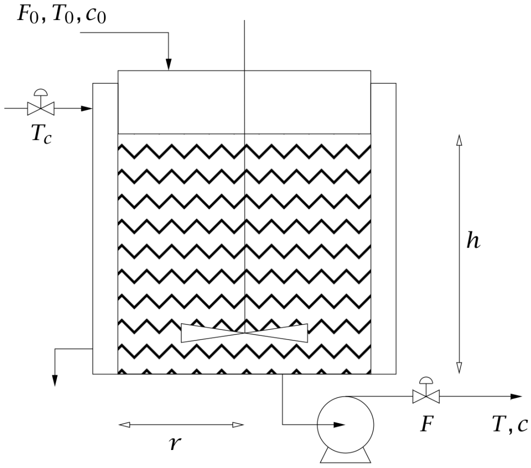
FrontSeat Summer School on *Optimization-based Control Systems: Theory & Applications*

Exercise 2: Numerical Optimal Control and NMPC with `MPC-code` and `acados`

Jonathan Frey, Marco Vaccari

This exercise PDF and all accompanying Python template code can be downloaded or cloned from the course repository https://github.com/CPCLAB-UNIPI/FrontSeatSummerSchool (please follow the instructions in README.md).

In this and some of the following exercises, we consider a nonlinear continuous stirred-tank reactor (CSTR).[1]



An irreversible, first-order reaction $A \to B$ occurs in the liquid phase and the reactor temperature is regulated with external cooling. Mass and energy balances lead to the following nonlinear model:

$$\begin{aligned}
\dot{c} &= \frac{F_0(c_0 - c)}{\pi r^2 h} - k_0 \exp\left(-\frac{E}{RT}\right) c \\
\dot{T} &= \frac{F_0(T_0 - T)}{\pi r^2 h} - \frac{\Delta H}{\rho C_p} k_0 \exp\left(-\frac{E}{RT}\right) c + \frac{2U}{r \rho C_p}(T_c - T) \\
\dot{h} &= \frac{F_0 - F}{\pi r^2}
\end{aligned}$$

with states $x = (c, T, h)$ where c is the concentration of substance A, T is the reactor temperature and h is the height. The controls $u = (T_c, F)$ are the coolant liquid temperature T_c and the outlet flowrate F.

Exercise 2.1: `MPC-code`

Description of closed-loop simulation environment

- `MPC_code.py` is the main file that has to be run containing the simulation of the closed-loop system and plot the trajectories
- `Target_calc.py` defines the steady-state target optimization module
- `Control_calc.py` defines the dynamic optimization module, i.e. the OCP
- `Estimator.py` contains all the possible state estimators

[1] The example as well as the figure have been adopted from Example 1.11 in Rawlings, J.B., Mayne, D.Q. and Diehl, M., 2017. *Model predictive control: theory, computation, and design (Vol. 2).* Madison, WI: Nob Hill Publishing.

- `Utilities.py` contains support functions used in all the other modules
- `Default_Values.py` contains the default values of many options the user can specify in the example file.
- `SS_JAC_ID.py` contains a tool for an automatic system linearization
- `Ex_NMPC_FSSS.py` defines the example of non-linear MPC containing the model equations stated above with values for all the parameter values and the tuning values for building the NMPC. This is the only file to be coded

Tasks

1. simulate the system with a constant reference control input and nominal NMPC by launching `MPC_code.py`
2. introduce an error in the initial flow rate F_0 of 10% in the process equations (`User_fxp_Cont(x,t,u,pxp,pxmp)`) and check how the behavior of the system has changed
3. introduce a non linear disturbance in the model to compensate the offset; use `offree="nl"` and set the dimension of the disturbance `"d"` to 2, then insert `"d"` into the model equations (`User_fxm_Cont(x,u,d,t,px)`).
4. Check how the OCP is created in `Control_calc.py`→ `opt_dyn`. Specifically, what is the main difference between the trajectory formulation in here with respect to the one you saw in Exercise 1.2, i.e. in `OCPsolver`?

Exercise 2.2: NMPC with `acados`

Description of closed-loop simulation environment

- The `main.py` file is prepared to simulate the closed-loop system with `acados` and plot the trajectories.
- The file `cstr_model.py` defines the model equations stated above with values for all the parameter values
- The file `setup_acados_integrator.py` uses this model to generate an integrator of the type `AcadosSimSolver` which we use as our plant model.

Tasks

Uncontrolled simulation:

1. simulate the system with a constant reference control input.

Exact NMPC with acados and CasADi:

2. Set `with_nmpc_controller = True` in `main.py` to also create an `AcadosOcpSolver` and run it in a closed loop simulation.
3. Set `with_casadi_nmpc_controller = True` in `main.py` to also create an `CasadiOcpSolver`.
4. Compare the trajectories with the one in the previous exercise and the computation times of the solvers. Note: the differences are documented in the documentation of `CasadiOcpSolver`.

Fast and approximate controllers:

5. From now on, to save CPU time, we omit the `CasadiOcpSolver` implementation again: set `with_casadi_nmpc_controller = False`

6. Set `with_rti_controller = True` in `main.py` to create an `AcadosOcpSolver` that uses the real-time iteration (RTI) algorithm. This algorithm performs one SQP iteration at each sampling time.

7. Set `with_linear_mpc_controller = True` in `main.py` to create an `AcadosOcpSolver` with a model linearized at the steady state.

8. Compare the resulting closed loop trajectories and the runtime of their controllers

NOTE: If you don't see much difference between LMPC and the constant reference input, run again in a clean `ipython` session or `jupyter` notebook.

Additional exercise (if desired): NMPC with model plant mismatch

- Note that the model parameter F_0 is implemented as a parameter in the `AcadosModel`.
- Task: Introduce a mismatch between the OCP model and the plant, by increasing F_0 in the OCP model by 5%. How well are the references tracked?