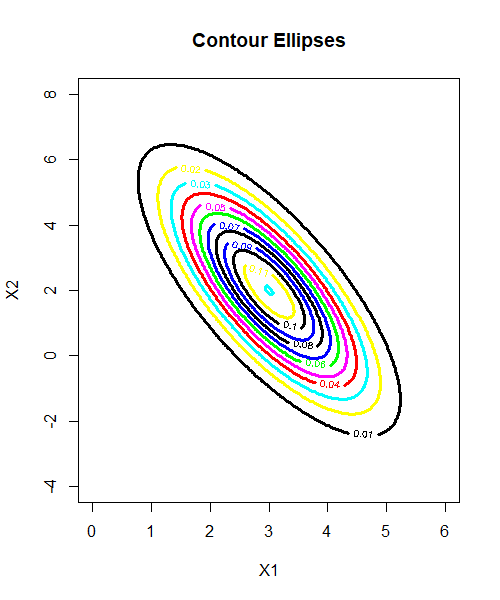
**Exercise 4.1** Assume that the random vector *Y* has the following normal distribution: . Transform it according to (4.49) to create with mean and . How would you implement the resulting formula on a computer?

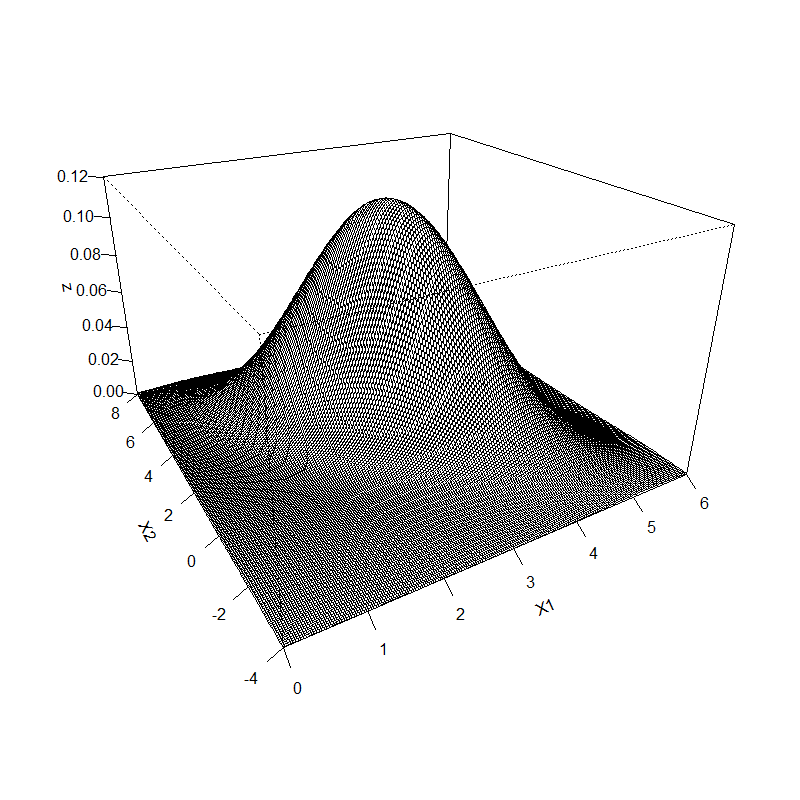
**Sol.**

用 做線性轉換得到，多變量常態母體 意味它的變數 服從卡方分配，自由度為p。



等高線圖中，圖形中心點為平均數 ，橢圓內的每個值與的距離都小於，數學式可表示為，當越靠近橢圓中心，表示它的probability也越高。

另外，搭配3D surface plot可以觀察出X只有一個高峰，而且等高線圖的每條等高線間距沒有特別離很遠或很近的，整體呈現對稱狀態，可以推測為常態分布。



**Exercise 5.1** Consider with and and the matrices , . Show that and are independent.

**Sol.**

根據多變量常態分配定理，若，則的任意線性組合皆為常態分配。

R Code:

# clear all variables

rm(list = ls(all = TRUE))

graphics.off()

# 4.1

# install and load packages

libraries = c("MASS", "mnormt")

lapply(libraries, function(x) if (!(x %in% installed.packages())) {

install.packages(x)

})

lapply(libraries, library, quietly = TRUE, character.only = TRUE)

# parameter settings

n1 = 200 # number of draws

mu1 = c(3, 2) # mean vector

sig1 = matrix(c(1, -1.5, -1.5, 4), ncol = 2) # covariance matrix

# bivariate normal sample

set.seed(1024)

x = mvrnorm(n1, mu1, sig1, 2)

# bivariate normal density

xgrid = seq(from = (mu1[1] - 3 \* sqrt(sig1[1, 1])), to = (mu1[1] + 3 \* sqrt(sig1[1, 1])),

length.out = 200)

ygrid = seq(from = (mu1[2] - 3 \* sqrt(sig1[2, 2])), to = (mu1[2] + 3 \* sqrt(sig1[2, 2])),

length.out = 200)

z = outer(xgrid, ygrid, FUN = function(xgrid, ygrid) {

dmnorm(cbind(xgrid, ygrid), mean = mu1, varcov = sig1)

})

# contour ellipses

contour(xgrid, ygrid, z, xlim = range(xgrid), ylim = range(ygrid), nlevels = 10, col = c("blue",

"black", "yellow", "cyan", "red", "magenta", "green", "blue", "black"), lwd = 3,

cex.axis = 1, xlab = "X1", ylab = "X2")

title("Contour Ellipses")

# surface plot

persp(xgrid, ygrid, z, theta=-30, phi=25, expand=0.6, ticktype='detailed', xlab="X1", ylab = "X2" )

# 5.1

# parameter settings

n2 = 200 # number of draws

mu2 = c(2, 2) # mean vector

sig2 = matrix(c(1, 0, 0, 1), ncol = 2) # covariance matrix

A <- t(matrix(c(1, 1)))

B <- t(matrix(c(1, -1)))

# bivariate normal sample

set.seed(1024)

x2 = mvrnorm(n2, mu2, sig2, 2)

# 先證AX,BX常態，再做共變異數=0

m1 <- A%\*%mu2

m2 <- B%\*%mu2

s11 <- A%\*%sig2%\*%t(A)

s12 <- A%\*%sig2%\*%t(B)

s21 <- B%\*%sig2%\*%t(A)

s22 <- B%\*%sig2%\*%t(B)

S <- matrix(c(s11, s21, s12, s22), ncol = 2)