Mesoscale Building Blocks of Pedestrian Mobility: a Discrete Vector Field Approach

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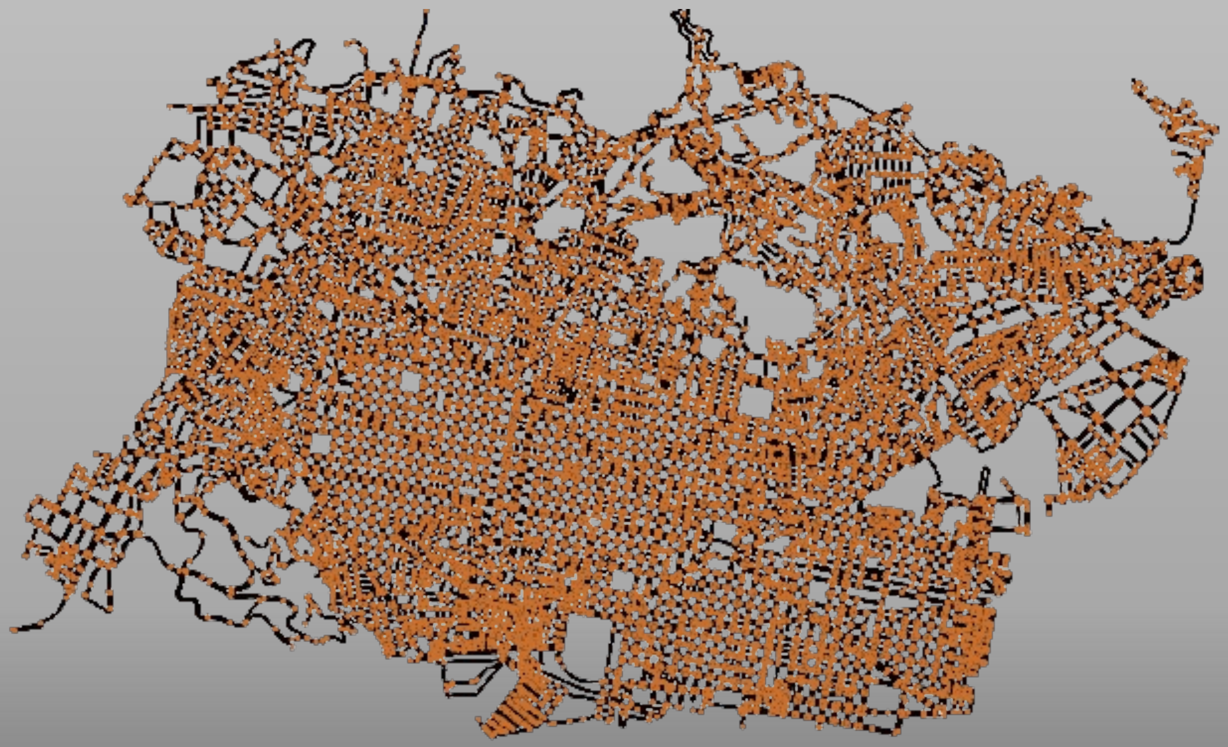

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Outline

- Motivation and Introduction to Mobility
- Studying Mobility: A Vector Field Approach
  - Continuous and Discrete Vector Fields
- Analysing Vector Fields: the Helmholtz-Hodge Decomposition (HHD)
- Random Walk Models for Pedestrian Behaviour
  - Discrete-time random walks (DTRW)
  - Continuous-time random walk models
- Results
  - Validity of the Deterministic Models
  - Attractive/repulsive behaviors
  - Origin of the cyclic components
- Conclusions

Introduction to Urban Mobility: Motivation

WHY URBAN MOBILITY?

- **Traffic forecasting**
- **Urban planning**
- **Sustainability**
- **Global health**

Introduction to Urban Mobility: Cities as a Complex System

Underlying Network/Infrastructure

Entity dynamics

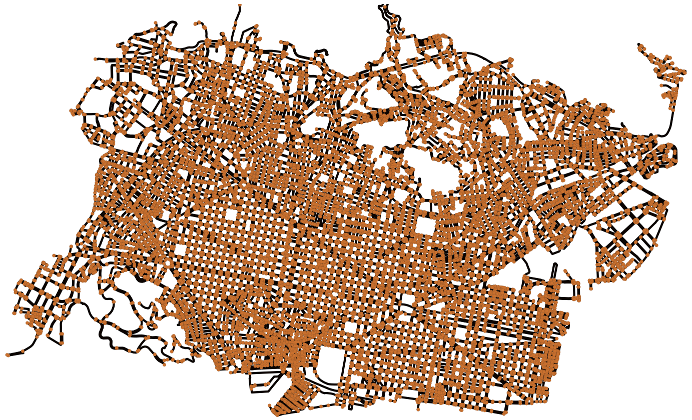
Many features of Complex Systems:

- Many interacting agents
- **Nonlinearity** in the interactions
- **Emergent Behaviour** (cultural trends, congestion, social dynamics)
- Etc.

Introduction to Urban Mobility: Objectives

We will focus on **pedestrian mobility** → Idealized **unbiased** random walk dynamics

Urban infrastructure and geometry



Sidewalk network of Barcelona

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Pedestrian dynamics

The **objective** is to see how the urban infrastructure and sidewalk network **geometry** affect pedestrian mobility:

- Appearance of **mesoscale patterns**?
- How are these patterns **structured**?
- Characteristic behaviours?

Simulating Pedestrian Behaviour: Mobility models

Different approaches have been presented over the years:

- Fluid dynamics
- Agent-based models (simulation software)
- Network-based models (betweenness)
- Random Walk (RW) approaches

The random walker jumps from node to node with a given **probability distribution**.

Markovian

- Memoryless
- Simple

Non-Markovian

- Memory
- Intricate dynamics

In this work:

Markovian → We want a **baseline** simple model

Unbiased → To remove artifacts from the dynamics and isolate the effect of the geometry of the network

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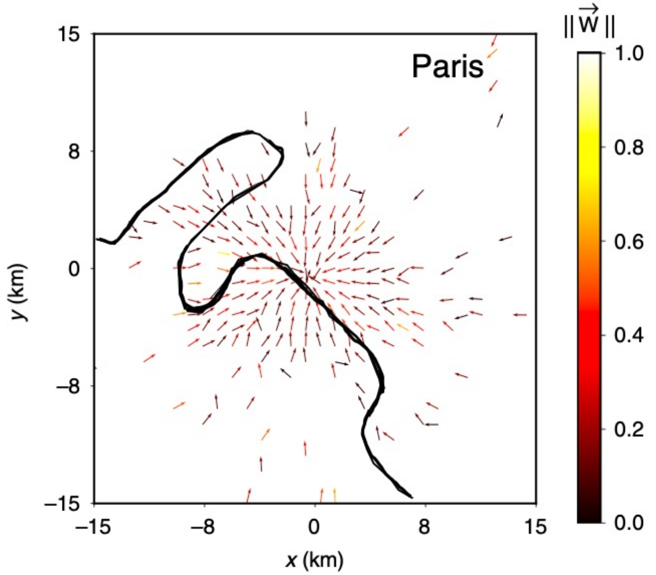
A Vector Field Approach to Study Mobility Flows

