KING'S College LONDON

Psalter: Particle Spectrum for Any Tensor Lagrangian

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GRTL meeting

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Plan:

1. What?
2. How?
3. Why?
4. Where?
5. Let’s give it a go!

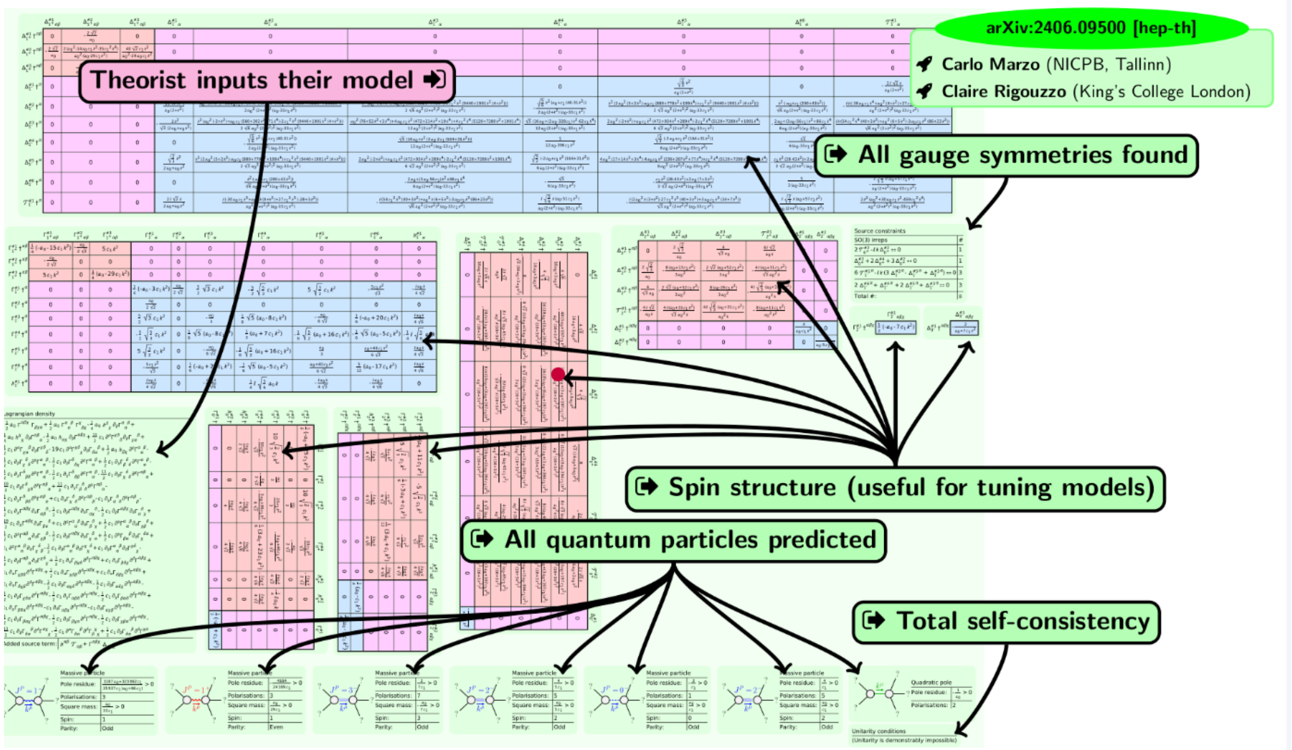
I. What?

A new Mathematica package that allows you to explore particle content of Lagrangians.

(part of the xAct package)

[1] *PSALTer: Particle Spectrum for Any Tensor Lagrangian*, W. Barker (Cambridge U), C. Marzo (NICPB, Tallinn), C. Rigouzzo (King's Coll. London), 2406.09500

I. What?



Slide courtesy of Will Barker

I. What?

Any Lagrangian?

Yes and No

- ✓ Abstract indices.
- ✓ Any tensor (up to rank 3).
- ✓ Any symmetry of the tensor.
- ✓ Agnostic about formulation of gravity.

I. What?

Any Lagrangian?

Yes and No

- ✕ No parity breaking terms (as of now)
- ✕ Maximum quadratic in tensors (as of now)
- ✕ Expansion around flat background (as of now)

II. How?

II. How?

- Based on an interesting piece of physics: Spin Projection Operators (SPOs).

- Remember group theory:

The Lorentz algebra is the direct sum of independant SU(2) algebras

$$\mathfrak{so}(1,3) \simeq \mathfrak{su}(2)_- \oplus \mathfrak{su}(2)_+$$

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II. How?

