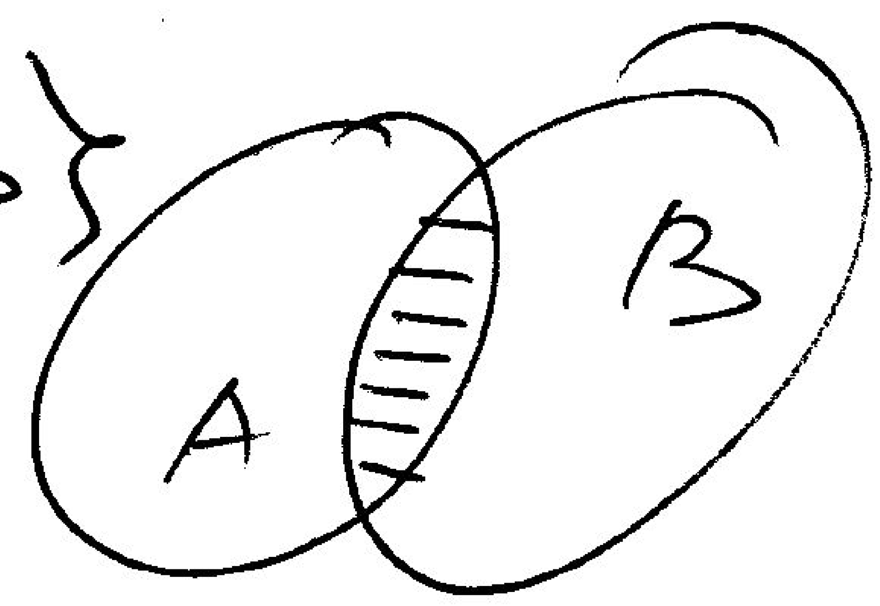
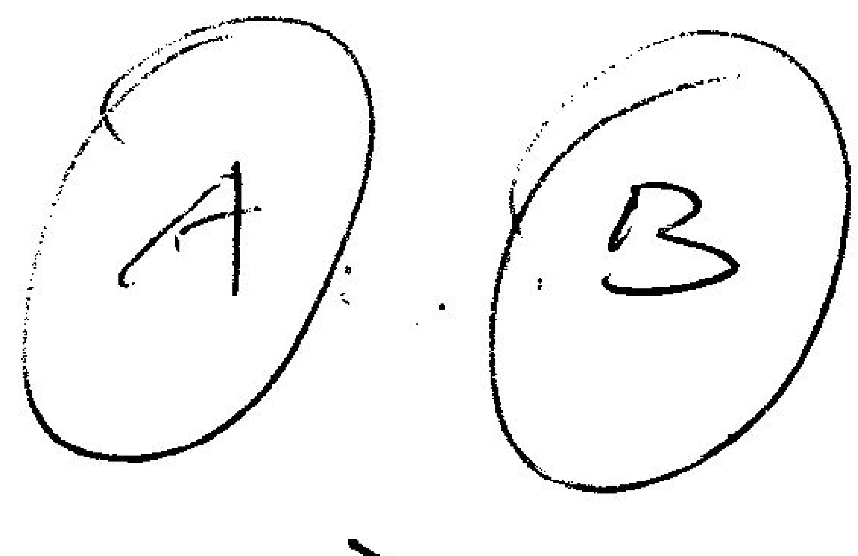
Def. 1) Siano A e B ins. Si dice <u>intersezione</u> di A e B l'insieme

$$A \cap B = \{x \text{ \underline{tali} \underline{che} } x \in A \text{ e } x \in B\}.$$

