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Summary

人们经常在充满热水的浴缸里得到清洁和放松。本文针对只有一个简单的热水龙头的浴缸，建立一个多目标优化模型，通过调整水龙头流量大小和流入水的温度来使整个泡澡过程浴缸内水温维持基本恒定且不会浪费太多水。

首先分析浴缸中水温度变化的具体情况。根据能量转移的特点将浴缸中的热量损失分为两类情况：沿浴缸四壁和底面向空气中丧失的热量根据傅里叶导热定律求出；沿水面丧失的热量根据水由液态变为气态的焓变求出。因涉及的参数过多，将系数进行回归分析的得到一个一元二次函数。结合两类热量建立了温度关于时间的微分方程。加入阻滞因子考虑环境温湿度升高对水温的影响，最后得到水温度随时间的变化规律（见图\*\*）。优化模型考虑保持水龙头匀速流入热水的情况。将过程分为浴缸未加满和浴缸加满而水从排水口溢出的两种情况，根据能量守恒定律优化上述微分方程，建立一个有热源的情况下水的温度随时间变化的分段模型，（见图\*\*）

接下来考虑人在浴缸中对水温的影响。我们从各个方面进行分析：人的体温恒定在37℃左右，能量仅因人的生理代谢而丧失，这一部分数量过小可以不考虑；而人在水中人的体积和运动都将引起浴缸中水散热面积和总质量的变化，从而改变了热量的损失情况。因人的运动是连续且随机的，利用MATLAB生成随机数表示人进入水中的体积变化量，将运动过程离散化。为体现其振荡的特点，我们利用三角函数拟合后离散的数据，以频率和振幅的变化来反映实际现象。将得到的函数与上述模型相结合，作图分析其变化规律（见图\*\*）。

利用以上温度变化的优化模型，结合用水量建立多目标优化模型。将热水浴与缸中水温差、浴缸水温偏离最适温度最值进行正向化和归一化再加权求和定义为舒适度。在流量维持稳定的情况下，要求舒适度越大而用水量越小。因该优化模型中的约束条件中含有微分方程，难以求解，则对其进行离散仿真，采用模拟退火算法求解全局最优解。最后讨论了加入泡泡剂后对模型的影响，求得矩形浴缸尺寸为长\*宽\*高=1.5m\*0.6m\*0.5m时，最优的热水温度、热水输入速率为。然后对浴缸形状体积和人的形状体积等影响因素进行灵敏性分析，发现结果受浴缸体积的影响最大。

# 问题回顾

这是一个关于能量变化的连续型问题。 家庭中普通的浴缸无法像SPA的高端浴缸那样控制水温也没有热水喷射系统，泡澡的时间过长则会导致水变凉。因此只能打开水龙头让热水源源不断的流进来，但是当水满了之后就打开排水口使得水从排水口流出，按照此种办法使得浴缸中的水保持相对恒定的温度。This is a continuous problem about energy changes. Households in the ordinary bathtub cannot be like the SPA's high-end bath tub that control water temperature，there is no hot water injection system, bath too long will cause the water cools. Therefore only open the faucet to let water flow in a steady stream, but when the tub reaches its capacity, excess water escapes through an overflow drain, in accordance with such an approach makes the water in the bathtub to maintain a relatively constant temperature.

我们知道水的温度是随着时间的变化而变化的，但是当加入热水后，浴缸中的水温将会发生变化，他不可以看作是一个恒温物体，因此我们需要建立一个浴缸中的水温关于位置和时间的变化而变化的模型，确定一个最佳的策略，决定水龙头入水量的大小和入水量的时间等因素，使得整个浴缸从头到尾的水都能够保持温度，还要不能浪费太多的水。We know that the water temperature varies over time, but when added to hot water, bath water temperature will change, it cannot be seen as a constant object, so we should develop a model of the temperature of the bathtub water in space and time to determine the best strategy the person in the bathtub can adopt to keep the temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water.

为了详细考虑现实生活中可能发生的情况，我们还要考虑人对温度变化过程的影响。除了上述涉及到的因素外，浴缸的大小和形状、人的大小和形状、以及人在浴缸中的动作等。例如可以考虑因人的运动使得水蒸发速率加快，在第一次加水时加入了泡泡剂来帮助清洁的等因素对模型的影响。For a detailed consideration of real-life situations that may occur, we have to consider the human impact on the process temperature changes. In addition to factors related to the above, the size and shape of the tub, human size and shape, as well as human action in the tub. For example, consider the person's movement so that the water evaporation rate accelerated in the first addition of water is added to help clean the bubble agent and other factors on the model

In addition to the required one-page summary for your MCM submission, your report must include a one-page non-technical explanation for users of the bathtub that describes your strategy while explaining why it is so difficult to get an evenly maintained temperature throughout the bath water.

# 问题分析：

这个问题是一个连续型优化问题，我们应该从最基础的问题分析起，逐步完善和优化，得到最优解。最开始我们找到一些关于数据，包括一般情况下室温为25℃，洗澡的最适合的温度为39℃，浴缸取的是市面上发展较好的玻璃钢材料的中空保温的浴缸。

This problem is a continuous optimization problem, we start from the most basic problem analysis, and gradually improve and optimize to get the optimal solution. To start, we collected some data, including general room temperature 25 ℃, the most suitable temperature bath for 39 ℃, and the bathtub is FRP material.

首先我们考虑的是影响浴缸中水温度变化的因素。将浴缸中的热量损失分为两类情况，沿浴缸四壁和底面的热量丧失和沿水面因水的蒸发丧失，两类情况设计的计算方法不同。对于缸壁和底面丧失的热量，在水温达到恒定后，热量丧失主要是因浴缸上的热量向空气中散发，因此可以确定这一部分散发的热量。而沿水面蒸发的部分我们不能仅仅考虑热对流引起的热量损失，还应该考虑水由液态变为气态，发生了物态变化所吸收的热量，因此我们需要考虑焓变的因素。其中涉及到的变量过多，对于一些类似于相对湿度和大气压等值可以假定为定值，而对于一些随时间变化的值，因最终考虑的是温度与时间的关系，我们将各参数整合在一起，以一个关于时间的函数来表示。最后将得到的二者丧失的能量相加，获得丧失的总能量，结合水的质量和比热容，将能量的损失转换成温度的变化。限定浴缸尺寸大小，做出温度随时间的变化规律。同时我们也必须考虑空气的温度随时间的变化，而不是恒定在25℃，对模型优化加入阻滞因子，考虑环境温湿度升高对水温的影响。

First, we considered the factors affecting the water in the bathtub temperature. There are two forms of bathtub water heat loss, heat conduction along the walls and bottom of bathtub and water evaporation along surface of the water. For the heat loss of heat conduction along the walls and bottom of bathtub, it is mainly due to the heat bath on the circulated air, so we can determine this part of the heat. For the heat loss of water evaporation along surface of the water, we not only considered the heat loss caused by thermal convection, but also the heat loss caused by water phase change, which is from a liquid to a gaseous state, so we consider the enthalpy change. To obtain the relationship between temperature and time, we sum the energy loss of the two together to get the total energy loss, combined with the mass of water and the specific heat capacity, the energy loss is converted into a change in temperature, and draw temperature variation with time figure. At the same time, we have to consider the air temperature changes over time, rather than constant at 25 ℃, and the environmental effects of elevated temperature and humidity on the water temperature, retardation factor was added to the model,

因洗澡时可以一边洗一边加入热水，此时我们要考虑在有热源引入的情况下水温的变化。结合实际人泡澡时不可能一开始就将浴缸放满，我们假设已经放了80%的39℃的水，从此时考虑加热水的情况。因此此时情况要分为浴缸未加满和浴缸加满而水从排水口溢出的情况，利用能量守恒定律可得到热量流入流出的关系，建立微分方程，求解作图分析其变化规律。

Because the person adds a constant trickle of hot water from the faucet to reheat the bathing water, we consider water temperature changes in the case of hot water introduced in. In real life, the bath tub can not be filled outset, we assume that it has been put 80% of the 39 ℃ water, from this point consider the case of add heating water. Therefore, these two situation are considered, bath tub is not filled up and filled up and excess water escapes through an overflow drain, using the energy conservation law to give the heat flow in and out of the relationship, the establishment of differential equations, solving and graphing analyze the variation.

接下来需要考虑人在浴缸中对水温的影响。我们从以下方面进行分析：对于人的体温，因人的温度恒定在37℃左右，能量仅因人的生理代谢而丧失，这一部分数量过小可以不考虑；而人在水中人的体积和运动都将引起浴缸中水的体积的变化，这样将改变其散热面积，从而改变其热量的丧失。对于人运动的问题，这是一个连续且随机的变量，我们无法直接将他的影响引入温度方程，于是我们考虑将运动过程离散化，每20s为一个周期，利用MATLAB生成的随机数体现其随机性，结合生活实际将随机数与三角函数结合，考虑频率和振幅的变化来将离散的过程连续化，与以前模型相结合。规定具体条件作图分析其变化规律。

Next, we need to consider the impact of people in the bathtub on water temperature. We analyze the following aspects：First, for the human body temperature, the human constant temperature is around 37 ℃, the energy loss come from physiological metabolism, which is too small and can be ignored. Second, the volume of people and movements will cause a change in the volume of water in the bathtub, which will change its cooling area, thus affecting the heat loss. Third, for the motions made by the person in the bathtub, this is a continuous and random variables, we can not directly introduce the affect into temperature equation, so we consider the person movements discrete, each for a period of 20s, using the random number generated by MATLAB to reflect its randomness, combining the actual life with random numbers and trigonometric functions, considering the change in frequency and amplitude of the person motions. Finally, we plot and analyze the variation.

