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Week 3_Tutorial_Introduction to Probability

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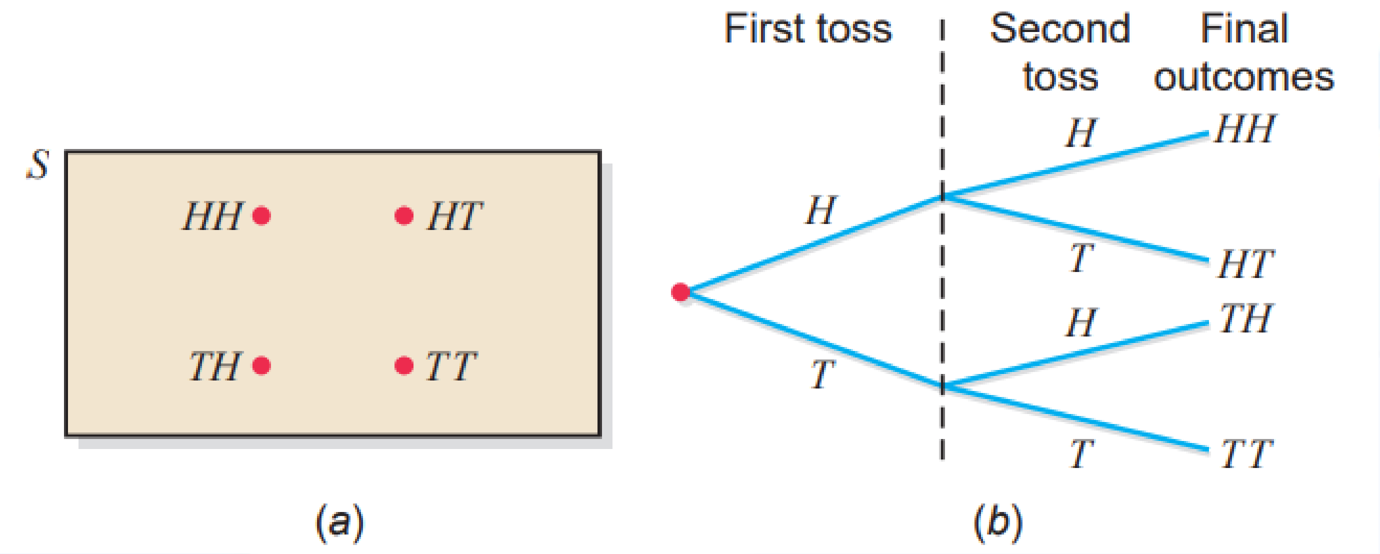
Experiments, Outcomes ,Sample space

Experiment	Outcomes	Sample Space
Toss a coin once	Head, Tail	$S = \{\text{Head, Tail}\}$
Roll a die once	1, 2, 3, 4, 5, 6	$S = \{1, 2, 3, 4, 5, 6\}$
Toss a coin twice	*HH*, *HT*, *TH*, *TT*	$S = \{HH, HT, TH, TT\}$
Play lottery	Win, Lose	$S = \{\text{Win, Lose}\}$
Take a test	Pass, Fail	$S = \{\text{Pass, Fail}\}$
Select a worker	Male, Female	$S = \{\text{Male, Female}\}$

Experiment of tossing a coin twice

- Both the the Venn and tree diagrams can show the sample space for this experiment.



- S= {HH, HT, TH, TT}

Example

- The probability of getting two heads when two unbiased coins are tossed is:

Answer =1/4 or 0.25

Explanation:

Sample space when two coins are tossed

= (H,H), (H,T), (T,H), (T,T)

n(S) = 4

The event "E" of getting two heads (H,H) = 1

So, the probability of getting two tails,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

Example…..

- Suppose you throw two dice, what is the probability of getting a sum of 5?

