CPCFI

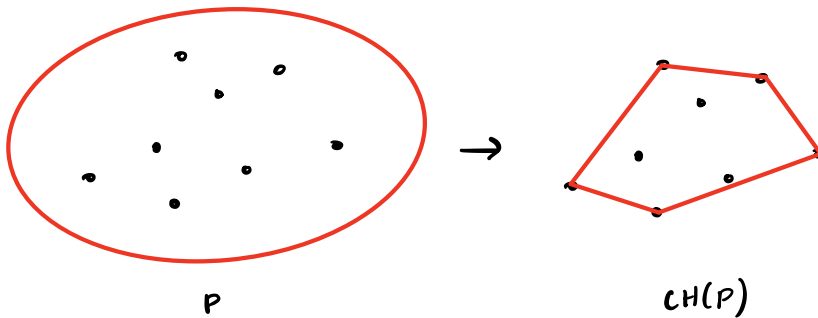## CONVEX HULL

→ [MIXING THINGS]
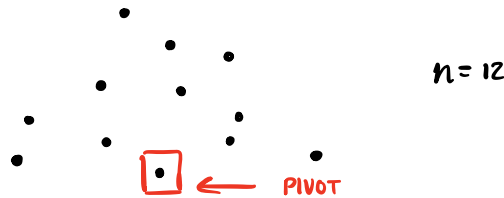
FINDING THE CONVEX HULL OF A SET OF POINTS P: → CH(P)

↳ SMALLEST POLYGON CH(P) FOR WHICH EACH POINT IN P
IS EITHER ON THE BOUNDARY OF CH(P) OR IN ITS INTERIOR
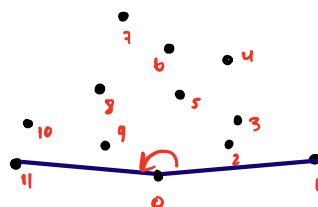

P   CH(P)

GRAHAM'S SCAN ALGORITHM $O(n \log n)$:

a) SORTS ALL n POINTS OF P (FIRST POINT DOESN'T NEED TO BE REPLICATED
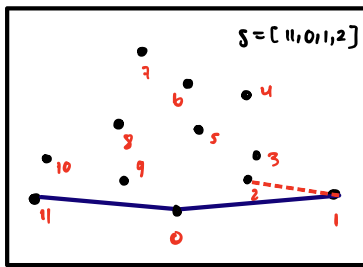AS LAST POINT) BASED ON THEIR ANGLES WITH RESPECT TO A PIVOT POINT

$n = 12$

← PIVOT

b) MAINTAIN A STACK S OF CANDIDATE POINTS

c) EACH POINT OF P IS PUSHED ONCE ONTO S AND POINTS THAT WON'T BE PART
OF CH(P) GET POPPED FROM S

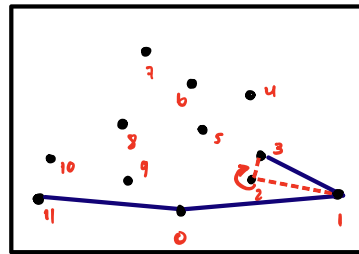d) WE FIRST INSERT POINTS $n-1, 0, 1$ ONTO S (THIS FORMS A LEFT TURN)

$S = [11, 0, 1]$

e) INSERT NEXT POINT i, IF TOP THREE ELEMENTS IN S MAKE A LEFT
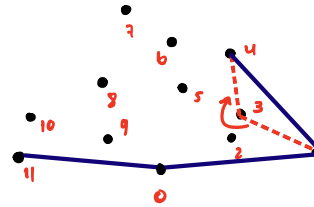TURN, THEN WE KEEP POINT i IN THE CH(P)

$S = [11,0,1,2]$

WE TRY TO INSERT POINT 2 TO S, SINCE 0-1-2 MAKE A LEFT TURN, WE KEEP 2 IN CP(H)
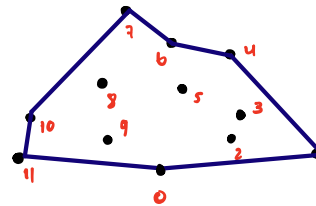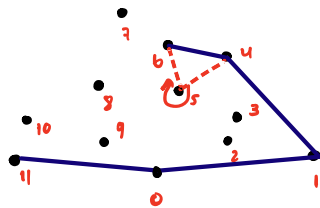
$S = [11,0,1,3]$

0-1-3 NOW FORMS A LEFT TURN

WE TRY TO INSERT POINT 3, BUT 1-2-3 MAKE A RIGHT TURN, THUS WE NEED TO POP 2

$S = [11,0,1,4,5]$

$S = [11,0,1,4]$

$S = [11,0,1,4,6]$

$S = [11,0,1, 4,6,7, 10,11]$
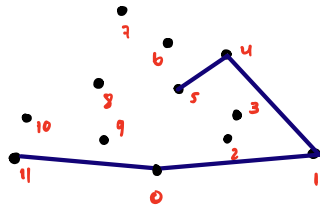
CONVEX HULL OF P

$\hookrightarrow$ SEARCHING FOR A PAIR OF INTERSECTING SEGMENTS :

[MAP OVERLAY PROBLEM]
(CITIES, STREETS, RIVERS)

