Function.

Every element in domain have a unique image in co-domain

Function.

- Algebric fun.
  - polynomial.
  - Rational.
  - Irrational.
  - Piece-wise fun.
    - Modulus fun.
    - Signum fun.
    - Greatest int. fun.
    - Least int fun.
    - Franctional part fun.
- Transcendental fun.
  - Trignometric fu
  - Inverse Trignom
  - Exponential fu
  - Logarithmic fun
  - Geometrical fo

Number Line Rule / Wary curve Method:

Used only when $<, >, \leq, \geq$.

(1) Plotted only odd power of x (odd power in Numerator or Denominator).

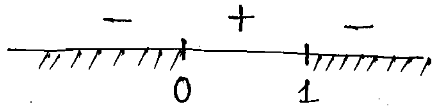
(2) Make the co-efficient of x +ve.

(3) Starting number line taking the sign outside the expression from right to left ~ alternatively.

E.g. $\frac{1}{x} < 1 \Rightarrow \frac{1}{x} - 1 < 0 \Rightarrow \frac{1-x}{x} < 0 \Rightarrow \frac{-(x-1)}{x} < 0.$

i.e $x < 0$ or $x > 1$

or $x \in (-\infty, 0) \cup (1, \infty)$

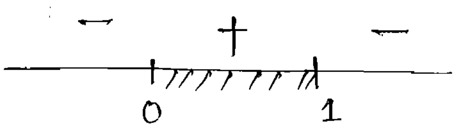


E.g. $\frac{1}{x} > 1$

$\Rightarrow \frac{1}{x} - 1 > 0$

$\Rightarrow \frac{1-x}{x} > 0$

$\Rightarrow \frac{-(x-1)}{x} > 0$



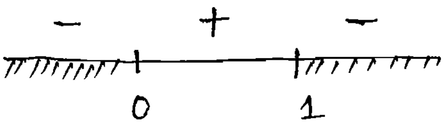
$\Rightarrow x \in (0, 1).$

$\frac{1}{x} \leq 1 \quad , x \neq 0$

$\Rightarrow \frac{1}{x} - 1 \leq 0$

$\Rightarrow \frac{1-x}{x} \leq 0$

$\Rightarrow \frac{-(x-1)}{x} \leq 0.$



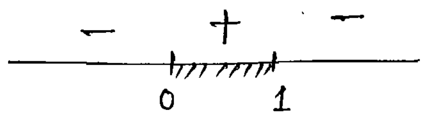
$x \in (-\infty, 0) \cup [1, \infty).$

$\frac{1}{x} \geq 1 \quad , x \neq 0$

$\Rightarrow \frac{1}{x} - 1 \geq 0$

$\Rightarrow \frac{1-x}{x} \geq 0$

$\Rightarrow \frac{-(x-1)}{x} \geq 0.$

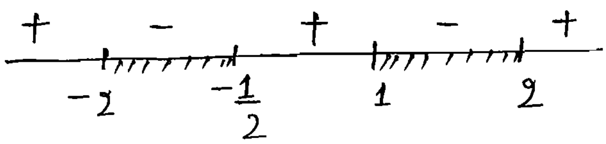


$x \in (0, 1].$

Q $\frac{(x-1)(x-2)}{(2x+1)(x+2)} < 0.$

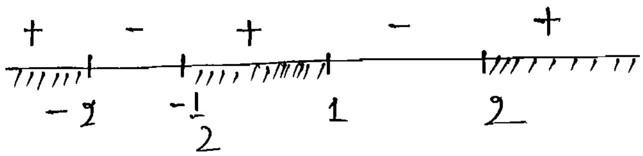
$x = 1, \quad x = 2$

$x = -\frac{1}{2}, \quad x = -2$



$x \in (-2, -\frac{1}{2}) \cup (1, 2)$

Q $\frac{(x-1)(x-2)}{(2x+1)(x+2)} > 0$



$x \in (-\infty, -2) \cup (-\frac{1}{2}, 1) \cup (2, \infty).$

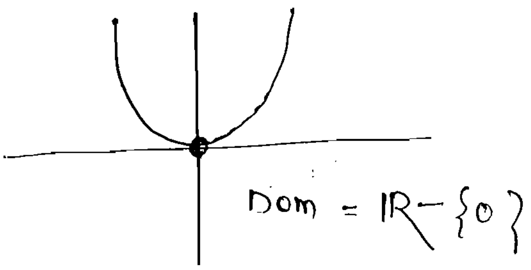
Q $\frac{(x-1)(x-2)}{(2x+1)(x+2)} \leq 0$

$x \in (-2, -\frac{1}{2}) \cup [1, 2]$

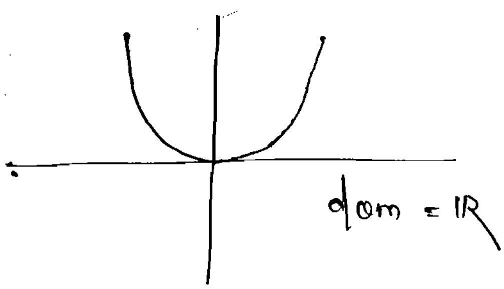
Q $\frac{(x-1)(x-2)}{(2x+1)(x+2)} \geq 0$

$x \in (-\infty, -2) \cup (-\frac{1}{2}, 1] \cup [2, \infty)$

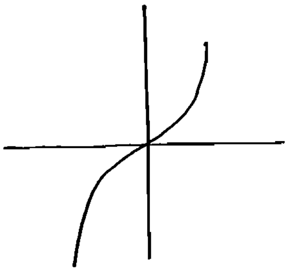
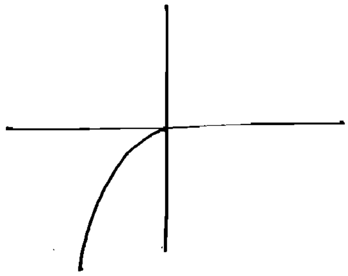
Q $f(x) = x^2 > 0$



$f(x) = x^2 \geq 0.$

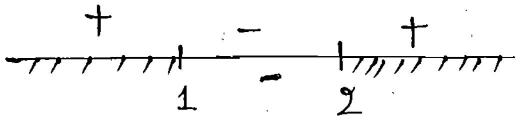


$f(x) = x^{o} \quad \forall\, x > 0$ $\qquad\qquad$ $f(x) = x^{o} < 0$

Q. $$\frac{x^{100}(x-2)^{67}}{(x-1)^{77}(x+2)^{200}} \geq 0 \quad , \; x \neq 1 , \; x \neq -2 .$$

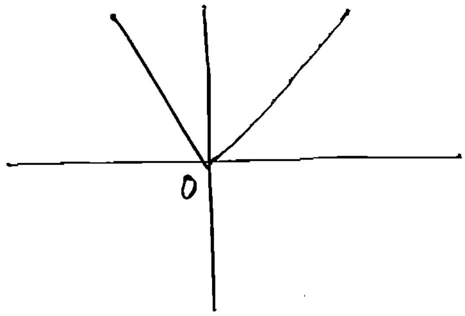
$x = 2, \; x = 1$ (consider only the odd power).



$x \in (-\infty, 1) \cup [2, \infty)$.

Modulus Function.

→ $y = |x| = \begin{cases} x \; ; & x \geq 0 \\ -x \; ; & x < 0 . \end{cases}$



dom = $\mathbb{R}$

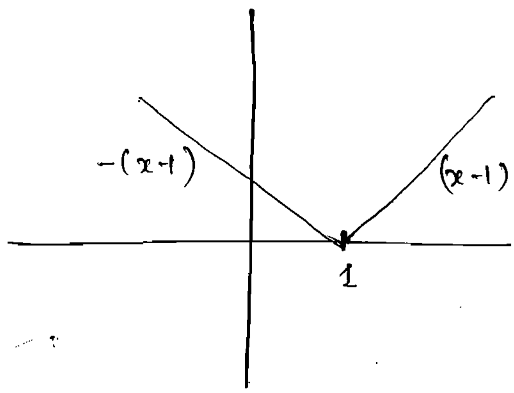
Range = $[0, \infty)$

local max. = ∞

local min. = 0 .