为了考虑在浴缸中泡澡的最佳方式，需要建立多目标优化模型。保持温度的恒定实际上是为了使得人更加舒适，因此我们可以从舒适度的角度上来看温度条件，温度必须恒定在39℃左右，而且最大温度和最小温度不能相差过大。而用水量方面也必须越小越好，根据实际情况分配权重和确定约束条件，建立一个多目标优化模型。最后可以用离散仿真的方法将最优解求解出来。

To determine the best strategy, we need to establish a multi-objective optimization model. Keeping the temperature constant aims to make people more comfortable. We can consider the temperature conditions in terms of comfort. The temperature must be constant at about 39 ℃. There are not too much difference between the maximum and minimum temperatures. The water also have to be as small as possible. According to the actual situation and assigning a weight to determine the constraints, we established a multi-objective optimization model. Finally, we use a discrete simulation method to get the optimal solution.

后面还有接上一段

用离散仿真的方法将连续问题离散化，利用模拟退火的办法求解最优解。最后分析各个因素对模型的影响：泡泡剂影响了水的蒸发速度和面积，而对于浴缸的大小体积、人的大小体积和人的运动等因素进行灵敏性分析，得到其影响程度，根据具体情况可以得到各种情况下的最优解。

# 模型假设：

这个优化模型考虑到了人的形状、体积以及人在浴缸中的运动，浴缸的形状、体积，是否添加泡泡剂，还有时间和空间等方面的因素，要探求保持浴缸中水的用量一定时，用水量最少的情况。同时考虑所有的约束条件问题复杂，而且有些问题条件我们可以事先作如下假设：

The optimization model takes into account the factor of the shape and volume of the tub, the shape/volume/temperature of the person in the bathtub, and the motions made by the person in the bathtub, a bubble bath additive, space and time and so on. To keep the temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water. If we consider all the constraints, problems become complex and difficult to solve, so we make the following assumptions in advance:

* 对于空气和水，因温度差异大时对流的速度会特别快，我们事先假设温度是分布均匀的，因此未考虑水的空间分布差异。
* For air and water, when the temperature difference is large, convection speed particularly fast, so we assume that the temperature is evenly distributed in advance, that is, without considering the differences in the spatial distribution of water.
* 我们知道温度达的情况下水蒸发的速度也是非常快的，此处我们需要根据水的蒸发情况考虑能量损失，但是相对于整体来说损失的量是非常小的，可以仅仅忽略掉水质量的损失。
* The higher the temperature, the faster evaporation of water, so we need to consider the energy loss of water evaporation. Compared to the amount of water in the bathtub, the water mass loss caused by evaporation is very small, so we ignore the loss of water quality.

# 符号定义

|  |  |
| --- | --- |
| Symbol | Meaning |
| q | 单位时间下丧失的热量Heat loss per unit of time |
| Q | 热量损失总量Total heat loss |
|  | 水体表面温度Water surface temperature |
|  | 热传导散热面积Heat transfer area |
|  | 蒸发散热面积Evaporation cooling area |
|  | 水面蒸发系数Water evaporation coefficient |
|  | 水蒸气饱和分压力Water vapor saturation partial pressure |
| k | 定义的简化参数Defined reduced parameters |
| C | 水的比热容Specific heat capacity of water |
| m | 水的质量mass of water |
|  | 浴缸底面周长Bottom circumference of bathtub |
|  | 热水流入的流量Flow of hot water |
|  | 浴缸底面积Bottom area of bathtub |
|  | 流入热水的温度temperature of hot water flowing into |
|  | 浴缸高度 Bath height |
|  | 浴缸中温度与39℃差值的绝对值the absolute value of the difference between the temperature of bath water and 39 ℃ |
|  | 舒适度Comfort |
|  | 浴缸中水开始溢出的时间the start time that water in the bathtub overflow |
|  | 浪费的水量Waste water |
| z | 温度和水量综合优化目标函数The optimization objective function of water temperature and waste water |

## 4.1----------

这是一个关于控制水温的连续型问题，首先我们讨论没有热水输入的情况，分析其热量丧失的情况。热水水面向大气中丧失的方式有三种：对流散热、传导散热和辐射散热。由于辐射散发的热量很小，对于沐浴这一小段时间来说，辐射散发的热量可以忽略不计，我们先不考虑浴缸内水对流的情况，将热量损失分为两部分，分别是沿浴缸四壁和底面的热量丧失和沿水面因水的蒸发丧失。This is a continuous problem about how to control the water temperature, first we discuss without hot water input, analyzing the heat loss. There are three ways that hot water loss energy to the atmosphere: Convective heat transfer, heat conduction and radiation. Due to the heat coming from the radiation is very small, for bath lasting a short period of time, the heat coming from the radiation can be neglected, we won't consider the waters convection in of bath crock, heat loss can be divided into two parts, respectively is along the walls and the bottom of the bathtub heat loss and along the surface of the water because of the evaporation loss of water.

Step one 沿缸壁和底面丧失的热量：Along the walls and the bottom of the bathtub heat loss

对于此方面的热量，因其通过热传导的方式丧失，其与接触面积、材料的导热系数、温度差和浴缸厚度等因素有关，得到热量的计算公式为【】： As for this Heat, because of the loss by means of heat conduction with the contact area. It is related to the contact area, the thermal conductivity of the material, the temperature difference and the thickness of the bath tub. Get the calculating formula for heat:

式中：

—热传导散热面积;

—导热系数，取市面常见玻璃钢导热系数0.4 W/(m·K);在此处我们应该考虑浴缸在制作时一般在制成中间夹层夹空气的情况，假设是两层玻璃钢之间夹一层空气，各层的厚度均相等，因此对于导热系数我们应该加以修正。空气一般的导热系数为0.023 W/(m·K)，修正后的导热系数为0.021 W/(m·K)。

—水体表面温度;

—空气干燥温度，取为室内常温25℃;

—缸壁厚度，取常见值1.5cm。

In this formula:

- Heat dissipation area in heat conduction;

- Thermal conductivity, take common market 玻璃钢 sheet thermal conductivity 0.4K-1m-1 market;

T - Water surface temperature

- Air drying temperature, taken as indoor ambient temperature 25 ℃;

- The thickness of the cylinder wall, take common values 1.5cm

Step two 沿水面随蒸发丧失的热量：Along with the evaporation of the water heat dissipated

沿水面热量的丧失是因为水和空气的对流引起的，水的蒸发携带热量的丧失，此处除了热量的变化外还存在物态的变化，因此在此需要考虑焓变的问题。晗是热力学中表征物质系统能量的一个重要状态参量，是具有能量的量纲，一定质量的物质按定压可逆过程由一种状态变为另一种状态，焓的增量便等于在此过程中吸入的热量，也就是焓变【】。除此之外能量损失还与散热面积和散热系数有关，因此得到丧失热量关于焓变的关系式【】如下： The loss of heat from the surface of the water is caused by the convection of water and air, and the loss of heat from the evaporation of water. There is a change of matter’s state in addition to the change of heat. So in the problems need to consider the change. Enthalpy in thermodynamics is an important state parameters to present the material system energy, is a dimension of energy, a certain mass of material according to the constant pressure reversible process from one state to another state, the enthalpy increment is equal to the heat in the process of inhaled, namely enthalpy change 【 】.Besides energy losses associated with the cooling area and the coefficient of heat transfer, result in loss of heat on the relation between the enthalpy change 【 】 as follows

式中: In this formula:

— 该地大气压The atmospheric pressure，取标准大气压Take the standard atmospheric pressure ；

— 水的汽化热Heat of vaporization of water， 其值为Its value is 40.8kJ/mol，The same as2260kJ/kg；

--和水温t相应的饱和空气焓；Corresponding saturated air enthalpy with temperature t

-- 空气焓Air enthalpy；

— 散热面面积Area of heat radiating surface；

—水面蒸发系数； Evaporation coefficient of water surface

上述参量均不是固定值，它会随着时间、温度和其他一些因素的改变而改变，因此以下对各因素一一作分析。The above parameters are not a fixed value, it will with the change of the time, temperature and other factors, so the following make analysis on the factors one by one

* 水面蒸发系数计算公式【】： Calculation formula of water surface evaporation coefficient

W— 水面上的风速Wind speed on the surface of the water， Indoor bath W＝ 0.2~0.5 m/s

* 对于的计算，查阅文献可得，湿空气焓值的计算有以下公式【】：For the calculation of , get from the literature available, wet air enthalpy calculation with the following formula

d—空气含湿量，可由相对湿度及水蒸气饱和分压力换算得到Air moisture content, relative humidity and the saturated water vapor partial pressure conversion

式中：o—相对湿度，对于饱和水蒸气取为100%，对于浴室取常用湿度40%-70%中较大值70%。Relative humidity, for saturated vapor was 100%, in the bathroom to take larger humidity 40% to 70% of value of 70%

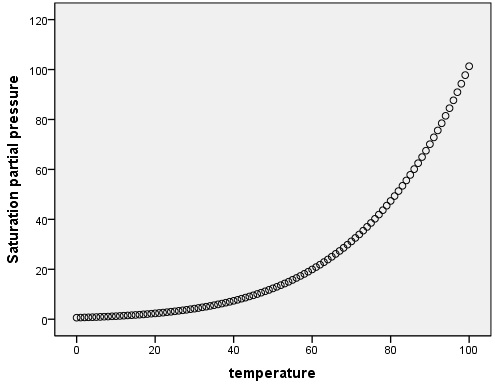
P—水蒸气饱和分压力，它是一个与温度相关的值，为得到饱和分压力p与温度T的函数关系，我们从文献上得到0~100℃下各温度对应的饱和分压力【】，具体数据见表\*\*，将得到的数据绘制成散点图，进行一元线性回归分析。P - saturated water vapor partial pressure, it is a value associated with temperature, In order to get the function relation between the saturation pressure P and temperature T, We get from the corresponding literature the saturated partial pressure at each temperature 0 ~ 100 ℃ 【 】, specific data are shown in table \* \*, data will be mapped into a scatterplot, monadic linear regression analysis.

