COSC1107/1105 - Computing Theory
Assignment 1 (15%)

Total marks: 73

Deadline: Sunday August 18th 2019, 11:59 PM

Please read all the following information before attempting your assignment. This is an *individual* assignment. You may not collude with any other individual, or plagiarise their work. Students are expected to present the results of their own thinking and writing. Never copy another student's work (even if they "explain it to you first") and never give your written work to others. Keep any conversation high-level and never show your solution to others. Never copy from the web or any other resource. You are meant to generate the solution to the questions *by yourself*. Suspected collusion or plagiarism will be dealt with according to RMIT policy.

Submission Instructions

- You are to **submit** all your files inside a .zip file and & **certify your submission** using this Google Form:

  http://tinyurl.com/ct19-ass1-sub

- Answers to questions must be submitted in individual text or JFLAP files. The exact file names for all files must be used (including exact capitalization and file extension), as indicated in each question. All such files must be put inside a *single zip file* called `<your student number>.zip`. For example, `3234212.zip` In total, this zip file should contain 25 `.txt` and 5 `.jff` files.
- Place all files in the *root* of the zip file; *do not* place the files in a folder inside your submission file.
- Files with extension `.txt` must be text ASCII/UTF-8 files (not Word or any other format). If in doubt, please check the type of your files, for example using `file` command in Unix-based systems. Confirm it is in ASCII or UTF-8 format, and not in other formats like Little-endian UTF-16 or Rich Text Format.
- When requested to submit a file with one or more words/strings (e.g., regexp or words), you should submit a *file with each string on a new line, and nothing else.* Do not include your name, quotes, dummy lines, or any other information or character (e.g., do not type "`w1=abc`", but just "`abc`" without quotes).
- When asked to submit a "`.jff`" file, it should be in proper JFLAP format. Your JFLAP files *must* run error-free in **version 7** of JFLAP. It is your responsibility to test your files in JFLAP 7 before submitting.
- Question 5 is a *bonus* question. You can still get full marks without answering this question.

Submissions not compatible with these instructions will attract zero marks and do not warrant a re-submission.

Corrections: From time to time, students or staff find errors in the assignment specification. In that case, a corrected version will be uploaded to the course web page as quickly as possible, an announcement will be made on the course web page as well as in the forum. Check the version date on the bottom left.

Forum postings on assignment: **Never** post any information on the forum that may disclose how to solve a question or what the solution may be. You may only post assignment related questions for *clarification*, for example, to clarify certain notation. Any post discussing possible solutions or strategies may directly be considered plagiarism, see above. **If in doubt, do not post** and ask your tutor the question instead.

Silence Policy: A silence policy will take effect 48hrs before this assignment is due. This means no questions about this assignment will be answered, whether they are asked on the discussion board, by email, or in person.

Late Submissions/Extensions: A penalty of 10% per day is applied to late submissions up to 5 days, after which you will lose ALL the assignment marks. Extensions will be given only in exceptional cases; please refer to Special Consideration process. Special Considerations given after solutions have been released (between 1 and 2 weeks after deadline) will automatically result in an **equivalent assessment in the form of a test**, assessing the same knowledge and skills of the assignment (location and time to be arranged by the instructor).

Exercise 1: Regular Expressions . . . . . **20 marks**

Regular Expressions Syntax: The + operator in regular expressions is used in some books to denote one or more applications of Kleene star. However, in other places, such as JFLAP, + denotes the alternation operator, equivalent to |. For the purpose of this assignment, we follow JFLAP and use operator + to denote an alternation. In JFLAP, you can use λ or ϵ to denote the empty string (see Preferences menu).

For parts (i) to (iii), please type each word on a new line. The notation $w_i \cdot w_j$ denotes concatenation of words w_i and w_j, and w^r denotes the word obtained by reversing w.

