UNIVERSITY OF PRETORIA

Week 4: Tutorial

UNIVERSITEIT VAN PRETORIA
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YUNIBESITHI YA PRETORIA
Denkleiers • Leading Minds • Dikgopolo tša Dihlalefi

Independent and mutually exclusive events

- Two events A and B are **independent** if the knowledge that one occurred does not affect the chance the other occurs.
- A and B are **mutually exclusive** events if they cannot occur at the same time. This means that A and B do not share any outcomes and $P(A \text{ AND } B) = 0$.
- The following is true for independent events:

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

$$P(A \text{ AND } B) = P(A) \times P(B)$$

- Events are independent if they satisfy any one of the above conditions

Question 1

Using the events and given probabilities of C and D,

C = taking an English class; D = taking a speech class.

Suppose $P(C) = 0.75$, $P(D) = 0.3$, $P(C|D) = 0.75$ and

$P(C \text{ AND } D) = 0.225$.

Justify your answers to the following questions numerically.

a. Are C and D independent?

b. Are C and D mutually exclusive?

c. What is $P(D|C)$?

Solution Question 1

a) Yes, because $P(C|D) = P(C)$.

b) No, because $P(C \text{ AND } D)$ is not equal to zero.

c) $P(D|C) = \frac{P(D\ AND\ C)}{P(C)}$ = P(D) **(for independent events)**

$$= \frac{0.225}{0.75} = 0.3$$

Contingency tables for probability

- A table in a matrix format that displays the aggregated data of variables.

- The table helps in determining probabilities and conditional probabilities from summarised data.

Question 2

- The table shows a random sample of 200 cyclists and the routes they prefer. Let M = males and H = hilly path.

Gender	Lake Path	Hilly Path	Wooded Path	Total
Female	45	38	27	110
Male	26	52	12	90
Total	71	90	39	200

a) Out of the males, what is the probability that the cyclist prefers a hilly path?

b) Are the events “being male” and “preferring the hilly path” independent events?

Solution; Question 2

a) Out of the males, what is the probability that the cyclist prefers a hilly path?

Gender	Lake Path	Hilly Path	Wooded Path	Total
Female	45	38	27	110
Male	26	52	12	90
Total	71	90	39	200

Event H = cyclist prefers hilly path and

Event M= cyclist is male

P(H/M)= $\frac{P(H \cap M)}{P(M)} = 52/90$

= 0.5778

Solution: Part 2b)

b) Are the events “being male” and “preferring the hilly path” independent events?

- Let H = cyclist prefers hilly path
- M = cyclist is male
- Given P(H|M) = 0.5778
- Which condition must we test?
- For independent events:
  - ✓ P(H/M)=P(H)
  - ✓ P(M/H)=P(M)
  - ✓ P(H and M)= P(H).P(M)

P(H/M) = P(H) , but P(H)=(90 /200) = 0.45

But Earlier calculated value P(H|M) = 0.5778 Event A and B are not independent events

Conditional probability

- Conditional probability for two events A and B
- The probability that event A will occur given that the event B has already occurred.
- Written as P(A|B)

$$P(A|B) = \frac{P(A \text{ } AND \text{ } B)}{P(B)}$$

- where P(B) is greater than zero.
- A conditional probability reduces the sample space

Question 3

Forty percent of the students at a local college belong to a club and 50% work part time. Five percent of the students work part time and belong to a club.

a) Draw a Venn diagram showing the relationships. Let C = student belongs to a club and PT = student works part time

b) Find the probability that the student belongs to a club given that the student works part time

c) The probability that the student belongs to a club OR works part time.