→ $y = |x - 1| = \begin{cases} (x-1) \; ; & x-1 \geq 0 \\ -(x-1) \; ; & x-1 < 0 . \end{cases}$

$= \begin{cases} x-1 \; ; & x \geq 1 . \\ -(x-1) \; , & x < 1 \end{cases}$

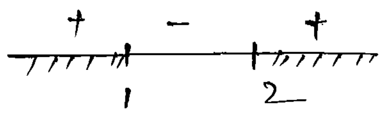


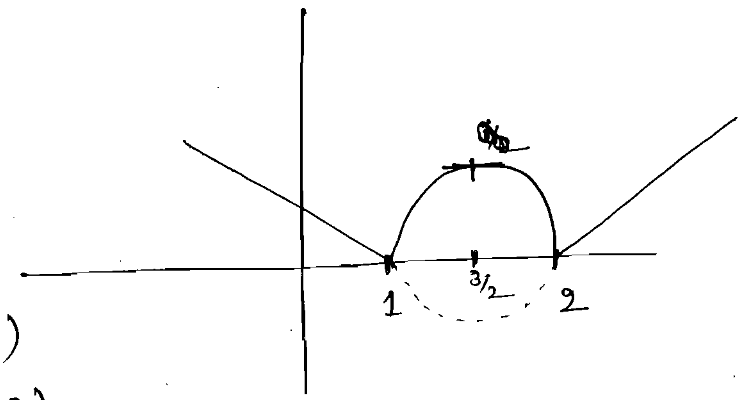
Dom = $\mathbb{R}$

Range = $[0, \infty)$

$y = |(x-1)(x-2)| = \begin{cases} (x-1)(x-2) & ; (x-1)(x-2) \geq 0 \\ -(x-1)(x-2) & ; (x-1)(x-2) < 0. \end{cases}$

$$= \begin{cases} (x-1)(x-2) & ; x \in (-\infty, 1] \cup [2, \infty) \\ -(x-1)(x-2) & ; x \in (1, 2). \end{cases}$$





Dom = $\mathbb{R}$

Range = $[0, \infty)$

local $\max^m$ at $\frac{3}{2}$ in $(1,2)$

local $\min^m$ at 0 in $(1,2)$

but global $\max^m = \infty$

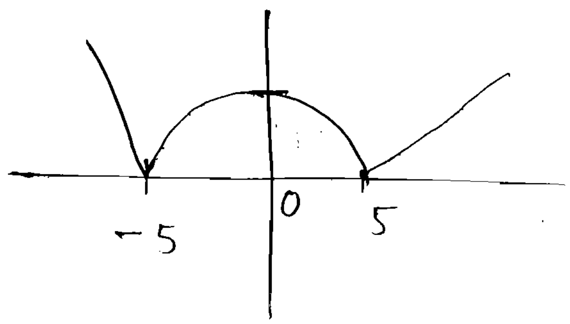
" $\min^m = 0$.

JAM-2016. Let $f(x) = |x^2 - 25| \ \forall x \in \mathbb{R}$., the total no. of points of $\mathbb{R}$ at which 'f' attains its extremum (local $\max^m$ or local $\min^m$.)

(a) 1 (b.) 2 (c) 3 (d) 4.

$$f(x) = |x^2 - 25| = |(x+5)(x-5)|$$

$$= \begin{cases} (x+5).(x-5) & ; x \in (-\infty, -5] \cup [5, \infty) \\ -(x+5)(x-5) & ; x \in (-5, 5). \end{cases}$$



Properties.

1. $|x|^2 = x^2$

2. $|x| < a \Rightarrow -a < x < a.$ (a is +ve)

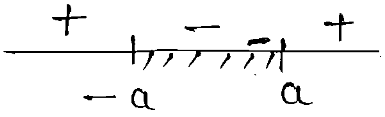
$\because |x| < a$

$\Rightarrow |x|^2 < a^2$

$\Rightarrow x^2 < a^2$

$\Rightarrow x^2 - a^2 < 0$

$\Rightarrow (x-a)(x+a) < 0.$



$x \in (-a, a)$

$\Rightarrow -a < x < a.$

3. $|x| < a$ (a is −ve)

$\Rightarrow$ No soln.

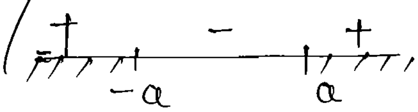
4. $|x| > a$ (a is +ve)

$\Rightarrow x > a$ or $x < -a.$

$\because |x| > a$

$\Rightarrow |x|^2 > a^2$

$\Rightarrow (x+a)(x-a) > 0.$



$\Rightarrow x \in (-\infty, -a) \cup (a, \infty)$

$\Rightarrow x < -a$ or $x > a.$

5. $|x| > a$ (a is −ve)

$\Rightarrow \forall x \in \mathbb{R}$, always true.

6. $a < |x| < b$ (a, b are +ve)

$\Rightarrow a < x < b$ or $-b < x < -a$

(7) ~~|a|+|b|~~

$\because |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$

if $x \geq 0$,	$x < 0$
$a < x < b$	$a < -x < b$
	$\Rightarrow -b < x < -a$

$\Rightarrow x \in (a, b) \cup (-b, -a).$

(7) if $|a| + |b| = |a+b|.$

$\Rightarrow a \cdot b \geq 0.$

$\because (|a| + |b|)^2 = |a+b|^2$

$\Rightarrow a^2 + b^2 + 2|ab| = (a+b)^2$

$\Rightarrow \cancel{a^2} + \cancel{b^2} + 2|ab| = \cancel{a^2} + \cancel{b^2} + 2ab$

$\Rightarrow 2|ab| = 2ab$

$\Rightarrow |ab| = ab$

$\Rightarrow \boxed{ab \geq 0}$ ($\because |x| = x \Rightarrow x \geq 0$)

(8) $|a|+|b| = |a-b|$

$\Rightarrow a \cdot b \leq 0.$

$\because (|a|+|b|)^2 = (|a-b|)^2$

$\Rightarrow \cancel{a^2} + \cancel{b^2} + 2|ab| = \cancel{a^2} + \cancel{b^2} - 2ab$

$\Rightarrow \cancel{2}|ab| = -\cancel{2}ab$

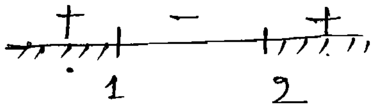
$\Rightarrow -ab \geq 0 \qquad (\because |x| = -x \Rightarrow x < 0).$

$\Rightarrow \boxed{ab \leq 0.}$

E.g. Find the soln. of the eqn. $\dfrac{|x|-1}{|x|-2} \geq 0$; $x \neq \pm 2$.

Soln. put $|x| = y$.

$\Rightarrow \dfrac{y-1}{y-2} \geq 0.$



$y \in (-\infty, 1] \cup (2, \infty)$

$y \leq 1$

$\Rightarrow |x| \leq 1$

$\Rightarrow -1 \leq x \leq 1$

$\Rightarrow x \in [-1, 1]$

~~and~~ OR

$y > 2$

$\Rightarrow |x| > 2$

$\Rightarrow x < -2$ or $x > 2$,

$\Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

$\therefore \Rightarrow x \in (-\infty, -2) \cup [-1, 1] \cup (2, \infty)$ □

FIELD.

F is said to be a field if it satisfies the following properties ;

i) $(\mathbb{F}, +)$ is an abelian group.

ii) $(\mathbb{F}, \cdot)$ closure property.

iii) $(\mathbb{F}, \cdot)$ associative.

iv) Unity.

v) Inverse of non-zero elements exists.

$\Rightarrow (\mathbb{F}^*, \cdot)$ is an abelian group.

vi) $\left.\begin{array}{l} a\cdot(b+c) = a\cdot b + a\cdot c \\ (a+b)\cdot c = a\cdot c + b\cdot c \end{array}\right] \forall\, a, b, c \in \mathbb{F}$.

E.g $(\mathbb{R}, +, \cdot)$, $(\mathbb{Q}, +, \cdot)$, $(\mathbb{C}, +, \cdot)$, $(\mathbb{Z}_p, +_p, \cdot_p)$ etc.

Cardinality of a finite field will be p^n ; $n \in \mathbb{N}$.

(can't be divisible by two distinct primes.)

LINEAR ALGEBRA:

It is nothing but finite dimensional functional analysis.

Internal Composition:

If V be non-empty set then a composition $*$ is said to be internal composition on V, if $v * v \in V$.

External Composition:

If V be non-empty set, then a composition '$\cdot$' is s.t.b external composition in V over $\mathbb{F}$ if $\alpha \in \mathbb{F}$ and ~~v~~ $v \in V$ then $\alpha \cdot v \in V$.