$$= \{x \mid x \in A \wedge x \in B\}$$



Si dice A e B disgiunti se $A \cap B = \emptyset$

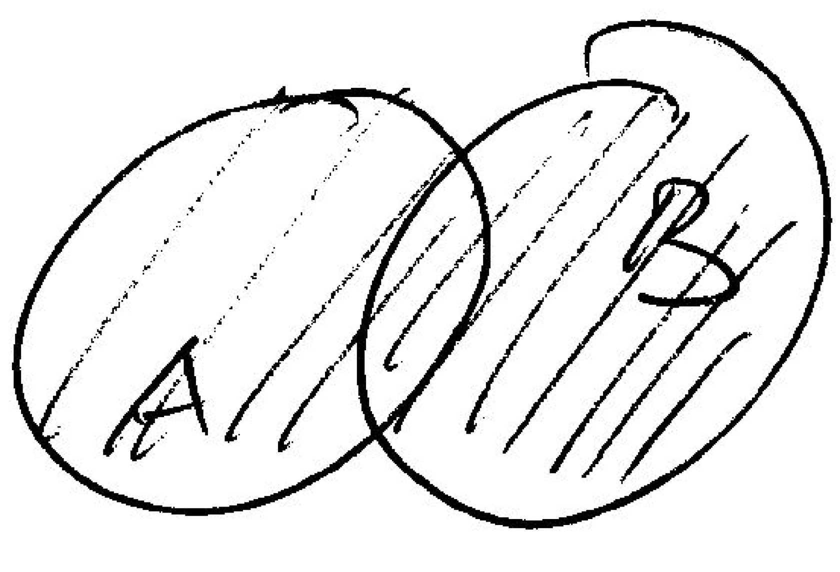


2). <u>l'unione</u> di A e B è

$$A \cup B = \{x \mid x \in A \text{ oppure } x \in B\}$$

e.g. $A = \{1, 2, 3\}$

$B = \{2, 3, 4\}$



$A \cup B$

$$A \cap B = \{2, 3\}$$

$$A \cup B = \{1, 2, 3, 4\}$$

Def. Siano $A_1, A_2, \dots, A_n$, insiemi.

l'insieme intersez.

$$A_1 \cap A_2 \cap \dots \cap A_n = \{x \mid x \in A_i,\ 1 \leq i \leq n\}$$

$$\bigcap_{1 \leq i \leq n} A_i$$

l'unione:

$$A_1 \cup A_2 \cup \dots \cup A_n = \{x \mid x \in A_i, \text{ per qualche } i,\ 1 \leq i \leq n\}$$

$$\bigcup_{1 \leq i \leq n} A_i, \quad \bigcup_{i=1}^{n} A_i$$

Prop: 1): proprietà assoc.

Dati A, B e C insiemi.

$$(A \cap B) \cap C = A \cap (B \cap C).$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

2) prop. idempotente:

$$A \cup A = A, \quad A \cup \emptyset = A;$$

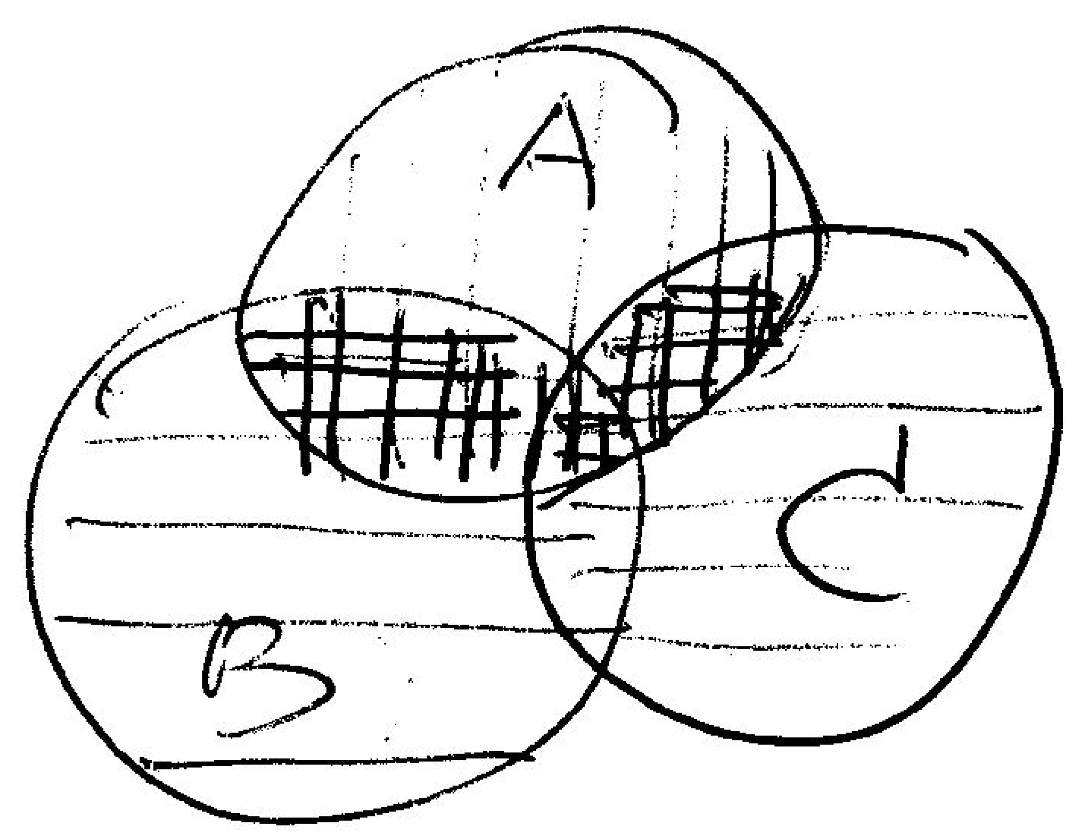
$$A \cap A = A, \quad A \cap \emptyset = \emptyset$$