Fig.1 温度与水蒸气饱和分压力散点图Fig. 1 temperature and water vapor saturation pressure scatter plot

散点图图显示随着温度的增大，水蒸气饱和分压力也随着增大，大致可以看出成三阶幂函数分布规律。Scatter tutu showed with the increase of temperature, water vapor saturated with partial pressure increases, which can be roughly into three order power function distribution.

因此对原始数据进行逐阶差分Stepwise difference，结果显示原始数据的三阶差分数据较接近，故可认为水蒸气饱和分压力与温度呈三次函数关系So Stepwise difference the raw data, results show that the original data of the third order differential data is the most close to, It can be considered saturated with water vapor partial pressure with temperature cubic function relationship我们采用三次函数 对原始数据进行拟合，计算各各参数的最小二乘估计值，借助SPSS进行一元非线性回归分析，得到拟合曲线方程为： We use cubic function raw data were fitted to calculate the least squares estimates of the parameters of Calvary performed, by means of SPSS monovalent non-linear regression analysis, curve fitting equation as follows:

对得到的回归方程进行残差检验：残差平方和:SSE=23.226，已更正平方和:SST=76563.275，样本决定系数make the resulting regression equation residual tested: the sum of squared residuals: SSE = 23.226, has been corrected sum of squares: SST = 76563.275, the coefficient of sample decision

因此该回归模型充分利用了指标量t的信息，拟合度很好。So the regression model makes full use of the index quantity t information, fitting is very good.

* 因计算公式过于复杂，此处我们主要关注热量散失随温度的的变化规律，因此我们定义一个函数：Because of its complex calculation formula, here we focus on heat losses with the temperature change rule, so we define a function:

运用MATLAB求解，得到k-t图像（见图\*\*），并计算k在T为25℃~100℃时的值（见附件表\*\*）。Using MATLAB to solve, getting k-t image (see figure \* \*), and calculate the value of k when T is 20 ℃ ~ 100 ℃ (see annex table \* \*)

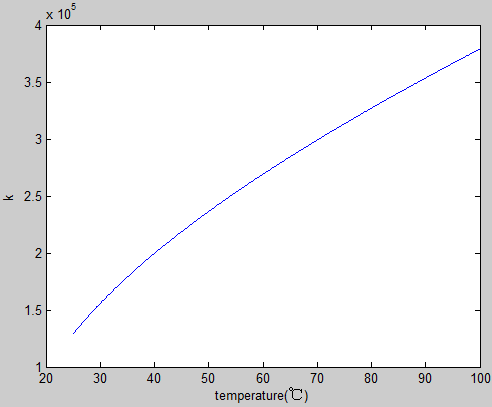


Fig 2 参数k随温度t的变化规律Fig 2 parameters k changing with the temperature t

由图可以看出，k值随着时间的增加逐渐增大，增长率逐渐降低，最后趋近于直线，可以近似看作一条开口向下的抛物线的左侧部分。By the graph, you can see that k value is gradually increasing with the increase of time, growth rate gradually reduced, finally tend to be linear, can be approximately regarded as a country opening to the left of the parabola.

由于与T的函数关系式过于复杂，用它建立微分方程求解后面的问题不仅加大了算法的复杂度，而且不一定能找到可行解。因此我们采用其他的简单函数对其进行拟合。对k逐阶差分，结果表明二阶差分值基本相等，因此采用一元二次函数逼近原函数。借助SPSS，对T=1,2,3,…,100时的k值进行拟合，得到是拟合残差平方和最小的曲线方程Because of the function relation with T is too complex, and use it to establish differential equations to solve the problem not only increase the complexity of the algorithm, but also cannot find a feasible solution. So we use other simple function on the fitting. To k line by order difference, the result showed that the second order differential basic equal score, so using a yuan quadratic function approximation function. With the help of SPSS, T = 1, 2, 3... when 100 k values for fitting, get is minimum residual sum of squares fitting curve equation

对拟合方程进行残差检验：残差平方和:SSE=0.023，已更正平方和:SST=37.259，样本决定系数Residual test was carried out on the fitting equation: the sum of squared residuals: SSE = 0.023, has been corrected sum of squares: SST = 37.259, the coefficient of sample decision

因此该回归模型充分利用了指标量t的信息，拟合优度很好。So the regression model makes full use of the index quantity t information, goodness of fit is very good.

为了使结果更加明显，我们将拟合得到的曲线和实际曲线绘制在一个图表内，观察二者的变化规律，如下所示：In order to make the results more obvious, we will get the fitting curve and the actual curve painted in a chart, observe the change rule, as shown below:

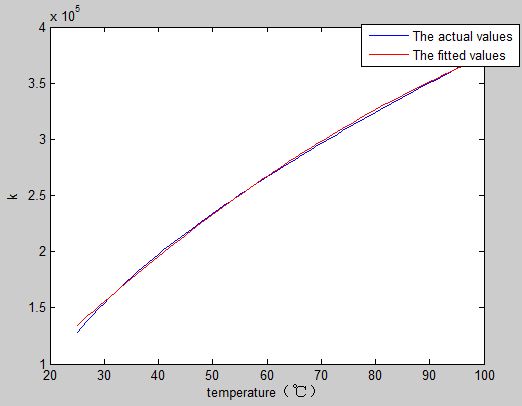


Fig 3 拟合k曲线与实际曲线对比图fitting k curve and the actual curve comparison chart

图中只列出了25℃-100℃范围内的曲线值，由图可以看出拟合程度非常好。虽然用二次函数拟合的k曲线对于T值远大于100℃时可能会偏离实际较大，但是洗浴用的水温基本只会在25~100℃范围内，故用拟合曲线代替实际曲线具有一定可行性。Figure lists only 25 ℃ to 100 ℃ values within the scope of the curve, can be seen from figure fitting degree is very good. Although the use of a quadratic function curve fitting for the time T s is much larger than 100 ℃ may deviate from the actual large, but bathing water temperature basic will only within the scope of the 25 ~ 100 ℃, so the fitted curve instead of the actual curve has certain feasibility.

最终得到的单位时间散热量q的表达式为：The resulting unit time expressions of heat q are as follows:

即

结合上述对公示的计算过程，我们分析可以得到，在静止状态和无热水输入的情况下，热量丧失情况和水与空气的温度差、相对湿度、大气压、接触面积、浴缸的材料、空气的风速等因素有关。Combining with the calculating process of the public, we analysis can be obtained, under the condition of the stationary state and there is no hot water input, heat loss and the water temperature and air temperature, relative humidity, atmospheric pressure, contact area, the material of bath crock, air, wind speed and other factors.

我们已经得到的能量丧失的变化规律，但现实生活中是难以定量感受到能量的变化的，因此我们要将能量的变化转换为温度的变化，由能量守恒定律得：Variation of energy loss we've got, but in real life it is difficult to feel the energy of the quantitative changes, so we have to change in energy is converted to temperature changes, by the energy conservation law was

In this formula:

Q— 损失的热量Loss of heat；

C—水的比热容； Specific heat capacity of water

m— 水的质量 Water mass；

我们定义We define：

,,

。

And then get：

验证此方程Verify this equation：

因此therefore

use(1)=(2) and get 得出水温T与时间t的关系式The relationship between water temperature T and time t。

取一常见矩形底面浴缸尺寸长×宽×高=1.7m×0.8m×0.7m，设水深为浴缸深度的80%，即0.56m；水温为正常人体适宜的沐浴温度39℃，即。运用MATLAB可求解得该浴缸水温与时间的关系曲线：Take a rectangular bottom common bathtub size L× W ×H = 1.7m × 0.8m×0.7m, set at 80% depth of the bath depth, namely 0.56m; normal human body temperature is appropriate bath temperature 39 ℃, that is . MATLAB can be solved using the bathtub curve obtained temperature and time:

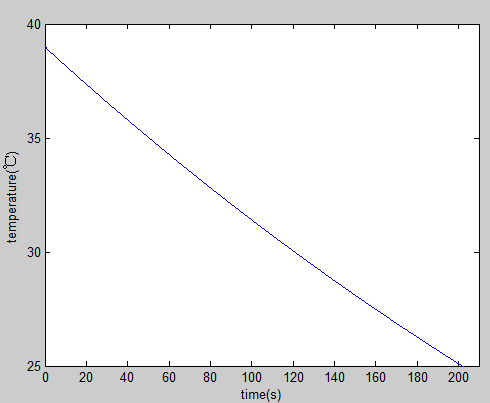


Table \*\*静置浴缸内温度随时间变化规律

图像显示，水温由39℃降至25℃大约需要210秒。这比实际中浴缸水中的温度下降快得多，其原因有两个方面，第一，该模型中没有考虑随时间的变化，空气湿度将越来越大，蒸发散热的速率会降低；第二浴缸缸壁导热只能将热量传递给周围的空气和地面，随着时间的增加，周围物质温度降增加，缸壁热量输入将大于输出，缸壁自身将升温且热传导速率会下降。Image display, the water temperature dropped to 25 ℃ from 39 ℃ takes about 210 seconds. That decline than the actual temperature of the bath water is much faster. Which for two reasons. First, the model does not consider that: with the changes over time, the air humidity will increase and evaporation cooling rate will be reduced. Second, cylinder wall thermal bath can only transfer heat to the surrounding air and the ground. As time increases the temperature of the surrounding material drop increases the cylinder wall heat input greater than the output, the cylinder wall itself will be raised temperature and the heat transfer rate will decline.