E.g-1 Is $\times$ an external composition in $\mathbb{C}$ over $\mathbb{C}$?

Ans. Yes.

E.g Is $\times$ an external composition in $\mathbb{C}$ over $\mathbb{R}$? (Yes.)

E.g " $\times$ " " " $\mathbb{C}$ over $\mathbb{Q}$? (Yes.)

E.g " $\times$ " " " " $\mathbb{R}$ over $\mathbb{R}$? (Yes.)

E.g " $\times$ " " " " $\mathbb{R}$ over $\mathbb{C}$?

Ans. NO. (i.e $i \in \mathbb{C}$, $1 \in \mathbb{R}$ but $i \cdot 1 = i \notin \mathbb{R}$)

E.g Is $\times$ ~~is~~ an external composition in $\mathbb{R}$ over $\mathbb{Q}$? (Yes.)

E.g " " " " $\mathbb{Q}$ over $\mathbb{Q}$? (Yes.)

E.g " " " " $\mathbb{Q}$ over $\mathbb{R}$?

Ans. NO, (i.e $\sqrt{2} \in \mathbb{R}$, $1 \in \mathbb{Q}$ but $1\sqrt{2} = \sqrt{2} \notin \mathbb{Q}$)

E.g Is $\times$ an external comp. in $\mathbb{Q}$ over $\mathbb{C}$?

Ans.- NO. (bcz, $i \in \mathbb{C}$, $1 \in \mathbb{Q}$ but $1 \cdot i = i \notin \mathbb{Q}$)

VECTOR SPACE :

Let V be any non-empty set and Let $(\mathbb{F}, +, \cdot)$ be any field, define two operations,

$+ : V \times V \longrightarrow V$ (vector addⁿ)

$\cdot : \mathbb{F} \times V \longrightarrow V$ (scalar multⁿ)

Then V together with '+' and '·' i.e $(V(\mathbb{F}), +, \cdot)$ is said to be a vector space if it satisfies the following properties.

(1) $(V, +)$ is an abelian group.

(2) $(\alpha + \beta) \cdot v = \alpha v + \beta v . \quad \forall \alpha, \beta \in \mathbb{F}$ and $v \in V$.

(3) $\alpha \cdot (u + v) = \alpha u + \alpha v \quad \forall \alpha \in \mathbb{F}$ and $u, v \in V$.

(4) $(\alpha \beta) \cdot v = \alpha \cdot (\beta v)$

(5) $1 \cdot u = u \quad \forall u \in V$.

where 1 is the unity of the field.

(i) Elements of V are said to be vectors.

(ii) Elements of $\mathbb{F}$ are said to be scalars.

Properties.

Let $V(\mathbb{F})$ be a vector space over a field $\mathbb{F}$. then,

(1) $0 \cdot v = 0 \quad \forall v \in V$.

(2) $\lambda \cdot 0 = 0 \quad \forall \lambda \in \mathbb{F}$.

(3) $\lambda \cdot v = 0 \Rightarrow \lambda = 0$ or $v = 0$.

(4) $\lambda \cdot (-v) = (-\lambda) \cdot v = -(\lambda \cdot v)$.

$\forall \lambda \in \mathbb{F}, v \in V$.

Extension of $\boxed{\mathbb{R} \to \mathbb{R},\ \mathbb{R} \to \mathbb{C}}$

" " $\boxed{\mathbb{C} \to \mathbb{C}}$.

SOME EXAMPLES OF VECTOR SPACE :

E.g-1 $V = \mathbb{R}^+$, $\mathbb{F} = (\mathbb{R}, +, \cdot)$.

Define ; $* : V \times V \longrightarrow V$.

$$u * v = uv.$$

$$\circ : \mathbb{F} \times V \longrightarrow V$$

$$\alpha \circ u = u^\alpha \quad \forall\, u, v \in V \text{ and } \alpha \in \mathbb{F}.$$

Check $V(\mathbb{F})$ is a vector space or not ?

Soln. i) clearly, $(\mathbb{R}^+, *)$ is an abelian group.

ii) $(\alpha+\beta)\circ v = v^{\alpha+\beta} = v^\alpha \cdot v^\beta = (\alpha \circ v) * (\beta \circ v)$

iii) $\alpha \circ (u * v) = \alpha \circ (uv) = (uv)^\alpha = u^\alpha \cdot v^\alpha$
$= (\alpha \circ u) * (\alpha \circ v)$.

iv) $(\alpha \cdot \beta)\circ u = u^{\alpha\beta} = (u^\beta)^\alpha = \alpha \circ (u^\beta) = \alpha \circ (\beta \circ u)$.

v) $1 \circ u = u^1 = u$.

Hence, $V(\mathbb{F})$ is a vector space.

E.g-2 $V = \mathbb{R}^n$; $\mathbb{F} = (\mathbb{R}, +, \cdot)$

$* : V \times V \longrightarrow V$

$(u_1, u_2, \text{—}, u_n) * (v_1, v_2, \text{—}, v_n) = (u_1 + v_1, \text{—}, u_n + v_n)$

$\circ : \mathbb{F} \times V \longrightarrow V$

$\alpha \circ (u_1, u_2, \text{—}, u_n) = (\alpha u_1, \alpha u_2, \text{—}, \alpha u_n.)$

Soln. i) $(\mathbb{R}^n, *)$ is an abelian group.

ii) $(\alpha+\beta)\circ v = ((\alpha+\beta)v_1, (\alpha+\beta)v_2, \text{—}, (\alpha+\beta)v_n)$
$= (\alpha v_1 + \beta v_1, \alpha v_2 + \beta v_2, \text{—}, \alpha v_n + \beta v_n.)$
$= (\alpha v_1, \alpha v_2, \text{—}, \alpha v_n) * (\beta v_1, \beta v_2, \text{—}, \beta v_n)$

iii) $\alpha \circ (u * v) = \alpha \circ (u_1+v_1, u_2+v_2, \text{---}, u_n+v_n)$ (3)

$= (\alpha(u_1+v_1), \alpha(u_2+v_2), \text{---}, \alpha(u_n+v_n))$

$= (\alpha u_1 + \alpha v_1, \alpha u_2 + \alpha v_2, \text{---}, \alpha u_n + \alpha v_n)$

$= (\alpha u_1, \alpha u_2, \text{---}, \alpha u_n) * (\alpha v_1, \alpha v_2, \text{---}, \alpha v_n)$

$= (\alpha \circ u) * (\alpha \circ v)$.

iv) $(\alpha\beta) \circ u = (\alpha\beta u_1, \alpha\beta u_2, \text{---}, \alpha\beta u_n)$

$= \alpha \circ (\beta u_1, \beta u_2, \text{---}, \beta u_n)$

$= \alpha \circ (\beta \circ u)$.

v) $1 \circ u = (1.u_1, 1.u_2, \text{---}, 1.u_n) = u$.

Hence, $V(\mathbb{F})$ is a vector space. □.

E.g. $V = \mathbb{R}^n$; $\mathbb{F} = (\mathbb{C}, +, \cdot)$

$* : V \times V \longrightarrow V$

$(u_1, u_2, \text{---}, u_n) * (v_1, v_2, \text{---}, v_n) = (u_1+v_1, \text{---}, u_n+v_n)$

$\circ : \mathbb{F} \times V \longrightarrow V$

$\alpha \circ u = (\alpha u_1, \alpha u_2, \text{---}, \alpha u_n)$

Soln. Take, $(u_1, u_2, \text{---}, u_n) = (1, 1, \text{---}, 1) \in \mathbb{R}^n$.

$\alpha = i \in \mathbb{C}$.

Now, $\alpha \circ u = (\alpha u_1, \alpha u_2, \text{---}, \alpha u_n)$

$= (i, i, \text{---}, i) \notin \mathbb{R}^n$.