Human flows can be studied through **continuous** or **discrete vector fields:**

CONTINUOUS VECTOR FIELDS

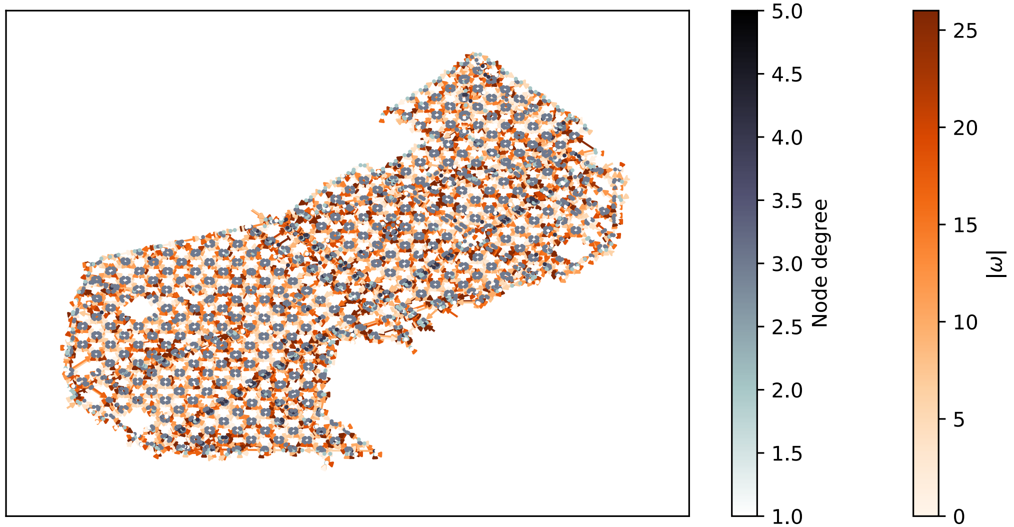
- An averaged field value and direction for every point in space
- Visual and understandable
- The city structure does not play a direct role
- Usually show trivial attractive patterns towards the city centre



Mazzoli, M., Molas, A., Bassolas, A. *et al.* Field theory for recurrent mobility. *Nat Commun* **10**, 3895 (2019). https://doi.org/10.1038/s41467-019-11841-2

DISCRETE VECTOR FIELDS

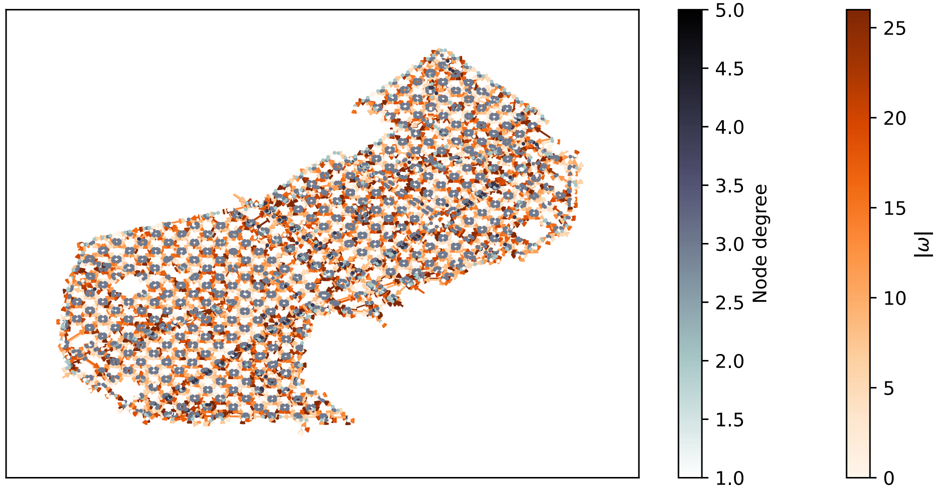
- Embedded in an **underlying structure** (**network**)
- **Graph representation**: a pedestrian flow value is assigned at each edge
- **Detailed view** of the flows at each street
- More difficult to interpret



Discrete vector field representing the random walk flow across the pedestrian network of l'Eixample district in Barcelona

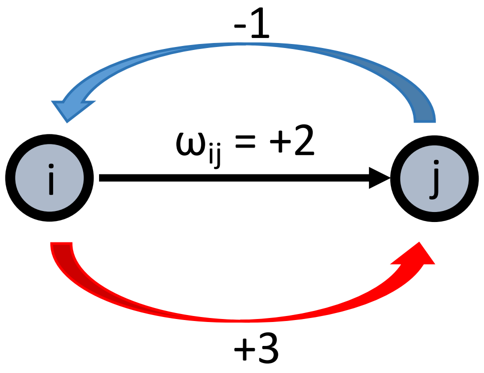
Obtaining the Discrete Vector Fields

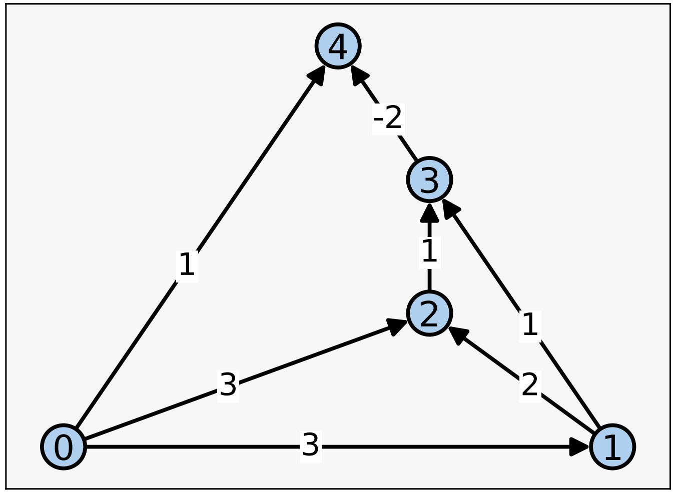
In this work we have obtained **discrete vector fields** from the **net random walk flow** on the edges:



Discrete vector field representing the random walk flow across the pedestrian network of l'Eixample district in Barcelona

Given an edge (i,j), we have calculated the total number of forwards minus backwards crossings during the simulation time, t_1, to obtain the net edge random walk flow during t_1.





Discrete vector field or **weighted graph**

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Analysing Vector Fields: the Helmholtz-Hodge Decomposition (HHD)

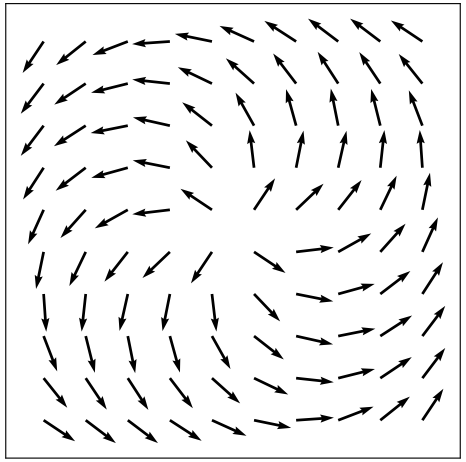
How do we find patterns in vector fields? ⇒ In **continuous** vector fields the **Helmholtz decomposition**

According to the **Helmholtz decomposition**, any well behaved continuous vector field can be decomposed into 2 orthogonal components:

$$\mathbf{v} = \boxed{-\nabla\phi} + \boxed{\nabla \times \mathbf{A}}$$

- **Gradient component**: Sinks & sources
- **Solenoidal component**: circulations

ORIGINAL VECTOR FIELD



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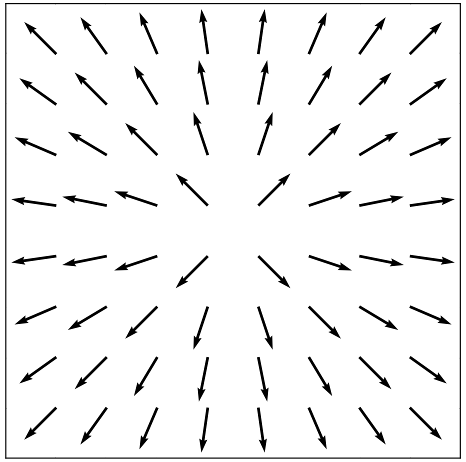
GRADIENT COMPONENT



Irrotational

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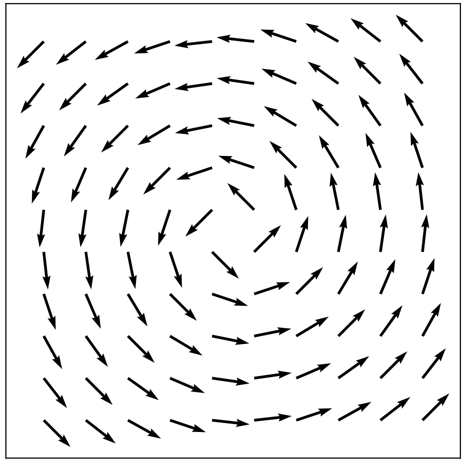
SOLENOIDAL COMPONENT



