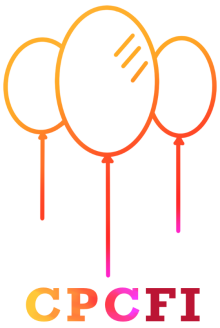
Monday, February 21, 2022 1:01 AM



Lecture: ASAP Floyd-Warshall
Unit: 7
Instructor: Carlos C.L

APSP Floyd–Warshall (Published 1962)

Inspiration

Given a graph G(V, E) and two nodes s, t find the best meeting node v,

d[s][v]+d[v][t] is minimum

In order to solve this problem, we need to know the minimum distance between any pair of nodes

Trivial solution using Dijkstra's

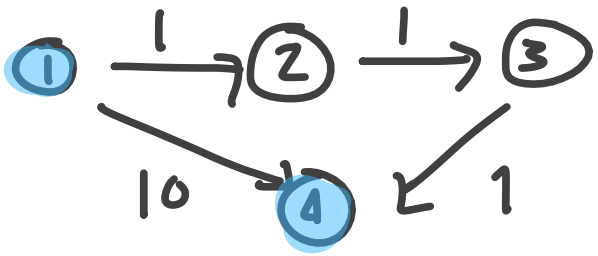
- Run Dijkstra's SSSP for every node
- Time: O(VELogV)
- Space: O(V^2)
- Worst Time: O(V^3LogV)

Better solution using Floyd-Warshall

Idea

Given 1 <= k <= V, find shortest path but use only internal vertices from [1,...,k]

1. **Definition:**
   d[i][j][k] := shortest path between i and j given only internal vertices are from [1,...,k]



d[1][4][0] = 10

d[1][4][1] = 10

d[1][4][2] = 3

2. **Base case:**
   d[i][j][0] = adj[i][j]
3. **Recursive function:**
   d[i][j][k] = min(d[i][j][k-1], d[i][k][k-1] + d[k][j][k-1])
   not use node k | use node k
4. **Improvements:**
   Since we only use the k-1 state, we can drop this dimension to reduce space consumption:
   d[i][j] = min(d[i][j], d[i][k] + d[k][j])
5. **Recover path**
   p[i][j] := last node before j in a path from i to j (i -> ... -> p[i][j] -> j)

   Initial values for p[i][j]
   p[i][j] = i if there is an edge from i to j

   if (d[i][k][k-1] + d[k][j][k-1] < d[i][j][k-1]):
   p[i][j] = p[k][j]

Complexity

- Time: O(V^3)
- Space: O(V^2)

Applications:

- **Finding (cheapest/negative) cycle in graph**
  Solution: run Floyd-Warshall, check d[i][i] for 1 <= I <= V, if all are negative then there exists a negative cycle

- **Diameter of a graph (maximum shortest path between any pair of nodes)**
  Solution: take de max of d[i][j]

- **Finding SCCs in directed graph**
  Solution: take a node v, for any 0 <= w <= V, if d[v][w] != INF && d[w][v] != INF, then they belong to the same SCC

Notes:

Graph Criteria	BFS $O(V+E)$	Dijkstra's $O((V+E)\log V)$	Bellman Ford's $O(VE)$	Floyd Warshall's $O(V^3)$
Max Size	$V, E \leq 10M$	$V, E \leq 300K$	$VE \leq 10M$	$V \leq 400$
Unweighted	Best	Ok	Bad	Bad in general
Weighted	WA	Best	Ok	Bad in general
Negative weight	WA	Our variant Ok	Ok	Bad in general
Negative cycle	Cannot detect	Cannot detect	Can detect	Can detect
Small graph	WA if weighted	Overkill	Overkill	Best

Problems:

- **UVa 1056 - Degrees of Separation (easiest ICPC World Finals problem)**
- **UVa 11463 - Commandos**
- **UVa 10171 - Meeting Prof. Miguel**