Here, $V(\mathbb{F})$ is not a vector space bcz, it fails scalar multn. □

※ We know; $\boxed{\mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}}$. Check vector space or not.

i) $\mathbb{Q}^n(\mathbb{Q})$ ✓

ii) $\mathbb{Q}^n(\mathbb{R})$ ×

iii) $\mathbb{R}^n(\mathbb{R})$ ✓

iv) $\mathbb{R}^n(\mathbb{Q})$ ✓

v) $\mathbb{R}^n(\mathbb{C})$ ×

vi) $\mathbb{C}^n(\mathbb{C})$ ✓

vii) $\mathbb{C}^n(\mathbb{R})$ ✓

viii) $\mathbb{C}^n(\mathbb{Q})$ ✓

E.g. $V = \mathbb{R}$, $\mathbb{F} = (\mathbb{R}, +, \cdot)$

Define; $* : V \times V \longrightarrow V$.

$$u * v = u + v + 1$$

$$\circ : \mathbb{F} \times V \longrightarrow V$$

$$\alpha \circ u = \alpha u + \alpha - 1.$$

Soln. i) $(\mathbb{R}, *)$ is an abelian group.

ii) LHS $(\alpha+\beta)\circ v = (\alpha+\beta)v + (\alpha+\beta) - 1$

$$= \alpha v + \beta v + \alpha + \beta - 1.$$

RHS $(\alpha \circ v) * (\beta \circ v) = (\alpha v + \alpha - 1) * (\beta v + \beta - 1)$

$$= \alpha v + \alpha - 1 + \beta v + \beta - 1 + 1$$

$$= \alpha v + \beta v + \alpha + \beta - 1.$$

$\therefore$ RHS = LHS.

iii) $\alpha \circ (u * v) = \alpha \circ (u + v + 1)$

$$= \alpha u + \alpha v + \alpha + \alpha - 1.$$

$$= \alpha u + \alpha v + 2\alpha - 1.$$

RHS. $(\alpha \circ u) * (\alpha \circ v) = (\alpha u + \alpha - 1) * (\alpha v + \alpha - 1)$

$$= \alpha u + \alpha - 1 + \alpha v + \alpha - 1 + 1$$

$$= \alpha u + \alpha v + 2\alpha - 1.$$ LHS = RHS.

Cartesian Product:- Def^n

Let A and B are two sets, then the set

$A\times B = \{(a,b) : a\in A, b\in B\}$, is called Cartesian product of A and B. Here (a,b) is called an ordered pair.

⊙ If $|A|=m$ and $|B|=n \Rightarrow$ $\boxed{|A\times B| = m\times n = |B\times A|}$

e.g. $A = \{1,2,3\}$, $B = \{a,b\}$.

$A\times B = \{(1,a),(1,b),(2,a),(2,b),(3,a),(3,b)\}$

$B\times A = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$.

$\therefore |A\times B| = |B\times A| = m\times n$.

e.g. $A = \{1,2,3\}$, $B = \{2,3\}$.

$A\times B = \{(1,2),(1,3),(2,2),(2,3),(3,2),(3,3)\}$.

$B\times A = \{(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$.

Here $|A\cap B| = 2$.

$|(A\times B)\cap(B\times A)| = 2^2 = 4$.

NOTE

$\boxed{\text{IF } |A\cap B| = r, \text{ then } |(A\times B)\cap(B\times A)| = r^2.}$

proof:⇒

Let $|A\cap B| = |D| = r$ (hypothesis)

$\because D\subseteq A$ and $D\subseteq B$

$\Rightarrow D\times D\subseteq A\times B$ and $D\times D\subseteq B\times A$

$\Rightarrow D\times D\subseteq A\times B\cap B\times A$ ——— ①

$\Rightarrow |D\times D| \leq |A\times B\cap B\times A|$

$\Rightarrow r^2 \leq |A\times B\cap B\times A|$ ——— ②.

Again let $(a,b)\in(A\times B)\cap(B\times A)$

$\Rightarrow (a,b)\in A\times B$ and $(a,b)\in B\times A$

$\Rightarrow a\in A, b\in B$ and $a\in B, b\in A$

$\Rightarrow a\in A\cap B$ ~~and~~, $b\in B\cap A = A\cap B$

$\Rightarrow a\in D$, $b\in D$

$\Rightarrow (a,b)\in D\times D$

$\Rightarrow (A\times B \cap B\times A) \subseteq D\times D$ —— ③

$\Rightarrow |A\times B \cap B\times A| \leq |D\times D| = r^2$

$\Rightarrow |A\times B \cap B\times A| \leq r^2$ —— ④

Now, from eqⁿ ② and ④ we conclude that

$$|A\times B \cap B\times A| = r^2. \quad \square.$$

Ordered Pair

Let $a, b \in X$ be any two elements of a nonempty set X.
Then (a,b) is defined as; $\boxed{(a,b) = \{\{a\},\{a,b\}\}.}$

⊙ $(a,b) = (b,a)$ iff $a = b$.

proof: $\Rightarrow$ Let $(a,b) = (b,a)$

$\Rightarrow \{\{a\},\{a,b\}\} = \{\{b\},\{b,a\}\} = \{\{b\},\{a,b\}\}.$

$\Rightarrow \{a\} = \{b\}, \quad \{a\} = \{a,b\}$

$\Rightarrow a = b, \quad a = b.$

$\Rightarrow a = b.$

conversly let $a = b$.

$\because (a,b) = \{\{a\},\{a,b\}\}$

$\Rightarrow (a,b) = \{\{b\},\{b,a\}\} \quad (\because a = b).$

$= (b,a). \quad \square.$

Relation:-

A subset of $A\times B$ is called a relation from A to B, or it may be called as a binary relation from A to B.

⊙ Number of relations from A to B, $= \boxed{2^{m\times n}}$, where $|A| = m$ and $|B| = n$.

$\because |A| = m$ and $|B| = n$.

$|A\times B| = m\times n$

$\Rightarrow |P(A\times B)| = 2^{m\times n}$ = Total no. of relations from A to B.

⊙ Binary Relation: (or Relation on A)

A subset R of $A\times A$ is called a binary relation on A or simply, a relation on A if $a, b \in A$ and we write aRb and 'a' is related to 'b'.

⊙ Number of binary relation on a set A $= |2^{|n|}| = 2^n$.

let $|A| = n$, $\Rightarrow$ $|A \times A| = n^2$

$\Rightarrow |P(A \times A)| = 2^{n^2} =$ no. of relations on A.

Types of Relations :-

→ **Empty Relation** :-

As ϕ is a subset of every set, hence $\phi \subset A \times A$.

$\therefore$ ϕ is a relation on A, called empty relation.

i.e. no any pair of elements satisfies the given condition.

→ **Universal Relation** :-

As $A \times A$ is a subset of $A \times A$

$\therefore$ $A \times A$ is a relation on A, called universal relation.

→ **Identity Relation** :-

A subset I of $A \times A$ is called Identity relation on A if $a \in A$, then $(a,a) \in I$ and $(a,b) \notin I$ if $a \neq b$.

e.g. Let $A = \{a_1, a_2, \ldots, a_n\}$ be any set of 'n' elements.

Then $I = \{(a_1,a_1), (a_2,a_2), \ldots, \ldots (a_n,a_n)\}$ is a subset of $A \times A$ is called identity relation.

$\boxed{\text{NOTE}}$ (i) $|A| = |I| = n$.

(ii) On a set, a relation is said to be identity if every element of A is related to itself only.

(iii) Identity relation is unique for any set A.

→ **Reflexive Relation** :-

A relation R on a set A is called reflexive if every element of A must related to itself, i.e. a subset R of $A \times A$ is called reflexive relation on A, if $\forall a \in A \Rightarrow (a,a) \in R$.

e.g. let $A = \{1, 2, 3\}$

$R = \{(1,1), (1,2), (2,2), (3,3)\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$

Here both R and R_1 are reflexive.

We may say $R_1 = I$, $\therefore$ $\boxed{I \subseteq R.}$

If $|A| = n$ and R is reflexive on A, then $\boxed{|R| \geq n}$.

→ **Irreflexive Relation:-**

A relation R on A is said to be irreflexive if $\forall a \in A, (a,a) \notin R$.

i.e. R is a irreflexive relation on A if no element of A is related to itself.

Properties:-

- Irreflexive is not the exact negation of reflexive.
- There exist some relations which are both reflexive and irreflexive. e.g. 'ϕ'.
- There exist some relations which are neither reflexive nor irreflexive.

  e.g. $R = \{(a,a), (a,b)\}$ where R is a relation on $A = \{a,b\}$.

  Here R is neither reflexive nor irreflexive.

→ **Symmetric Relation:-**

A relation R on a set A is symmetric if $(a,b) \in R \Rightarrow (b,a) \in R$.

Properties:-

- ϕ is symmetric (As $\nexists$ any $(a,b) \in \phi$ s.t. $(b,a) \notin \phi$) on any set A.
- $I \subseteq R$, where R is symmetric.

→ **Asymmetric Relation:-**

A relation R defined on A is called asymmetric whenever $(a,b) \in R \Rightarrow (b,a) \notin R$.

