 Analysis of question three

Nowadays, different inspection sites have different number of ports, some sites have less people to wait with many ports while some have too many people with less ports. This phenomenon is frequent in a few hospitals which definitely causes the waste of medical resource and cost of time of both patient and nurse. As a result, we would like to figure out the right number of ports for different sites based on the realistic situation. According to the question 2, we discuss X-ray, B-ultrasound, Computed Tomography(hereinafter called CT),Blood routine examination(hereinafter called blood RT), Urine sample detection all 5 inspection sites. Using queuing system finds the proper ports for every inspection site.

2.5 Analysis of question Four

Within the scope of the medical system, private hospitals and public hospitals face the same objects of service and provide services in deep coordination. So we study the cooperative decision-making of resource sharing and utilization between them and establish a decision model of medical service product output. We compare the situation of medical service market and the optimal decision output of medical service products which are provided by private hospitals and public hospitals when they make joint decisions and make independent decisions through this model.

2.6 Analysis of question five

Based on the 4 questions, we find that there are many things could do in the future. So, we offer the advice to the government about how to reform the relative organization.

The detail answer in the 8(Conclusion)

3.Symbol and Assumptions

3. 1 Symbol Description

Symbol	Meaning
X_1	Total population at the end of the year
X_2	Consumption level of residents
X_3	Gross national income
X_4	Gross domestic product
X_5	Gross domestic product
Y	The proportion of people over 65
π_1, π_2, π	Profit(private hospitals, public hospitals, cooperate together)
P_1, P_2	Price(private hospitals, public hospitals)
P_3	Price of medical shared resources($-b_1 \le P_3 \le d_2$)
Q_1, Q_2	Optimal supply(private hospitals, public hospitals)
a_i, b_i	Demand function coefficient
λ	The number of people will come to the system during the unit time
μ	The number of people will leave the system during the unit time
c	The number of servers
P_0	the probability of the line with no person
L_q	the average queue length

3.2 Fundamental assumptions

3.2.1 Assumptions of question three

1. Every queue is prior rule (come first, serve first)
2. patient will wait in line until he or she get service.
3. The separation time between different patients coming into line obeying Poisson distribution
4. The serving time of every patient obeying Poisson distribution.
5. No one will cut in line.

3.2.2 Assumptions of question four

1.The medical resources of private hospitals can be used as the basis of diagnosis and treatment or resource sharing of public hospitals. Private hospitals and public hospitals can form a symbiotic system around medical shared resources or utilization.
2.It is assumed that the medical services provided by private hospitals and public hospitals are both price elastic, and their prices and demands are linear. It means that private hospitals pay a certain fee to public hospitals to help them purchase medical resources. That is to say, the cost paid by private hospitals to public hospitals will not be higher than the cost of their own purchase, and the purchase price of public hospitals will not be higher than the price of their alternative medical resources. There is complete information between private hospitals and public hospitals.

4. Model

4.1 Model of question one

Set the original data of each influencing factor as: $x^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right)$

Calculating the rank ratio of a sequence $\lambda(k)=\dfrac{x^{(0)}(k-1)}{x^{(0)}(k)},k=2,3\cdots,n$,And check whether the grade ratio is within $\left(e^{-\frac{2}{n+1}},e^{\frac{2}{n+2}}\right)$, if in this scope, then grey forecasting model can be carried out, if not, we will take translation transformation in order to let the grade ratio change to optimal value $y^{(0)}(k)=x^{(0)}(k)+c,k=1,2\cdots n$

Take 2009-2018 influencing factors as basic variables, predict them through grey forecasting model. Set the original data of each influencing factor， after one accumulation: $x^{(1)}=\left(x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(10)\right)$, $x^{(1)}(k)=\sum_{i=0}^{k}x^{(0)}(i)(k=1,2,\cdots,10)$.

The generated sequence satisfies whitening differential equation

Set $\mu=(a,b)^T$, $Y=\left(x^{(0)}(2),x^{(0)}(3)\cdots,x^{(0)}(10)\right)^T$

Then get $\mu=(a,b)^T=\left(B^TB\right)^{-1}B^TY$ by the least square method, then we solve the whitening differential equation $x^{0(1)}(k+1)=\left(x^{(0)}(1)-\frac{b}{a}\right)e^{-k}+\frac{b}{a}$,Then reduce the original sequence by subtraction: $x^{0(1)}(k+1)=\left(1-e^{a}\right)\left(x^{(1)}(1)-\frac{b}{a}\right)e^{-ak}$

We use residual test, set residual is $\varepsilon(k)$, so $\varepsilon(k)=\frac{x^{(0)}(k)-x^{0(1)}(k)}{x^{(0)}(k)}$, if $\varepsilon(k)<0.2$, can be seen as to satisfy requirements.

We can use the G, M (1,1) model to predict the influencing factors in 2019-2028.

Set Multiple linear regression model. The correlation coefficient between some factors and the proportion of population over 65 years old is high, so multiple linear regression model can be established:

Set

$$Y_i=\beta_0+\beta_1\ln X_{1i}+\beta_2\ln X_{2i}+\cdots+\beta_k\ln X_{ki}+\mu_i,\quad i=1,2\cdots n$$

$$Y=\left[Y_1\,Y_2\cdots Y_n\right]^T,\ln X=\left[1\ \ln X_{22}\cdots\ln X_{k2}\right],\beta=\left[\beta_1\,\beta_2\cdots\beta_k\right]^T,\mu=\left[\mu_1\,\mu_2\cdots\mu_k\right]$$

So we can get the matrix form $Y=\ln X\beta+\mu$.