3) prop. distributiva

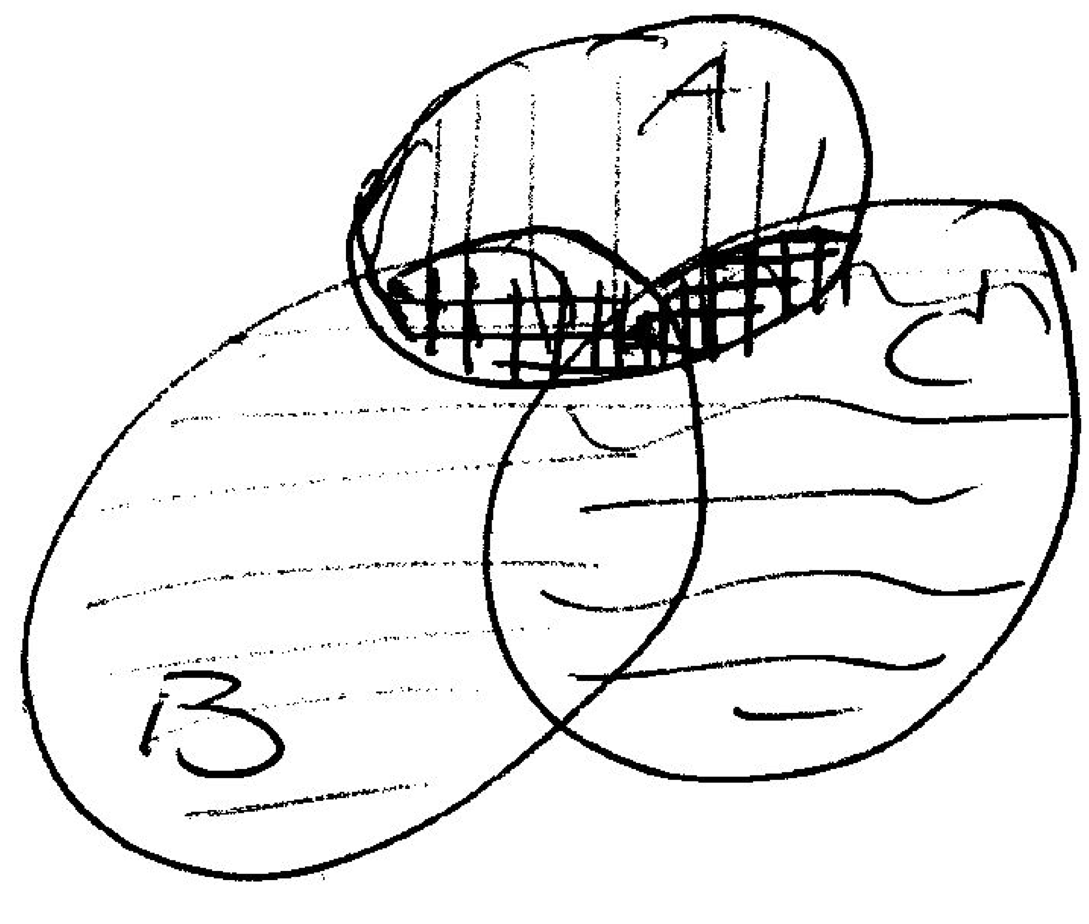
① $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

