

Study on the Security Problems of UAVs in Yangpu District Under G20 in 2016

Abstract

We have established four models in this paper : the optimal model based on graph theory, Fussy synthetic evaluation model, the optimal model based on regional time difference, and the optimal model of the monitoring range of UAVs. Then, we use MATLAB, EXCEL, and other softwares to solve the securities problems of UAVs in Yangpu District, thus obtaining comparatively reasonable results. We also analyze and conclude problems from multiple perspectives, and evaluate and promote those modelings, having certain significance on the security application of UAVs and the distribution of national security strategy.

In Problem 1, we firstly define the parameters of a UAV, contributing to the calculation of its monitoring range. Then, we divide the map of Yangpu District into grids according to the monitoring range of UAVs, store the statistics by using graph theory, and establish the optimal model based on graph theory. As target function is the least number of UAVs, we then constrain the monitoring time and the monitoring range so that one-way flight time in every UAV route shall be no more than 7.5 minutes and the union set of the label set of every route could cover all labels. At last, we use MATLAB to solve the model and obtain that 13 UAVs are needed to carry out the surveillance of all places of Yangpu District.

In Problem 2, considering that the definition of the amount of human traffic is a relative concept, we adopt Fuzzy synthetic evaluation model to evaluate human traffic level in every area, and use the levels to distribute monitoring time intervals. After that, we improve the model in Problem 1, mostly in terms of the monitoring time difference of every area, and then we obtain the improved optimal model based on regional time difference. At last, we obtain that the least number of UAVs needed is 20 in order to fulfill the prompt monitoring of every area of Yangpu District.

In Problem 3, which is based on Problem 1, we obtain that the available UAVs is 9. At the same time, single journey flight time of every route should be near to 7.5 minutes as much as possible to reach the largest monitoring range. Also, considering that the girds in the boundary areas of Yangpu District may not be intact, we make the UAVs in Problem 3 avoid passing the incomplete grids as much as possible. In consequence, three implied conditions can be summed up in the following:

- The number of available UAVs is 9.
- The one-way flight time of every UAV should be near to 7.5 minutes as much as possible.
- A UAV should avoid passing the incomplete grids as much as possible.

At last, we use MATLAB to solve the problem and obtain that 9 UAVs are used in this section, which can provide the largest monitoring range of 47.5 square kilometers, thus covering 78.37% of the whole Yangpu District.

The biggest highlight of this paper is the mathematical expression of constraints, for we adopt the union set and intersection set to fulfill the mathematical expression of covering all monitoring areas, and of not passing incomplete grids. At the same time, this paper solves the hot issue of the security problems of UAVs. And the results obtained are precise and innovative.

Keywords: Graph theory; optimal model; Fussy synthetic evaluation model; MATLAB; UAV

1. A Few Words About the Problems

Introduction

1.1 Background and Significance of the Problems

The Group of Twenty(also known as the G-20 or G20) is an international forum for the governments and central bank governors from 20 major economies. The 11th G20 meeting will be held in China two years later. We assume that the final conference center locates in the Yangpu District of Shanghai. The host city needs to invest plenty of manpower to stabilize the order of the city. Because of the development of unmanned aerial vehicle(UAV), it has been used in the field of the security.

1.2 Presentation of the Problem

Now the government hires us to design a surveillance plan for the entire borough of Yangpu by the UAVs. The current UAVs are relatively robust to complicated external environment, can fly up to 4 hours without need to refuel, and require no human being to monitor each of them. Instead, a sophisticated computerized controller can be programmed to follow any patrol strategy of one's choice. The government requires our team to accomplish the following different plans:

Requirement 1: All geographic point of Yangpu District should remain observed from the air for at least 15 minutes in a row. How many UAVs will us need to achieve this goal?

Requirement 2: Some parts of the district are more important, e.g. the neighborhood of Fudan University and the Wanda Plaza. Such areas should be observed at least once in each 5 minutes interval. On the other hand, some roads has lower density of people, and there is no need to observe it more than once in 20 minutes. How many UAVs will us need to provide the requested variable level of coverage?

Requirement 3: Assuming that all areas are equally important and should remain regularly observed, but some UAVs are not reliable and 30% of them become unusable. What kind of surveillance coverage will your plan provide?

2. Basic Assumptions

- Assume that the observed area of a UAV is like a square.
- Assume that the flight speed of a UAV is unchanging all the way.
- Assume that a UAV is unaffected in the execution phase.

3. Symbol Description

Serial number	Symbols	Symbol Description
1	m	The number of UAVs
2	$\vec{a}(x)$	The label vector of grids the x^{th} route has passed
3	S	The area of the observed places
4	n	Dimension of the label vector of grids
5	$t(j)$	The single flight cycle of the j^{th} route

6	$b(j)$	The label set the j^{th} has passed
7	$\vec{T}(k)$	The monitoring time interval of the k^{th} area

4. Problem Analysis

4.1 Analysis of Problem 1

It is required that all geographic point of Yangpu District should remain observed from the air for at least 15 minutes in a row and then the number of UAVs needed to satisfy this goal is to be calculated. Therefore, we divide the map of Yangpu District into grids, store the statistics by using graph theory ^[1], and establish the optimal model based on graph theory through determining target function, and constraining monitoring time and monitoring range, and finally solve the model.

4.2 Analysis of Problem 2

It is required that the importance degree of every area should be differentiated based on human traffic level, and then the monitoring time interval should be laid out based on the levels, and after that the least number of UAVs is require to be calculated. Considering that the definition of the amount of human traffic is a relative concept, we adopt Fuzzy synthetic evaluation model to evaluate human traffic level in every area, and use the levels to distribute monitoring time intervals. After that, we improve the model in Problem 1, mostly in terms of the monitoring time difference of every area, and then we obtain the improved optimal model based on regional time difference. At last, the least number of UAVs needed is obtained.

