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Title：Optimal Travel Package Design for Family Summer Vacation

Abstract

Nowadays, more and more parents choose to take their kids to a new city for traveling with the development of improvement of living standard. But different families have different requirements which includes expense of the travel package, time, number of family members, the weather and so forth. So, we choose Jinan as a city for traveling and comprehensively consider the different requirements of different families. Then we design an optimal travel package for families with certain specific requirements.

We have selected the typical combinations of family requirements to analyze using control variate method and referring to the knowledge of psychology. And we choose expense, number of sight spots and time as the three basic requirements, the relationships of which is considered from the first model to the third. Model four mainly considers the number of family members while the fifth is about weather.

In model one, two, three we focus on different family requirements listed as follows: 1.cover all the sight spots with limitless expense and a minimum amount of time. 2.cover a maximum amount of sight spots with limited expense and time. 3.use minimum money to travel with limited time or enough spots to go to. Considering all these requirements, we choose the shortest route and use the fastest means of transportation. Then we use Lingo to process the data. Next, we establish a triune requirement model combining the three factors which includes expense, time, number of sight spots using greedy method and graph theoretic approach to satisfy the needs of families.

Model four and five focus on the condition that several families will go together and the effects of rainy days based on the aforesaid models. The models are trying to save more money to improve the travelers’ satisfaction. We come up with several-family-travel-together strategy and define the loss index of rainy days. Finally, we get the optimal result with a minimum loss.

Key words：travel route design, greedy method, graph theoretic approach, requirement model, degree of satisfaction

Optimal Travel Package Design for Family Summer Vacation

1. Restatement of the problem

The summer vacation is drawing near. Lots of parents seize the chance to take their kids to a new city for a wonderful holiday. Apparently, however, different families have different requirements, such as the expense of the travel package, time, number of family members and the weather. Choose a city and comprehensively consider the different requirements of different families. Then design a travel package for families with certain specific requirements.

2. Analysis

According to the our understanding of the problem, we need to design a optimal travel package by considering different family requirements in order to minimize the expense, to shorten the time, to maximize the number of sight spots or satisfy other requirements. Taking a variety of factors, we divide the requirements into five categories listed as follows: expense of travel package, numbers of sight spots, time, number of family members, the weather.

We have selected the typical combinations of family requirements to analyze using control variate method and referring to the knowledge of psychology. And we choose expense, number of sight spots and time as the three basic requirements, the relationships of which is considered from the first model to the third. Model four mainly considers the number of family members while the fifth is about weather.

Taking various factors into account, we establish an efficient mathematical model choosing Jinan as the travel city.

Model one focuses on requirements listed as follows: limitless expense, covering all the sight spots and a minimum amount of time. Having satisfied all these requirements, we can choose the route with the shortest length and faster means of transportation. After analysis, we design an optimal route to satisfy the requirements using a software named Lingo.

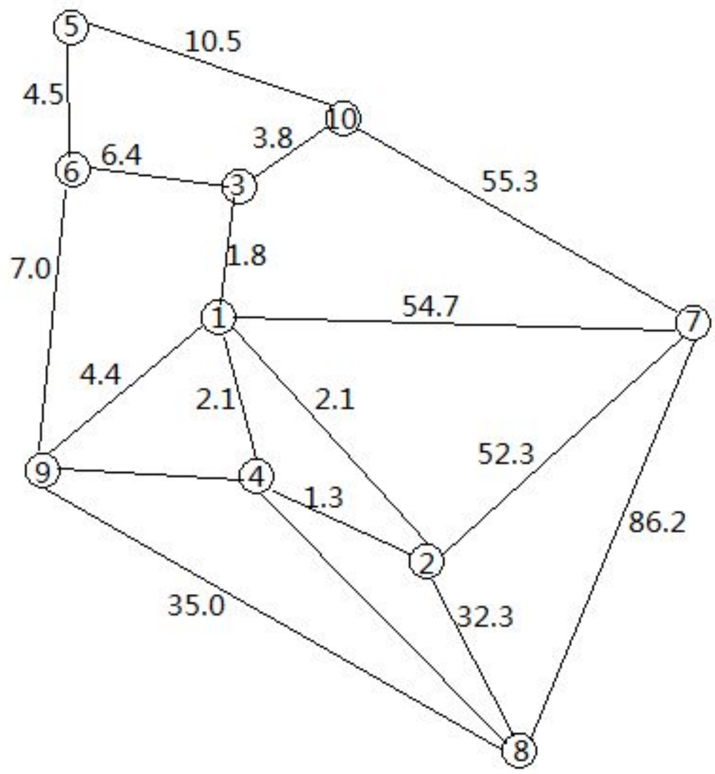
Model two focuses on requirements listed as follows: limited expense, limited time and more sight spots. Because of the two limited factors, we analyze one factor at a time and then consider both. The first one is about the control of time. Specifically speaking, we need to find a travel package which covers the most spots with limited expense and limitless time. Then we build the objective function and condition constraint formulas. The second one is about the control of expense. To be specific, we need to find the optimal strategy which covers the most spots with limited time and limitless expense. Then comes the function and formulas. Finally we consider the both aforesaid factors and obtain the final objective function.

Consequently we are able to design an optimal route by greedy method and graph theoretic approach.

Model three focuses on the requirements of the minimum expense, under the condition of which we analyze the combination of numbers of spots and time. Firstly, we consider a family who wants to travel the most spots with limited time and a minimum amount of expense. To get a best package, we decide the number of spots at the beginning and the calculate the minimum amount of expense under corresponding condition constraint. We can get several resultant strategies so that families are able to choose one according to their own interests. Secondly, we consider the requirements listed as follows: a minimum amount of expense, covering all spots and limitless time. Actually it is based on the analysis the first one for the only difference between them is the time. So we can use the model above.

Model four focuses on the number of the family members. When two families choose to go together but their time does not fit we need to find a strategy that costs least. As we know from the assumptions that more travelers at a spot means less total expense, we need to take it into account in order to cut the expense.

Model five focuses on the requirements of time. It is obvious that bad weather will lead to a less satisfying vacation, so we try our best to minimize the effect of the weather. We define the loss index first and then add the two objectives including total expense and total loss by their weights. Finally, we get the optimal strategy with a minimum loss.



1---Baotu Spring park+Quancheng Square

2---Qianfo Mount

3---Daming Lake
4---Quancheng Park
5---Yellow River Forest Park
6---Jinan Zoo
7---Zhujia Yu
8---Jinxiangshan Amusement Park
9---Yingxiongshan Memorial Park
10---Hongjialou Square

3. Assumptions

1.There is no accident during the travel.
2.Assume that the length of the summer vacation is no longer than two months.
3.The ability of each spot to serve travelers is excellent which means that it is possible to serve travelers from different routes.
4.More travelers means lower total expense.
5.The speed of the bus is 50km per hour, and the cost is 3RMB per kilometer.
6.Going directly from one spot to another means the spots in the middle of the route is just a station instead of a real spot.
7.Quancheng Square is the beginning and the ending as well. There is no travelers who leave the family during the travel or those who don't go inside the spot.

4. Definitions and notations

T : total time a family spends;

p_1 : time that a family spends on the road;

p_2 : time that a family spends at the sight spots;

p_3 : time that a family spent on possible accommodation;

e_i : possible accommodation time that a family may stay at i spot sight i;

n : the number of sight spots;

r_{ij} : a 0-1 variable thar demonstrates whether a family goes from sight spot i to sight spot j;

t_{ij} : the time that a family spends from sight spot i to sight spot j;

c_{ij} : expense that a family spends on transportation from sight spot i to sight spot j;

m : total expense that a family spends on the travel;

m_1 : total expense that a family spends on transportation;

m_2 : total expense that a family spends on all the sight spots;

m_3 : total expense that a family probably spends(accommodation, food and so on);

$\lambda_i^{'}$: the weight of sight spot i for the first family;

$\lambda_i^{''}$: the weight of sight spot j for the first family;

$m_1^{'}$: total expense that the first family spends on transportation;

$m_1^{''}$: total expense that the second family spends on transportation;

$m_2^{'}$: total expense that the first family spends on all the sight spots;

$m_2^{''}$: total expense that the second family spends on all the sight spots;

m_3 **:** the amount of saved expense when two families go to the same spot at the same time;

α_i : a 0-1 variable thar demonstrates whether two families go to the same spot at the same time;

P_{is} : possibility that it is overcast and rainy at sight spot i;

T_i : time that a family arrive at sight spot i;

V : set of sight spots where a family will go. For example, V ={1， 8， 5， 7}means a family has gone to sight spot 1, 8, 5, 7.

5.Modeling and making a solution

Based on requirements of the subject,we comprehensively consider travel route,time,quantity,weather factors and so on when we design a family travel package during summer holiday to adapt to different families' different requirements.

