Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
A		2113656

Fungus: Who Moved My Woody Fibers

Summary

The carbon cycle includes the decomposition of plant material and woody fibers. In nature, fungi are the main decomposers. The decomposition rate of fungi is determined by two traits: the growth rate of the fungus and their tolerance to moisture. In this paper, our goal is to model the decomposition of woody fibers and analyze the interactions between different species.

In view of **a single type of fungus** living in America, firstly, we use the exponential decay model to describe the decomposition process. Then we fit the experimental data related to growth rate and moisture resistance to obtain the function between the decomposition rate and these two traits. Finally, we establish the decomposition model of a single species of fungus. Taking five types of fungi as examples, we get their decomposition processes respectively shown in Figure 6.

In view of **various types of fungi** living in America, we analyzed the short-term and long-term trends respectively. **In a short period of time**, various species grow independently. The total average decomposition rate is the sum of each. Then we establish the decomposition model of various species in short-term. The simulated decomposition process is shown in Figure 7. **In a long period of time**, the species will influence on each other. We improved the Logistics differential equation model and established a competition model of various species. After using the Runge-Kutta method, we get their succession and decomposition process with interactions shown in Figures 8 and 9.

Based on the above two models, we consider the impact of global warming and climate conditions changes on the model. First, according to the time series *ARIMA (0,1,1)*, we predict the future annual average temperature in America. Compared with 2020, temperature conditions in 2060 will cause the extinction of three species (Figure 13). Next, we bring the environmental conditions of five representative climates into the competition model. Results show different climates lead to different endings of competition (Figure 15).

Using the above two models, we find that as the number of fungal species increases, the overall decomposition efficiency of ground litter continues to increase. And we analyze that biodiversity plays a huge role in maintaining system stability.

Keywords: Fungus The carbon cycle Biodiversity of fungi The decomposition model The Competition model of various species Differential equation

Contents

1 Introduction

1.1 Background

The carbon cycle is an important part of life on earth. It describes the process of the exchange of carbon throughout the geochemical cycle of the earth and it maintains the balance of carbon dioxide in the atmosphere. Part of the carbon cycle includes the decomposition of compounds, and the key components of this part are the decomposition of plant materials and woody fibers. In nature, the key component in decomposing woody fibers are fungi. We know that the decomposition rate of woody fibers is determined by various traits of fungi through the background of the research. In this problem, we would focus on just two traits of the fugus: the growth rate and its tolerance to moisture.

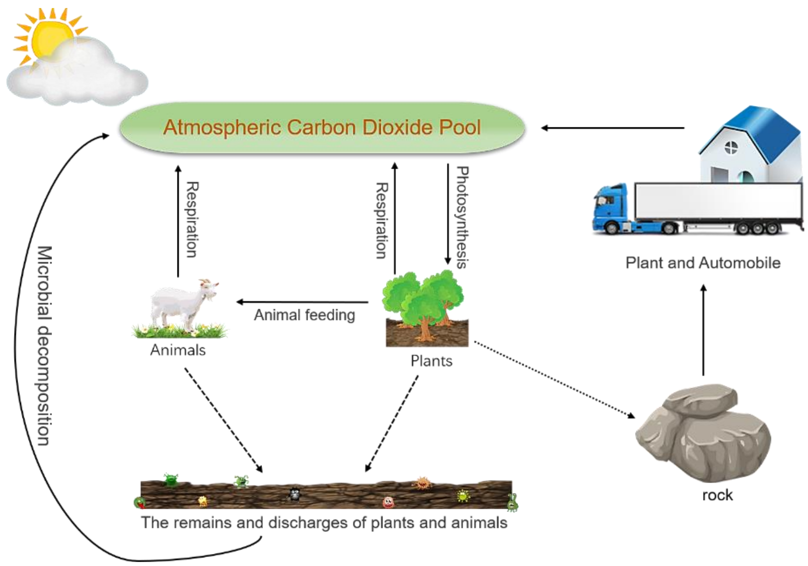


Figure 1: The process of carbon cycle

1.2 Restatement of the Problem

Our main goals are to study the effect of the growth rate of the fungus and the fungus' tolerance to moisture on decomposition rate, and to build a model about the decomposition of woody fibers on a given land. For this, we need to solve the following problems:

- Build a mathematical model to describe the specific decomposition process of litter and wood fiber through fungal activity.
- In the above model, when there are many different species of fungi, which have different growth rates and different moisture resistance, we need to incorporate the interactions between different species of fungi.
- On the basis of the above, we need to build the model which describes the interaction

between different types of fungi, and the different performance of the dynamic characteristics of the interaction in short-term and long-term trends should be characterized and described. When the environment fluctuates rapidly, we need to test the sensitivity of the model and determine the overall impact of changing atmospheric trends, so as to assess the impact of local weather changes on the model.

- We need to predict the relative advantages and disadvantages for each species and combinations of species that may last for a period of time, and do so for the different environments of arid, semi-arid, temperate, arbor, and tropical rainforest.
- Analyze how the diversity of fungal communities of a system affects the overall efficiency of ground waste decomposition. When there are varying degrees of changes in the local environment, predict the importance and role of biodiversity.
- By establishing and solving above models, we need to write a two-page article to discuss the latest progress in our understanding of the role of fungi play in ecosystems.

2 Analysis of the Problem

- Based on the Mitscherlich equation, we build a litter attenuation model conforming to the negative exponential form. From the figures of the paper, we believe that the average decomposition rate of the plant material and woody fibers is affected by the growth rate and moisture resistance of the fungus (fungi). We fit this set of experimental data with a power function and an exponential function, and found the equivalence relation between the growth rate, the moisture resistance and average decomposition rate. Based on this, we show the change of remaining proportion over time (single species) (Figure 6).
- When a variety of fungi gather together, there is interactions between them, which is mainly competition. We divide the decomposition process of various species into short-term and long-term. In a short period of time, we believe that each fungus decomposes litter independently, when the decomposition of litter by multiple groups can be considered as the sum of individual's. Taking five groups as an example, we show the change of remaining proportion (various species, short-term trends) (Figure 7).
- Over a long period of time, the result of fungal competition can be measured by the steady state of the extension length of the hyphae. Based on the Logistics model and the Lotka-Volterr model, we establish a competition model of various species, and measure the contribution of different fungi in decomposition with the steady-state dominant species-inferior species (Figure 8), and show the above five the change of remaining proportion with competitions (various species, long-term trends) (Figure 9).

- Since the external environment will determine the succession of the community, we separately consider the impact of global warming and climate change on the model. According to the time series *ARIMA(0,1,1)*, we predict the annual average temperature of North America in 2060 (Figure 12), which is 1.2℃ higher than 2020. We find that such a small temperature change will cause three of the five species perish (Figure 13). Under the different environments including arid, semi-arid, temperate, arboreal, and tropical rain forests, the competition results of populations vary (Figure 15).
- Considering the fungus and different combinations of fungi, we find that when the extension length increases, the overall efficiency of ground garbage decomposition continues to increase (Table 4). As for single fungus, environment change may cause the growth rate greatly reduced, which hinder decomposition. But as for multiple groups, even if environment change result in different results in the steady state of competition, the decomposition rate is impacted little.

