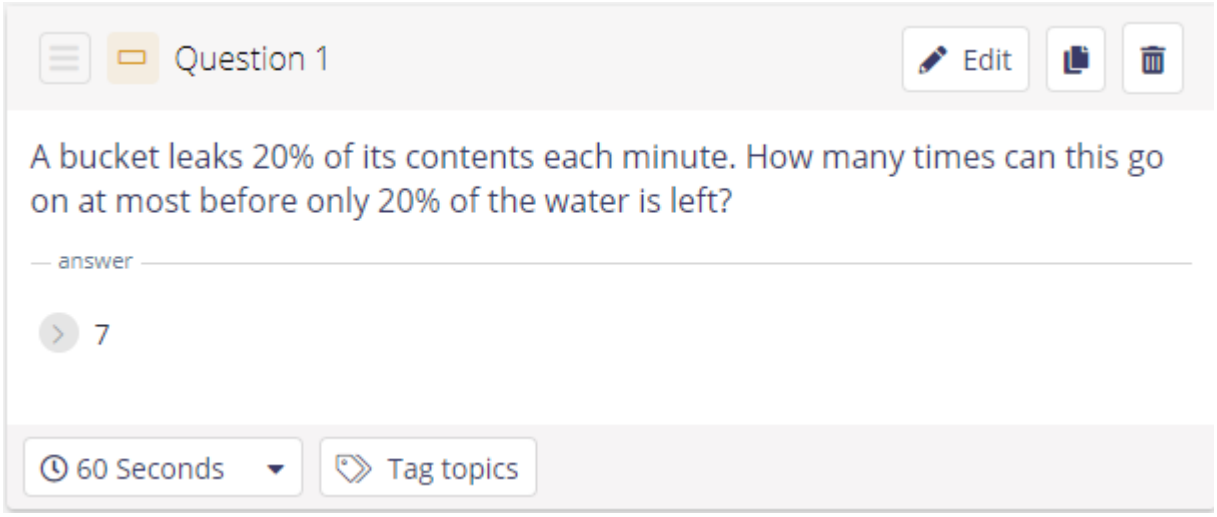
Solutions to the Introduction to CPSTA Quiz

Note that the question orders and options will vary in the actual event.

Question 1

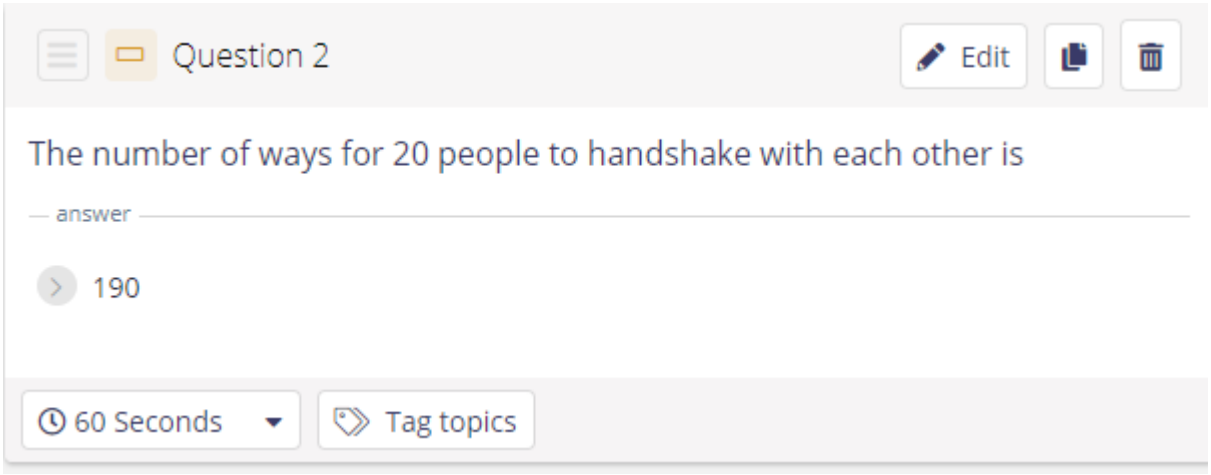


Notice that this is compounded. So after the first minute, the bucket has leaked 20%. After the second minute, it has leaked 20% (of 20%). Hence, after the n^{th} minute, the bucket has $\left(\frac{4}{5}\right)^n$ of water left, and we need to find *n* such that it's equal to 1/5. This means that