4.2 Model of question two

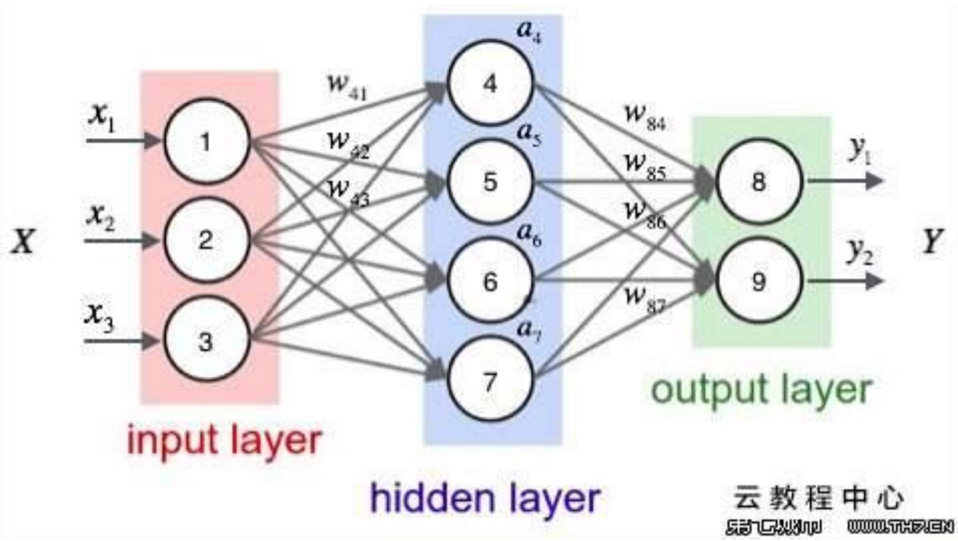
Downloading the relevant data from inter net (table 1). We expect to find a prediction model for ever data of disease, which means we can get the data of special disease in the future by giving the year. Here, we take Neural Network to find the proper model to fit the data that we can predict the future number.

Table1: Number of 8 Different Disease In Beijing From 2008 To 2018

Year	hepatitis B	syphilis	influenza	Tuberculosis	Malignant tumor	Heart disease	Cerebral vascular disease	Chronic respiratory disease
2008	5579	1816	8	7164	119	112	116	58

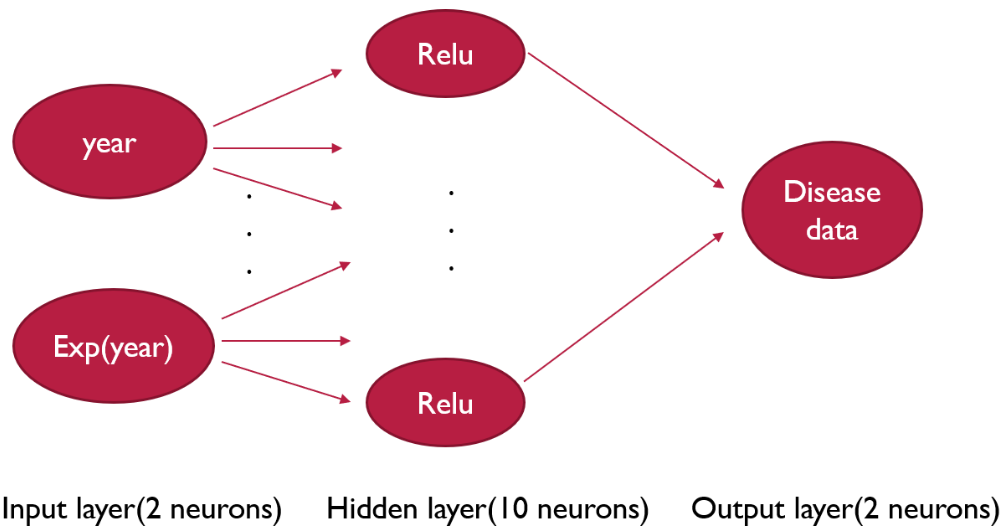
2009	6999	2714	120	8805	124	121	119	61
2010	9593	3581	290	7889	132	124	121	58
2011	6671	3973	221	8215	138	137	123	57
2012	4564	3786	335	9576	142	141	134	60
2013	3821	4002	5147	8029	152	146	133	58
2014	3172	4382	830	8021	159	157	142	61
2015	3116	4671	391	8099	158	146	129	62
2016	2612	4438	1003	8219	161	151	129	59
2017	1683	5310	3439	9724	165	157	131	58
2018	1696	4975	20279	6731	169	158	129	63

Neural Network include 3 layers: input layer, hidden layer and output layer (Picture 1). Input layer and output layer represent independent variable and dependent variable respectively. Hidden layer includes activate function such as $tanh$ or $sigmoid$ to solve un-linear problem. There is a weight value w_{ij} between every neuron as multiplier to connect the input and output.



Picture 1 Neural Network

In a traditional way, for a certain disease, we just need to use a single neuron as input layer (year data) and corresponding data as output layer. For instance, to predict the number of influenza in the future, $X^0 = (2008,2009 \dots ,2018)^T$ represent year data as input layer while $Y^0 = (1816,2714 \dots ,4975)^T$ represent disease data for the certain year as output layer. We use 10 neurons as hidden layer and $relu$ function as activation function. However, it is difficult to describe and fit some special disease data. After observation, we found that the exponential function seems a good function to fit the data. So we use 2 neuron as input layer, specifically, we change the input layer X^0 to $X = (X^0, X^1)$, $X^1 = \exp(X^0)$ (picture 2)



