Supplementary information to paper Social impact of CAVs – coexistence of machines and humans in the context of route choice

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1 Appendix

In which settings can we reasonably evaluate the impact of sudden introduction of CAVs?
We propose to evaluate the impact of CAVs only in settings in which HDV choices stabilize. For this purpose, we conducted exhaustive experiments supporting our selection of the ϵ-Gumbel model, which are described in the following.

1.1 Stabilization in HDV-only systems

We present the results of a systematic experimental (in silico) study of equilibrium properties in the case of two routes. The default parameters as well as their alternative values are given in Supplementary Table 1. As logit models are the most important models accounting for human choice in transportation settings we set the default *Model Type* to 'Logit'. The *Dispersion Parameter* is by default $\gamma = 1/\beta = 0.2$. The 'Free Flow' *Initial Knowledge* seems to be reasonable, see Methods, however we also tested optimistic (0 min) and pessimistic (25 min) initial travel time estimates. Given the *Initial Knowledge*, we set 'Random' as the default *Initial Choice*, however 'ArgMin' also can be used. Our experiments assume that agents learn from experience only, however we compare this to learning from full knowledge of the travel times. The number of HDVs is fixed at 1000, which corresponds to moderate congestion. Finally, the learning rate is typically 0.2 [1] and exploration rate 0.1.

In every experiment, we vary two parameters and display the travel times on routes A and B. We conducted the following simulations and arrived at the following conclusions:

1. The impact of learning rate and dispersion in the logit model (Suppl. Fig. 1). We conclude that the default parameters $\gamma = 0.2$, $\alpha = 0.2$ result in the system stabilizing (with travel times on both routes substantially different). However, for certain other combinations of parameters the system either exhibits oscillations or fails to converge to a state similar to SUE.
2. The effect of oscillations is particularly pronounced in the full information case. Suppl. Fig. 2 demonstrates this effect, which was theoretically studied in [2]. We note that in the full information case equation (6) is replaced by
$$T_r(i) \leftarrow (1-\alpha)T_r(i) + \alpha t_r(i)$$
for $r \in \{A, B\}$, i.e. estimates on *both* the used and unused routes are updated.
3. By default we assume that the initial knowledge of every agent corresponds to free flow times. What might happen if this assumption is altered is shown in Suppl. Fig. 3. Interestingly, pessimistic initial knowledge results in agents not learning at all for high γ. However, both FreeFlow and Optimistic cases seem to converge to a situation when the agents have learnt the equilibrated travel times.
4. We generally assume that the initial choice of the route is random. Suppl. Fig. 4 demonstrates that the choice Argmin (selecting the route with a better initial estimate) leads to the same results.
5. Suppl. Fig. 5 shows that the Logit and ϵ-Gumbel models (note we use the names ϵ-Gumbel and GumbelEps interchangeably) exhibit similar behaviour in general. The Gumbel model, on the other hand, sometimes converges to a state where the equilibrium travel times remain 'unlearned'. The Gumbel model differs from the default ϵ-Gumbel model by replacing equation (8) by
$$r(i) = \underset{r \in \{A,B\}}{\arg\min}\, U_r(i), \tag{A1}$$
i.e. by removing the extra random exploration. The Logit model, in turn, differs from the default ϵ-Gumbel

model by replacing (8) with (A1) and by sampling $\varepsilon_r(i)$ in (7) from $Gumbel(\mu, \beta)$ every day anew, independently for every agent i and route r (instead of keeping them fixed and sampled once on day 1). Equivalently [13], equation (8) can be replaced by choosing A with probability given by the logit formula

$$P_A = \frac{\exp(-T_A(i)/\beta)}{\exp(-T_A(i)/\beta) + \exp(-T_B(i)/\beta)}, \qquad \text{(A2)}$$

and $P_B = 1 - P_A$.

This standard logit model has a different interpretation to ϵ-Gumbel. Namely, the heterogeneity of choices is not caused by fixed different tastes which on average produce logit proportions on alternatives, but rather by random everyday fluctuations of driving conditions or agent preferences.

6. Suppl. Fig. 6 shows the behaviour of ϵ-Greedy Model for various values of parameters α and γ. ϵ-Greedy model is the limit of ϵ-Gumbel for $1/\gamma = \beta \to 0$. More precisely, we replace equation (8) by

$$r(i) = \begin{cases} \arg\min_{r\in\{A,B\}} T_r(i) & \text{with probability } 1-\epsilon, \\ \text{uniformly random} & \text{with probability } \epsilon. \end{cases}$$

It can be seen that wild oscillations prevail in the case of full knowledge and some instability can be observed for learning from experience only. We conclude that if learning never stops, ϵ-Greedy choice might become unstable.

7. This inherent instability is shown in Suppl. Fig. 7. Note that for $\alpha = 0.2$ and $\epsilon = 0.1$ what appeared as a stable pattern during the first 200 days turned out to be unstable in a longer-term horizon. We conclude that in order to consider greedy choice as a baseline, we need to either halt human learning at some point or be very cautious in order not to mistake the instability of the model for the impact of CAVs. Because of this instability as well as its unrealistic interpretation of humans with no biases whatsoever, we do not consider this model in further experiments.