$$\left(\frac{4}{5}\right)^n = \frac{1}{5}$$
$$n \ln\frac{4}{5} = \ln\frac{1}{5}$$
$$n = \frac{\ln\frac{1}{5}}{\ln\frac{4}{5}} \cong 7.2$$

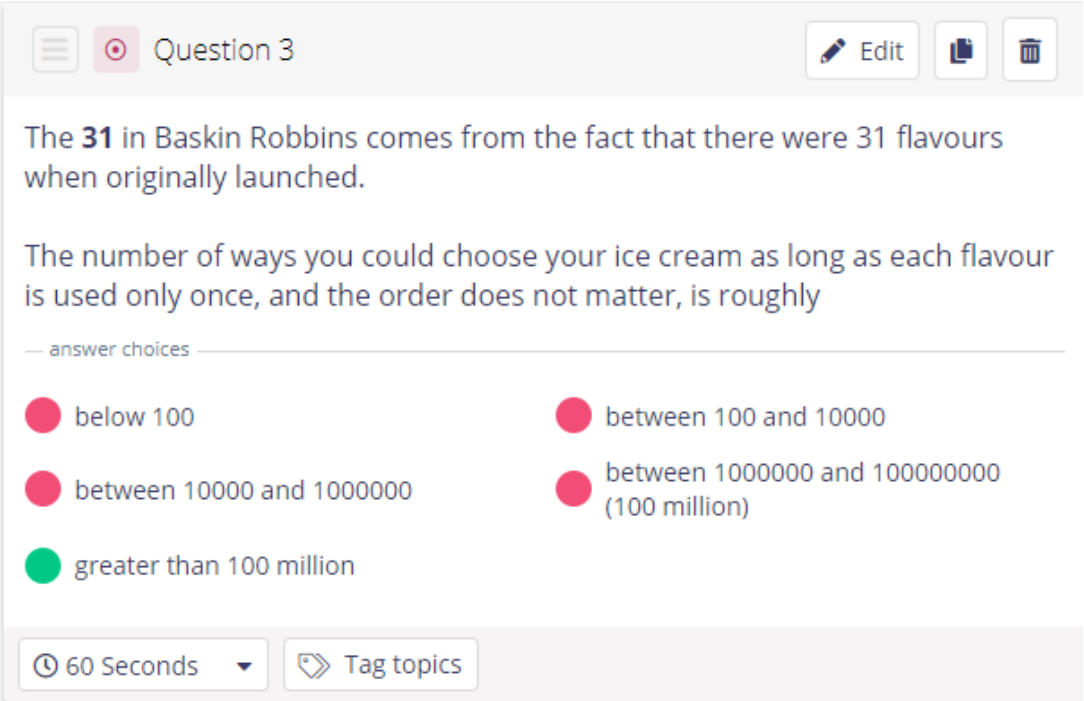
As we are looking for the number of times this can go on at **most,** the answer is 7.

Question 2



Each of the 20 people will handshake with 19 people. But then there will be duplicates – for example A handshaking B is the same has B handshaking A. Hence, we divide the result by 2: 380/2 = 190.