5.1 Model 2 building and making a solution

5.1.1 Object function building

Firstly,we consider requirement as visiting all sight spots and spend the minimum of money without limitation of time.Based on our understanding of the subject ,we define the spending time involve three parts,namely those spent on the road,staying at the sight spots and possible accomdation.Here,we define the following symbols:

T ----total time a family spends

p_1 --time that a family spends on the road

p_2 --time that a family spends at the sight spots

p_3 --time that a family spent on possible accomdation

e_i --possible accomdation time that a family may stay at i spot sight

n ---the number of sight spots

In conclusion,we can draw the general object function:

$$MinT = p_1 + p_2 + p_3$$

(1)Time that a family spends on the road

Because t_{ij} represents the time that a family spends from sight spot i to sight spot j; r_{ij} represents a 0-1 variable thar demonstrates whether a family go from sight spot i to sight spot j.Because a family wants to visit all ten sight spots and go through these spots just once.So we can derive the follwing function to demonstrate the time a family spends on the road:

$$p_1 = \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times t_{ij}$$

(2)Time that a family spends at the sight spots

We use t_i to represent the time a family spends at sight spot i and r_{ij} to represent a 0-1 variable thar demonstrates whether a family go from sight spot i to sight spot j.So we can derive the follwing function to demonstrate the time a family spends at the sight spots:

$$p_2 = \frac{1}{2}\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}(t_i + t_j)$$

(3)Time that a family spends at possible accomdation

We use e_i to represent the possible accomdation time that a family spends at sight spot i and r_{ij} to represent a 0-1 variable thar demonstrates whether a family go from sight spot i to sight spot j.So we can derive the follwing function to demonstrate the time a family spends at possible accomdation:

$$p_3 = \frac{1}{2}\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}(e_i + e_j)$$

In conclusion ,we can derive the total object function:

$$MinT = p_1 + p_2 + p_3$$

$$= \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times t_{ij} + \frac{1}{2}\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}(t_i + t_j) + \frac{1}{2}\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}(e_i + e_j)$$

5.1.2 Constraint conditions

(1)The constraint of sight spots

Based on the requirements of the subject and the assumed conditions,we use $\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}$ to represent the quantity of sight spots.So we assume the quantity of sight spots as

$n(n = 2,3,4,5,6,7,8,9,10,11)$ .Based on the assumption that we use Quancheng Park as the start and end point,so $n = 11$ .Finally,we can derive the quantity constraint of sight spots.

(2) The constraint of 0-1 variables

To achieve the goal of spending the minimum of time,we neeed to choose from different travel routes. Based on the assumption that we use a circuit route and travel sight spots on the route once,we can draw a circle to connect up all sight spots and view every sight spot as a point on that circle.For every point,it’s only allowed to let one edge come out at most,and let one edge come in at most.What’ more,it requires that once one edge come in then one edge come out.So we can derive thi constraint function:

$$\sum_{i}^{11} r_{ij} \le \sum_{j}^{11} r_{ij} \le 1 \ (i, j = 1,2,3\ldots\ldots 9,10,11)$$

When i=1, $\sum_{i=1} r_{ij} = 1$ --based on the assumption that Quancheng Park is start point ;

When j=1, $\sum_{j=1} r_{ij} = 1$ --based on the assumption that a family has to return back to the start point(Quancheng Partk);

Based on the description of the subject,we can draw conclusions as follows:

$$\sum_{i} r_{ij} = \sum_{j} r_{ij} \le 1$$

$$\sum_{i=1} r_{ij} = 1$$

$$\sum_{j=1} r_{ij} = 1$$

$$(i, j = 1,2,3\ldots\ldots 9,10,11)$$

In the same way,when $i, j \geq 2$,it's impossible to get $r_{ij} = r_{ji} = 1$,that is thre is no possibility that a family make a round trip between two sight spots based on the assumption that any sight spot can only be visited once.In conclusion,we can draw a constraint function as follows:

$$r_{ij} \times r_{ji} = 0$$

5.1.3 modeling

Based on our understanding of the subject,we can derive the overall model as folllows:

$$MinT = p_1 + p_2 + p_3$$

Constraint condition:

$$\begin{cases} \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} = 11 & (i, j = 1,2,3\ldots\ldots 9,10,11) \\ \sum_{i=1} r_{ij} = 1, \quad \sum_{j=1} r_{ij} = 1 & (i, j = 1,2,3\ldots\ldots 9,10,11) \\ r_{ij} \times r_{ji} = 0 & (i, j = 2,3\ldots\ldots 9,10,11) \end{cases}$$

5.1.4 The result analysis of the modeling solution domain

Here we introduce the following symbols:

d_{ij} --the distance between sight spot i to sight spot j

v ---the average speed per hour of the tourist bus that a family takes, v=50km/h;

m --the average expense of the tourist bus that a family takes, h=0.25 元/h;

By searching the Internet,we can get the specific value of d_{ij} ,and we can get the corresponding value of t_{ij} from formula $t_{ij} = d_{ij} / v$ .In the same way,we can get the corresponding value of c_{ij} （i， j=1，2，……， 11) from formula $c_{ij} = d_{ij} \times m$ (the specific values of d_{ij} , t_{ij} and c_{ij} can be seen in the appendix)

In the same way,by consulting some travel agencies in Jinan,we cxan get the best sojourn time and total consumption at sight spot i:

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
36	24	5	4	24	8	18	10	6	32

(Unit:hour)

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}

240	180	130	50	400	170	320	90	148	450

(Unit:Yuan)

The result of model 1:

Based on the model,use Lingo programming to obtain the shortest path and adjust to get a better travel route considering the actual conditions.

Number of sight spots n	10
Total expenses per person$c$ （Unit:Yuan）	1380
Route	Baotu Spring Park+ Quancheng Square->Quancheng Park->Yinxiongshan Memorial Park->Hongjialou Square->Yellow River Forest Park->Jinxiangshan Amusement Park->Zhujia Yu->Qianfo Mount->Jinan Zoo->Baotu Park+Quancheng Square

5.2 Model 2 building and making a solution

5.2.1Object function building

Here we consider a better way to define expense:within certain time and expenses limitation,visit sight spot as much as possible.So the designed route must satisfy the requirement of time and expense,that is the time and expense become the constraint condition.We consider employing control variate method to seperate up the constraint conditions.And finally obtain the object function.

(1) limited expenses,visiting the maximum sight spots,unlimited time(controling the time variable)

The decision variable: $r_{ij} = \begin{cases} \text{a family goes from sight spot i to sight spot j} \\ 0 \quad else \end{cases}$

The object function: $\min \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij} r_{ij}$

The constraint condition: $\sum_{i=1} r_{ij} = 1, \sum_{j=1} r_{ij} = 1$ （i, j =1，2，3……9,10,11）

x_{ij} must be a round route

Because the fact that Hamiltonian cycle problem is a NP complete problem and it’s unrealistic to get a optimum solution for a graph with 11 points(need to add m-1 more points),so we consider employing heuristic algorithm based on algebra to obtain the approximate optimal solution.

The first thing to do is deteriming the number of sight spots under corresponding conditions.

The total travel expenses consist of two parts-travel expenses and sight-seeing expenses.We can get the object fnction as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} r_{ij} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}\left(b_i + b_j\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}\left(d_i + d_j\right) \le 2000$$

①Total travel expenses

Considering that c_{ij} represents the travel expenses from sight spot i to sight spot j,we can easily derive the function of total travel expenses:

$$a_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij} \times c_{ij}$$

②Total sight-seeing expenses

As mentioned above, b_i and d_i represents the consumption that a family make at sight spot i, r_{ij} can represent whether a family goest to sight spot i and sight spot j and the entire tavel route is circular.Consideing those conditions,we actually comput the sight-seeing expenses twice.So we can derive the function of sight-seeing expenses as follows:

$$b_i + d_i = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}\left(b_i + b_j\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}\left(d_i + d_j\right)$$

③Constraint condition:0-1 variables constraint

Based on the assumption that the travel route starts and ends at the same sight spot--Quancheng Square,.we can draw a circle to connect up all sight spots and view every sight spot as a point on that circle.For every point,it's only allowed to let one edge come out at most,and let one edge come in at most.What' more,it requires that once one edge come in then one edge come out.The proof has been done in the first modeling.So we can directly derive the constraint function:

$$\sum_{i}^{11} r_{ij} \le \sum_{j}^{11} r_{ij} \le 1 \ (i, j = 1,2,3\ldots\ldots 9,10,11)$$

When i=1, $\sum_{i=1} r_{ij} = 1$ --based on the assumption that Quancheng Park is start point ;

When j=1, $\sum_{j=1} r_{ij} = 1$ --based on the assumption that a family has to return back to the start point(Quancheng Square);

Based on the description of the subject,we can draw conclusions as follows:

$$\sum_{i} r_{ij} = \sum_{j} r_{ij} \le 1$$

$$\sum_{i=1} r_{ij} = 1$$

$$\sum_{j=1} r_{ij} = 1$$

$$(i,j = 1,2,3\ldots\ldots 9,10,11)$$

In the same way,when $i,j \geq 2$,it's impossible to get $r_{ij} = r_{ji} = 1$,that is thre is no possibility that a family make a round trip between two sight spots based on the assumption that any sight spot can only be visited once.In conclusion,we can draw a constraint function as follows:

$$r_{ij} \times r_{ji} = 0$$

(2) unlimited expenses,visiting the maximum sight spots,limited time(controling the expenses variable)

When designing a family travel package in this condition,it has to satisfy the requirements that visiting sight spots as many as possible with limited time under no limitation of expenses.

Because the constraint conditions in practical problems,such as mismath of transportation,the close of sight spot,accommodation,it's not possible to obtain the best travel route from the overal situation.So we employ greedy method to solve the problem from partial situation.

Obviously,time limitation and visting sight spots as many as posssible serve as the two goals of this problem.

$$MinT = m_1 + m_2 + m_3 = \sum_{i=1}^{11}\sum_{j=1}^{11} t_{ij} r_{ij} + + \frac{1}{2}\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}\left(t_i + t_j\right) + \frac{1}{2}\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}\left(e_i + e_j\right)$$

Constraint condition:

$$\begin{cases} \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} = 11 & (i,j = 1,2,3\ldots\ldots 9,10,11) \\ \sum_{i=1} r_{ij} = 1, \quad \sum_{j=1} r_{ij} = 1 & (i,j = 1,2,3\ldots\ldots 9,10,11) \\ r_{ij} \times r_{ji} = 0 & (i,j = 2,3\ldots\ldots 9,10,11) \end{cases}$$

Greedy method[3]:

Use greedy method to derive partial optimal solutin by comprehensively considering some aspects as travel time,the distance of travel route and time at possible accommodation.Firstly,we delette some most time-consuming sight spots and some relatively distant sight spots:Zhujia Yu,Jinxiangshan Amusement Park.The left sight spots are relatively centralized and the staying time is relatively less.Everytime we choose from the adjacent sight spots considering picking up the least time-consuming sight spots and there is no-mismatch between transportation and sight-seeing.This choice satisfy the aim of partialy optimize slot time allocation.And the like,we can obtain a travel route that contain some sight spots as many as possible in 5 days.

