Stochastic inflation in numerical relativity

& the SIGRid suite project

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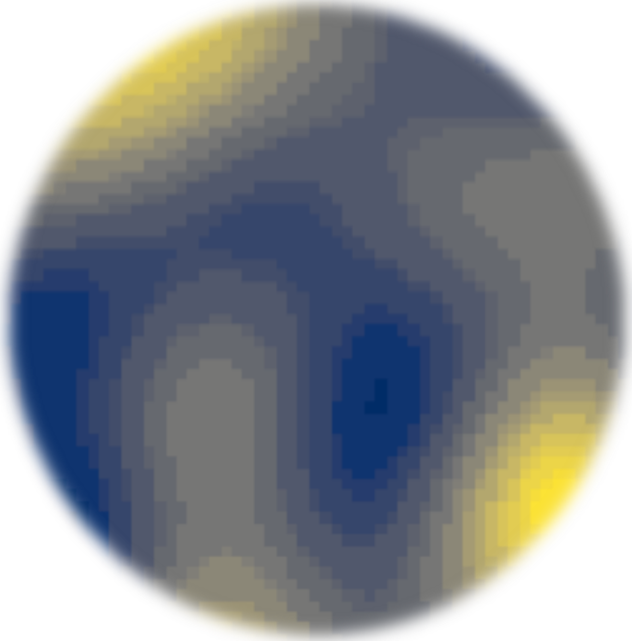
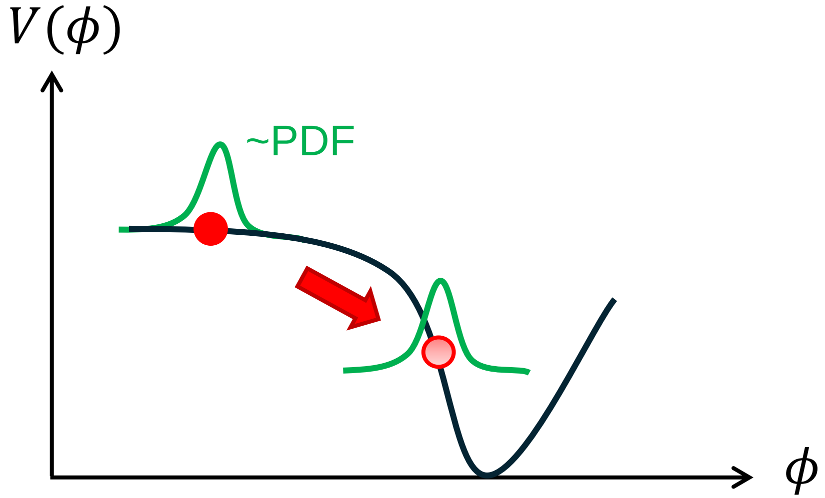
GRTL collaboration meeting, Thursday, June 27th, 2024