Question 3



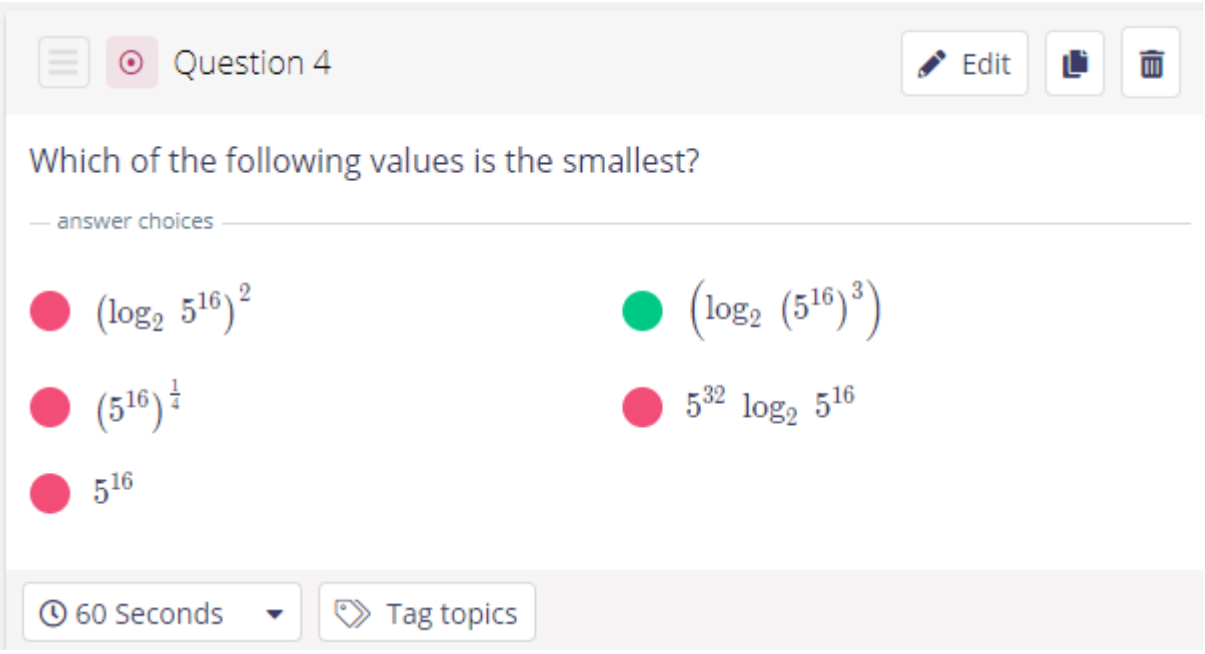
The number of ways to choose r items from n is $\binom{n}{r}$, also called as n choose r.

We can also show that (proof not included)

$$\sum_{r=1}^{n} \binom{n}{r} = 2^n$$

Notice that this is the number of ways to choose an ice cream with at least one flavour, up to choosing all 31 flavours of them. And hence the answer is 2^{31}, which is over two billion.

Question 4



The values being asked are simply too large, and hence it would help to write them down in terms of n:

A: $(\log_2 n)^2$

B: $\log_2 n^3$

C: $n^{\frac{1}{4}}$

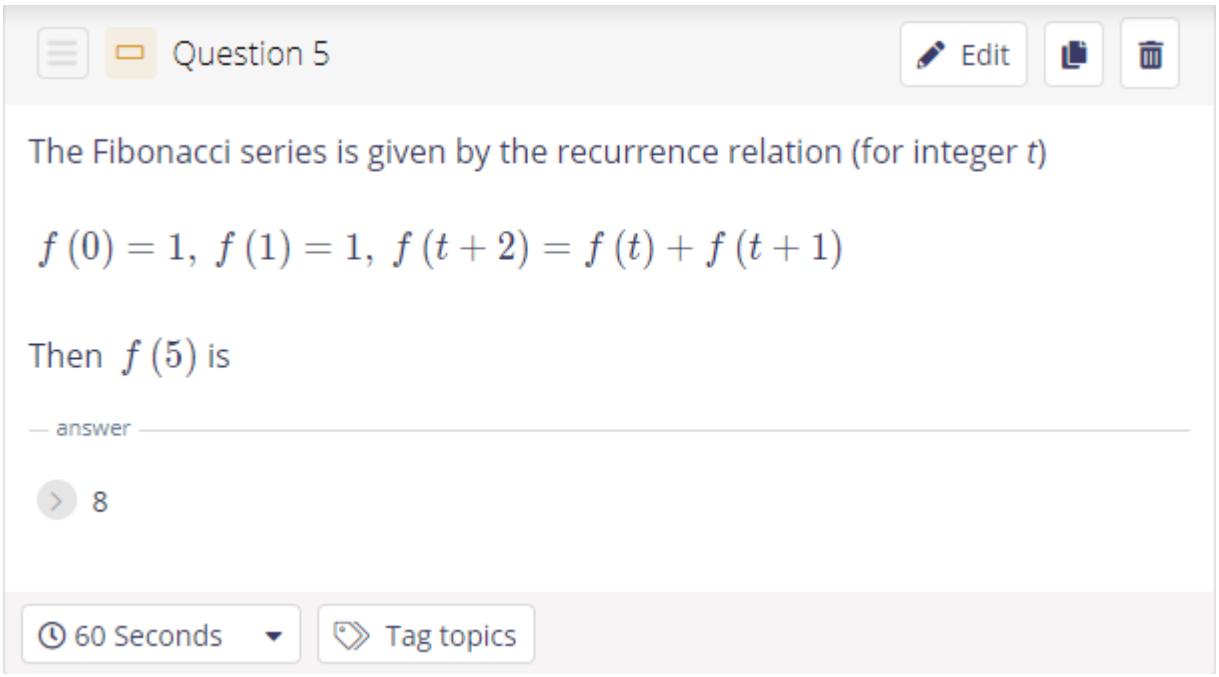
D: $n^2 \log_2 n$

E: n

Firstly, $\log n < n$, and hence we can concentrate on A and B. Now notice that B can be reduced to $3 \log_2 n$, but that results in the comparison $\log_2 n > 3$, which happens when $n > 8$. Hence for larger n, A > B and hence B is the smallest (for $n = 5^{16}$).

(this is the essence of the "Big-O" notation)

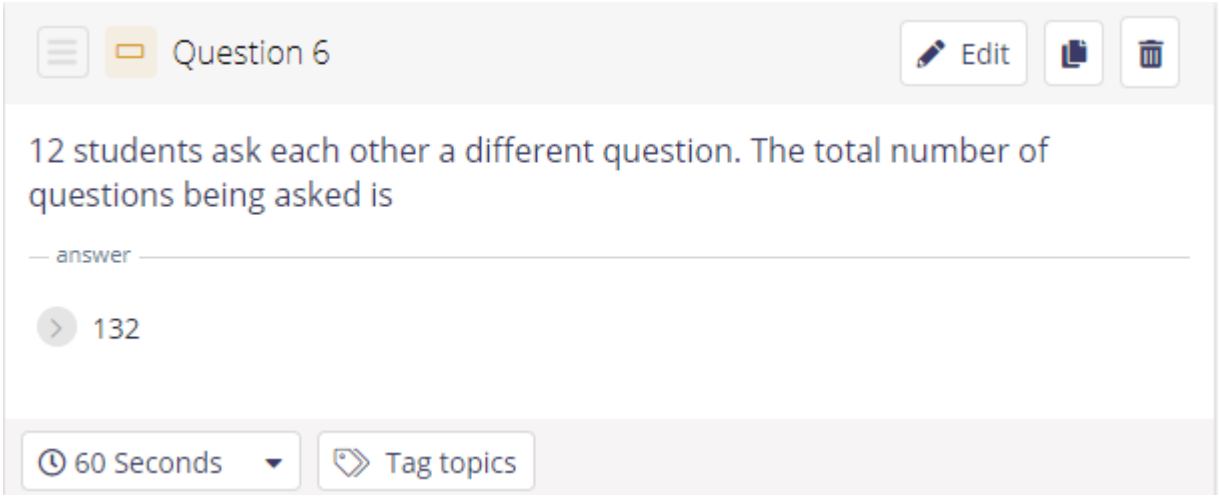
Question 5



Just run through the sequence:

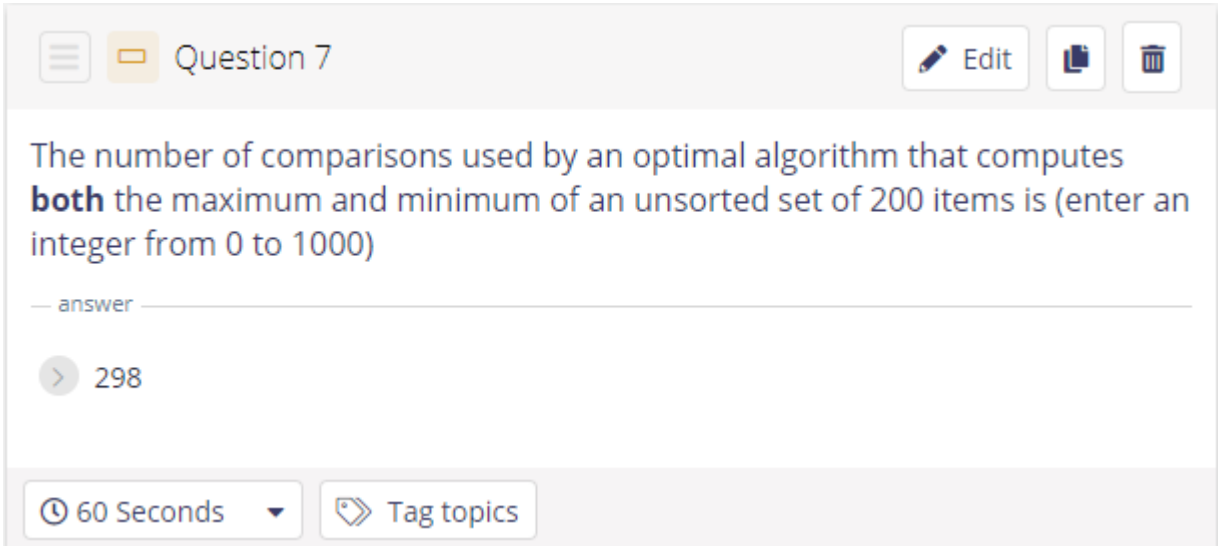
$$f(2) = 1 + 1 = 2$$
$$f(3) = 2 + 1 = 3$$
$$f(4) = 3 + 2 = 5$$
$$f(5) = 5 + 3 = 8$$

Question 6



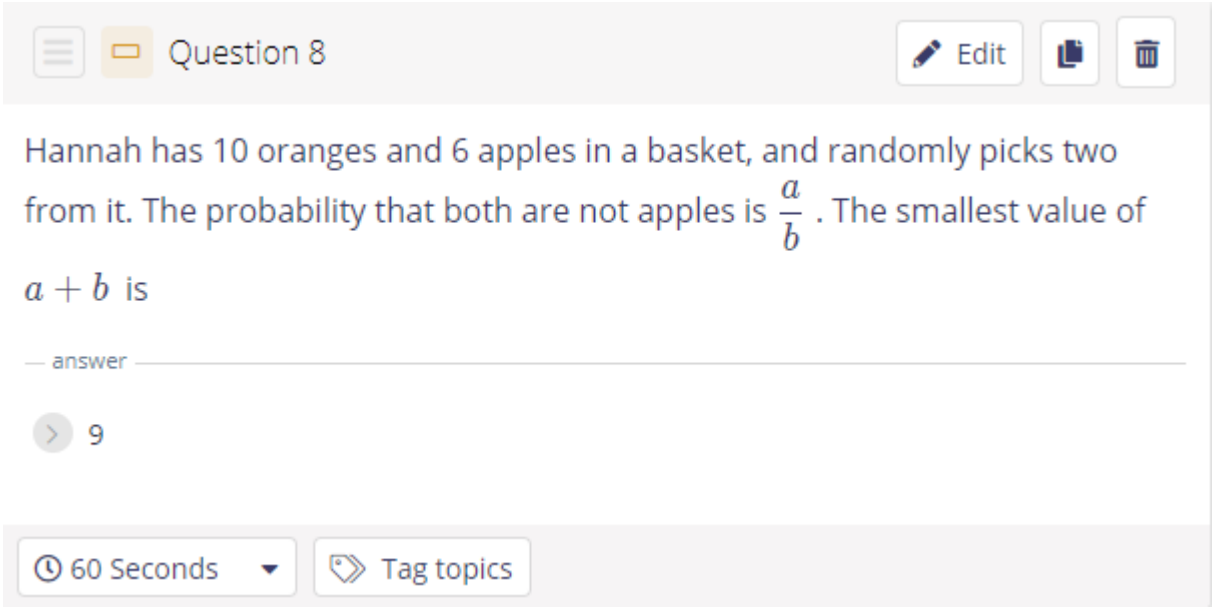
The 12 students ask the 11 remaining students a question, and hence the total number of questions being asked is 12*11 = 132.