(j_-, j_+)	dim	Type	Example
$(0,0)$	1	Scalar	$\pi^0, \pi^\pm$, Higgs
$(1/2, 0)$	2	Left-handed spinor	Neutrinos
$(0, 1/2)$	2	Right-handed spinor	Anti-neutrinos
$(1/2, 0) \oplus (0, 1/2)$	4	Dirac spinor	$e^\pm, p, n$
$(1/2, 1/2)$	4	Vector	$\gamma, W^\pm, Z^0, g$
$(1, 1)$	9	Traceless metric tensor	"Gravity"

II. How?

- Clebsch-Gordan decomposition of a vector:

$$\left(\frac{1}{2}, \frac{1}{2}\right) \to (1^-) \oplus (0^+)$$

II. How?

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- Clebsch-Gordan decomposition of a rank two tensor :

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left(\frac{1}{2},\frac{1}{2}\right)\rightarrow(2^+)\oplus(1^+)\oplus 2(1^-)\oplus 2(0^+)$$

II. How?

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- Clebsch-Gordan decomposition of a rank two tensor :

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left(\frac{1}{2},\frac{1}{2}\right)\rightarrow(2^+)\oplus(1^+)\oplus 2(1^-)\oplus 2(0^+)$$

Conclusion: possible to have all sorts of propagating spinorial sectors, and we want to keep track of them!

II. How?

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- Clebsch-Gordan decomposition of a rank two tensor :

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left(\frac{1}{2},\frac{1}{2}\right)\rightarrow(2^+)\oplus\cancel{(1^+)}\oplus 2\cancel{(1^-)}\oplus 2\cancel{(0^+)}$$

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II. How?

- Idea 1: First decompose into each spin subsector

→ Use Spin Projection Operators (SPOs).

- Idea 2: massage the equation to obtain **propagator**

→ All the information (mass? Tachyon? Ghost?) is encoded in it.

II. How?

Particles	Propagator
Spin 0 (scalar fields (Higgs, pions , ..)	$\frac{i}{q^2 - m^2}$
Spin ½	$\frac{i}{\not{p}^2 - m^2} = i\frac{\not{p}^2 + m}{p^2 + m^2}$
Spin 1 massive (W,Z weak boson)	$\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m^2}\right)}{q^2 - m^2}$
Spin 1 massless(photon)	$\frac{-ig_{\nu\mu}}{q^2}$

II. How?

- Example: massive spin 0 field

$$Propagator \propto \frac{A}{k^2 - m^2}$$

- No ghosts: $A < 0$ (break causality/unitarity)
- No tachyons: $m^2 > 0$ (subluminal speed)

III. Why?

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III. Why?

- Was first motivated to explore the zoo of modified theory of gravity.

- In Palatini formulation, the metric and affine connection are independent: $(h_{ij}, \Gamma_{l[mn]})$

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left(\frac{1}{2},\frac{1}{2}\right)\oplus\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left((1,0)\oplus(0,1)\right)$$
$$\to 2\left(2^{+}\right)\oplus\left(2^{-}\right)\oplus 3\left(1^{+}\right)\oplus 4\left(1^{-}\right)\oplus 3\left(0^{+}\right)\oplus\left(0^{-}\right)$$

III. Why?

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- Real need for computers to do this analysis…

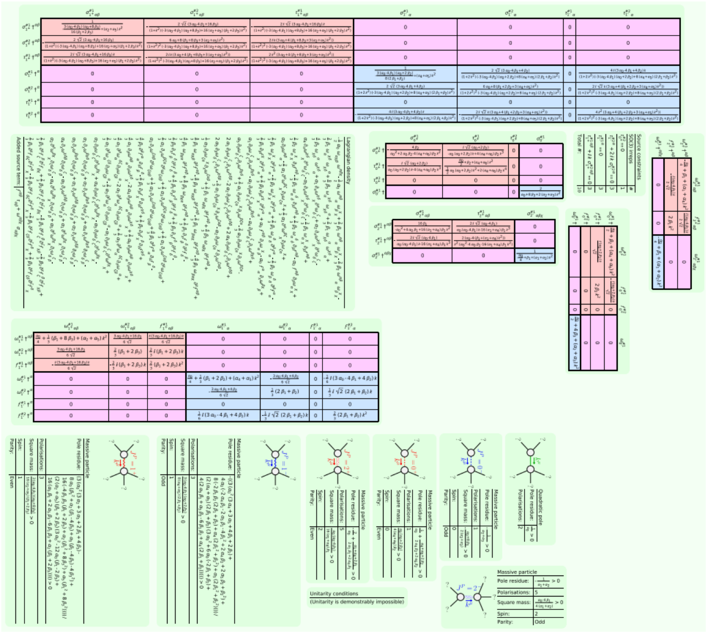

FIG. 21. The particle spectrum of the most general parity-preserving PGT. The fields $f^{i}{}_{\mu}$ and $\omega^{ij}{}_{\mu}$ contain respectively 16 and 24 d.o.f. The Poincaré symmetry eliminates 2×10 d.o.f, and two are accounted for by the graviton polarisations. The remaining 18 d.o.f are partitioned amongst the six massive species shown above. As is well known, only for special cases of the PGT action in Eq. (8) do the masses and pole residues of these species allow for unitarity: the general case shown here is sick. All quantities are defined in Figs. 18 and 19.