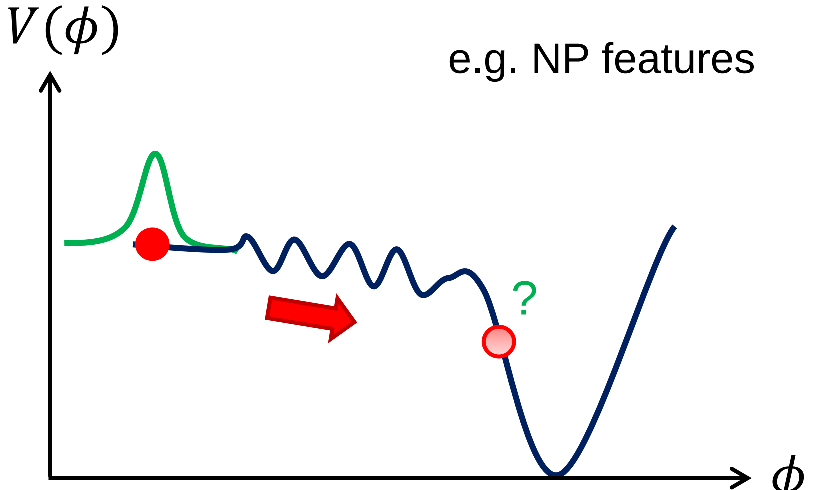


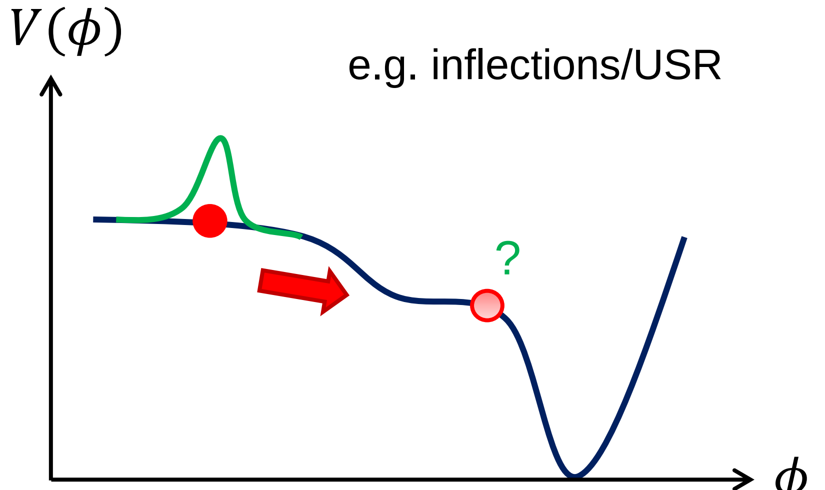
Statistical imprints from all scenarios

Inflation: ~constant ϕ / ~constant $\varepsilon_0 = H$,
gives a ~cosmological constant behavior (~dS)
+quantum perturbations

Non-dS behavior ⇔ $\varepsilon_{i+1} = -\frac{d\ln\varepsilon_i}{dN} \neq 0$







= info about interactions (high order statistics)

What if couplings $g \propto \{\varepsilon_i\}$ blow up?
What if perturbations blow up?

No such thing as non-perturbative QFTCS.

~dS+pert. is only required on the (very gaussian/linear) scales/signals.
New experiments get us closer to non-linear/ potentially non-perturbative scales:
desperate need for predictions on these scales → desperate for NR!!!

A tour of numerical inflation *to get those inhomogeneities' behaviour…*

- **Background** equations (Friedman's): ✓
- **Linear** equations (Mukhanov-Sasaki's): ✓

  see e.g. oscode Public
- **Higher order perturbations** correlations (in-in or transport equations): ✓

  see e.g. [many,…, Clarke21] (basis separation of 3pt tree-level in-in)

  or PyTransport Public CosmoFlow Public (theoretically Npts, no loops though)
- **Lattice Cosmology on big/classical scales** cosmolattice Public STOLAS_dist Public ✓
- **Full GR**

  Mostly looking at inflation's robustness
  - *Background*: lemon squeezy
  - *Arbitrary initial conditions, perturbative or not* [Clough17,Aurrekoetxea20/23, Joana21, Elley24, Joana24]. Using heuristic ICs or a solver. ✓
  - *Full Bunch-Davies vacuum noise and **spectral studies*** → us! +Ericka's work on gravitonic ICs.
  - *& more surpises*

Physical initial conditions: *quantum and gauge fights*

Illustration: *Breaking the ice between theorists*, deepai.org.

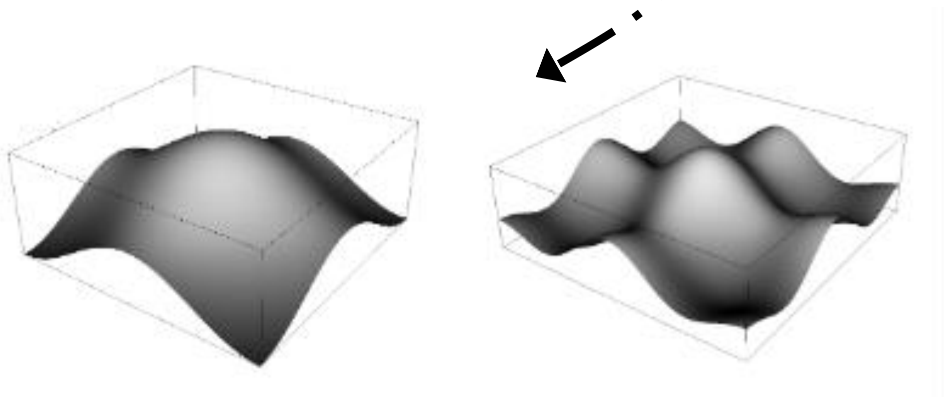
Initial Value Problem

$$\begin{cases} {}^3R + \frac{2}{3}K^2 - \tilde{K}_{ij}\tilde{K}^{ij} - 2M_{Pl}^{-2}\rho = 0, \\ \tilde{K}_{i|j} - \frac{2}{3}K_{|i} - M_{Pl}^{-2}\mathcal{J}_i = 0 \end{cases}$$

To 'start' a spacetime, you need to find a solution to constraints

→Analytically difficult

Inhomogeneous inflation might require a simplified analytical framework [Clough17] or a solver [Aurrekoetxea20,22,23]