4.3 Analysis of Problem 3

It is required that the largest surveillance coverage should be calculated under the condition that all areas are equally important and 30% UAVs are unusable. This problem is based on Problem 1, from which we obtain that the available UAVs is 9. At the same time, single journey flight time of every route should be near to 7.5 minutes as much as possible to reach the largest monitoring range. Also, considering that the grids in the boundary areas of Yangpu District may not be intact, we make the UAVs in Problem 3 avoid passing the incomplete grids as much as possible.

5. Modeling Establishment and Solution of Problem 1

We define the whole monitoring range by determining the parameters of a UAV combined with its own monitoring areas. In order to study more precisely, we divide the Yangpu District map into grids, make statistical storage of graph theory, establish the optimal model based on graph theory, constraint monitoring time and monitoring area, and at last solve the modeling problem. The steps are as follows.

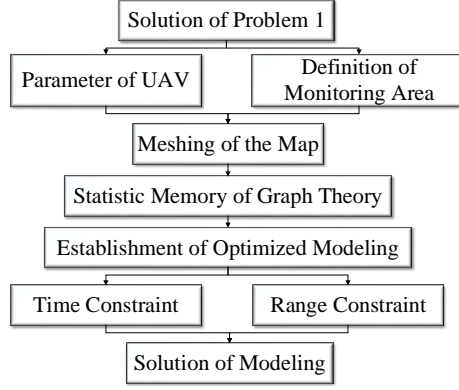


Figure 1. Flow diagram of solution process of Problem 1

5.1 Preparations Before Modeling

5.1.1 Determination of the Parameters of a UAV and Its Monitoring Area

As a high-tech product, UAV is gradually applied in national defense and fields of security ^[2]. It is composed of flight control system, ground station system, communication link system, payload system, takeoff and recovery system. Usually, small UAVs refer to those having the maximum takeoff weight less than 10 kilograms. They are used extensively in more and more fields, like security and environmental monitoring, because of their light weight, small size, low cost, and being easy to operate. The ground monitoring system of UAV, also called GCS(Ground Control Station), has the functions of data link communication, flight conditions, revelation and control of flight path, memory and recovery of flight statistics, and also the revelation and control of payload statistics and the like. After referring to some literature, we determine the parameters of a UAV, as indicated in **Table 1**:

Table 1. The parameters of a UAV

Semidiameter (m)	Viewing angle ($^{\circ}$)	Height(m)	Speed (m/s)
500	60	288	10

When flying overhead while monitoring, a UAV radiates with its viewing angle to be 60 degrees($\varphi = 60^{\circ}$). Its monitoring area appears to be a circular shape with its semidiameter to be 500 meters calculated by given parameters from the above table, as indicated in **Figure 2**:

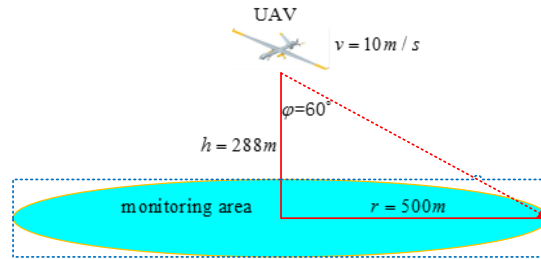


Figure 2. The space surveillance diagram of a UAV

Considering that the monitoring range of a UAV might not be strictly a real circle, however, in order to simplify the solution process, we assume the observed area to be the circumscribing square of the circle. That is, when flying over the mid-point of the square area, it can monitor the whole square area with its side length to be 1000 meters. The plan sketch of monitoring is presented in **Figure 3**.

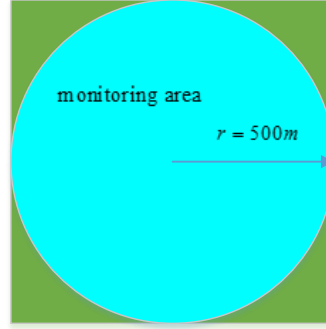


Figure 3. Surveillance plan of a UAV

All in all, the above is the definition of the parameter of a UAV and its monitoring area.

5.1.2 Meshing of Yangpu District

Yangpu District covers an area of 60.61 square meters. To simplify the calculation process, the grid division of Yangpu District is carried out combined with the monitoring area of a UAV. Every grid stands for the monitoring area of a UAV, which is the mentioned square with the side length to be 1000 meters. Besides, the map scale is 1:119408, based on which we make a division of grids on the map. And the division result is as follows:



Figure 4. The map of Yangpu District

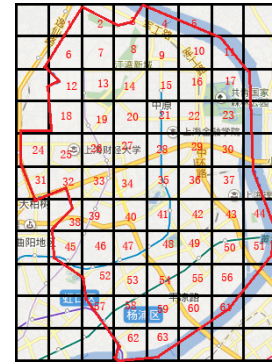


Figure 5. Division of grids

As for the boundary areas of Yangpu District, only when at least a half area of any grid of them is within Yangpu District, can that grid be labeled as a full grid; otherwise, it would be excluded.

Every grid is labeled. From **Figure 5**, we can see that altogether Yangpu District is divided into 63 grids.

5.1.3 Statistic Memory Based on Graph Theory

We build weight graph $G=(V, E, \mathbf{W})$ to better store the statistical information of Yangpu District, of which vertex set is $V(G)=\{v_1, v_2, \dots, v_n\}$, and v_1, v_2, \dots, v_n correspond with the labels of grids ($n=1, 2, 3, \dots, 63$), and $E(G)$ is the set of sides, while $\mathbf{W}(G)$ is adjacent matrix. The definition of $\mathbf{W}(G)=(w_{ij})_{n \times n}$ is as follows:

$$w_{ij} = \begin{cases} 1.67 \text{ min} & v_i \text{ adjacent to } v_j \\ 0 & v_i \text{ nonadjacent to } v_j \end{cases}$$

where 1.67 minutes is obtained by calculation, the midpoint distance of two adjacent grids is 1000 meters, and the speed of a UAV is 10 m/s, therefore, flight time is 100 seconds, that is, 1.67 minutes.

5.2 Establishment of the Optimal Model Based on Graph Theory

5.2.1 Target Analysis

The number of UAVs is required to be the least, therefore the objective function is:

$$\min m \quad (1)$$

where m stands for the number of UAVs.