- ϕ is asymmetric.
- Every asymmetric relation is irreflexive but the converse is not always true.

→ **Anti-symmetric Relation:-**

A relation R on A is called anti-symmetric if $(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b$ for $a,b \in A$.

properties:-

- ϕ is anti-symmetric.
- A relation R on A is said to be anti-symmetric iff $\exists$ no pairs of distinct elements $a, b \in A$ such that $(a,b) \in R$ and $(b,a) \in R$.
- e.g. $R = \{(a,a), (b,b)\}$, $A = \{a,b\}$.

  Here R is both symmetric and anti-symmetric.

e.g $A = \{1,2,3\}$, $R = \{(1,1),(2,2)\}$

Then R is anti-symmetric but not asymmetric.

→ Transitive Relation:-

A relation R on a set A is called transitive if $(a,b) \in R$ and $(b,c) \in R$ $\Rightarrow (a,c) \in R$. for $a,b,c \in A$.

⊙ ϕ is transitive.

⊙

→ Equivalence Relation:-

A relation R on a set A is called an equivalence relation if it is reflexive, symmetric and transitive.

e.g. Relation of $\parallel^{el}$ lines on a set of lines in a plane.

Equivalence Class:-

let R be an equivalence relation on a set A.

let $a \in A$, then the set defined and denoted as;

$\boxed{cl(a) = \{x \in A \mid \underbrace{(x,a) \in R}_{\text{i.e. } xRa}\}}$ is called an equivalence class of $a \in A$ by the relation R.

Imp ⊙ Equivalence classes are either disjoint or identical.

Quotient Set:-

let A be a non-empty set and 'R' be an equivalence relation on; Then the set of all disjoint equivalence classes is called quotient set of A by R.

& denoted by $\frac{A}{R} = \{\bar{a} \mid a \in A\}$ is quotient set of A by R.

NOTE

① $\frac{A}{R} \subseteq$ ~~P(A)~~ A and $\frac{A}{R} \in P(A)$.

② If $A = \phi$, then ~~$\frac{A}{R} \not\subseteq P(A)$~~. $\frac{A}{R} \subsetneq A$.

e.g. on $\mathbb{R}$, $aRb \Leftrightarrow [a] = [b]$.

$\bar{1} = [1,2)$, $\bar{2} = [2,3)$ ....

$f: \{\bar{a} \mid a \in \mathbb{R}\} \to \mathbb{Z}$ given by i.e. $f: \frac{\mathbb{R}}{R} \to \mathbb{Z}$.

$f(\bar{a}) = a$,

$\Rightarrow$ f is one-one and onto.

$\Rightarrow$ $\frac{\mathbb{R}}{R} \sim \mathbb{Z}$.

Here total no. of equivalence classes is countably infinite.

then $\left|\frac{\mathbb{R}}{R}\right|$ = countanbly infinite, i.e. $(\aleph_0)$.

Both-way Relation:-

A relation R on a set A is called bothway if $R \subseteq A \times B$ and $R \subseteq B \times A$, both.

$\because R \subseteq A \times B$ and $R \subseteq B \times A$

$\Rightarrow R \subseteq A \times B \cap B \times A$

$\Rightarrow R \subseteq D \times D$, where $D = A \cap B$ and $|D| = r$ (say).

$\Rightarrow$ Total no. of bothway relation $= 2^{r \times r} = 2^{r^2}$.

Ques. If n-couples are invited to a party with the condition that husband has to be accompanied by his wife, however wife mayn't be accompanied by her husband. Then how many gathering are possible?

Ans sol. $\Rightarrow$ No. of choices for a 1-couple = 3.

as the appearance may be {HW, W,H, W}

Hence n-couples have 3^n choices.

Alterantively. $\{\underbrace{a_1, a_2, \ldots, a_n}_{n\text{-husbands}}, \underbrace{b_1, b_2 \ldots, b_n}_{n\text{-wives}}\}$.

then the appearance is not based on the choice of husbands, as they depend on the choice of their respective wives.

Now, each wife has 3 choices $\longrightarrow$ Go to party with husband.

$\searrow$ Go to party without "

$\downarrow$ Doesn't go to party.

$\therefore$ n wives will have 3^n choices.

Ques. How many possible pairs (A,B) are there, such that $A, B \subseteq S$, and $A \cap B = \phi$, where $S = \{1,2,3,4,5\}$.

IIT-JAM SYLABUSS.

ODE :→

(1) Introduction.

(2) Order and Degree.

(3) Formation of ODE. → (1)

(4) Types of solutions.

(5) Methods of solutions.

~~(6)~~ (i) Separation of Variable.

(ii) Reducible to separation of var.

(iii) Homogeneous.

(iv) Reducible to homogeneous eq^n.

(V) Exact diff. eq^n + I.F → (1)

(Vi) 1st order, 1st deg. ~~d~~ Linear diff. eq^n.

(Vii) Reducible to Linear Diff. eq^n.

(Viii) Bernoulie's eq^n.

Topic-2

(6) Linear diff. eq^n with constant co-efficients. → (1)

(7) Linear diff. eq^n with Variable co-efficients.

(8) (i) Variation of parameters → (1)

(ii) Cauchy-Euler's eq^n.

(8) Wronskian.

(9) Orthogonal Trajectory.

Dependent Variable and Independent Variable:

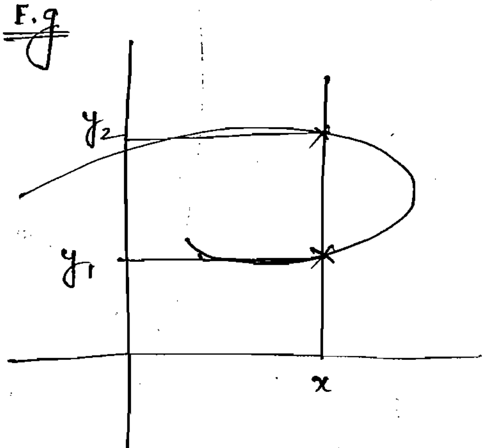
The variable whose value is assigned is called independent variable and the variable whose value is obtained corresponding to assign value is called dependent variable.

Function.

(i) Every element in domain have a unique image in the co-domain.

(ii) Mathematical defn. : → $f: A \to B$ is said to be a fun. if $\forall\, x \in A\ \exists$ unique $y \in B$ such that $y = f(x)$.

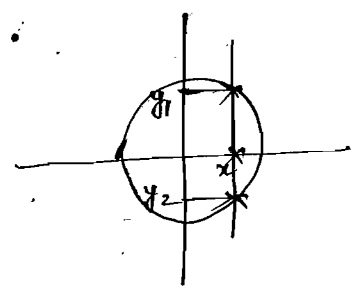
E.g



Here, $\left.\begin{array}{l} y_1 = f(x) \\ y_2 = f(x) \end{array}\right]$ not unique img. of x.

⇒ not a fun.

E.g

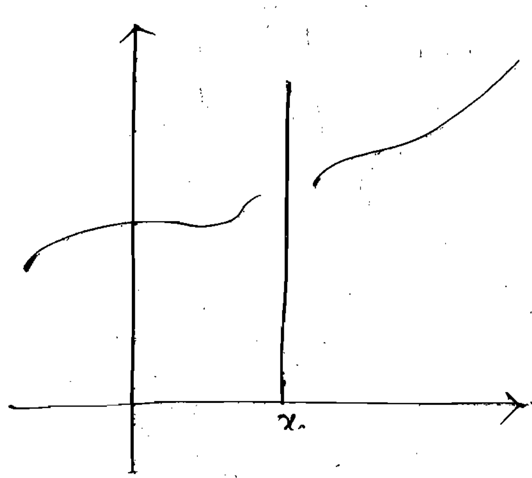


$y_1 = f(x)$

$y_2 = f(x)$

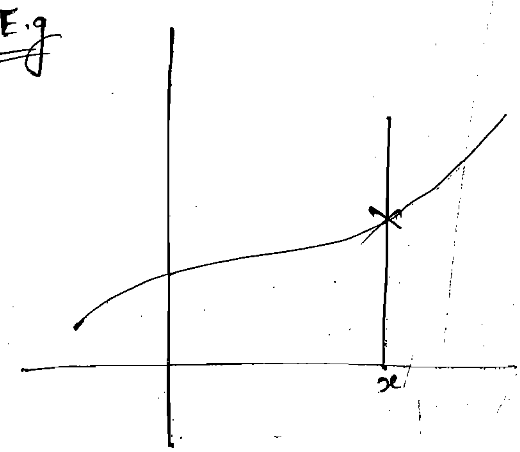
⇒ It is not a fun.