		White Die					
		1	2	3	4	5	6
Red Die	1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
	3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
	5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
	6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

Sample space: S=36

P(sum of 5)= $\frac{4}{36} = \frac{1}{9}$

Tree diagrams

- A tree diagram is a special type of graph used to determine the outcomes of an experiment.
- It consists of "branches" that are labelled with either frequencies or probabilities.
- Tree diagrams make probability problems easier to visualize and solve.
- Random sampling from the sample space can be done with replacement or without replacement.

Example , Sampling with replacement

- In an urn, there are 11 balls. Three balls are red (R), and eight balls are blue (B). Draw two balls, one at a time, with replacement.

What would be the probabilities at each branch?

- Note: "With replacement" means that you put the first ball back in the urn before you select the second ball.

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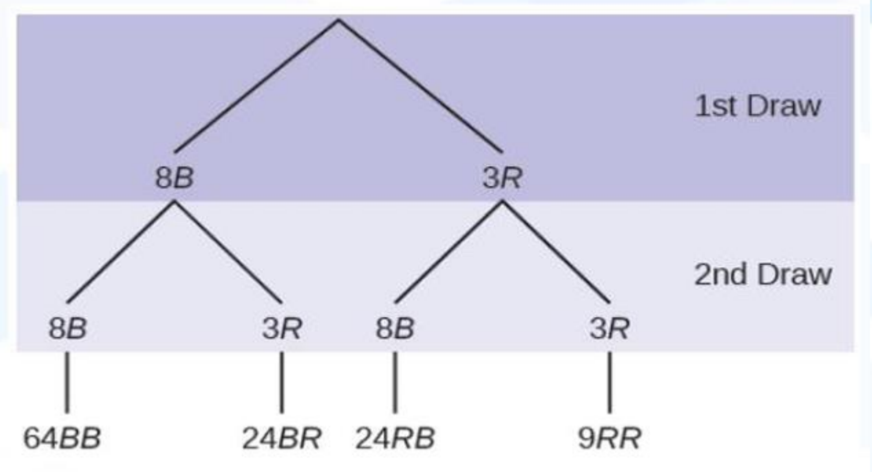
Example , Sampling with replacement cont…….

- Using the tree diagram, calculate

(i) P(RR)

(ii) P(RB OR BR)



Solution:

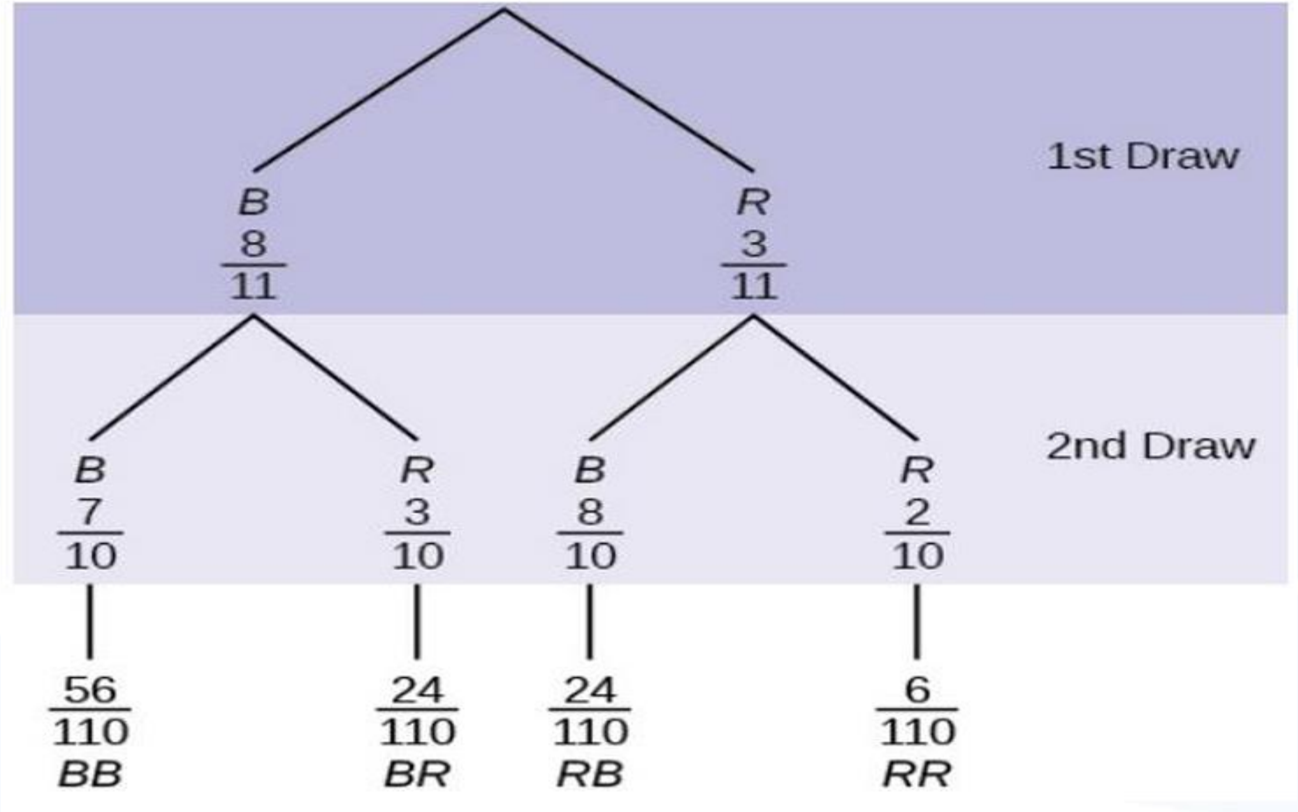
$$P(RR) = \left(\frac{3}{11}\right)\left(\frac{3}{11}\right) = \frac{9}{121}$$

$$P(RB \text{ OR } BR) = \left(\frac{3}{11}\right)\left(\frac{8}{11}\right) + \left(\frac{8}{11}\right)\left(\frac{3}{11}\right) = \frac{48}{121}$$

Example: Sampling without replacement

- An urn has three red marbles and eight blue marbles in it. Draw two marbles, one at a time, this time without replacement, from the urn. Construct a tree diagram for the above scenario
- Note: "Without replacement" means that you do not put the first ball back before you select the second marble.

Example: Sampling without replacement cont…..



Example: Sampling without replacement cont…..

- Calculate the following probabilities using the tree diagram.