Incompressible

Where:

- ϕ : Scalar potential
- **A :** Vector potential

Analysing Vector Fields: the Helmholtz-Hodge Decomposition (HHD)

How do we find patterns in **discrete** vector fields? → The **Hodge decomposition**

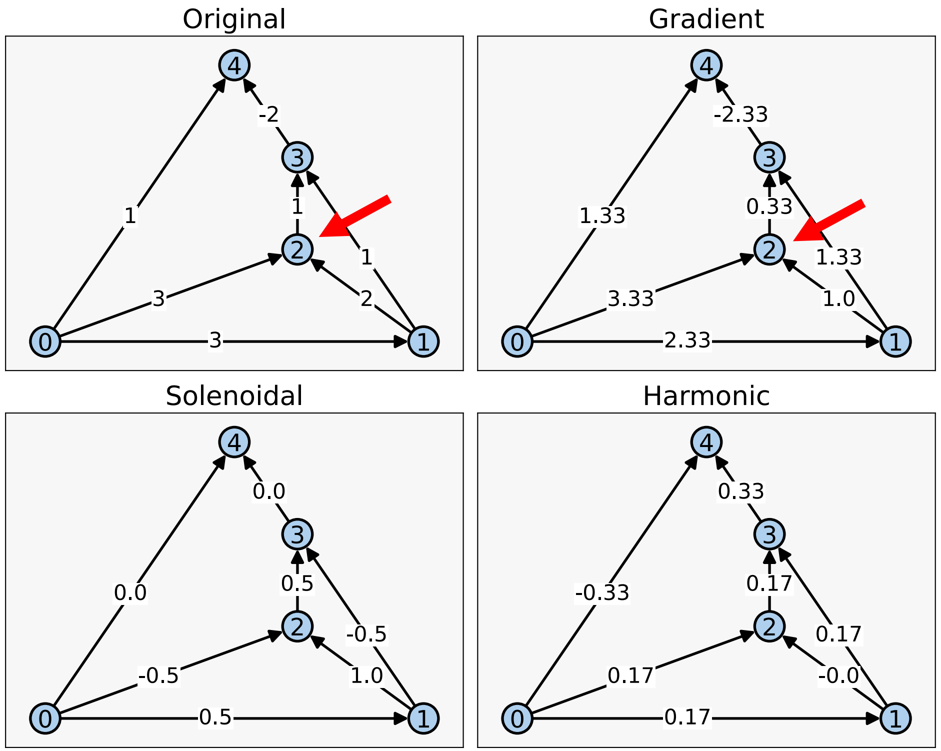
$$\mathbf{v} = \boxed{-\nabla\phi} + \boxed{\nabla \times \mathbf{A}}$$ Helmholtz (continuum)

$$\omega = \boxed{\omega_g} + \boxed{\omega_s + \omega_h}$$ Hodge (discrete)

Gradient component → Accounts for the **divergence** of ω. **Irrotational**

Solenoidal component → Circulations along **triangles** → **Incompressible**

Harmonic component → Circulations along **Larger cycles** → **Incompressible**



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Random Walk Models for Pedestrian Behaviour

Which **Random Walk dynamics** have we used to generate these vector fields?

The **simplest approach** is the unweighted **Discrete-Time Random Walk** (**DTRW**) model.

- At each time step the walker performs a jump to an adjacent node with a given **transition proabability**

$$\boxed{T_{ij} = 1/k_i}$$

Walkers “**spend the same**” time crossing each edge (1 iteration)
Edge length has **no effect** on the dynamics

Metric effects (geometry) of the network not captured. Only **topology**.

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The **walking time** or **distance** are important in pedestrian mobility

CTRW

Random walk at a **constant velocity**

DTRW

Random Walk Models for Pedestrian Behaviour

To consider the lengths of the edges, the DTRW has been upgraded to a random walker that walks at the **typical walking** speed $v = 1.32\ m/s$ or $v = 4.75\ km/h$

Random walk at a **constant velocity**

To account for the walking time, a **time budget** has been introduced.

DTRWs

The budget is **equivalently** spent among all edges

Constant Velocity

The budget is spent according to the **length of each edge**

CTRW

Random walk at a **constant velocity**

DTRW

Problem: The constant velocity model cannot be described analytically since the time of the walkers will depend on the given path, which is random.

This model can only be **simulated**

Random Walk Models for Pedestrian Behaviour

- We need an **analytical description** to validate the resilts.

- We have taken the constant speed model as a **benchmark** and found **analytically describable approximations** through **Continuous-Time Random Walks** (**CTRW**).

CTRWs

Transition probability $T_{ij} = 1/k_i$

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Exponentially distributed waiting times between jumps with **rate** λ_i.

Transition Rates from node i to node j**:**

$$R_{ij} = \frac{A_{ij}\,\lambda_i}{k_i}$$

CTRW

Random walk at a **constant velocity**

DTRW

The **waiting times** have been reinterpreted as the **time spent to go through each link**.

Two CTRW models:

- **Node-centric CTRW**: Node to node jumps
- **Edge-centric CTRW**: Edge to Edge jumps

<u>Mean waiting times:</u>

$$1/\,\lambda_i = \frac{1}{k_i}\sum_j d_{ij}/v$$

$$1/\,\lambda_{ij} = di_j/v$$

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Results: Analytical vs. Simulated Random Walk Flows

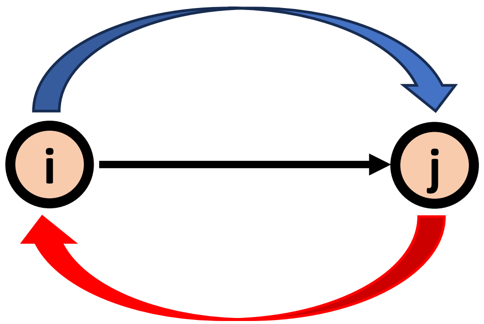
- We have derived the **expected value** of the net edge random walk flows in CTRWs:

- **DTRW:**

$$\omega' = \boxed{p_i(t)T_{ij}} - \boxed{p_j(t)T_{ji}}$$

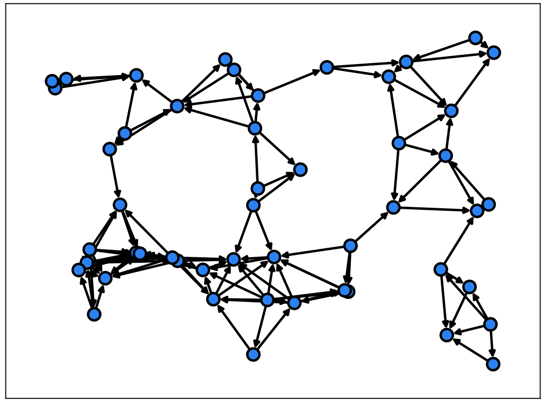
Expected net flow from i to j between steps t and t+1