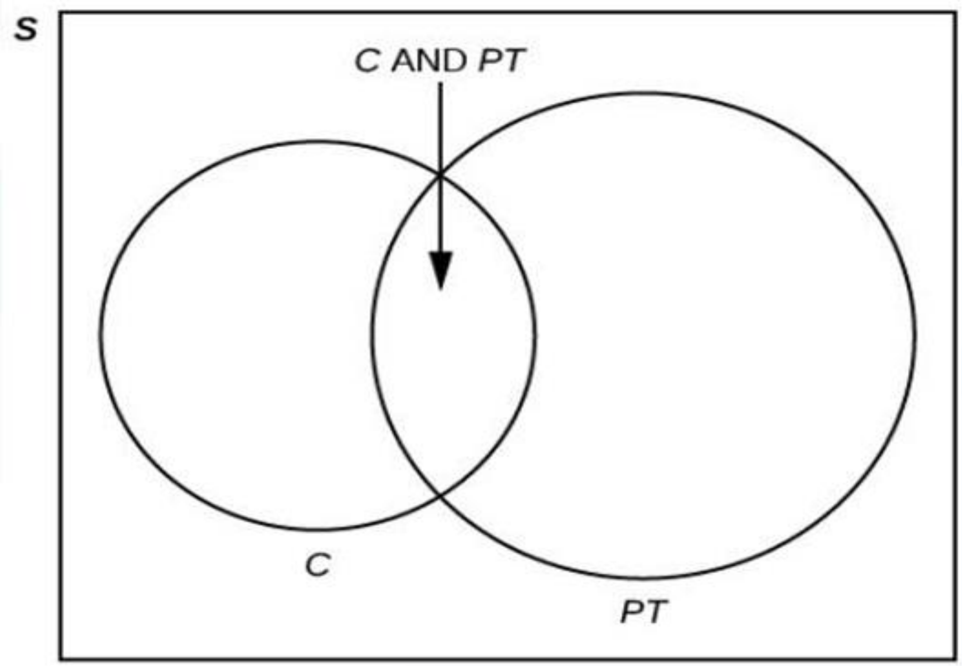
Solution: Question 3

- Draw the Venn diagram showing the relationships
- Let C = student belongs to a club PT = student works part time.

  P(C)= 0.4

  P(PT)=0.5

  P(C AND PT)=0.05



Solution: Part b)

a) Find the probability that the student belongs to a club given that the student works part time

- Conditional probability

$$P(C/PT) = P\frac{(C\ AND\ PT)}{P(PT)}$$

=0.05/50

=0.1

b) The probability that the student belongs to a club **OR** works part time.

$$P(C\ OR\ PT) = P(C) + P(PT) - P(CAND\ PT)$$

= 0.40 + 0.50 - 0.05 = 0.85

Question 4

A chain of video stores sells three different brands of DVD players. Of its DVD player sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's DVD players require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

a) What is the probability that a randomly selected purchaser has bought a brand 1 DVD player that will need repair while under warranty?

b) What is the probability that a randomly selected purchaser has a DVD player that will need repair while under warranty?

Question 4

- A chain of video stores sells three different brands of DVD players. Of its DVD player sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's DVD players require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

Let Ai = Brand i is selected;

B = Player needs repair

Given:

P(A1) = 0.5	P(B|A1) = 0.25
P(A2) = 0.3	P(B|A2 = 0.2
P(A3) = 0.2	P(B|A3) = 0.

Solution: Question 4

Let A_i = Brand i is selected; B = Player needs repair

Given:

P(A1) = 0.5

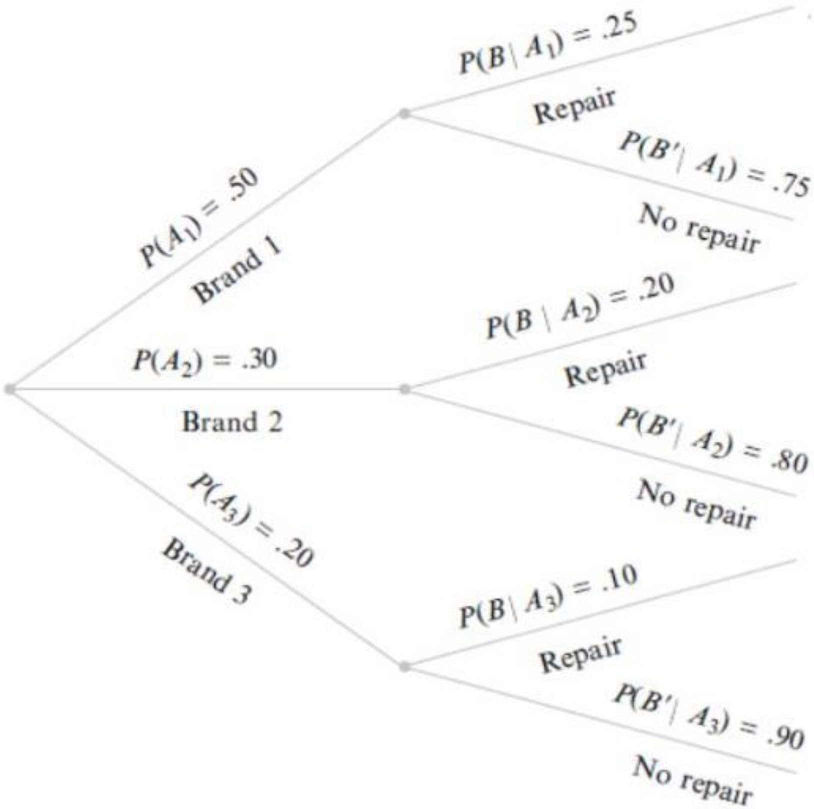
P(A2) = 0.3

P(A3) = 0.3

P(B|A1) = 0.25

P(B|A2) = 0.2

P(B|A3) = 0.1



$P(A_1 \cap B) = 0.125$

$P(A_1 \cap B') = 0.375$

$P(A_2 \cap B) = 0.06$

$P(A_2 \cap B') = 0.24$

$P(A_3 \cap B) = 0.02$

$P(A_3 \cap B') = 0.18$

Solution: Question 4

a) What is the probability that a randomly selected purchaser has bought a brand 1 DVD player that will need repair while under warranty?

Ans = 0.125

b) What is the probability that a randomly selected purchaser has a DVD player that will need repair while under warranty?

Ans= 0.125+0.06+0.02= 0.205

Mean and variance of continuous random variables.

- If (x), is a continuous random variable with pdf $f(x)$,
- Then the Expected value or mean of (x) is given by

$$\mu = \mu X = E[X] = \int_{-\infty}^{\infty} x \cdot f(x)dx.$$

$$Var(X) = E[x^2] - \mu^2 = \left(\int_{-\infty}^{\infty} x^2 \cdot f(x)dx\right) - \mu^2$$

Question 5

The number of potholes fixed per month in Hatfield by the municipality is a random variable with the probability density function given by

$$f(x) = \frac{x(x-1)}{270}; \;\; 4 \leq x \leq 10.$$

1. Determine the mean and standard deviation of the number of potholes fixed per month in Pretoria?

Solution: Question 5

- Given $f(x) = \frac{x(x-1)}{270}$; $4 \leq x \leq 10.$

- $Mean \;=\; \int_4^{10} (\frac{x^2-x}{270}).x$

$$= \frac{1}{270} \int_4^{10} (x^3 - x^2)$$

$$= \frac{1}{270} \left[\frac{x^4}{4} - \frac{x^3}{3} \right]_4^{10}$$

= 7.88 potholes

Solution continued…….

- Given $f(x) = \frac{x(x-1)}{270}$; $4 \le x \le 10.$

- The variance = $\frac{1}{270}\int_4^{10}(x^2 - x)x^2 \; - \; 7.88^2$

$$= \frac{1}{270}\left[\frac{x^5}{5} - \frac{x^4}{4}\right]_4^{10} - \; 7.88^2$$

$$= \frac{1}{270}\left[\left[\frac{10^5}{5} - \frac{10^4}{4}\right] - \left[\frac{4^5}{5} - \frac{4^4}{4}\right]\right] - \; 7.88^2$$

= 3.645 potholes

- Standard deviation = $\sqrt{3.645}$

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Thank you!

Happy studying ☺