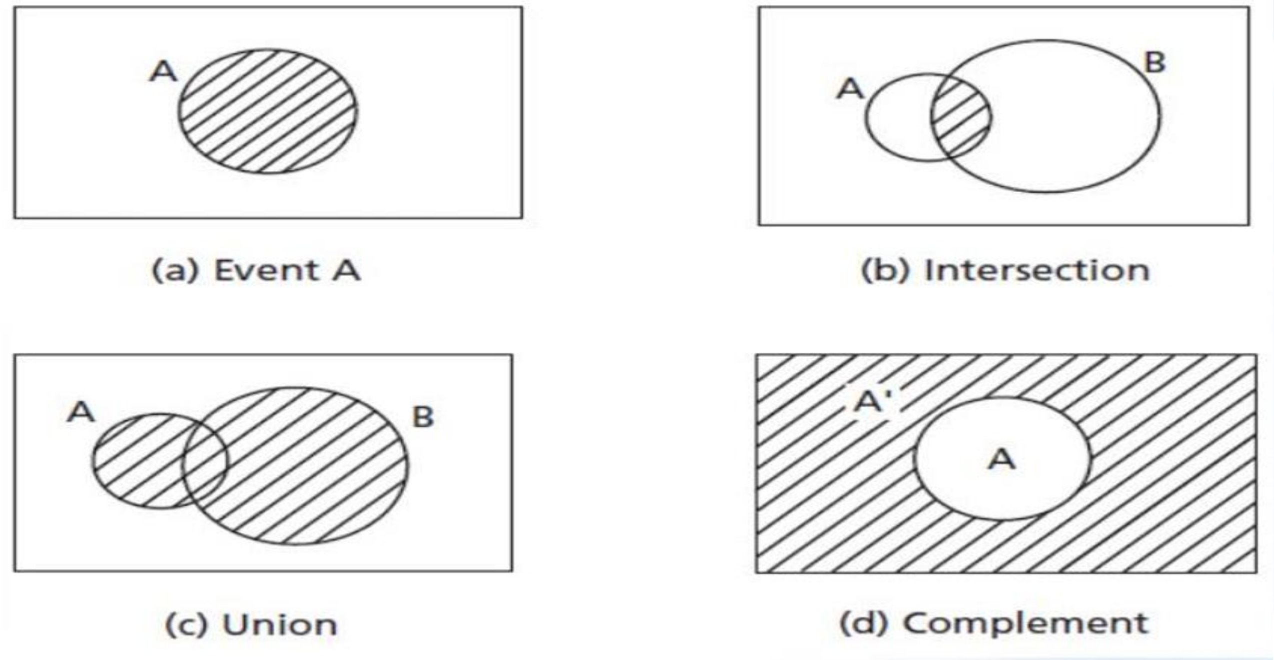
a) P(RR)……………..Ans= $\frac{6}{110}$

b) P(RB OR BR)………..Ans= $\frac{48}{110}$

c) P(BB)…………………Ans= $\frac{56}{110}$

Venn diagrams explained……..

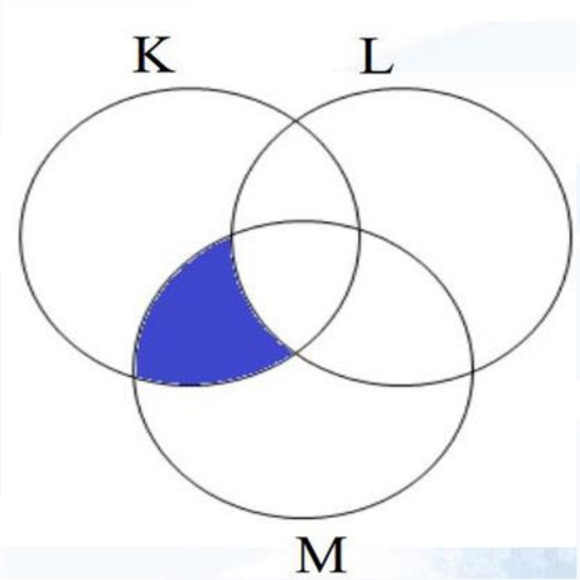
- A Venn diagram is a picture that represents the outcomes of an experiment.
- The sample space is represented by a square and the events are represented by circles.
- How to use Venn diagrams



Interpreting Venn diagrams………….

- **N:B,** shorthand for "AND" is ∩ and shorthand for "OR" is ∪.

Question: Identify the correct interpretation of the Venn diagram.



Answer = b)