Picture 2: Frame of Network

The principle of Neural Network here is like Least Square Method——finding Minimum Square Error(hereinafter called MSE). We suppose e as loss, and e is defined as:

$$e(w) = \frac{1}{2}\sum_{i=1}^{n}\left(d_i^2 - y_i^2\right)^2$$

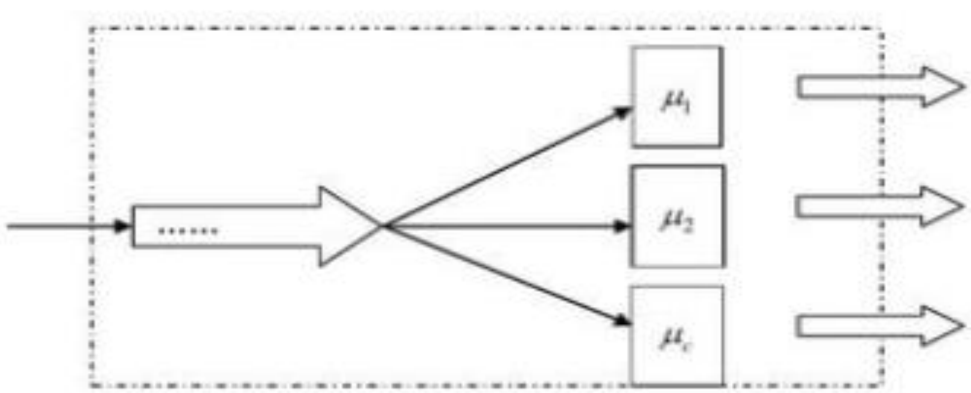
Neural Network take gradient descent stagey to figure out the w corresponding to the minimum $e(w)$. That is:

$$w = w - learning\_rate * \frac{\partial e(w)}{w}$$

After training, we could find the special w in every connection. Eventually, after training 1000 epochs, for the special disease, giving future year data will get the corresponding disease data with the model.

4.3 Model of question three

According to the analysis, for every inspection site, it will have different number of ports for patient. Consequently, multi-server queuing system is introduced here to describe the waiting process.



Picture 3 multi-server queuing system

Multi-server queuing system is a classical model. Custom coming to the line will wait until be served. The system has more than 1 number of servers so custom will choose the server which is free first.
According to **3.2.4**, we assume that the custom come to line with Poisson distribution and the parameter is λ. Parallelly, we also assume that the serving time for different customs obeying the Poisson distribution with parameter μ. And the service intensity of the system ρ is:

$$\rho = \frac{\lambda}{\mu}$$

If we assume the number of servers is C, we could get the average queue length (L_q) of the system:

$$L_q = \frac{1}{C!}\frac{(C\rho)^C\rho}{(1-\rho)^2}P_0(C)$$

Here, P_0 is the probability of the line with no person with the following expression:

$$P_0(C) = \left[\sum_{k=0}^{C-1}\frac{1}{k!}(\rho)^k + \frac{1}{C!}\frac{1}{1-\rho}(\rho)^C\right]^{-1}$$

We expect to have less L_q, that means, more port should be open. However , there is more cost for open a serve, so we introduced the Z_{min} as our optimum target. Z_{min} is defined following:

$$\min_C Z(C) = L_q(C) + \gamma C$$

Here, γ is a parameter to balance the cost of L_q and the cost of every port. Its value depends on the type of the inspection.
In fact, Z is a discrete value. So, we could figure out Z by the following equation:

$$\begin{cases} Z(C) \le Z(C+1) \\ Z(C) \le Z(C-1) \end{cases}$$

That is: $$L_q(C) - L_q(C+1) \le \gamma \le L_q(C-1) - L_q(C)$$
As a result, we could use this way to find the optimum C.

4.4 Model of question four

Private hospitals and public hospitals take the maximization of benefits as their decision-making objectives. Therefore, here is the symbiotic decision model of private hospitals and public hospitals:

$$\pi_1(Q_1) = P_1Q_1 + P_3\min(\delta Q_2, \beta Q_1) - c_1Q_1 - d_1\max(\beta Q_2 - \delta Q_1, 0)$$
$$\pi_2(Q_2) = P_2Q_2 + P_3\min(\delta Q_2, \beta Q_1) - c_2Q_2 - d_2\max(\beta Q_2 - \delta Q_1, 0)$$
$$\pi(Q_1, Q_2) = P_1Q_1 + P_2Q_2 - c_1Q_1 - c_2Q_2 - d_1\max(\beta Q_1 - \delta Q_2, 0) - d_2\max(\beta Q_2 - \delta Q_1, 0)$$
$$P_1 = \frac{(a_1, Q_1)}{b_1}$$
$$P_2 = \frac{(a_2, Q_2)}{b_2}$$

5. Test the Models

5.1 Test the model of question one

G, M (1,1)'s prediction results of influencing factors:

Most of the original data of the influencing factors pass the level ratio test, so the grey forecasting model can be carried out, and the residual error of the simulated value of each factor is less than 10%, which shows that the prediction result has certain rationality, and the prediction result of the grey prediction model is shown in table 2.

Table 2:The prediction result of the grey prediction model

year	X_1(ten thousand people)	X_2(yuan)	X_3(billion yuan)	X_4(billion yuan)	X_5(yuan)
2019	14033	27896	980421.3	982201.3	70192
2020	14104	30651	1071957.4	1073542.8	76335
2021	14176	33678	1172064.1	1173325.3	83014
2022	14248	37004	1281462.7	1282436.8	90276
2023	14321	40659	140172.5	1401572.6	98163
2024	14394	4674	1531962.3	1531816.9	106751
2025	14467	49086	1674982.6	167428.3	116093
2026	14541	53934	1831375.2	1829869.2	126247
2027	14615	59261	2002371.4	1999805.6	137286
2028	14693	65113	2189216.3	2185743.7	149298

