Ventilator Pooling: Formulation and Data Sources

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1 Introduction

In the midst of the COVID-19 pandemic, health care providers and policymakers are facing sharp rises in demand for medical supplies in order to treat affected patients and combat the spread of the disease. In particular, the disease's potentially pronounced respiratory symptoms have led to increased demand for medical ventilators—expensive machines providing oxygen to patients with difficulty breathing. For COVID-19 patients in severe condition, access to a ventilator can mean the difference between life and death. As a result, the spread of the disease has led to a strong growth in demand for ventilators, ultimately increasing the risk of ventilator shortages, both in the United States and abroad [1, 2, 3, 4].

Still, local shortages of ventilators do not necessarily imply global shortages of ventilators. In the United States, the total supply of ventilators exceeds projected demand from COVID-19 patients—and several manufacturers are actively building extra ventilators. But the pandemic does not affect each state in the same way at the same time, leading to potential shortages at the local level (e.g., within a state, within a hospital). Such shortages can be alleviated by pooling the ventilator supply: states with a peak of infections in the past or weeks away can help other states facing an immediate shortage at basically no cost to their own constituents. Such inter-state transfers are already happening in practice—for instance, the states of Oregon, Washington and California have recently sent ventilators to the state of New York [5, 6, 7]. We propose an optimization formulation to study the benefits and tradeoffs of ventilator pooling, in the hope that policymakers will find it helpful as they grapple with the scarcity of life-saving devices.

2 Problem statement

The ventilator-pooling problem is formulated as a multi-period resource allocation problem. Let us consider a planning horizon of D days, indexed by $d = 1, \cdots, D$. We define a set of S states, indexed by $s = 1, \cdots, S$. For each day d and each state s, the demand for ventilators is denoted by $v_{s,d}$ (obtained, for example, from an epidemiological forecasting model). The goal is to determine where to allocate the federal stockpile of ventilators and how many ventilators to transfer between states, so that the supply of ventilators in each state is sufficient to meet the forecasted demand (or at least to minimize the local shortages).

The key objective, of course, is to maximize health outcomes for COVID-19 patients. We operationalize this objective by minimizing the costs of ventilator shortages. If we assume that the demand forecast is perfectly accurate and that states can perfectly allocate ventilators to the patients that need them the most, then minimizing ventilator shortages is a reasonable objective. To cope with demand uncertainty and with within-state inefficiencies, however, we also introduce a buffer, thus minimizing the cost of projected shortages (based on the demand forecast) and the cost of worst-case shortages (based on the pessimistic demand estimate with the additional buffer). We describe next the model formulation in greater detail.

3 Formulation

3.1 Basic formulation

Data. We consider the allocation problem over S states and D days. We are given the following data:

- $v_{s,d}$ is the demand for ventilators in state s on day d ($1 \leq s \leq S$, $1 \leq d \leq D$).
- b_s is the base supply of ventilators starting in each state s.
- n_d is the surge supply of ventilators distributed by the federal government on day d.
- $d_{s,s'}$ is the distance between state s and state s'.
- $\tau_{s,s'}$ is the lead time between state s and state s'.

Three comments are noteworthy. First, the surge supply n_d corresponds to the number of ventilators that are actually distributed by the federal government on day d—the question of how to manage a federal stockpile falls beyond the scope of this model. Second, the consideration of distances aims to encourage transfers between proximal states, to the extent possible. Third, we calibrate the distances such that $d_{s,s} > 0$ for each state s. This ensures that the model will not propose any transfer from any state into itself.

Decisions. We define integer decision variables as follows:

- $x_{s,d} \in \mathbb{Z}^+$ is the supply of ventilators in state s on day d ($1 \leq s \leq S$, $0 \leq d \leq D$).
- $y_{s,s',d} \in \mathbb{Z}^+$ is the number of ventilators sent from state s to state s' on day d.
- $w_{s,d} \in \mathbb{Z}^+$ is the shortage of ventilators in state s on day d relative to the demand $v_{s,d}$.
- $\Delta_{s,d} \in \mathbb{Z}^+$ is the shortage of ventilators in state s on day d relative to the demand with a buffer.
- $z_{s,d} \in \mathbb{Z}^+$ is the additional supply state s receives from the federal government on day d.

