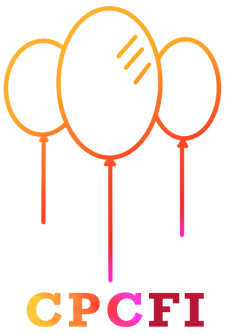
Tuesday, March 1, 2022 11:07 AM

Lecture: Network Flow
Unit: 7.4
Instructor: Carlos C.L



4. Max Flow

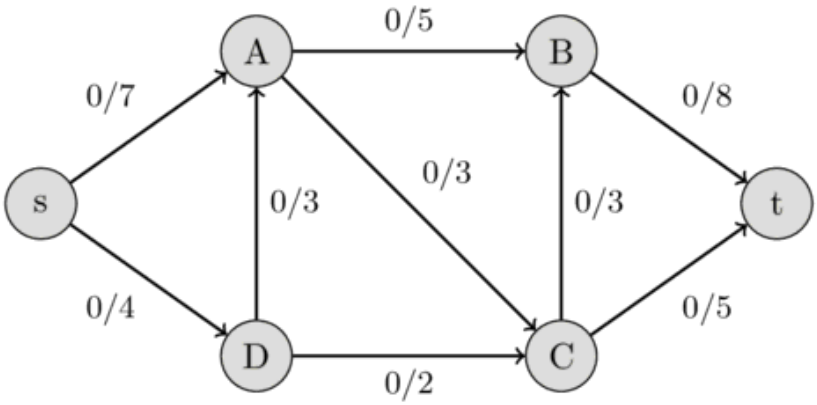
Flow network

A directed graph G(V, E) combined with a capacity function c: E -> {R > 0} and two vertices, a source(s) and a sink(t).

Flow

A function f: E -> {R > 0}. It must satisfy the following conditions:

$$f(e) \leq c(e)$$
$$\Sigma f((u, v) = \Sigma f((u, v))$$



We can prove the following holds given this conditions:

$$\Sigma f((s, u) = \Sigma f((u, t)) = \text{flow of the network}$$

Ford-Fulkerson - Edmonds Karp (published1956)

Our problem is to find the flow function f, that maximizes the flow of the network.

Residual Graph

The residual graph is the same graph but with reverse edges and a different capacity function, the residual capacity.

Residual capacity:

How much we can increase the flow for a given edge:

$$rc(e) = c(e) - f(e)$$

Reverse edges:

For any edge (u, v) in the original graph, we define a reverse edge (v, u) in the residual graph

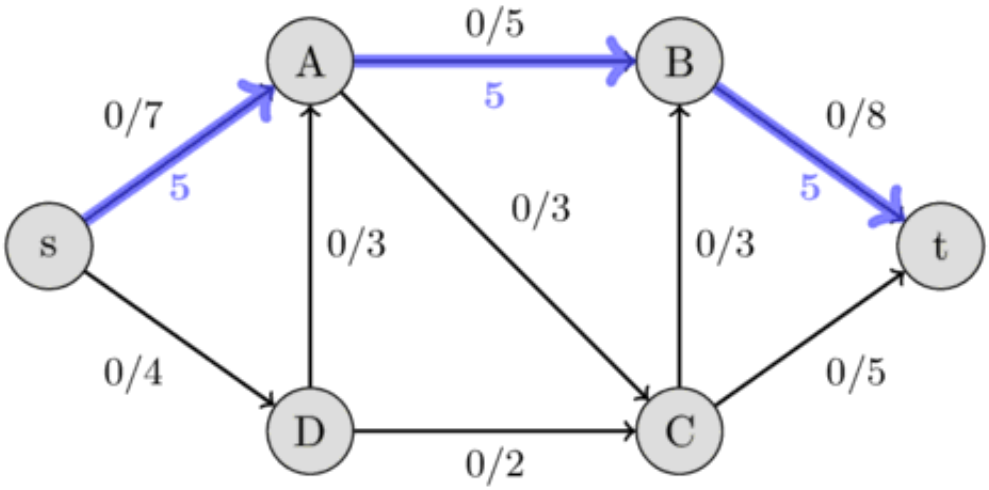
$$f((v, u)) = - f((u, v))$$
$$c((v, u)) = 0$$
$$rc((v, u)) = f(u, v)$$

Augmented path:

It's a path on the residual graph with all edges having positive residual capacity.

Bottleneck capacity:

The lowest residual capacity on an augmented path.



Algorithm:

1. Find augmented path from s to t (use BFS or DFS)
2. Find bottleneck capacity
3. Augment each edge on the path
   c((u, v)) -= C
   c((v, u)) += C
4. Repeat until no augmented paths can be achieved

If we use BFS for step one then we are solving with Edmonds Karp.

Complexity:

Time: O(EF), F = Max flow

Time: O(EV^2)

Space: O(V+E)

Problems:

[CSES Download Speed]

Dinic's Algorithm

Solves max flow problem.