5.2.2 Constraint Analysis

1) Constraint of Monitoring Time

It is required that the monitoring time of all places be no more than 15 minutes, which requires every UAV to have its own fixed route, for if its route changes every time, arrangement work is likely to be disorganized, based on which, we can see that the one-way time of every UAV route must be no more than 7.5 minutes so that the requirement can be satisfied. Therefore, constraint is established as follows:

$$\sum_{i=2}^n w_{\vec{a}(x)_{i-1} \vec{a}(x)_i} \leq 7.5 \min \quad (2)$$

where

$\vec{a}(x)$ is the labeling vector of grids the x^{th} flight course has passed; $x=1,2,\dots,m$,

$w_{\vec{a}(x)_{i-1} \vec{a}(x)_i}$ is an element of adjacent matrix, and

n is the dimension of labeling vector of grids.

Examples are as follows:

If the labels the third UAV route has passed is 19,26,27,28,35, therefore, it is marked as $\vec{a}(3)=(19,26,27,28,35)$, and vector dimension $n=5$, $\vec{a}(3)_1=19$, $\vec{a}(3)_2=26$, $\dots, \vec{a}(3)_5=35$, therefore, it is obtained that $w_{\vec{a}(3)_1 \vec{a}(3)_2} = w_{19,26}$, and other values can be obtained when referring to adjacent matrix.

2) Constraint of Monitoring Range

All places are required to be monitored, therefore this section implements a constraint in terms of set. Assume the label set the x^{th} route has passed is $b(x)$; then we obtain:

$$b(x) = \{\vec{a}(x)_1, \vec{a}(x)_2, \dots, \vec{a}(x)_n\} \quad (3)$$

where

$x=1,2,\dots,m$,

m is the number of UAVs, and

n is the dimension of the label vector of grids.

To monitor all grids, the union set of the label set of every route should cover all labels, that is:

$$b(1) \cup b(2) \cup \dots \cup b(m) = \{1,2,3,4,\dots,62,63\}$$

5.2.3 Establishment of Modeling

According to the above objective function and constraints, the optimal modeling based on graph theory is established:

$$\min m \quad (4)$$

$$s.t. \begin{cases} \sum_{i=2}^n w_{\vec{a}(x)_{i-1} \vec{a}(x)_i} \leq 7.5 \min \\ b(1) \cup b(2) \cup \dots \cup b(m) = \{1,2,3,4,\dots,62,63\} \end{cases}$$

5.3 Solution of Optimized Modeling Based on Graph Theory

5.3.1 Introduction of Algorithm

Step1: Use MATLAB to read adjacent matrix \mathbf{W} , and define 63-dimension vector \mathbf{a} as the number of UAVs able to be detected in the i^{th} region and its original state is 0, that is, $\mathbf{a} = \mathbf{0}$.

Step2: Look for $\mathbf{a}_i = 0$ from the 1^{th} grid to the 63^{th} one, and make the flight time(t) of UAVs equal to 0, that is, $t = 0$, $\mathbf{a}_i = \mathbf{a}_i + 1$, and then enter Step3. If $\mathbf{a}_i = 0$ does not exist, the procedure ends.

Step3: When $t > 7.5$, enter Step2, otherwise, entering Step4.

Step4: Look for $w_{ij} \neq 0$ and $w_{ij} \neq \infty$ from the 1^{th} grid to the 63^{th} one, and j minimizing the value of \mathbf{a}_j , is considered to be UAVs' next monitoring region. If $t + w_{ij} < 7.5$, make $i = j$, $t = t + w_{ij}$, $\mathbf{a}_j = \mathbf{a}_j + 1$, and then return to Step3, otherwise, entering Step 2.

5.3.2 Solution Result

According to the above algorithm steps and MATLAB program, the results obtained are shown in **Table 2**:

Table 2. Distribution of flight course

UAV(number)	Flight course	UAV(number)	Flight course
1	1->2->3->4->5	2	6->7->8->9->10
3	11->17->16->15->14	4	12->13->19->18->25
5	20->21->22->23->30	6	24->31->32->33->26
7	27->28->29->36->35	8	34->40->39->38->45
9	37->43->42->41->48	10	44->51->50->49->55
11	46->47->53->52->57	12	54->59->58->62->63
13	56->61->60->55->49		

Distribution diagram of flight courses is shown in **Figure 6**:



Figure 6 Distribution diagram of flight course

where there are 13 routes, which means 13 UAVs are needed to satisfy the prompt monitoring of all places of Yangpu District.

5.4 Result Analysis

Firstly, we define the parameters of a UAV and its monitoring areas; secondly, the map of Yangpu District is divided into grids, and the statistics of graph theory is stored; thirdly, the optimal model based on graph theory is established and monitoring time and monitoring range are constrained; lastly, 13 UAVs needed are obtained and

the prompt monitoring of Yangpu District is achievable. The optimal model adopted is not only intuitive, but can precisely calculate the number of UAVs monitoring all places of Yangpu District.

6. Modeling Establishment and Its Solution of Problem 2

6.1 Problem Analysis and Choice of Modeling

Problem 2 requires us to differentiate the importance degree of every area of Yangpu District according to human traffic level, and to define monitoring time intervals based on the importance degree, and finally to calculate the least number of UAVs needed when satisfying the first two requirements. Considering that the amount of human traffic is a relative concept, we couldn't quantitatively define importance degree based on it, and it is hard to calculate the human traffic of every area of Yangpu District. However, we can vaguely use some iconic places, for example, shopping malls, universities, to define the relative amount of human traffic. Therefore, the important places mentioned in this section are a fussy concept, because of which we adopt Fuzzy synthetic evaluation model to evaluate human traffic level in every area, and use the levels to distribute monitoring time intervals.

After getting the monitoring time intervals, we can improve the modeling adopted in Problem 1 to obtain the optimized modeling based on regional time difference. Besides, the least number of UAVs can be obtained. The flow chart of the solution process of Problem 2 is presented in **Figure 7**.

