Lecture: 6: SINGLE SOURCE SHORTEST PATHS II

Unit: 6 - GRAPHS I

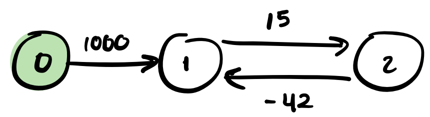
Instructor: WÉSTON

I. SSSP ON GRAPH WITH NEGATIVE WEIGHTS

MOTIVATING PROBLEM: GIVEN A WEIGHTED DIRECTED GRAPH G WITH n VERTICES AND m EDGES AND SOME SOURCE VERTEX s, FIND THE LENGTH OF THE SHORTEST PATHS FROM s TO ANY OTHER VERTEX

* IMPORTANT: G CAN CONTAIN NEGATIVE WEIGHT EDGES OR EVEN NEGATIVE CYCLES

DIJKSTRA'S ALGORITHM DOESN'T WORK BECAUSE IT WILL KEEP ADDING NEW VERTICES TO THE SET OR PRIORITY QUEUE PRODUCING AN INFINITE LOOP



SHORTEST PATH TO VERTEX 2 IS $-\infty$ BECAUSE WE COULD KEEP TRAVERSING THE GRAPH REDUCING ON EACH STEP D[2] UNTIL $-\infty$

$D[2] = 1015 \rightarrow 988 \rightarrow 961 \rightarrow \dots \rightarrow -\infty$

HOW DO WE SOLVE THIS PROBLEM? (AND BY LIMITING THE # OF ITERATIONS)

$\Downarrow$

$\boxed{\text{BELLMAN - FORD'S ALGORITHM}}$ $\rightarrow$ LIST OF EDGES AS GRAPH REPRESENTATION

- LET'S ASSUME NO NEGATIVE WEIGHT CYCLE IS PRESENT

0
|
0

0 CREATE AN ARRAY OF DISTANCES D[0, ..., n-1] —o —o —o

- D[s] = 0

- $D[v] = \infty \quad \forall v \in G \mid v \neq s$

① ITERATE n-1 TIMES :

- ON EACH ITERATION ITERATE OVER ALL m EDGES :
  - TRY TO PRODUCE RELAXATION OF EDGE (A, B) WITH WEIGHT C
    → TRY TO IMPROVE D[B] USING D[A] + C

$$D[B] = \min(D[B],\ D[A] + c)$$

BELLMAN - FORD FOR NEGATIVE CYCLES

- INTERESTED IN REACHING A NEGATIVE CYCLE FROM VERTEX S
  → DISTANCES TO VERTICES IN THIS CYCLE ARE NOT DEFINED → $-\infty$

- MEASURE AGAINST NEGATIVE DISTANCES DUE TO THE PRESENCE OF A NEGATIVE WEIGHT CYCLE :
  → (WATCH OUT FOR INTEGER OVERFLOW) :

$$D[v] = \max(-\text{INF},\ D[u] + \text{WEIGHT}(u,v))$$

- CRITERION FOR DETECTING NEGATIVE CYCLES : [FINDING NEGATIVE CYCLES]

① STARTING FROM NODE S

② RUN BELLMAN - FORD FOR n-1 ITERATIONS

③ RUN B.F. FOR 1 MORE ITERATION AND IF IT PERFORMS ONE MORE RELAXATION THEN GRAPH CONTAINS A NEG. CYCLE REACHABLE FROM S

→ REMEMBER LAST VERTEX u FOR WHICH THERE WAS A RELAXATION IN THE n-TH PHASE

RETRIEVE THIS CYCLE AS A SEQUENCE OF VERTICES CONTAINED IN IT

•) u WILL EITHER LIE IN THE CYCLE OR IS REACHABLE FROM IT

••) START FROM u PASS THROUGH ITS PREDECESSORS n TIMES

...) WE WILL GET TO VERTEX W (VERTEX IN THE CYCLE EARLIEST REACHABLE FROM THE SOURCE)

::) GO FROM VERTEX W ITERATING OVER ITS PREDECESSORS UNTIL WE REACH W AGAIN