Question 7



It can be shown (using stacks) that for a given *n*, the minimum number of comparisons required is $\frac{3n}{2} - 2$. For $n = 200$, that results in the answer 298.

Question 8

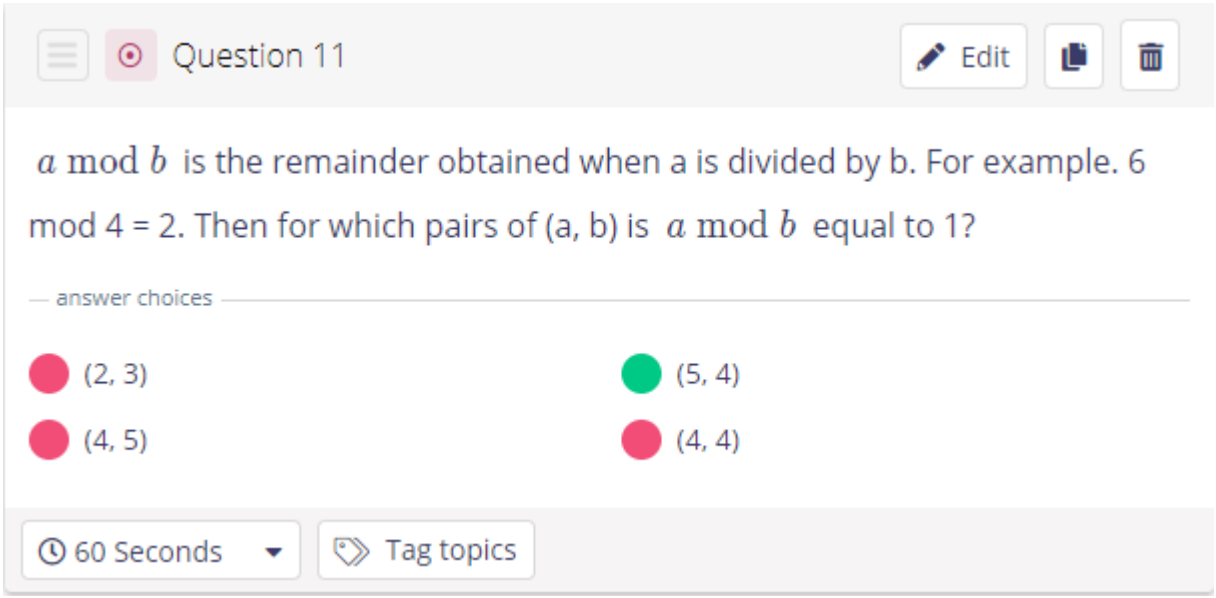


The probability that she does not pick an apple the first time is 6/16, and 5/15 the second time. Hence the probability that she does not get an apple **both** times is simply

$$\frac{6}{16}\left(\frac{5}{15}\right)=\frac{2}{5}\left(\frac{5}{16}\right)=\frac{1}{8}$$

Since a = 1 and b = 8, $a+b=9$.

Question 9



5 gives remainder 1 when divided by 4.