1.2 Conclusion

We select the multi-agent ϵ-Gumbel model with default parameters listed in Table 2 as our benchmark motivated as follows. On the one hand, it allows for certain diversity among agents and renders the agents' decisions deterministic up to ϵ-size exploration (note that ε_r determining human tastes are sampled only once, while exploration with probability ϵ may occur every day), which seems to be a reasonable tradeoff between simplicity and realisticity. On the other hand, it stabilizes across a wide range of reasonable parameters. Importantly, the ϵ-Gumbel model and the more common standard Logit model result in similar proportions of choices as confirmed by our experiments. However, as motivated above, we prefer ϵ-Gumbel as it seems to be a simple model with sounder behavioural underpinnings.

1.3 Time-dependent plots of CAV-HDV interaction

In Suppl. Fig. 8 and 9 we present the time dependent plots for a selection of models and various parameters upon which the plots in the main text are based (for Model = GumbelEps). In particular, we see that the logit model and the ϵ-Gumbel model behave similarly with a slight difference in variance of vehicle counts or travel times. Moreover, for the default parameters the flows and travel times eventually stabilize after Day-M (day 200). Nonetheless, for certain CAV shares, the malicious and disruptive strategies can be executed most efficiently keeping the system permanently out of equilibrium by introducing oscillations visible in the plots.

1.4 Reproducibility and statistical significance

For parameters used in Fig. 3 we reran the experiments independently 10 times obtaining satisfactory reproducibility, Suppl. Fig. 10. In particular, for the combinations of parameters reported in Fig. 2 we obtained statistical significance of the difference between mean HDV travel time averaged over days $(101-200)$ and mean HDV travel time averaged over days $(301-400)$ as well as between mean HDV travel time averaged over days $(101-200)$ and mean CAV travel time averaged over days $(301-400)$. We used the paired two-sided t-test to test the null hypothesis of equality of the respective averaged mean travel times and obtained in every case reported in Fig. 2 p-values less than 0.001.

References

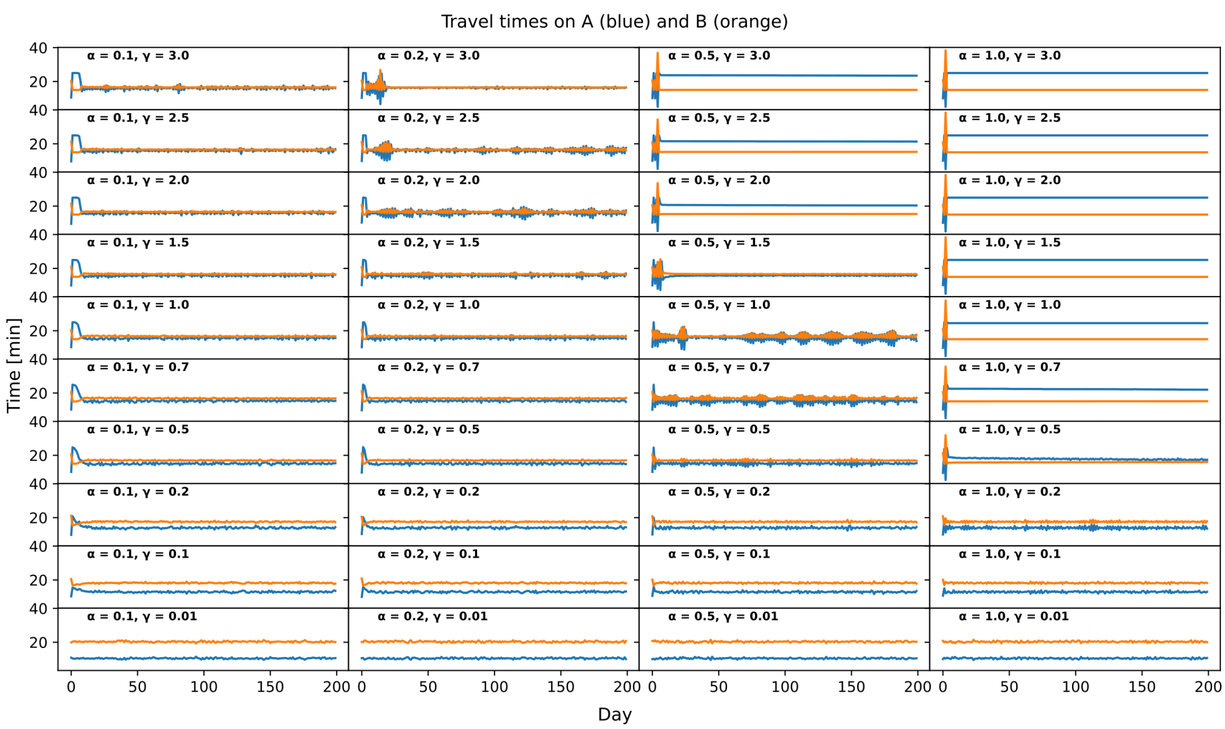
[1] Bogers, E. A. I., Bierlaire, M. and Hoogendoorn, S. P. (2007). Modeling Learning in Route Choice. Transportation Research Record, 2014(1), 1-8. https://doi.org/10.3141/2014-01

[2] Horowitz, Joel L. (1984). The stability of stochastic equilibrium in a two-link transportation network. Transportation Research Part B: Methodological, Elsevier, vol. 18(1), pages 13-28, February.