(3) limited expenses,visiting the maximum sight spots,limited time

Based on the analysis above,we can derive the ultimate object function:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} r_{ij} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}\left(b_i + b_j\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}\left(d_i + d_j\right) \le 2000$$

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}\left(e_i + e_j\right)$$

5.2.2 model solution and result analysis

Based on the model and by using Lingo programming,we can draw the following conclusion:

Number of sight spots n	6
Total expenses per person c （Unit:Yuan）	790
Route	Baotu Spring Park+Quancheng Square→Daming Lake→Quancheng Park→Yingxiongshan Memorial park →Hongjialou Square→Baotu Spring Park+Quancheng Square

5.3 Model 3 building and making a solution

5.3.1 Object function building

In this model,we need to meet the requirement that spending the minimum amout of time to visit some sight spots as many as possible within 10 days.Obviously, spending the minimum amount of money and visiting some sight spots as many as possible serve as the two goals.

Here,firstly we derive the number of sight spots to visit under corresponding constraint conditions.And then compute the minimum amount of expenses under this condition.Finally,we can obtain several travel routes and the family can choose from them based on their practical situation.

The total visiting expenses consist of three parts:total transportation expense,sight-seeing expense and possible expenses.Here we define the following symbols:

m --the total expenses that a family spends

m_1 --the total transportation expenses that a family spends

m_2 --the total sight-seeing expenses that a family spends

m_3 --the possible expenses that a family spends(such as accommodation and eating)

(1)The total transportation expenses

Because c_{ij} represents transportation expenses of going from sight spot i to sight spot j; r_{ij} represents a 0-1 variable thar demonstrates whether a family go from sight spot i to sight spot j.So we can easily derive the follwing function to demonstrate the total transportation expenses that a family spends:

$$m_1 = \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times c_{ij}$$

(2)Sight-seeing expenses that a family spends at the sight spots

We use c_i to represent the total expenses that a family spends at sight spot i and r_{ij} to represent a 0-1 variable thar demonstrates whether a family go from sight spot i to sight spot j.And the entire travel route is circular.So the formula $\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times \left(c_i + c_j\right)$ actually compute the total sight-seeing expenses twice.So we can derive the follwing function to demonstrate the sight-seeing expenses a family spends at the sight spots:

$$m_2 = \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times \left(c_i + c_j\right)$$

(3)Possible expenses that a family spends

We use d_i to represent the possible expenses that a family spends at sight spot i,which include accommodation expenses,meal expenses and other aspects; r_{ij} to represent a 0-1 variable thar demonstrates whether a family go from sight spot i to sight spot j.Based on the analysis of the subject,the travel route is circular and all sight spots are just visited once.So we can derive the follwing function to demonstrate the possible expenses a family spends :

$$m_3 = \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times \left(f_i + f_j\right)$$

In conclusion ,we can derive the object function:

$$\text{Min} \quad m = m_1 + m_2 + m_3$$

$$= \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times c_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times \left(c_i + c_j\right) + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times \left(f_i + f_j\right)$$

5.3.2 Constraint conditions

(1)The constraint of time

Based on the subject and the assumption,the time that a family need to stay at Jinan should be less than 10 days (we consider that only 12 hours in a day are spent on sight-seeing),that is 240 hours.The total time includes transportation time and stay time at sight spot.Because t_{ij} represents the transportation time from sight spot i to sight spot j,the total transportation time is $\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times t_{ij}$.

Because t_i represents the stay time at sight spot i,the stay time that a family spends at sight spots is $\frac{1}{2}\times\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times (t_i + t_j)$ .In conclusion,the total time constraint can be represented as follows :

$$\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times t_{ij} + \frac{1}{2}\times\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times (t_i + t_j) \le 720$$

(2)The constraint of sight spots

Based on the requirements of the subject and the assumed conditions,the entire travel route is circular,that is Quancheng Park serves as the start and end point.So $\sum_{i=1}^{11} 1 \sum_{j=1}^{11} r_{ij}$ represemts the number of sight spots that a family visits.Here we assume the quantity of sight spots as $n(n = 2,3,4,5,6,7,8,9,10,11)$ .Finally,we can derive the quantity constraint of sight spots:

$$\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} = n \qquad (n=2，3，……，11)$$

(3)The constraint of 0-1 variables

We can draw a circle to connect up all sight spots and view every sight spot as a point on that circle.For every point,it’s only allowed to let one edge come out at most,and let one edge come in at most.What’ more,it requires that once one edge come in then one edge come out.So we can derive thi constraint function:

$$\sum_{i} r_{ij} = \sum_{j} r_{ij} \le 1 \qquad (i，j=1，2，……，11)$$

When i=1, $\sum_{i=1} r_{ij} = 1$ --based on the assumption that Quancheng Park is start point ;

When j=1, $\sum_{j=1} r_{ij} = 1$ --based on the assumption that a family has to return back to the start point(Quancheng Partk);

In conclusion,we can derive the following formula:

$$\sum_i r_{ij} = \sum_j r_{ij} \le 1 \qquad (i, j = 1,2,3\ldots\ldots 9,10,11)$$

$$\sum_{i=1} r_{ij} = 1$$

$$\sum_{j=1} r_{ij} = 1$$

In the same way,when $i, j \ge 2$,it's impossible to get $r_{ij} = r_{ji} = 1$,that is thre is no possibility that a family make a round trip between two sight spots.Because it obviously doesn't meet the requirement of visiting some sight spots as many as possible.In conclusion,we can draw a constraint function as follows:

$$r_{ij} \times r_{ji} = 0 \qquad (i,\ j=2,\ 3,\ \ldots\ldots,\ 11)$$

5.3.3 modeling

In conclusion,we can derive the overall model as folllows:

$$\text{Min} \quad m = m_1 + m_2 + m_3$$

$$= \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times c_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times (c_i + c_j) + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times (f_i + f_j)$$

Constraint condition:

$$\begin{cases} \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times t_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times (t_i + t_j) \le 120 \\ \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} = n \qquad (n=2,\ 3,\ \ldots\ldots,\ 11) \\ \sum_i r_{ij} = \sum_j r_{ij} \le 1 \qquad (i,\ j=1,\ 2,\ \ldots\ldots,\ 11) \\ \sum_{i=1} r_{ij} = 1 \qquad \sum_{j=1} r_{ij} = 1 \\ r_{ij} \times r_{ji} = 0 \qquad (i,\ j=2,\ 3,\ \ldots\ldots,\ 11) \end{cases}$$

5.3.4 The result analysis of the modeling solution domain

Based on the model and using Lingo programming,we can obtain results as the following table:

Number of sight spots n	2	3	4
Total expeses per person m (Unit:Yuan)	200	340	560

Route	1→3→1	1→4→3→1	1→4→3→5→1

Number of sight spots n	5	6
Total expeses per person m (Unit:Yuan)	670	740
Route	1→3→9→7→4→1	1→3→10→7→9→8→1

Number of sight spots n	7
Total expeses per person m (Unit:Yuan)	890
Route	1→3→10→4→7→9→8→1

Based on the above results,our recommendation can be described as follows:

Route 1:Baotu Spring Park+Quancheng Square->Quancheng park->Daming Lake->Huanghe Forest Park->Baotu Spring Park+Quancheng Square

Quantity:4 Expenses per person:￥560

Route 2:Baotu Spring Park+Quancheng Square->Daming Lake->Yingxiongshan Memorial Park->Zhujia Yu->Quancheng Park->Baotu Spring Park+Quancheng Square

Quantity:5 Expenses per person:￥670

Route 3:Baotu Spring Park+Quancheng Square->Daming Lake->Hongjialou Square->Zhujia Yu->Yingxiongshan Memorial Park->Baotu Spring Park+Quancheng Square

Quantity:6 Expenses per person:￥860

5.3.5 The improvement of the model

Under time limitation and considering the length of summer vacation,we can broaden the requirement of time in order to visit all sight spots.We may as well assume the limited time as two months(720 hours).Likely,we can get the following formula:

$$\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times t_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times \left(t_i + t_j\right) \leq 720$$

The model doesn't change,while the constraint conditions are changed into the following fromula:

$$\begin{cases} \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times t_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times (t_i + t_j) \le 720 \\ \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} = 11 \quad (i,\ j=1,2,\ldots\ldots,11) \\ \sum_i r_{ij} = \sum_j r_{ij} \le 1 \quad (i,\ j=1,2,\ldots\ldots,11) \\ \sum_i r_{ij} = 1 \quad \sum_j r_{ij} = 1 \quad (i,\ j=1,2,\ldots\ldots,11) \\ r_{ij} \times r_{ji} = 0 \quad (i,\ j=2,3,\ldots\ldots,11) \end{cases}$$

After the improvement,based on the model and using Lingo programming,we can obtain results as follows:

Number of sight spots n	8
Total expeses per person m (Unit:Yuan)	920
Route	Baotu spring park+Quancheng Square->Daming Lake->Hongjialou Square->Jinan Zoo->Zhujia Yu->Huanghe Forest Park->Yingxiongshan Memorial Park->Jinxiangshan Park->Baotu spring park+Quancheng Square

5.4 Model 4 building and making a solution

5.4.1 Analysis of the question

When designing a family travel package,we need to consider the quantity factor.Assume that each family has five members.If two families want to go sight-seeing in a group and one family has to delay for a few days to go sight-seeing.In this case,based on the asssumption,the more people are visiting a sight spot,the less every person is charged.In order to achieve the goal of using the minimum amount of money,we have to arrange the trip to make the two families can visit the same sight spots in those days the can both go sight-seeing.