**模型优化**

随着时间的变化，空气湿度和环境温度也会发生变化，二者均使得蒸发散热速率变慢。为了将此因素考虑进模型中，得到更接近实际情况的降温模型。我们知道浴缸中水温度下降的下限应为室温为，当温度T= 时，水温不再下降，即下降率为零。因此对温度下降速率加上阻滞因子（1-/T）,显然温度T越小，水温下降速率越小，并逐渐趋近于0. As time changes, the air humidity and ambient temperature will change, so both evaporation cooling rate slower. In order to consider this factor into the model to give more realistic cooling model. We know that the bathtub water temperature drops lower limit should be room temperature, when the temperature T=, the water temperature is no longer declining, that decline rate is zero. So the temperature drop rate plus retardation factor (1-/T), is clearly smaller the temperature T the smaller the rate of fall in temperature and gradually approaches zero.

其它参数设置不变，得到浴缸水温与时间的关系曲线： Other parameter settings unchanged, get the relation curve between bathtub water temperature and time

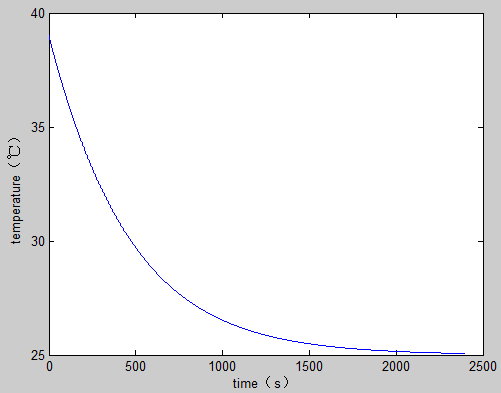


Fig \*\*优化后浴缸内温度随时间变化曲线 Time optimization curve of temperature in the optimized bath

由图像可以看出温度降低速度开始的时候快，后面速度越来越慢，最后趋近于室温25℃，可见若室温恒定在25℃，最后将达到一个平衡状态，温度将不再降低，而浴缸内水温的变化在10分钟内降低约为11℃，比较符合现实情况。

As can be seen by the image begins when the temperature decreases fast speed the back more slowly at room temperature finally reaching 25 ℃, when the room seen in a constant temperature 25 ℃, and finally will reach a state of equilibrium the temperature will not decrease. Changes in the bath tub water temperature within 10 minutes down about 11 ℃, more in line with reality.

## 4.2 有热水输入的温度变化模型

### 4.2.1模型假设与定义

接下来我们考虑在匀速加入热水的条件下温度随时间的变化。我们知道热水的流入是动态连续的随时间变化的函数，其温度和流量时刻影响着水的整体温度。在上文中静态水温度变化的模型基础上，我们继续考虑能量输入的情况。对此我们要做出几项合理的假设与定义：Next we consider under the condition of uniform add hot water temperature changes over time. We know that hot water inflow is a function of the dynamic continuous changes over time, its temperature and flow rate affects the overall temperature of the water. In the model based on static water temperature change in this paper, we continue to consider the situation of the energy input. We are going to have to make several reasonable assumptions and definitions

* 假设水从一开始就匀速地，连续不断地加入到浴缸中We assume that water from the beginning of uniform, continuously added to the bath，且水缸中水温在每一时刻都是一个常数，即忽略水温度的空间分布不均。Assumes that the water from the start, uniform, continuously added to the bathtub, and water temperature in the tank in every moment is a constant, which ignore the spatial distribution of water temperature.
* 同时在此我们也未考虑人在浴缸内的情况，仅考虑在注水情况下水温在有热水注入的情况下随时间的变化规律。At the same time in this we are not considering the situation of people in the bathtub, consider only the water temperature changes in the case of the hot water injection over time.
* 我们从上述模型中已经注入了80%的情况下开始讨论，前一段时间注入水而水未溢出，后一段时间水超过容器则水溢出，两方面的热量丧失规律是不同的，应该分类讨论。根据人体要求，取39℃时人适合泡澡的时长20min。From the above model, we have started to discuss injected 80% of cases, some time ago was poured into water and the water does not overflow, the water over a period of time after the water overflows the container, the heat loss of two laws are different and should be classified discussions. According to the requirements of the human body, when people take 39 ℃ bath for the duration 20min

符号的定义：— 浴缸底面周长Bottom circumference of bathtub

— 浴缸底面积Bottom area of bathtub

— 热水流入的流量Flow of hot water

—水密度water density

—流入热水的温度temperature of hot water flowing into

—浴缸高度 Bath height

— 初始水位高度Initial height of water level

### 4.2.2 模型的建立The establishment of the model

由于浴缸内水温的下降不能保证人在浴缸中的舒适度，而浴缸又没有自身的加热系统，只能从外界加入热水以维持浴缸中水温的恒定。假设水从一开始就匀速地，连续不断地加入到浴缸中，且水缸中水温在每一时刻都是一个常数，即忽略水温度的空间分布不均。浴缸的体积是一定的，而热水是在源源不断地往浴缸里流，因此如果时间足够长，总能找到一个时间点使得浴缸装满水，缸中水开始溢出。所以讨论有热水输入的温度变化需要分时间段讨论，建立分段函数。Due to the drop in the temperature of the bath cannot guarantee the comfort of people in the bathtub, And the bath tub has no heating system of its own, only from the outside world to join the hot water to maintain constant water temperature in the bathtub. Assumes that the water from the start, uniform, continuously added to the bathtub, and water temperature in the tank in every moment is a constant, which ignore the spatial distribution of water temperature. The volume of bath crock is certain, and hot water is in continuously flows into the bathtub, so if the time is long enough, will always find a point in time the tub filled with water, water in the tank began to overflow. So there was discussion of the hot water input temperature variation need time to discuss, establish a piecewise function.

* 水未溢出时，热水的加入将带来三方面的影响：热量的输入；水质量的增加；水体积增加，即散热面积的增加。When the water not overflow, the addition of hot water will lead to three aspects: the influence of heat input, the increase of water mass, the increase of water volume increases, just the heat dissipation area.
* 对于整体水而言，整体内部是满足能量守恒的，整体温度的变化率与其输入热量与输出热量有关：For the whole water, full interior is to meet energy conservation, the rate of change of the whole temperature is related to the input quantity of heat and the quantity of heat.

水的质量影响了水的整体温度和蒸发散热状况，因此我们需要考虑在水充满浴缸之前水质量的变化。Water mass affect the overall temperature and evaporation of water cooling condition, Therefore, we need to consider changes in water mass in a bathtub before filled with water.

而能量的输入是以热水的输入的形式实现的，根据热学能量变化规律求得有如下规律：And energy input in the form of hot water input, according to the laws of the thermal energy change law obtained has the following

因为此时我们是从浴缸中本来就含有80%体积的水的情况开始讨论的，加水的量为，应使其小于剩余20%体积量，即。Because we from already contains 80% volume of water in the bathtub, began to discuss the amount of water for, should be less than 20% residual volume, that is

我们知道水的通过热传导丧失的情况下与水域浴缸的接触面积有关，而随着水量的增加会增大散热的接触面积：We know that under the condition of water through heat conduction loss associated with the contact area of water bath crock, and with the increase of the amount of water will increase the heat dissipation of the contact area:

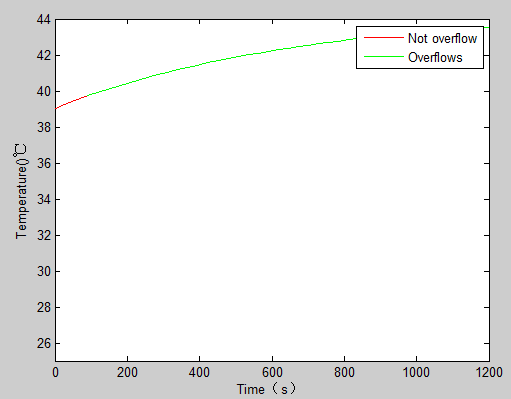
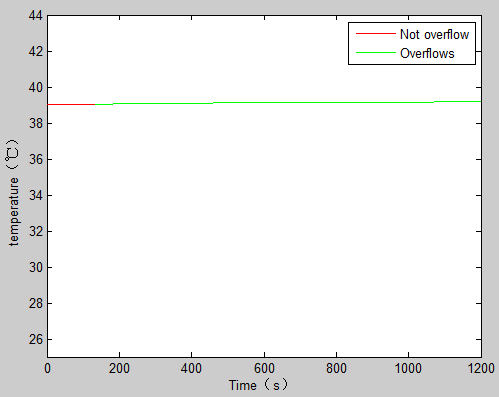
最终得到在注满浴缸之前温度随时间变化的模型如下：Before finally get filled bath temperature change with time as a model

* 水溢出时，不考虑水密度和体积随温度的变化，热水的加入将不再带来质量和体积的改变，只有热量的进入，则以上一步中能量守恒和水量输入的条件下，质量和接触面积已经保持恒定，即，为初始条件微分方程组: When water overflow, does not consider water density and volume changes with temperature, the addition of hot water will no longer bring mass and volume change, only the heat into the conservation. In the above step input conditions, the mass and the contact area is constant, i.e., differential equations for the initial conditions:

### 4.2.3 模型求解

我们假设浴缸用的实际依旧是长×宽×高=1.7m×0.8m×0.7m的方形浴缸，热水放入的流速为0.5L/s，约为是国家标准水平。但是如果按照此标准计算时将浴缸注满水需要的时间达到32min，在这种情况下结合上述散热规律，将热水放满后水温就已经冷却至一个偏冷的水平，不能够满足泡澡的要求。因此完善考虑浴缸的厚度，实际内部容积接近于长×宽×高=1.5m×0.6m×0.5m，因此内部容积为450L。We assume that the actual use is still a bathtub L × W × H = 1.7m × 0.8m × 0.7m square bathtub, hot water into the flow rate of 0.5 L/s, is about the national standard level. But if the bathtub filled with water when calculated according to this standard requires time to 32 min, combined with the heat dissipation in this situation, when the hot water is filled, the water temperature is cooled to a level of cold. Will not be able to meet the requirements of the bath. Therefore consider the thickness of the bath, close to the actual internal volume L × W × H = 1.5m × 0.6m × 0.5m, so the internal volume of 450 L.

