CPSTA Quiz 2 solutions

List of questions

1. If $x^3 - 1 = 7$, then what is *x*?

☐ a) 2 ☐ b) -2

☐ c) 3 ☐ d) -3

2. There are 10 balls in a basket with an equal number of red and blue balls. One ball is taken out of the basket. The probability that it is a red ball is

☐ a) 0.1 ☐ b) 0.3

☐ c) 0.5 ☐ d) 0.7

3. Nguyen and Wang are at (0,0). Nguyen goes four units north and five units east. Then the shortest distance between Wang and Nguyen is

☐ a) between 6 and 7 ☐ b) between 7 and 8

☐ c) between 8 and 9 ☐ d) exactly 9

4. A shopkeeper gives a discount of 20%, only to raise its price by 20%. Then in the end, the product is

☐ a) getting cheaper ☐ b) getting costlier

☐ c) it's the same either way ☐ d) the price of the product is needed

5. The numbers in the set [2,4,6,8,10] are changed to [4,8,12,16,20]. Then which of the following is **not** doubled?

☐ a) Mean ☐ b) Mode

☐ c) Standard deviation ☐ d) Variance

6. Consider the quadratic $y = x^2 + 6x + k$. What is the maximum value of *k* beyond which *y* is always positive?

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7. The sum of two numbers is *p* and their product being *q*. Then the sum of the reciprocals of the two numbers is closest to

☐ a) p/q ☐ b) q/p

☐ c) pq/(p + q) ☐ d) (p + q)/(pq)

8. There are 50 toys and you need to pick *r* of them, and the other does not matter. For which value of *r* would the number of ways that you could get your toys be maximised? Enter a number from 1 to 100.

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9. Find the value of k in the below equation:-

$$\frac{1+2+3+4+\ldots+n}{1-2-3-4-\ldots-n}=\frac{n(n+1)}{k-n(n+1)}$$

☐ a) 4 ☐ b) 2

☐ c) 3 ☐ d) 1

10. It can be shown that for $0 \le k < \frac{\pi}{2}$ and n being a natural number, that $\int_0^k \left(\tan^n x - \tan^{n-1} x\right) dx > 0$. Then

☐ a) $k < \frac{\pi}{4}$ ☐ b) $k > \frac{\pi}{4}$

☐ c) $k < \frac{\pi}{3}$ ☐ d) $k > \frac{7\pi}{16}$

Question 1

$x^3 = 8 \rightarrow x = 2$.

Straightforward, well answered.

Question 2

There are 5 red balls and 5 green balls. Hence the probability is $\frac{5}{10} = \frac{1}{2}$.

Trivial, and well answered again.

Question 3

Nguyen is at (4,5), and the shortest distance is determined by the Pythagoras theorem. $\sqrt{4^2+5^2} = \sqrt{41}$, which is between 6 and 7.

This question was also well answered.

Question 4

Let the price of the product be *x*. Then after a 20% discount is given, the price is $\frac{4}{5}x$. But if the price is raised by 20%, the new price is

$$\frac{6}{5}\left(\frac{4x}{5}\right) = \frac{24}{25}x < x$$

Hence the product ends up being cheaper (by 4% precisely).

This was also well answered, with barely anyone getting confused or faillling into the expected trap that the price would be the same.

Question 5

Variance is quadrupled (in fact as it is the square of standard deviation, the S.D doubles as expected).

This was poorly answered, with many getting confused with the concept of standard deviation and how they relate to variance.

Question 6

We have $y = x^2 + 6x + k$. We know that $(x + 3)^2 = x^2 + 6x + 9$, and this means that for $k > 9$, we can write *y* as $(x + 3)^2 + p$, where $p > 0$, and hence it's always positive then. Hence $k = 9$ is the answer.

(in fact, this can be quickly verified by finding the discriminant: $D = 36 - 4k$ and we want $D \geq 0$ because otherwise *y* is always positive)

This was reasonably well answered, with many getting the idea behind it, especially since this was a numeric answer problem.

Question 7

Let the two numbers be *a* and *b*. We have $a + b = p$ and $ab = q$, and we want $\frac{1}{a} + \frac{1}{b}$. Then

$$\frac{p}{q} = \frac{a+b}{ab} = \frac{a}{ab} + \frac{b}{ab} = \frac{1}{a} + \frac{1}{b}$$

This was "in the middle", with many people mistakenly picking $\frac{p+q}{pq}$. *The problem with that answer is that* p *and* q *are not the numbers themselves, rather they represent the sum and the product of the numbers. This was a common trap to fall to.*

Question 8

Notice that we want to maximise 50 choose *r*, or

$$\binom{50}{r} = \frac{50!}{(50-r)!\,r!}$$

This happens when $r = \frac{50}{2} = 25$.

(notice that n choose r is equal to n choose (n – r)), which further lends credence to the fact that the maximum value is in the middle)

This was reasonably well answered for a question towards the end of the quiz.