② $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

①



$A \cap (B \cup C) = \#$



$\# = (A \cap B) \cup (A \cap C)$

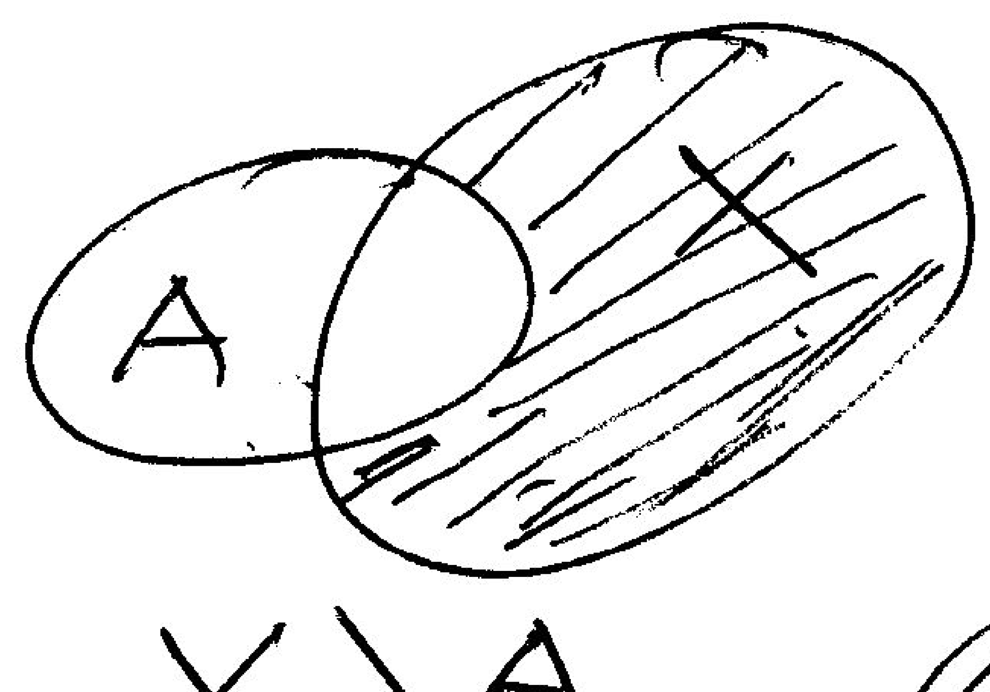
② eser.

Def. Dati due insiemi A e X

si dice <u>differenza</u> di X ed A è

un sottoins. di X.