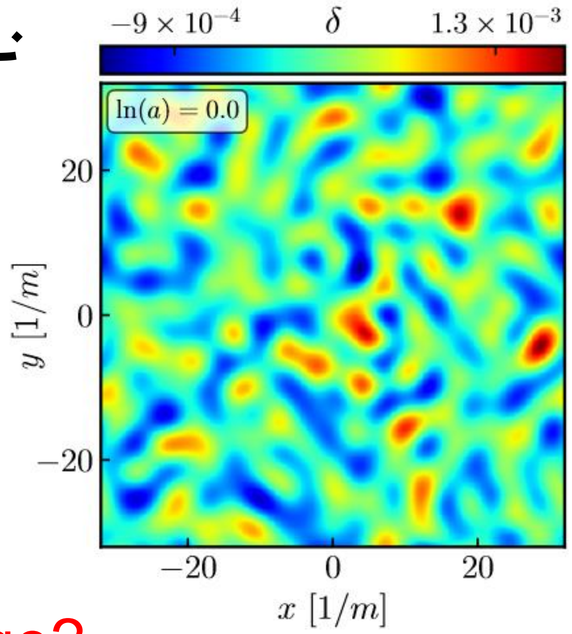
e.g. give

$$\phi(\mathbf{x}) = \phi_{\text{reh}} + \delta\phi(\mathbf{x})$$

$$\dot{\phi}(\mathbf{x}) = \dot{\phi}_{\text{reh}} + \delta\dot{\phi}(\mathbf{x})$$

$$\gamma_{ij}(\boldsymbol{x}) = a_{\text{reh}}^2\delta_{ij} + 0$$

& solve for K_{ij}



'Spacetime filling': what about false minima, uniqueness, & gauge?

Cosmo. perturbations?

REMINDER $\left(\begin{aligned} ds^2 &= -\alpha_b^2(1+2\Psi)dt^2 + 2a^2 B_{,i} dt dx^i \\ &+ a^2\left[(1-2\Phi)\delta_{ij} + 2E_{,ij}\right] dx^i dx^j \end{aligned} \right)$ REMINDER

Defining the curvature perturbation on comoving hypersurfaces, à la cosmologist

$$\mathcal{R} = \Phi + \frac{H}{\dot{\phi}_b}\delta\phi \qquad \alpha_b = 1$$

Gives a linear gauge-invariant quantity with a known evolution

$$\boxed{\ddot{\mathcal{R}}_k + H(3-\varepsilon_2)\dot{\mathcal{R}}_k + \frac{k^2}{a^2}\mathcal{R}_k = 0}$$

Mukhanov-Sasaki evolution eqn.

→ only one DOF for scalar perturbations, including backreaction

→ Note – *hence QFT in the title* ☺ – initial quantum conditions

i.e. far past Minkowski

$$\frac{a\dot{\phi}}{H}\mathcal{R}_k(\tau \longrightarrow -\infty) = \frac{1}{\sqrt{2k}}e^{-ik\tau}$$

& quantification

$$\hat{\mathcal{R}}_k = \mathcal{R}_k \hat{a}_{\vec{k}} + \mathcal{R}_k^* \hat{a}^\dagger_{-\vec{k}}$$

Note that you can do it all with tensor perturbations too (gravitons).

Initial value linear solution

$$\begin{cases} {}^3R + \frac{2}{3}K^2 - \tilde{K}_{ij}\tilde{K}^{ij} - 2M_{Pl}^{-2}\rho = 0, \\ \tilde{K}_{i|j} - \frac{2}{3}K_{|i} - M_{Pl}^{-2}\mathcal{J}_i = 0 \end{cases}$$

To 'start' a spacetime, you need to find a physical solution to constraints, but that's difficult. Also, NR types of gauges are very unnatural for cosmologists…

At linear order $\mathcal{R}$ is the only input you need in any gauge, satisfying constraints indirectly

In particular in a generalized synchronous gauge (NR-compatible): [see PhysRevD.109.123523]