5.4.1.1 The processing of data

Like the above question,we can define the following symbols:

λ_i' --sight spot i's weight for the first family

λ_i'' --sight spot i's weight for the second family

5.4.1.2 Object function building

In this question,we introduce the following symbols:

m --the total expenses

$m_1^{''}$ --the total transportation expenses of the first family

$m_1^{''}$ --the total transportation expenses of the second family

$m_2^{'}$ --the total sight-seeing expenses of the first family

$m_2^{''}$ --the total sight-seeing expenses of the second family

m_3 --the saved expenses when two families go sight-seeing at the same sight spots than go seperate sight-seeing

5.4.2 The object function

Based on above assumption and symbols,we can easily get the total object function:

$$\text{Min} \quad m = m_1^{'} + m_1^{''} + m_2^{'} + m_2^{''} - m_3$$

The corresponding total expenses for each family of the derived results:

$$m = \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times c_{ij} + \frac{1}{2} \times \frac{90}{100} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times \left(c_i + c_j\right)$$

Here we define:

$$r_{ij}^{'} = \begin{cases} 1 \text{ the first family goes directly from sight spot i to sight spot j} \\ 0 \quad else \end{cases}$$

$$r_{ij}^{''} = \begin{cases} 1 \text{ the second family goes directly from sight spot i to sight spot j} \\ 0 \quad else \end{cases}$$

Here we can derive the following formula:

$$\begin{cases} m_1^{'} = 5 \times \sum_{i=1}^{11}\sum_{j=1}^{11} \lambda_i^{'} \times r_{ij}^{'} \times c_{ij} \\ m_1^{''} = 5 \times \sum_{i=1}^{11}\sum_{j=1}^{11} \lambda_i^{''} \times r_{ij}^{''} \times c_{ij} \end{cases}$$

Based on the assumption that every one more people at the sight spot, every traveler's sight-seeing expense can be cut by 1‰ of the original price.So we can derive the following formula:

$$\begin{cases} m_2^{'} = \frac{1}{2} \times 5 \times 0.95 \times \sum_{i=1}^{11}\sum_{j=1}^{11} \lambda_i^{'} \times r_{ij}^{'} \times \left(c_i + c_j\right) \\ m_2^{''} = \frac{1}{2} \times 5 \times 0.95 \times \sum_{i=1}^{11}\sum_{j=1}^{11} \lambda_i^{''} \times r_{ij}^{''} \times \left(c_i + c_j\right) \end{cases}$$

5.4.3 The saved expenses

Here we define:

$$\alpha_i = \begin{cases} 1 & \text{two families go sight-seeing at sight spot i} \\ 0 & \text{else} \end{cases}$$

Because of the fact that each person is charged 95% per person when go seperate sight-seeing and 90% per person when two families go sight-seeing at the same sight spot,it means that the latter is cheaper than the former by 5%.Here we can derive the following formula:

$$m_3 = 100 \times 0.05 \times \frac{1}{2} \times \sum_{j=1}^{11}\sum_{i=1}^{11} \gamma_{ij} \times \left(\alpha_i \times \lambda_i \times c_i + \alpha_j \times \lambda_j \times c_j\right)$$

5.4.4 The constraint conditions

(1)The constraint of time

Based on the subject,the time that a family need to stay at Jinan should be less than 2 months(we consider that only 12 hours in a day are spent on sight-seeing),that is 720 hours.The total time includes transportation time and stay time at sight spot.Because t_{ij} represents the transportation time from sight spot i to sight spot j,the total transportation time for the two families is respectively $\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times t_{ij}$ and $\sum_{i=1}^{11}\sum_{j=1}^{11} r''_{ij} \times t_{ij}$ .Because t_i represents the stay time at sight spot i,the stay time for two families is respectively $\frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} \times \left(t_i + t_j\right)$ and $\frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}'' \times \left(t_i + t_j\right)$ .In conclusion,the total time constraint can be represented as follows :

$$\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}' \times t_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}' \times \left(t_i + t_j\right) \le 720$$

$$\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}'' \times t_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}'' \times \left(t_i + t_j\right) \le 720$$

(2)The constraint of sight spots

Based on the requirements of the subject and the assumed conditions,the entire travel route is circular,that is Quancheng Park serves as the start and end point.So $\sum_{i=1}^{11} 1 \sum_{j=1}^{11} r_{ij}$ represemts the number of sight spots that a family visits.Here we assume the quantity of sight spots as $n(n = 2,3,4,5,6,7,8,9,10,11)$ .Finally,we can derive the quantity constraint of sight spots:

$$\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} = n \quad （n=2，3，……，11）$$

(3)The constraint of 0-1 variables

We can draw a circle to connect up all sight spots and view every sight spot as a point on that circle.For every point,it's only allowed to let one edge come out at most,and let one edge come in at most.What' more,it requires that once one edge come in then one edge come out.So we can derive thi constraint function:

$$\sum_i r_{ij} = \sum_j r_{ij} \le 1 \qquad \sum_i r_{ij}' = \sum_j r_{ij}' \le 1$$

$$\sum_i r_{ij}'' = \sum_j r_{ij}'' \le 1 \quad （i，j=2，\cdots\cdots，11）$$

When i=1, $\sum_{i=1} r_{ij} = 1$ --based on the assumption that Quancheng Park is start point ;

When j=1, $\sum_{j=1} r_{ij} = 1$ --based on the assumption that a family has to return back to the start point(Quancheng Partk);

In conclusion,we can derive the following formula:

$$\sum_i r_{ij} = \sum_j r_{ij} \le 1 \qquad (i,j = 1,2,3\ldots\ldots 9,10,11)$$

$$\sum_{i=1} r_{ij} = 1$$

$$\sum_{j=1} r_{ij} = 1$$

In the same way,when $i, j \ge 2$,it's impossible to get $r_{ij} = r_{ji} = 1$,that is thre is no possibility that a family make a round trip between two sight spots.Because it obviously doesn't meet the requirement of visiting some sight spots as many as possible.In conclusion,we can draw a constraint function as follows:

$$r_{ij} \times r_{ji} = 0 \quad （i，j=2，3，……，11）$$

5.4.5 Modeling

In conclusion,we can derive the overall model:

$$\text{Min} \quad m_\lambda = m_1{}' + m_1{}'' + m_2{}' + m_2{}'' - m_3$$

$$\begin{cases} m_1{}' = 5 \times \sum_{i=1}^{11}\sum_{j=1}^{11} \lambda_i{}' \times r_{ij}{}' \times c_{ij} \\ m_1{}'' = 5 \times \sum_{i=1}^{11}\sum_{j=1}^{11} \lambda_i{}'' \times r_{ij}{}'' \times c_{ij} \\ m_2{}' = \frac{1}{2} \times 5 \times 0.95 \times \sum_{i=1}^{11}\sum_{j=1}^{11} \lambda_i{}' \times r_{ij}{}' \times \left(c_i + c_j\right) \\ m_2{}'' = \frac{1}{2} \times 5 \times 0.95 \times \sum_{i=1}^{11}\sum_{j=1}^{11} \lambda_i{}'' \times r_{ij}{}'' \times \left(c_i + c_j\right) \\ m_3 = 10 \times 0.05 \times \frac{1}{2} \times \sum_{j=1}^{11}\sum_{i=1}^{11} \gamma_{ij} \times \left(\alpha_i \times \lambda_i \times c_i + \alpha_j \times \lambda_j \times c_j\right) \end{cases}$$

The constraint condition:

$$\begin{cases} \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}{}' \times t_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}{}' \times \left(t_i + t_j\right) \le 720 \\ \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}{}'' \times t_{ij} + \frac{1}{2} \times \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}{}'' \times \left(t_i + t_j\right) \le 720 \\ \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}{}' = \sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}{}'' = n \qquad (n=2,\ 3,\ \ldots\ldots,\ 11) \\ \sum_i r_{ij}{}' = \sum_j r_{ij}{}' \le 1 \qquad \sum_i r_{ij}{}'' = \sum_j r_{ij}{}'' \le 1 \ (i,\ j=1,\ 2,.,\ 11) \\ \sum_{i=1} r_{ij}{}' = 1 \qquad \sum_{i=1} r_{ij}{}'' = 1 \\ \sum_{j=1} r_{ij}{}' = 1 \qquad \sum_{j=1} r_{ij}{}'' = 1 \\ r_{ij}{}' \times r_{ji}{}' = 0 \qquad r_{ij}{}'' \times r_{ji}{}'' = 0 \qquad (i,\ j=2,\ 3,\ \ldots\ldots,\ 11) \end{cases}$$

5.4.6 Solving the model and analyzing

By employing the Lingo programming,we can derive the best results:

Number of sight spots n	5
Total expenses per person c (Unit:Yuan)	750

Route	The frist family: Baotuquan Park+Quancheng Squate->Daming Lake->Qianfo Mount->Hongjialou Square->Zhujia Yu->Baotuquan Park+Quancheng Squate The second family: Baotuquan Park+Quancheng Squate->Hongjialou Square->Zhujia Yu->Jinxiangshan Park+Quancheng Squate

That means one family go sight-seeing in advance.After visiting DamingLake and Qianfo Mount,this family goes to Hongjialou Square to meet with the second family.The two families visit HongjialouSquare and Jinxiangshan Park together.Then the first family returns back to Quancheng Square and the second family goes to Jinxiang Mount.

5.5 Model 5 building and making a solution

5.5.1 Analysis of the model

This question is based on the last model and considering the weather factor,that is there is one more aim--reduce the loss of rainy weather to the minimum. We define traveling loss as the sum of stay time at sight spots and the corresponding probability of precipitation.