$$\omega_{ij} = \sum_{t=0}^{t_1} \omega'_{ij}(t)$$

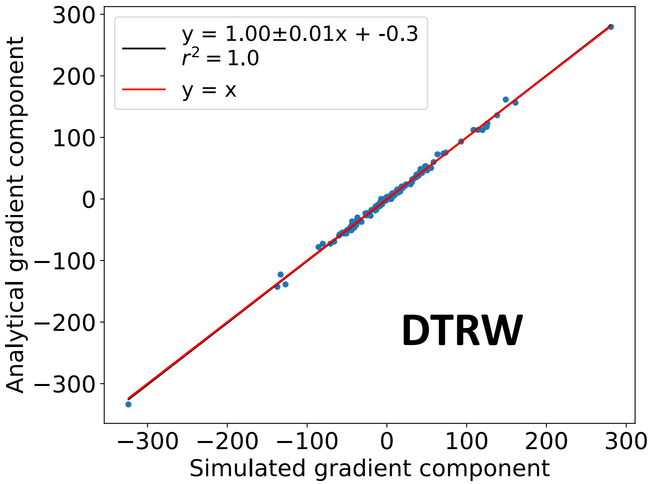


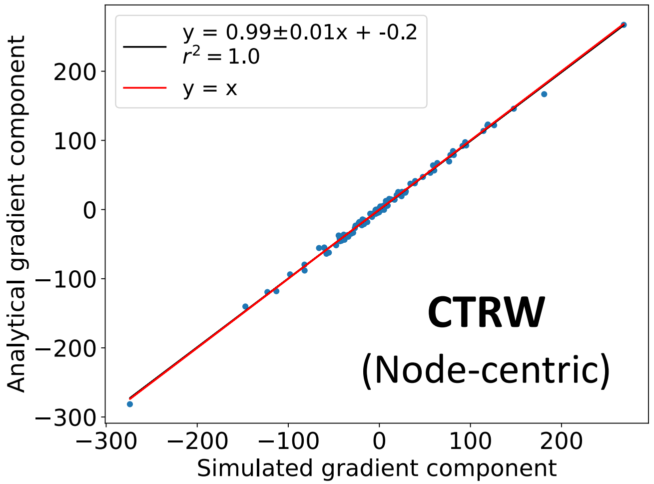
- **CTRW** (node-centric): $\omega_{ij} = \int_0^{t_1} [\boxed{p_i(t)R_{ij}} - \boxed{p_j(t)R_{ji}}]\, dt$

Analytical flows only have **gradient component. Correlation** with the **gradient component of the simulations**



Random Geometric graph 50 nodes and r = 0.2

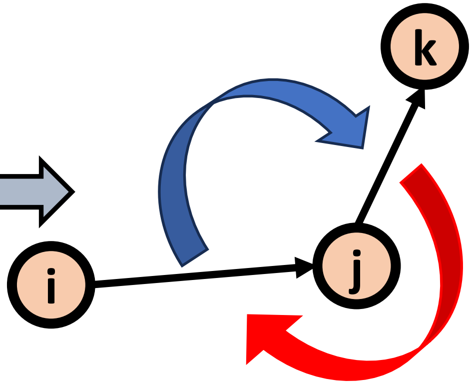




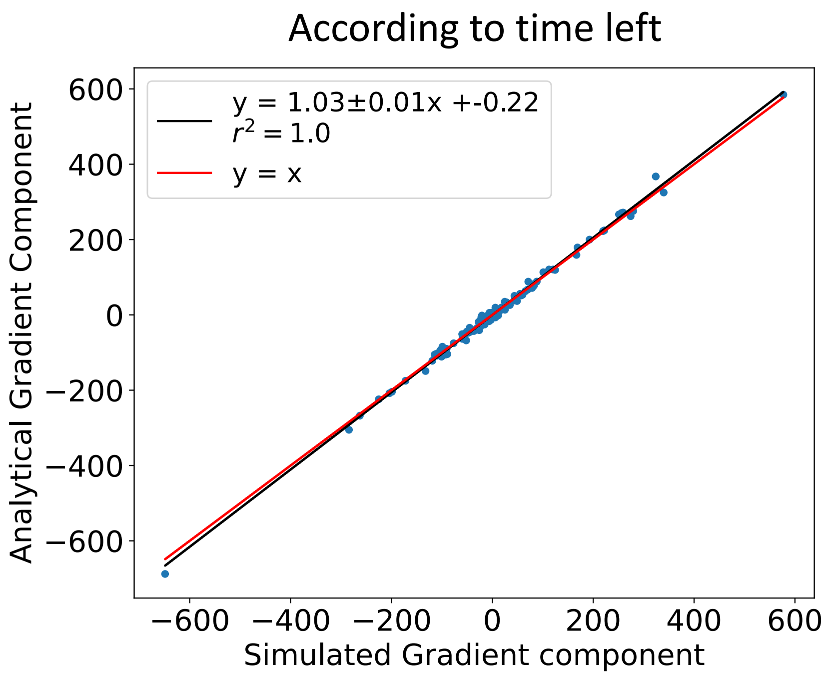
Results: Analytical vs. Simulated Random Walk Flows

- For the **edge-centric** CTRW the same formalism can be used to find the **flow between two edges** $\omega_{ij\to jk}$.

$$\omega_{ij\to jk} = \int_0^{t_1} \left[p_i(t)R_{ij} - p_j(t)R_{ji}\right] dt$$



From the flow between links we can get the net flow along the edge.

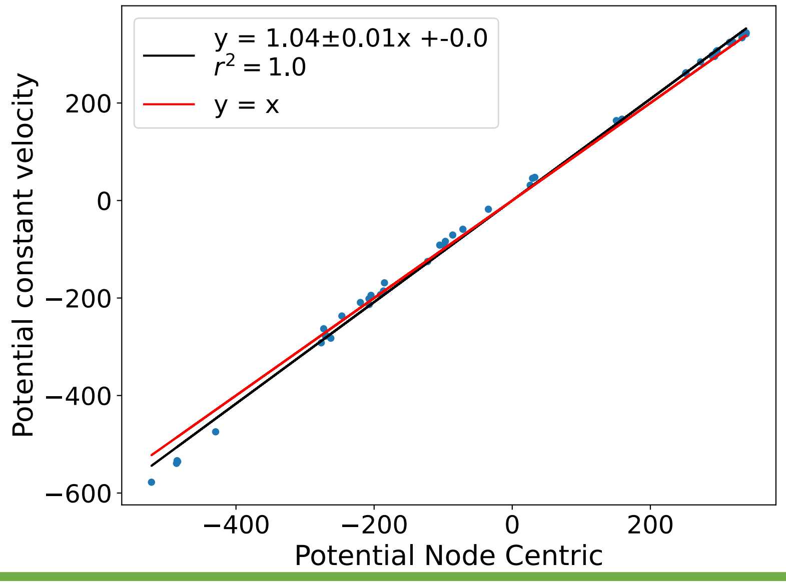


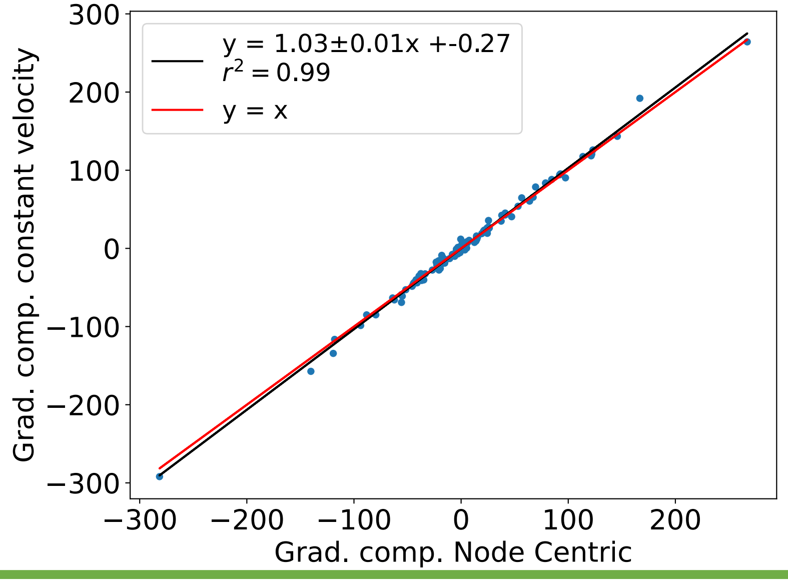
Problem: When a walker starts or ends at an edge, does it go through it?

- It does half of the times in the first step and depending on the time left on the last step.

