

Deterministic Push Down Automata

In the Deterministic Push Down Automata has at most one choice means zero or one no more than one.

A Dpda $M = \langle Q, q_0, \Sigma, \delta, F, \Gamma, Z \rangle$ is same as npda except the following two condition.

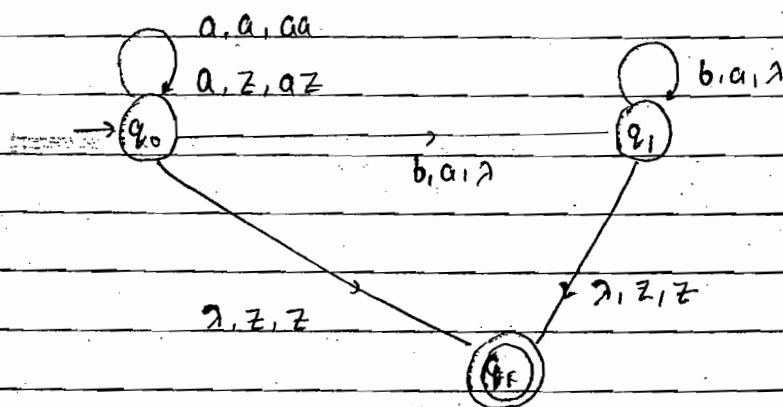
- (a) $\delta(q, a, b)$ has ATMOST 1 choice no more than one.
- (b) If $\delta(q, \lambda, b)$ is defined then $\delta(q, c, b)$ is empty $\forall c \in \Sigma$ (not defined)

→ If in npda a transition is perform then cannot assign the transition to other symbol.

$$\delta : Q \times \Sigma \cup \{\lambda\} \times \Gamma \longrightarrow Q \times \Gamma^*$$

$\delta(q_0, \lambda, b)$	and	$\delta(q_0, c, b)$	Wrong
$\delta(q_0, \lambda, b)$	and	$\delta(q_0, \lambda, c)$	Right
$\delta(q_0, \lambda, b)$	and	$\delta(q_0, c, a)$	Right

eg. $L = \{a^n b^n \mid n \geq 0\}$



$$\delta(q_0, a, z) = \{(q_0, az)\}$$

$$\delta(q_0, a, a) = \{(q_0, aa)\}$$

$$\delta(q_0, b, a) = \{(q_1, \lambda)\}$$

$$\delta(q_1, b, a) = \{(q_1, \lambda)\}$$

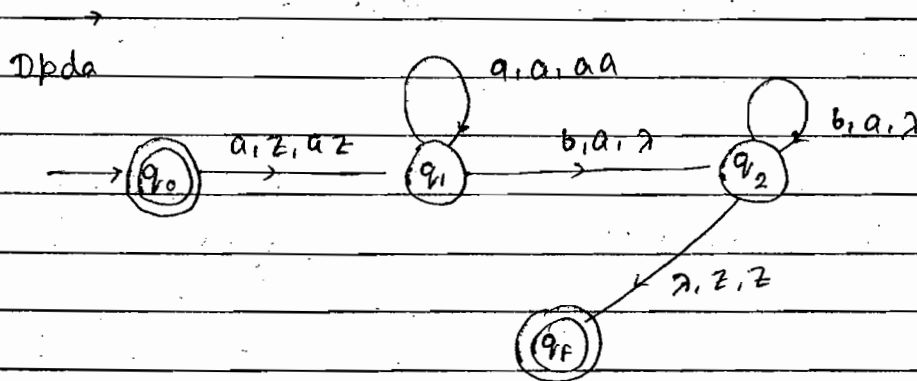
$$\delta(q_1, \lambda, z) = \{(q_f, z)\}$$

$$\delta(q_0, \lambda, z) = \{(q_f, z)\}$$

These transition breaking the rule 2

So this machine is not valid for dpda.