(a) Let $R_1 = 1(1^* + 2^*)3^*(2^* + 3)1^*2(1+3)^*2^*$ and $R_2 = 1^*3(2+3)^*2^*(1+3)^*(1+3)^*$ be two regular expressions. Whenever asked to provide words, these must be non-empty ones (and different if more than one).

i. (2 marks) `[E1-Qa1.txt]` Give two non-empty words w_1 and w_2 such that $\{w_1, w_2\} \subseteq L(R_1) \cap L(R_2)$.

ii. (2 marks) `[E1-Qa2.txt]` Give two non-empty words w_1 and w_2 such that $\{w_1, w_2\} \subseteq L(R_1) \setminus L(R_2)$.

iii. (2 marks) `[E1-Qa3.txt]` Give two non-empty words w_1 and w_2 such that $\{w_1, w_2\} \subseteq L(R_2) \setminus L(R_1)$.

iv. (1 mark) `[E1-Qa4.txt]` Give a non-empty word $w \in L(R_1)$ such that $w \cdot w^r \in L(R_1)$.

v. (1 mark) `[E1-Qa5.txt]` Give a non-empty word $w \in L(R_1)$ such that $w \cdot w^r \notin L(R_1)$.

vi. (1 mark) `[E1-Qa6.txt]` Give a regular expression R such that $L(R) = L(R_1) \cup L(R_2)$.

vii. (3 marks) `[E1-Qa7.txt]` Give a regular expression R such that $L(R) = L(R_1) \cap L(R_2)$.

(b) Give regular expressions for the following languages.

i. (2 marks) `[E1-Qb1.txt]` $L_1 = \{a^{3n+2}b^{m+1} \mid n \geq 1, m \geq 0, m \bmod 2 = 1\}$.

ii. (2 marks) `[E1-Qb2.txt]` $L_2 = \overline{L_1}$, where $\overline{L}$ is the complement of L (assume alphabet $\Sigma = \{a, b\}$).

iii. (2 marks) `[E1-Qb3.txt]` $L = \{w \mid w \in \{a,b\}^*, |w| \bmod 3 = 2\}$, where $|w|$ denotes the length of w.

iv. (2 marks) `[E1-Qb4.txt]`
$L = \{b^n w_1 \mid n > 1, w_1 \in (\{a,c\}^* \cap \{a,b,c\}^*) \setminus (\{b\}^* \cup \{c\}))\} \cap \{w_2 c \mid w_2 \in \{a,b,c\}^*\}$.

Exercise 2: Grammars . . . . . **18 marks**

(a) Provide regular expressions for the following languages.

i. (3 marks) `[E2-Qa1.txt]` Give a regular expression R such that $L(R) = L(G_1)$, where G_1 is:

$$
\begin{aligned}
S &\rightarrow AcB \\
A &\rightarrow ac \mid bC \mid \epsilon \\
B &\rightarrow baB \mid caB \mid D \\
C &\rightarrow bC \mid \epsilon \\
D &\rightarrow aD \mid b
\end{aligned}
$$

ii. (3 marks) `[E2-Qa2.txt]` Give a regular expression R such that $L(R) = L(G_2)$, where G_2 is:

$$
\begin{aligned}
S &\rightarrow ACS \mid \epsilon \\
A &\rightarrow aA \mid bA \mid bB \\
B &\rightarrow cB \mid \epsilon \\
C &\rightarrow bcC \mid acC \mid D \\
D &\rightarrow bD \mid \epsilon
\end{aligned}
$$

iii. (2 marks) `[E2-Qa3.txt]` Give a regular expression R such that $L(R) = L(G_1) \cup L(G_2)$.

iv. (2 points (bonus)) `[E2-Qa4.txt]` Give a regular expression R such that $L(R) = L(G_1) \cap L(G_2)$.

