

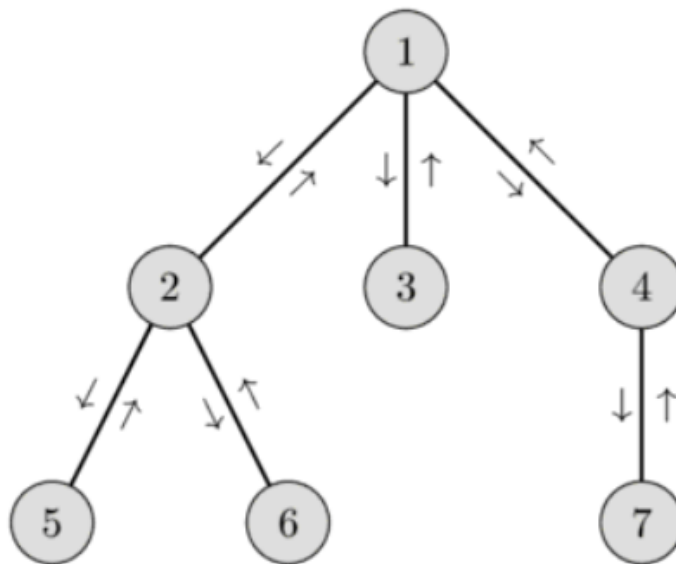
Lecture: Lowest Common Ancestor
Unit: 7.3
Instructor: Carlos C.L



Lowest Common Ancestor

Given a tree, and two nodes v, w , their lowest common ancestor is the **first node in the intersection of paths** from v and w to the root

Transform to RMQ



Vertices:	1	2	5	2	6	2	1	3	1	4	7	4	1
Heights:	1	2	3	2	3	2	1	2	1	2	3	2	1

Idea:

- Get Euler tour

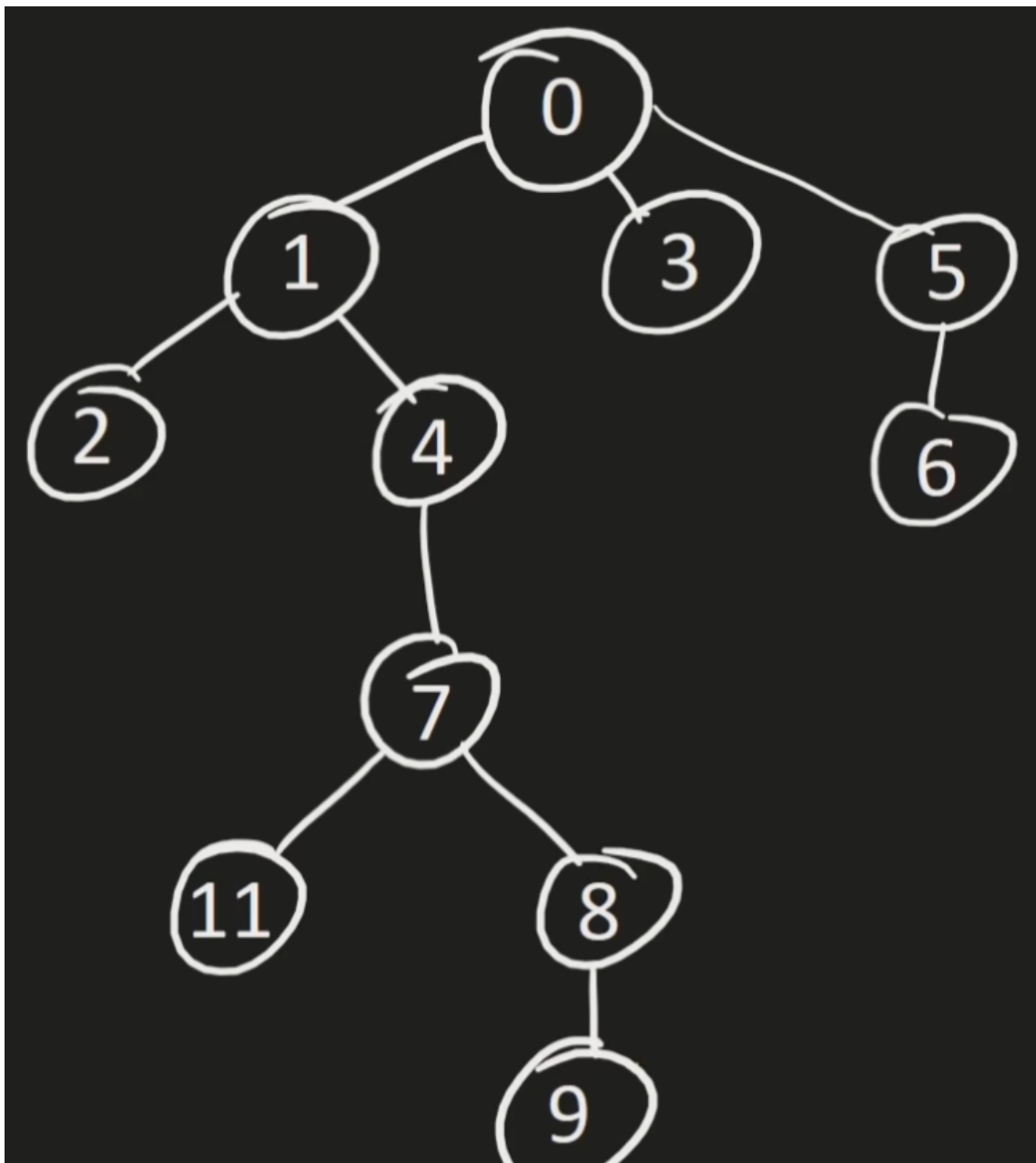
- Get nodes height
- For any nodes v, w their LCA will be the node with the lowest height between the indices of nodes in the Euler tour array.

Complexity:

- Time:
 - Preprocessing: $O(N)$
 - Queries: $O(\log N)$
- Space: $O(N)$

Binary Lifting

Given a tree and v, k , find the k -th ancestor of v .





10

Trivial Solution:

- Run $v = \text{par}[v]$ k times.
- Time complexity: $O(k)$

Idea:

1. Function definition

$\text{up}[v][j] = 2^j\text{-th ancestor of } v$

2. Base case

$\text{up}[v][0] = \text{par}[v]$

3. Recursive relation

$$2^j = 2^{(j-1)} + 2^{(j-1)}$$

$$\text{up}[v][j] = \text{up}[\text{up}[v][j-1]][j-1]$$

- Every number has a binary representation:

$$k = 2^4 + 2^6 + \dots$$

- Move to ancestors in power of two

- $v1 = \text{p}[v][4]$

- $v2 = \text{p}[v1][6]$

....

Complexity:

- Time:
 - Preprocessing: $O(N \log N)$
 - Queries: $O(\log N)$
- Space: $O(N \log N)$

Problems:

- [SPOJ LCA \(test your algorithm\)](#).
- [CSES Company Queries](#)
- [CSES Distance Queries](#)