Brute Force: Common Strategies

Keep it simple, keep it neat

UTEC - Competitive Programming

Fixing Variables

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- One of the most intuitive techniques.
- Usually this problems involve an equation of m variables.
- You will *fix* a variable by making it constant and then try to find the solution under this new constrain.
- You will iterate through all possible values of the fixed variable and see how this fixed solution contributes to the general solution of the problem.

Fixing Variables - Example

Given numbers n, a, b and c find:

$$|\{(x, y, z)|x, y, z \in \mathbb{Z}^{+0} \wedge x \leq a \wedge y \leq b \wedge z \leq \wedge \frac{x}{2} + y + 2z = n\}|$$

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Simulation with Brute Force

Simulation

- Problems that can be solved using this strategy usually have an underlying pattern or rules that we can compute easily.
- To find the answer we just need to let our simulation run until we get the expected results.
- **Problem:** Consider the following function:

$$f(n) = \begin{cases} \frac{n}{2} & n \equiv 0 \pmod 2 \\ 3n + 1 & \text{otherwise} \end{cases}$$

  The cycle of a number n is the number of times we need to apply function f so that $n = 1$. Find the maximum cycle between two given numbers i and j. $i, j < 10^5$

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- Run that function for all x between i and j and save the maximum.
- Complexity? Smallest cycle smaller than $10^5 \approx 400$, therefore $O(400m)$. Where $m = max\{i - j\}$

Weak Constrains

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- This problems usually seem hard until you realize an underlying constrain that reduces significantly the search space.
- Even the problem might have some constrains, this might be very far from the constrains that can be deduced by the statement.
- After finding this new constrains we can use simple brute-force.

Weak Constrains: Example

Given n and s ($n, s \leq 10^{18}$), find:

$$|\{x | x \leq n \wedge x - f(x) \geq s\}|$$

Where $f(x)$ is the sum of digits of x.

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- Observation 3: $x \geq s + \leq 9 \cdot 18 \rightarrow x - f(x) \geq s$
- We only need to check $x \in [s, s + 9 \cdot 18]$!
- Answer will be $\texttt{count}(s, s + 9 \cdot 18) + n - (s + 9 \cdot 18)$.

Thanks for Listening!