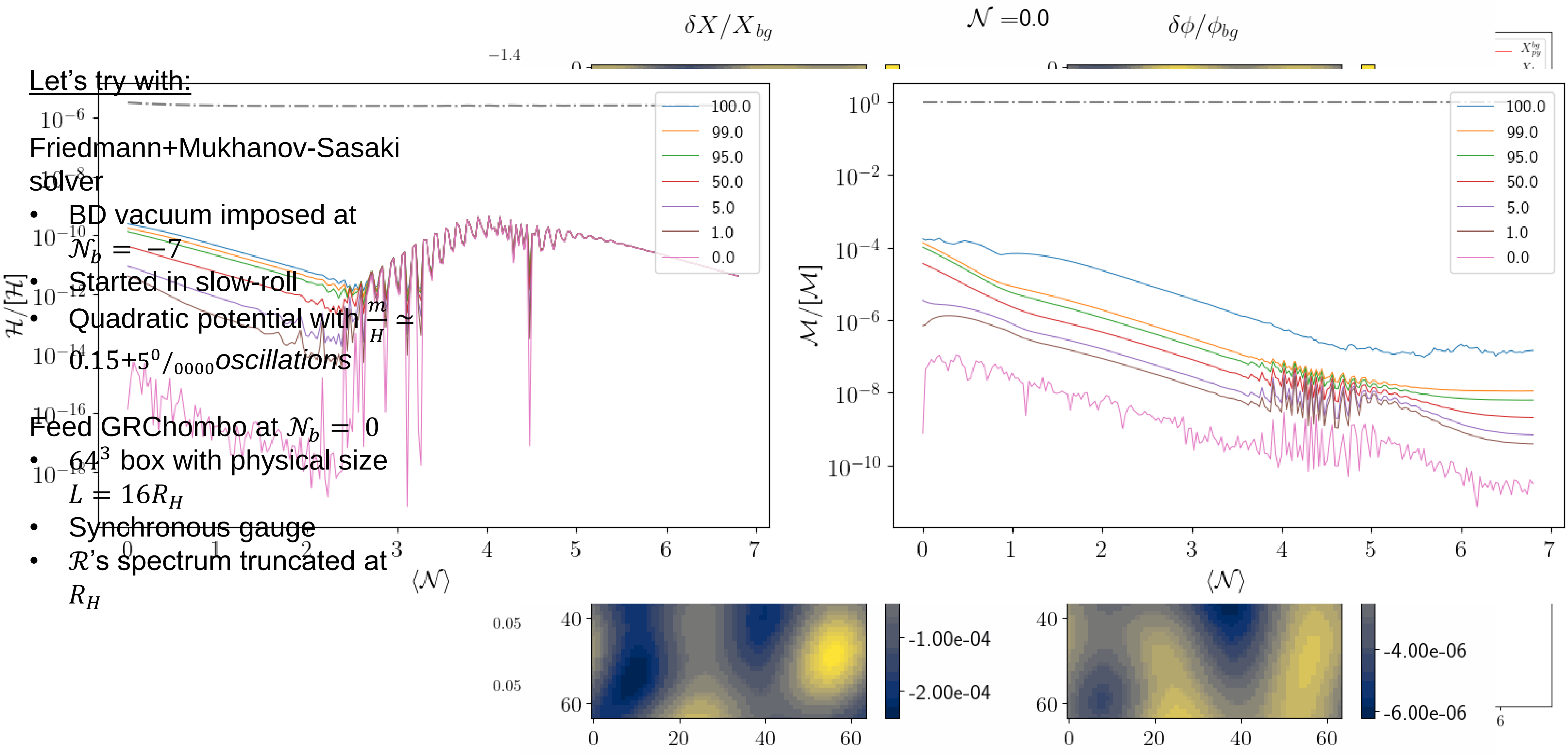
$$\begin{cases} \chi &= \int^t (\Psi^* - \Psi_B[\mathcal{R}])dt' + \chi_0, \\ E &= \int^t (B^* + a^{-2}\chi[\Psi^*, \mathcal{R}])dt' + E_0, \\ \Phi &= \Phi_B[\mathcal{R}] - H\chi[\Psi^*, \mathcal{R}], \\ \delta\phi &= \sqrt{2\varepsilon_1}M_{Pl}\left(\mathcal{R} - \Phi[\Psi^*, \mathcal{R}]\right), \end{cases}$$

where

$$\begin{cases} \Phi_B &= -\varepsilon_1 H a^2 k^{-2}\dot{\mathcal{R}}, \\ \Psi_B &= \varepsilon_1 \mathcal{R} + \varepsilon_1 a^2 k^{-2}\left[\ddot{\mathcal{R}} + H(2-\varepsilon_2)\dot{\mathcal{R}}\right], \end{cases}$$

Converting cosmological perturbations to NR is not a mystery/approximate anymore! Neither is conformal flatness!