取流速,流入热水的温度运用MATLAB得到浴缸水温与时间的关系曲线Take The flow rate, and the temperature of hot water flow in. Using MATLAB to get the relationship curve between the water temperature and the time of the bathtub



图\*\*在有热水注入的条件下温度随时间变化规律Is hot water injection in figure \* \* under the condition of temperature variation with time

如上图所示，在加热水的情况下，水温随着时间的增加而显著增加，开始时增加速度快，而后速度会有所降小。红色部分表示在水注满浴缸之前温度的变化规律，可见因散热面积的变化，其温度增大的较快，而后来考虑主要是室温影响的情况，最后温度增长的速率减缓。此时是没有考虑人在浴缸中的变化规律，实际上是不满足人的舒适度的要求的。因此需要对模型进行进一步的完善。As shown in the above, in the case of heating water, the water temperature is increased with the increase of time increased significantly, the speed increased at the beginning, and speed will be reduce. Red part shows the change in the law before top up the water bath temperature, its temperature increases quickly, and then considering the effects of mainly at room temperature, the temperature of the rate of growth slowed. At this point is not to consider people change rule in bath crock, actually it is not meet the requirements of human comfort. Therefore need to further improve the model.

## 4.3 有人存在的温度变化模型Temperature model of human presence

### 4.3.1 模型影响因素的讨论Discussion influencing factors of the model

上述模型中我们考虑的是仅浴缸中洗澡水的能量的变化，但是现实生活中人是在浴缸中的，人的温度与水之间会发生热传导，人体形状的大小将影响接触面积，而人的运动不仅会加速蒸发，还将影响水的总体积，这些都将影响水的温度。The above model we consider only the energy change in the bath tub, but the reality of life is human in the tub, it will happen thermal conductivity between man and water temperature, the shape of body will affect the body contact area, and people the movement will not only accelerate the evaporation will also affect the total volume of water, which will affect the water temperature.

* 人体的温度：一般所称的体温是指机体内部或深部的温度，不管外界温度如何变化，人的体温都保持在37℃左右，我们成为“恒温”。当外界温度低时，骨骼肌的活动增强，组织氧化代谢作用增强等措施使产热增加；当外界温度高时，散热中枢通过皮肤血管植物神经的活动而使皮肤血管扩张，于是皮肤温度升高，促进辐射、对流、传导以散热中枢还能通过神经传导使汗腺分泌，加快蒸发散热，使体温保持恒定。【体温与发热】Body temperature: commonly known as body temperature refers to the internal temperature of the body or the deep, regardless of the outside temperature changes, the human body temperature maintained at around 37 ℃, we become " constant temperature " When the outside temperature is low, the activity of skeletal muscle enhancement, enhanced oxidative metabolism of other measures to increase the heat; when the outside temperature is high, heat the central autonomic nervous activity through the skin blood vessels leaving the skin blood vessels to dilate, so skin temperature promoting radiation, convection, heat conduction to the central nerve conduction but also through the sweat glands secretion, accelerate evaporation heat, so that the temperature remains constant

但是当人在沐浴时，其体温或许会有稍许波动，但其幅值不会很大，人体温波动幅值超过1℃就会引发身体的不适。因此，对于人体的体温影响可以忽略不计，而把它假设成恒温体。But when people in the shower, body temperature may be slightly fluctuate, but the amplitude will not be great, man magnitude than 1 ℃ temperature fluctuations will lead to physical discomfort. Therefore, the temperature can affect the human body is negligible, but it is assumed that a constant temperature body.

这样具体能量转移过程就是水温传热给人体，人体通过基本代谢运动使得热量丧失。一般来说，成人每天至少[需要](http://baike.baidu.com/view/215827.htm)1500千卡的能量来维持身体机能，这是因为即使你躺着不动，你的身体仍需能量来保持体温、心肺功能和大脑运作。换算成能量为6276kJ，而20分钟损失的总能量为87.16kJ。而在前文中我们考虑静止状态下水能从39℃降到26℃，20分钟损失的能量达到51979.2kJ，人体代谢损失的能量仅占水损失的0.17%。因数值过小，且变化规律无法预测，所以此处忽略人的体温对水温的影响。

The energy transfer process is that water transfers heat to the human body, the basic metabolism of the body's movement results in the heat loss. In general, adults need at least 1500 kcal per day to maintain bodily functions. This is because even if you lie still, your body still needs energy to maintain body temperature, to ensure the functioning of the brain and heart and lung function. Converted into energy is 6276kJ, the total energy loss for 20 minutes is 87.16kJ. The previous article we consider in the stationary state water from 39 ℃ down to 26 ℃, the energy loss of 20 minutes to reach 51979.2kJ, the energy loss result from metabolism in the body accounted for only 0.17% of water. Because the value is too small, and the variation can not be predicted, we ignore the effects of water temperature on the human body.

* 人体的形状大小：根据上面的恒温假设，人体在浴缸中所起的作用就是占有一部分的体积，使得水的总量变少。由于热对流的存在，再加上人本身在水中的活动。热水携带的热量在水体中扩散非常快，因此讨论水体中温度的空间分布意义不大。因此在这里不讨论人体存在对水体温度空间分布的影响。一般人的体积大概为60L, 正常情况下，人沐浴时头部总在水面以上，浸入水中体积约为自身体积的90%。因此在总的初始体积中需要减去这一部分体积才能得到水的实际体积

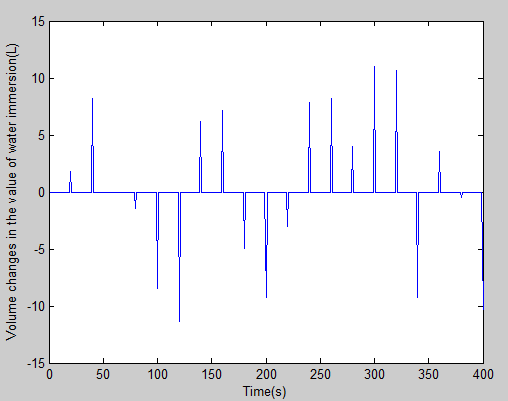
Body shape and size: according to constant temperature assumption above, the role of the human body is to occupy space partly in the tub, and the total amount of water is reduced. Due to thermal convection, as well as human activities in the water itself. The diffusion of heat that hot water carried in the water was very fast, so the discussion of water temperature distribution in the space is of little significance. Therefore, we do not discuss the impact on the water temperature spatial distribution of human existence. In general, the volume of people is approximately 60L, when people are taking shower, their heads are always above the water, and people’s volume that immersed in water is about 90% of its own volume. Therefore, this part of the volume should be subtracted to obtain the actual volume of water.

* 人的运动 The movement of people：人在浴缸中的运动主要分为两个方面：一是在浴缸内运动引起的水的运动。因为我们不考虑水的温度的影响，因此对接触面积变化的影响忽略不计。因我们是将水视为均匀温度的液体，对于内部温度变化忽略不计。而是因为人的起立或者其他运动导致的水的质量变化将会导致蒸发量的变化。此处人的运动对水温的影响主要表现在对水位升降的影响，人在运动过程中必然伴随着身体的某一部分露出水面或进入水体，从而导致浴缸中水位的改变。人在沐浴时进出水面频率和引起的体积变化幅值都是随机的，但从人们沐浴的一般规律还是能得到其运动的一些规律。在刚开始进入浴缸和快要出浴缸的时人的运动会比较频繁，中间时段的运动会比较少。

the motions made by the person is divided into two areas：First, the movement of people caused by the movement of water in the bath tub. we do not consider the effect of water temperature, so ignore the impact of changes in the contact area. Because we treat the liquid as a homogeneous temperature, so the internal temperature variation is negligible. Variation of water mass that results from stand and other motion leads to the evaporation variation. Affect on water temperature that caused by people's motion is mainly on water level fluctuations, motion of people in the process must be accompanied by a certain part of the body out of the water or into the water, causing the water level in the tub change. The frequency of people’s movement and amplitude of the volume change of the water in the shower are random, however, we can get some law from the motion of people bathing. When people enter and come out of the bath tub, their motions are more frequent, less movement in the middle period.