$$X \setminus A = \{x \in X \mid x \notin A\} \qquad \text{"} X - A \text{"}$$

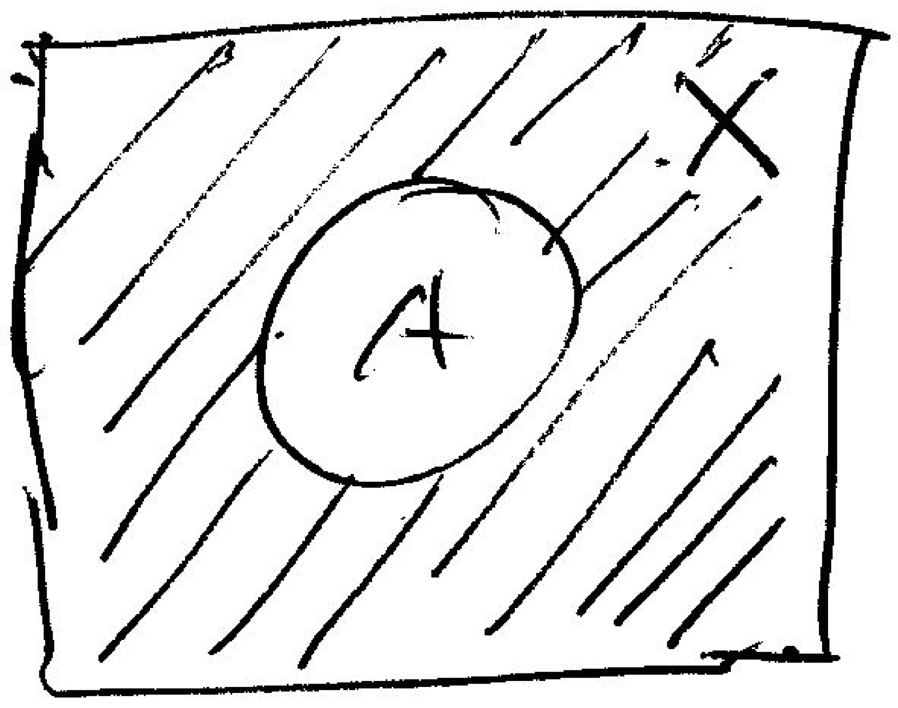


$X \setminus A =$ 

Def: Sia A un ~~sottoins.~~ di X.

Un <u>complementare</u> di A in X è un ~~sottoins.~~ $C_X(A)$ di X.