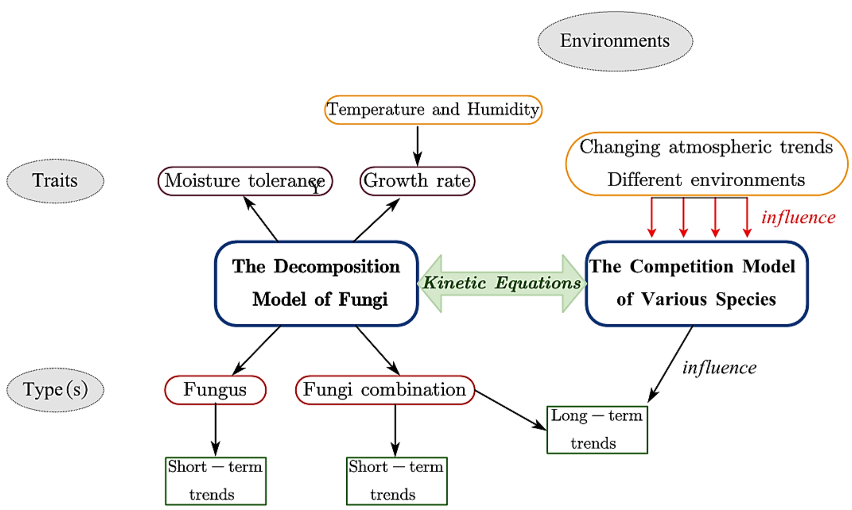


Figure 2: Problem solving ideas

3 Assumptions

- **In a short period of time, there is no interaction between different species of fungi living in the same environment.** In the early stages of growth, for all kinds of species, resources such as nutrients, space, and dissolved oxygen are sufficient. Therefore, in the short term, the adverse effects from other species on the growth of fungi can be ignored.
- **The interaction between different species of fungi is only equal to the competitive relationship.** When different types of fungi live in the same environment, there are complex interrelationships between them, such as competition, cooperation, symbiosis and so on, but competition is the most important and common relationship, so we can ignore other types of

interrelationships [1].

- **The data used in this article are true and credible.** In order to apply and solve the mathematical model which have been established, we collected the relevant data of 34 kinds of fungi[2]. To ensure the objectivity and accuracy of the results, we make this assumption.

4 Symbols

Descriptions	Definitions
Mass of the material at time t	m_t
Hyphal extension rate	v
Moisture resistance	w
Temperature	T
Humidity	S
Average decomposition rate	k

5 The Decomposition Process of Fungi

According to the relevant research, it is known that certain traits of fungi determine the decomposition rate of plant materials and woody fibers.

Figure 3: The decomposition process of fungi

In this problem, we only need to focus on two traits of fungi: fungal growth rate (Hyphal Extension Rate) and moisture resistance (the difference between a fungus' competitive ranking and its moisture niche width), and we assume these two traits are independent of each other. In view of the existence of a single type of fungus or multiple types of fungi on a given land, we have established a short-term decomposition model of fungus as follows:

5.1 The Decomposition Model of Single Species

According to the information in the title, when the ambient temperature is 22℃, the relationship between mycelial extension rate and decomposition rate is shown in Figure 1, and the relationship between moisture resistance and decomposition rate is shown in Figure 2:

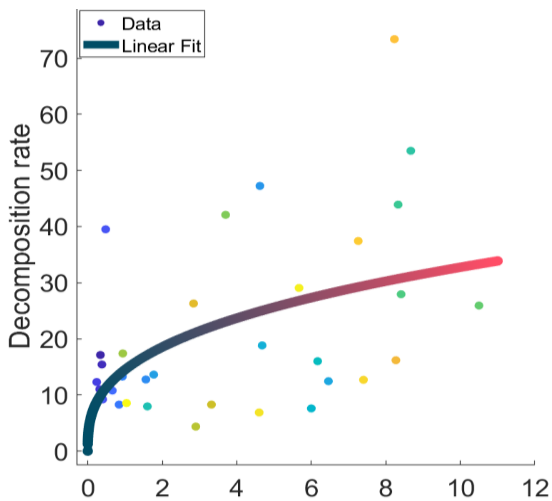


Figure 4: The relationship between the hyphal extension rate (mm/day) of various fungi and the decomposition rate (% mass loss over 122 days at 22℃)

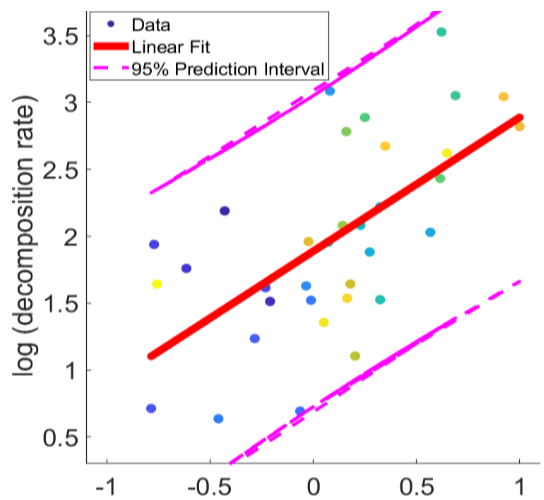


Figure 5: The relationship between the moisture (both scaled to [0,1]) of various fungi and the wood decomposition rate (% mass loss over 122 days at 22℃, log transformed)

Since Figure 2 is the test result under the optimal external conditions, it is assumed that the temperature condition of Figure 2 is 22°C. That is to say, the above two figures reflect the relationship between the decomposition rate and the hyphal extension rate (fungus growth rate) and moisture resistance when the temperature is 22°C.

We think Δm is the reduced mass of the plant material and woody fiber in a period of time, m_0 is the mass of the initial plant material and wood fiber, m_t is the mass of the initial plant material and wood fiber after a period of decomposition. Then the ordinate of Figure 1 and Figure 2 is the resulting decomposition rate:

$$\frac{\Delta m}{m_0}=\frac{m_0-m_t}{m_0}=1-\frac{m_t}{m_0} \tag{1}$$

Now using the principle of least squares method, combined with the prompt of the title, it is essential to fit the data in Figure 1. Then we get that when the temperature is 22℃, the relationship between the resulting decomposition rate and the hyphal extension rate is in the form of a power function, and the relationship and its core function is shown as follows:

$$\frac{\Delta m}{m_0}=a\bullet v^b=14.5\bullet v^{0.3538}\propto v^b \tag{2}$$

Next, we process the data in Figure 2. Firstly, we take the logarithm of the decomposition rate, and then fit the processed data to obtain the relationship between the decomposition rate and moisture resistance. The result and the kernel function at 22°C are shown as follows:

$$\begin{aligned} &\ln\frac{\Delta m}{m_0}=kw+t=1.0004k+1.8878 \\ \Rightarrow\quad &\frac{\Delta m}{m_0}=e^{kw}\bullet e^{t}=e^{1.0004w}\bullet e^{1.8878}\propto e^{kw} \end{aligned} \tag{3}$$

Since the decomposition rate $\Delta m/m_0$ is only directly related to the fungal growth rate v (hyphae elongation rate) and moisture resistance w, the decomposition rate is a linear function of the product of the two kernel functions. From Figure 1, we know that $v=0$, $\Delta m/m_0=0$. Then we get:

$$\frac{\Delta m}{m_0}=p\bullet e^{kw}\bullet v^b \tag{4}$$

Now we take any decomposition rate from the two figures, and bring the corresponding fungal growth rate and moisture resistance into the formula (4). Through calculation, we get the coefficient as:

$$p=12.1785 \tag{5}$$

According to the existing literature[3], we know that the decomposition process of the plant material and woody fibers and other litters mentioned in this article follows the Mitscherlich equation[4][5], that is, as time goes by, the litters are gradually decomposed. The decomposition tends to be slow, and the final decomposition rate becomes a constant value.