E.g



x has no image in

E.g



It is a fun.

(iii) A mapping $f: A \to B$ is called a fun. if any line passing through domain and ∥ to y-axis should intersect the curve $y = f(x)$ exactly one.

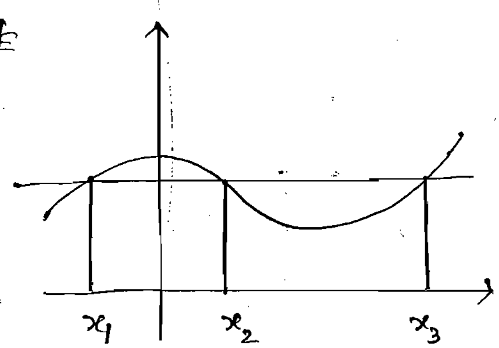
One-one Function:

$f: A \to B$ is called one-one fun;

if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

or $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

E.g



Here, $f(x_1) = f(x_2) = f(x_3)$

$\not\Rightarrow x_1 = x_2 = x_3$:

$\Rightarrow$ It is not one-one fun.

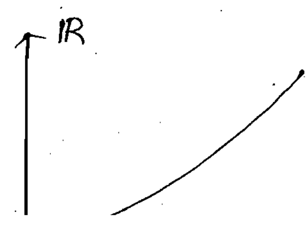
Graphical Defn:

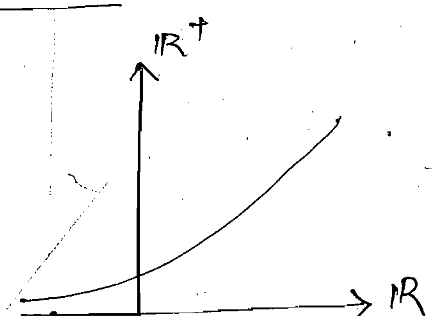
A fun. $f: A \to B$ is called one-one if any line passing through co-domain and ∥ to x-axis should intersect the curve $y = f(x)$ at-most one.

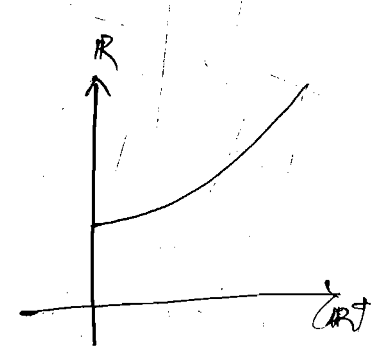
Onto Function:

A fun. $f: A \to B$ is called onto if any line passing through co-domain and ∥ to x-axis should intersect the curve $y = f(x)$ atleast one.

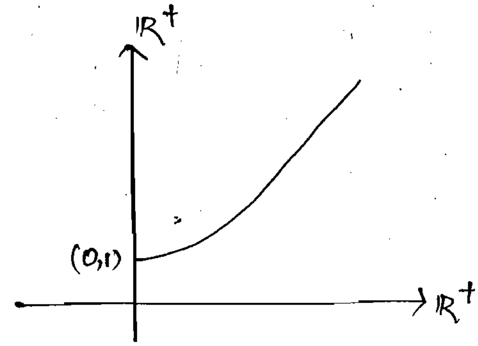
E.g $f(x) = e^x$.





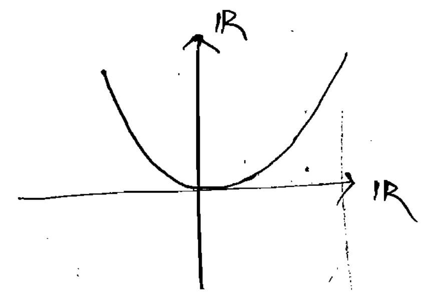


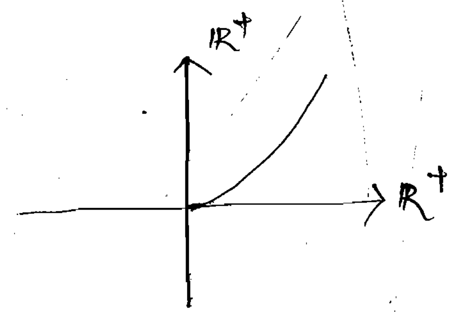
	function	One-One	Onto.
$f: \mathbb{R} \to \mathbb{R}$	✓	✓	×
$f: \mathbb{R} \to \mathbb{R}^+$	✓	✓	✓
$f: \mathbb{R}^+ \to \mathbb{R}$	✓	✓	×
$f: \mathbb{R}^+ \to \mathbb{R}^+$	✓	✓	×



Eg $f(x) = x^2$.

	fun.	One-One	Onto.
$f: \mathbb{R} \to \mathbb{R}$	✓	×	×
$f: \mathbb{R} \to \mathbb{R}^+$	×	×	×
$f: \mathbb{R}^+ \to \mathbb{R}$	✓	✓	×
$f: \mathbb{R}^+ \to \mathbb{R}^+$	✓	✓	✓





\# $\Delta y = y_2 - y_1 \Rightarrow$ dist. b/w y_1 and y_2.

$\Delta x = x_2 - x_1 \Rightarrow$ dist. b/w x_1 and x_2.

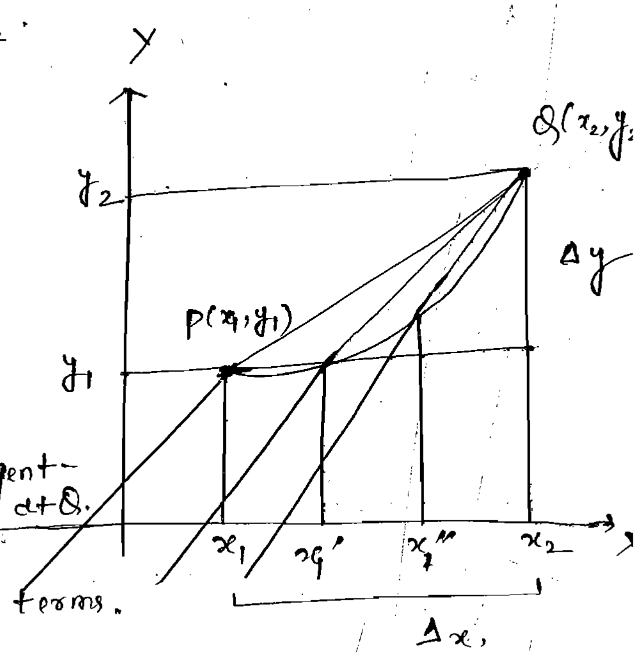
$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ = slope of L.

$\therefore$ Total rate of change

$= \frac{dy}{dx} = \lim_{x_1 \to x_2} \frac{\Delta y}{\Delta x}$ = Slope of tangent at Q.

$\downarrow$ alg. term.

$\downarrow$ graphical terms.



\# If $y = f(x)$

→ In these cases,

y → dependent Variable

Differential Equation:

Any eq^n bet^n dependent Variable, indp. variable and derivative of dependent variable with respect to indp. var. is called differential equation.

NOTE. In partial derivative, y should be fun. of 2 or more than 1 indp. variable.

E.g. $y = f(x)$

(i) $\frac{dy}{dx} + e^x y = \sin x$.

(ii) $\frac{d^2y}{dx^2} + y = \sin x$

Simple ODE.

$y_1 = f(x)$
$y_2 = g(x)$

$\frac{d^2y_1}{dx^2} + \frac{d^2y_2}{dx^2} = e^x$

$\frac{dy_1}{dx} + \cos x . \frac{dy_2}{dx} = 0$

System of ODE

$z = f(x,y)$

$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = e^x$

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin(x+y)$

Simple PDE

$z_1 = f(x,y)$
$z_2 = g(x,y)$

$\frac{\partial z_1}{\partial x} + \frac{\partial z_2}{\partial y} = 0$

$\frac{\partial^2 z_1}{\partial x^2} + \frac{\partial^2 z_2}{\partial x \partial y} = 0$

System of PDE.

Ordinary Differential Equation:

Any differential eq^n in which unique indp. variable and derivative of dependent var. dependent variable and total derivative of dependent variable is called ordinary diff. eq^n. w.r.t indp. variable

Partial Differential Equation:

Any diff eq^n which contains partial derivative is called partial diff. eq^n.