$$C_X(A) = \{x \in X \mid x \notin A\} \qquad \text{"}\bar{A}\text{"}$$



$\Rightarrow X = A \cup C_X(A)$

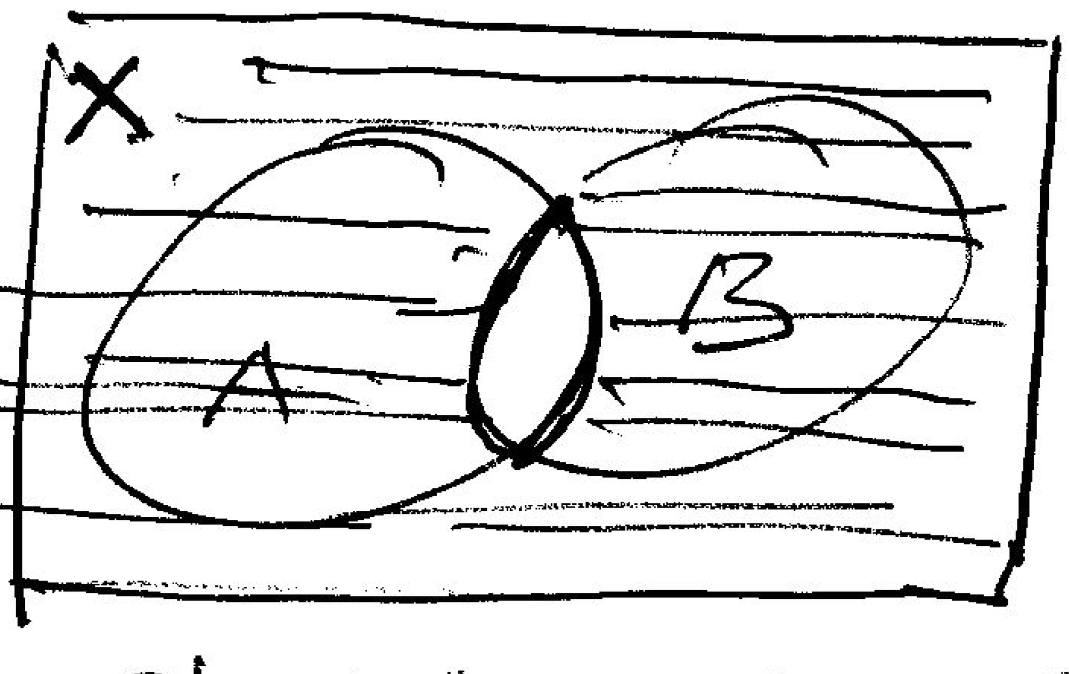
$X \setminus A$

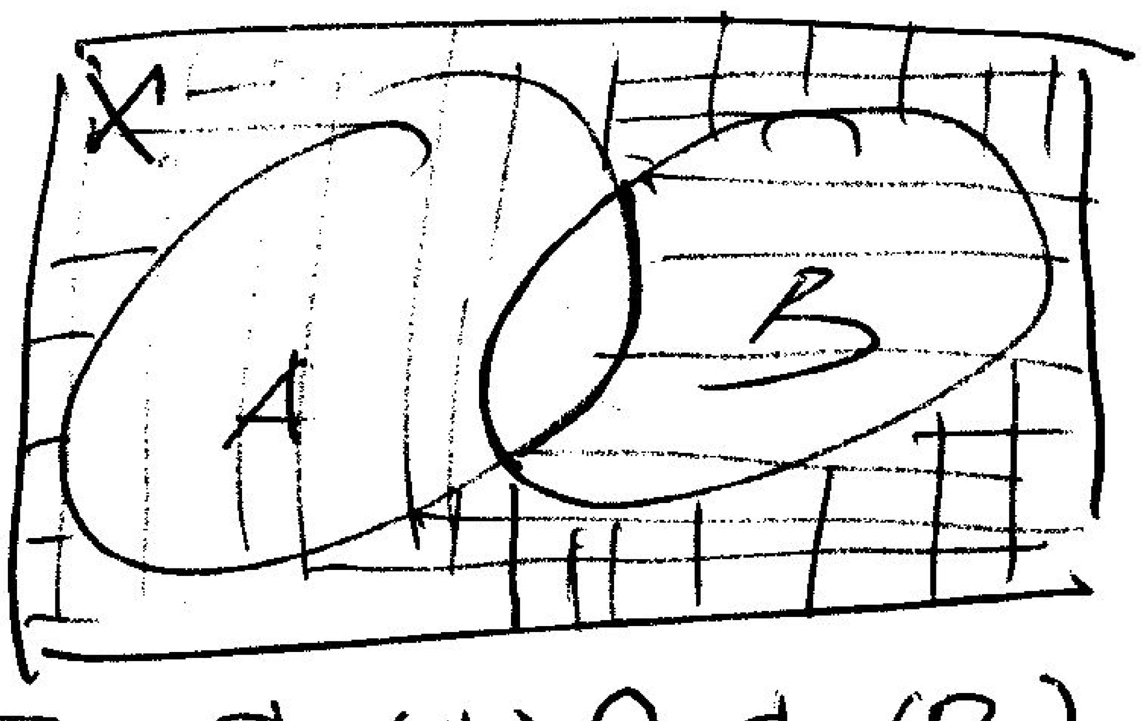
Teo: (Leggi di De Morgan).

Siano A e B ~~sottoins.~~ di X.

(1) $C_X(A \cap B) = C_X(A) \cup C_X(B)$ | $\overline{A \cap B} = \bar{A} \cup \bar{B}$

(2) $C_X(A \cup B) = C_X(A) \cap C_X(B)$ | $\overline{A \cup B} = \bar{A} \cap \bar{B}$

(1) 



(2) (eser)

$C_X(A \cap B) = \boxed{\equiv} = \boxed{\ } = C_X(A) \cap C_X(B)$

Def: Sia X un insieme. Sia $A = \{A_i\}_{i \in I}$ una famiglia di sottoins. di X, dove I è un ins. d'indice ...

Se
$$\bigcup_{\forall i \in I} A_i = X$$
Si dice A è un ricoprimento di X.

e.g. $A_1 = \{$numer. inter. posit.$\}$.