Therefore, the exponential decay model[5] is the best model to describe the decomposition process of plant material and woody fibers. The model is:

$$\frac{m_t}{m_0} = e^{-kt} \tag{6}$$

Where k is the average decomposition rate. Combined with the above analysis, we can get:

$$\begin{aligned} 1 - e^{-kt} &= p \bullet e^{kw} \bullet v^b \\ \Rightarrow k &= -\frac{1}{t} \bullet \ln\left(1 - p \bullet v^b e^{kw}\right) \end{aligned} \tag{7}$$

Because the data shown in Figure 1 and Figure 2 are experimental results after 122 days, $t = 122$. In summary, we finally get a decomposition model of single species of fungus as follows:

$$\begin{cases} \dfrac{m_t}{m_0} = e^{-kt} \\ k = -\dfrac{1}{122} \bullet \ln\left(1 - 12.1785 \bullet v^{0.3538} \bullet e^{1.0004w}\right) \end{cases} \tag{8}$$

In this model, k is the average decomposition rate of plant material and woody fibers by a certain species of fungus, which is directly related to only the growth rate v (hyphae extension rate) and moisture resistance w.

5.2 The Decomposition Model of Various Species

In the natural ecosystem, there are multiple types of fungi living on a given land, rather than a single species. They live and decompose plant material and woody fibers together. Next, we incorporate the activities of different species of fungi, which have different growth rates and different moisture resistance. Then we establish a decomposition model of various species. In this model, we only consider the short-term situation. According to the assumptions, different types of fungi grow independently in the short-term.

According to the above analysis, we get that k is the average decomposition rate of a certain type of fungi in the exponential decay model $m_t / m_0 = e^{-kt}$,. If various species of fungi live together and their growth processes are independent of each other, the total decomposition rate is the sum of their respective decomposition rates. From this, we can get that the exponential decay model when there are various species living on a given land is as follows:

$$\frac{m_t}{m_0} = \exp\left(-\sum_{i=1}^{n} k_i t\right) \tag{9}$$

Where k_i is the average decomposition speed of type i, which is determined by growth rate and moisture tolerance. Combining the decomposition model of single species, we finally get the multi-type fungi competition model as:

$$\begin{cases} \dfrac{m_t}{m_0} = \exp\left(-\sum_{i=1}^{n} k_i t\right) \\ k_i = -\dfrac{1}{122} \cdot \ln\left(1 - 12.1785 \cdot v_i^{0.3538} \cdot e^{1.0004 w_i}\right) \end{cases} \tag{10}$$

5.3 Simulations Based on The Decomposition Model

Combining the title and the mathematical model we have established, after collecting some information, we have obtained relevant data about five types of fungi, as shown in the following table:

Table 1: The data about five types of fungi

Fungus species	Extension rate (mm/ day, at 22℃)	moisture trade-off (scaled to [0,1])
Lentinus crinitus PR2058 C1B	6.17	0.228833608
Phlebiopsis flavidoalba FP150451 A8G	10.5	0.614882787
Phlebia acerina DR60 A8A	8.23	0.922073267
Pycnoporus sanguineus PR SC 95 A11C	7.60	0.348555452
Schizophyllum commune PR1117	4.60	0.051616603

To obtain the specific decomposition process of ground litter and woody fibers, we bring the data corresponding to the above five types of fungi into the decomposition model of single species, and get the corresponding remaining mass ratios as shown in the Figure 6.

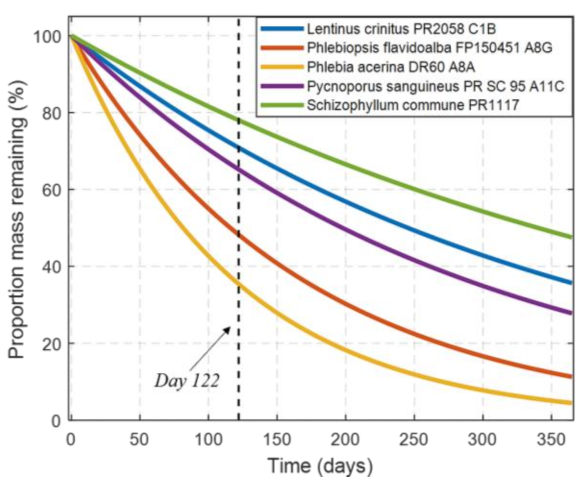


Figure 6: The remaining proportion of single type

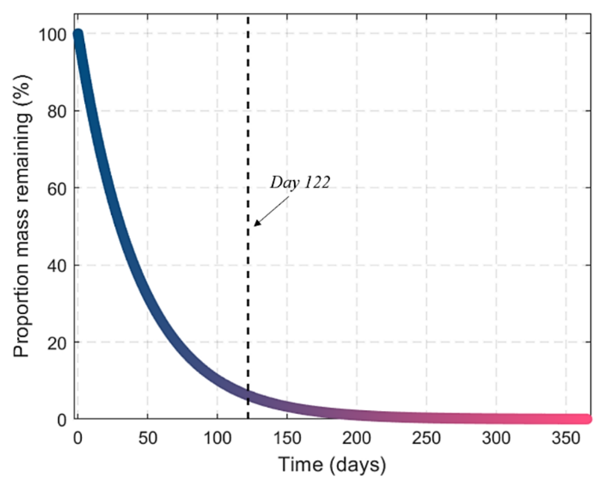


Figure 7: The remaining proportion of various type

When there are various species of fungi, we incorporate the activities of the above five types of fungi, and bring the specific data into the multi-type fungi decomposition model to obtain the decomposition process of litter and woody fibers as shown in the Figure 7.

Observing the above two figures, taking 122 days as the boundary, it is obvious that compared to the decomposition activity of any single type of fungus, when the five types of fungi live together, the decomposition process of ground litter and woody fibers proceeds faster. It is consistent with the actual situation of the ecosystem and also verifies that the fungal decomposition model is scientific and accurate.

6 The Interactions Between the Various Species of Fungi

According to the hypothesis, it is known that the interaction between different types of fungi is just the relationship of competition, while ignoring the relationship of cooperation and symbiosis. And, in a short period of time, there is no interaction between different species, which means that they grow independently. Only in a long period of time, we consider the influence of competition[4][6] on the their growth. Next, we plan to establish a competition model between various species, and then analyze the decomposition process of fungi with the relationship of competition.