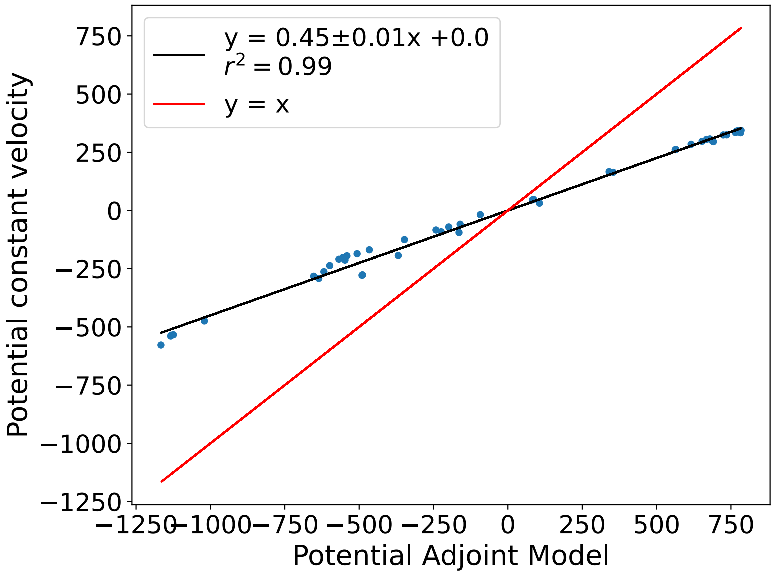
Results: Best Approximation to the Constant Velocity Model

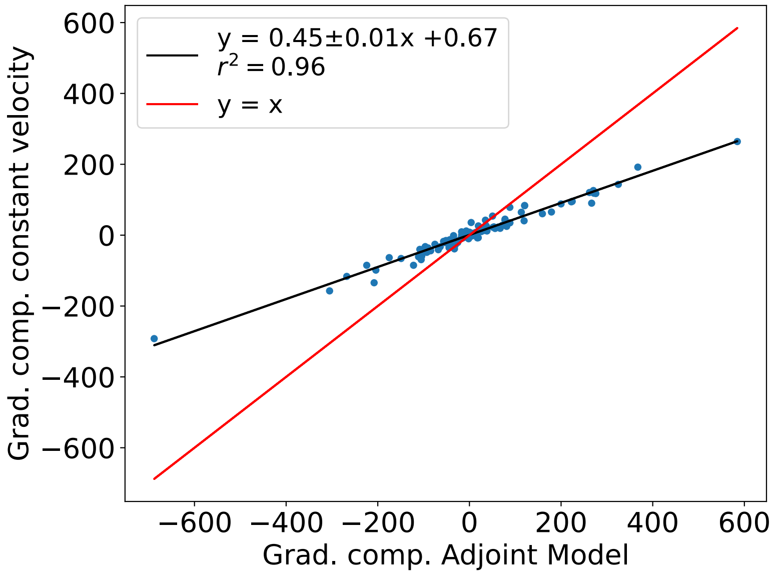
Node-centric





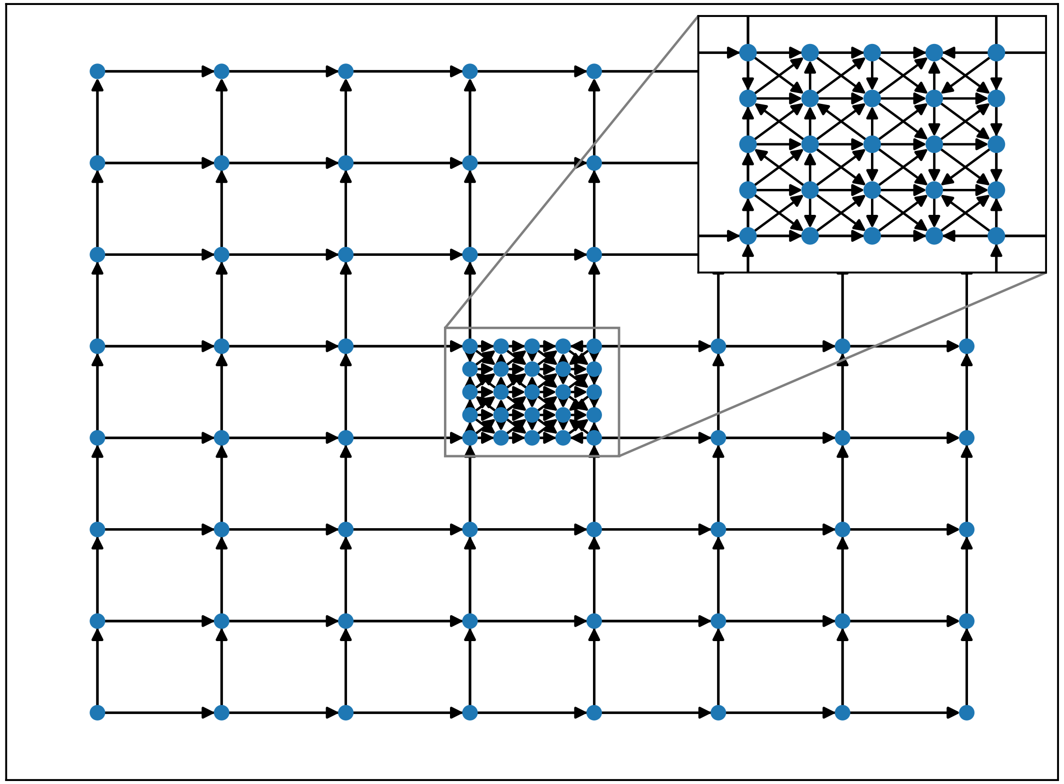
Edge-centric





Results: The Attractiveness of Connected Regions

We want to see the dynamical effect of **densely connected** city centres. We modified a lattice with periodic-boundary conditions (PBC) such that two clearly distinguishable regions are present:



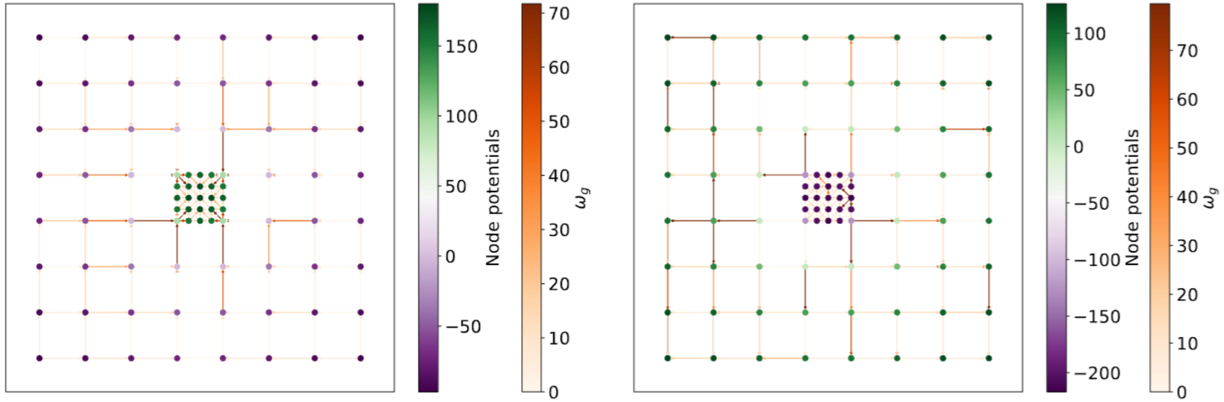
Modified PBC lattice

- Regular region: Barcelona (Eixample), Spain

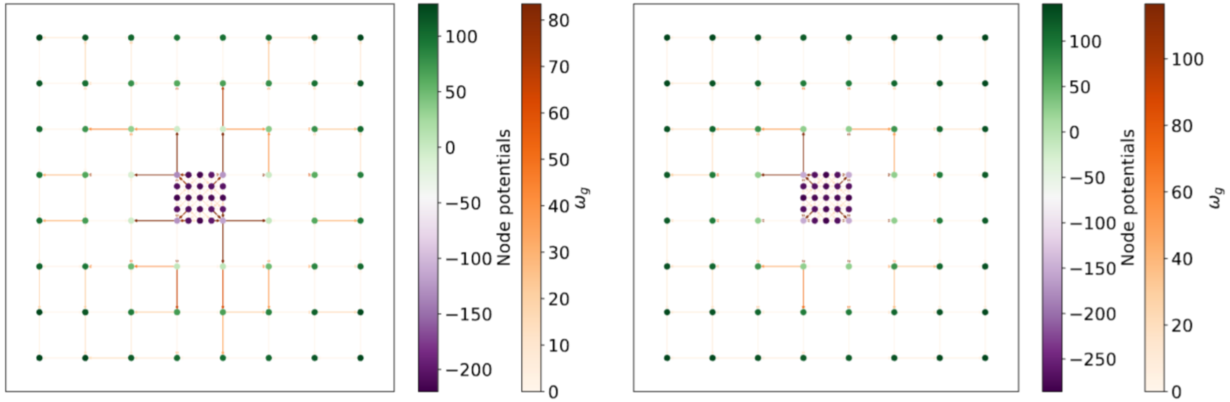
- Connected core: Ciutat-Vella, Barcelona, Spain

Results: The Attractiveness of Connected Regions

We studied the **potentials** to see attractive/repulsive patterns



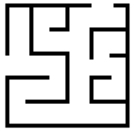
(a) DTRW (b) Node-centric CTRW



(c) Constant v CTRW (d) Edge-centric CTRW

In DTRWs close and connected regions show **attractiveness** (high potentials) while all CTRW models show **repulsiveness.**

Why aren't both dynamics equivalent?

- **Cluster** ≡ 
- **Time budget** ≡ Exploring time
- **DTRWs**: short edges = long edges
- **CTRWs**: short edges have a **lower cost**

With the **same time budget** CTRWs can perform **more jumps** and have a **larger probability** to **escape** the cluster.

Results: A transition between DTRWs and CTRWs

Consequently, we have **two competing factors** in the attractiveness of centres:

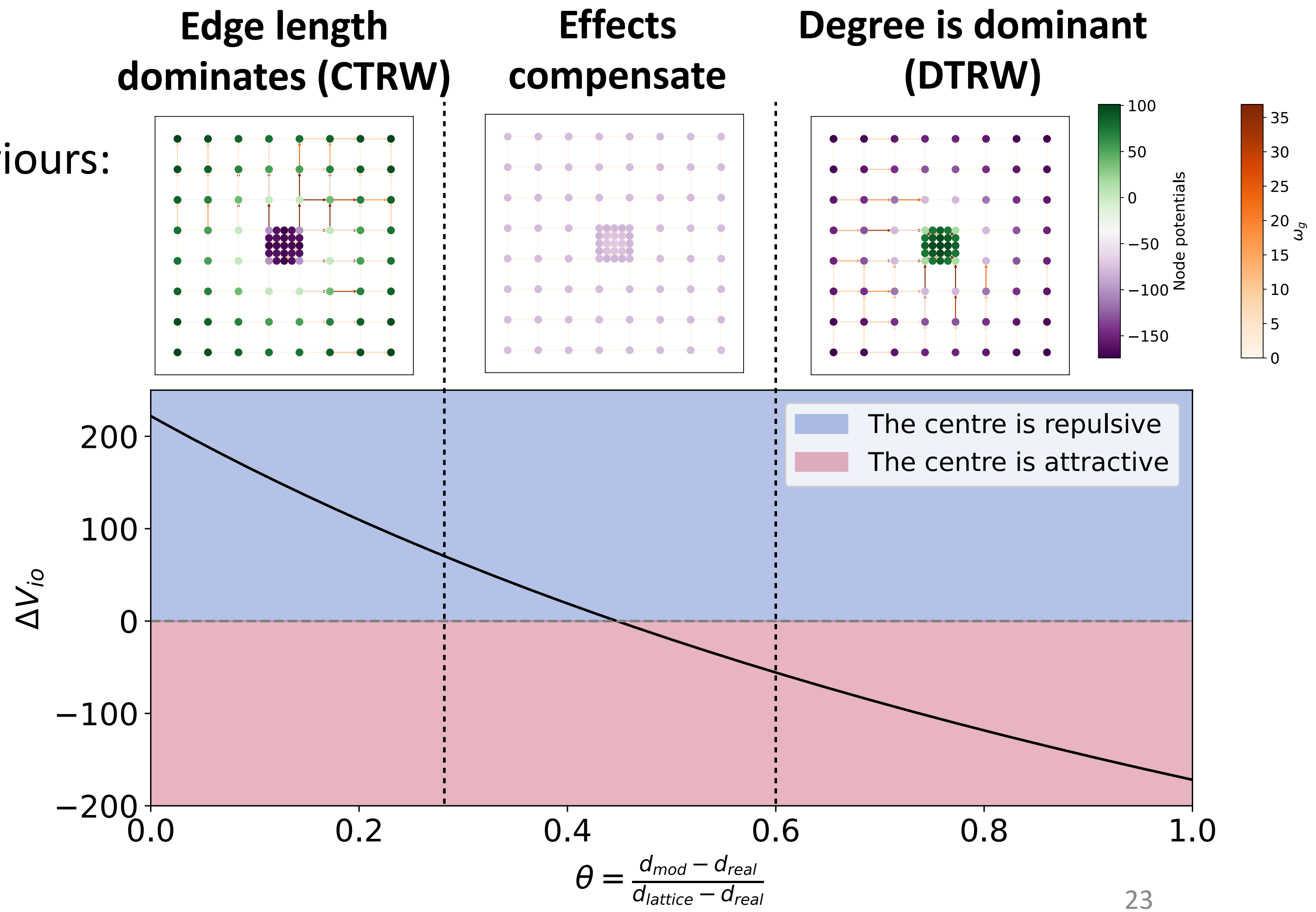
- **Node degree**
- **Edge length**

Transition between attractive/repulsive behaviours:

We continuously modified the inner edge lengths from the real ones (CTRW) to the same as the outer links (effective DTRW).

⇩

We observe a **dominance shift** in the effect of the **degrees** and **edge-lengths**



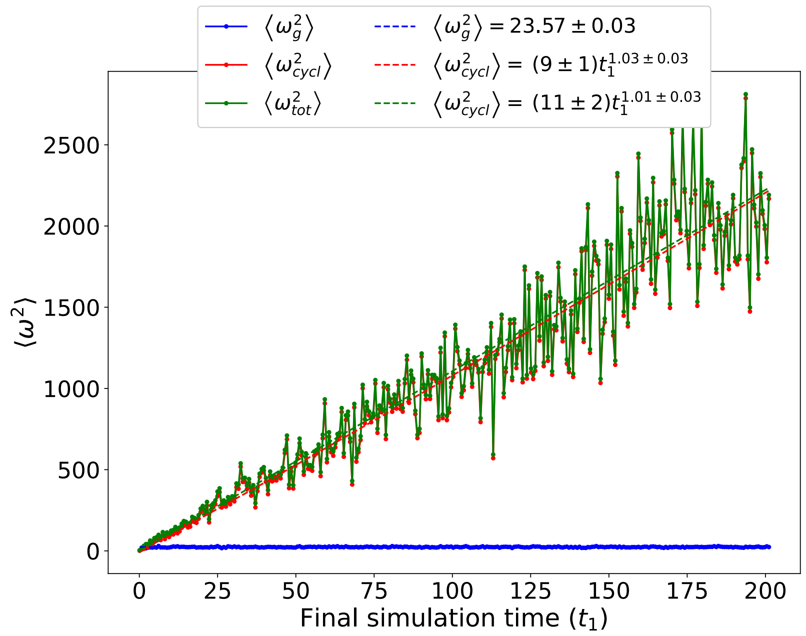
Results: Orgin of the Cyclic Components

The cyclic components were present in the simulations of both the **node-centric** and **constant velocity** RWs, but **there are no expected cyclic flows.**