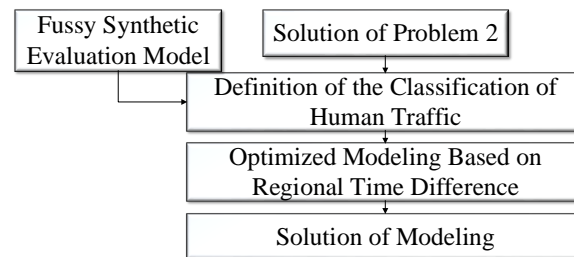


Figure 7. Flow chart of the solution process of Problem 2

6.2 Gathering of Fussy Evaluation Statistics

We use Experts Grading Method to obtain the rating situation of human traffic in every area. One grid is labeled as an area. The original statistics is obtained by distributing questionnaires on line and by scoring tables. The experts should be familiar with the changing situation of the topography and human traffic of Yangpu District.

6.3 Establishment and Solution of Fuzzy Synthetic Evaluation Model

Because it is hard to calculate the human traffic of every area of Yangpu District. Therefore, we can vaguely use some iconic places, for example, shopping malls, universities, to define the relative amount of human traffic ^[3]. In consequence, the important places mentioned in this section are a fussy concept, because of which we adopt Fuzzy synthetic evaluation model to evaluate memberships of human traffic in every area according to maximum membership principle so as to define the monitoring time intervals of UAVs.

6.3.1 Definition of Membership Function

Step1. Establishment of Index Set and Comment Set

Human traffic of a place includes fixed population and floating population. Therefore, the index set is $u = \{\text{fixed population}, \text{floating population}\}$.

When analyzing the above index set, we see that every index can be measured by “more or less”. Therefore, according to the amount of human traffic, we divide the comment set into four degrees, that is $v = \{\text{more, general, less, very less}\}$. And we label them as the first, second, third, and fourth level separately. If an area belongs to the first level--“more”, its monitoring time interval is 5 minutes; the second level--“general” stands for 12 minutes, the third level-- “less” for 18 minutes, and the fourth level “ very less” for 25 minutes.

Evaluation data is obtained by expert grading on evaluation objects carried out by Shanghai’s experts and other professionals studying on human traffic according to assessment level, and by our calculation of scoring results. The scoring criteria of every level is $(S_1, S_2, S_3, S_4) = (10, 7, 5, 3)$, that is, when a score m_{ij} satisfies $7 \leq m_{ij} \leq 10$, it meets the first level; when a score satisfies $5 < m_{ij} < 7$, it satisfies the second level, and the like.

Step2. Definition of Fussy Synthetic Evaluation Matrix

As the scores of every index are fussy, it is reasonable to mark out gradational boundary by employing membership degree. Therefore, single factor assessment matrix is composed by the row element r which is single factor membership. Membership degree is defined by calculating membership function. According to the characteristics of data distribution, we choose “trapezoid” function and we establish every function based on the four standards of every factor.

The first level: when $k = 1$, the function is:

$$R_{ik} = \begin{cases} 1, S_{k+1} < m_{ij} < S_k \\ \frac{m_{ij} - S_{k+3}}{S_{k+1} - S_{k+3}}, S_{k+3} < m_{ij} < S_{k+1} \\ 0, m_{ij} \leq S_{k+3} \end{cases} \quad (5)$$

The second level: when $k = 2$, the function is:

$$R_{ik} = \begin{cases} \frac{S_{k-1} - m_{ij}}{S_{k-1} - S_k}, S_k \leq m_{ij} \leq S_{k-1} \\ 1, S_{k+1} < m_{ij} < S_k \\ \frac{S_{k+1} - m_{ij}}{S_{k+1} - S_{k+2}}, S_{k+2} < m_{ij} < S_{k+1} \\ 0, m_{ij} < S_{k+2} \end{cases} \quad (6)$$

The third level: when $k = 3$, the function is:

$$R_{ik} = \begin{cases} 0, S_{k-1} \leq m_{ij} \leq S_{k-2} \\ \frac{S_{k-1} - m_{ij}}{S_{k-1} - S_k}, S_k < m_{ij} < S_{k-1} \\ 1, S_{k+1} < m_{ij} < S_k \\ \frac{m_{ij}}{S_{k+1}}, m_{ij} < S_{k+1} \end{cases} \quad (7)$$

The fourth level: when $k = 4$, the function is:

$$R_{ik} = \begin{cases} 0, S_{k-2} < m_{ij} < S_{k-3} \\ \frac{S_{k-2} - m_{ij}}{S_{k-2} - S_k}, S_k < m_{ij} < S_{k-2} \\ 1, m_{ij} \leq S_k \end{cases} \quad (8)$$

Bring certain scores of some area made by experts into the previous definite membership function, and then membership degree can be calculated. Moreover, single factor fuzzy evaluation matrix of every area is established, that is R_{ik} ($i=1,2;k=1,2,3,4$); when assembling each of them, we obtain fuzzy synthetic evaluation matrix \mathbf{R} .

6.3.2 Definition of Weight

Weight is a relative concept, measuring the relative amount of the influence of certain index of index set on human traffic [2]. The bigger the weight coefficient is, the more influence this index has on human traffic. When analyzing all scores, we find that when the scores of similar index made by different experts convergence, which means experts agree with each other on that index, and that the qualitative nature of it specified by them is more accurate, and that the weight of it should be bigger. Instead, when experts evidently give different scores on the same index, which means they somewhat disagree with each other on that index, and that the qualitative nature of it given by them is comparatively fuzzy, and that the weight of it should be made smaller. As a result, we choose the reciprocal of variance as the weight of every index, and then normalize every weight. Finally, we get:

$$w_j' = \frac{1}{s_j}, w_j = \frac{w_j'}{\sum_{j=1}^m w_j'}, j=1,2,\dots,m \quad (9)$$

where $s_j = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_{ij} - \bar{x})^2}$, $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_{ij}$ ($j=1,2,\dots,m$),

where

j is the number of index. The number of index is 2 in this section, that is, $m=2$, from which we obtain the weight set $A=(w_1, w_2)$, and use it in fuzzy synthetic evaluation.