Proof by intimidation

III. Why?

- It is also an useful tool for scalarisation/ vectorisation.

- **Definition:** Scalarisation is a mechanism that endows self-gravitating bodies, such as BH and neutron stars, with a scalar-field configuration [1]

[2] *Spontaneous scalarization*, Daniela D. Doneva, Fethi M. Ramazanoğlu, Hector O. Silva, Thomas P. Sotiriou, and Stoytcho S. Yazadjiev, Rev. Mod. Phys. 96, 015004

III. Why?

- At the perturbative level, spontaneous scalarisation is signalled by a tachyon.

→ Can use Psalter to find tachyons.

[3] *Structure of neutron stars in massive scalar-tensor gravity*, R. Rosca-Mead, C. J. Moore, U. Sperhake, M. Agathos, D. Gerosa, 2007.14429 [gr-qc], Symmetry 12 (2020) 9, 1384

III. Why?

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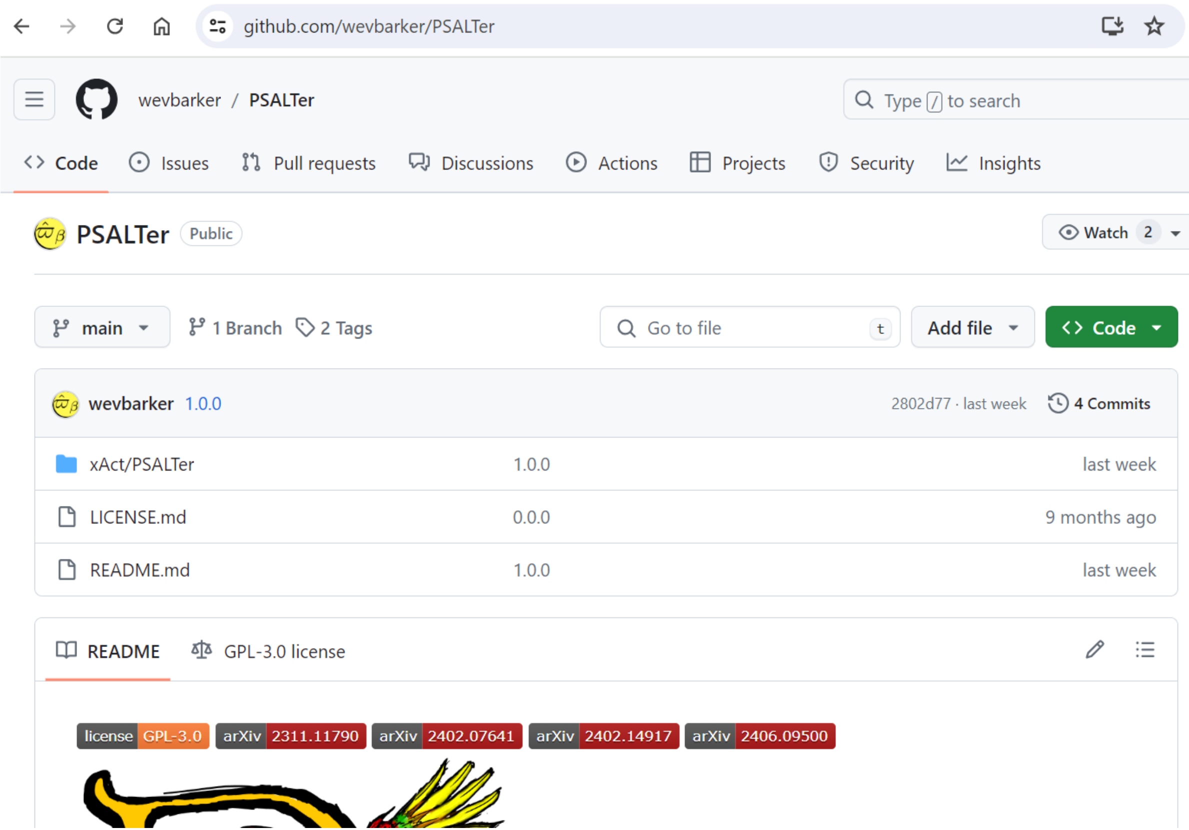
- Can also do $R^2, R_{\mu\nu} R^{\mu\nu}$ ... in **any** formulation of gravity

IV. Where?

IV. Where?

IV. Where?



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IV. Where?