Jay Jagannath.

SETS AND ITS FUNDAMENTALS.

SET. Collection of well-defined distinct objects is called set.

NOTE. (1) By well-defined we mean, there is no confusion regarding inclusion or exclusion of any objects.

(2) The word set is mathematical form of the word collection.

(3) A set itself is considered as an object, hence eligible for collection to form a set.

(4) Generally, sets are denoted by capital letters $X, Y, Z, \ldots$ etc and the objects included in the set called elements are denoted by small letters $x, y, z, \ldots$ etc.

(5) If X is a set and 'a' is an object collected in X, we say 'a' belong to X and denoted by $a \in X$.

(6) Empty collection is well-defined as every object being tested has to be excluded. Hence, it is a set and is called void set, null set and denoted by ϕ or $\{\}$

Axiom of Regularity:

"No set belongs to itself", i.e if A is a set then $A \notin A$.

Ordinary Set: A set X is said to be ordinary if $X \notin X$.

Extraordinary set:

A set X is called extra-ordinary set if $X \in X$.

E.g $X = \{x \mid x \text{ is not a marker}\}$.

$\because$ X is well-defined

$\Rightarrow$ X is a set & X is not a marker.

$\Rightarrow$ $X \in X$ $\Rightarrow$ X is an extra-ordinary set.

Collection/ Set Builder Notation:

$X = \{$ Type of object ; Rule for collection $\}$.

e.g $X = \{ x$ is a natural no. : $2 < x < 9 \}$.

Russel's Paradox:

"There is no set of all sets" i.e collection of all the sets doesn't form a set.

OR "There is no set of all ordinary sets."

i.e collection of all ordinary sets is not a set.

Proof. Let $X = \{ A \mid A$ is an ordinary set $\}$.

If not let, X is a set.

Case-1 If X is ~~not~~ an ordinary set

$\Rightarrow X \in X$ (def^n)

$\Rightarrow X$ is an extra-ordinary set $\times$.

Case-2 If X is extra-ordinary set.

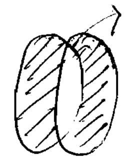
$\Rightarrow X \in X$ (Def^n)

$\Rightarrow X$ is ordinary set. $\times$.

$\Rightarrow X$ is not a set. $\square$.

NOTE. Throughout onwards, we will use the ordinary sets for analysis, but not extraordinary set.

<u>NOTES.</u> Let A and B are sets.



(i) $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

(ii) $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

(iii) $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

(iv) $A \Delta B = (A \cup B) - (A \cap B)$

$= (A - B) \cup (B - A)$.

<u>SUBSET.</u> Let A and B are two sets.

→ if $x \in A \Rightarrow x \in B$

$\Rightarrow$ A is a subset of B, denoted by $A \subset B$.

→ if $\nexists x \in A$ s.t $x \notin B$ then A is called a subset of B.

→ if $A = B$.

$\Rightarrow x \in A \Rightarrow x \in B = A \Rightarrow x \in A$

$\Rightarrow$ $\boxed{A \subset A \text{ for any set } A}$ → Any set is subset of itself.

→ if $A = \phi$

$\Rightarrow \nexists x \in A$ s.t $x \notin B$.

$\Rightarrow A \subset B$

$\Rightarrow$ $\boxed{\phi \subset B \text{ for any } B}$ → ϕ is subset of any set.

<u>POWER SET.</u>

$P(X) = \{A : A \subset X\}$.

= The set of all the subsets of X.

$\Rightarrow X \in P(X)$

$\Rightarrow \phi \in P(X)$

* $X = \{a, b, \{a, b\}\}$

$Y = \{a, b\}$ Here, $Y \in X$ & $Y \subset X$ $\Rightarrow Y \in P(X)$.

Cartesian Product:

Let A and B are 2 sets.

$A \times B = \{ (a, b) : a \in A \text{ and } b \in B \}$.

$=$ cartesian product of A and B.

Where, (a, b) is called ordered pair.

Function:

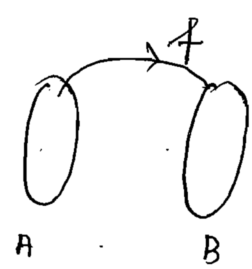
Let A and B are 2 non-empty sets. Then a rule by which every elements of A is assigned to some unique element of B, defines a fun. from $A \to B$.

→ We denote it by, $f : A \to B$.

→ If $x \in A$ is assigned to $y \in B$, then y is unique for x, and we denote it by $y = f(x)$.

→ Moreever, y is called the image of x and x is called a pre-image of y.

NOTES.



(1)→ A = Domain

(2) B = co-domain.

(3) $f(A) = \{ f(x) \mid x \in A \} \subset B \Rightarrow$ Range of 'f' $\subset B$

(4) Let $Y \subset B$

then $f^{-1}(Y) = \{ x \in A \mid f(x) \in Y \} \subset A$.

Types of Functions.

(1) One-One (Injection)

if $f(x) = f(y)$

$\Rightarrow x = y$ OR $x \neq y \Rightarrow f(x) \neq f(y)$.

(2) Onto (surjection)

if $f(A) = B$

i.e Range of A = co-domain

i.e Every elements of B has a pre-image in the set A.

(3) Bijection.

$f: A \to B$ s.t f is one-one and onto. both,

then f is called a bijection from A to B.

→ If f is both one-one and onto from $A \to B$

i.e $f: A \xrightarrow[\text{onto}]{1-1} B$

we can define, $g: B \to A$

Let $g(s) = t$ if $f(t) = s$.

⇒ g is called the inverse of f.

and denoted by f^{-1}.

and we say that f is invertible.

Similar Sets.

Two non-empty sets are said to be similar if ∃ a bijection betⁿ them. The words like equivalent, equinumorous, equipotent are also used in place of similar.

Finite Set:

A non-empty set is said to be finite, if it ~~is~~ ~~similar to~~ has finite no. of elements.

i.e the set having finite cardinality.

Eg A = Set of days ~~too~~ in a month. (Provided 30 days)

Here, $|A| = 30$ ⇒ A is finite.

* By extension of defn;

Empty set is also finite set, and its cardinality is zero.

i.e $\boxed{card(\phi) = 0}$

NOTE $\mathbb{N} = \{1, 2, 3, \ldots, n, n+1, \ldots\}$

$\mathbb{N}_0 = \{0, 1, 2, \ldots, n, n+1, \ldots\}$.

are sets of finite cardinals.

Where $\mathbb{N}$ is set of cardinality of all non-empty finite sets. and,

$\mathbb{N}_0$ is the set of cardinalities of all finite sets,

Infinite Set:

The set which is not similar to $S_{(n)}$ for any n.

i.e the set which has infinite no. of element.

E.g $\mathbb{N}$, $\mathbb{N} \times \mathbb{N}$, $P(\mathbb{N})$, ...

Imp.

NOTE. To compare potential of 2 sets we use functions.

If (i) from A to B onto fun. can't be defined, we say B has more potential than that of A. and we write $card(A) < card(B) \Rightarrow |A| < |B|$.

If (ii) from A to B, one-one fun. can't be defined, we say A has more potential than that of B. and we write $card(A) > card(B) \Rightarrow |A| > |B|$.

VECTOR CALCULUS :

SYLLABUS .

1 Basic

(i) Dot product
(ii) Cross product
(iii) Scalar Triple Product
(iv) Vector Triple product . ※

2 (i) Gradiant , Divergence , Curl .

(ii) Tangent vector .
(iii) Unit tangent vector .
(iv) Normal vector . → 100% 1 question.
(v) Eqn of tangent plane .
(vi) Eqn of normal .
(vii) Directional Derivative .

(viii) Irrotational vector .
(ix) Solenodial vector .

→ 2 questions.

3 (i) Line Integral . ⟶ 100% . 1 question .
(ii) Surface Integral . ⟶ ※ Top question. (1)
(iii) Volume Integral .
(iv) Work Done .
(v) Conservative vector field .

→ 2 questions.

4 . (i) Green's Thm. ⟶ 1 question.
(ii) Stock's Thm. ⟶ 1 question.
(iii) Gauss – Divergence Thm. and their properties → 1 question

[illegible] $\boxed{\vec{a} = |a| \cdot \hat{a}}$ → dirⁿ.

↓ magnitude.