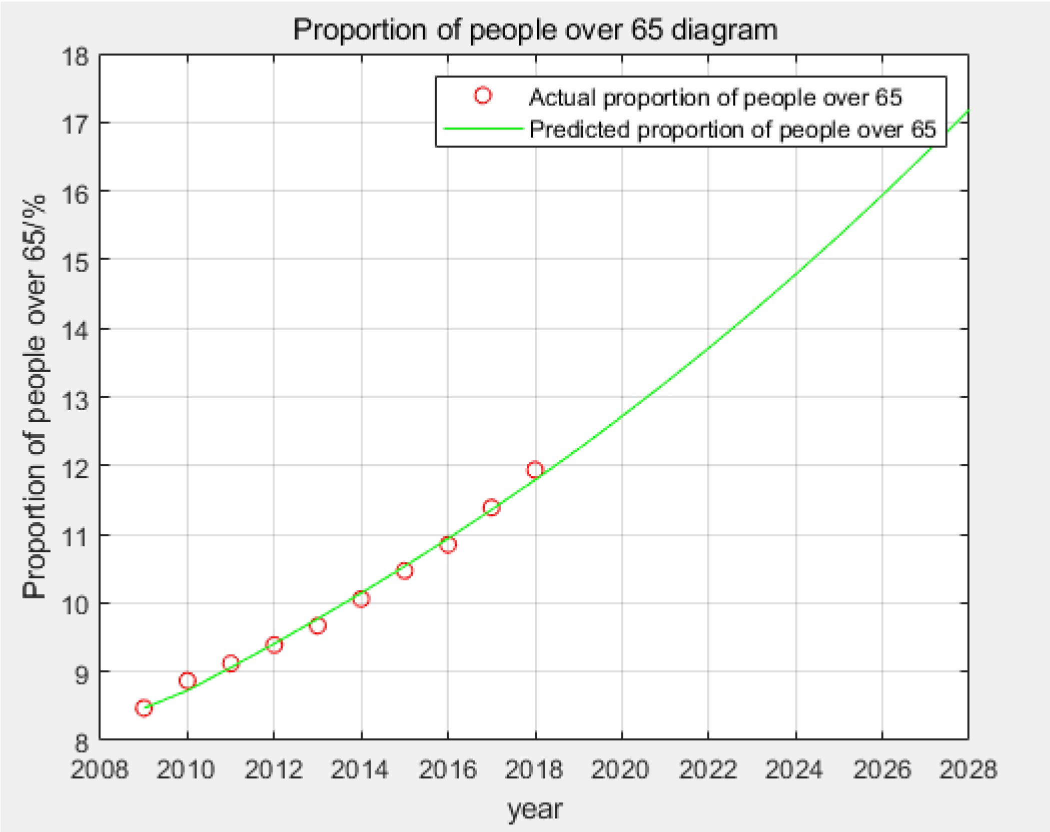
Forecast results of insurance demand based on multiple linear regression model:

Using multiple linear regression model to predict the proportion of the population over 65 years old, the error between the real value and the predicted value is very small, so using this model to predict 2019-2028 can get more accurate results in theory.

After testing and eliminating multicollinearity, heteroscedasticity and self phase, the results of multiple linear regression model can be obtained:

$$Y = -23.635 - 0.244 \ln X_1 + 6.060 \ln X_2 - 2.086 \ln X_5$$

Picture 4 shows the results of predicted and actual values:



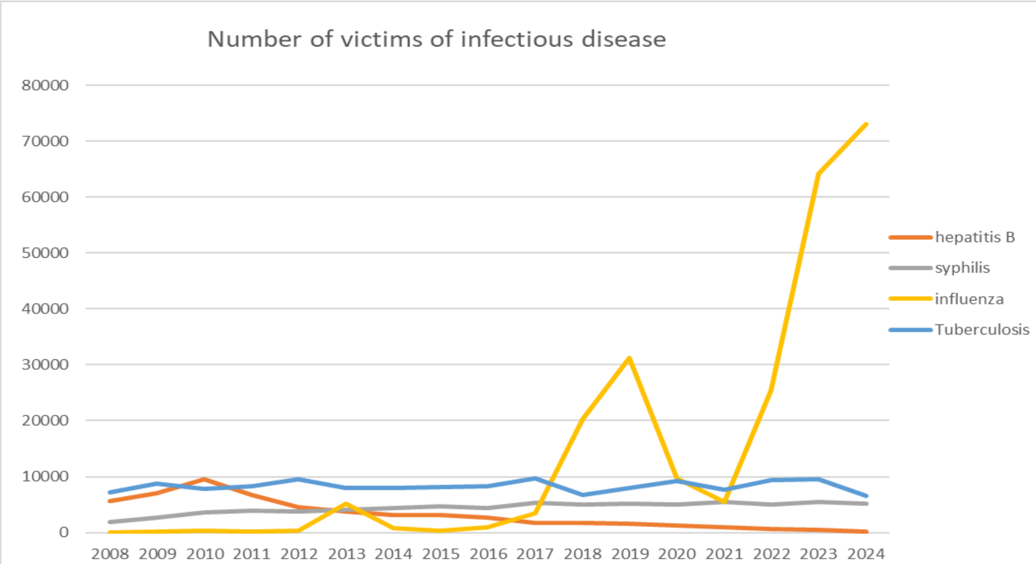
Picture 4:The actual and predicted proportion of people over 65