6.1 The Competition Model of Various Species

When there is only one kind of fungus living on a given land, the logistic model can be used to describe the process of its growth[7]. That is, as time goes by, the extension of fungal hyphae will reach the final steady state:

$$\frac{dx(t)}{dt} = rx\left(1 - \frac{x}{x_m}\right) \tag{11}$$

Where $x(t)$ is the hypha length of this species at time t, r is the inherent hyphal extension rate, x_m is the maximum hypha length allowed by environmental conditions.

If there are two or more types of fungi living on a given land, then in a long period of time, they will compete for the same kind of food and living space. The difference in competitiveness determines their ending[8]. Now we consider that there are n types of fungi living together on a given land. When they live in a natural environment alone, their growth laws satisfy the logistic model. That is, for each type of fungus:

$$\frac{dx_i(t)}{dt} = r_i x_i\left(1 - \frac{x_i}{x_{mi}}\right) \quad i = 1, 2, \cdots, n \tag{12}$$

Where $x_i(t)$ is the hypha length of the type i at time t, r_i is the inherent hyphal extension rate of the type i, and x_{mi} is the maximum hyphal length of the type i allowed by the environment. The factor $\left(1-x_i / x_{mi}\right)$ reflects the retarding effect of the type i on its growth due to its consumption of limited resources.

When there are multiple types of fungi living on a given land, considering the impact of other types of fungi on the growth of the certain type of fungi, it is reasonable to subtract a number of items in the factor $\left(1-x_i / x_{mi}\right)$, which is simplified as:

$$\frac{dx_i(t)}{dt}=x_i\left(r_i-\frac{r_i}{x_{mi}}x_i-\sum_{j\neq i}\frac{\sigma_i}{x_{mj}}x_j\right) \tag{13}$$

The meaning of σ_i is: the decomposition mass of a unit quantity of type j fungi is σ_i times bigger than a unit quantity of type i fungus[8]. We suppose that the inherent average decomposition rate of the type i fungi is $\tilde{k}_i$, so we can consider:

$$\sigma_i=\frac{\tilde{k}_j}{\tilde{k}_i} \tag{14}$$

$$\tilde{k}_i=-\frac{1}{122}\bullet\ln\left(1-12.1785\bullet r_i^{0.3538}\bullet e^{1.0004w_i}\right) \tag{15}$$

Where r_i is the inherent hyphal extension rate of the type i, which is related to temperature T and humidity S in the environment.

And the temperature and humidity have the same influence on the inherent hyphal extension rate of fungi, which highlights the close consistency between the moisture and heat niche width[2]. Let $f_i(T)$ and $g_i(S)$ denote the functional relationship about temperature and humidity respectively, so:

$$r_i=\frac{1}{2}\bullet f_i(T)+\frac{1}{2}\bullet g_i(S) \tag{16}$$

In summary, we can get the competition model of various species as:

$$\begin{cases}\dfrac{dx_i(t)}{dt}=x_i\left(r_i-\dfrac{r_i}{x_{mi}}x_i-\sum\limits_{j\neq i}\dfrac{\sigma_i}{x_{mj}}x_j\right)\\ \sigma_i=\dfrac{\tilde{k}_j}{\tilde{k}_i} & i=1,2,\cdots,n\\ \tilde{k}_i=-\dfrac{1}{122}\bullet\ln\left(1-12.1785\bullet r_i^{0.3538}\bullet e^{1.0004w_i}\right)\\ r_i=\dfrac{1}{2}\bullet f_i(T)+\dfrac{1}{2}\bullet g_i(S)\end{cases} \tag{17}$$

Combined with the model we have established before, in order to simulate the decomposition process of different types of fungi with the competitive relationship, we can finally get the decomposition model of various species of fungi in the long-term process as:

$$\begin{cases} \dfrac{m_t}{m_0} = \exp\left(-\sum_{i=1}^{n} k_i t\right) \\ k_i = -\dfrac{1}{122} \bullet \ln\left(1 - 12.1785 \bullet v_i^{0.3538} \bullet e^{1.0004 w_i}\right) \\ \dfrac{dx_i(t)}{dt} = x_i\left(r_i - \dfrac{r_i}{x_{mi}} x_i - \sum_{j \neq i} \dfrac{\tilde{k}_j}{\tilde{k}_i x_{mj}} x_j\right) \\ \tilde{k}_i = -\dfrac{1}{122} \bullet \ln\left(1 - 12.1785 \bullet r_i^{0.3538} \bullet e^{1.0004 w_i}\right) \\ r_i = \dfrac{1}{2} \bullet f_i(T) + \dfrac{1}{2} \bullet g_i(S) \end{cases} \tag{18}$$

6.2 Solutions of The Competition Model

In the fungal decomposition model of problem 1, the hyphal extension rate calculated in the equation is the value measured over 122 days, so we can think:

$$v_i = \left(\frac{dx_i}{dt}\right)\Bigg|_{t=122} \tag{19}$$

By using the basic knowledge of biology, we set the hyphal length at the initial moment as $x(0) = 5\,mm$. Assuming that the maximum hyphal length allowed by the environment of each type is the same, and $x_{mi} = 1000\,mm$. When environmental conditions are fixed as 20°C, we calculate and get the relevant data of the above five types of fungi as shown in the table:

Table 2: The data about 5 types of fungi

Fungus species	Inherent extension rate	Moisture trade-off (scaled to [0,1])	Extension rate (over 122 days)
Lentinus crinitus PR2058 C1B	5.16	0.228833608	0.67
Phlebiopsis flavidoalba FP150451 A8G	9.48	0.614882787	7.82
Phlebia acerina DR60 A8A	8.04	0.922073267	4.81
Pycnoporus sanguineus PR SC 95 A11C	5.74	0.348555452	1.09
Schizophyllum commune PR1117	3.09	0.051616603	0.11

Next, we substitute the data in Table 2 into the competition model between different types of fungi. Then we obtain the evolution of the five fungi over time at 20°C as shown in the figure below:

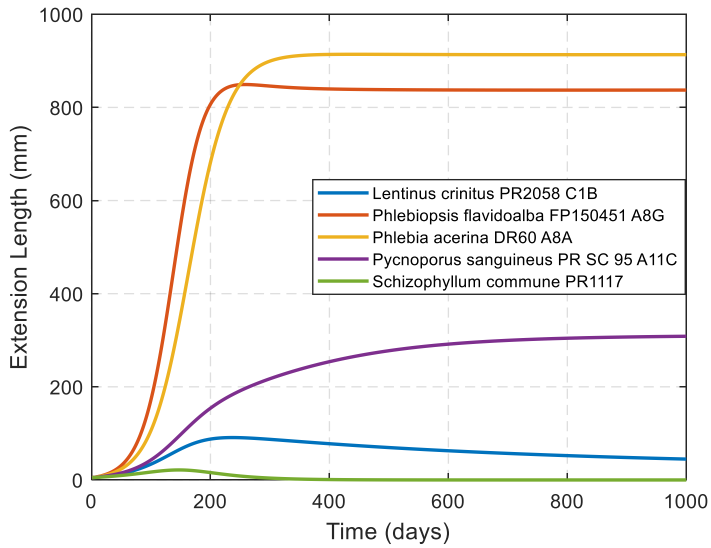


Figure 8: The succession at 20℃

From the Figure 5, we get that Phlebiopsis flavidoalba FP150451 A8G has the strongest competitiveness over time at 20°C. It has the longest hypha length when it reaches a steady state. While Schizophyllum commune PR1117 has the weakest competitiveness, which will die out eventually.