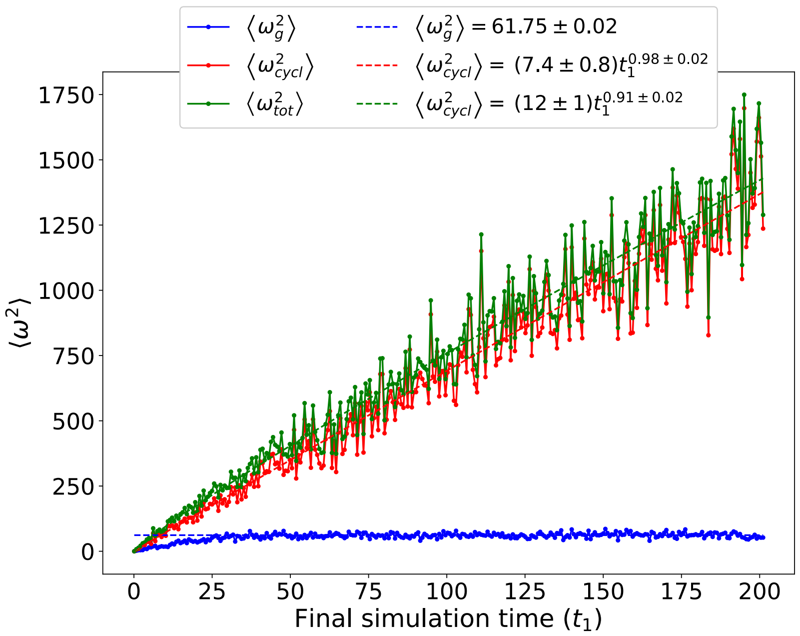
Do the cyclic components reach any stationary value?

<u>Analysis of the Mean-Squared Field (MSF)</u>

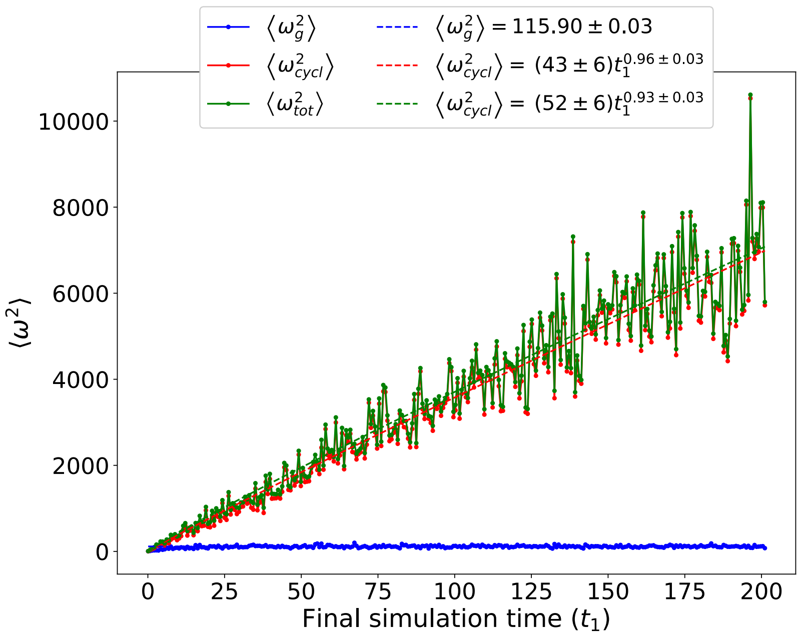
$$\langle \omega_g^2 \rangle = \frac{1}{E} \sum_{i,j} (\omega_{ij}^g)^2 \quad ; \quad \langle \omega_{cycl}^2 \rangle = \frac{1}{E} \sum_{i,j} (\omega_{ij}^{cycl})^2$$



Spatially-embedded Erdös-Rényi graph with 50 nodes and p = 0.1



PBC modified lattice



Random geometric with 50 nodes and r = 0.2

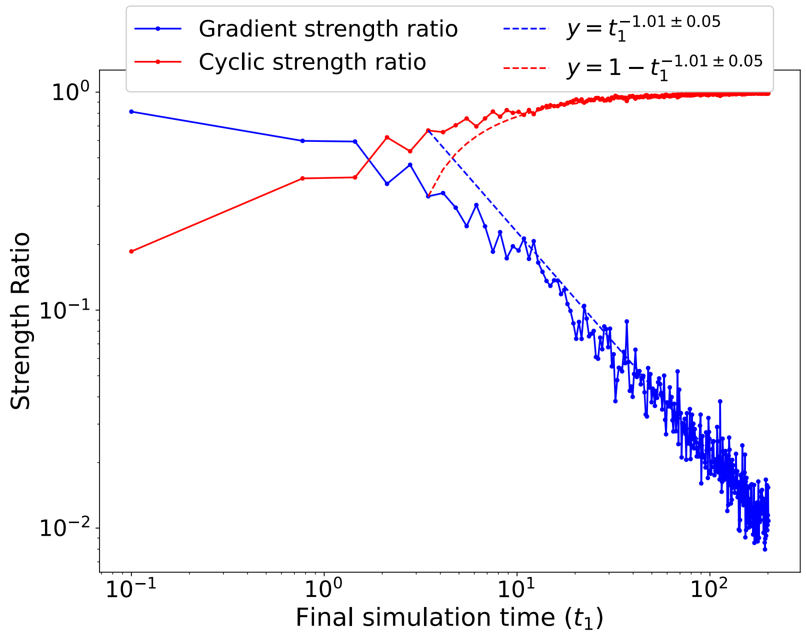
The cyclic MSF **grows linearly** with the total walking time ⟶ MSD in geometric RWs?
The gradient MSF reaches a **stationary value** as expected

Results: Orgin of the Cyclic Components

The cyclic components were present in the simulations of both the **node-centric** and **constant velocity** RWs, but **there are no expected cyclic flows.**

Importance of each component?

Analysis of the **strength ratios:** $1 = \frac{\|\omega_g\|^2}{\|\omega\|^2} + \frac{\|\omega_s\|^2}{\|\omega\|^2} + \frac{\|\omega_h\|^2}{\|\omega\|^2} = \eta_g + \eta_s + \eta_h$



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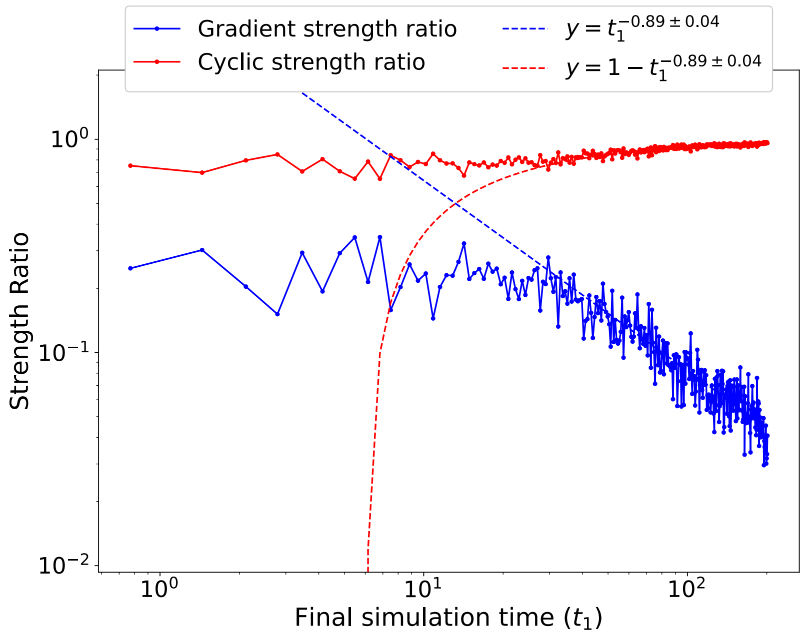

