# Brute Force: Common Strategies

Keep it simple, keep it neat

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UTEC - Competitive Programming

# Fixing Variables

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- One of the most intuitive techniques.
- Usually this problems involve an equation of $m$ variables.
- You will *fix* a variable by making it constant and then try to find the solution under this new constrain.
- You will iterate through all possible values of the fixed variable and see how this fixed solution contributes to the general solution of the problem.

# Fixing Variables - Example

Given numbers $n$, $a$, $b$ and $c$ find:

$$|\{(x, y, z)|x, y, z \in \mathbb{Z}^{+0} \land x \leq a \land y \leq b \land z \leq c \land \frac{x}{2} + y + 2z = n\}|$$

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- This soultion has a complexity of $O(ab)$. Can we do better?

# Simulation with Brute Force

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- Problems that can be solved using this strategy usually have an underlying pattern or rules that we can compute easily.
- To find the answer we just need to let our simulation run until we get the expected results.
- **Problem:** Consider the following function:

$$f(n) = \begin{cases} \frac{n}{2} & n \equiv 0 \pmod 2 \\ 3n+1 & \text{otherwise} \end{cases}$$

The cycle of a number $n$ is the number of times we need to apply function $f$ so that $n = 1$. Find the maximum cycle between two given numbers $i$ and $j$. $i, j < 10^5$

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- Complexity? Smallest cycle smaller than $10^5 \approx 400$, therefore $O(400m)$. Where $m = max\{i - j\}$

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- This problems usually seem hard until you realize an underlying constrain that reduces significantly the search space.
- Even the problem might have some constrains, this might be very far from the constrains that can be deduced by the statement.
- After finding this new constrains we can use simple brute-force.

## Weak Constrains: Example

Given $n$ and $s$ ($n, s \leq 10^{18}$), find:

$$|\{x|x \leq n \wedge x - f(x) \geq s\}|$$

Where $f(x)$ is the sum of digits of $x$.

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- Observation 2: $f(x) \leq 9 \cdot 18$
- Observation 3: $x \geq s + 9 \cdot 18 \rightarrow x - f(x) \geq s$
- We only need to check $x \in [s, s + 9 \cdot 18]$!
- Answer will be $\texttt{count}(s, s + 9 \cdot 18) + n - (s + 9 \cdot 18)$.

# Thanks for Listening!