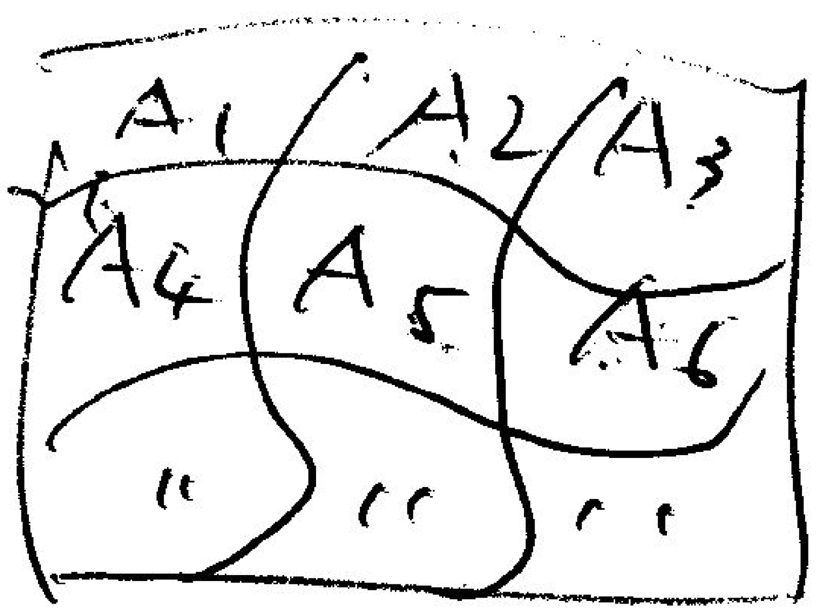
$A_2 = \{$ ... ... negat. $\}$.

$A = \{A_1, A_2\}$ e
$$A_1 \cup A_2 = \mathbb{Z}.$$

Def. Una partizione di un insieme X è un ricoprimento $\{A_i\}_{i \in I}$ di X, t.c.

1) $A_i \neq \emptyset$

2) $A_i \cap A_j = \emptyset$, $\forall i \neq j$, $i, j \in I$.



X

e.g. $A_1 = \{\text{numeri pari in } \mathbb{Z}\}$.

$= \{0, \pm 2, \pm 4, \dots\}$

$= 2\mathbb{Z} = \{2a \mid \forall\, a \in \mathbb{Z}\} = \text{"}\bar{0}\text{"}$

$A_2 = \{\text{Numeri dispari di } \mathbb{Z}\}$.

$= \{2a+1 \mid \forall\, a \in \mathbb{Z}\} = \text{"}\bar{1}\text{"}$.

$= 2\mathbb{Z}+1$

$$\left.\begin{array}{l} A_1 \cup A_2 = \mathbb{Z} \\ A_1 \cap A_2 = \emptyset \end{array}\right\} \{A_1, A_2\} \text{ è una partizione di } \mathbb{Z}.$$

Def. ~~Data~~ una partizione $\{A_i\}_{i \in I}$ di X, l'ins. $\{A_i\}_{i \in I}$ si chiama un ins. quoziente di X.

e.g.: $\{2\mathbb{Z}, 2\mathbb{Z}+1\}$ è un quoziente di $\mathbb{Z}$.