Every dpda is by default ndpda.



$$\delta(q_0, a, z) = \{(q_0, az)\}$$

$$\delta(q_1, a, a) = \{(q_1, aa)\}$$

$$\delta(q_1, b, a) = \{(q_2, \lambda)\}$$

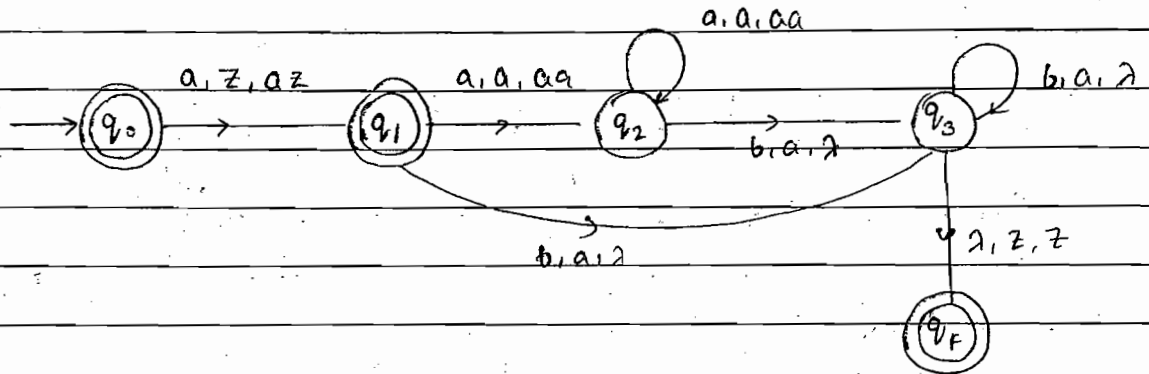
$$\delta(q_2, b, a) = \{(q_2, \lambda)\}$$

$$\delta(q_2, \lambda, z) = \{(q_f, z)\}$$

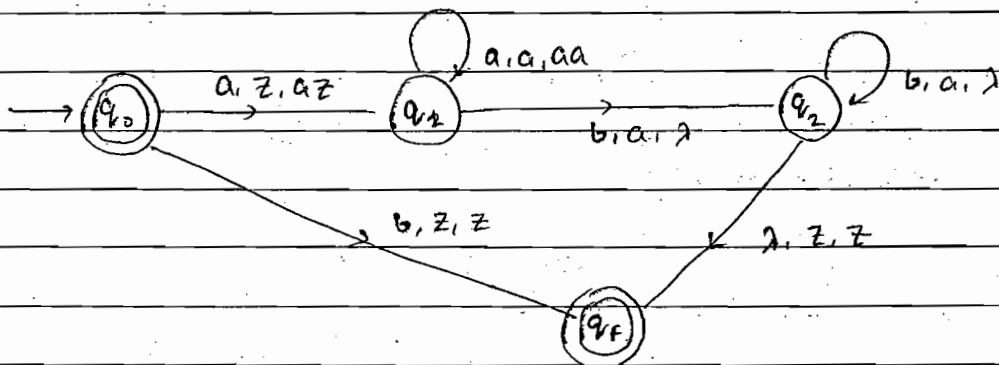
This machine is dpda

This language is $L = \{a^n b^n \mid n \geq 0\}$ is Deterministic C.F.L as well as Non-Deterministic C.F.L

① $L = \{a^n b^n \mid n \geq 0\} \cup \{a\}$

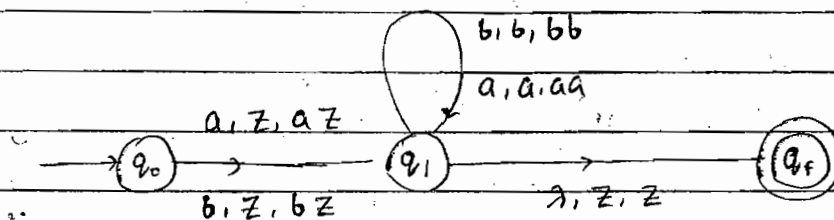


② $L = \{a^n b^n \mid n \geq 0\} \cup \{b\} = \{b, \lambda, ab, a^2 b^2, \dots\}$



③ $L = \{w \in \{a, b\}^* \mid N_a(w) \neq N_b(w)\}$

$$L = \{w \in \{a,b\}^* \mid N_a(w) = N_b(w)\}$$



$L = \{ww^R \mid w \in \{a,b\}^*\}$ not a deterministic C.F.L so we can't make a dpda.