Table3 :shows the predicted proportion of people over 65

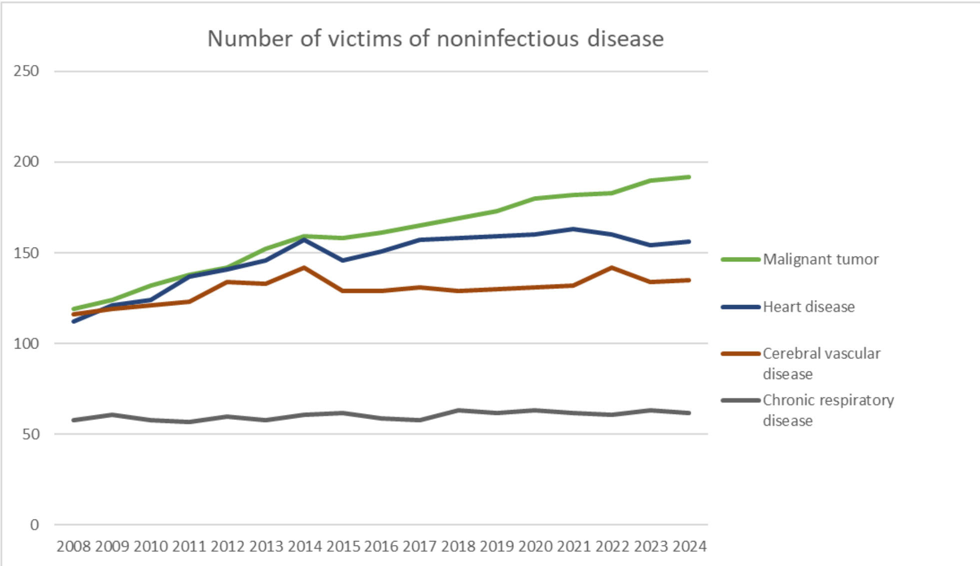
year	2019	2020	2021	2022	2023
the predicted proportion of people over 65(%)	12.25	12.72	13.20	13.71	14.24
year	2024	2025	2026	2027	2028
the predicted proportion of people over 65(%)	14.78	15.35	15.94	16.55	17.19

5.2 Test the model of question two

The following pictures dividing to infectious disease and noninfectious disease contain 4 curves which represent the special disease getting from the model which include the observation data between 2008 and 2018 while prediction data for the next 5 years.



Picture5: data of infectious disease



Picture6: data of noninfectious disease

Table 4: Prediction data of 8 Disease In Beijing From 2019 To 2024

Year	hepatitis B	syphilis	influenza	Tuberculosis	Malignant tumor	Heart disease	Cerebral vascular disease	Chronic respiratory disease
2019	1543	5210	31231	8021	173	159	130	62
2020	1234	4936	9742	9201	180	160	131	63
2021	983	5410	5421	7631	182	163	132	62
2022	632	5003	25464	9412	183	160	142	61
2023	421	5521	64213	9572	190	154	134	63
2024	213	5213	73123	6573	192	156	135	62

Compared with different data, it is obvious that hospital should make a plan to cope with for the coming large number of victims who have influenza, which seems the most popular disease among the infectious diseases. Enough mask should be prepared to avoid the separation of sickness by coughing or sneezing. In addition, as for malignant tumor which is the serious noninfectious disease in the future by prediction. Hospital should arrange significance rooms for the victims who have cancer. In addition, doctors and nurse should be trained for how to relive the stress of victims and his or her family who got malignant tumor.

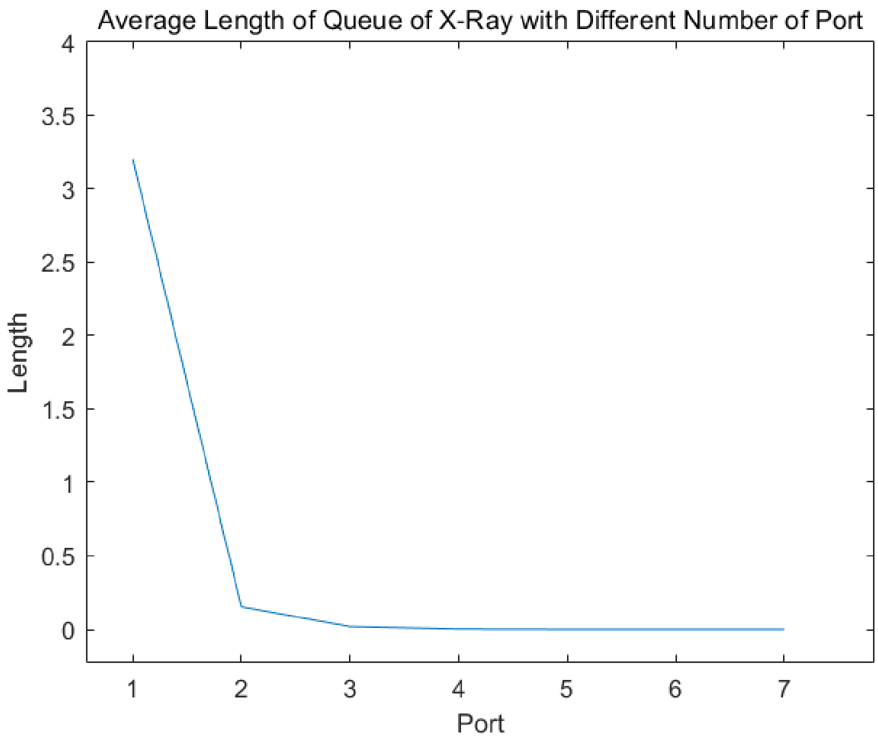
5.3 Test the model of question three

Giving the parameter of 5 inspection following:

Table5. The paraments of 5 inspection

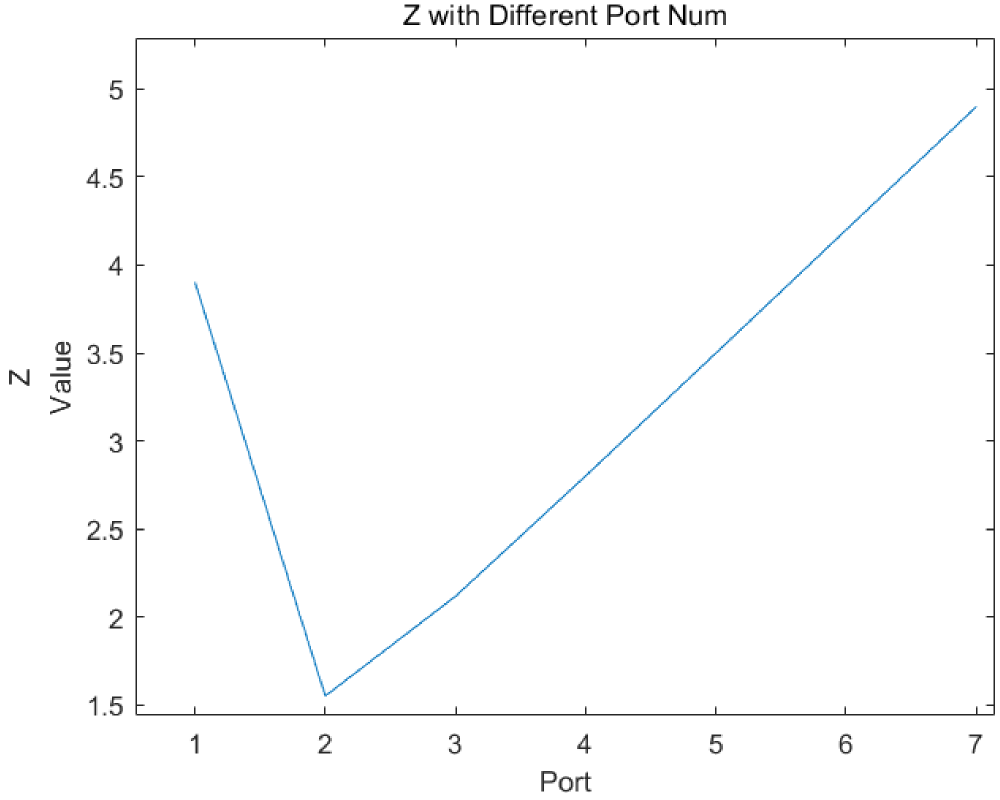
Inspection	λ	μ	ρ	γ
X-ray	2	3	0.67	0.7
B-ultrasound	0.5	0.2	2.5	0.8
Computed Tomography	0.2	0.1	2	1
Blood routine examination	12	15	0.8	0.3
Urine sample detection	15	25	0.6	0.4

Here, we take X-ray as example. We compute the $\boldsymbol{L_q}$ and the result is following:



Pcture7: Average Length-Port relationship image

And the target of Z is:



Picture8: Z_{min} value-port relationship image

As we can see, it is better to open 2 ports for X-Ray inspection.
To conclude, we could get the proper number of ports for every inspection following:

Table6. The proper number of ports for every inspection

Inspection	proper port number
X-ray	2
B-ultrasound	3
Computed Tomography	1
Blood routine examination	5
Urine sample detection	4

5.4 Test the model of question four

5.4.1 The supply of medical shared resources is more than the demand

Note: $*$ is the mark of symbiotic decision making, -is the mark of independent decision making, Δ is the mark of amount of change.
The benefit function is as follows:

$$\pi_1(Q_1) = P_1Q_1 + P_3\delta Q_2 - c_1Q_1 - d_1(\beta Q_2 - \delta Q_1)$$
$$\pi_2(Q_2) = P_2Q_2 + P_3\delta Q_2 - c_2Q_2$$
$$\pi(Q_1, Q_2) = P_1Q_1 + P_2Q_2 - c_1Q_1 - d_1(\beta Q_1 - \delta Q_2)$$

The private hospitals and public hospitals are fully cooperative in decision-making. When they cooperate completely, the quantity and price of medical services will be decided in coordination, so as to maximize the market benefits of medical and health services.

When we make sure Q_1, through the conditions of π to achieve maximum utility, when the first-order condition of $\pi(Q_1)$ is zero, we can infer that:

$$Q_1^* = \frac{(a_1 - b_1c_1 - b_1d_1\beta)}{2} \qquad Q_2^* = \frac{(a_2 - b_2c_2 - b_2d_1\delta)}{2}$$

So the optimal profit of system composed of private hospitals and public hospitals is:

$$\pi^* = \frac{\left(\frac{a_1}{b_1} - c_1 - d_1\beta\right)(a_1 - b_1c_1 - b_1d_1\beta)}{2} + \frac{\left(\frac{a_2}{b_2} - c_2 + d_1\delta\right)(a_2 - b_2c_2 - b_2d_1\delta)}{2} - \frac{(Q_1^*)^2}{b_1} - \frac{(Q_2^*)^2}{b_2}$$

The private hospitals and public hospitals are not cooperative in decision-making. In order to better explore the impact of private hospitals and public hospitals on the medical market after independent decision-making, when private hospitals know the public hospitals' demands of resources is δQ_2, the private hospitals can adjust P_3 and Q_3 to maximize the profits of private hospitals.

When we make sure Q_1, it can be found that $\pi_1(Q_1)$ is an increased function of P_3, so the private hospitals will improve the price of medical shared resources as much as possible. When $\overline{P}_3 \equiv d_2$, we can infer that:

$$\overline{Q}_1 = \frac{(a_1 - b_1c_1 - b_1d_1\beta)}{2} \qquad \overline{Q}_2 = \frac{(a_2 - b_2c_2 - b_2d_2\delta)}{2}$$

So the optimal profit of system composed of private hospitals and public hospitals:

$$\overline{\pi} = \left(\frac{a_1}{b_1} - c_1 - d_1\beta\right)\overline{Q}_1 + \left(\frac{a_2}{b_2} - c_2 + d_1\delta\right)\overline{Q}_2 - \frac{\overline{Q}_1^2}{b_1} - \frac{\overline{Q}_2^2}{b_2}$$

5.4.2 The supply of medical shared resources is less than the demand

The benefit function is as follows:

$$\pi_1(Q_1) = P_1Q_1 + (P_3\beta - c_1)Q_1$$
$$\pi_2(Q_2) = P_2Q_2 + (d_2\beta - P_3\beta)Q_1 - (c_2 - d_2\delta)Q_2$$
$$\pi(Q_1, Q_2) = P_1Q_1 + P_2Q_2 + (d_2\beta - c_1)Q_1 - (c_2 + d_2\delta)Q_2$$

The private hospitals and public hospitals are fully cooperative in decision-making.

When they cooperate completely, the quantity and price of medical services will be decided in coordination, so as to maximize the market benefits of medical and health services.