Question 10

Question 13 Edit

Three real numbers in interval [0,1] are chosen independently and at random. Then the probability they are the side lengths of a triangle with positive area is

answer choices

- 1/4
- 1/3
- 1/2
- 2/3
- 1

60 Seconds Tag topics

Given two numbers *a* and *b,* if *c* is chosen randomly, then there are two possibilities:

$$a + b > c$$
$$a + b < c$$

But the second option is impossible due to the Triangle Inequality. Hence out of the two options, only one works -> the probability is ½.

Question 11

Question 14 Edit

The formula that correctly represents the (h)our and (m)inute when the hands of a clock coincide is

answer choices

- 60h=11m
- h=5m
- 20h=3m
- 60h=13m

60 Seconds Tag topics

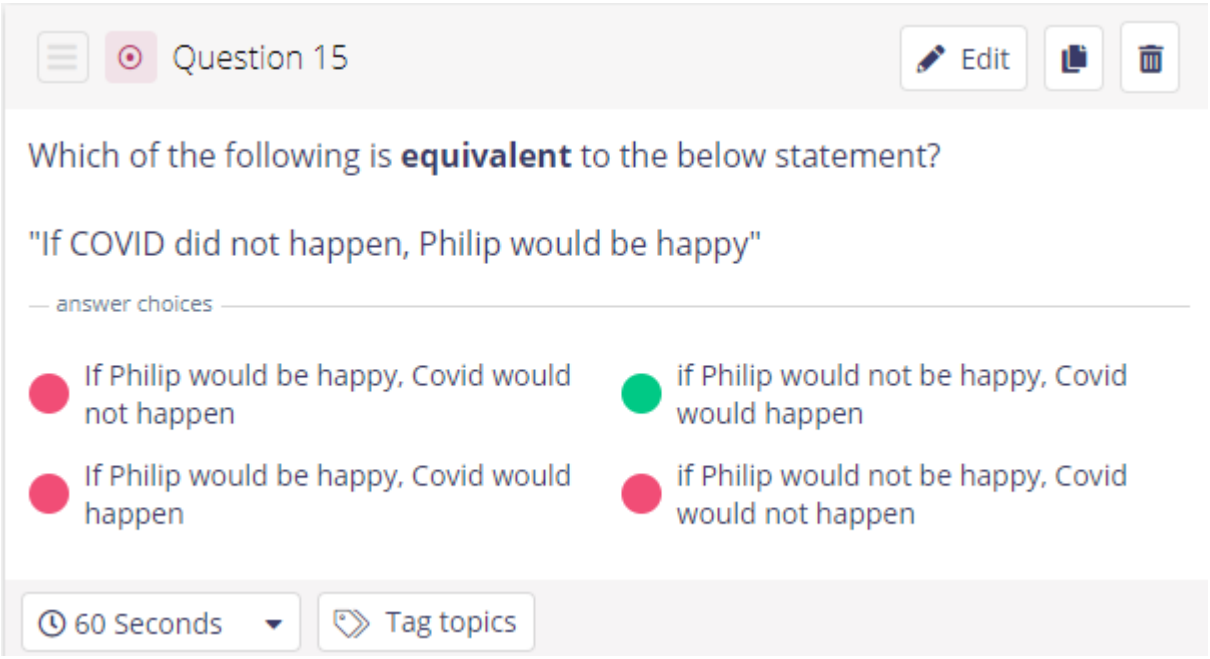
The hands of the minute clock move 6 degrees for every minute.
Hence the total angle by the minute hand is $6m$.

The hour hand moves $\frac{30}{60} = \frac{1}{2}$ degrees as the minute clock moves every minute.
Additionally, the hour hand moves by 30 degrees for every hour that moves past.
Hence the total angle by the hour hand is $\left(30h + \frac{m}{2}\right)^{\circ}$

As both hands coincide,

$$6m = 30h + \frac{m}{2}$$
$$\frac{11m}{2} = 30h$$
$$60h = 11m$$

Question 12



We are looking for the contrapositive of the statement.