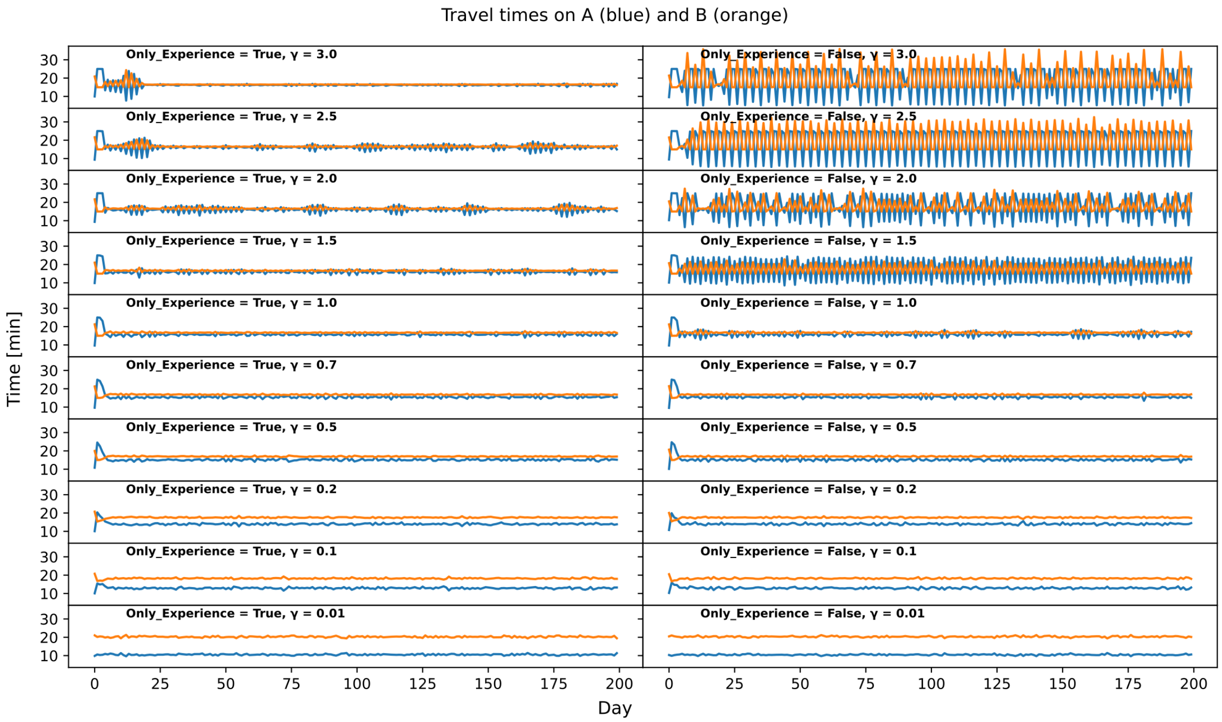
2 Supplementary Tables and Figures

Supplementary Table 1: HDV parameter values used to study the properties of equilibria.

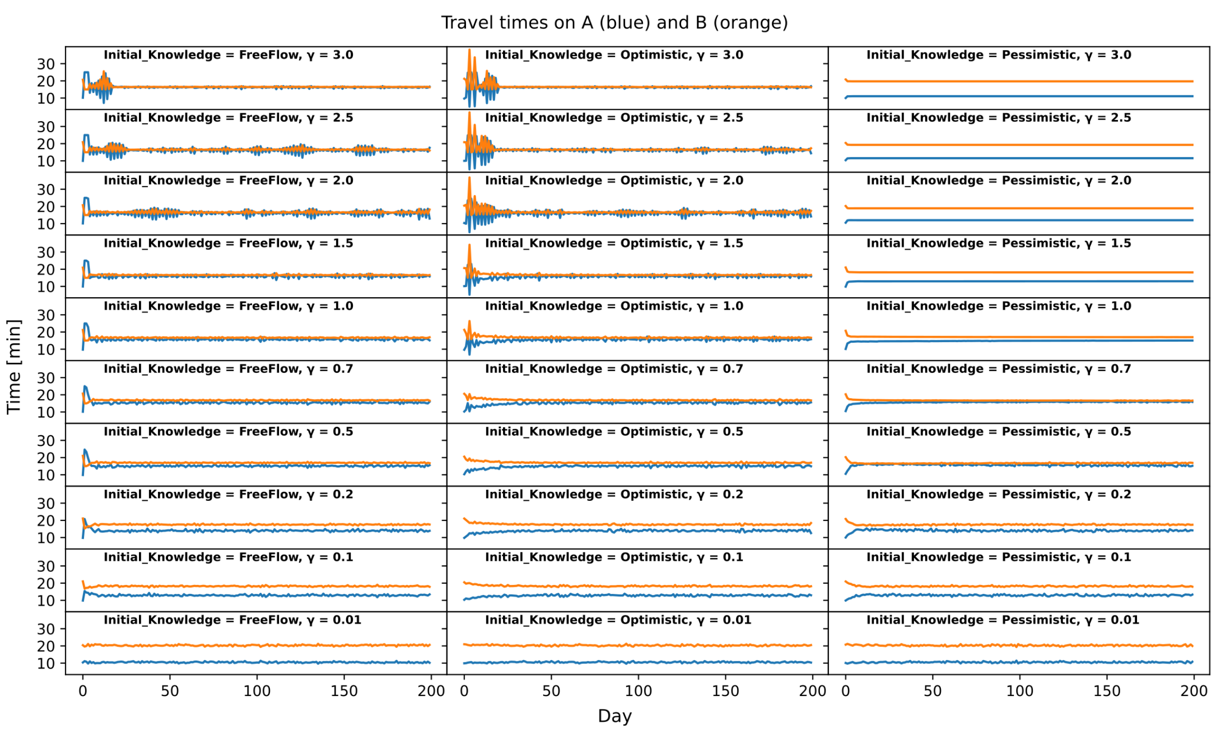
Parameter	Default Value	Alternative Values
Model Type	Logit	Gumbel, ϵ-Gumbel, ϵ-Greedy
Model Parameter γ (Logit parameter by default)	0.2	0.01 – 3.0
Initial Knowledge	Free Flow	Optimistic, Pessimistic
Initial Choice	Random	ArgMin
Learning From Experience Only	True	False
HDV number	1000	–
Learning rate of HDVs (α)	0.2	0.1 – 1.0
Exploration rate of HDVs (ϵ)	0.1	0.01 – 0.5