Parameters. We define the following parameters, which control the tradeoffs of the optimization problem:

- $f_{\max} \in [0, 1]$ is the maximum fraction of its base supply that each state is willing to share.
- $\alpha \in [0, \infty)$ is the percentage of projected demand that states would like to plan for (with buffer supply).
- $\lambda \in [0, \infty)$ is the LASSO regularization parameter on the transfers. *Default Value = 0.1.*
- $t_{\min} \in \mathbb{Z}^+$ is the minimum number of days a ventilator is in use. *Default Value = 10.*
- $V_{\max} \in \mathbb{Z}^+$ is the maximum number of ventilators a state can ship per day. *Default Value = 3000.*
- $\rho \in [0, 1]$ is a relative cost parameter. Each unit of supply that falls short of the projected demand is assigned a cost of 1. Each unit of supply that exceeds the demand but does not exceed the state's desired buffer supply is assigned a cost of ρ. *Default Value = 0.25.*

Constraints. We now present the constraints for the optimization problem:

- Initial supply for each state s:
$$x_{s,0} = b_s, \quad \forall s = 1, \cdots, S.$$
- Maximum amount of supply that each state s is willing to share on each day d:
$$x_{s,d} \geq (1 - f_{\max}) b_s, \quad \forall s = 1, \cdots, S, \; d = 1, \cdots, D.$$
- The surge supply must be distributed among states on each day d:
$$\sum_{s=1}^{S} z_{s,d} \leq n_d, \quad \forall d = 1, \cdots, D.$$
- The shortage variable is defined as the difference between demand and supply, if positive, for each state s and day d:
$$w_{s,d} \geq v_{s,d} - x_{s,d}, \quad \forall s = 1, \cdots, S, \; d = 1, \cdots, D.$$
- The buffer shortage variable is defined such that the total (actual plus buffer) shortage corresponds to the difference between the buffered demand and the supply, if positive, for each state s and day d:
$$w_{s,d} + \Delta_{s,d} \geq (1 + \alpha) v_{s,d} - x_{s,d}, \quad \forall s = 1, \cdots, S, \; d = 1, \cdots, D.$$
- Conservation of flow for each state s and day d. It ensures that today's supply is equal to yesterday's supply plus what is received today from the government and the other states, minus what is sent to

the other states (the term $\tau_{s',s}$ reflects the shipments' lead times):

$$x_{s,d} = x_{s,d-1} + z_{s,d} + \sum_{s'=1}^{S} y_{s',s,d-\tau_{s',s}} - \sum_{s'=1}^{S} y_{s,s',d}, \quad \forall s = 1, \cdots, S,\ d = 1, \cdots, D.$$

- No shipment from states facing shortages, for each state s and day d. It imposes that a state will not send ventilators to another state if it currently faces a shortage. To write these constratints, we define auxiliary binary variables $a_{s,d}$ indicating if there is a shortage in state s on day d.

$$w_{s,d} + \Delta_{s,d} \leq v_{s,d}(1+\alpha)a_{s,d}$$
$$\sum_{s'=1}^{S} y_{s,s',d} \leq V_{\max}(1 - a_{s,d})$$

- Minimum days a ventilator is in use, for each state s and day d. It imposes that any incoming ventilator, either from another state or from the federal government, cannot be shipped out for at least $t_{\min}$ days.

$$\sum_{d'=\max(1,d-t_{\min})}^{d-1} \left(z_{s,d'} + \sum_{s'=1}^{S} y_{s',s,d'} \right) \leq x_{s,d}, \quad \forall s = 1, \cdots, S,\ d = 1, \cdots, D.$$