Hubble numerics, perturbative regime



Let's try with:

Friedmann+Mukhanov-Sasaki solver

- BD vacuum imposed at $\mathcal{N}_b = -7$
- Started in slow-roll
- Quadratic potential with $\frac{m}{H} \simeq 0.15+5^0/_{0000}$ *oscillations*

Feed GRChombo at $\mathcal{N}_b = 0$

- 64^3 box with physical size $L = 16R_H$
- Synchronous gauge
- $\mathcal{R}$'s spectrum truncated at R_H

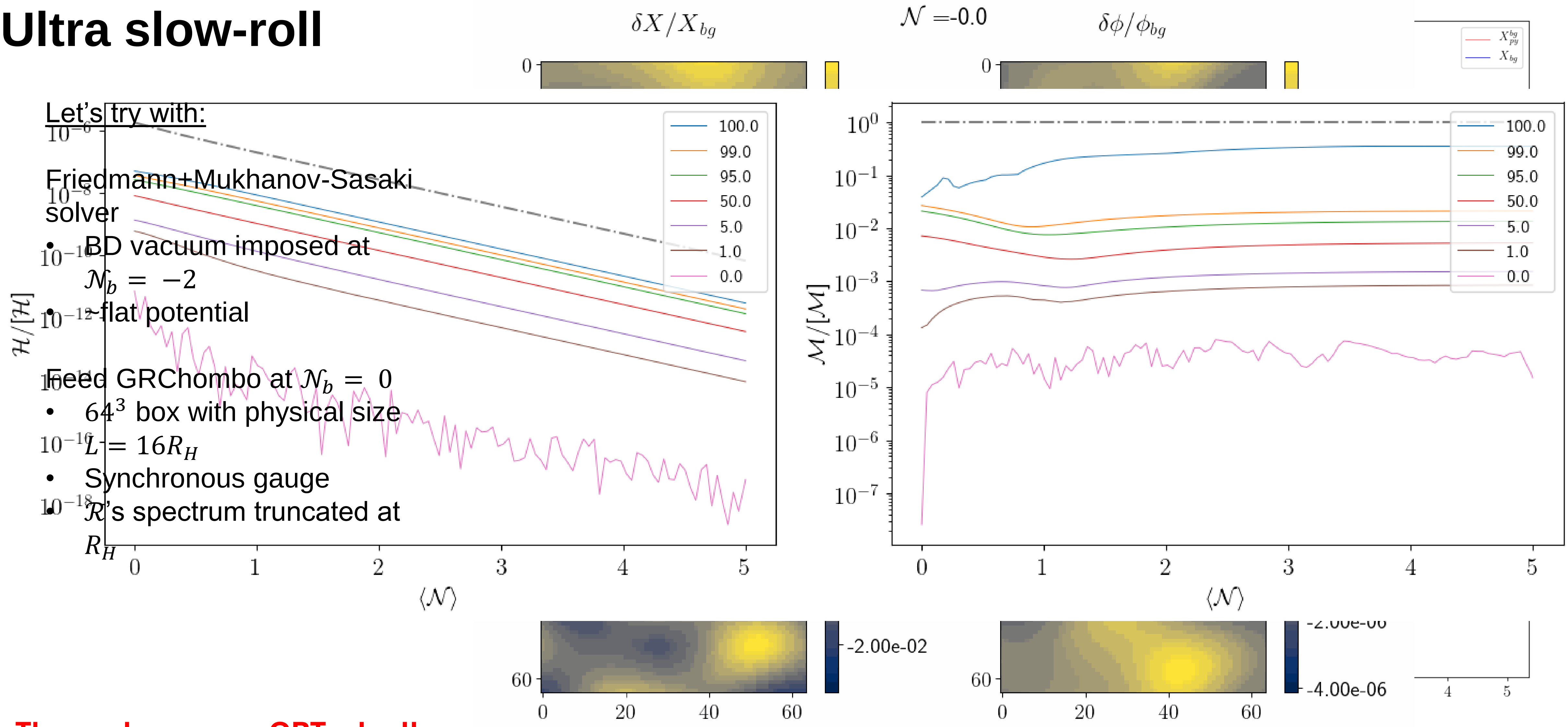
Hubble numerics, non-perturbative regime

Ultra slow-roll

Let's try with:

Friedmann+Mukhanov-Sasaki solver
- BD vacuum imposed at $\mathcal{N}_b = -2$
- ~flat potential

Feed GRChombo at $\mathcal{N}_b = 0$
- 64^3 box with physical size $L = 16R_H$
- Synchronous gauge
- $\mathcal{R}$'s spectrum truncated at R_H



These also run on GRTeclyn!!

This is a slide just to say that it’s officially running perfectly on GRTeclyn with at least the same speed and minor porting efforts!!

#chooseyourteam

Deepai.org ‘s GRChombo vs GRTeclyn

Incoming (py) package: STOchastic Inflationary Initial Conditions for GR

This generator will be open access and will provide

- ❑(BD vac) Random inflationary spacetime grids for different cosmological & NR gauges
- ❑Predictions from background and linear theory
- ❑GRC and GRT input files/ checkpoint files? See Cristian's.
- ❑**a diagnostic of how bad it is to assume classicality of the modes**

Future extensions:

- ❑Initial guess for solvers to go beyond first order in perturbation theory
- ❑Merge with Ericka's tensorial BD ICs.

