GAME THEORY

Shreyans Garg

Topics to cover

1. Nim Game
2. Grundy Numbers
3. Problems

Two friends Dev and Deepak play a game.

Initially, there is one pile consisting of n stones on the table. During one move one pile should be taken and divided into an arbitrary number of piles consisting of $a_1 > a_2 > \ldots > a_k > 0$ stones. The piles should meet the condition $a_1 - a_2 = a_2 - a_3 = \ldots = a_{k-1} - a_k = 1$. Naturally, the number of piles k should be no less than two.

The friends play in turns. The player who cannot make a move loses. Dev makes the first move. Who will win if both players play optimally?

$1 \le n \le 10^5$

https://codeforces.com/contest/88/problem/E
https://codeforces.com/contest/88/submission/279992369

Shreyans and Tanmay play a game. The table has n piles of coins lying on it, the i-th pile has a_i coins. They move in turns, Shreyans moves first. In one move you are allowed to:

- choose some pile, let's denote the current number of coins in it as x;
- choose some integer y ($0 \le y < x$; $x^{1/4} \le y \le x^{1/2}$) and decrease the number of coins in this pile to y. In other words, after the described move the pile will have y coins left.

The player who can't make a move, loses. Your task is to find out, who wins in the given game if both play optimally well.

$1 \le n \le 10^5$, $1 \le a_i \le 10^{12}$

https://codeforces.com/problemset/problem/255/E
https://codeforces.com/contest/255/submission/313282555

Shobhit and Shashwat are playing a game. They have a list of n numbers in the game.

In a player's turn, he chooses a number p^k (where p is a prime number and k is a positive integer) such that p^k divides at least one number in the list. For each number in the list divisible by p^k, call it x, the player will delete x and add to the list.

The player who can not make a valid choice of p and k loses.
Shobhit starts the game and the players alternatively make moves. Determine which one of players will be the winner if both players play optimally.

$1 \le n \le 100$, $1 \le a_i \le 10^9$

https://codeforces.com/problemset/problem/850/C
https://codeforces.com/contest/850/submission/275088675

Tanay is playing a game with his friend Sushant. He brought a tree with n nodes numbered from 1 to n and rooted at node 1. The i-th node has a_i apples. This tree has a special property: the lengths of all paths from the root to any leaf have the same parity (i.e. all paths have even length or odd length).

Tanay and Sushant will take turns to play. Sushant will make the first move. The player who can't make a move loses.
In each move, the current player will pick a single node, take a non-empty subset of apples from it and do one of the following two things:

1. eat the apples, if the node is a leaf.
2. move the apples to one of the children, if the node is non-leaf.

Before Sushant comes to start playing, Tanay will make exactly one change to the tree. He will pick two different nodes u and v and swap the apples of u with the apples of v.
Can you help Tanay count the number of ways to make the swap (i.e. to choose u and v) after which he will win the game if both players play optimally? (u, v) and (v, u) are considered to be the same pair.

$2 \le n \le 10^5$, $1 \le a_i \le 10^7$

https://codeforces.com/contest/812/problem/E
https://codeforces.com/contest/812/submission/313287186

Reading Material

https://codeforces.com/blog/entry/66040

https://www.isid.ac.in/~antar/Teaching/Fall-2013/Books/Ferguson_31DEC1969.pdf

https://cp-algorithms.com/game_theory/sprague-grundy-nim.html

Thank You