Objective. As described previously, the problem is formulated as a bi-objective problem, minimizing total shortage costs and the amount of ventilator transfers. Ventilator shortages are assigned a weight of 1 for shortages relative to the projected demand and a weight $\rho \leq 1$ for shortages relative to the buffered demand. We formalize the bi-objective problem by means of a LASSO penalty on transfers.

$$\min \sum_{s=1}^{S} \sum_{d=1}^{D} (w_{s,d} + \rho\Delta_{s,d}) + \lambda \sum_{s=1}^{S} \sum_{s'=1}^{S} \sum_{d=1}^{D} d_{s,s'} y_{s,s',d}.$$

3.2 Planned extensions

Inter-state fairness. It is essential to ensure that the allocating mechanism does not disproportionately penalizes one state by favoring another state. We propose two fairness considerations.

- We penalize the maximum per-state shortage in the objective function, thus creating a tradeoff between individual state welfare and nationwide welfare.

$$\min \sum_{s=1}^{S} \sum_{d=1}^{D} (w_{s,d} + \rho\Delta_{s,d}) + \lambda \sum_{s=1}^{S} \sum_{s'=1}^{S} \sum_{d=1}^{D} d_{s,s'} y_{s,s',d} + \mu \max_{s=1,\cdots,S} \left\{ \sum_{d=1}^{D} (w_{s,d} + \rho\Delta_{s,d}) \right\}.$$

- We guarantee that no state is left much worse off than it would be on its own. To implement this constraint, we compute W_s as the shortage cost of state s in the absence of inter-state ventilator transfers—obtained by solving the model with transfers from the federal government but without inter-state transfers (by fixing the variables $y_{s,s',d}$ to 0). We then impose the following constraint, which ensures that, with inter-state transfers, each state s bears a cost of at most W_s plus $\beta\%$.

$$\sum_{d=1}^{D} w_{s,d} \leq W_s(1+\beta), \quad \forall s = 1, \cdots, S.$$

State participation. States are more willing to participate in a ventilator sharing program if it is guaranteed their interests can be protected. The additional constraint below ensures that, whenever a state shares a ventilator with another state, it guarantees that it will receive a corresponding ventilator when it will need one itself. This is written as follows, for each state s and day d:

$$w_{s,d} \leq \max\left(0, W_s - \sum_{s'=1}^{S}\sum_{d'=1}^{d-1} y_{s,s',d'}\right).$$

Uncertainty management. There is significant uncertainty on the exact number of ventilators needed in every state (obtained from epidemiological models) and the amount of ventilators available to the federal government (obtained from publicly available sources and news articles). We shall address this uncertainty in order to ensure the robustness of the proposed allocation to variations from the "best guess" values.

4 Parameter choices and data sources

4.1 Data Sources

The key data for the ventilator sharing problem relate to the supply and demand of ventilators, and we now describe our methodology in collecting this data.

Demand. Forecasting demand for ventilators has been the focus of significant recent efforts. We single out two particular models:

- our own epidemiological forecasting model (available on `covidanalytics.io`), which publishes daily state-by-state forecasts for cases, hospitalizations and ventilator usage.
- a forecasting model built by researchers at the Insitute for Health Metrics and Evaluation (IHME) at the University of Washington.

The two forecasting models agree on fundamental characteristics of the COVID-19 pandemic, but the first model tends to be a bit more conservative, and demand estimates can differ on a state-by-state basis. As a result, it is of interest to study the ventilator sharing problem under both demand forecasts, and we allow the user to select the demand data source of their choice to understand the impact on the recommended allocation. To the best of our ability, we are updating the demand forecasts daily.

Supply. As is often the case with medical equipment, ventilator supply in the United States is mostly decentralized, and it can be difficult to estimate how many ventilators are available in each state as well as at the federal level. For the base supply b_s in each state, we use inventory levels from a 2010 American Medical Association report [8], adjusted for population growth under the assumption that the number of ventilators per capita has remained constant in each state.

