Complete Search: Introduction

Try everything

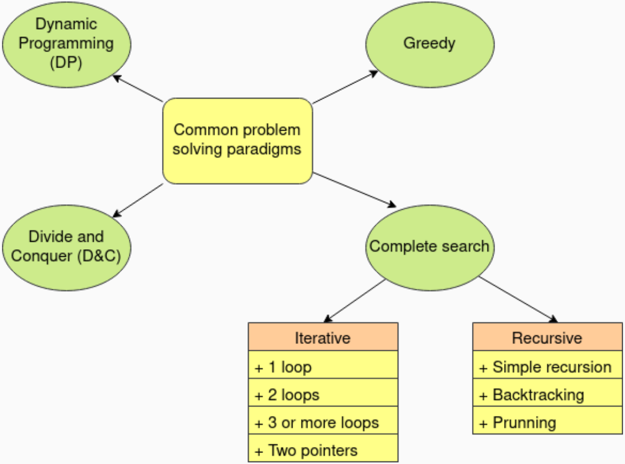
UTEC - Competitive Programming

Problem Solving Paradigms

- A programming paradigm is a common pattern that can be used to solve problems.
- The following are the problem paradigms commonly used in competitve programming:
- Paradigms = Algorithm?

Commonly used Paradigms in Competitive Programming

The four problem solving paradigms commonly used in contests:



Complete Search

- Complete search is a method that can to solve almost any problem.
- As the name suggests, we will iterate through all possible solutions.
- Complete search comes in two ways: iterative and recursive.
- When realizing complete search in a iterative way, it is calles *brute force*.

Motivation

A pythagorean triple is a 3-tuple (x, y, z) that satisfies the equation:

$$x^2 + y^2 = z^2$$

Given an integer n ($n \leq 10^3$), find the number of pythagorean triples such that $1 \leq x, y, z \leq n$.

- Any ideas?

First Approach

We check all possible values of x, y, z and see if they are a valid Pythagorean Triplet.

```
int solution1 (int n) {
    int cnt = 0;
    for (int x = 1; x <= n; x++) {
        for (int y = 1; y <= n; y++) {
            for (int z = 1; z <= n; z++) {
                if (x * x + y * y == z * z) {
                    cnt++;
                }
            }
        }
    }
    return cnt;
}
```

Second Approach

We can fix x, y and check if there exists a vule of z that satisfies the equation. This will improve our efficiency significantly.

```
int solution2 (int n) {
    vector <bool> is_sq(n * n + 1, false);
    for (int z = 1; z <= n; z++) {
        is_sq[z * z] = true;
    }
    int cnt = 0;
    for (int x = 1; x <= n; x++) {
        for (int y = 1; y <= n; y++) {
            int z2 = x * x + y * y;
            if (z2 <= n * n and is_sq[z2]) {
                cnt++;
            }
        }
    }
    return cnt;
}
```

Third Approach

- $x^2 + y^2 = z^2 \rightarrow (kx)^2 + (ky)^2 = (kz)^2$
- If (x, y, z) is a Pythagorean triple and $gcd(x, y, z) = 1$, we say it is a *primitive Pythagorean triple*.
- Thanks to Euclids Formula we know that every primitive PT can be represented as:

$$x = a^2 - b^2$$

$$y = 2ab$$

$$z = a^2 + b^2$$

- So how is this useful?

Third Approach

Here is the implementation of the solution described above:

```
int solution3 (int n) {
    int cnt = 0;
    for (int a = 1; a * a < n; a++) {
        for (int b = 1; b < a; b++) {
            if (__gcd(a, b) != 1) continue;
            if (a % 2 and b % 2) continue;

            int x = a * a - b * b;
            int y = 2 * a * b;
            int z = a * a + b * b;
            int add = min({n / x, n / y, n / z});
            cnt += 2 * add;
        }
    }
    return cnt;
}
```