Question 9

Quickfire solution

If you're not sure, just take $n = 1$. Then we get

$$\frac{1}{1} = \frac{2}{k-2}$$

And it's easy to show that $k = 4$ right there.

Full solution

Notice that

$$\frac{1+2+3+4+\cdots+n}{1-2-3-4-\cdots-n} = \frac{1+2+3+4+\cdots+n}{2-1-2-3-4-\cdots-n} \to \frac{1+2+3+4+\cdots+n}{2-(1+2+3+\cdots+n)}$$

We know that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$.

Hence,

$$\frac{1+2+3+4+\cdots+n}{2-(1+2+3+\cdots+n)} \to \frac{\frac{n(n+1)}{2}}{2-\frac{n(n+1)}{2}} = \frac{n(n+1)}{4-n(n+1)}$$

Comparing with RHS of question, we note that $k = 4$.

This question was very poorly answered with barely any correct attempts, perhaps due to the tricky sequence structure. This was despite an easy way out by just setting n = 1.

Question 10

First, it helps to understand that when $k < \frac{\pi}{4}$, $0 < \tan^n x < \tan^{n-1} x < 1$, and hence

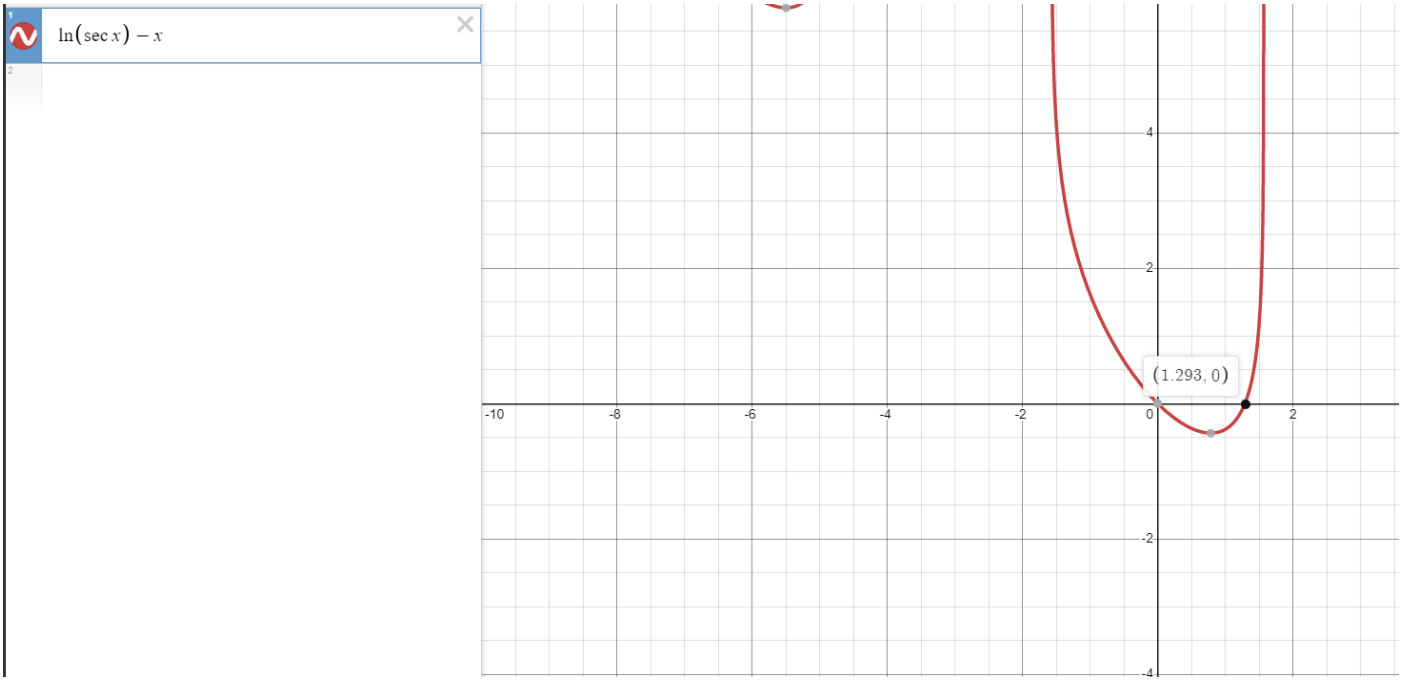
$$\int_0^k \tan^n x - \tan^{n-1} x < 0$$

This means that *k* must always be greater than $\frac{\pi}{4}$, and can rule out two options immediately. Then set $n = 1$.

Then we get

$$\int_0^k (\tan x - 1)\, dx = [|\ln \sec x| - x]_0^k = \ln \sec k - k > 0$$

Then graph the resulting equation. We see that a zero is obtained at 1.293 radians, which is less than $\frac{7\pi}{16}$. Hence the answer is D.



This was poorly answered as expected for a question at the end of the quiz. Surprisingly, many realised the connection with $\frac{\pi}{4}$*, but for some reason chose that* k *is less than it.*