6.3.3 Definition of the Level of Human Traffic

After defining fuzzy synthetic evaluation matrix \mathbf{R} and weight set \mathbf{A} , we make a fuzzy synthetic evaluation on human traffic of every area, that is:

$$\mathbf{B} = \mathbf{A} \cdot \mathbf{R} \quad (10)$$

After obtaining evaluation vector \mathbf{R} , we can determine the level of human traffic according to maximum membership principle, based on which we then determine monitoring time intervals.

6.3.4 Solution of Fuzzy Synthetic Evaluation Model

Yangpu District is altogether divided into 63 areas. Here, we take the human traffic levels of 2 areas as solution examples. Now, take Area Number One and Area Number 25 as examples.

The rating situation in Area One (10 points system):

Table 3. The rating situation of Expert 1

Fixed population (points)	5	6	5	6	4
Floating population (points)	4	7	6	6	5

Take the mean value of the above five groups of scores made by 5 experts into membership function, and then we obtain the fuzzy synthetic evaluation matrix \mathbf{R} of Area 1, which is as follows:

$$R_1 = \begin{bmatrix} 0.33 & 0.67 & 0.25 & 0 \\ 0.5 & 1 & 0.5 & 0 \end{bmatrix}$$

Evaluate the variance of every index and take its reciprocal as weight; then normalize all weights, and we obtain:

$$s_1 = 0.56, \quad s_2 = 1.04, \quad w_1' = \frac{1}{0.56} = 1.79, \quad w_2' = \frac{1}{1.04} = 0.96$$

therefore:

$$w_1 = \frac{1.79}{1.79 + 0.96} = 0.65, \quad w_2 = \frac{0.96}{1.79 + 0.96} = 0.35$$

and:

$$A_1 = [0.65 \quad 0.35]$$

The evaluation vector obtained is as follows:

$$B_1 = A_1 \cdot R_1 = [0.65 \quad 0.35] \cdot \begin{bmatrix} 0.33 & 0.67 & 0.25 & 0 \\ 0.5 & 1 & 0.5 & 0 \end{bmatrix} = [0.39 \quad 0.79 \quad 0.34 \quad 0]$$

The human traffic of this area belongs to the second level--“general” according to maximum membership degree, therefore, its monitoring time interval is 12 minutes.

The rating situation of Area 25:

Table 4. The rating situation of Expert 2

Fixed population (points)	7	8	8	7	8
Floating population (points)	6	7	8	6	8

Likewise, take the mean value of the above five groups of scores made by 5 experts into membership function, and then we obtain the fuzzy synthetic evaluation matrix R of Area 25, which is as follows:

$$R_{25} = \begin{bmatrix} 1 & 0.67 & 0.33 & 0 \\ 0.33 & 0.5 & 0.25 & 0 \end{bmatrix}$$

Consider the reciprocal of variance of every index as weight, and normalize every weight. Therefore, we obtain:

$$s_1 = 0.24, \quad s_2 = 0.80, \quad w_1' = \frac{1}{0.24} = 4.17, \quad w_2' = \frac{1}{0.80} = 1.25$$

therefore:

$$w_1 = \frac{4.17}{4.17 + 1.25} = 0.77, \quad w_2 = \frac{1.25}{4.17 + 1.25} = 0.23$$

and:

$$A_{25} = [0.77 \quad 0.23]$$

The evaluation vector obtained is as follows:

$$B_{25} = A_{25} \cdot R_{25} = [0.77 \quad 0.23] \cdot \begin{bmatrix} 1 & 0.67 & 0.33 & 0 \\ 0.33 & 0.5 & 0.25 & 0 \end{bmatrix} = [0.85 \quad 0.63 \quad 0.31 \quad 0]$$

Evaluate the monitoring time interval of every area of Yangpu District, we obtain the vector of the regional monitoring time interval \vec{T} ,

$$\vec{T} = (12, 12, 18, 25, \dots, 25, 25)_{1 \times 63}$$

Using different colors to differentiate monitoring time intervals, we obtain the distribution map of human traffic level, which is as follows:

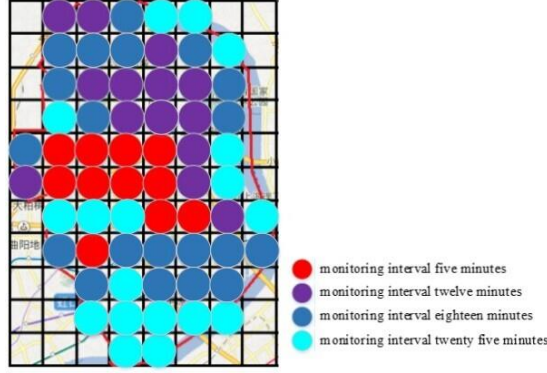


Figure 6. Distribution map of human traffic level

6.4 The Optimal Model Based on Regional Time Difference

6.4.1 Target Analysis

The least number of UAVs is required, therefore the target function is as follows:

$$\min m \quad (11)$$

where m is the number of UAVs.

6.4.2 Constraint Analysis

1) Constraint of Monitoring Time

We obtain the vector of regional monitoring time intervals according to Fussy synthetic evaluation model. It stands for the least time interval every area needs. When many UAVs pass the same area, the time devoid of monitoring in that area is that of any UAV with the shortest flight route cycle. Therefore, the constraint can be established as follows:

$$\sum_{i=2}^n w_{\vec{a}(x)_{i-1} \vec{a}(x)_i} \leq t(j), \quad j=1,2,\dots,m \quad (12)$$

$$t(j) = \min \frac{\bar{T}(k)}{2}, k \in b(j) \quad (13)$$

where

$\vec{a}(x)$ is the labeling vector of grids the x^{th} flight course has passed; $x=1,2,\dots,m$,

$w_{\vec{a}(x)_{i-1} \vec{a}(x)_i}$ is an element of adjacent matrix,

n is the dimension of labeling vector of grids,

$t(j)$ is the one-way flight cycle of the j^{th} flight course,

$b(j)$ is the labeling set the j^{th} flight course has passed, and

$\bar{T}(k)$ is the monitoring time interval of the k^{th} area.

