Lecture: BITWISE OPERATIONS

Unit: 5 - DYNAMIC PROGRAMMING II

Instructor: Néstor

INTEGERS → BINARY

$$23 = 16 + \underline{7} = 16 + \underbrace{4 + 3} = 16 + 4 + 2 + 1$$

$$= 2^4 + 2^2 + 2^1 + 2^0$$

$$= \underline{10111}_{(2)}$$

BINARY → INTEGER

$$\underset{2^4}{1}\ \underset{2^3}{0}\ \underset{2^2}{1}\ \underset{2^1}{1}\ \underset{2^0}{1} = 2^4 + 2^2 + 2^1 + 2^0 = 16 + 4 + 2 + 1 = 23$$

LOGICAL OPERATIONS

A	B	!A	A && B	A || B	A ^ B
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	1	1
1	1	0	1	1	0

* NOTE: A != B IS EQUIVALENT TO A ^ B

A	B	!=	^
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

BITWISE OPERATIONS

↳ COMPUTE LOGICAL OPERATIONS BIT BY BIT

$23 = 10101_{(2)}$

$69 = 1000101_{(2)}$

$23 \,\&\&\, 69$

$$\begin{array}{r} 0010101 \\ \&\& \; 1000101 \\ \hline 0000101 \end{array} \rightarrow 5$$

BITWISE SHIFTS

$\boxed{① \text{ LEFT SHIFT : } a << b \implies a \cdot 2^b}$

$a = 5 = 101_{(2)}$

$b = 3 = 011_{(2)}$

$5 << 3 = 101_{(2)} << 3 = 101000_{(2)} = \underline{40}$

$5 << 3 = 5 \cdot 2^3 = \underline{40}$

$128 = 1000000_{(2)} = 1 << 7 = \underline{\underline{1 \cdot 2^7}}$

$\boxed{② \text{ RIGHT SHIFT : } a >> b \implies \left\lfloor \frac{a}{2^b} \right\rfloor}$

FLOOR FUNCTION

$a = 5 = 101_{(2)}$

$b = 3 = 011_{(2)}$

$5 >> 3 = 101_{(2)} >> 3 = 0\cancel{1}\cancel{0}\cancel{1}$

$5 >> 3 = \left\lfloor \frac{5}{2^3} \right\rfloor = 0$

$a = 23 = 10111_{(2)}$

$b = 3 = 011_{(2)}$

$23 >> 3 = 10111_{(2)} >> 3 = 10\cancel{1}\cancel{1}\cancel{1}_{(2)} = 2$

$23 >> 3 = \left\lfloor \frac{23}{2^3} \right\rfloor = \left\lfloor \frac{23}{8} \right\rfloor = 2$

BINARY REPRESENTATION OF A NUMBER

$x = 101001$ $\qquad\qquad$ $y = 100101$

$$\boxed{\begin{array}{c} m = 8 = 1 \ll 3 \\ = 1000_{(2)} \end{array}}$$

x & m $\qquad\qquad$ y & m

$$\begin{array}{r} 101001 \\ \& \ 001000 \\ \hline 001000 \end{array} \qquad\qquad \begin{array}{r} 100101 \\ \& \ 001000 \\ \hline 000000 \end{array}$$

$\boxed{x \,\&\, (1 \ll i)} \rightarrow$ is <u>ith BIT OF X ON</u>?

IMPORTANT : OVERFLOW

$1 \ll i$ $\rightarrow$ i AT MOST 30 SINCE <u>1</u> IS TREATED AS AN INTEGER

$\Downarrow$ SOLUTION

$1LL \ll i$ $\rightarrow$ i AT MOST 62

$\Downarrow$

$1ULL \ll i$

<u>BETTER SOLUTION</u>: (AVOID OVERFLOWS)

$\hookrightarrow$ CHECK IF $\boxed{(x \gg i) \,\&\, 1}$

BITSETS

↳ A BITMASK CAN ONLY HANDLE AS MUCH INTEGERS AS THE COMPUTER WORD SIZE

↳ ~ 64

∴ WE CAN AT MOST REPRESENT (1 << 63)

SOLUTION ?

↳ BITSETS → $\underbrace{\texttt{bitset<64> a;}}_{\text{NO PROBLEM}}$

⇓

- "LONGER BINARY MASK"
- "SET WITH QUICK INTERSECTIONS"

→ bool A[20] : 20 BYTES

→ bitset<20> A : 20 BITS ✓✓