$L = \{w \in \{0,1\}^* \mid w \text{ is a palindrome}\}$

$\{w \mid w = w^R \text{ \& \# } |w| \text{ is even}\} \cup$

$\{w \mid w = w^R \text{ \& \# } |w| \text{ is odd}\}.$

Not make dpda

\Rightarrow npda is more powerful than dpda

$$n\text{dpda} > \text{dpda} \Rightarrow n.c.f.l \supset d.c.f.l$$

- ① $L = \{ww \mid w \in \{a,b\}^*\}$ is not C.F.L. so can't make pda. with one stack.
We will make it with 2-stack pda.

★ Two stack pda is more powerful than one-stack pda.

- ② $L = \{a^n b^m c^n d^m \mid n, m \geq 0\}$

Converting a C.F.G to npda

Requirement - C.F.G. has to be in G.N.F.

Steps -

1. Push S (start symbol) on the stack.

$$\delta(q_0, \lambda, z) = (q_1, Sz)$$

2. $A \rightarrow a\lambda$ $A \in V, a \in T, \lambda \in V^*$

\downarrow
top of
stack

\downarrow
reading
the symbol

\swarrow
replacement

3. $\delta(q_1, a, A) = (q_1, \lambda)$ [λ can be λ]

4. $\delta(q_1, \lambda, z) = (q_f, z)$

ex- $S \rightarrow asb$

$S \rightarrow ab$

$S \rightarrow aSB$

$S \rightarrow aB$

$B \rightarrow Bb$

1. $\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

2. $\begin{cases} \delta(q_1, a, S) = \{(q_1, SB)\} \\ \delta(q_1, a, S) = \{(q_1, B)\} \end{cases}$

Combine both

$$\delta(q_1, a, S) = \{(q_1, SB), (q_1, B)\}$$

$$\delta(q_1, b, B) = \{(q_1, \lambda)\}$$

Asymptotic Notation : Machine Notation Independent

1. Big 'O' A straight line which touches the vertical curve on the infinite
2. Big 'Ω'
3. Theta (Θ)
4. little 'o'
5. little 'ω'

- Abstract representation of time complexity
- It (time complexity) depends upon
- The dominant term in the expression.

eg. $5n^2 + 6n + 3 = f(n)$ $n = \text{input size}$

Growth rate depend on the dominant term
(Highest term) $5n^2$.

$f(n)/g(n) \rightarrow$ Time complexity (Total count) of A
↳ Execution time.

Assumption -

- ⊙ $f(n)$ & $g(n)$ both are non-negative because time is always +ve

- ⊙ $f(n)$ & $g(n)$ both are increasing function of n
- ⊙ n is very large number.

◆ Big 'O' - Represents the WORST case behaviour of the algo.

- Gives the UPPER BOUND of the running time of the algo.

Upper Bound : Maximum time but the algo can take for any value of n (Input size)

★ $f(n) = O(g(n))$ the value of $g(n)$ is more than OR equal to the value of $f(n)$

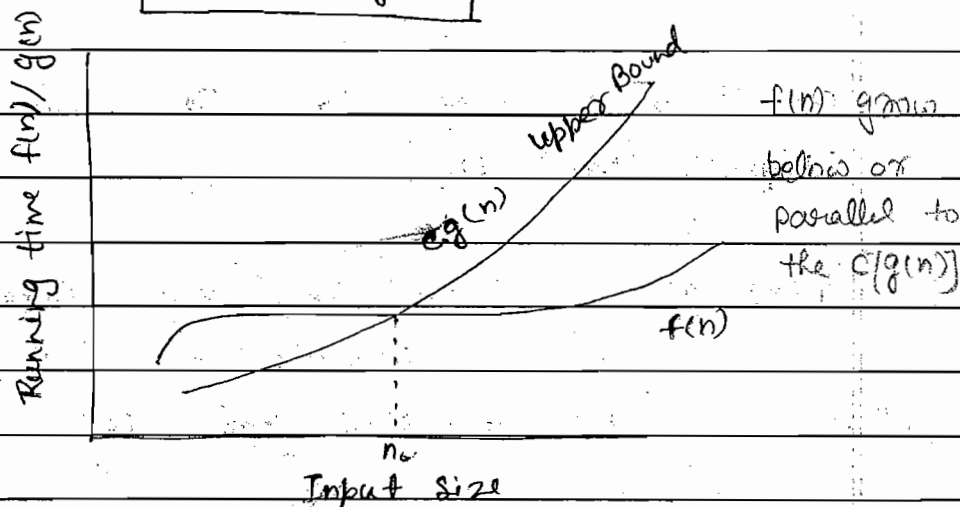
→ $(n^2) + 3 = O(n^2)$ TRUE complexity depends on n^2