+Incoming (py/YT) package: VIZualizer for Inflationary Relativity

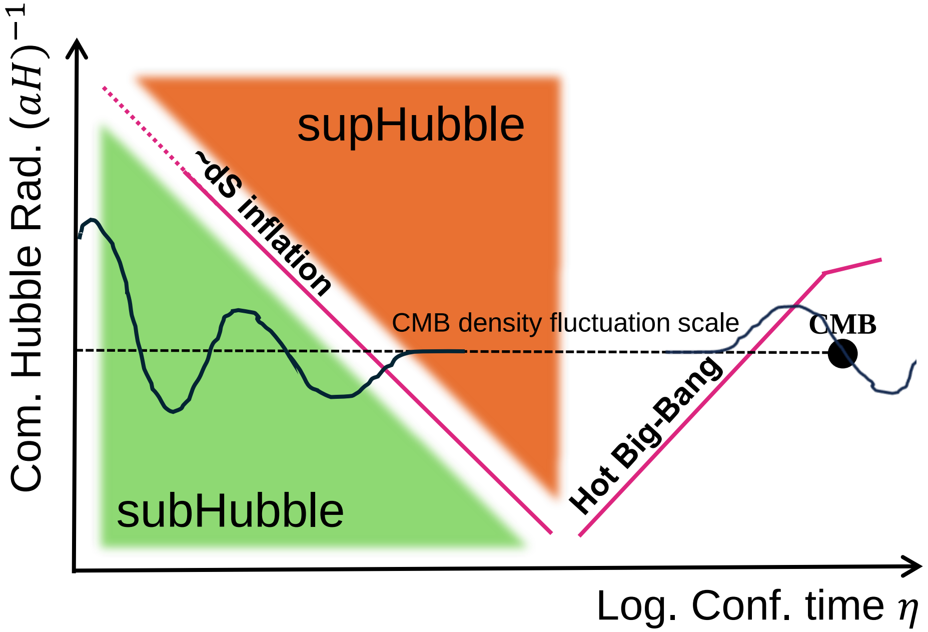
What’s next?