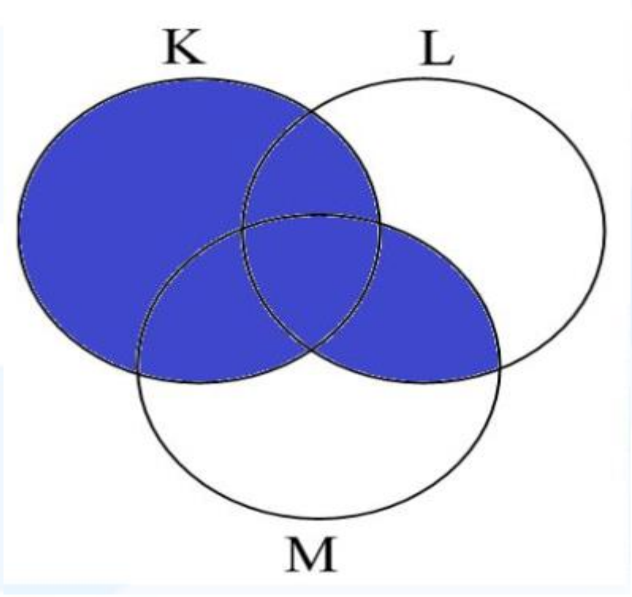
a) $K \cap L \cap M'$

b) $K \cap L' \cap M$

c) $K' \cap L \cap M$

Interpreting Venn diagrams………….

Question: Identify the correct interpretation of the Venn diagram.



Answer = c)

a) $K \cap (L \cup M)$

b) $K \cup L' \cup M'$

c) $\boxed{K \cup (L \cap M)}$

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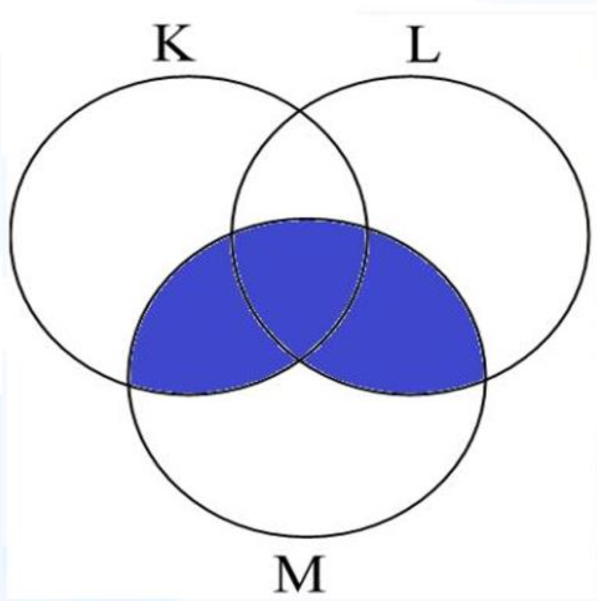
Interpreting Venn diagrams………….

Question: Identify the correct interpretation of the Venn diagram.



Answer = c)

a) $(K \cap L \cap M) - (K \cap L)$

b) $(K \cap L \cap M) \cup M$

c) $\boxed{(K \cup L) \cap M}$

Venn diagrams

Example:

Suppose an experiment has the outcomes 1, 2, 3, ... , 12 where each outcome has an equal chance of occurring.

Let event A = {1, 2, 3, 4, 5, 6} event B = {6, 7, 8, 9}.

1) Draw a Venn diagram showing the sample space and events.
2) Find, i) (A AND B)

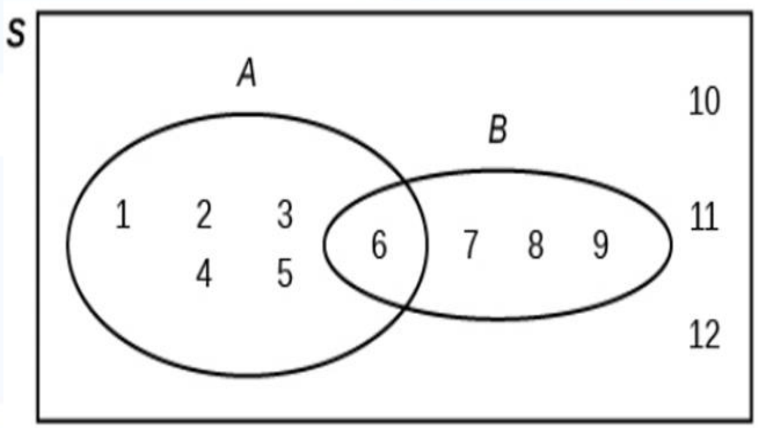
   ii) (A OR B)

Solution: Venn Diagram

- Given event A = {1, 2, 3, 4, 5, 6}

  event B = {6, 7, 8, 9}.