- A part of the official xAct packages :

www.xAct.es

xAct

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Packages

- *xCore*
- *xPerm*
- *xTensor*
- *xCoba*
- *xPert*
- Harmonics
- *Invar*
- *Spinors*
- *xPrint*
- *SymManipulator*

xAct: Efficient tensor computer algebra for the Wolfram Language

José M. Martín-García, GPL 2002-2024

Main collaborators: Alfonso García-Parrado, Alessandro Stecchina, Barry Wardell, Cyril Pitrou, David Brizuela, David Yllanes, Guillaume Faye, Leo Stein, Renato Portugal, Teake Nutma, Thomas Bäckdahl.

Introduction

xAct is a suite of free packages for tensor computer algebra in the Wolfram Language. *xAct* implements state-of-the-art algorithms for fast manipulations of indices and has been modelled on the current geometric approach to General Relativity. It is highly programmable and configurable. Since its first public release in March 2004, *xAct* has been intensively tested and has solved a number of hard problems in GR.

There are four packages acting as a kernel for the rest:

- *xCore*: generic programming tools
- *xPerm*: manipulation of large groups of permutations
- *xTensor*: abstract tensor computations, the flagship of the system
- *xCoba*: component tensor computations

Other packages include:

- *xPert*: high-order perturbation theory in GR, by David Brizuela, JMMG and Guillermo Mena Marugán.
- Harmonics: tensor spherical harmonics, by David Brizuela, JMMG and Guillermo Mena Marugán.
- *Invar*: polynomial invariants of the Riemann tensor, by JMMG, David Yllanes, Renato Portugal and Leon Manssur.
- *Spinors*: spinor computations in GR, by Alfonso García-Parrado and JMMG.
- *xPrint*: Formatted input for *xAct*, by Alessandro Stecchina.
- *SymManipulator*: Symmetrized expressions in *xAct*, by Thomas Bäckdahl.
- *AVF*: Exterior calculus with algebra-valued forms, by Hugo D. Wahlquist.
- *xTras*: Additions to *xAct*, by Teake Nutma.
- *TexAct*: Tex code to format *xAct* expressions, by Thomas Bäckdahl, JMMG and Barry Wardell.
- *xPand*: Cosmological perturbation theory, by Cyril Pitrou, Xavier Roy and Obinna Umeh.
- *xTerior*: Exterior calculus, by Alfonso García-Parrado and Leo C. Stein.
- *SpinFrames*: NP and GHP form of spinor equations, by Thomas Bäckdahl and Steffen Aksteiner.
- *xIST/COPPER*: General scalar-tensor theories and perturbations, by Johannes Noller.
- *EFTofPNG:*: Effective field theory of post-Newtonian gravity, by Michele Levi and Jan Steinhoff.
- *bimEX*: 3 + 1 Bimetric relativity, by Francesco Torsello.
- *FieldsX*: Fermions, gauge fields and BRST cohomology, by Markus B. Fröb.
- *xPPN*: Parametrized post-Newtonian formalism, by Manuel Hohmann.
- *SymSpin*: Symmetric spinors, by Steffen Aksteiner and Thomas Bäckdahl.
- *HiGGS*: Hamiltonian analysis of Poincaré gauge theory, by Will Barker.
- xBrauer: Brauer algebra for tensor calculus, by Thomas Helpin.
- *PSALTer*: Particle spectrum for any tensor Lagrangian, by Will Barker, Carlo Marzo and Claire Rigouzzo.

A single file with the current versions (17 October 2021) of all packages can be downloaded: xAct_1.2.0.tgz for linux/unix/mac, or xAct_1.2.0.zip for windows. See the Installation notes.

V. Let's give it a go!

- Let's work out a quick example together ☺
  - Start with the usual graviton:

$$L \propto \int d^4x \sqrt{-g} R$$

  - Massive gravity:

$$L \propto \int d^4x \sqrt{-g} \left[\alpha R + \beta\left(h_{ab}h^{ab} - hh\right)\right]$$

  - Sick massive gravity:

$$L \propto \int d^4x \sqrt{-g} \left[\alpha R + \beta h_{ab}h^{ab} - \gamma\, hh\right]$$

V. Let's give it a go!

- Just R^2?:

$$L \propto \int d^4x\sqrt{-g}\,\alpha R^2$$

Conclusion and outlook:

- Application:
  - ✓ Deals with alternative formulation of gravity: you give a Lagrangian, you get the particle content (and more).
  - ✓ Scalarisation
- Outlook:
  - ❑ Extend the code to deal with parity odd Lagrangian (c.f Karananas).
  - ❑ Extend the code to higher interaction terms/ deal with loop computations.
  - ❑ Extend the code to deal with non-trivial background (dS, AdS, torsion condensate…)

Questions? Comments?