FIND BRIDGES POSITIONS

- GIVEN n LINE SEGMENTS ON A PLANE, IT'S REQUIRED TO CHECK WHETHER AT LEAST TWO OF THEM INTERSECT WITH EACH OTHER. REPORT ALL INTERSECTING SEGMENTS
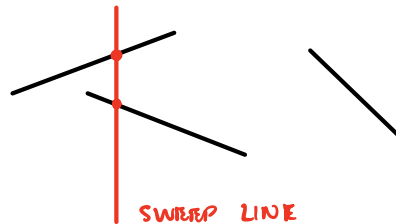
NAIVE SOLUTION RUNS IN $O(n^2)$

CHECK ALL PAIRS OF LINES AND FIND THEIR POSSIBLE INTERSECTION POINT

SWEEP LINE ALGORITHM : $O(n \log n)$

$\hookrightarrow$ IMAGINE A VERTICAL LINE THAT MOVES FROM $X = -\infty$ TO $X = \infty$. THIS LINE WILL INTERSECT WITH THE LINE SEGMENTS ON THE PLANE :

SWEEP LINE

$\hookrightarrow$ WE'RE INTERESTED IN THE ORDER OF THE SEGMENTS ALONG THE VERTICAL. WE'LL STORE THE SEGMENTS CROSSING THE SWEEP LINE AT ANY GIVEN POINT AND STORE THEM BY y-COORDINATE :

SWEEP LINE
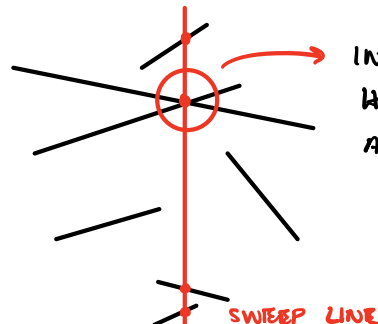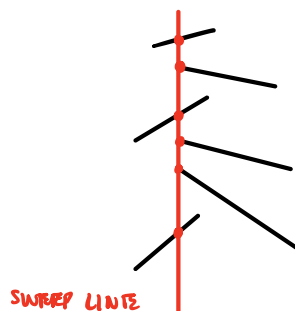
SWEEP LINE

INTERSECTING SEGMENTS HAVE THE SAME y-COORDINATE AT ONE TIME

KEEP IN MIND :

- SWEEP LINE ALGORITHM GIVES AN ORDER OF COMPARISON OF SEGMENTS
- WE ONLY COMPARE ADJACENT SEGMENTS AT FIXED POSITION OF THE S.L.
- CONSIDER S.L. ONLY IN POSITIONS WHEN NEW SEGMENT APPEAR AND OLD ONE DISAPPEAR
- WE KEEP A LIST OF ACTIVE SEGMENTS
- IF A NEW SEGMENT APPEARS, WE INSERT IT INTO THE LIST AND CHECK WITH UPPER AND LOWER NEIGHBORS IF THEY INTERSECT

ORDERED BY x-COORDINATE IN A SET
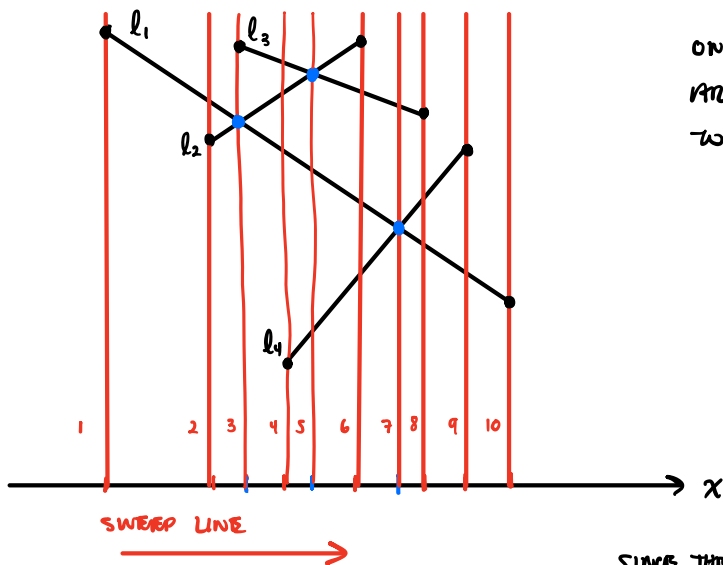
ON EACH ITERATION OF THE SWEEP LINE, WHAT ARE THE **ACTIVE SEGMENTS** THAT WE HAVE TO COMPARE?

ℓ_1

ℓ_3

ℓ_2

ℓ_4

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

x

SWEEP LINE

SINCE THEY ARE ADJACENT BASED ON THEIR x - COORDINATE, WE CHECK FOR AN INTERSECTION POINT

SWEEP LINE ITERATION	ACTIVE LINE SEGMENTS
1	ℓ_1
2	ℓ_1, ℓ_2
3	ℓ_1, ℓ_2, ℓ_3
4	$\ell_1, \ell_2, \ell_3, \ell_4$
5	$\ell_1, \ell_2, \ell_3, \ell_4$
6	ℓ_1, ℓ_3, ℓ_4
7	ℓ_1, ℓ_3, ℓ_4
8	ℓ_1, ℓ_3, ℓ_4
9	ℓ_1, ℓ_4
10	ℓ_1

AT THIS POINT, ℓ_4 AND ℓ_1 INTERSECT, AND WE FIND ITS INTERSECTION POINT BECAUSE AT x_4, ℓ_4 AND ℓ_1 ARE ADJACENT