→ $(5n^2) + 3n + 6 = O(n^2)$ TRUE dominating term

There exists two +ve constants c and n_0 such that

$$f(n) \leq c g(n)$$

$$\forall n \geq n_0$$



e.g $f(n) = 5n^2 + 3n + 6$ represent in terms of Big O notation.

The growth rate is depend on the higher degree term.

$$5n^2 \leq 5n^2$$

$$3n \leq 3n^2$$

$$6 \leq 6n^2$$

$$n \geq 1$$

$$5n^2 + 3n + 6 \leq 5n^2 + 3n^2 + 6n^2$$

$$n \geq 1$$

$$f(n) \leq 14n^2$$

$$n \geq 2$$

$$c = 14$$

$$g(n) = n^2$$

$$f(n) = O(g(n)) = O(n^2)$$

Limit Representation

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0 \text{ / finite quantity (Equal value)}$$

eg. $n = O(n^2)$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{n}{n^2} = \lim_{n \rightarrow \infty} \frac{1}{n}$$

$$= \frac{1}{\infty} = 0$$

eg. $2^n = O(2^{n+1})$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{2^n}{2^{n+1}} = \lim_{n \rightarrow \infty} \frac{1}{2} = \frac{1}{2}$$

eg. $2^{n+1} = O(2^n) \Rightarrow 2^n \cdot 2^1 = O(2^n)$

$$\lim_{n \rightarrow \infty} \frac{2^{n+1}}{2^n} = \underline{\underline{2}}$$

eg. $(n+a)^b = O(n^b)$

$$\lim_{n \rightarrow \infty} \frac{(n+a)^b}{n^b} = \lim_{n \rightarrow \infty} \left(\frac{n+a}{n} \right)^b$$

$$= \lim_{n \rightarrow \infty} \left(\frac{n}{n} + \frac{a}{n} \right)^b = \lim_{n \rightarrow \infty} \left(1 + \frac{a}{n} \right)^b$$

$$= \left(1 + \frac{a}{\infty} \right)^b = (1+0)^b = \underline{\underline{1}}$$

$$x = 0$$

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$$\rightarrow (2^2)^n = 4^n$$

1. $2^{2n} = O(4^n)$ TRUE

$$\lim_{n \rightarrow \infty} \frac{2^{2n}}{4^n} = \lim_{n \rightarrow \infty} 2^{2n} \cdot 4^{-n}$$

Ans

UGC Net
Dec 2014

2. $\log n = O(n)$ TRUE

Ans

3. $\sqrt{n} = O(n)$ TRUE

4. $n! = O(n \log n)$ FALSE

Net June
2014

Big 'O' estimate for factorial function & logarithm of factorial function i.e. $n!$ & $\log n!$

$$n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1 \quad n \geq 1$$

$$n! \leq n \times n \times n \times \dots \times n \times n \times n$$

$$n! \leq n^n$$

$$n! \leq n^n \quad \boxed{n! = O(n^n)}$$

Taking log both sides

$$\log n! \leq \log n^n$$

$$\log n! \leq n \log n$$

$$\boxed{\log n! = O(n \log n)}$$

Ans. $O(n^n)$ & $O(n \log n)$

UGC Net
June 2013

Give a good Big 'O' estimate for the following function.

$$(n \log n + n^2)(n^3 + 2) \text{ \& \& } (n! + n^2)(n^3 + \log(n^2 + 1))$$

e.