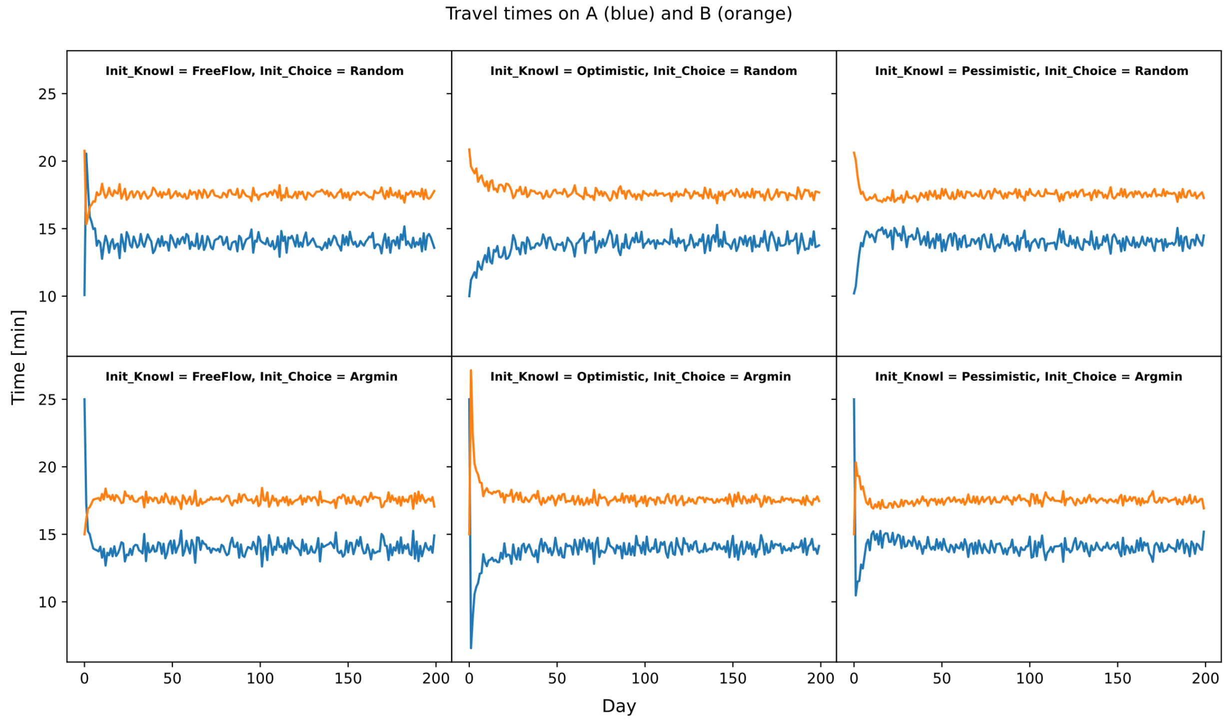
Supplementary Figure 1: Travel time in the logit model for different learning rates and logit parameters.