5.5.1 The processing of data

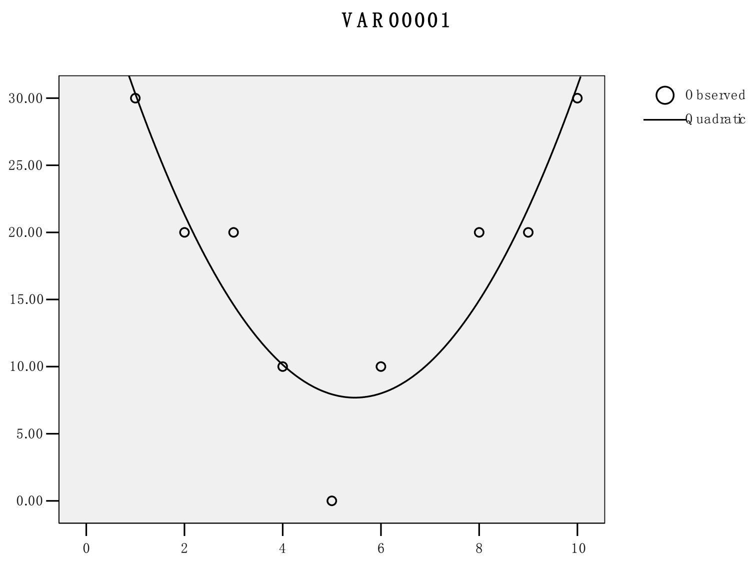
(1)By searching website Shandong Jinan Weather we can get the appendix II,and we can process the data.

I.revise the data more than 100% to 100%

II.For those missing data in the appendix,we can use SPSS to do time series prediction as follows:

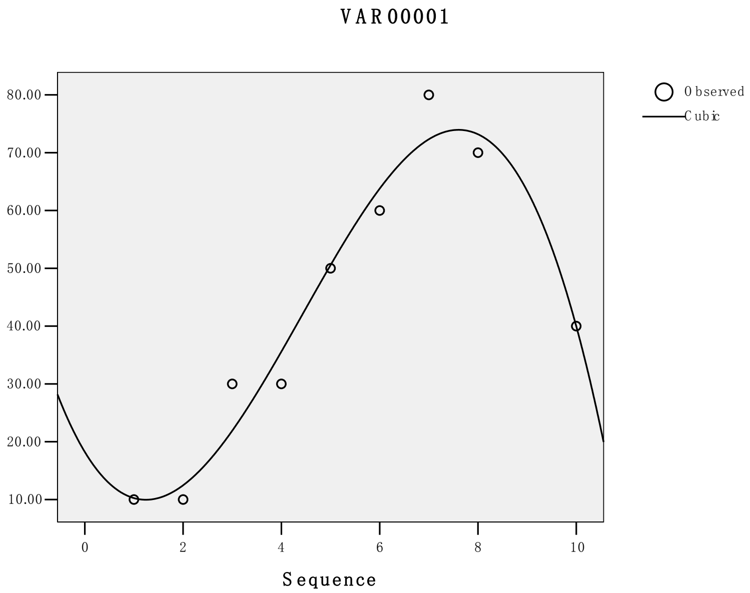
For the probability of precipitation in Jinxiang Mount,the best fitting curve is a conic curve.The fitting result is:The probability of precipitation in Jinxiang Mount on the 7th day is 10.33898%.Here we view it as 10%.

The fitted curve:



For the probability of precipitation in Zhujia Yu,the best fitting curve is a cubic curve.The fitting result is:The probability of precipitation in Zhujia Yu on the 9th day is 63.39119%.Here we view it as 63%.

The fitted curve:



In conclusion,we can obtain the ultimate matrix: $[P_{is}]_{11\times 5}$ (see at appendix)

(1)Normalization processing of the data

Observing the data,we can see that the total expenses and the travel loss that rainy weather brings has a big gap.When combining the two factorsm,in order to avoid the influence of this big gap,we employ the commonly used method-range transformation to normalize the data.That is:

$$C=\frac{c-c(n)_{\min}}{c(n)_{\max}-c(n)_{\min}} \quad ; \quad L=\frac{l-l(n)_{\min}}{l(n)_{\max}-l(n)_{\min}}$$

(2)Object function building

Following the above train of thoughts,firstly we obtain the quantity of sight spots under corresponding limitations.Secondly,we compute the corresponding travel expenses and travel loss of rainy weather.Thirdly,we process the data with normalization method and then weight to choose the smallest one.In that way,we can derive some best routes and the family can choose one based on practical situation.The ultimate object function can be described as follows:

$$\text{Min} \quad Q=\gamma_1\times C+\gamma_2\times L$$

(C,L are as mentioned above， γ_1 γ_2 are weithts and $\gamma_1+\gamma_2=1$)

I.For C

We can draw $c = m_1' + m_1'' + m_2' + m_2'' - m_3$ from the last model.The corresponding $c(n)_{\min}$ and $c(n)_{\max}$ can be computed from the above 4 models(see specific values from the appendix),that is we can use known result to represent C.

II.For L

In the formula of symbol L,the key point is representing L.After representing L, $l(n)_{\min}$ and $l(n)_{\max}$ can be easily computed by programming according to corresponding models.Because the fact that l represents the travel loss of rainy weather and we define this loss as the sum of stay time and the corresponding probability precipitation.Here we define the folliwing symbols:

P_{is} --the probability of percipitation of sight spot i on the sth day;

（s=1，2，……10）

T_i --the time arrving at sight spot i

V -the set of visited sight spots,for example, $V=\{1,8,5,7\}$ represents the family vistis sight spot 1,8,7,5.

So the family will stay at sight spot i from $[\mathrm{T_i}]$th day to $[\mathrm{T_i+t_i}]$ day ($[\ ]$ means rounding).

In conclusion,l can be represented as $\sum_{i\in V}\sum_{s=[T_I]}^{[T_I+t_i]} P_{is}$.

5.5.2Constraint conditions

(1)The constraint of time

Based on the subject,the time that a family needs to stay at Jinan should be less than 2 months(we consider that only 12 hours in a day are spent on sight-seeing),that is 720 hours.The total time includes transportation time and stay time at sight spot.Because t_{ij} represents the transportation time from sight spot i to sight spot j,the total transportation time is $\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}\times t_{ij}$.

Because t_i represents the stay time at sight spot i,the stay time that a family spends at sight spots is $\frac{1}{2}\times\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}\times(t_i+t_j)$.In conclusion,the total time constraint can be represented as follows :

$$\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}\times t_{ij}+\frac{1}{2}\times\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij}\times(t_i+t_j)\le 720$$

(2)The constraint of sight spots

Based on the requirements of the subject and the assumed conditions,the entire travel route is circular,that is Quancheng Park serves as the start and end point.So

$\sum_{i=1}^{11} 1 \sum_{j=1}^{11} r_{ij}$ represemts the number of sight spots that a family visits.Here we assume the quantity of sight spots as $n(n = 2,3,4,5,6,7,8,9,10,11)$ .Finally,we can derive the quantity constraint of sight spots:

$$\sum_{i=1}^{11}\sum_{j=1}^{11} r_{ij} = n \qquad (n=2, 3, \ldots\ldots, 11)$$

(3)The constraint of 0-1 variables

We can draw a circle to connect up all sight spots and view every sight spot as a point on that circle.For every point,it's only allowed to let one edge come out at most,and let one edge come in at most.What' more,it requires that once one edge come in then one edge come out.So we can derive thi constraint function:

$$\sum_i r_{ij} = \sum_j r_{ij} \le 1 \qquad (i, \ j=1, 2, \ldots\ldots, 11)$$

When i=1, $\sum_{i=1} r_{ij} = 1$ --based on the assumption that Quancheng Park is start point ;

When j=1, $\sum_{j=1} r_{ij} = 1$ --based on the assumption that a family has to return back to the start point(Quancheng Partk);

In conclusion,we can derive the following formula:

$$\sum_i r_{ij} = \sum_j r_{ij} \le 1 \qquad (i, j = 1,2,3\ldots\ldots 9,10,11)$$

$$\sum_{i=1} r_{ij} = 1$$

$$\sum_{j=1} r_{ij} = 1$$

In the same way,when $i, j \ge 2$,it's impossible to get $r_{ij} = r_{ji} = 1$,that is thre is no possibility that a family make a round trip between two sight spots.Because it obviously doesn't meet the requirement of visiting some sight spots as many as possible.In conclusion,we can draw a constraint function as follows:

$$r_{ij} \times r_{ji} = 0 \qquad (i, \ j=2, 3, \ldots\ldots, 11)$$

(4)The constraint of T_i

Because T_i represents the time to get to sight spot i, T_i is the sum of the time to get to sight spot j and the transportation time form sight spot i-1 to sight spot i.That is :

$$T_i = T_{i-1} + t_{i-1,i} \qquad (i=2, 3, \ldots\ldots, 10)$$

5.5.3Modeling

In conclusion,we can obtain the overall model sa follows:

$$\text{Min} \quad Q = \gamma_1 \times C + \gamma_2 \times L$$

Constraint conditon:

$$\begin{cases} \sum_{i=1}^{10}\sum_{j=1}^{10} r_{ij} \times t_{ij} + \frac{1}{2} \times \sum_{i=1}^{10}\sum_{j=1}^{10} r_{ij} \times (t_i + t_j) \le 720 \\ \sum_{i=1}^{10}\sum_{j=1}^{10} r_{ij} = k \\ \sum_{i} r_{ij} = \sum_{j} r_{ij} \le 1 \qquad (i, \ j=1, 2, \ldots\ldots, 10) \\ r_{ij} \times r_{ji} = 0 \qquad (i, \ j \ge 2) \\ T_i = T_{i-1} + t_{i-1,i} \qquad (i=2, 3, \ldots\ldots, 10) \end{cases}$$

5.5.3The results of the modeling

Because the data is enormous,we just list the situation when $n = 5$.