$$n^2 \cdot n^3 = O(n^5) \text{ \& \& } n! \cdot n^3 = O(n! \cdot n^3)$$

Ans

$$O(n^5) \quad \& \quad O(n^3 \cdot n!)$$

Use Net
Dec 2013

What is the Big 'O' estimate for

$$f(x) = (x+1) \log(x^2+1) + 3x^2$$

$$x \log x^2 + 3x^2 = 2x \log x + 3x^2$$

Ans.

$$f(x) = O(x^2)$$

◆ Big 'Ω'

- It represents the best case behaviour of al
- It gives the lower bound on the running time of algo.

$$f(n) = \Omega(g(n))$$

$g(n)$ is smaller or equal to $f(n)$

$n \geq 1$

$$n = \Omega(n)$$

$$n^2 = \Omega(n^2)$$

$$n^2 = \Omega(n)$$

$$\log n = \Omega(\log \log n)$$

$$n = \Omega(\sqrt{n})$$

$$n^2 + 2n + 5 = \Omega(n^2)$$

$$2^n = \Omega(4^n) \quad \text{False}$$

TRUE

(n)

There exist two +ve constants c & n_0 such that

$$f(n) \geq c \cdot g(n)$$

$\forall n \geq n_0$

$n^2 + 1)$

e.g

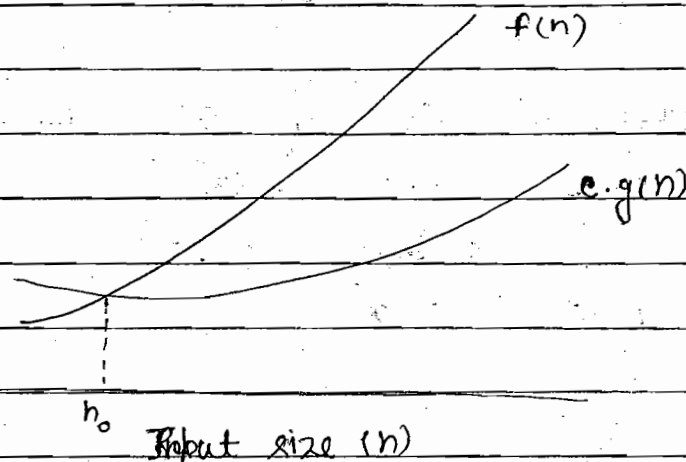
$$2^{2n} = O(2^{n+1}) \quad \text{false}$$

$$2^{2n} = 2^n \cdot 2^n$$

,3)

$$2^{2n} > 2^n$$

Running time



e.g.

e.g. $f(n) = 5n^2 + 6n + 3$
 $= \Omega(n^2)$

$f(n) = 5n^2 + 6n + 3 > 5n^2 \quad n \geq 0$

$f(n) > 5n^2 \quad \forall n \geq 0$

$c = 5, \quad n_0 = 0$

$g(n) = n^2$

e.g.

e.g. $f(n) = n^4 + 3n^3 + 4n^2 + 10$
 $= \Omega(n^4)$

$f(n) = \Omega(g(n))$

$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty/a$ finite Quantity other than 0

$n! = 1 \times 2 \times 3 \times \dots \times (n-1) \times n$
 $= O(n^n)$

$n! = 1 \times 2 \times 3 \times 4 \times \dots \times (n-1) \times n$

$n! > 2 \times 2 \times 2 \times 2 \times \dots \times 2 \times 2$

$n! > 2 \times 2 \times 2 \times \dots \times 2$ n times

$n! > 2^n$

Set :- Set is a "well defined collection,
of distinct object,"

↓
for every person
answer must same

(Vowels : States of India)

$$\{a\} = \{a, a\} = \{a, a, a\}$$

eg. set of english vowels = $\{a, e, i, o, u\}$
= $\{e, a, o, i, u\}$

Representation of Set -

1. Roster form : Set enclosed in $\{ \}$
elements separated by $,$.

eg. $A = \{a, e, i, o, u\}$
 $B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

2. Set Builder form : $A = \{ \text{variable} \in \text{Domain} : \text{Condition} \}$

: such that 'OR' | such that

eg. $A = \{x \in \text{english alphabet} : x \text{ is vowel}\}$

eg. $A = \{x \in \mathbb{N} : -1 \leq x \leq 3\}$
 $A = \{1, 2, 3\}$

eg. $A = \{x : x^2 - 25 = 0\}$
 $A = \{-5, 5\}$

Evolution of Number System

- ◆ Natural Number (N) / Counting Number —

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

★

- ◆ Whole Number (W) —

$$\mathbb{W} = \{0, 1, 2, 3, \dots\} = \{0, \mathbb{N}\}$$

Zero — Zerifo "How can nothing be something"

- ◆ Integer Number (I) or (Z) [Zahlen] German words —

$$\mathbb{I} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

- ◆ Set of Rational Number (Q) — [Quotient]

$$\mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0 \right\}$$

Repeating or terminating decimal Number.