Def: Dati ins. A e B, un prodotto cartesiano di A e B è l'insieme

$$A\times B = \{(a,b) \mid \forall a\in A, \forall b\in B\}.$$

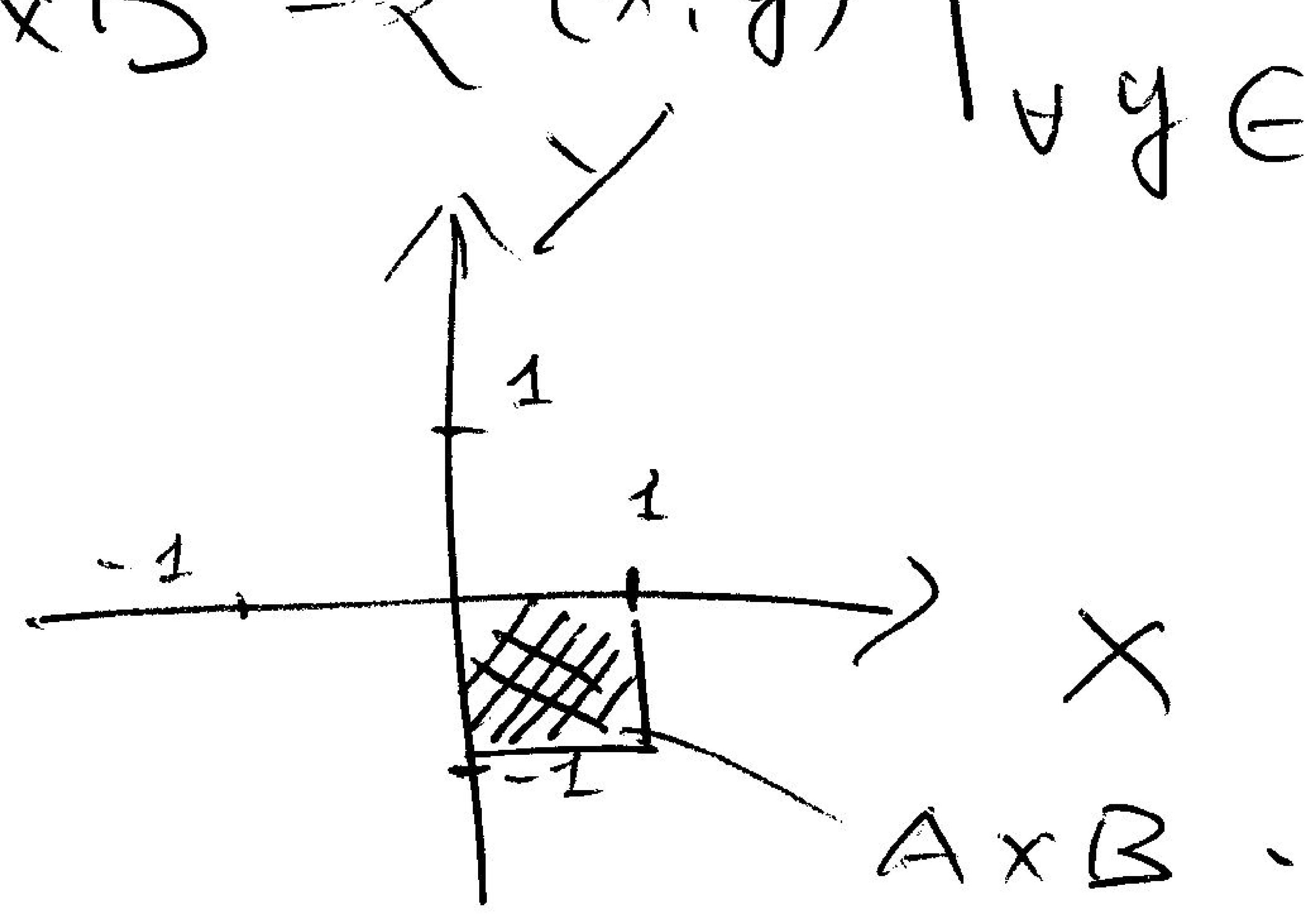
e.g.: $A=\{1,2,3\}$, $B=\{4,5\}$.

$$A\times B = \left\{\begin{matrix}(1,4), & (1,5) \\ (2,4), & (2,5) \\ (3,4), & (3,5)\end{matrix}\right\}.$$

e.g. $[0,1] = \{x\in\mathbb{R} \mid 0\le x\le 1\} = A$

$[-1,0] = \{y\in\mathbb{R} \mid -1\le y\le 0\} = B.$

$$A\times B = \left\{(x,y) \;\middle|\; \begin{matrix}\forall x\in[0,1] \\ \forall y\in[-1,0]\end{matrix}\right\}.$$



Compiti:

Leggere il libro:

L.M.D.

1. Capt. 1. tutto.

2. Eser. capt. 1. tranne 1.19

Da eser 1.1 — 1,18.