For a long time, there is competitive relationship between different species. At this time, we bring the data in Table 2 into the fungal decomposition model for simulation. Finally, we get the decomposition process of plant material and woody fibers by fungi in a long period of time as shown in the following figure:

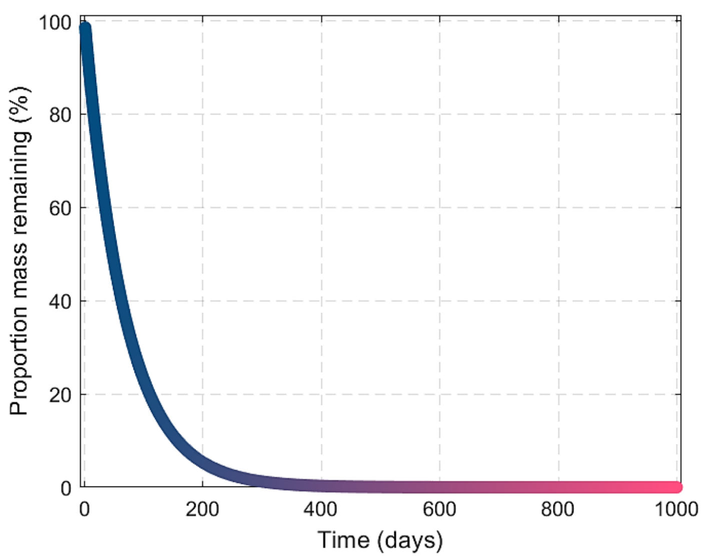


Figure 9: The change of remaining proportion with competitions

Compared with the decomposition curve when the fungus grows independently shown in the

Figure 4, it is obvious that the decomposition process of litter and woody fibers proceeds more slowly with the relationship of competition. Because there are contradictions between the species which is consistent with the actual situation of the ecosystem. And it also verifies that the fungal decomposition model we have established is scientific.

6.3 Sensitivity to Environmental Fluctuations

When the environment fluctuates rapidly, that is, when the temperature and humidity change, the succession results of different species of fungi will change over time, and the dominant species may also change. We have collected and obtained some relevant data about five types of fungi. Different temperatures, humidity and corresponding inherent hyphal rate are shown respectively as Figure 10 and 11[2]. In fact, the following two figures are the visual representation of the function $f_i(T)$ and $g_i(S)$:

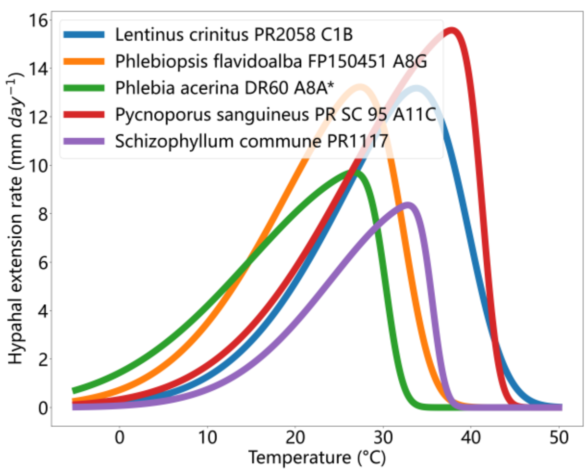


Figure 10: The relationship about temperature

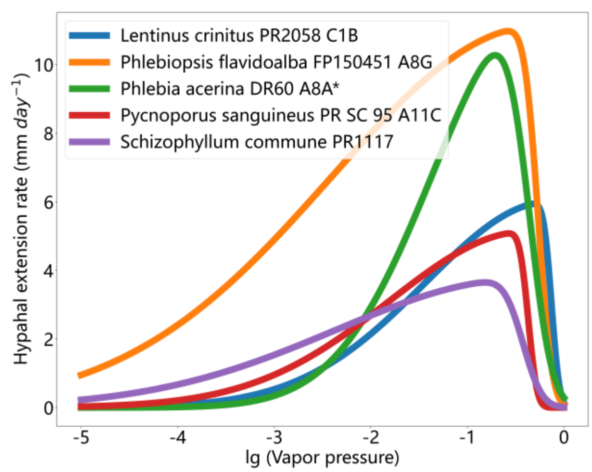


Figure 11: The relationship about temperature

In fact, the abscissa of the Figure 6 represents the water vapor pressure (MPa) in the air. The water vapor pressure in the air can be converted to humidity by the Goff-Gratch equation[9].

6.3.1 Impact of Changing Atmospheric Trends

In recent years, due to the continuous accumulation of the greenhouse effect, energy continues to accumulate in the earth-atmosphere system, resulting in temperature rise and global warming. This atmospheric trend[2][3] also directly led to changes in the living environment of fungi. Similarly, under such an overall trend, local weather patterns in the United States will also change. We have collected annual average temperature data from NOAA in the United States from 1960 to 2020. Combined with time series methods *ARIMA(0,1,1)*, corresponding predictions are made as below:

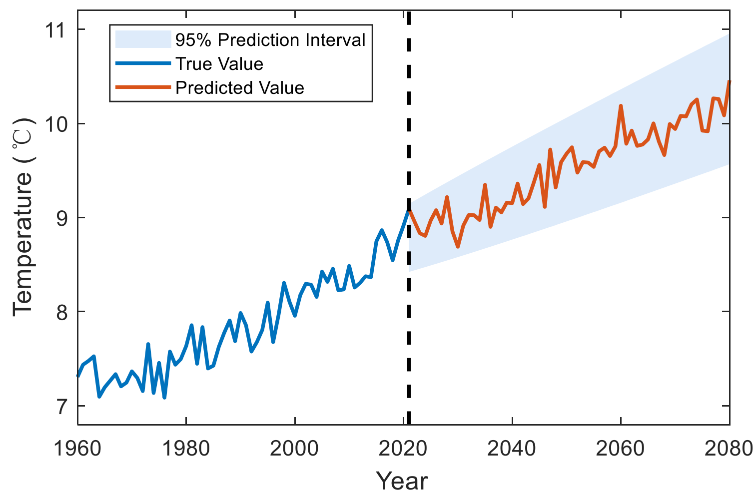


Figure 12: The forecast of future temperature in America

Taking 2020 and 2060 as examples, we substitute the environmental conditions of these two years into the fungal competition model. Over a long period of time, the succession process of five types of fungi is as follows:

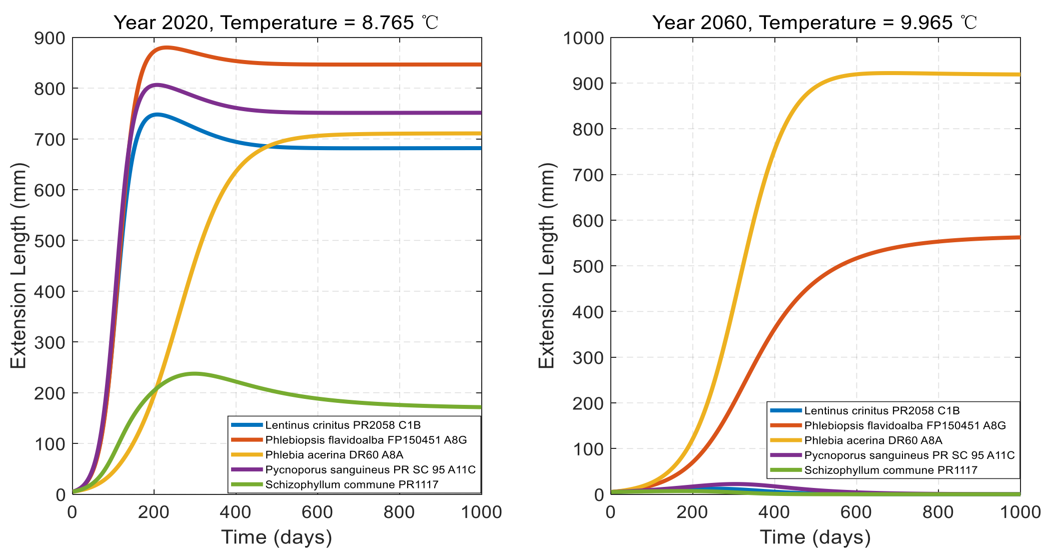


Figure 13: Fungal succession in 2020 and 2060

Observing the above two pictures, we find that with the trend of global warming, the growth rate and succession process of the five types of fungi have changed, and even the dominant fungi species have also changed. This shows that the current overall atmospheric trend has a great influence on the growth of fungi, which proves the model is extremely sensitive.

6.3.2 Impact of Different Geographical Environment

When the geographical environment changes, the climatic conditions will also change immediately. We have selected several representative climates including: arid, semi-arid, temperate, arboreal, and tropical rain forests. Then we selected 5 countries corresponding to the climates, which are Saudi Arabia, Sudan, America, China and Brazil[10], as shown in the figure:

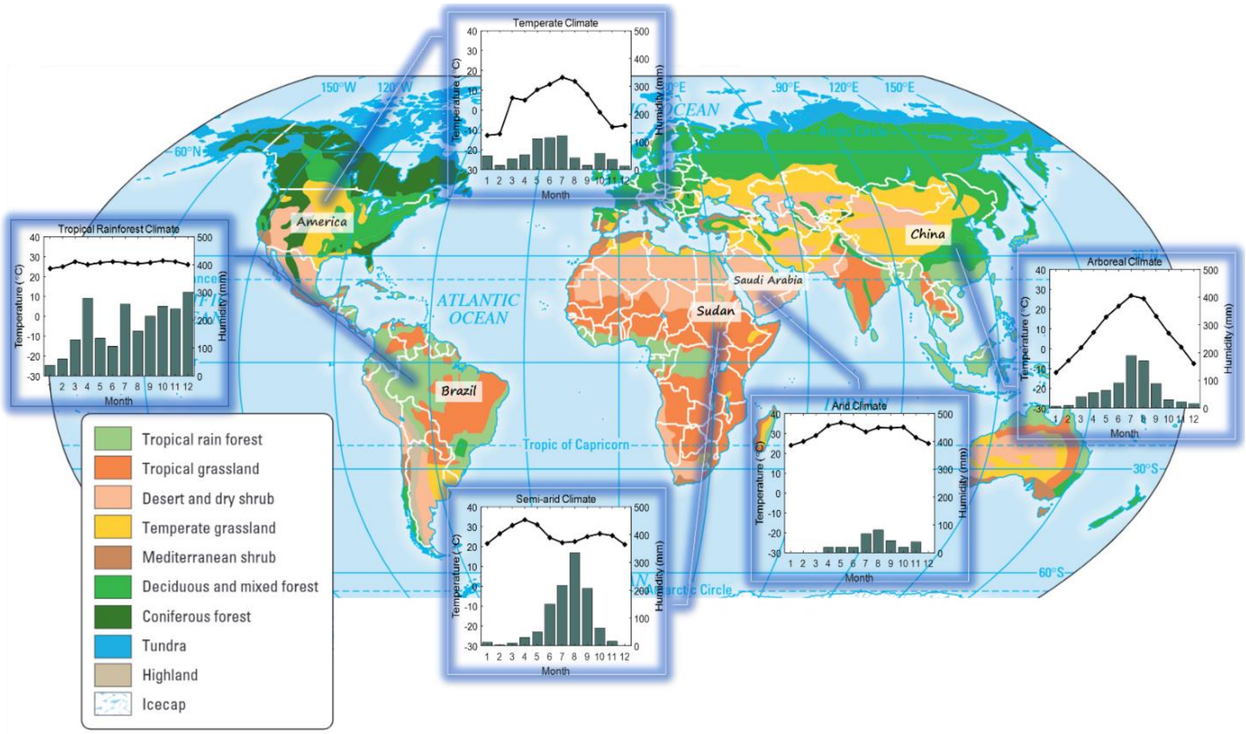


Figure 14: The distribution of representative climatic conditions

Through the built-in Wolfram database, we have obtained the specific environmental conditions of the countries and regions corresponding to the above five types of climates as shown in the following table:

Table 3: The data about 5 types of climates

Climate	Country	Average temperature (°C)	Relative humidity	Vapor pressure in the air (MPa)
Arid	Hart Arab	28.7220	0.32308	7.76×10^{-3}
Semi-arid	Sudan	20.5201	0.74436	3.10×10^{-2}
Temperate	America	8.9162	0.67365	1.29×10^{-2}
Arboreal	China	16.5114	0.75267	1.81×10^{-2}
Tropical rain forests	Brazil	27.2025	0.82752	1.43×10^{-2}

Using the data in Table 2 to calculate the inherent growth rate of the five fungi under the above different climatic conditions, combined with the competition model, we get the succession of the five fungi over a long period of time under these representative climatic conditions The process is as follows:

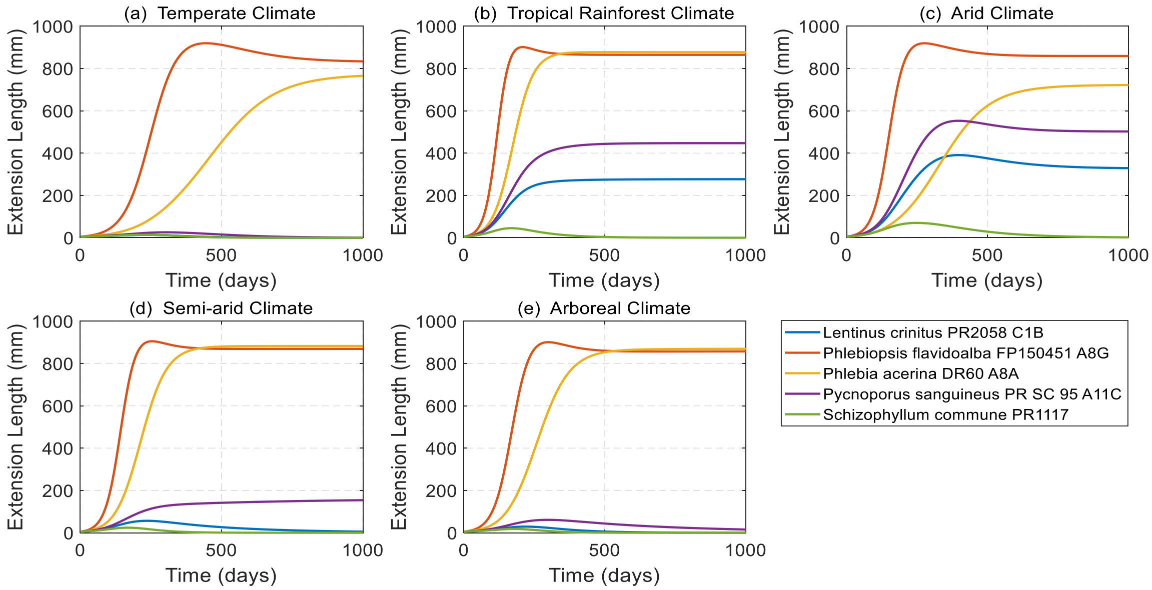


Figure 15: The prediction of fungal succession in different climates

Observing the above five pictures, we find that the fungal growth rate and succession process of the five types of fungi have changed for the climatic conditions of arid, semi-arid, temperate, arboreal, and tropical rain forests. This verifies that the competition model we have built is extremely sensitive to environmental conditions.

7 The Impact of Biodiversity

7.1 The Impact on Decomposition Efficiency

Whether in a long period or a short period of time, when the number of fungal species on a given land increase, the decomposition rate of plant material and woody fibers should be quicker than that when a single type of fungus exists[4].

Taking the above five types of fungi as an example, using the competition model and the decomposition model, we do the following work: When different combinations of species live on a given land at 20℃, we could calculate the time when the proportion mass remaining is 30%. Then we could compare how the diversity of fungal communities of a system impacts the overall efficiency of a system with respect to the breakdown of ground litter. After calculation, we get the results as shown in the following table:

Table 4: The comparison of decomposition efficiency

Fungus or the combination of fungi	Time when the proportion mass remaining is 30% (days)
A	842
A, B	181
A, B, C	95
A, B, C, D	85
A, B, C, D, E	82

In Table 3, **A** refers to Lentinus crinitus PR2058 C1B, **B** refers to Phlebiopsis flavidoalba FP150451 A8G, **C** refers to Phlebia acerina DR60 A8A, **D** refers to Pycnoporus sanguineus PR SC 95 A11C, **E** refers to Schizophyllum commune PR1117.

Obviously, from Table 3, we can see that as there are more and more fungal species living in a given land, the decomposition efficiency becomes higher and higher. This indicates that the biological diversity of the fungal community promotes the improvement of decomposition efficiency, and then promotes the carbon cycle efficiency in the ecosystem. In summary, biodiversity has a positive effect on the survival of various organisms on the earth.

7.2 The Importance and Role of Biodiversity

The rate of the fungal community decomposes ground litter and wood fiber is related to its growth rate traits and water tolerance, while the growth rate is affected by environmental factors (traits are related to genes, and the expression of traits is restricted by the environment). Obviously, different kinds of fungi have different optimum growth temperature and humidity, and in a certain environment, the maximum growth rate of fungi is constant.

If there is only one type of fungus in the system, changes, especially large changes in the environment, are likely to affect its growth rate and cause the community to grow slowly or even die, thereby affecting the decomposition efficiency of the fungus in the carbon cycle of the system.

However, if there are multiple types of fungi in the system and the environment undergoes the same change, the probability that there are still fungi in a more suitable growth environment is greatly increased, thus ensuring the continuous and effective progress of the carbon cycle.

It can be seen that species diversity can reduce the risk of environmental changes destroying the carbon cycle balance, thereby improving the stability of the ecosystem.

8 Model Test

8.1 Analysis of Sensitivity

When fungi are decomposing plant material and woody fibers, changes in environmental conditions will result in changes in the growth rate of fungi, which will lead to changes in the decomposition rate of fungi, and ultimately affect the efficiency of the entire decomposition process. Here, we think that environmental conditions include temperature and humidity. Next, we make the temperature and humidity fluctuate rapidly, and substitute the environmental conditions into the fungal decomposition model in a long period of time. Considering the competitive relationship, we observe the influence of environmental condition changes on the fungal decomposition process to test the sensitivity of the model.

Under the optimal humidity conditions, we calculated and obtained the decomposition rate of 5 types of fungi at 10℃, 20℃ and 30℃ respectively, as shown in the left figure below. Under the optimal temperature, calculate the decomposition rate of fungi when lg (Vapor pressure) is -3, -2, -1, as shown in the figure below.

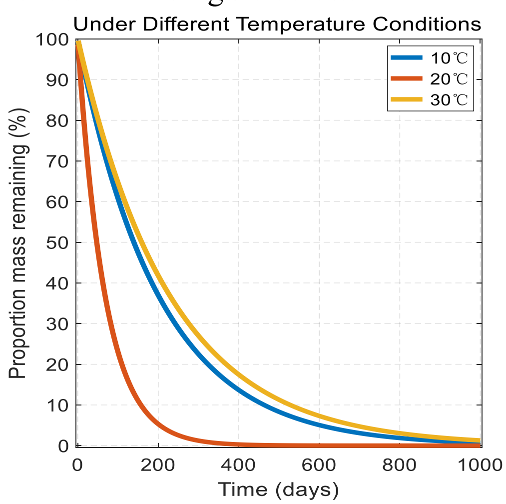


Figure 16: Sensitivity test about temperature

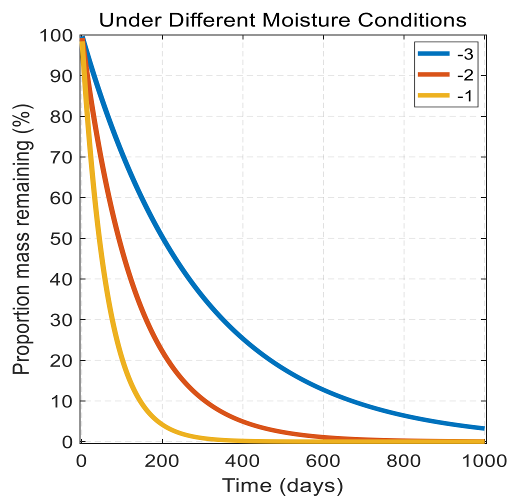


Figure 17: Sensitivity test about humidity

According to the results in the above figure, it can be concluded that when the temperature and humidity changes, the decomposition efficiency of the five types of fungi will also change significantly. It conforms to the rule of the actual situation, indicating that the model has high sensitivity.