Examples are as follows:

If the labels the third UAV route has passed is 19,26,27,28,35, therefore, it is marked as $\vec{a}(3) = (19, 26, 27, 28, 35)$, and vector dimension $n=5$, $\vec{a}(3)_1 = 19$, $\vec{a}(3)_2 = 26$, $\dots \vec{a}(3)_5 = 35$, therefore, it is also obtained that $w_{\vec{a}(3)_1 \vec{a}(3)_2} = w_{19,26}$, and other values can be obtained when referring to adjacent matrix. The monitoring time interval of the mentioned 5 areas is respectively 12 minutes, 5 minutes, 18 minutes, 25 minutes, 12 minutes from which we take the shortest time interval, which is $t(3) = 5/2 = 2.5\text{min}$, indicating that the one-way flight cycle of that route is 2.5 minutes. Only in this way can this constraint satisfy the given requirements of Problem 2.

2) Constraint of Tonitoring Range

All areas of Yangpu District are required to be observed. The same as Problem 1, assume that the label set the x^{th} route has passed is $b(x)$, and then we obtain:

$$b(x) = \{\vec{a}(x)_1, \vec{a}(x)_2, \dots, \vec{a}(x)_n\} \quad (14)$$

where

$x = 1, 2, \dots, m$,

m is the number of UAVs, and

n is the dimension of labeling vector of grids.

The union set of every label set of flight course should cover all labels in order to observe all areas, that is:

$$b(x) = \{\vec{a}(x)_1, \vec{a}(x)_2, \dots, \vec{a}(x)_n\} \quad (15)$$

6.4.3 Establishment of Model

Synthesize the above target functions and constraints, and then the improved optimized model is established:

$$\begin{aligned} & \min m \quad (16) \\ s.t. & \begin{cases} \sum_{i=2}^n w_{\vec{a}(x)_{i-1}\vec{a}(x)_i} \leq t(j), j=1, 2, \dots, m \\ t(j) = \min \frac{\bar{T}(k)}{2}, k \in b(j) \\ b(1) \cup b(2) \cup \dots \cup b(m) = \{1, 2, 3, 4, \dots, 62, 63\} \end{cases} \quad (17) \end{aligned}$$

6.5 Solution of Model

6.5.1 Introduction of Algorithm

Step1: Use MATLAB to read adjacent matrix \mathbf{W} , and 63-dimension vector \mathbf{T} used to denote the monitoring time interval that every area needs, and define 63-dimension vector \mathbf{a} to be the number of UAVs able to be detected in i^{th} region and its original state is 0, that is, $\mathbf{a} = \mathbf{0}$.

Step2: Use Floyd algorithm to calculate the shortest route between adjacent matrixes, and store it in matrix of the shorted routes.

Step3: Look for the equation--“ $\mathbf{a}_i = 0$ ” from the 1^{st} grid to the 63^{rd} one according to t_i ; make the flight time of a UAV equal to 0, that is, $t = 0$, $\mathbf{a}_i = \mathbf{a}_i + 1$, and record the required patrolling time t_i ; and then enter Step4; if the equation--“ $\mathbf{a}_i = 0$ ” does not exist, the procedure ends.

Step4: When $t > t_i$, enter Step3, otherwise entering Step5.

Step5: Look for $w_{ij} \neq 0$ and $t_j = t_i$ from the 1^{st} grid to the 63^{rd} one. If $D_{ij} < t_i$, every correspondent a along the journey is equal to $a + 1$, that is, $a = a + 1$, and make $i = j$, $t = t + D_{ij}$, and then returns to Step3; if that does not exist, then enter Step6.

Step6: Look for $w_{ij} \neq 0$ and $w_{ij} \neq \infty$ from the 1^{st} grid to the 63^{rd} one, and j minimizing the value of \mathbf{a}_j is considered as the next patrolling area of that UAV. If $t + w_{ij} < t_i$, make $i = j$, $t = t + w_{ij}$, and $\mathbf{a}_j = \mathbf{a}_j + 1$, and then returns to Step4, otherwise entering Step3.

6.5.2 Solution Result

According to the above algorithm steps and MATLAB program, we obtain the following results:

Table 5. Distribution of flight course

UAV (number)	Flight course	UAV (number)	Flight course
1	1->2->3->9	2	5->11->16
3	6->7->8->4->10	4	13->14->15->16
5	17->23->30->37->44	6	19->25
7	20->21->22->29	8	25->26
9	27->28	10	31->24->18->12
11	32->33	12	34->35
13	36->43->49	14	38->39->40->33
15	41->42	16	46->45
17	47->48->54->55->50	18	51->56->60->59->53
19	52->57->58->62->63	20	61->55

Intuitive distribution map of flight courses is as follows:

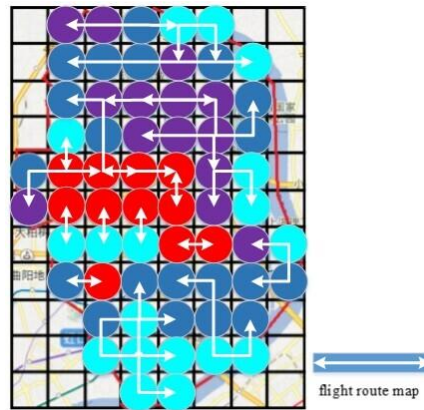


Figure7. Distribution of flight course

There are 20 routes in the map, meaning 20 UAVs are required to successfully perform the prompt monitoring of human traffic in every place of Yangpu District.

6.6 Result Analysis

As different areas have different human traffic, thus requiring different monitoring time, based on which we establish the Fussy synthetic evaluation model to better arrange for the number of UAVs. And the human traffic level obtained from this model corresponds with the reality, meaning the result is comparatively reasonable. After the definition of the target function and the constraint of monitoring time and monitoring areas, we obtain that 20 UAVs are required to successfully perform the prompt monitoring of human traffic in every place of Yangpu District.