Of course, ventilators are a critical piece of medical equipment that can be used to treat both COVID-19 patients and non-COVID-19 patients. We currently assume that 50% of ventilator supply across the board is unavailable due to non-COVID-19 usage.

In addition, our model takes into account the daily availability n_d of ventilators at the federal level that can be distributed to the states based on need. Estimating the number of ventilators that can be available from publicly-available sources is both difficult due to limited data and politically fraught. We propose one estimate, described below; yet, given the underlying uncertainty, we allow the user to adjust the level of federal government availability in our interface, in order to explore the model's sensitivity to this input—the interplay between interstate transfers and federal intervention underpins critical decision-making.

The federal government maintains a strategic stockpile of medical equipment in warehouses around the country. The Society for Critical Medicine estimates that this stockpile contains at least 12,700 ventilators in its COVID-19 ICU Resource Availability Report [9]. Some recent news reports suggest a lower estimate of 10,000 based on some defects in the stockpiled equipment [10], while others suggest an estimate of 16600 based on older model repairs [11]. Based on these reference points, we estimate roughly 13500 available ventilators and assume that they can be deployed evenly over the course of the next month. In other words, we allow 450 ventilators to be deployed each day for 30 days (starting on day 4 to allow for lead times). This gradual release reflects potential operational constraints and strategic considerations of controlling the release of inventory in case of unexpected outbreaks.

Distances and lead times. We compute the interstate distance $d_{s,s'}$ as the Euclidean distance between the centers of states s and s', and we let the lead time parameter $\tau_{s,s'}$ equal 3 days for every pair of states. Our choice of a conservative uniform lead time for shipments is motivated by simplicity concerns. This could be improved, in future work, to better reflect efficiencies in the US shipping infrastructure.

4.2 Parameter calibration

User-selected parameters. Many of the parameters in our model reflect complex and important decisions in the hands of policymakers today. As a result, we allow users to modify some parameters to understand their interplay. To avoid overwhelming users, we restrict the set of user-modified parameters to the following:

- The fraction $f_{\max}$ of its base supply that each state is willing to share. We allow users to select $f_{\max}$ from $\{5\%, 10\%, 15\%, 20\%\}$. We observe diminishing returns when increasing $f_{\max}$ past 20%.
- The percentage α of projected demand that states would like to plan for with buffer supply. We allow users to select this parameter from $\{0\%, 5\%, 10\%, 20\%\}$. We note that this parameter can model different operational considerations, including uncertainty in the demand forecast as well as robustness to inefficiency in ventilator allocation within each state.
- As mentioned before, we let users reduce the federal daily supply by 10%, 25%, or 50%, since this amount is both difficult to estimate from public data and subject to many policy considerations.

Fixed parameters. We listed default values for other parameters in Section 3, but we now expand on the default value choices. The most important fixed parameter is the LASSO regularization parameter λ, which penalizes transfers based on distance. As λ tends to zero, transfers incur no cost other than rendering the ventilators unavailable during shipment, while as λ tends to infinity, transfers become heavily discouraged, starting with transfers between more distant states. We choose a value of $\lambda = 0.1$ in our current model, which corresponds to a small penalty on transfers (enough to discourage large or superfluous transfers, but not enough to materially impact the optimal allocation). A priority of our future efforts is to include the Pareto curve for the two terms in our objective to explore the tradeoff between ventilator shortages and inter-state ventilator transfers/.

The other fixed parameters do not have a strong effect on the optimal solution (typically less than 1% of the objective). We set $V_{\max}$ to 3000, exceeding the forecasted shortage of any state, ρ to 0.25, and $t_{\min}$ to 10 days. The latter value is also used as the average time on ventilation in the demand forecast. The constraint that ventilators cannot be transferred too many times in a 10-day period is almost never binding, as shipping a ventilator from state to state means it is unavailable for several days at a time while in transit.

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