8.2 Analysis of Robustness

When simulating the competition process between different types of fungi, we set the hypha length of each type of fungus at the initial moment $x_i(0) = 5\,mm$. However, in actual natural ecosystems, the initial hyphal length is unknown. In order to be more consistent with reality, we changed the initial length as $x_i(0) = 50\,mm$ and substituted it into the model to calculate the competition process of the five types of fungi. At 22℃, the comparison figures of the two cases are as follows:

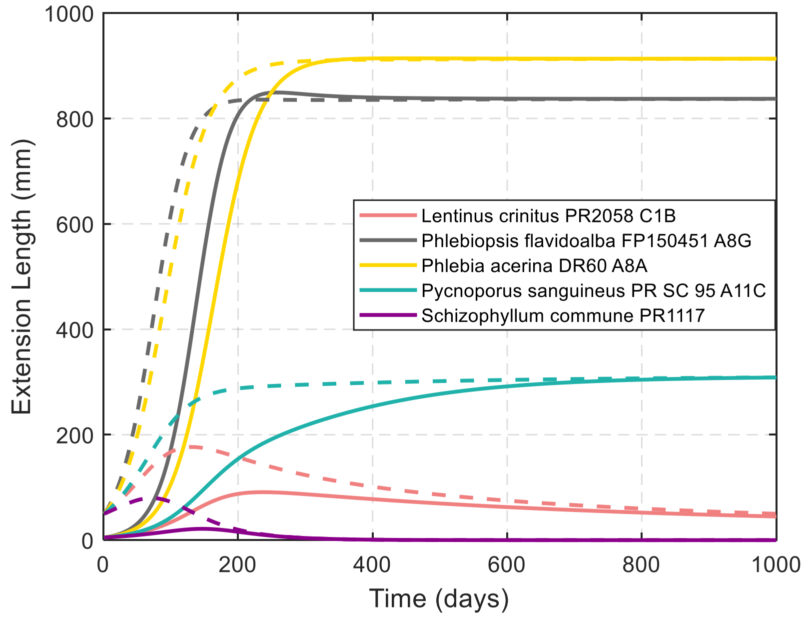


Figure 18: Competition process when the initial length of mycelium changes

According to this picture, it can be concluded that even if the initial hyphal length of the fungi has changed, when the time is long enough, the ending of the competition and the dominant species have not changed. This proves that the model we have established is robust.

9 Strengths and Weaknesses

9.1 Strengths

- After searching relevant documents and databases, we have found some complete and real data including "the geographical distribution of fungi", "the relationship between fungal growth rate and temperature / humidity", and "the temperature in North America in recent 60 years", which are strong foundations for the model's solution.
- We have established a decomposition model of a single type and multiple types of fungi, considering many kinds of situations, in line with the actual situation. So the model has excellent universality and flexibility.
- We have established a short-term and long-term competition model for multiple types of

fungi to analyze the competitive relationship among species. According to it, we can obtain the advantages and disadvantages for combinations of species under different environmental conditions.

9.2 Weaknesses

- When considering the environmental conditions, we only use temperature and humidity, while ignoring the influence of other factors such as pH value. It may cause some errors in the results of the fungal decomposition model.
- In the competition model, we only consider the competition among different species, while ignoring the influence of intraspecific struggle on the decomposition efficiency. It may also make the results inaccurate.
- In order to simplify the solving process of models, some parameter settings in the model lack scientific basis, so we must estimate them based on common sense, which are too subjective.

10 An Article for Fungi

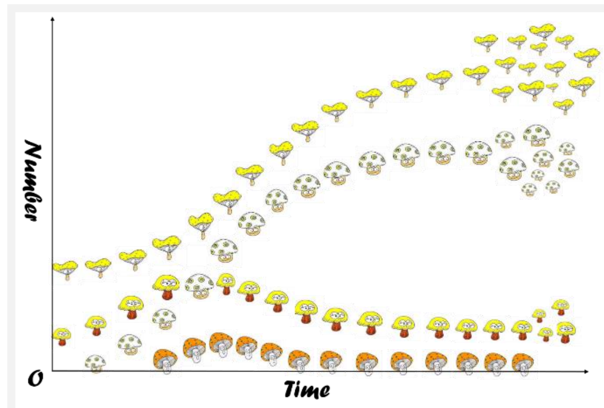
Exploring Fungi

The carbon cycle refers to the cyclic state of carbon in nature. The carbon cycle in the biosphere is mainly indicated that green plants absorb carbon dioxide from the air, convert it into glucose through photosynthesis, and release oxygen. The carbon cycle is a chain of material cycles between the inorganic environment and organisms, which is important to maintain the carbon balance at the atmosphere.

As everyone knows, as decomposers in the ecosystem, fungi participate in the carbon cycle by decomposing plant material and woody fibers. There are many types of fungi that can decompose plant material and woody fibers, and the decomposition rate of them is different. The rate of fungal decomposition is affected by its growth rate and moisture resistance. The factors that affect the growth rate are divided into internal and external ones. The internal factors include the type of fungal species. And fungal species with high growth rate are likely to grow quicker than others. External environmental factors include natural climate conditions and environmental capacity. Every kind of fungus has its optimal temperature, humidity, and pH value. Once deviating from these optimum conditions, even the species with high growth gene cannot grow quickly. In addition, when the resources are limited or inter-species struggle fiercely, the growth rate of various types of fungi will be limited in the presence of different degrees of variability.

We measure the degree of decomposition by using the quality retention rate of plant materials (that is, the ratio of the retained quality of plant materials to the initial quality at a certain moment). We have established a fungal decomposition model and believe that the change in the quality retention rate of plant materials meets the negative exponential form. As for various species, the competition among them has become a factor that cannot be ignored.

As time goes by, limited resources are constantly being consumed, and the competition among populations becomes more and more intense. Therefore, the decomposition of multiple groups of fungi can be divided into short-term and long-term trends. In a relatively short period of time, various species decompose plants at their own rates without interfering with each other. At this time, the total decomposition amount is equal to the sum of the decomposition amounts of all fungi under their independent growth. In a relatively long period of time, the competition between different populations has gradually emerged, and the decomposition rate of each type of fungus on plants is limited by other types of fungi. After establishing a competition model of various species, we simulate the growth process of various types. The result of competition is that the growth of each type is mostly inhibited in the presence of variability degrees, and there will be dominant species and inferior species.



As mentioned above, the decomposition rate of fungi is limited by natural conditions. Global warming speeds up the succession of communities, the decomposition rate of wood materials, which will in turn accelerate the carbon cycle. This increase in the concentration of carbon dioxide in the atmosphere will trigger the greenhouse effect and aggravate global warming. This shows that we must control carbon dioxide and prevent global warming.

In response to changes in natural conditions, a single species is always more dramatic than various species. When temperature or humidity greatly deviates from the optimum temperature and humidity for a certain species, the decomposition model of multiple species of fungi remains robust, then the efficiency of decomposition is greatly affected, and the carbon cycle is not affected. This reflects biodiversity plays an important role in the carbon cycle: to maintain the stability of the ecosystem, to make the carbon cycle continue to be effective.

In summary, fungi play an extremely essential role in the carbon cycle. The biodiversity of fungi is an important factor in keeping the carbon cycle stable. It is sure that protecting biodiversity is of great significance to us.

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