[Dinic's Algorithm | Network Flow | Graph Theory]

Complexity:

Time complexity: O(V^2E)

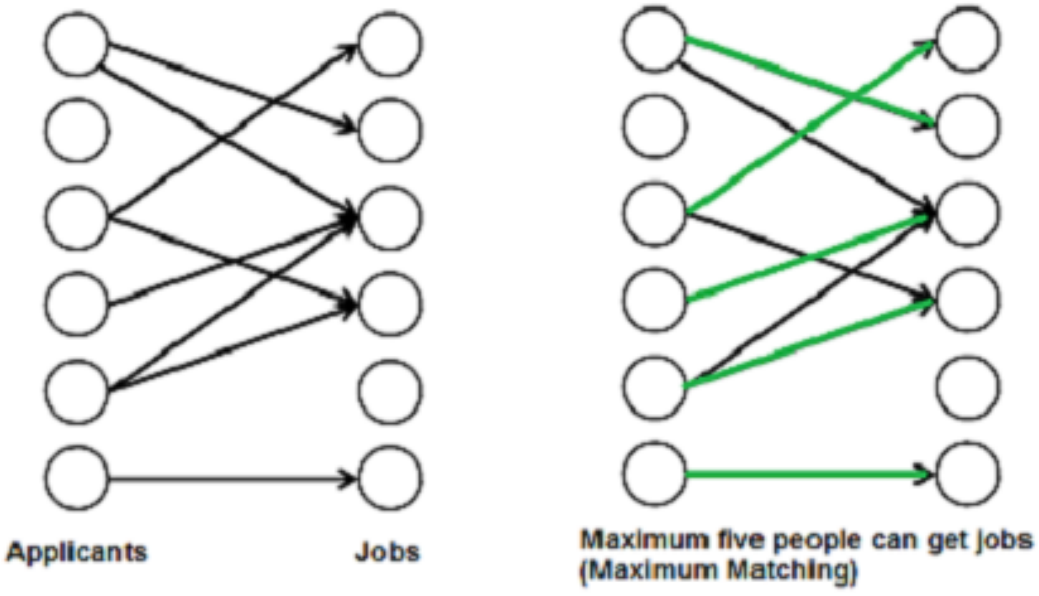
For unit networks:

A unit network is a network where all edges have unit capacity.

Time complexity: O(EV^1/2)

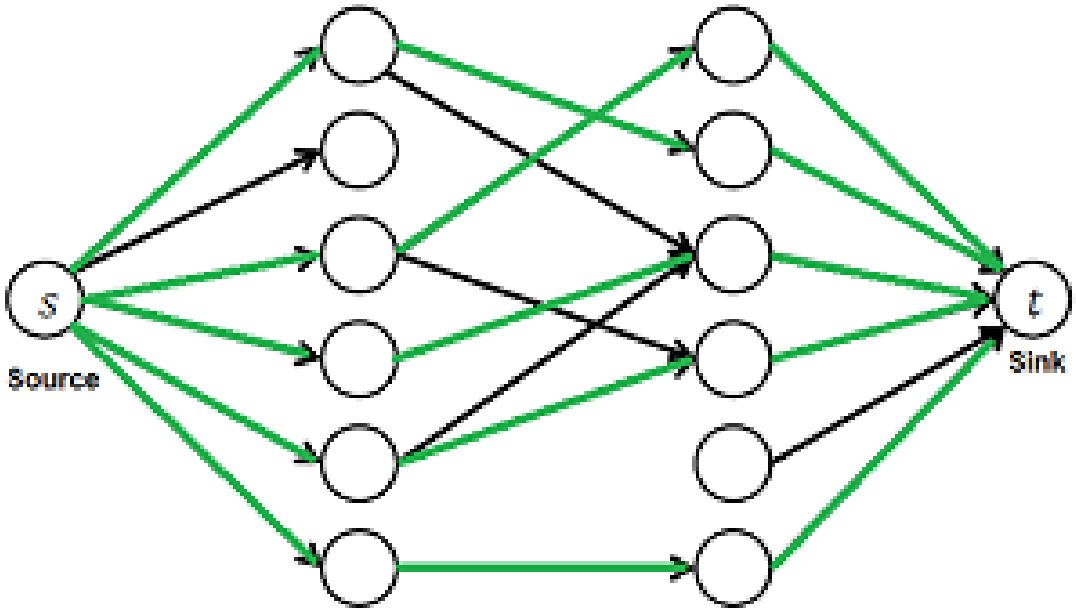
Unweighted Bipartite Matching

We have a bipartite graph representing applicants and jobs. There is an edge (applicant, job) if the applicant has the requirements to do the job.



We want to maximize the number of applicants that get a job.