(b) Let $G_3 = (\{S\}, \{a, b\}, \Gamma, S)$ be a grammar, where the set of rules Γ is defined as follows:

$$
\begin{aligned}
S &\rightarrow aSbSa \\
S &\rightarrow bSbSa \\
S &\rightarrow aSbSb \\
S &\rightarrow \epsilon
\end{aligned}
$$

i. (4 marks) `[E2-Qb1.txt]` Does there exist a regular expression R such that $L(R) = L(G_3)$? If it exists, provide such R; otherwise, simply put 0.

(c) Let $L = \{c^{2n-1}a^m c^3 b^{2m+1}a^n \mid n, m > 0\}$.

i. (3 marks) [E2-Qc1.txt] Complete the following *context-free* grammar G such such $L(G) = L$.

$$S \rightarrow ccSa \mid < \text{missing string} >$$
$$A \rightarrow aAbb \mid aBbb$$
$$B \rightarrow ccc$$

Provide the missing string as a single line in the given text file (e.g., if your response is $xSSy$, submit a file with a single line containing string "xSSy" (of course, without the quotes)).

ii. (3 marks) [E2-Qc2.txt] Does there exist a regular expression, DFA, regular grammar or PDA over the alphabet $\Sigma = \{a, b, c\}$ which is equivalent to the language L? (Answer the following question as a string of bits that translate to 1 for *yes* and 0 for *no*. For example, if your answer is "no, no, yes, no" give your response as 0010).

Exercise 3: Automata . . . . . **25 marks**

(a) Answer the following questions based on the finite state automaton M_1 present in the JFLAP file FA-3.a.jff available in Assessments section of the course website. Assume alphabet $\Sigma = \{a, b, c, d, e, f\}$.

i. (2 marks) [E3-Qa1.txt] Give four strings of length 4 accepted by M_1. Please type each string on a new line.

ii. (2 marks) [E3-Qa2.txt] Give four strings of length 4 rejected by M_1. Please type each string on a new line.

iii. (4 marks) [E3-Qa3.txt] Give the language of this machine M_1 as a regular expression.

iv. (2 marks) [E3-Qa4.jff] Remove any redundant states from M_1 and adjust the transitions accordingly. Give your answer as a .jff JFLAP file.

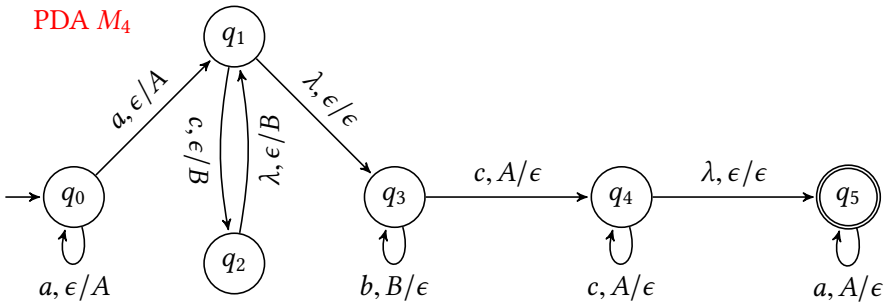
v. (4 marks) [E3-Qa5.jff] Create an automaton M_2 (deterministic or non-deterministic) such that it accepts the language L_1 where $L_1 = L(M_1) \cap L\big((aca + aba + bab)^*(a(ba)^* + c^*a)\big)$. Your machine should *not* accept words that are *not* in this language. Give your answer in a .jff JFLAP file.

(b) Let $L_1 = \{w \mid w \in \{0, 1\}^*, w \text{ does not contain the substring } 101101\}$.

i. (4 marks) [E3-Qb1.jff] Give the DFA M_3 where $L(M_3) = L_1$.

ii. (3 marks) [E3-Qb2.txt] Give the regular expression R such that $L(R) = L_1$.

(c) Consider the pushdown automaton M_4 over the alphabet $\Sigma = \{a, b, c\}$ as shown below.