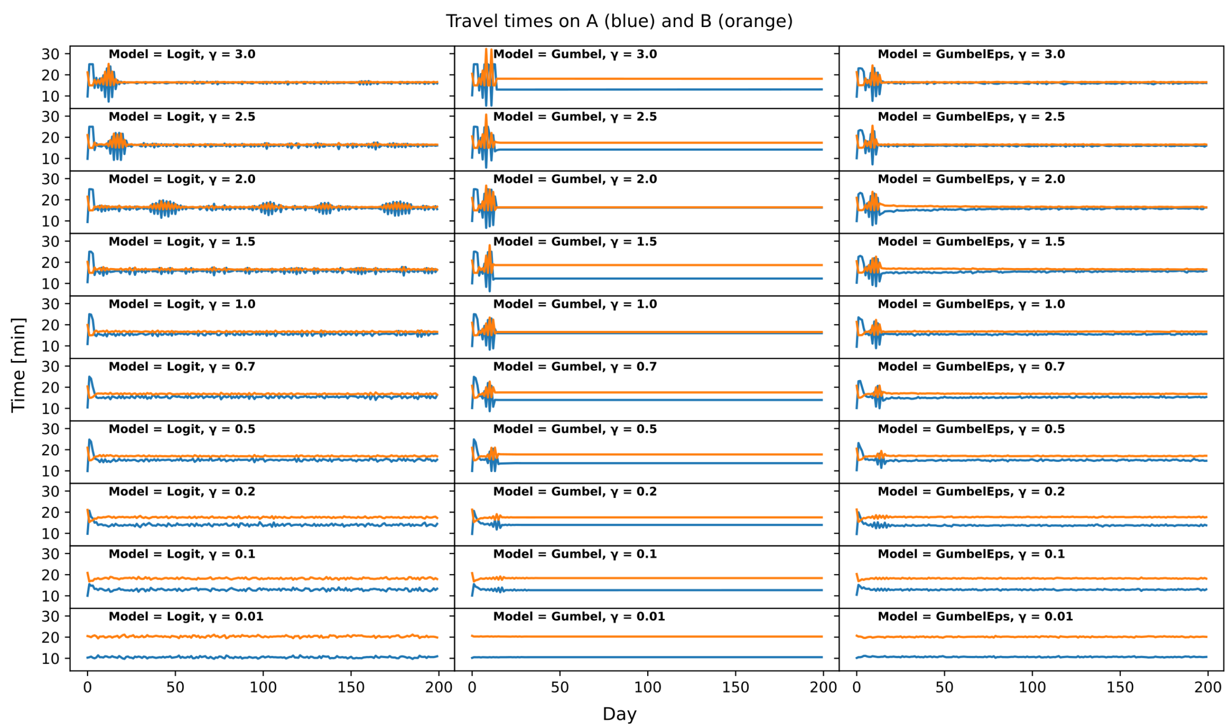
Supplementary Figure 2: Oscillations in the logit model for different dispersions in the case of learning from experience only and learning from full information.



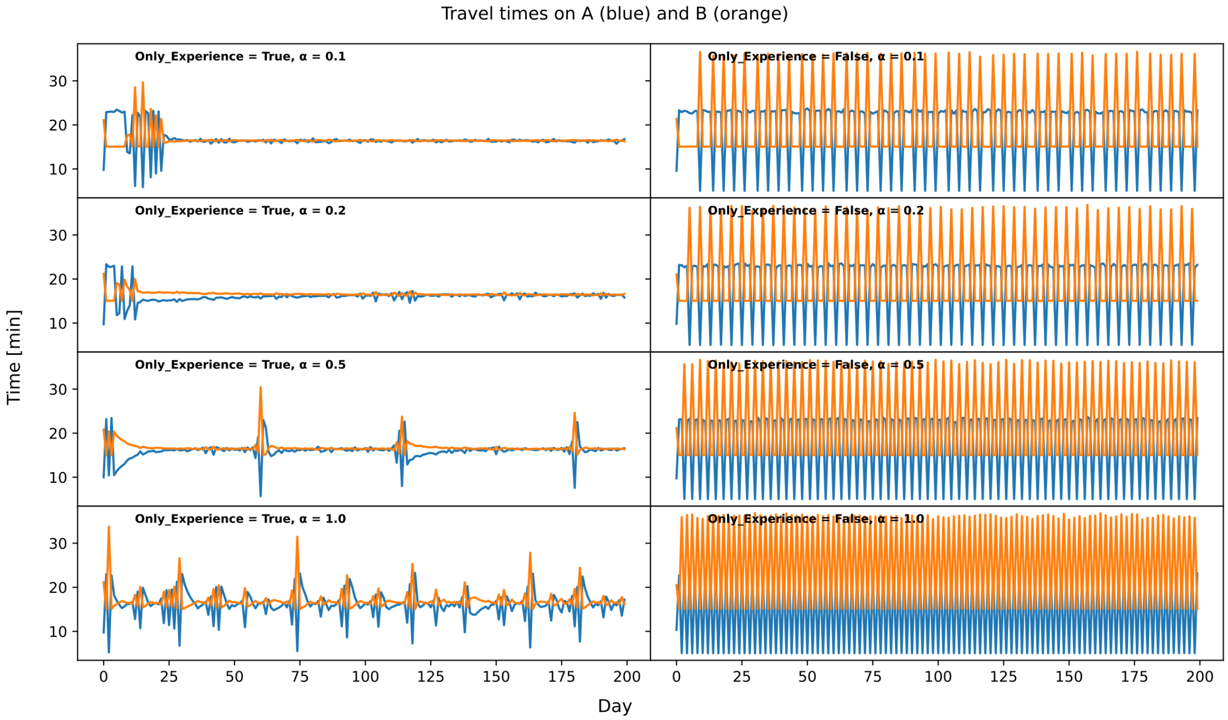
Supplementary Figure 3: Travel time in the logit model for different Initial Knowledge and logit parameters.