When $\gamma_1 = 0.6, \gamma_2 = 0.4$

Total expenses per person C(Unit:Yuan)	loss
660	1.7
Route:1→10→3→4→8→1	

When $\gamma_1 = 0.4, \gamma_2 = 0.6$

Total expenses per person C(Unit:Yuan)	loss
730	1.6
Route:1→7→9→8→4→1	

When $\gamma_1 = 0.7, \gamma_2 = 0.3$

Total expenses per person C(Unit:Yuan)	loss
640	1.8
Route:1→10→3→7→4→1	

When $\gamma_1 = 0.3, \gamma_2 = 0.7$

Total expenses per person C(Unit:Yuan)	loss
780	0.9
Route:1→10→9→8→4→1	

When $\gamma_1 = 0.5, \gamma_2 = 0.5$

Total expenses per person C(Unit:Yuan)	loss
680	1.6

Route:1→10→9→8→4→1

Based on the results above,we recommend the following route:

Baotu Spring Park+Quancheng Square->Hongjialou Square->Yingxiongshan Memorial Mount->Jinxiangshan Park->Quancheng Park-> Baotu Spring Park+Quancheng Square.The corresponding expenses per person is ￥9680.The loss of rainy weather is 1.6.

6. Evaluation and popularization

6.1 Merits and drawbacks of the model

Merits:

1. The thread of this paper is clear and straight with appropriate models and optimal proposal.

2. It is a success that the model and the program function well due to a use of 0-1 variables.

3.We use TCP algorithm to simplify the process of calculating the result.

4.We transform the problem of finding an optimal route into optimum Hamiltonian path problem, by which we get the best result that is perfectly close to the theoretical one using greedy method. Moreover, the complexity of the algorithm is $O(n^2)$.

Drawbacks:

1.The result is not so accurate because we do not get enough data. Also, we do not further study the final result.

2.When it is overcast and rainy, the program need to function as well as possible to handle the large amount of data. The result is not so accurate though it has been proven. So the program has space for improvement.

6.2 Improvements and popularization

1. There is more means of transportation in the real situation, such as airplane, train and so forth. The result will be more sensible is we take them into account.

2. When considering the requirements listed as follows: limited time, limited expense and maximum number of spots, we can optimize our analysis and transform it into satisfying the travelers as much as possible with limited expense. The reason for this is that travel is for entertainment and relaxation and to make the family enjoy their time is not only depends on the how many spots they go to but on various factors listed as follows:

(1) The degree of satisfaction of different sight spot is different

(2) Traveling sometimes causes troubles for travelers. For example, people will get upset when the time spent on transportation is too long or the road is too rugged. Thus we need to minimize the effects of such negative factors.

(3) From the perspective of psychology, a traveler will be so much sorry is he

does not make it to the spot where he is desired to go. So hope to design an optimal route which are able to satisfy the family the most.

3. We can also take into account the open time of all the sight spots and the categories of them because of the fact that some families want to go to spots that they are fond of while others do not.

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8.Appendix

Program and running results(we only choose part of them because the data is too enormous)

(Program)

```
sets:
jingdian/1..10/:c,t,l;
links(jingdian,jingdian):r,cc,tt;
endsets
data:
t=7 24 18 12 36 30 12 9 15 24 17;
c=720 423 300 135 378 390 175 90 148 303 241;
tt=0 8.54 4.74 2.82 3.44 5.08 8.4  1.32 1.54 6.14 6.6
8.54 0   1.22 11.52   12.14   10.9 13.1 8.84 8.98 14.84   15.54
4.74 1.22 0   11.22   11.82   9.38 11.58   7.66 7.46 13.44   13.9
2.82 11.52   11.22   0  0.88 7.78 8.08 4.02 4.24 5.84 6.3
```

```
3.44 12.14 11.82 0.88 0 8.42 8.24 4.66 4.88 6 6.46
5.08 10.9 9.38 7.78 8.42 0 2.18 4.24 4.04 5.98 6.74
8.4 13.1 11.58 8.08 8.24 2.18 0 6.08 6.22 3.86 2.86
1.32 8.84 7.66 4.02 4.66 4.24 6.08 0 0.3 6.28 6.74
1.54 8.98 7.46 4.24 4.88 4.04 6.22 0.3 0 6.08 6.54
6.14 14.84 13.44 5.84 6 5.98 3.86 6.28 6.08 0 2.08
6.6 15.54 13.9 6.3 6.46 6.74 2.86 6.74 6.54 2.08 0;
cc=0128.1 71.1 42.3 51.6 76.2 126 19.8 23.1 92.1 99
128.1 0 18.3 172.8 182.1 163.5 196.5 132.6 134.7 222.6 233.1
71.1 18.3 0 168.3 177.3 140.7 173.7 114.9 111.9 201.6 208.5
42.3 172.8 168.3 0 13.2 116.7 121.2 60.3 63.6 87.6 94.5
51.6 182.1 177.3 13.2 0 126.3 123.6 69.9 73.2 90 96.9
76.2 163.5 140.7 116.7 126.3 0 32.7 63.6 60.6 89.7 101.1
126 196.5 173.7 121.2 123.6 32.7 0 91.2 93.3 57.9 42.9
19.8 132.6 114.9 60.3 69.9 63.6 91.2 0 4.5 94.2 101.1
23.1 134.7 111.9 63.6 73.2 60.6 93.3 4.5 0 91.2 98.1
92.1 222.6 201.6 87.6 90 89.7 57.9 94.2 91.2 0 31.2
99 233.1 208.5 94.5 96.9 101.1 42.9 101.1 98.1 31.2 0;
n=?;
enddata
min=@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(cc(i,j)+0.5*(c(i)+c(j)))));
@for(jingdian(i):r(i,i)=0);
@for(jingdian(i)|i#ge#2:@for(jingdian(j)|j#ge#2:r(i,j)+r(j,i)<1));
a=@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(tt(i,j)+0.5*(t(i)+t(j)))));
@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(tt(i,j)+0.5*(t(i)+t(j)))))<720;
@for(jingdian(i):@sum(jingdian(j):r(i,j))=@sum(jingdian(j):r(j,i)));
@for(jingdian(i)|i#eq#1:@sum(jingdian(j):r(i,j))=1);
@for(jingdian(i)|i#ne#1:@sum(jingdian(j):r(i,j))<1);
@for(links:@bin(r));
@sum(jingdian(j):@sum(jingdian(i):r(i,j)))=n;
@for(jingdian(i):@for(jingdian(j)|j#gt#1#and#j#ne#i:l(j)>=l(i)+r(i,j)-(n-2)*(1-r(i,j))+(n-3)*r(j,i)));
@for(jingdian(i)|i#gt#1:l(i)<n-1-(n-2)*r(1,i);l(i)>1+(n-2)*r(i,1));
Global optimal solution found at iteration:        2042
   Objective value:                                   670
                          Variable        Value        Reduced Cost
                             N          5.000000        0.000000
                          R( 1, 4)      1.000000             120
                          R( 4, 7)      1.000000             130
                          R( 7, 9)      1.000000             170
                          R( 8, 1)      1.000000             110
                          R( 9, 8)      1.000000             140
```

sets:

```
jingdian/1..10/:c,t,l;
links(jingdian,jingdian):r,cc,tt;
endsets
data:
t=7 24 18 12 36 30 12 9 15 24 17;
c=720 423 300 135 378 390 175 90 148 303 241;
tt=0  8.54 4.74 2.82 3.44 5.08 8.4   1.32 1.54 6.14 6.6
8.54 0      1.22 11.52       12.14       10.9 13.1 8.84 8.98 14.84       15.54
4.74 1.22 0      11.22       11.82       9.38 11.58       7.66 7.46 13.44       13.9
2.82 11.52       11.22       0      0.88 7.78 8.08 4.02 4.24 5.84 6.3
3.44 12.14       11.82       0.88 0      8.42 8.24 4.66 4.88 6      6.46
5.08 10.9 9.38 7.78 8.42 0      2.18 4.24 4.04 5.98 6.74
8.4   13.1 11.58       8.08 8.24 2.18 0      6.08 6.22 3.86 2.86
1.32 8.84 7.66 4.02 4.66 4.24 6.08 0      0.3   6.28 6.74
1.54 8.98 7.46 4.24 4.88 4.04 6.22 0.3   0      6.08 6.54
6.14 14.84       13.44       5.84 6      5.98 3.86 6.28 6.08 0      2.08
6.6   15.54       13.9 6.3   6.46 6.74 2.86 6.74 6.54 2.08 0;
cc=0128      71        42    52    77    126  20    23    92    99
128  0      18      173        182    164        197  133  135  223  233
71    18    0      168  177  141  174  115  112  202  209
42    173  168  0      13    117      121      60.   64    88    95
52    182  177  13    0      126  124  70    73    90    97
76    164    141        117  126  0      33    64    61    90    101
126  197          174  121  124  33    0      91    93    58    43
20    133  115  60    70    64    91    0      5      94    101
23    135  112  64    73    61    93    5      0      91    98
92    223  202  88    90    90    58    94    91    0      31
99    233  209  95    97    101  43    101  98    31    0;
enddata
min=@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(cc(i,j)+0.5*(c(i)+c(j)))));
@for(jingdian(i):r(i,i)=0);
@for(jingdian(i)|i#ge#2:@for(jingdian(j)|j#ge#2:r(i,j)+r(j,i)<1));
a=@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(tt(i,j)+0.5*(t(i)+t(j)))));
@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(tt(i,j)+0.5*(t(i)+t(j)))))<360;
@for(jingdian(i):@sum(jingdian(j):r(i,j))=@sum(jingdian(j):r(j,i)));
@for(jingdian(i)|i#eq#1:@sum(jingdian(j):r(i,j))=1);
@for(jingdian(i)|i#ne#1:@sum(jingdian(j):r(i,j))<1);
@for(links:@bin(r));
@sum(jingdian(j):@sum(jingdian(i):r(i,j)))=11;
@for(jingdian(i):@for(jingdian(j)|j#gt#1#and#j#ne#i:l(j)>=l(i)+r(i,j)-(n-2)*(1-r(i,j))+(n-3)*r(j,i)));
@for(jingdian(i)|i#gt#1:l(i)<n-1-(n-2)*r(1,i);l(i)>1+(n-2)*r(i,1));
```

