Lecture: 5 - COMPUTATIONAL GEOMETRY II

Unit: 9 - STRING PROCESSING + COMPUTATIONAL GEOMETRY

Instructor: WÉSTON

1. POLYGONS

↳ PLANE FIGURE BOUNDED BY A CLOSED PATH COMPOSED OF A FINITE SEQUENCE OF STRAIGHT LINE SEGMENTS

↳ EDGES / SIDES

- POLYGON'S VERTEX / CORNER: POINT WHERE TWO EDGES MEET

REPRESENTATION OF A POLYGON:

→ POLYGON IS AN ENUMERATION OF VERTICES (2D) IN EITHER CLOCKWISE OR COUNTER CLOCKWISE ORDER WITH FIRST AND LAST VERTEX BEING EQUAL

PERIMETER OF A POLYGON:

→ POLYGON P OF n VERTICES

$$\text{PERIMETER} = \sum_{i=1}^{n-1} \text{EUCLIDEAN\_DISTANCE}(P[i], P[i+1])$$

AREA OF A POLYGON:

→ POLYGON P OF n VERTICES

$$A = \frac{1}{2} \begin{bmatrix} x_0 & y_0 \\ x_1 & y_1 \\ \vdots & \vdots \\ x_{n-1} & y_{n-1} \end{bmatrix} = \frac{1}{2}\left(x_0 y_1 + x_1 y_2 + \dots + x_{n-1} y_0 - x_1 y_0 - x_2 y_1 - \dots - x_0 y_{n-1} \right)$$

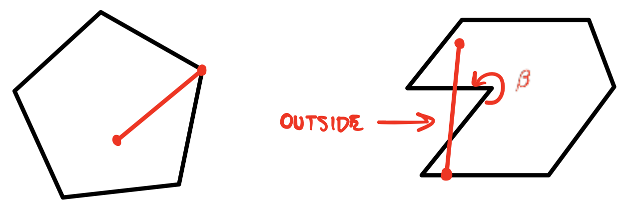
$$= \frac{1}{2} \sum_{(p,q) \in \text{EDGES}} (p_x - q_x)(p_y + q_y)$$

CHECKING IF A POLYGON IS CONVEX:

→ A POLYGON IS SAID TO BE $\boxed{\text{CONVEX}}$ IF ANY LINE SEGMENT DRAWN INSIDE THE POLYGON DOES NOT INTERSECT ANY EDGE OF THE POLYGON.

→ OTHERWISE, THE POLYGON IS $\boxed{\text{CONCAVE}}$

A POLYGON IS CONVEX IF THE LINE SEGMENT BETWEEN TWO POINTS OF THE POLYGON REMAINS INSIDE OR ON THE BOUNDARY



CONVEX

CONCAVE

- EVERY INTERNAL ANGLE IS STRICTLY LESS THAN 180°
- WILL ALWAYS CONTAIN AT LEAST ONE ANGLE $\beta \in [180°, 360°)$

→ TEST FOR CONVEXITY : •) CHECK WHETHER ALL THREE CONSECUTIVE VERTICES OF THE POLYGON FORM THE SAME TURNS

a) ALL LEFT TURNS IF VERTICES WERE LISTED IN COUNTER CLOCKWISE ORDER

b) ALL RIGHT TURNS, OTHERWISE

••) IF A TRIPLET EVALUATES TO FALSE, THE POLYGON IS CONCAVE

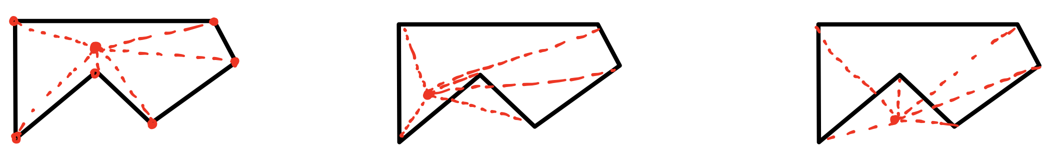
CHECKING IF A POINT IS INSIDE A POLYGON :

→ POLYGON P AND POINT PT
→ WINDING NUMBER ALGORITHM

•) COMPUTES THE SUM ANGLES BETWEEN THREE POINTS :