7. Modeling Establishment and Its Solution in Problem 3

7.1 Problem Analysis

Problem 3 asks us to calculate the largest monitoring range based on two assumptions that all areas are equally important, and 30% UAVs are unusable. This problem is based on Problem 1 which has obtained that 13 UVAs are needed to guarantee that the absence of monitoring time in every place of Yangpu District shall be no more than 15 minutes, from which we can see that if 30% UAVs were not available, the number of usable UAVs would be 9. At the same time, single journey flight time of every route should be near to 7.5 minutes as much as possible. Also, considering that the girds in the boundary areas of Yangpu District may not be intact,

we make the UAVs in Problem 3 avoid passing the incomplete grids as much as possible. In consequence, three implied conditions can be summed up in the following:

- The number of available UAVs is 9.
- The one-way flight time of every UAV should be near to 7.5 minutes as much as possible.
- A UAV should avoid passing the incomplete grids as much as possible.

7.2 The Optimal Model of the Monitoring Range of UAVs

7.2.1 Target Analysis

The observed area is required to be the largest, hence the target function is:

$$\max S \quad (18)$$

where S is the area of the observed fields.

7.2.2 Constraint Analysis

1) Constraint of the Number of UAVs

From Problem 1, we have obtained that 13 UAVs are needed to guarantee that the absence of monitoring time in every place of Yangpu District shall be no more than 15 minutes. Hence, if 30% UAVs were not available in this section, the number of available UAVs would be 9. As a consequence, we obtain:

$$m \leq 9 \quad (19)$$

2) Constraint of Monitoring Time

Each single journey flight time of every route should be near to 7.5 minutes to reach the largest monitoring range. Hence, the constraint can be established as follows:

$$\sum_{i=2}^n w_{\vec{a}(x) \vec{a}(x_i)} \rightarrow 7.5 \text{ min} \quad (20)$$

where $\vec{a}(x)$ is the labeling vector of grids the x^{th} flight course has passed ($x=1,2,\dots,m$), $w_{\vec{a}(x) \vec{a}(x_i)}$ is an element of adjacent matrix, and n is the dimension of labeling vector of grids

3) Constraint of Monitoring Range

Considering that the grids in the boundary areas of Yangpu District may not be intact, we make the UAVs in Problem 3 avoid passing the incomplete grids as much as possible. This section implements a constraint in terms of set. Assume that the label set the x^{th} flight course has passed is $b(x)$; then we obtain:

$$b(x) = \{\vec{a}(x)_1, \vec{a}(x)_2, \dots, \vec{a}(x)_n\} \quad (21)$$

where

$x=1,2,\dots,m$,

m is the number of UAVs, and

n is the dimension of label vector of grids.

From **Figure 4**, we can see that the small grids in the boundary areas of Yangpu District are 1,2,3,5,31,38,57,61, whose set form is as follows:

$$D = \{1,2,3,5,31,38,57,61\}$$

To make the UAVs avoid passing the incomplete grids as much as possible requires them not to pass any of these incomplete ones. Hence, the constraint is as follows:

$$[b(1) \cup b(2) \cup \dots \cup b(m)] \cap D = \emptyset$$

7.2.3 Establishment of Model

According to the above target functions and constraints, the improved optimal model is established:

$$\max S \quad (22)$$

$$s.t. \begin{cases} m \leq 9 \\ \sum_{i=2}^n w_{a(x)_{i-1} \bar{a}(x)_i} \rightarrow 7.5 \min \\ [b(1) \cup b(2) \cup \dots \cup b(m)] \cap D = \emptyset, D = \{1, 2, 3, 5, 31, 38, 57, 61\} \end{cases} \quad (23)$$

7.3 Solution of Model

7.3.1 Introduction of Model

Step1: Use MATLAB to read adjacent matrix \mathbf{W} , and define 63-dimension vector \mathbf{a} to be the number of UAVs able to be detected in i^{th} region and its original state is 0, that is, $\mathbf{a} = \mathbf{0}$.

Step2: Manually select those boundary places whose areas are less than 1 square kilometers and make $\mathbf{a} = 1$.

Step3: Look for the equation--“ $\mathbf{a}_i = 0$ ” from the 1^{st} grid to the 63^{rd} one according to t_i ; make the flight time of a UAV equal to 0, that is, $t = 0$, $\mathbf{a}_i = \mathbf{a}_i + 1$; then record the required patrolling time t_i ; and then enter Step4; if the equation--“ $\mathbf{a}_i = 0$ ” does not exist, enter Step6.

Step4: When $t > 8.5$, enter Step3, otherwise entering Step5.

Step5: Look for $w_{ij} \neq 0$ and $w_{ij} \neq \infty$ from the 1^{st} grid to the 63^{rd} one, and j minimizing the value of \mathbf{a}_j is considered as the next patrolling area of that UAV. If $t + w_{ij} < 8.5$, make $i = j$, $t = t + w_{ij}$, and $\mathbf{a}_j = \mathbf{a}_j + 1$, and then returns to Step4, otherwise entering Step3.

Step6: Mark the last side of the sixth position of every route as half-side.