Result（take n=5 as example）：

Local optimal solution found at iteration: 390

Objective value: 920

Variable	Value	Reduced Cost
R(1, 4)	1.000000	0.000000
R(2, 3)	1.000000	0.000000
R(3, 1)	1.000000	0.000000
R(4, 5)	1.000000	0.000000
R(5, 10)	1.000000	0.000000
R(6, 9)	1.000000	0.000000
R(7, 6)	1.000000	0.000000
R(8, 2)	1.000000	0.000000
R(9, 8)	1.000000	0.000000
R(10, 11)	1.000000	0.000000
R(11, 7)	1.000000	0.000000

```
sets:
jingdian/1..11/:c,rrv,t,w,w2,l,l2,v,v2,b,b2,y,tv,tv2,sv;!其中：gailv/1..10/;
links(jingdian,jingdian):r,r2,cc,tt,ssv,rrr,u1,u2,u3,u4,u5,u6,u7,u8,u9,u10,u11;
links2(jingdian,gailv):pp,pv;
links1(jingdian,jingdian,jingdian):x,x2,bb,bb2,ppv;
endsets
data:
t=7 24 18 12 36 30 12 9 15 24 17;
c=720 423 300 135 378 390 175 90 148 303 241;w=0    1 1 1.058    1.058    0.942
0.942 1.058    1.058    0.942 0.942;
w2=0 0.843 0.843 1.104 1.104 1.024 1.024 1.004 1.004 1.024 1.024;
cc=0     128     71     42   52   77   126 20   23   92   99
128 0     18      173     182     164     197 133 135 223 233
71   18   0     168 177 141 174 115 112 202 209
42   173 168 0     13   117      121   60.  64   88   95
52   182 177 13   0     126 124 70   73   90   97
76   164     141     117 126 0     33   64   61   90   101
126 197        174 121 124 33   0     91   93   58   43
20   133 115 60   70   64   91   0     5     94   101
23   135 112 64   73   61   93   5     0     91   98
92   223 202 88   90   90   58   94   91   0     31
99   233 209 95   97   101 43   101 98   31   0;tt=0 8.54   4.74      2.82      3.44
     5.08       8.4  1.32       1.54       6.14       6.6
8.54 0   1.22       11.52      12.14      10.9       13.1       8.84       8.98       14.84      15.54
4.74 1.22       0     11.22      11.82      9.38       11.58      7.66       7.46       13.44      13.9
2.82 11.52      11.22      0     0.88       7.78       8.08       4.02       4.24       5.84       6.3
3.44       12.14      11.82      0.88       0     8.42       8.24       4.66       4.88       6     6.46
5.08 10.9       9.38       7.78       8.42       0     2.18       4.24       4.04       5.98       6.74
```

```
8.4  13.1       11.58     8.08       8.24       2.18       0     6.08       6.22       3.86       2.86
1.32       8.84       7.66       4.02       4.66       4.24       6.08       0     0.3  6.28       6.74
1.54       8.98       7.46       4.24       4.88       4.04       6.22       0.3  0     6.08       6.54
6.14       14.84     13.44     5.84       6     5.98       3.86       6.28       6.08       0     2.08
6.6  15.54     13.9       6.3  6.46       6.74       2.86       6.74       6.54       2.08       0;
z=0.95;
n2=6;
n=6;
enddata
min=@sum(jingdian(j):@sum(jingdian(i):w(j)*r(i,j)*(cc(i,j)+0.95*c(j)*(1-rrv(j)*(1-z))
)))+@sum(jingdian(j):@sum(jingdian(i):w2(j)*r2(i,j)*(cc(i,j)+0.95*c(j)*(1-rrv(j)*(1-
z)))));
feiyong=@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(cc(i,j)+0.95*c(j)*(1-rrv(j)*(1-z))))
);
feiyong2=@sum(jingdian(j):@sum(jingdian(i):r2(i,j)*(cc(i,j)+0.95*c(j)*(1-rrv(j)*(1-z)
))));
@sum(jingdian(j):@sum(jingdian(i):w(j)*r(i,j)*(cc(i,j)+0.95*c(j)*(1-rrv(j)*(1-z)))))+
@sum(jingdian(j):@sum(jingdian(i):w2(j)*r2(i,j)*(cc(i,j)+0.95*c(j)*(1-rrv(j)*(1-z)))))
>1493;
@for(jingdian(i):r(i,i)=0);
@for(jingdian(i):r2(i,i)=0);
@for(jingdian(i)|i#ge#2:@for(jingdian(j)|j#ge#2:r(i,j)+r(j,i)<1))；
@for(jingdian(i)|i#ge#2:@for(jingdian(j)|j#ge#2:r2(i,j)+r2(j,i)<1));
@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(tt(i,j)+t(j))))<720;
@sum(jingdian(j):@sum(jingdian(i):r2(i,j)*(tt(i,j)+t(j))))<720;
@for(jingdian(i):@sum(jingdian(j):r(i,j))=@sum(jingdian(j):r(j,i)));
@for(jingdian(i):@sum(jingdian(j):r2(i,j))=@sum(jingdian(j):r2(j,i)));
@for(jingdian(i)|i#eq#1:@sum(jingdian(j):r(i,j))=1);
@for(jingdian(i)|i#eq#1:@sum(jingdian(j):r2(i,j))=1);
@for(jingdian(i)|i#ne#1:@sum(jingdian(j):r(i,j))<1);
@for(jingdian(i)|i#ne#1:@sum(jingdian(j):r2(i,j))<1);
@for(links:@bin(r));
@for(links:@bin(r2));
@for(jingdian:@bin(rrv));
@for(links:@bin(rrr));
@sum(jingdian(j):@sum(jingdian(i):r(i,j)))=n;
@sum(jingdian(j):@sum(jingdian(i):r2(i,j)))=n2;
!@sum(jingdian(j):@sum(jingdian(i):r(i,j)))>n-0.5;
!@sum(jingdian(j):@sum(jingdian(i):r(i,j)))<n+0.5;
!@sum(jingdian(j):@sum(jingdian(i):r2(i,j)))<n2+0.5;
!@sum(jingdian(j):@sum(jingdian(i):r2(i,j)))>n2-0.5;
@for(jingdian(i):@for(jingdian(j)|j#gt#1#and#j#ne#i:l(j)>=l(i)+r(i,j)-(n-2)*(1-r(i,j))+(
n-3)*r(j,i)));
@for(jingdian(i):@for(jingdian(j)|j#gt#1#and#j#ne#i:l2(j)>=l2(i)+r2(i,j)-(n2-2)*(1-r2
```

```
(i,j))+(n2-3)*r2(j,i)));
@for(jingdian(i)|i#gt#1:l(i)<n-1-(n-2)*r(1,i);l(i)>1+(n-2)*r(i,1));
@for(jingdian(i)|i#gt#1:l2(i)<n2-1-(n2-2)*r2(1,i);l2(i)>1+(n2-2)*r2(i,1));
@for(jingdian(i)|1#eq#i:v(i)=1;b(i)=0);
!@for(jingdian(k)|1#lt#k#and#k#le#n:@for(jingdian(i):@for(jingdian(j):x(k,i,j)=@if(
r(i,j)#eq#1#and#v(k-1)#eq#i,j,0)));!v(k)=@sum(jingdian(j):@sum(jingdian(i):x(k,i,j)))
);
@for(jingdian(k)|1#lt#k#and#k#le#n:@for(jingdian(i):@for(jingdian(j):x(k,i,j)=@if(0.
5#le#r(i,j)#and#r(i,j)#le#1.5#and#(i-0.5)#le#v(k-1)#and#v(k-1)#le#(i+0.5),j,0)));v(k)
=@sum(jingdian(j):@sum(jingdian(i):x(k,i,j))));
@for(jingdian(i)|1#eq#i:v2(i)=1;b2(i)=0);
!@for(jingdian(k)|1#lt#k#and#k#le#n2:@for(jingdian(i):@for(jingdian(j):x2(k,i,j)=@
if(r(i,j)#eq#1#and#v2(k-1)#eq#i,j,0)));!v2(k)=@sum(jingdian(j):@sum(jingdian(i):x2
(k,i,j))));
@for(jingdian(k)|1#lt#k#and#k#le#n2:@for(jingdian(i):@for(jingdian(j):x2(k,i,j)=@i
f(0.5#le#r2(i,j)#and#r2(i,j)#le#1.5#and#(i-0.5)#le#v2(k-1)#and#v2(k-1)#le#(i+0.5),j,0)
));v2(k)=@sum(jingdian(j):@sum(jingdian(i):x2(k,i,j))));
@for(jingdian(i)|2#le#i#and#i#le#(n):@for(jingdian(k):@for(jingdian(j):bb(k,i,j)=@if
(v(i)#eq#j#and#v(i-1)#eq#k,t(k)+tt(k,j),0)));b(i)=@sum(jingdian(j):@sum(jingdian(k):
bb(k,i,j))));
@for(jingdian(k)|k#le#n:tv(k)=@sum(jingdian(i)|i#le#(k):b(i))/12);
@for(jingdian(i)|2#le#i#and#i#le#(n2):@for(jingdian(k):@for(jingdian(j):bb2(k,i,j)=
@if(v2(i)#eq#j#and#v2(i-1)#eq#k,t(k)+tt(k,j),0)));b2(i)=@sum(jingdian(j):@sum(jing
dian(k):bb2(k,i,j))));
@for(jingdian(k)|k#ne#1#and#k#le#n2:tv2(k)=@sum(jingdian(i)|i#le#(k):b2(i))/12+4)
;
@for(jingdian(k)|k#eq#1#or#k#gt#n2:tv2(k)=0);
@for(jingdian(k)|k#le#n2:@for(jingdian(j)|j#le#n:rrr(j,k)=@if(v(j)#eq#v2(k)#and#(tv
2(k)-tv(j))#lt#1#and#(tv(j)-tv2(k))#lt#1,1,0)));
@for(links(j,k)|(j#gt#n#and#k#le#n2)#or#(j#le#n#and#k#gt#n)#or#(j#gt#n#and#k#gt
#n):rrr(j,k)=0);
@for(jingdian(j)|j#ge#2#and#j#le#n2:rrv(j)=@sum(jingdian(k):rrr(j,k)));
@for(jingdian(k)|k#eq#1#or#k#gt#n2:rrv(k)=0);
End
```