人在浴缸中运动是一个连续的动态过程，我们这里将他按照时间进行离散化。一般人的体积大概为60L,沐浴时间为20min适宜。除去开始沐浴进入水体和沐浴完离开水体两个大的水位变化，其他的人体运动一起的浸入水体积的均值为0，最大值为人自身体积的20%，即12L。正常情况下，人沐浴时头部总在水面以上，浸入水中体积约为自身体积的90%。用MATLAB产生一组随机数模拟人在水中400s内的运动，假设400s内共运动了20次且各时刻运动频率相同，得到该段时间内由人体运动导致的人体浸入水中体积的变化：

The movement of people in the tub is a continuous dynamic process, we will discrete the process by time. The general volume of is approximately 60L, suitable bath time is 20min. Apart from two large water level changes which results from entering and go out of the bath tub, other human motion immersion average volume of water is zero, the maximum volume of its own people by 20%, that is 12L. Normally, a person's head and parts of body are always above the water, so the body that immersed in the water is about 90% of its own volume. We use MATLAB simulation generates a set of random numbers represent the motion of people in water during 400s, assuming a total of 20 times and the motion of the same frequency each time the action, and get the volume changes that body immersed in water within the period of time caused by the body motion:



图\*\*人体浸入水中体积的变化量

上图表示人体进入水中带来的体积变化，为正表示人体进入水中，浸入水中的体积增大，为负表示人体从水中出来，这是由MATLAB生成的随机数，取前400s为例探讨的变化。事实上我们是规定了运动周期为20s，将时间离散化来考虑的情况。反映出的人体运动产生的体积变化，满足了随机性的这个条件

The figure represents the volume change brought by the body into the water. A positive shows body into the water and the increase that volume immersed in water. A negative shows body out of the water. These are random numbers generated by the MATLAB, and we take the first 400s for case to study the changes. In fact, we require a motion cycle time for the 20s and discrete the time. This can reflect the volume variation caused by the movement of the body, and this condition is satisfied randomness.

### 4.3.2模型的建立

4.3.2 Model Establishment

上述人体运动对水量变化的影响满足了随机这个条件，但是并不能反映沐浴时的真实状况，在沐浴时如果人体在某个时刻坐起，即人体一部分脱离水面，那么他的下一个动作必定是躺下也就是人体一部分浸入水体，且两次运动产生的体积变化值大致相等。这个规律表明相邻的两次体积变化值大小相近但正负号相反，上图这组随机数显然不满足。

Affect of the body's motion on the amount of water’s variation meets the randomness of this condition, but does not reflect the real situation when bathing. If the person in the shower at a time to sit up, that part of the body out of the water, his next move must be to lie down, which is part of the human body is immersed in of water, and the volume change is substantially equal to twice movement. This principle states that the volume change of the two adjacent values of similar size but opposite sign, these set of random numbers in figure are clearly not satisfied.

另外由于前面的讨论运用的都是连续的模型，如果加入这种离散的随机变量，势必造成微分方程难以求解。

In addition, as the foregoing discussion is continuous model, if the addition of such a discrete random variable is bound to cause difficult to solve differential equations.

为解决这两方面的矛盾，我们很自然地想到三角函数中的正余弦函数。一方面它的变化满足与一个波峰相邻的必定是一个波谷；另一方面它很好地解决了离散数列难以引入到微分方程的难题。另外，它还有一个得天独厚的优点——频率和振幅可调。由于正弦函数中心对称的性质能保证一个波峰相邻的必定是一个波谷，则人体运动引起的浸入水中的体积随时间的变化关系为：

To solve the contradiction between these two aspects, we naturally think of trigonometric functions of sine and cosine functions. On the one hand, it changes to meet with a crest must be adjacent to a trough; on the other hand, it solves the discrete series difficult to introduce into the differential equation problem. In addition, it also has a unique advantage - the frequency and amplitude are adjustable. The centrosymmetric nature of the sine function can guarantee a peak adjacent to a trough must be immersed in water. So the relationship between the change caused by the body movement and time is:

* 频率g(t)
* Frequency

根据上面对沐浴规律的分析，取沐浴开始和结束时的频率都为1/40，周期为40s,此时g(t)=。当t=600s时也就是中间时刻人几乎不做任何运动，频率都为0，此时g(t)=0.对这之间的时刻，假定频率和时间以二次函数变化， g(t)的表达式为：

According to the analysis of the law of bath, we take the motion frequencies at the beginning and end of bath for 1/40, a period of 40s, and g(t)=. When t=600s, which is intermediate in time, people hardly do any movement, the frequency is zero, and g(t)=0. So the expression of g(t) is:

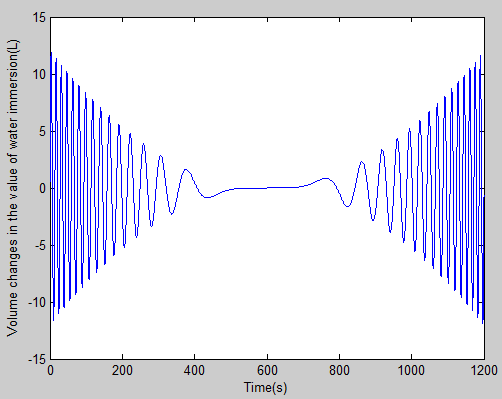
* 幅值f(t)
* Amplitude

与频率一样，认为初始时刻和结束时刻的的幅值为12L，中间时刻为0L，按二次函数变化，则：

The same as frequency, We believe that the magnitude of the initial time and end time for the 12L, intermediate time for 0L, according to a quadratic function, then:

按照上面的参数设定，得到与时间的函数图像

Accordance with the parameters set above, we get the image of function and time



图\*\*优化后人体进入浴缸体积变化量

上图结果大于零表示人体浸入水中的体积增加量，小于零表示人体从浴缸中出来的体积减少量。我们将离散的问题连续化，考虑三角函数的振荡性质，同时结合现实生活中泡澡运动的实际情况，得到一个人的运动对水体积的影响的图。从图中可以看出，在开始阶段和最后阶段人洗澡的运动频率和幅度都比较大，而中间过程人在浴缸中运动不明显。Above result is greater than zero indicates the body is immersed in water to increase the amount of volume, less than zero indicates the volume decrease body come out from the tub. We will make the discrete problem continuous, considering the nature of the oscillation trigonometric functions, combined with the actual situation in real life movement bath to give affect a person's exercise on the volume of water figure. As can be seen from the figure, at the initial stage and the final stage of the movement of people bathing frequency and amplitude are large, while the middle movement of people is not obvious process in the tub.

人在水中的情况下，水的温度变化仍然要分两个阶段，即水位未超过浴缸高度和水位超过浴缸高度。水位未超过浴缸高度时，人的运动将导致水位升降，进一步导致热传导的导热面积变化，其变化量为。。因此应对热水输入温度变化模型中的热传导面积进行修正：

When people in the water, the water temperature variation is still to be divided into two stages, namely, the water level does not exceed more than the height of overflow drain and exceed. When the water level does not exceed the height of the bathtub, the movement of people will lead to change water level, leading to further change heat transfer area of the heat transfer process, the variation amount . Therefore, we should amend the heat transfer area in the model of input hot water temperature variation:

修正后散热速率q的表达式为：

The expression of cooling rate q after being amended

建立微分方程组：

Establishment of differential equations

During this period, the heat change is depends on the difference of two discharge water temperature

当水位超过浴缸高度后，人体浸入水中的体积增加将不会引起热倡导面积的增加，而是会导致相同体积的水溢出；人体浸入水中体积减少则会导致热传导面积的减小。前面已经设定了浸入水体积变化的频率和幅值，在幅值最大处运动一次，会产生12L的体积变化，下一次的运动会在10s内到来，这10s内热水会不断注入以填补水体积的减少，10s后人将回到水中，此时浸入水的体积又将增加，浴缸中将排出与10s内注入的热水体积相等的水。这期间的热量变化就在于两次排出水的温度存在差异，但是从前面得到的温度曲线可以发现，10s对应整个水体温度的变化值时非常微小的，可以忽略不计。或许有的人会注意到在频率很小时，中间的时间间隔会远大于10s，但你可以发现此时人体浸入水中体积变化的幅值很小，对应的热量变化依然微乎其微。When the water level exceeds the height of the bathtub, increase in the volume of human body immersed in water will not lead to increase in the area of heat advocate, but will result in the same volume of water overflow; Reducing the volume of the human body into the water will lead to a decrease in the heat conduction area. As already set up immersed in water volume change of frequency and amplitude, Moving once at the maximum amplitude, will produce changes in the volume of 12L, the arrival of the next motion in 10s, in the 10s it would continue to inject hot water to fill the water volume reduction, after 10s person will return to the water, At this point, the volume of water in the water will increase, bath will be discharged within 10s injected with an equal volume of hot water. During this period, the heat change is depends on the difference of two discharge water temperature, but the temperature curve obtained from the front can be found, 10s when the corresponding change in the value of the entire water temperature is very small, negligible. Perhaps some people will notice that the frequency is very small, in the middle of the interval will be much larger than the 10s, but you can see the magnitude of this time the body is immersed in water volume change is small, the corresponding change in the heat is still minimal.

因此，当水位超过浴缸高度后，我们依然认为导热面积的变化符合式（1）。得到水位超过浴缸高度后水温变化微分方程组：

Therefore, when the water level exceeds the height of the bathtub, we continue to believe that change of heat transfer area according to formula (1). We obtained the differential equations of temperature change after the water level exceeding the height of the bathtub:

### 4.3.3 Solving model

Solution

以矩形底面浴缸，长\*宽\*高=1.5m\*0.6m\*0.5m，初始水深0.4m，水温为39℃为例，取流量,流入热水的温度，运用MATLAB得到浴缸水温与时间的关系曲线

Taking the bathtub with rectangular base, L \* W \* H = 1.5m \* 0.6m \* 0.5m, initial water depth 0.4m, the water temperature 39 ℃ for case, and flow rate , temperature of hot water flowing into , We use MATLAB to draw a bath temperature against time curve:

**5.1 优化目标的确定**

Optimization objectives determination

从前面的讨论可以看出，当浴缸形状和体积等都确定的前提下，沐浴过程中的水温变化主要受到加热水的温度和流量的影响。要想确定最佳的策略，使浴缸里的人可以用这个模型来让整个浴缸保持或尽可能接近初始的温度，而不浪费太多的水。其目的是要得到合适的热水温度和流量值，使人的舒适度尽可能大而浪费的水量都尽可能地小，这便是一个多目标优化问题。

As it can be seen from the foregoing discussion, when the premise bathtub shape and volume are all determined during the bath water temperature changes are mainly affected by the heating water temperature and flow. To determine the best strategy the person in the bathtub can adopt to keep the temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water. The aim is to get the right water temperature and flow rate, to increase the comfort of people as much as possible and minimize water wastage, which is a multi-objective optimization problem.