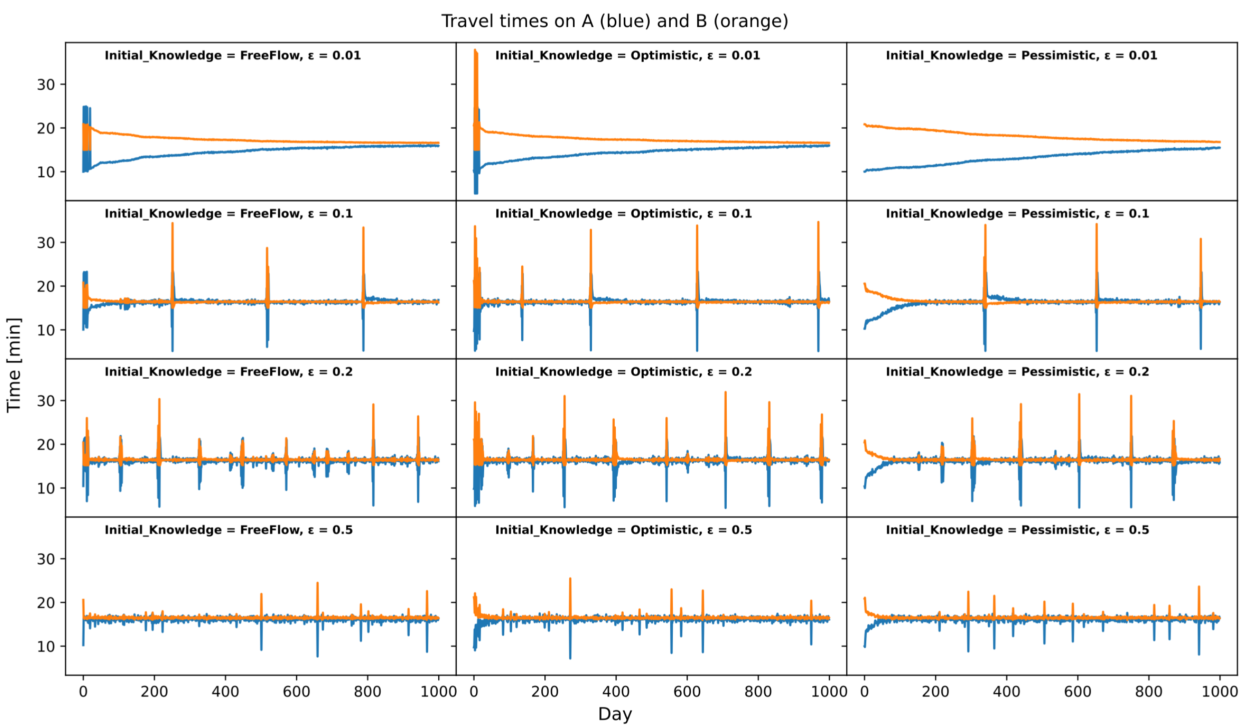
Supplementary Figure 4: Travel time in the logit model for different Initial Knowledge and Initial Choice modes.



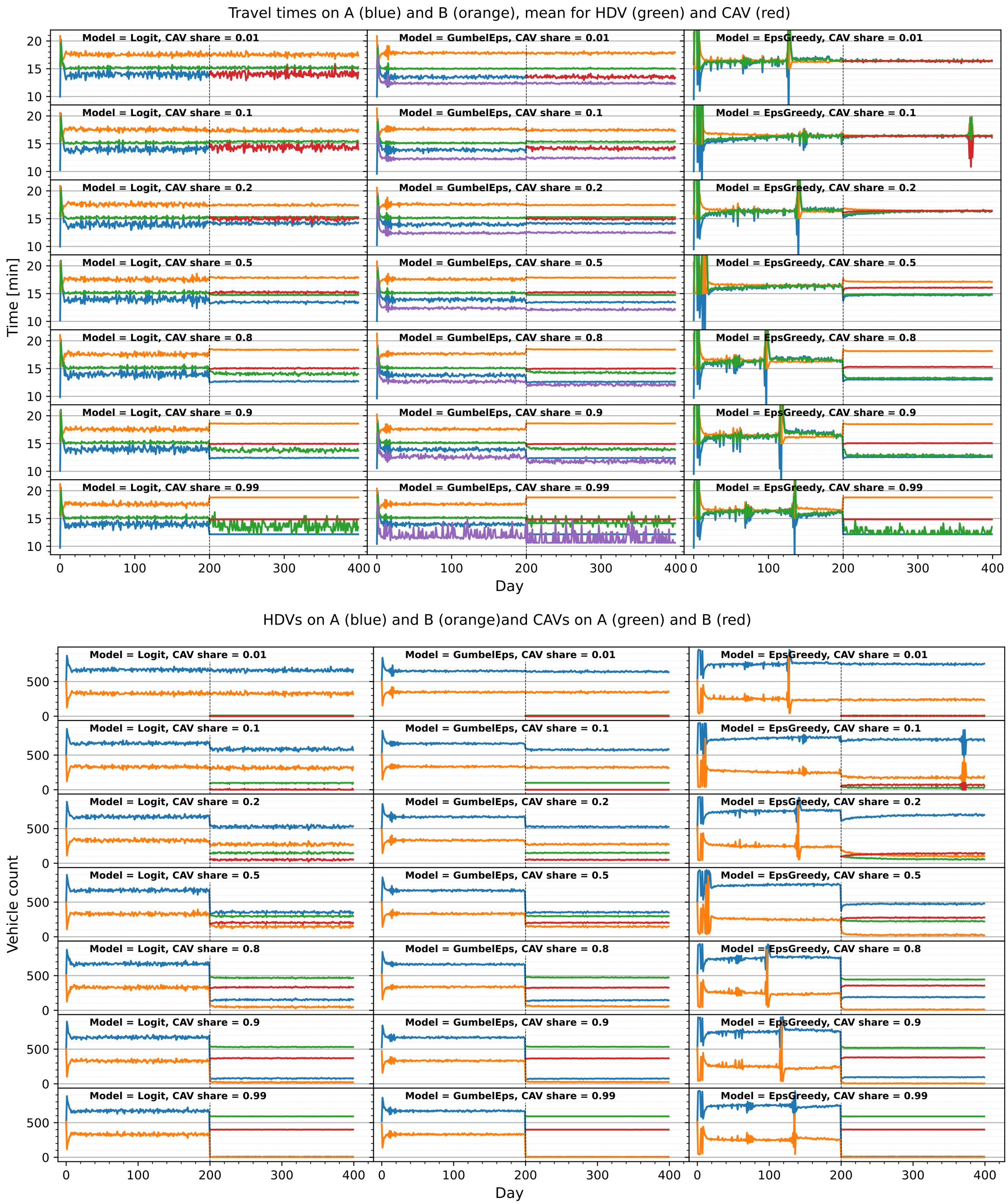
Supplementary Figure 5: Travel time for different dispersions in different logit-type models.