Result（take n=5 as example）：

```
Local optimal solution found at iteration:            5831
   Objective value:                                       750

                      Variable           Value        Reduced Cost
                             Z       0.9500000            0.000000
                            N2        5.000000            0.000000
                             N        5.000000            0.000000
                       FEIYONG        960.6000            0.000000
```

```
FEIYONG2        980.4500        0.000000
V( 1)        1.000000        0.000000
V( 2)        0.000000        0.000000
V( 3)        0.000000        0.000000
V( 4)        0.000000        0.000000
V( 5)        0.000000        0.000000
V( 6)        0.000000        0.000000
V( 7)        0.000000        0.000000
V( 8)        0.000000        0.000000
V( 9)        0.000000        0.000000
V( 10)        0.000000        0.000000
V( 11)        0.000000        0.000000
V2( 1)        1.000000        0.000000
V2( 2)        0.000000        0.000000
V2( 3)        0.000000        0.000000
V2( 4)        0.000000        0.000000
V2( 5)        0.000000        0.000000
V2( 6)        0.000000        0.000000
V2( 7)        0.000000        0.000000
V2( 8)        0.000000        0.000000
V2( 9)        0.000000        0.000000
V2( 10)        0.000000        0.000000
V2( 11)        0.000000        0.000000
```

Program（take n=5 as example）：

```
sets:
jingdian/1..11/:c,t,w,l,v,b,y,tv;
gailv/1..10/;
links(jingdian,jingdian):r,cc,tt;
links2(jingdian,gailv):pp,pv;
links1(jingdian,jingdian,jingdian):x,bb,ppv;
endsets
data:
t=7 24 18 12 36 30 12 9 15 24 17;
c=720 423 300 135 378 390 175 90 148 303 241;w= 0 0.185 0.185 0.217 0.217 0.196
0.196 0.206 0.206 0.196 0.196;
tt=0 8.54	4.74	2.82	3.44	5.08	8.4 1.32	1.54	6.14	6.6
8.54 0	1.22	11.52	12.14	10.9	13.1	8.84	8.98	14.84	15.54
4.74 1.22	0	11.22	11.82	9.38	11.58	7.66	7.46	13.44	13.9
2.82 11.52	11.22	0	0.88	7.78	8.08	4.02	4.24	5.84	6.3
3.44	12.14	11.82	0.88	0	8.42	8.24	4.66	4.88	6	6.46
5.08 10.9	9.38	7.78	8.42	0	2.18	4.24	4.04	5.98	6.74
8.4 13.1	11.58	8.08	8.24	2.18	0	6.08	6.22	3.86	2.86
```

```
1.32	8.84	7.66	4.02	4.66	4.24	6.08	0	0.3	6.28	6.74
1.54	8.98	7.46	4.24	4.88	4.04	6.22	0.3	0	6.08	6.54
6.14	14.84	13.44	5.84	6	5.98	3.86	6.28	6.08	0	2.08
6.6	15.54	13.9	6.3	6.46	6.74	2.86	6.74	6.54	2.08	0;
cc=0	128	71	42	52	77	126	20	23	92	99
128	0	18	173	182	164	197	133	135	223	233
71	18	0	168	177	141	174	115	112	202	209
42	173	168	0	13	117	121	60.	64	88	95
52	182	177	13	0	126	124	70	73	90	97
76	164	141	117	126	0	33	64	61	90	101
126	197	174	121	124	33	0	91	93	58	43
20	133	115	60	70	64	91	0	5	94	101
23	135	112	64	73	61	93	5	0	91	98
92	223	202	88	90	90	58	94	91	0	31
99	233	209	95	97	101	43	101	98	31	0;
n=5;
pp=0.15	0.1	0.3	0.8	0.7	0.5	0.6	0.3	0.1	0
1	0.8	0.8	0.5	0.5	0.4	0.5	0.6	0.3	0.2
0.5	1	0.9	1	0.3	0	0.2	0	0.4	0.4
0.1	0	0.1	0.5	0.5	0.7	0.3	0.3	0.1	0.1
0.3	0.4	0.4	0.6	0.3	0.3	0.2	0.4	0.6	0.6
0.6	0.6	0.6	0.5	0.8	0.3	0.1	0.1	0.1	1
0.3	0.2	0.2	0.1	0	0.1	0.1	0.2	0.2	0.3
0.4	0.3	0.3	0.2	0.4	0.9	0.9	0.9	0.8	0.8
0.5	0.3	0.3	0.4	0.3	0.8	1	0.9	0.9	0.9
0.2	0.6	0.6	0.4	0.1	0	0.8	0.7	0.6	0.4
0.1	0.1	0.3	0.3	0.5	0.6	0.8	0.7	0.63	0.4;
enddata
min=0*(p-0.9)/3.3+0.9*(quan- 137)/173;
quan=@sum(jingdian(j):@sum(jingdian(i):w(j)*r(i,j)*(cc(i,j)+0.45*(c(i)+c(j)))));
q=@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(cc(i,j)+0.45*(c(i)+c(j)))));
@for(jingdian(i):r(i,i)=0);
@for(jingdian(i)|i#ge#2:@for(jingdian(j)|j#ge#2:r(i,j)+r(j,i)<1));

@sum(jingdian(j):@sum(jingdian(i):r(i,j)*(tt(i,j)+0.5*(t(i)+t(j)))))<720;

@for(jingdian(i):@sum(jingdian(j):r(i,j))=@sum(jingdian(j):r(j,i)));
@for(jingdian(i)|i#eq#1:@sum(jingdian(j):r(i,j))=1);
@for(jingdian(i)|i#ne#1:@sum(jingdian(j):r(i,j))<1);
@for(links:@bin(r));
@sum(jingdian(j):@sum(jingdian(i):r(i,j)))=n;
@for(jingdian(i):@for(jingdian(j)|j#gt#1#and#j#ne#i:l(j)>=l(i)+r(i,j)-(n-2)*(1-r(i,j))+(
n-3)*r(j,i)));
@for(jingdian(i)|i#gt#1:l(i)<n-1-(n-2)*r(1,i);l(i)>1+(n-2)*r(i,1));
```

```
@for(jingdian(i)|1#eq#i:v(i)=1;b(i)=0);
@for(jingdian(k)|1#lt#k#and#k#le#n:@for(jingdian(i):@for(jingdian(j):x(k,i,j)=@if(0.
5#le#r(i,j)#and#r(i,j)#le#1.5#and#(i-0.5)#le#v(k-1)#and#v(k-1)#le#(i+0.5),j,0)));v(k)
=@sum(jingdian(j):@sum(jingdian(i):x(k,i,j))));
@for(jingdian(i)|2#le#i#and#i#le#(n):@for(jingdian(k):@for(jingdian(j):bb(k,i,j)=@if
(v(i)#eq#j#and#v(i-1)#eq#k,t(k)+tt(k,j),0))));
@for(jingdian(i)|2#le#i#and#i#le#(n):b(i)=@sum(jingdian(j):@sum(jingdian(k):bb(k,
i,j))));
@for(jingdian(k)|k#le#n:tv(k)=@sum(jingdian(i)|1#le#i#and#i#le#(k):b(i)/12));
!@for(jingdian(j)|j#le#n:@for(jingdian(i):ssv(i,j)=@if(v(j)#eq#i,t(i),0)));
!@for(jingdian(j)|j#le#n:sv(j)=@sum(jingdian(i):ssv(i,j)));
@for(jingdian(j)|2#le#j#and#j#le#n:@for(gailv(k):@for(jingdian(i):ppv(i,j,k)=@if(v(j)
#eq#i#and#(tv(j))#le#k#and#k#le#(tv(j)+t(i)/12),pp(i,k),0))));
@for(links2(j,k)|2#le#j#and#j#le#n:pv(j,k)=@sum(jingdian(i):ppv(i,j,k)));
@for(links2(j,k)|j#lt#2#or#j#gt#n:pv(j,k)=0);
p=@sum(links2(j,k):pv(j,k));
end
```

Result

Variable	Value	Reduced Cost
N	5.000000	0.000000
P	1.600000	0.000000
QUAN	157.8990	0.000000
Q	927.2000	0.000000
V(1)	1.000000	0.000000
V(2)	7.000000	0.000000
V(3)	9.000000	0.000000
V(4)	8.000000	0.000000
V(5)	4.000000	0.000000
V(6)	0.000000	0.000000
V(7)	0.000000	0.000000
V(8)	0.000000	0.000000
V(9)	0.000000	0.000000
V(10)	0.000000	0.000000
V(11)	0.000000	0.000000