  1)



2i) A AND B = {6}

2ii) A OR B = {1, 2, 3, 4, 5, 6, 7,8, 9}.

Example: Venn Diagrams

- It is known that 30% of a certain company's washing machines require service while under warranty, whereas only 10% of its dryers need such service. Assume that the two machines function independently of each other.
- a) If someone purchases both a washer and a dryer made by this company, what is the probability that both machines need warranty service?

Solution: Venn Diagrams

- Let event A = washer needs service under warranty,
- Let event B = dryer needs service under warranty

a)

We are given that P(A) = 0.30 and P(B) = 0.10

For independent events:

$$P(A\ AND\ B) = P(A) \times P(B)$$

$$P(A\ AND\ B) = 0.3 \times 0.1$$

$$P(A\ AND\ B) = 0.03$$

Solution: Venn Diagrams

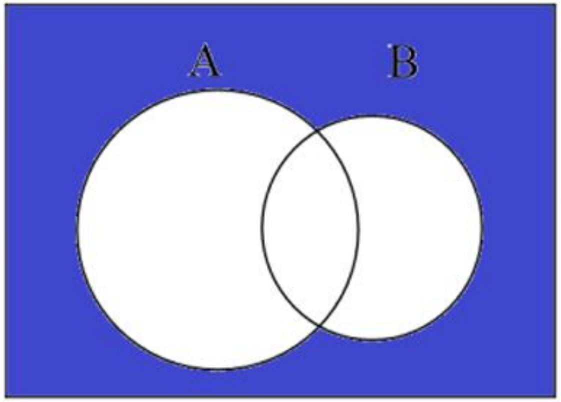
b) What is the probability that both machines will not require service during the warranty period?

Let A = washer needs service under warranty,

Let B = dryer needs service under warranty.

$$(A \cup B)' = (A' \cap B')$$

$$\begin{aligned} P(A' \text{ AND } B') &= P(A') \times P(B') \\ &= 0.7 \times 0.9 \\ &= 0.63 \end{aligned}$$



Question: Combinations

- A student is generating the tree diagram for a coin tossed 10 times.

(i) What is the total number of outcomes in the sample space?

(ii) How many of the outcomes would have exactly 4 heads?

(iii) What is the probability of having exactly 4 heads?

(iv) How many outcomes constitute the event “not more than 2 heads”?

Solution.

(i) S = 2^{10} = 1024

(ii) $10\,C_4 = \frac{10!}{6!\times 4!} = \ \ 210$

(iii) P(4 Heads) = $\frac{210}{1024}$ = 0.205

(iv) P($\leq 2\ heads) = \mathrm{P}\ (0) + \mathrm{P}(1) + \mathrm{P}(2)$

= $10C_0$ + $10C_1$ + $10C_2$

= 1+10+45 = 56

Question:

- A box contains 15 switches, and 4 of the switches are known to be bad. A technician picked four switches randomly. What is the probability that:

(i) 2 of the switches picked are bad

(ii) 3 of the switches are good

(iii) All switches picked are good

Solution:

$$(i)\ \frac{11_{C_2} + 4C_2}{15_{C_4}} = \frac{\frac{11!}{9!\times 2!} \times \frac{4!}{2!\times 2!}}{\frac{15!}{11!\times 4!}} = 0.241$$

$$(ii)\frac{11_{C_3} \times 4C_1}{15_{C_4}} = \frac{\frac{11!}{8!\times 3!} \times \frac{4!}{3!\times 1!}}{\frac{15!}{11!\times 4!}} = 0.4835$$

$$\text{(iii)}\ \frac{11C_4 \times 4C_0}{15_{C_4}} = \frac{\frac{11!}{7!\times 4!} \times \frac{4!}{4!\times 0!}}{\frac{15!}{11!\times 4!}} = 0.242$$

Thank you!

Happy studying ☺