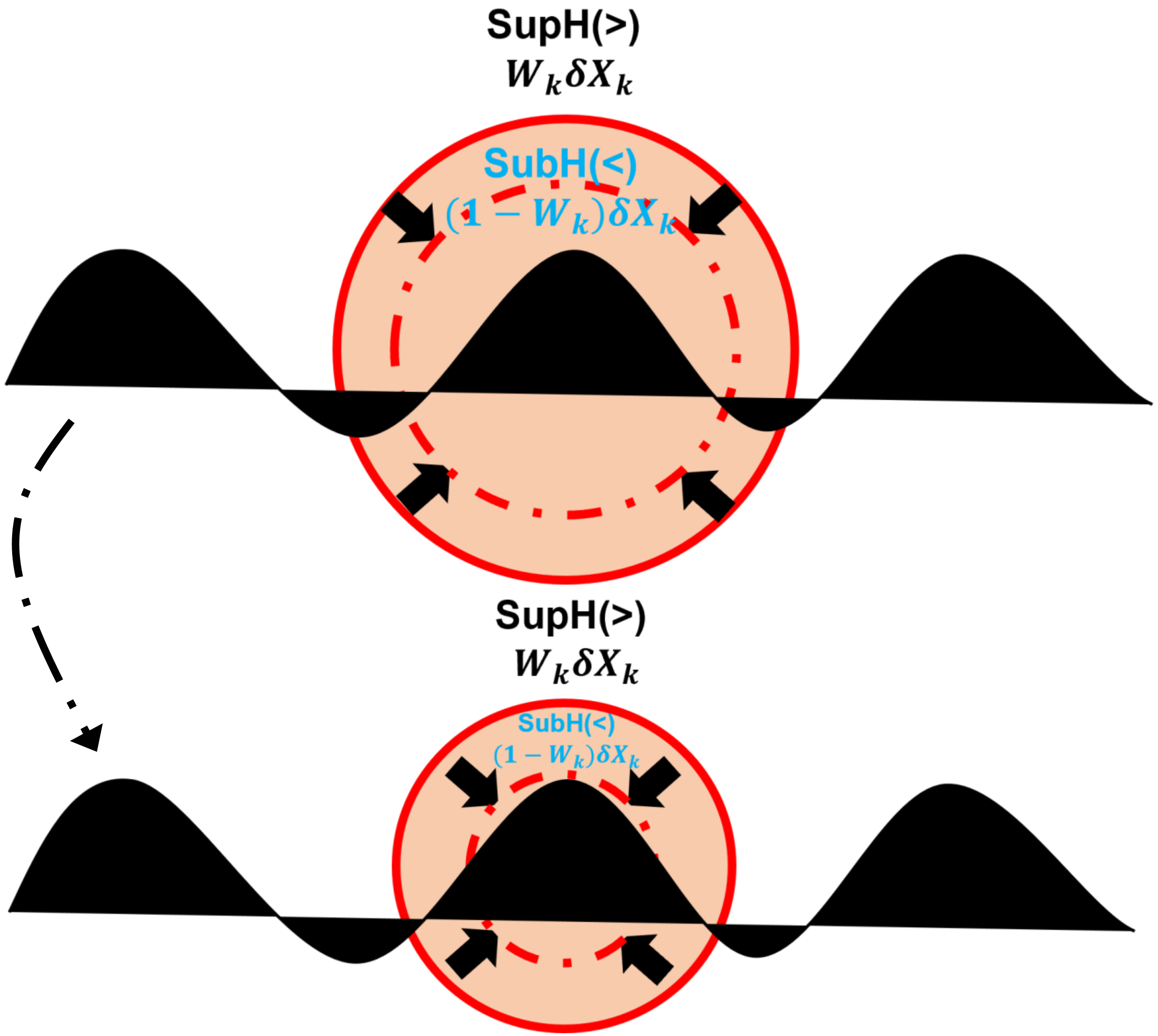
stochastic inflation

Illustration: *Stochastic General Relativity for Inflation*, deepai.org.

What is stochastic inflation and why necessary?

Initial entry of modes vs a continuous entry of modes…





…Effectively two systems; one slightly quantum, one classical.

+The IC technique is a bad windowing, too early to be classical for certain scales…

Langevin ADM equations

Previously in my PhD…

…How to source the 2pt function of all modes progressively (!= ICs)

$$
\begin{cases}
\dot{K} + \beta^i K_{,i} + \alpha^{|i}{}_{|i} - \alpha\left({}^3R + K^2\right) - M_{Pl}^{-2}\alpha\left(\frac{1}{2}S - \frac{3}{2}\rho\right) = \mathcal{F}^{-1}\{\mathcal{S}_K(\vec{k})\boldsymbol{\alpha}_{\vec{k}}\} + c.c., \\
\dot{\tilde{K}}_{ij} + 2\alpha\tilde{K}_{il}\tilde{K}^l{}_j + \beta^k\tilde{K}_{ij|k} - 2\beta_i{}^{|k}\tilde{K}_{jk} + \alpha_{|i|j} + \ldots = \mathcal{F}^{-1}\{\mathcal{S}_{\tilde{K}_{ij}}(\vec{k})\boldsymbol{\alpha}_{\vec{k}}\} + c.c., \\
\frac{1}{\alpha}\left(\dot{\Pi} + \beta^i\Pi_{|i}\right) - K\Pi - \frac{\alpha^{|i}}{\alpha}\phi_{|i} - \phi^{|i}_{|i} + \frac{dV}{d\phi} = \mathcal{F}^{-1}\{\mathcal{S}_\Pi(\vec{k})\boldsymbol{\alpha}_{\vec{k}}\} + c.c., \\
{}^3R + \frac{2}{3}K^2 - \tilde{K}_{ij}\tilde{K}^{ij} - 2M_{Pl}^{-2}\rho = \mathcal{F}^{-1}\{\mathcal{S}_{\mathcal{H}}(\vec{k})\boldsymbol{\alpha}_{\vec{k}}\} + c.c., \\
\tilde{K}^j{}_{i|j} - \frac{2}{3}K_{|i} - M_{Pl}^{-2}\mathcal{J}_i = \mathcal{F}^{-1}\{\mathcal{S}_{\mathcal{M}_i}(\vec{k})\boldsymbol{\alpha}_{\vec{k}}\} + c.c..
\end{cases}
$$

where
$$
\begin{cases}
\mathcal{S}_K = -\varepsilon_1\mathcal{S}_{\mathcal{R}} + c.c., \\
\mathcal{S}_{\tilde{K}_{ij}} = a^2\varepsilon_1\left(\frac{1}{3}\delta_{ij} - k^{-2}k_ik_j\right)\mathcal{S}_{\mathcal{R}} + c.c., \\
\mathcal{S}_\Pi = \sqrt{2\varepsilon_1}M_{Pl}\mathcal{S}_{\mathcal{R}} + c.c., \\
\mathcal{S}_{\mathcal{H}} = 0, \\
\mathcal{S}_{\mathcal{M}_i} = 0.
\end{cases}
$$

- **(small) Gauge-independent**
- **Constraints satisfied**
- **Long-λ in W only**
- **SUA limit → Standard SI**

All pointing to MS source:

$$\mathcal{S}_{\mathcal{R}}[\mathcal{R}_k, W_k, k] = \mathcal{R}_k W_k + [2\mathcal{R}_k + (3-\varepsilon_2)H\mathcal{R}_k]\dot{W}_k$$

Next order scalar induced gravitons! + BD sources see questions/paper.