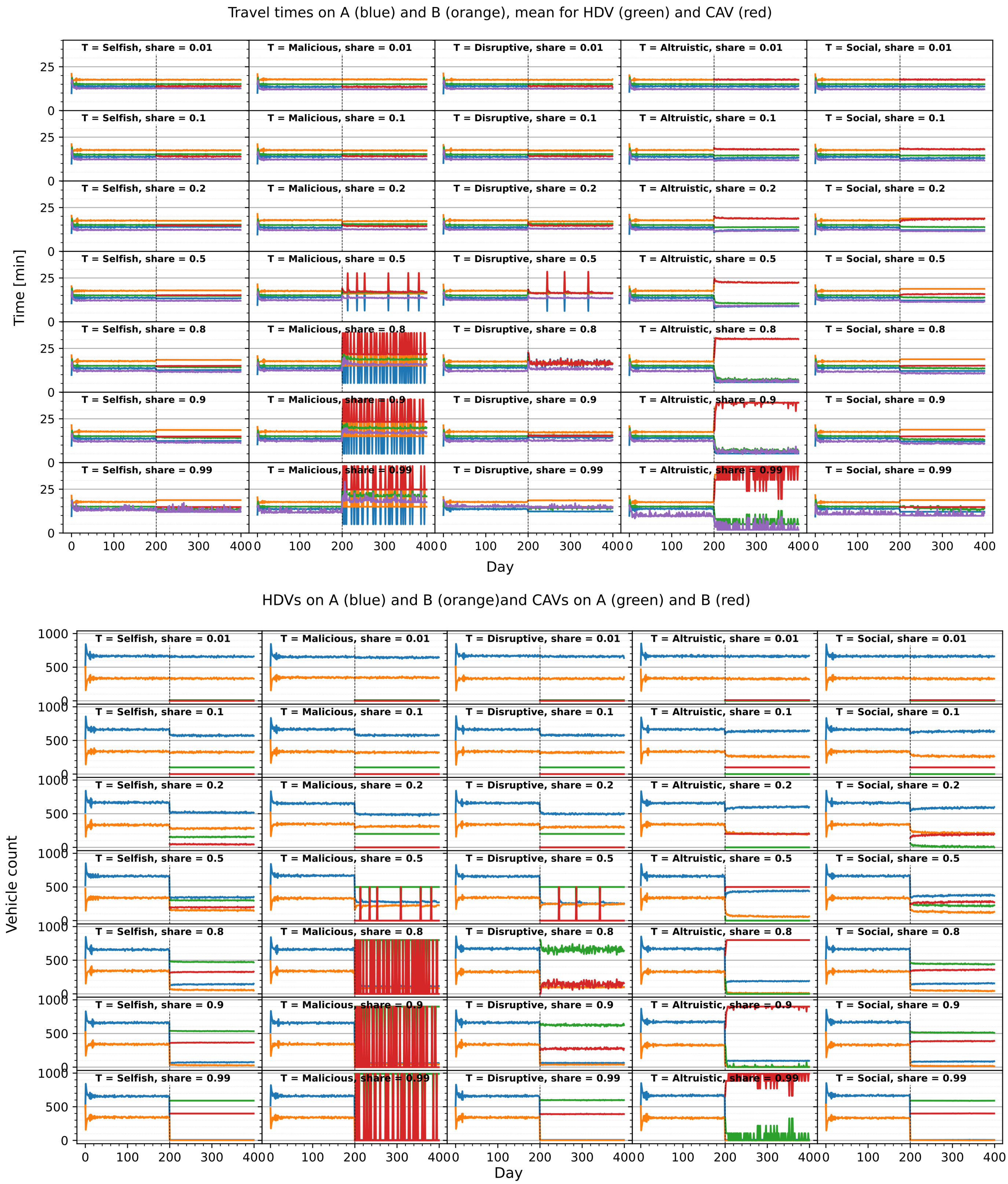
Supplementary Figure 6: Travel time in the ϵ-greedy model for different learning rates and experience.