7.3.2 Solution Result of Model

According to the above algorithm steps, MATLAB software, and relevant calculations, the results obtained are shown in **Figure 6**:

Table 6. Distribution of flight course

UAV (numbers)	Flight course	UAV (numbers)	Flight course
1	4->9->8->7->6-> (12)	2	10->11->17->16->15-> (14)
3	13->19->18->25->24	4	20->21->22->23->30-> (29)
5	26->27->28->35->34-> (33)	6	32->39->40->41->42
7	36->37->43->50->49-> (48)	8	45->46->47->53->52
9	54->55->56->60->59		

Place the flight courses in the map, which appears to be intuitive and objective, as shown in **Figure 8**:

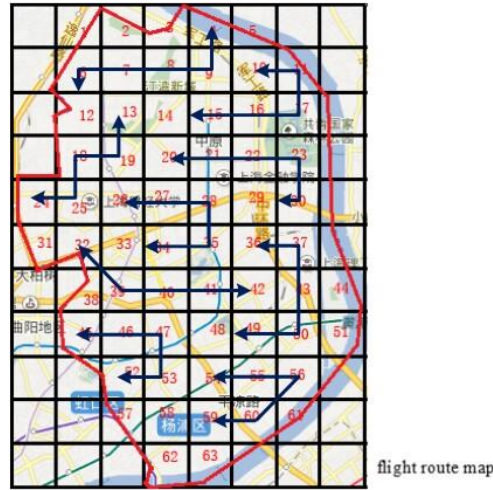


Figure 8. Distribution of flight course

Based on the above algorithm, when 9 UAVs are used in this section, they can provide the largest monitoring range of 47.5 square kilometers, covering 78.37% of the whole Yangpu District.

7.4 Result Analysis

30% UAVs are unusable in this section, therefore, the number of UAVs employed is 9. To reach the largest monitoring range, we have excluded those incomplete grids. Besides, after calculation, the one-way flight time of every route should be near to 7.5 minutes as much as possible and the UAVs should avoid passing the incomplete grids as much as possible, based on which the optimal model of UAVs' monitoring range is established. And after solving the algorithm, the monitoring area of 47.5 square kilometers is obtained, constituting 78.37% of Yangpu District. The method adopted is scientific and precise, and the result fits with the reality.

8. Model Summary

This paper establishes the optimal model based on graph theory, Fussy synthetic evaluation model, the optimal model based on regional time difference, and the optimal model of the monitoring range of UAVs to solve the security problems of UAVs. Moreover, this paper makes evaluation and promotion on these models.

8.1 Model Evaluation

8.1.1 Strengths of These Models

- In Problem 1, we divide Yangpu District into grids which are later labeled. And the distance between those grids is measured by the flight time of UAVs, by which method we change the given information into statistical information, and store those statistics by using graph theory. This step is the key of this paper, which is the basis of the establishment of constraint functions.
- The Fussy synthetic evaluation model is adopted in Problem 2. We use precise digital means to deal with the fussy evaluation objects, and make level divisions on the areas with higher density of people and those with lower density of people, which is close to actual quantitative evaluation, scientific and reasonable. And the results obtained is highly fit for the reality.
- The results can be intuitively shown through pictures, and especially in

Problem 2, we use different colors to differentiate different monitoring time intervals of different places, vivid and intuitive.

- The biggest highlight of this paper is the mathematical expression of constraints, which adopts the union set and intersection set to fulfill the mathematical expression of covering all monitoring areas, and of not passing incomplete grids. At the same time, this paper solves the hot issue of the security problems of UAVs. And the results obtained are precise and innovative.
- We adopt different models in this paper to satisfy the given different requirements, precise and innovative. In problem 1, the optimal model is established on the statistical storage of graph theory; in Problem 2, the optimal model based on regional time difference is established on the improvement of time constraint; at last, the optimal model of the UAV monitoring range is established. All in all, we establish different target functions and constraints according to different problems, solving the hot issue of the securities problems of UAVs, and the results obtained are comparatively ideal.

8.1.2 Weaknesses of the models

- 1) The optimal models adopted in this paper are subject to the following weaknesses:
 - When we consider the incomplete grids in the map as a complete one ideally to simplify calculations, there exists errors.
 - The solution of local optimal algorithm is not the global optimal one.
- 2) The fussy synthetic evaluation model is also subject to the following problems:
 - The calculation of this model is complex, and the definition of the vector of index weight is highly subjective.
 - When the index set U is comparatively larger, that is, if any number of index set is bigger, the weight coefficient of relative membership degree is often smaller under the constraint that the sum of weight vector is 1.
 - When weight vector does not fit with the fussy matrix, the result would be hugely vague with a bad resolution, unable to tell whose membership degree is much higher, or even resulting in evaluation failure. At this time, we can use hierarchical fussy evaluation method to improve it.

8.2 The Promotion of These Models

The three models and the fussy synthetic evaluation method adopted in this paper can be used in the following fields:

- Earthquake relief work: to search for the trapped people after an earthquake.
- Fire monitoring: to monitor the development of city fire, and to take measures to control it timely.
- Forest irrigation: to study irrigation district problems.
- Haze pollution control: to properly arrange for artificial rainfall according to pollution degree.

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- [4] Li Daqian. 1998. China Undergraduate Mathematical Contest in Modeling. Beijing: Higher Education Press.
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Appendix

Appendix list

Appendix one: evaluate the relevant data of Problem 1

1. procedures of question one

```
clc;clear;
```

[illegible]

- 20 -


```
                break;
            end
        end
    fprintf('\n');
end
```

Appendix Two : Evaluate the Relevant Data of Problem 2

2. Procedures of Question Two

[illegible]

[illegible]


```

        end
    end
    D=G;
    for i=1:1:n
        for j=1:1:n
            R(i,j)=j;
        end
    end
    for k=1:n
        for i=1:n
            for j=1:n
                if D(i,k)+D(k,j)<D(i,j)
                    D(i,j)=D(i,k)+D(k,j);
                    R(i,j)=R(i,k);
                end
            end
        end
    end
    while(1)
        for i=1:1:4
            s=0;
            for j=1:1:x(i)
                if bj(a(i,j))==0
                    s=a(i,j);
                    bj(a(i,j))=bj(a(i,j))+1;
                    t=i;
                    fprintf('%d',s);
                    break;
                end
            end
            if s~=0
                break;
            end
        end
        if s==0
            break;
        end
        time=0;
        tt=T(s)/2;
        if t==1
            min=inf;
            minx=0;
            for j=1:1:x(i)
                if time+D(s,a(i,j))<tt && bj(a(i,j))==0 &&
D(s,a(i,j))<min && a(i,j)~=s
                    min=D(s,a(i,j));
                    minx=a(i,j);
                end
            end
            if minx~=0
                break;
            end
        end
    end
end

```


[illegible]

[illegible]