该问题中的优化目标有两个：人的舒适度最高，浪费水量少。

There are two optimization objectives: Increasing the comfort of people as much as possible and minimizing water wastage

* 人的舒适度可以用浴缸中的整体水温变化、热水温度与最适温度差值来衡量。已知人体最适的沐浴温度是39℃，因此衡量水温变化可以用浴缸中温度与39℃差值的绝对值的最大值，即
* We used bath water temperature change in the overall, and the difference between the hot water temperature and optimum temperature to measure the comfort of people. We know the optimum body temperature bath is 39 ℃, and therefore what can be used to measure temperature changes is that the absolute maximum value of the difference between the temperature of bath water and 39 ℃, namely

热水是直接加入到浴缸中的，若加入的热水太烫，必然导致一定区域内的水温偏高，影响舒适度，用热水温度与人体最舒适的沐浴温度是39℃的差值来衡量。

Hot water is added directly to the tub. If added hot water temperature is too high, it will inevitably lead to a certain high temperature region, thereby affecting comfort of people, which is measured by the difference between hot water temperature and the most comfortable bathing temperature of body.

从人体舒适度的角度出发，整体水温的变化相对于局部水温变化更能影响舒适度，因此为，赋权值0.7,0.3。同时，为了使，具有可比性，对其进行归一化。热水温度最大值，水温42℃便不再适合沐浴，浴缸中水温的最大变化量。因此定义舒适度

From the perspective of human comfort point of view, compared to the local water temperature changes, the overall change in temperature have much more effect on comfort, so take the weighting value of ， for 0.7,0.3. Meanwhile, in order to make ， comparable, we normalize them. The hot water temperature maximum . Therefore, the definition of comfort is

一浴缸中温度与39℃差值的绝对值。

一the absolute value of the difference between the temperature of bath water and 39 ℃

* 浪费的水量可以根据水溢出的时间和热水流量来计算,沐浴总时间依然以20min计：
* Waste water is calculated by the product of water overflowed and the water flow, and the total bathing time is still 20min

式中：

—浴缸中水开始溢出的时间the time of water overflowed

—浪费的水量the waste water

和舒适度一样，应对浪费水量进行归一化处理，按最大流量算出最多的浪费水量，归一化后的浪费水量为

The same as comfort, normalizing the waste water, the most waste water is calculated according to the maximum flow rate , and the waste water after normalization is

考虑到当前的社会发展水平下，人们在洗澡时，先关心的应该是舒适度，其次才是浪费的水量最少。因此为两个优化目标引入优先因子、，得到目标函数：

Taking into account the current level of social development, it should be the first concern of comfort in the bath, followed by a minimum of wasted water. Therefore, we introduce priority factors , for two optimization objectives, and give the objective function

根据目标函数的要求，首先考虑带因子的实现最小化，满足这个条件的热水温度和热水流量f肯定不止一个。因此接下来考虑带因子的 实现最小，得到一组满足目标函数的、f值。

According to the requirements of the objective function, firstly we minimized the , and hot water temperature and water flow f that meet the conditions are more than one set, then we minimized the , finally get a group to meet the objective function , f value

**5.2 约束条件的确定**

Determination of constraints

对于约束条件我们要逐条利用上述模型中讨论的影响因素，

For the constraint we have to use factors discussed above model one by one,

* 目标约束
* Constrains

我们在文章的前面已经建立了浴缸中水温随时间变化的微分方程模型，也就是说DT, w,, f需要满足约束

We have built a model of the bath water temperature varies with time differential equations, DT, w,, f have to satisfy the constraint that

式中f为微分方程模型中确立的()与()的映射关系。

f is mapping between () and (), which is established in the differential equation model.

* 系统约束
* System constraints

在实际中水的温度是有上下限的，标准大气压下热水的温度最大为100℃，至于下限，可取为浴缸中水的初始温度39℃。流量f的取值范围可以根据市场上常见水龙头的最大流量确定，这里定为0.7L/s。

In practice, temperature of the water is limited, on the standard atmospheric pressure, temperature of hot water is up to 100 ℃, as for the lower limit, the initial temperature of the water bath is 39 ℃. The range of flow f is determined by the maximum flow of faucet in the market, which is 0.7L / s

根据上面的分析，写出相应的规划模型为

## 5.3模型的求解

该优化模型中的约束条件中含有微分方程，直接用lingo或者MATLAB无法求解。因此只能对其进行离散仿真，采用模拟退火算法求解全局最优解。Differential equations contained in the optimization model of constraint condition, directly using lingo or MATLAB, unable to solve.So only on the discrete simulation, using the simulated annealing algorithm for solving the global optimal solution.

模拟退火算法的基本思想是从一个给定的解开始,从邻域中随机产生另一个解,接受Metropolis准则允许目标函数在有限范围内变坏,它由一个控制参数决定*,* 其作用类似于物理过程中的温度*T,*对于控制参数的每一取值*,* 算法持续进行“产生—判断—接受或舍弃”的迭代过程*,*对应着固体在某一恒定温度下趋于热平衡的过程*.* 经过大量的解变换后*,* 可以求得给定控制参数*t*值时优化问题的相对最优解*.* 然后减小控制参数*t*的值*,* 重复执行上述迭代过程*,* 当控制参数逐渐减少并趋于0时,系统亦越来越趋于平衡状态,最后系统状态对应于优化问题的全局最优解【】The basic idea of simulated annealing algorithm begins with a given solution, another solution from the randomly generated neighborhood, accept the Metropolis criterion allows bad objective function in a limited range, it is determined by a control parameter, its action is similar to the temperature T, physical process for each value, the control parameters of the algorithm for generation "- judgment - to accept or abandon" iterative process, solid at a constant temperature corresponding to the thermal equilibrium state, respectively. After a lot of solution transformation, we can obtain the given control parameters when T value relative to the optimal solution of optimization problem. Then reduce the value of control parameter T, repeat the above iterative process, gradually reduced and tends to zero when the control parameters, the system is also more and more tends to equilibrium state and the system state corresponding to the global optimal solution of optimization problem 【 】

算法步骤如下

Step one给定模型参数变化范围,在这个范围内先用蒙特卡罗方法求得一个较好的初始解[,],并计算相应的目标函数值 . A given model parameter range, within the scope of the first to use the monte carlo method to get a good initial solution [and], and calculate the corresponding objective function values

Step two对当前模型进行扰动产生一个新解，计算相应的目标函数值,得到Disturbance to the current model to create a new, calculate the corresponding objective function values, get it

Step three 接受准则

若,则解被接受;若,则新解按概率进行接受。即用计算机产生一个[0,1]区间的随机数rand，若则接受。

Step four在温度T下,重复一定次数的扰动和接收过程,即重复步骤(2) 、(3) .

Step five降低温度，利用选定的温度系数进行降温，取新的温度为（这里的为上一步迭代的温度），这里选定。

Step six重复步骤(2) 、(5) ,直至收敛条件小于临界温度e为止.

我们将具体的步骤绘制成程序框图，具体步骤如下所示：If the solution is acceptable.If, then the new accept according to probability.Use the computer to produce a [0, 1] interval random number rand, if is acceptable.  
Step four under temperature T, repeated a certain number of disturbance and receiving process, namely, repeat steps (2), (3).  
Step five lower temperature, the use of the selected temperature coefficient for cooling, a new temperature for (here for the temperature of the Step iterative), selected here.  
Step six repeat Step (2), (5), until the convergence condition E is less than the critical temperature.  
Specific steps we will draw a program block diagram, as shown in the following steps

Input parameters

你哈O

Generate new solutions

No

Yes

No

Output f,

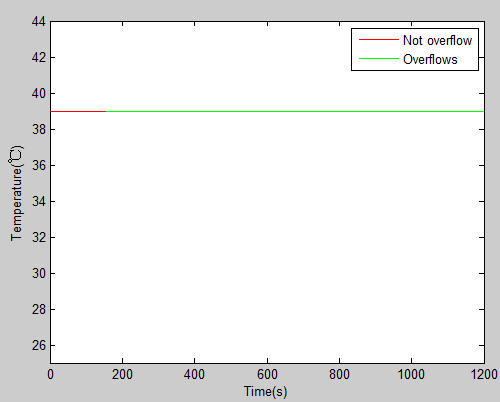
Yes

No

Yes

Generating initial solution

程序运行结果显示，矩形底面浴缸尺寸为长\*宽\*高=1.5m\*0.6m\*0.5m，水深0.4m，，人体积为60L的情况下，最优的热水温度、热水输入速率为，此时的浪费水量为，浴缸中水的温度-时间图像为Program is run, according to the results of the rectangular bottom bathtub size for length \* width \* height = 1.5 m \* 0.6 m \* 0.5 m, the depth of 0.4 m, and people of 60 l volume of cases, the most optimal input rate of hot water temperature, hot water, the waste water, the temperature of the water in the bathtub - time for images



图\*/\* 最优条件下水温变化规律

由图可见水温基本可以控制在不变的水平，既与初始温度相差不大，也能使人处在一个舒适度较高的环境中，而且浪费的水资源较少，模型比较合理。By the water temperature can control the basic figure visible at the same level, both with the initial temperature difference is not big, can also make a person in a higher comfort level of environment, and waste of water resources is less, the model is reasonable.