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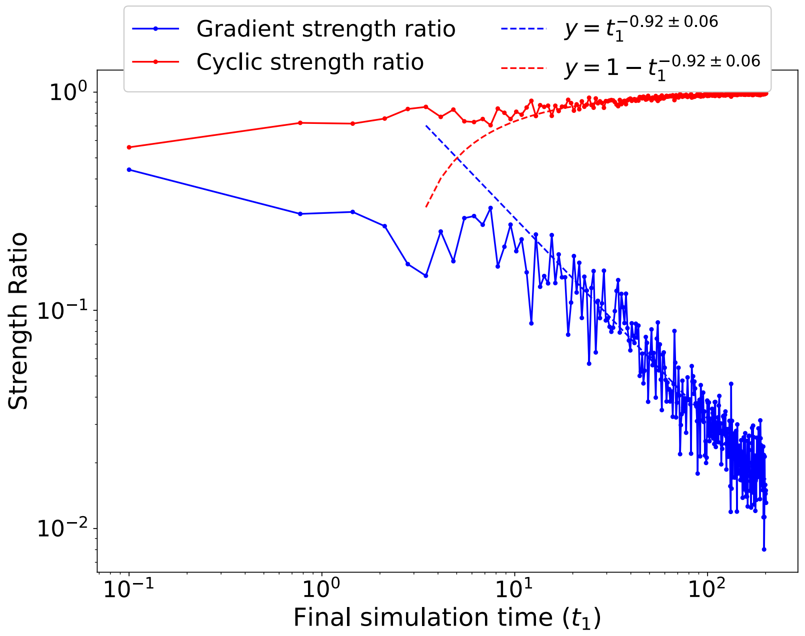
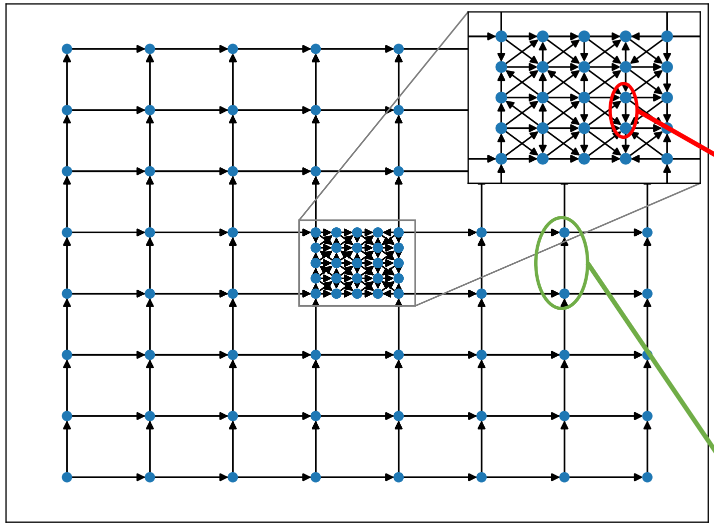


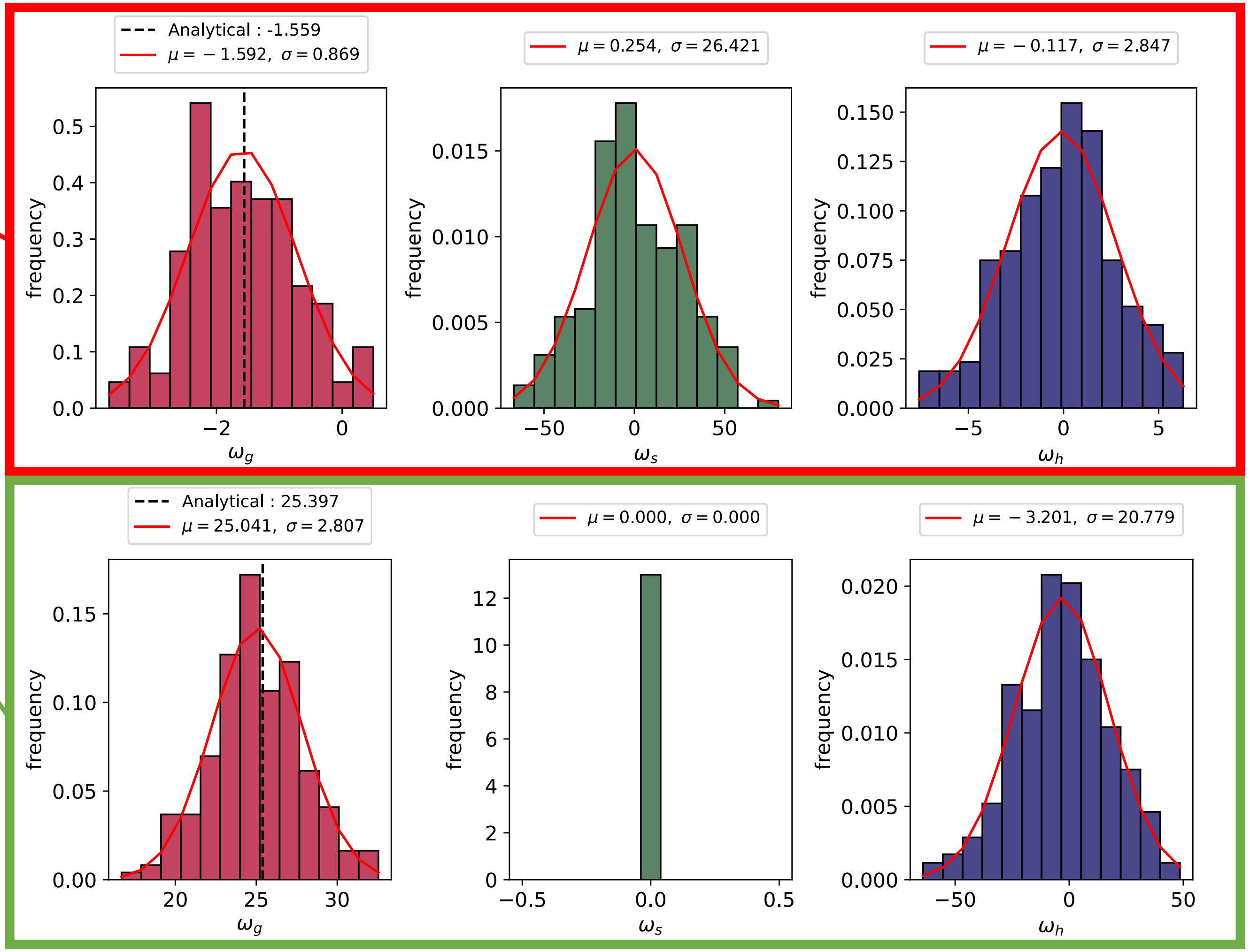
Random geometric with 50 nodes and r = 0.2

Transient initial phase followed by a power-law decay of the gradient strength ratio with exponent close to -1 as predicted by the gradient MSF

Results: Orgin of the Cyclic Components

We studied the **distribution** of each component to see if the **mean of the cyclic components** was actually 0:





- Histograms for the cyclic flows reveal that the cyclic component has **zero mean** (as the **expected flows** predict).
- The means of the gradient component coincide with the **expected value**.

The cyclic components seem to be a product of the **stochasticity**

Discussion and Conclusions

- Analysis of pedestrian mobility through Markovian unweighted **random walk** dynamics with **time budget**.
- Development of an **analytical expression** for the expected values of the total net random walk flow.
- Observed the transition from former **attractive** patterns in DTRW to **repulsiveness** in CTRW.
- The **cyclical components** seem capture the **stochasticity of the process**. Implications on mobility?
- Magnitude of the **time budget** has a big impact on the importance of the cyclical components.

Unresolved Questions

- Real pedestrian networks and other transport layers.
- Analytical expectation for the variance of the **cyclical components** and look for **correlations** with the **geometrical** or **topological** structures of the graph.
- Adjust the random walk models to more **realistic** ones to see if the observed effects are maintained.

Thank you for your Attention!

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