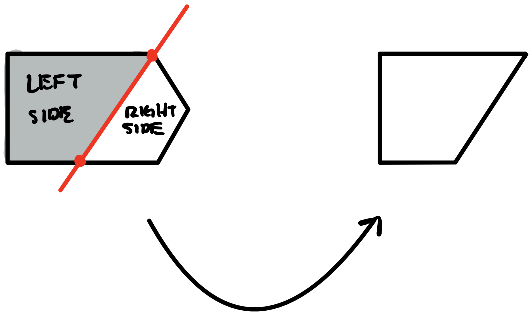
$$\{P[i],\ PT,\ P[i+1]\}$$

••) IF FINAL SUM = 360°, PT IS INSIDE
•••) OTW, PT IS OUTSIDE



INSIDE INSIDE OUTSIDE

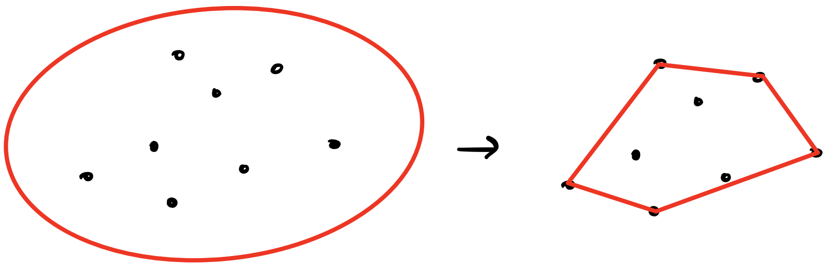
CUTTING A POLYGON WITH A STRAIGHT LINE:



TO FORM THE NEW POLYGON WE ONLY KEEP POINTS THAT ARE ON THE LEFT HAND SIDE OF THE RED LINE

FINDING THE $\boxed{\text{CONVEX HULL}}$ OF A SET OF POINTS P: → CH(P)

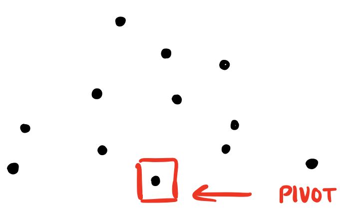
→ SMALLEST POLYGON CH(P) FOR WHICH EACH POINT IN P IS EITHER ON THE BOUNDARY OF CH(P) OR IN ITS INTERIOR



P CH(P)

GRAHAM'S SCAN ALGORITHM $O(n \log n)$:

a) SORTS ALL n POINTS OF P (FIRST POINT DOESN'T NEED TO BE REPLICATED AS LAST POINT) BASED ON THEIR ANGLES WITH RESPECT TO A PIVOT POINT

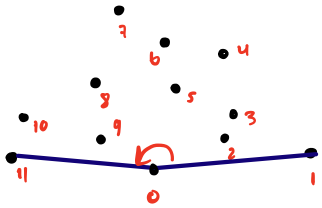


$n = 12$

b) MAINTAIN A STACK S OF CANDIDATE POINTS

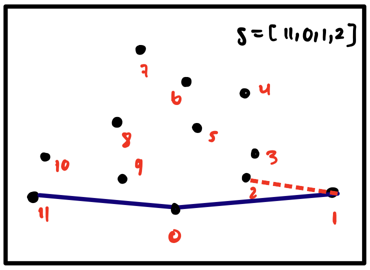
c) EACH POINT OF P IS PUSHED ONCE ONTO S AND POINTS THAT WON'T BE PART OF CH(P) GET POPPED FROM S

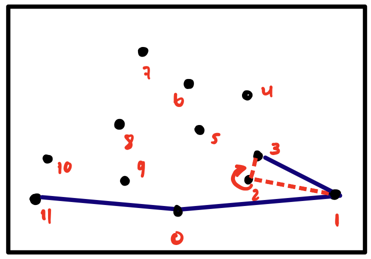
d) WE FIRST INSERT POINTS n-1, 0, 1 ONTO S (THIS FORMS A LEFT TURN)



S = [11, 0, 1]

e) INSERT NEXT POINT i, IF TOP THREE ELEMENTS IN S MAKE A LEFT TURN, THEN WE KEEP POINT i IN THE CH(P)





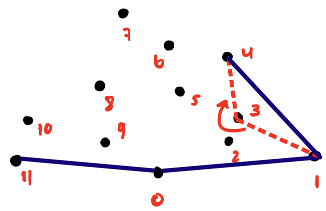
S = [11, 0, 1, 3]

0-1-3 NOW FORMS A LEFT TURN

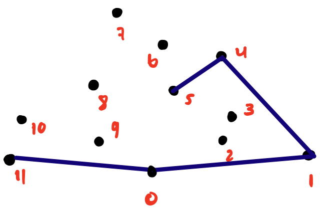
WE TRY TO INSERT POINT 2 TO S, SINCE 0-1-2 MAKE A LEFT TURN, WE KEEP 2 IN CP(H)

WE TRY TO INSERT POINT 3, BUT 1-2-3 MAKE A RIGHT TURN, THUS WE NEED TO POP 2

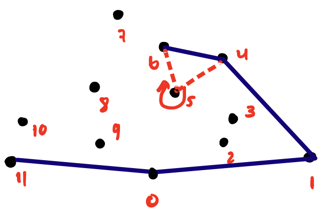
S = [11, 0, 1, 4]

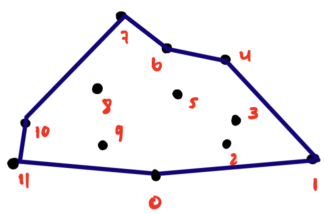


S = [11, 0, 1, 4, 5]



S = [11, 0, 1, 4, 6]





S = [11, 0, 1, 4, 6, 7, 10, 11]

CONVEX HULL OF P

[REFERENCES : HALIM S., HALIM F.. COMPETITIVE PROGRAMMING 3]