## 5.4 泡泡剂的影响

肥皂泡是非常薄的一个带虹彩表面的空心肥皂水的膜。某些类似界面活性剂具有长碳链的分子在水中的溶解度很低，但是在液体表层由于疏水力作用会均匀散布，形成非常薄的覆盖层。1932 年诺贝尔化学奖得主Fritz Pregl (1881-1957) 证明这类覆盖层只有一个分子的厚度，称为单分子，能有效地阻碍水分子穿透表层，抑制其下液体的蒸发，使蒸发速率降低40%以上，肥皂泡越厚，保温的效果越佳。另外肥皂泡的堆叠将使蒸发散热面积提升，考虑到泡沫堆叠高度不会太高，设定散热面积增大20%。

所以泡泡浴剂的加入影响的是蒸发散热速率和蒸发面面积，将蒸发散热系数取为原来的0.6倍，蒸发散热面积改为重新代入模型重新进行求解。

模型求解结果显示，矩形底面浴缸尺寸为长\*宽\*高=1.5m\*0.6m\*0.5m，水深0.4m，，人体积为60L的情况下，最优的热水温度、热水输入速率为。因此泡泡浴即的加入能够有效地减少浪费水量，提高舒适度。

## 5.5 灵敏度的分析

由于使用模拟退火算法求解，模型的灵敏度分析结果不能直接获取，只能改变参数值，进行数值仿真，讨论参数值改变的条件下目标函数的改变率。

Table\*\* 浴缸体积

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 浴缸体积（m3） | 热水温度（℃） | 热水流量（L/s） | z | 变化率（%） |
| 0.45 | 81 | 0.49 | 0.394 |  |
| 0.405 | 79 | 0.48 | 0.3821 | 3.02 |
| 0.36 | 73 | 0.52 | 0.3698 | 3.22 |
| 0.315 | 75 | 0.45 | 0.3569 | 3.49 |
| 0.27 | 73 | 0.43 | 0.3445 | 3.47 |
| 0.225 | 66 | 0.48 | 0.3239 | 5.98 |
| 平均变化率（%） | | | | 3.84 |

保持浴缸的长宽高比例不变，缩小浴缸体积，得到优化结果和目标函数的变化率。数据显示浴缸体积越小，目标函数值越小，也就是说浴缸体积越小，模型得出的策略越适用；浴缸体积变化10%，将导致目标函数值变化3.84%。

Table \*\* 形状

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 长：宽 | 热水温度（℃） | 热水流量（L/s） | z | 变化率（%） |
| 1 | 76 | 0.55 | 0.3855 |  |
| 1.4 | 74 | 0.58 | 0.387 | 0.39 |
| 1.8 | 79 | 0.52 | 0.3899 | 0.75 |
| 2.2 | 82 | 0.49 | 0.3906 | 0.18 |
| 2.6 | 79 | 0.53 | 0.3933 | 0.69 |
| 3 | 83 | 0.49 | 0.3951 | 0.46 |
| 平均变化率（%） | | | | 0.49 |

保持浴缸底面积不变，改变浴缸长宽比，数值仿真结果表明长宽比每增加0.4，目标函数值变化0.49%。同时可以看到目标函数值随长宽比的减小而减小，正方形的缸底最好，其原因是围成相同的面积，正方形的周长小，可以有效地减小侧面散热面积。依此类推，椭圆形底面的浴缸比矩形底面的好，圆形底面比椭圆形底面好。

Table \*\*人体积

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 人体积 | 热水温度（℃） | 热水流量（L/s） | z | 变化率（%） |
| 50% | 85 | 0.44 | 0.3902 |  |
| 60% | 85 | 0.44 | 0.3909 | 0.18 |
| 70% | 85 | 0.44 | 0.3917 | 0.20 |
| 80% | 85 | 0.44 | 0.3927 | 0.25 |
| 90% | 85 | 0.44 | 0.3937 | 0.25 |
| 100% | 85 | 0.49 | 0.394 | 0.08 |
| 110% | 81 | 0.49 | 0.3943 | 0.08 |
| 120% | 84 | 0.45 | 0.3946 | 0.08 |
| 平均变化率（%） | | | | 0.17 |

保持人体型不变，改变人体积，使它为原来的50%，60%,...,120%，数值仿真结果表明人的体积每增大10%，目标函数值增大0.17%。

Table \*\*人的运动

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 浸入水体积变化（L） | 热水温度（℃） | 热水流量（L/s） | z | 变化率（%） |
| 9 | 81 | 0.49 | 0.3939 |  |
| 12 | 81 | 0.49 | 0.394 | 0.02 |
| 15 | 81 | 0.49 | 0.3941 | 0.02 |
| 18 | 84 | 0.45 | 0.3946 | 0.13 |
| 21 | 84 | 0.45 | 0.3951 | 0.13 |
| 24 | 84 | 0.45 | 0.3956 | 0.13 |
| 27 | 84 | 0.45 | 0.3963 | 0.18 |
| 30 | 85 | 0.44 | 0.3982 | 0.48 |
| 平均变化率（%） | | | | 0.15 |

保持人体积不变，改变人运动过程中浸入水体积的变化，进行数值仿真，结果表明人浸入水体积变化每增大3L，目标函数值增大0.15%。

综上所述，目标函数值受浴缸体积的影响最大，浴缸形状的影响次之，受人的形状体积和运动的影响最小。我们根据其影响程度可以确定各种情况下的最优解

# 8 non-technical explanation of the bathtub

亲爱的用户 ：  
随着生活的发展，我们对生活品质的要求越来越高，洗澡也不例外。针对没有类似于spa那样的恒温系统和淋浴喷头的浴缸，我们推出了一种仅依靠一个热水龙头就可以保持水温大致恒定，且不浪费太多水资源的战略方案。

我们从能量的损失出发，考虑浴缸中水的各种散热方式，将热量的丧失转换成温度的变化，先考虑不加热水的情况，此时温度是下降的比较快的。然后考虑水龙头保持恒定速度流入，此时流入水的温度和流量将影响其变化规律。事实上我们还需考虑人存在时对水温度的影响。人存在时应考虑人的温度、形状、体积和运动等因素。最后分析浴缸尺寸和体积、是否加泡泡剂对温度的影响，来确定最佳策略。

对于温度的恒定在生活中我们仅仅可以考虑控制水龙头流速、流入水的温度和流入水的时间。为了使洗澡过程更加方便和舒适，我们假定水龙头一直在流，过控制流量在q=0.48L/s，入水温度在81.8℃时可以保证水温基本保持不变，且用水量较少的条件。这个解是假定了浴缸的尺寸是L × W × H = 1.7m × 0.8m × 0.7m的方形浴缸，根据不同形状和不同容积的浴缸可以做适当的调整，代入模型中可以计算出各种尺寸和条件下对应的最适温度和和流量。

我们的模型考虑的因素完善，除了上述的影响因素外，还考虑了浴缸的材料是夹层的保暖玻璃钢材料，而且还考虑了温湿度随温度变化的情况。分析的过程中每一步都与现实生活做了对比，都比较符合实际情况。而且我们优化的目标不仅仅是用水量最少，还要求温度与初始条件相差不大。总而言之我们的策略是比较优秀的。

但是在此必须说明，我们的模型没有考虑浴缸中的水空间分布不均的情况。现实生活中热水对流的速度是特别快的，而且考虑人的运动对水对流的影响，热水变均匀的时间非常短。如果考虑这段时间想要维持整体上的温度恒定将会变得非常困难。

Dear users:

With the development of life, our quality of life become increasingly demanding, a bath is no exception. For the bathtub without a spa-style tub with a secondary heating system and circulating jets, we introduce a strategy that can maintain water temperature substantially constant rely solely on a hot water faucet and without wasting too much water.

We depart from the loss of energy, consider various water bath cooling mode, convert the heat loss into a change of temperature, and obtain the relationship that water temperature changing with time under different conditions. First, we consider the case without adding hot water, when the temperature is falling fast. Then, we consider the case of hot water flowing at a constant velocity, when inflow water temperature and flow rate will affect its variation. In fact, we also consider the impact on the water temperature changes when human existence. At this point, we consider the human temperature, shape, volume, movement and other factors. Finally, we analyze influence caused by the size and volume of the bathtub, adding bubble on water temperature. Thereby we determine the best strategy.

To keep the temperature even throughout the bathtub and as close as possible to the initial temperature, we consider control the flow rate of water faucet, water temperature and the inflow time of water. We add a constant trickle of hot water from the faucet. The flow rate of water faucet is 0.48L/s. The hot water temperature is 81.8℃. Under these conditions, the temperature can be kept even throughout the bathtub and as close as possible to the initial temperature without wasting too much water. This solution assumes the square bathtub, whose size is L × W × H = 1.7m × 0.8m × 0.7m. Depending on the shape and volume of different bathtubs, we can make the appropriate adjustments. In other words, the model strategy can be applied to calculate the corresponding to the size and the optimum the flow rate of water faucet, water temperature under various sizes and conditions.

Compared to the hot water temperature and flow rate before optimization, Our strategy not ensure the comfort of people, but also save water resources.

However, our model does not consider the case of the spatial uneven distribution of water in the bathtub. Considering the impact of the movement of people on the water convection randomness and fast hot water convection speed in real life, it is so difficult to get an evenly maintained temperature throughout the bath water only by setting a fixed hot water temperature and flow rate.