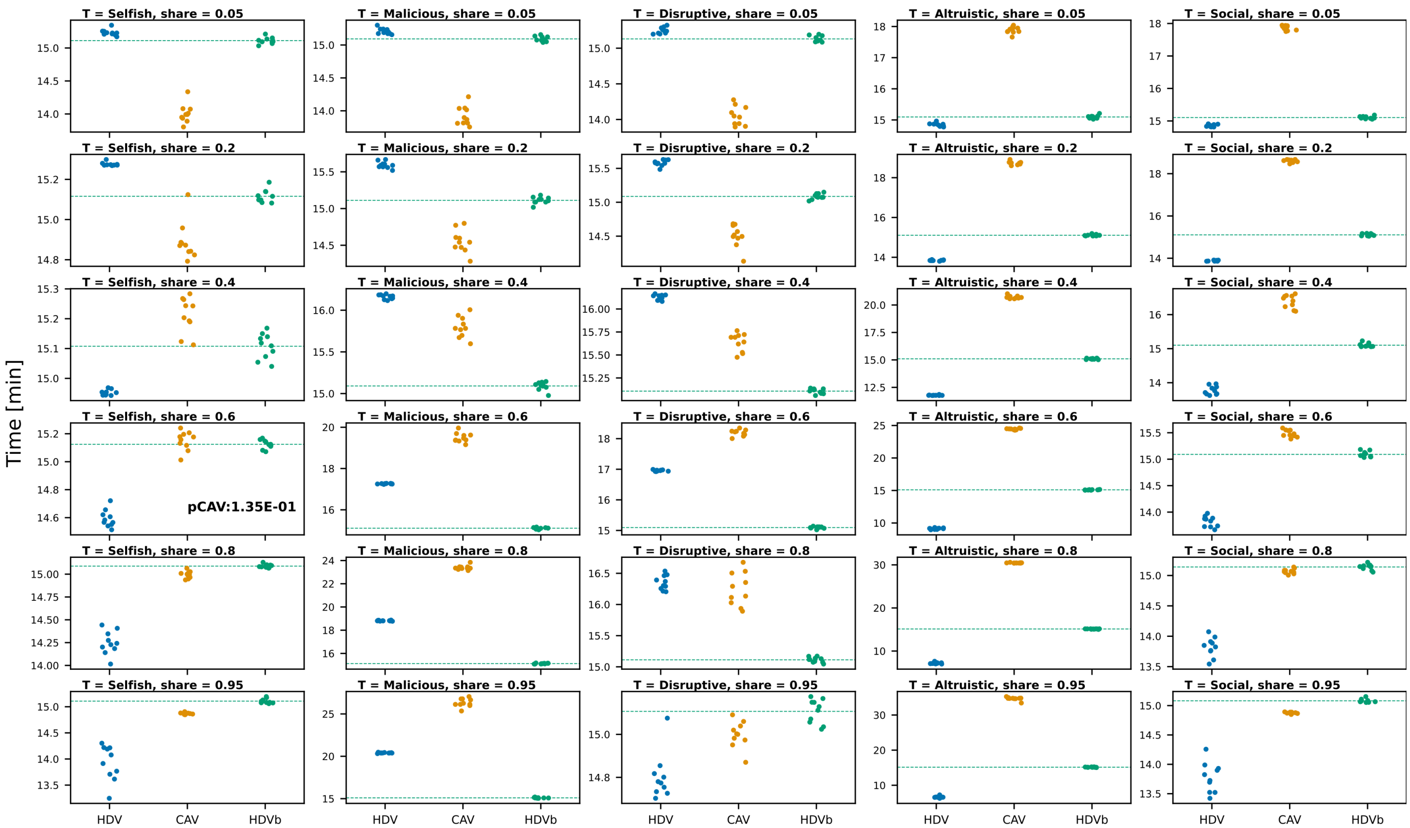
Supplementary Figure 7: Long term behaviour of the system with ϵ-greedy human choice.



Supplementary Figure 8: Travel times and vehicle counts for different human behaviour models and CAV shares and selfish strategy. Purple – mean perceived travel time.



Supplementary Figure 9: Travel times and vehicle numbers for different CAV optimization targets and CAV shares in the ϵ-Gumbel model. Purple – mean perceived travel time.



Supplementary Figure 10: Mean travel times of HDVs averaged over days $301 - 400$ (HDV), mean travel times of CAVs averaged over days $301 - 400$ (CAV) and mean travel times of HDVs averaged over days $101 - 200$ (HDVb) obtained in 10 independent experiments for different strategies and CAV shares. The green dotted line is the mean of the sample of HDVb. The mean of HDV vs. mean of HDVb as well as mean of CAV vs. mean of HDVb are all statistically different with $p < 0.001$ (two-tailed paired t-test) except for the selfish strategy for CAV share 0.6. Only the p-values larger than 0.001 are displayed. $pCAV$ – p-value for the hypothesis of equality of CAV and HDVb. $pHDV$ – p-value for the hypothesis of equality of HDV and HDVb.