And the calculation method is the same as **5.4.1**, and we can infer that:

$$Q_1^* = \frac{(a_1 - b_1c_1 + b_1d_2\beta)}{2} \qquad Q_2^* = \frac{(a_2 - b_2c_2 + b_2d_2\delta)}{2}$$

So the optimal profit of system composed of private hospitals and public hospitals is:

$$\pi^* = \left(\frac{a_1}{b_1} - c_1 + d_2\beta\right)Q_1^* + \left(\frac{a_2}{b_2} - c_2 - d_2\delta\right)Q_2^* - \frac{Q_1}{b_1} - \frac{(Q_2^*)^2}{b_2}$$

The private hospitals and public hospitals are not cooperative in decision-making. In order to better explore the impact of private hospitals and public hospitals on the medical market after independent decision-making, when private hospitals know the public hospitals' demands of resources is δQ_2, the private hospitals can adjust P_3 and Q_3 to maximize the profits of private hospitals.

And the calculation method is the same as **5.4.2**, and we can infer that:

$$\overline{Q}_1 = \frac{(a_1 + b_1P_3 - b_1c_1)}{2} \qquad \overline{Q}_2 = \frac{(a_2 - b_2c_2 - b_2d_2\delta)}{2}$$

So the optimal profit of system composed of private hospitals and public hospitals is:

$$\overline{\pi} = \left(\frac{a_1}{b_1} - c_1 + d_2\beta\right)\overline{Q}_1 + \left(\frac{a_2}{b_2} - c_2 - d_2\delta\right)\overline{Q}_2 - \frac{\overline{Q}_1^2}{b_1} - \frac{\overline{Q}_2^2}{b_2}$$

5.4.3 Analysis of symbiotic and collaborative decision-making between private hospitals and public hospitals

The comparison of strategies in the case that the supply of medical shared resources is more than the demand.

Compared with the independent decision-making of private hospitals and public hospitals, the final profit is expressed by the change between the total profit of complete information cooperation and the total profit of non cooperation independent decision-making:

$$\begin{aligned}\Delta\pi = &\left(\frac{a_1}{b_1} - c_1 - d_1\beta\right)Q_1^* + \left(\frac{a_2}{b_2} - c_2 - d_2\delta\right)Q_2^* - \frac{(Q_1^*)}{b_1} - \frac{(Q_2^*)^2}{b_2} \\ &-\left(\frac{a_1}{b_1} - c_1 - d_1\beta\right)\overline{Q}_1 + \left(\frac{a_2}{b_2} - c_2 - d_2\delta\right)\overline{Q}_2 - \frac{\overline{Q}_1^2}{b_1} - \frac{\overline{Q}_2^2}{b_2}\end{aligned}$$

The

increment of private hospitals' service supply $\Delta Q_1 = 0$, the increment of public hospitals' service supply $\Delta Q_2 = \frac{b_2\delta(d_1+d_2)}{2} \geq 0$. And the corresponding price increment of private hospitals and public hospitals is $\Delta P_1 = 0$ and $\Delta P_2 = \frac{-\delta(d_1+d_2)}{2} \leq 0$.

In summary, the comparison of strategies in the case that the supply of medical shared resources is more than the demand, the cooperative symbiotic decision-making between private hospitals and public hospitals can bring extra profits. At the same time, the supply of private hospitals remains unchanged, but the supply of public hospitals will increase to $\frac{b_2\delta(d_1+d_2)}{2}$. Besides, the prices of private hospitals remains unchanged, but the price of public hospitals will decrease because of the increase of supply. In fact, If the supply of private hospitals exceeds the demand of public hospitals, private hospitals have to provide more costs. From the perspective of medical service market, the increase of the supply of medical services in public hospitals will increase the demand for medical resource sharing, and increase the total profit. It can be seen that collaborative decision-making is better than independent decision-making. In that case, the cooperation between private hospitals and public hospitals depends on the efforts of public hospitals and whether the cooperation can be achieved depends on the encouragement of private hospitals.

The comparison of strategies in the case that the supply of medical shared resources is less than the demand. The influence of total profit improvement in medical service market is mainly reflected in the total profit under independent decision-making:

$$\begin{aligned}\Delta\pi = &\left(\frac{a_1}{b_1} - c_1 - d_2\beta\right)Q_1^* + \left(\frac{a_2}{b_2} - c_2 - d_2\delta\right)Q_2^* - \frac{(Q_1^*)^2}{b_1} - \frac{(Q_2^*)^2}{b_2} \\ &-\left(\frac{a_1}{b_1} - c_1 + d_2\beta\right)\overline{Q}_1 - \left(\frac{a_2}{b_2} - c_2 - d_2\delta\right) + \frac{\overline{Q}_1^{\,2}}{b_1} + \frac{\overline{Q}_2^{\,2}}{b_2}\end{aligned}$$

So the increment of supply and price is: $\Delta Q_1 = 0$, $\Delta Q_2 = 0$, $\Delta P_1 = 0$, $\Delta P_2 = 0$.

Based on the above analysis, at that time, whether private hospitals and public hospitals cooperate will not influence the total profits. In fact, medical substitutes provided by private hospitals are sold to public hospitals at the same price d_2 .For public hospitals, the price of purchasing shared resources is the same as that of other resources. So it is meaningless for private hospitals and public hospitals to cooperate together at this situation.

6. Sensitivity Analysis

Here, we take question 2 as example. To analyze the sensitivity, we produce some noisy data for every true label. For disease, we add noise data by:

$$X^i_{noise} = Rand(0,2) + X_{True}$$
$$Y^i_{nosie} = Rand(0,100) + Y_{True}$$
$$i \in (0,100)$$

After the operation, we got 100 noise data for every true data. We put the new data to the model.
With more epochs, we got the same result as picture 5 and 6.