Notation $x, A/X$ means a transition where x is the input symbol being read, A is the symbol on top of the stack that is popped, and X is the symbol pushed onto the stack. Here, λ stands for the "empty" input and ϵ stands for the "empty" stack symbol. Acceptance is by final state *and* empty stack.

i. (2 marks) [E3-Qc1.txt] Give 4 strings of length 11 over Σ that are accepted by PDA M_4. Remember to type each string on a new line.

ii. (2 marks) [E3-Qc2.txt] Give 4 strings of length 11 over Σ that are rejected by PDA M_4. Remember to type each string on a new line.

Exercise 2 continues on the next page...

Exercise 4: Telephone System Design . . . . . **10 marks**

Answer the following questions by submitting a JFLAP file for each.

(a) (5 marks) [E4-Qa1.jff] You have been tasked to design part of the internal phone system for a company that is expanding its operations from Melbourne to include a new Sydney office. Someone else in your the team will handle the internal routing of the calls; your job is to design a *deterministic* finite state machine that takes a number as input and determines whether it is valid (or should be rejected). The input to the system is a string composed in the following way:

- The first part is optional and carries the *area code* of the office you want to be connected to:
  - Melbourne is 03.
  - Syndey is 02.
- The next 2 digits are 94 (regardless of the city).
- The next 3 digits determine the *department* you want to be connected to:
  - Marketing is 555.
  - R&D is 528.
  - Sales is 552.
- The final three (meaningful) digits indicate the *personal extension* of whoever you are calling:
  - Extensions are all in the range 000-059.

Besides the above format for numbers, your automaton must adhere to the following rules:

- As with all phone numbers, you should be able to enter any digits after the personal extension digits and still be connected, however *you must end your string with a # symbol* to tell the system the input is complete; otherwise the whole number is deemed invalid.
- The area code is also optional. If you are in Melbourne, you should not have to dial 03 to connect to the Melbourne office, so your automaton should accept numbers *with* an area code and numbers *without* an area code. For example, to be connected to extension 029 in the Melbourne sales department you would dial 0394552029#. However, 94552029#, 0394552029123456# and 94552029123456# should also be accepted. Your string must terminate with a # symbol, so ***you cannot have 94552029 or 94552029123456, nor can you have 94552029#123678***.
- Finally, as the Sydney office is just being set up, they do not have an R&D department yet, any phone call directed explicitly to the Sydney R&D apartment should also be rejected. If there is no area code you do not need to worry about this.

Any number that does not fit the pattern, as explained above, is invalid (and should be rejected).

(b) (5 marks) [E4-Qb1.jff] Several months later your company is again contracted to make some modifications to the system. This time they want exactly the same functionality, however now they would like the user to be able to *enter their customer number after the # symbol*. The only catch is that the customer numbers have been designed by an ex-Computing Theory student and are needlessly complicated! The numbers are split up into two parts *of equal length*; with the first part being composed of the digits 0-4 and the second part composed of the digits 5-9. These parts can be any length, but they *must* be of equal size (and they must exist!). The rest of the machine should function exactly as before. The company asked you for the *simplest type of automaton possible* as they want to save money. Remember that whenever we use PDAs, the acceptance condition used is *both* final state *and* empty stack.

To use a similar example as before, if your customer number was 123678 and you wanted to be connected to extension 029 in the Melbourne sales department you would dial 0394552029#123678. However, 94552029#123678, 0394552029123456#123678 and 94552029123456#123678 should also be accepted.

Exercise 3 continues on the next page…

Exercise 5 (Bonus): Intersection of Regular Languages . . . . . 5 bonus marks

`[E5-Qa1.txt]` Find the regular expression R_3 such that $L(R_3) = L(R_1) \cap L(R_2)$, where:

- $R_1 = (ca)^*a(c + b)^*(ab + ca)^*$; and
- $R_2 = ((b + c)^*a(c + a)(c + a)^*(c + a + b)^*)^*$.

— End of assignment paper —

Question	Points	Bonus Points
Regular Expressions	20	0
Grammars	18	2
Automata	25	0
Telephone System Design	10	0
Intersection of Regular Languages	0	5
Total:	73	7