A glimpse of the ongoing development of ISTORIz: Inflationary STOchastic Relativity Integrator for zeta

A GRTeclyn story..

Implementation in progress of a stochastic BSSN

$$
\begin{cases}
\partial_t X - \frac{2}{3} X\alpha K + \frac{2}{3} X\partial_k\beta^k - \beta^k\partial_k X = \mathcal{F}^{-1}\boldsymbol{\mathcal{S}}^{BSSN}_X\} \\
\partial_t\tilde{\gamma}_{ij} + 2\alpha\tilde{A}_{ij} - \tilde{\gamma}_{ik}\partial_j\beta^k - \tilde{\gamma}_{jk}\partial_i\beta^k + \frac{2}{3}\tilde{\gamma}_{ij}\partial_k\beta^k - \beta^k\partial_k\tilde{\gamma}_{ij} = \mathcal{F}^{-1}\{\boldsymbol{\mathcal{S}}^{BSSN}_{\tilde{\gamma}_{ij}}\} \\
\partial_t K + \gamma^{ij}D_iD_j\alpha - \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right) - \beta^i\partial_i K - 4\pi\alpha(\rho + S) = \mathcal{F}^{-1}\{\boldsymbol{\mathcal{S}}^{BSSN}_K\} \\
\partial_t\tilde{A}_{ij} - X\left[-D_iD_j\alpha + \alpha\left(R_{ij} - M_{Pl}^{-2}\alpha S_{ij}\right)\right]^{TF} - \alpha\left(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l_j\right) \\
\qquad -\tilde{A}_{ik}\partial_j\beta^k - \tilde{A}_{jk}\partial_i\beta^k + \frac{2}{3}\tilde{A}_{ij}\partial_k\beta^k - \beta^k\partial_k\tilde{A}_{ij} = \mathcal{F}^{-1}\{\boldsymbol{\mathcal{S}}^{BSSN}_{\tilde{A}_{ij}}\} \\
\partial_t\tilde{\Gamma}^i - 2\alpha\left(\tilde{\Gamma}^i_{jk}\tilde{A}^{jk} - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K - \frac{3}{2}\tilde{A}^{ij}\frac{\partial_j X}{X}\right) + 2\tilde{A}^{ij}\partial_j\alpha - \beta^k\partial_k\tilde{\Gamma}^i \\
\qquad -\tilde{\gamma}^{jk}\partial_j\partial_k\beta^i - \frac{1}{3}\tilde{\gamma}^{ij}\partial_j\partial_k\beta^k - \frac{2}{3}\tilde{\Gamma}^i\partial_k\beta^k + \tilde{\Gamma}^k\partial_k\beta^i + 2M_{Pl}^{-2}\alpha\tilde{\gamma}^{ij}S_j = \mathcal{F}^{-1}\{\boldsymbol{\mathcal{S}}^{BSSN}_{\tilde{\Gamma}}\} \\
\partial_t\Pi - \beta^i\partial_i\Pi - \alpha\partial_i\partial^i\phi - \partial_i\phi\partial^i\alpha - \alpha\left(K\Pi - \gamma^{ij}\Gamma^k_{ij}\partial_k\phi - \frac{dV}{d\phi}\right) = \mathcal{F}^{-1}\{\boldsymbol{\mathcal{S}}^{BSSN}_{\Pi}\}
\end{cases}
$$

Because AMReX has:

- ❑ GPU speed
- ❑ Fourier transforms
- ❑ stochastic children
  (see FHDeX)

Road to the SIGRid suite project: conclusions

Stochastic Inflation with GR on a GRID

→ **STOIIC-GR,** random draws for realistic inflationary spacetime perturbations ☑

- ❑ still looking at non-perturbative regimes before open access
- ❑ investigating the classicality assumption

→ **GRTeclyn-ISTORIz,** stochastic sources ☐

- ❑ ported ICs to GRTeclyn
- ❑ implementing each step's random draw

Take-away, article incoming:
we have generated the first non-perturbative scalar phenomena in full GR from the BD vacuum and this is very promising!

Thank you!
Questions?