






Solutions to the Introduction to CPSTA Quiz

Note that the question orders and options will vary in the actual event.

Question 1




 Question 1

 Edit  

A bucket leaks 20% of its contents each minute. How many times can this go on at most before only 20% of the water is left?

— answer —

> 7



 60 Seconds   Tag topics




Notice that this is compounded. So after the first minute, the bucket has leaked 20%. After the second minute, it has leaked 20% (of 20%). Hence, after the n^{th} minute, the bucket has $\left(\frac{4}{5}\right)^n$ of water left, and we need to find n such that it's equal to $1/5$. This means that

$$\begin{aligned}\left(\frac{4}{5}\right)^n &= \frac{1}{5} \\ n \ln \frac{4}{5} &= \ln \frac{1}{5} \\ n &= \frac{\ln \frac{1}{5}}{\ln \frac{4}{5}} \cong 7.2\end{aligned}$$

As we are looking for the number of times this can go on at **most**, the answer is 7.

Question 2




 Question 2

 Edit  

The number of ways for 20 people to handshake with each other is






— answer —

> 190

 60 Seconds   Tag topics

Each of the 20 people will handshake with 19 people. But then there will be duplicates – for example A handshaking B is the same as B handshaking A. Hence, we divide the result by 2: $380/2 = 190$.

Question 3

  Question 3  Edit  

The **31** in Baskin Robbins comes from the fact that there were 31 flavours when originally launched.


The number of ways you could choose your ice cream as long as each flavour is used only once, and the order does not matter, is roughly


— answer choices —

☐ below 100 ☐ between 100 and 10000

☐ between 10000 and 1000000 ☐ between 1000000 and 100000000 (100 million)

☒ greater than 100 million

 60 Seconds ▾

 Tag topics






The number of ways to choose r items from n is $\binom{n}{r}$, also called as n choose r .

We can also show that (proof not included)

$$\sum_{r=1}^n \binom{n}{r} = 2^n$$

Notice that this is the number of ways to choose an ice cream with at least one flavour, up to choosing all 31 flavours of them. And hence the answer is 2^{31} , which is over two billion.

Question 4

  Question 4  Edit  


Which of the following values is the smallest?


— answer choices —

☐ $(\log_2 5^{16})^2$ ☒ $(\log_2 (5^{16})^3)$

☐ $(5^{16})^{\frac{1}{4}}$ ☐ $5^{32} \log_2 5^{16}$

☐ 5^{16}

 60 Seconds ▾

 Tag topics

The values being asked are simply too large, and hence it would help to write them down in terms of n :

A: $(\log_2 n)^2$

B: $\log_2 n^3$

C: $n^{\frac{1}{4}}$






D: $n^2 \log_2 n$

E: n

Firstly, $\log n < n$, and hence we can concentrate on A and B. Now notice that B can be reduced to $3 \log_2 n$, but that results in the comparison $\log_2 n > 3$, which happens when $n > 8$. Hence for larger n , $A > B$ and hence B is the smallest (for $n = 5^{16}$).

(this is the essence of the "Big-O" notation)

Question 5

 Question 5  Edit  




The Fibonacci series is given by the recurrence relation (for integer t)

$$f(0) = 1, f(1) = 1, f(t+2) = f(t) + f(t+1)$$

Then $f(5)$ is

— answer —

> 8

 60 Seconds   Tag topics

Just run through the sequence:



$$f(2) = 1 + 1 = 2$$




$$f(3) = 2 + 1 = 3$$

$$f(4) = 3 + 2 = 5$$

$$f(5) = 5 + 3 = 8$$

Question 6



 Question 6


 Edit  

12 students ask each other a different question. The total number of questions being asked is

— answer —

> 132


 60 Seconds 

 Tag topics

The 12 students ask the 11 remaining students a question, and hence the total number of questions being asked is $12 \cdot 11 = 132$.

Question 7



 Question 7

 Edit  

The number of comparisons used by an optimal algorithm that computes **both** the maximum and minimum of an unsorted set of 200 items is (enter an integer from 0 to 1000)

— answer —

> 298

 60 Seconds 

 Tag topics

It can be shown (using stacks) that for a given n , the minimum number of comparisons required is $\frac{3n}{2} - 2$. For $n = 200$, that results in the answer 298.

Question 8



Question 8



Edit



Hannah has 10 oranges and 6 apples in a basket, and randomly picks two from it. The probability that both are not apples is $\frac{a}{b}$. The smallest value of $a + b$ is

— answer —



9

⌚ 60 Seconds ▾



Tag topics

The probability that she does not pick an apple the first time is $\frac{6}{16}$, and $\frac{5}{15}$ the second time. Hence the probability that she does not get an apple **both** times is simply

$$\frac{6}{16} \left(\frac{5}{15} \right) = \frac{2}{5} \left(\frac{5}{16} \right) = \frac{1}{8}$$

Since $a = 1$ and $b = 8$, $a + b = 9$.

[Question 9](#)



Question 11



Edit



$a \bmod b$ is the remainder obtained when a is divided by b . For example. $6 \bmod 4 = 2$. Then for which pairs of (a, b) is $a \bmod b$ equal to 1?

— answer choices —



(2, 3)



(5, 4)



(4, 5)



(4, 4)

⌚ 60 Seconds ▾



Tag topics

5 gives remainder 1 when divided by 4.

[Question 10](#)



Question 13



Edit



Three real numbers in interval $[0,1]$ are chosen independently and at random. Then the probability they are the side lengths of a triangle with positive area is

— answer choices —

☐ $1/4$

☐ $1/3$

☒ $1/2$

☐ $2/3$

☐ 1

⌚ 60 Seconds ▾

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Given two numbers a and b , if c is chosen randomly, then there are two possibilities:

$$a + b > c$$

$$a + b < c$$

But the second option is impossible due to the Triangle Inequality. Hence out of the two options, only one works -> the probability is $1/2$.

[Question 11](#)



Question 14



Edit



The formula that correctly represents the (h)our and (m)inute when the hands of a clock coincide is

— answer choices —

☒ $60h=11m$

☐ $h=5m$

☐ $20h=3m$

☐ $60h=13m$

⌚ 60 Seconds ▾

🏷 Tag topics

The hands of the minute clock move 6 degrees for every minute. Hence the total angle by the minute hand is $6m$.

The hour hand moves $\frac{30}{60} = \frac{1}{2}$ degrees as the minute clock moves every minute.






Additionally, the hour hand moves by 30 degrees for every hour that moves past.

Hence the total angle by the hour hand is $\left(30h + \frac{m}{2}\right)^\circ$

As both hands coincide,

$$6m = 30h + \frac{m}{2}$$
$$\frac{11m}{2} = 30h$$
$$60h = 11m$$

Question 12



  Question 15  Edit  

Which of the following is **equivalent** to the below statement?

"If COVID did not happen, Philip would be happy"

— answer choices —

<input type="radio"/> If Philip would be happy, Covid would not happen	<input checked="" type="radio"/> if Philip would not be happy, Covid would happen
<input type="radio"/> If Philip would be happy, Covid would happen	<input type="radio"/> if Philip would not be happy, Covid would not happen

 60 Seconds  Tag topics

We are looking for the contrapositive of the statement.