1. Transform problem to max flow problem
2. Make an imaginary source and target, connect source to applicant and jobs to applicants
3. Find max flow in this graph

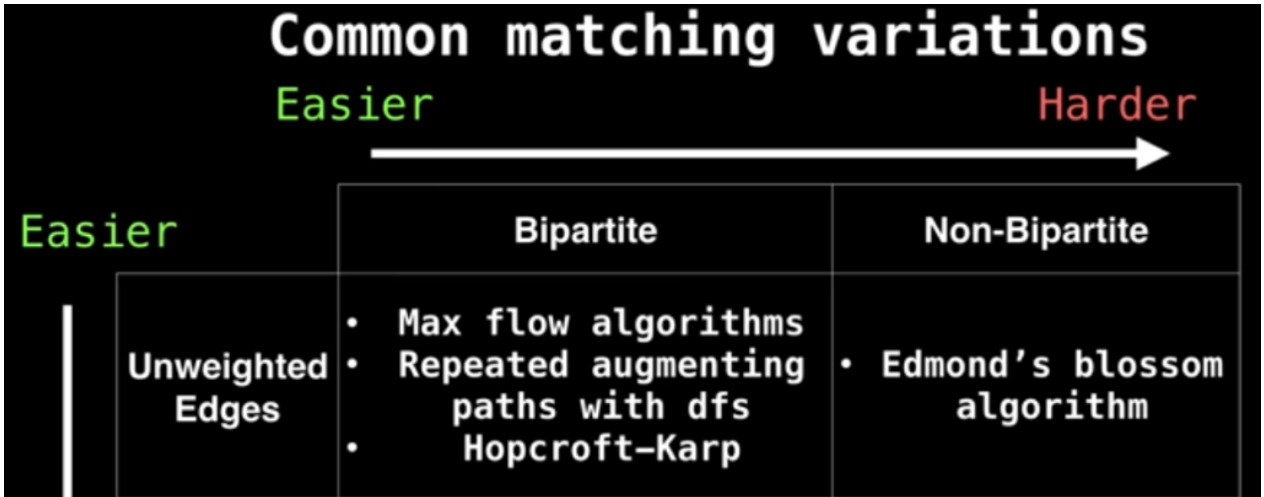


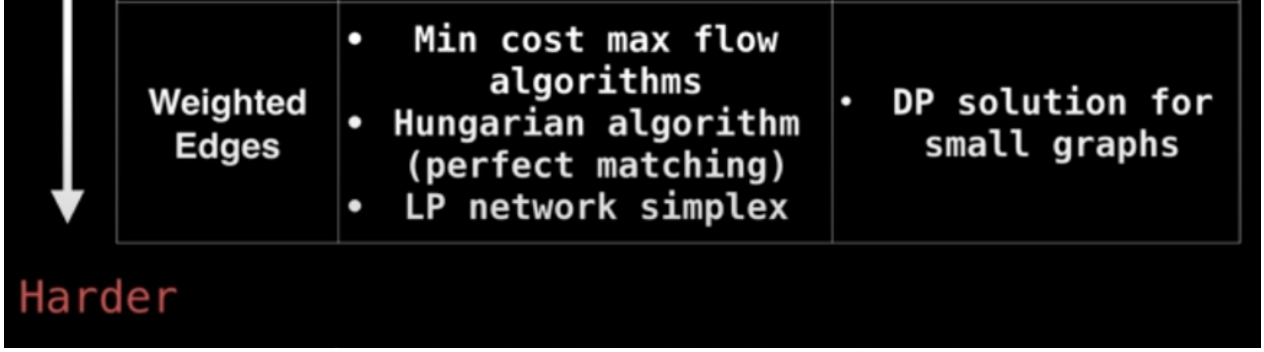
The maximum flow from source to sink is five units. Therefore, maximum five people can get jobs.

Problems:

[CSES School Dance]

Matching Problems



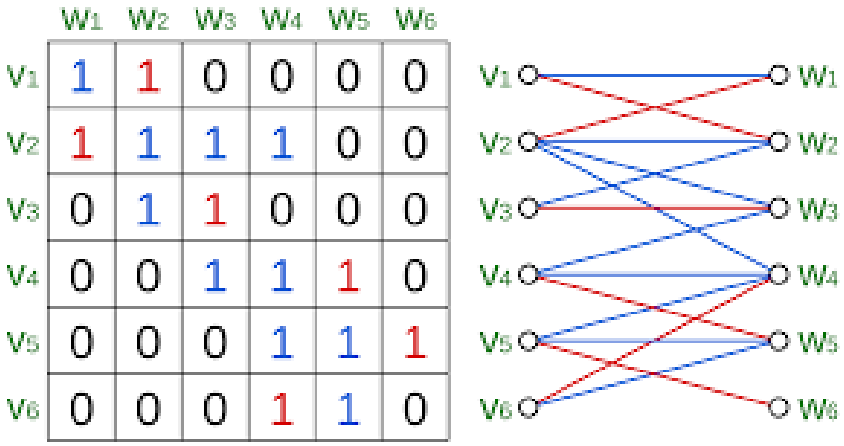


Resources:

[Unweighted Bipartite Matching | Network Flow | Graph Theory]

Assignment problem:

There are N jobs and N workers. Some jobs can only done by some workers, and we know the cost it takes for the worker to do the job. We need to assign all jobs so that the total cost is minimized.



Complexity:

Time: O(V^5)

Hungarian Algorithm

It can solve the assignment problem on O(V^3)

Problems:

CSES Police Chase