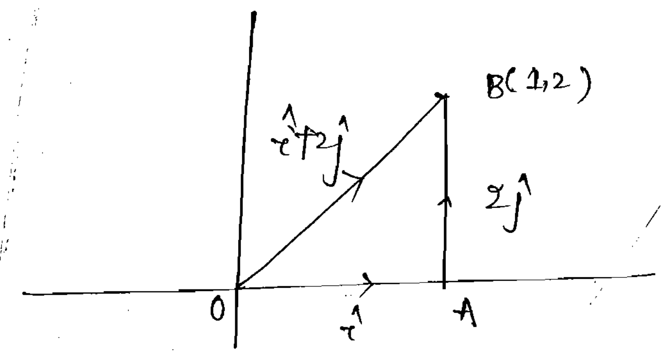
(1) **Scalar.** A scalar is a quantity which has only magnitude but doesn't have a dir^n.

E.g. Time, Mass, Distance, Temp. etc.

(2) **Vector.** A vector is a quantity which has magnitude, dir^n and follow the Triangle law of add^n.

E.g. Force, Displacement etc.

(3) **Position Vector.**



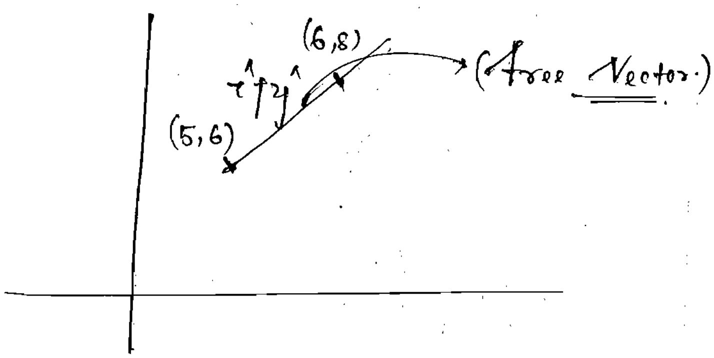
$\vec{a} = \hat{i} + 2\hat{j}$

$OA = \hat{i}$; $AB = 2\hat{j}$; $OB = \hat{i} + 2\hat{j}$

$\vec{OA} + \vec{AB} = \vec{OB}$

$\Rightarrow \boxed{\vec{AB} = \vec{OB} - \vec{OA}}$

E.g. $A = (5, 6)$ $B = (6, 8)$. (Free vector.)



$\vec{OA} = 5\hat{i} + 6\hat{j}$

$\vec{OB} = 6\hat{i} + 8\hat{j}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\Rightarrow \boxed{\vec{AB} = \hat{i} + 2\hat{j}}$

Types of Vectors:

(1) Equal Vector.

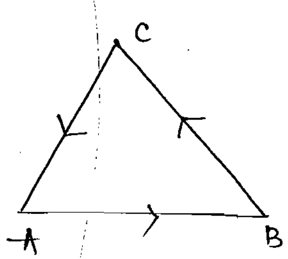
Two vectors are s.t.b equal iff they have equal magnitude and same dir^n.

$A \longrightarrow B$ $\quad |\vec{AB}| = |\vec{CD}|$.

$C \longrightarrow D$

(2) Zero Vector / Null Vector.

A vector whose initial and terminal pts. are same is called null vector.



$\vec{AB} + \vec{BC} + \vec{CA} = \vec{AA} = 0$.

$\Rightarrow \boxed{\vec{AB} + \vec{BC} + \vec{CA} = 0}$

(3) Like / UnLike Vector:

Two vectors are said to be

(i) like when they have same dir^n.

(ii) unlike when they have opposite dir^n.

$\vec{a}$ and $-\vec{a}$ unlike. $\boxed{\vec{a} \quad -\vec{a}}$

$\vec{a}$ and $\lambda\vec{a}$ if $\lambda > 0 \Rightarrow$ like.

$\lambda < 0 \Rightarrow$ unlike

(4) Unit Vector.

A unit vector is a vector whose magnitude is unity.

i.e $\boxed{\hat{a} = \frac{\vec{a}}{|\vec{a}|}}$

(5) **Position Vector :**

If p is any pt. in the space then the vector $\overrightarrow{op}$ is called position vector of the point p, where O is origin.

(6) **Co-initial Vector.**

Vectors having same initial point are called co-initial vector.

Distance Formaula.

$A = (x_1, y_1)$; $B = (x_2, y_2)$.

$\overrightarrow{OA} = x_1\hat{i} + y_1\hat{j}$; $\overrightarrow{OB} = x_2\hat{i} + y_2\hat{j}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$.

$|\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $\rightarrow$ (Dist is a scalar quantity.)

* $A = (x_1, y_1, z_1)$; $B = (x_2, y_2, z_2)$

$\overrightarrow{OA} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$; $\overrightarrow{OB} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$.

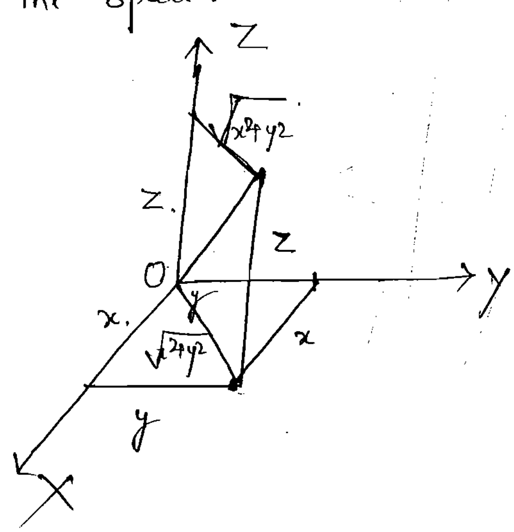
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}$.

$|\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

* Let $p(x,y,z)$ be any point on the space.

$\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$.

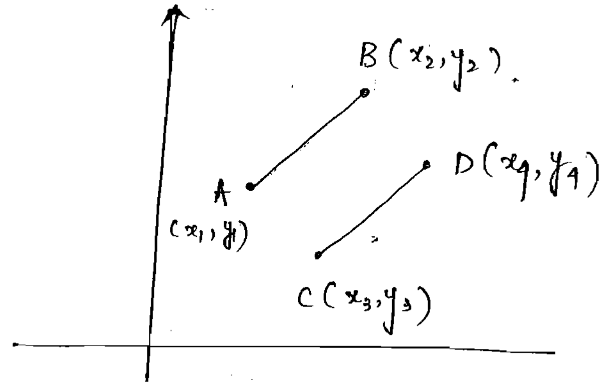
$|\overrightarrow{OP}| = \sqrt{x^2+y^2+z^2}$



* $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

$$\boxed{|\vec{r}| = r = \sqrt{x^2+y^2+z^2}}$$

Parallel Vector.



* $AB \parallel CD \Rightarrow m_1 = m_2$.

$$\Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_4 - y_3}{x_4 - x_3} = \lambda$$

$$\Rightarrow \frac{y_2 - y_1}{y_4 - y_3} = \frac{x_2 - x_1}{x_4 - x_3} = \lambda .$$

$\vec{OA} = x_1 \hat{i} + y_1 \hat{j}$; $\vec{OB} = x_2 \hat{i} + y_2 \hat{j}$

$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$\vec{CD} = (x_4 - x_3)\hat{i} + (y_4 - y_3)\hat{j}$

$\vec{AB} = \lambda (x_4 - x_3)\hat{i} + \lambda (y_4 - y_3)\hat{j}$

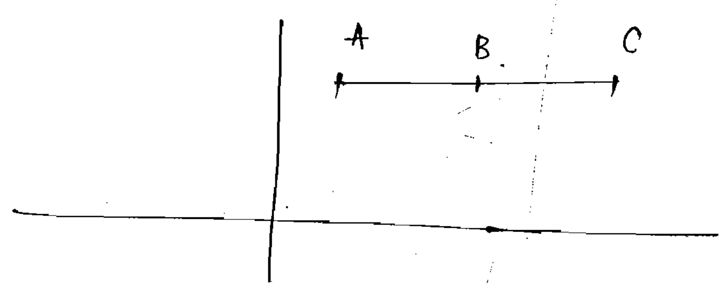
$\boxed{\vec{AB} = \lambda \vec{CD}}$ if $\vec{AB} \parallel \vec{CD}$.

* $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$

$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$.

$\boxed{\text{if } \vec{a} \parallel \vec{b} \text{ then } \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}}$ or. $\boxed{\vec{a} = \lambda \vec{b}}$

Point Co-linear:



$AB = \lambda . BC$.