7.Strengths and Weakness

7.1 Strengths

1. For question 1, the G,M(1,1) model can better predict the value of each factor, it also lets values that do not conform to the preliminary situation are corrected by translation transformation
2. For question 2, the model uses Neural Networks and consider the formation of the input variable which reduces the error of the loss and improve the robustness
3. For question 3, the model gives the optimum data for different inspection site, taking the cost of new port into consideration.
4. For question 4, we put independent decison and symbiotic decision by private hospitals and public hospitals efficiently.

7.2 Weakness

1. For question 1, the predicted value may be affected by emergencies.
2. For question 2, insufficient data causes the result do not fit well while can not predict the future data.
3. For question 3, the model does not consider the patient leaving in the line and other unforeseen factors.
4. For question 4, the technical and personnel elements are not considered.

8. Conclusion

Recommendation

With the development of the economics and technology, the life span of people has been extended and the living condition has been improved. However, it also causes loads of problem to cope with.
According to question 1, we can predict that the proportion of the aging people over 65 will increase to 17% in 2028. The whole society, especially the hospital, will have great stress to take care of the aging people. Besides, we can deduce that the proportion of people who have chronic disease will steadily increase. The malignant tumor, which takes account for the most percentage among the chronic disease is a big challenge for the society. Apart from the chronic disease and aging, infectious disease is also a

curious problem. Influenza, for example, will be the most popular disease in the future. We would like to offer some suggestion by solving the question 2,3,4. Firstly, government should invest more on public hospital including money, technology, experts and so on. And the private hospital could provide the medical resource to the public hospital while the public hospital could transfer some patients to the private hospital. It will make the joint progress. In addition, inner hospital has something to improve and reform. Some inspection sites could set the number of ports more sensitive like the conclusion in 5.3. Moreover, people should protect themselves from disease like influenza. They can exercise every day and keep clean to be healthier while they could wear a mask to prevent the germs.

The “14th Five-Year Plan” is the essential period for the socialist modernization drive. The level of hospital connecting to the domestic health level of people plays a key role in the development of the economics. For the coming decade, more money should be invested for the medical research. Balancing the medical research between private hospital and public hospital is also important by Ministry of Health Department. As for Propaganda Department of the CPC, they should advocate the healthy life style and correct disease knowledge. The hospital should check themselves to reform unreasonable service which is also a key way.

References

[1]Beijing Municipal Bureau of Statistics. Beijing Statistical Yearbook 2017[M]. Beijing: China Statistics Press, 2017:2-25

[2] Beijing Municipal People's Government. Report on health and population health in Beijing in 2016[M]. Beijing: People's Health Press, 2017:2-25.

[3]MA Gui-hua, WANG Yu-fei, ZHU Jia-ming. Forecast of Commercial Health Insurance Demand under the Background of Population Aging——Based on the Combined Forecasting Model[J]. Journal of Tianjin Sino German University of Applied Technology, 2019(4).

[4]ZHANG Ming-yue, GAO Xing. Prediction of risks of non-communicable diseases in population in Beijing area[J]. Practical preventive medicine, 2019(6).

[5]NING Zheng, NIU Hongxia, ZHANG Zhaoxin. Automatic operation regulation optimization model of metro train based on queuing theory[J]. Journal of Railway Science and Engineering, 2019(7).

[6]JING Ri-ze, FANG Hai. Comparative Study on the Total Factor Productivity Changes in Public Hospitals and Private Hospitals of Beijing[J]. Health economy in China, 2018(9).

[7]JING Ri-ze, ZHANG Lu-yu, ZHANG Hu-yang, XU Ting-ting, GAO Shan, YANG Shuo, FANG Hai. Comparative analysis of efficiency between public and private hospitals in Beijing ——based on DEA model[J]. Health economy research, 2018(6).

[8]LI Xi-Ping, LIU Hui. Benefit Decision-making Mechanism Research on the Symbiotic Cooperation between Private Hospitals and Public Hospitals Based on Optimized Theory[J].Health economy in China,2017(10).

[9]Maimaitiaizezi Aili. Further Research on the M/G/1 Queueing Model with Additional Optional Service and No Waiting Capacity[J]. Acta Analysis Functionalis Applicata, 2018(3).

[10]LI Bi. Production logistics enterprise competition and cooperation strategy earnings model and risk analysis[J]. Logistics engineering and management, 2009(1).

Appendix

```
clc;clear;
syms a b;
c = [a b]';
A = [8.47,      8.87,9.12,9.39,9.67,10.06,10.47,10.85,  11.39,11.94,];
n = length(A);
B = cumsum(A);
for i = 2:n
    C(i) = (B(i) + B(i - 1))/2;
end
```

```
C(1) = [];
B = [-C;ones(1,n-1)];
Y = A; Y(1) = []; Y = Y';
c = inv(B*B')*B*Y;
c = c';
a = c(1); b = c(2);
F = []; F(1) = A(1);
for i = 2:(n+10)
    F(i) = (A(1)-b/a)/exp(a*(i-1))+ b/a;
end
G = []; G(1) = A(1);
for i = 2:(n+10)
    G(i) = F(i) - F(i-1);
end
disp('预测数据为：');
G
H = G(1:10);
epsilon = A - H;
delta = abs(epsilon./A);
disp('相对残差 Q 检验：')
Q = mean(delta)
disp('方差比 C 检验：')
C = std(epsilon, 1)/std(A, 1)
S1 = std(A, 1);
tmp = find(abs(epsilon - mean(epsilon))< 0.6745 * S1);
disp('小误差概率 P 检验：')
P = length(tmp)/n
t1 = 2009:2018;
t2 = 2009:2028;
plot(t1, A,'ro'); hold on;
plot(t2, G, 'g-');
xlabel('year'); ylabel('Proportion of people over 65/%');
legend('Actual proportion of people over 65','Predicted proportion of people over 65');
title('Proportion of people over 65 Diagram');
grid on;
```