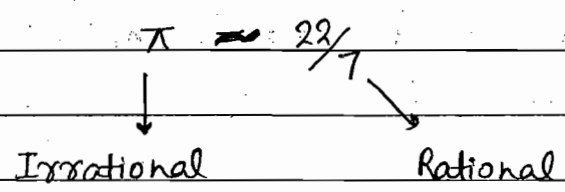
e.g. $10/3 = 3.33\dots$ Repeating decimal number
 $= 3.\bar{3}$

e.g. $5/2 = 2.5$ Terminating decimal number

- ◆ Irrational Number Neither repeating nor terminating

e.g. $\pi = 3.14\dots$, $\sqrt{2}$, $\sqrt{3}$

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$



e = euler No.
 ϕ = Golden Ratio

★ Irrational Numbers used in 'Cryptography'.

◆ Real Number -

Rational No + Irrational No. = Real No.

ething"

◆ Complex Number -

words -

$C = \{a + ib : a, b \in \mathbb{R}\}$

\downarrow
iota [imaginary]

- }

e.g. $Z = 2 + 3i$

\downarrow
Real part

\downarrow
Imaginary part

$$\left\{ \begin{array}{l} i = \sqrt{-1} \\ i^2 = -1 \\ i^3 = -i \\ i^4 = 1 \end{array} \right\}$$

Irrational Number - $\pi - pi$

or.

$\rightarrow e$ - euler No. = $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots = 2.7$

ber

Natural Logarithm. $2 < e < 3$

umber

$\rightarrow \phi$ - Golden Ratio $\phi = \frac{F_n}{F_{n-1}}$ $\phi = \frac{1 + \sqrt{5}}{2}$

$\phi = 1.618$

 $\phi = \lim_{n \rightarrow \infty} \frac{F_n}{F_{n-1}}$

Use in 'Fibonacci Series' / Human Face

Q. The golden ratio ϕ & its conjugate ϕ' both satisfy the eqⁿ.

4. Infi

eg. /

(a) $x^3 - x - 1 = 0$ ~~(b)~~ $x^2 - x - 1 = 0$

(c) $x^3 + x - 1 = 0$ (d) $x^2 + x - 1 = 0$

Corr

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{1 \pm \sqrt{5}}{2}$$

'Corr

set

set

Types of Set —

Rep

1. Empty Set / Null Set / Void Set : A set which contain no element.

eg. A

IF

$$\{ \} \text{ or } \phi$$

2. Singleton Set : A set containing only one element.

5. Eql

g. $A = \{a\}$, $B = \{\phi\}$, $C = \{1\}$

no

$$A = \{\{1, 2\}\}$$

$$\{1\} \neq \{1, \{1\}, \{\{1\}\}\}$$

eg. A

3. Finite Set : A set containing finite number of element.

B

C

D

eg $A = \{a, e, i, o, u\}$

F

$$B = \{1, 2, 3\}$$

B

C

4. Infinite Set: A set which is not finite.

eg. $\mathbb{N} = \{1, 2, 3, \dots\}$

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

Cardinality / Cardinal No. of Finite Set - n is called a 'cardinal number' or cardinality of a finite set A & it's is the no. of element in set A .

It is also called 'order of set'
Represented by

$$|A| = n(A) = n(\text{no. of element})$$

in

eg. $A = \{a, e, i, o, u\}$

$|A| = n(A) = 5$

5. Equivalent Set: Two finite set A & B are equivalent if their cardinal no. are same. i.e.

$$|A| = |B| \text{ or}$$

$$n(A) = n(B)$$

eg. $A = \{a, b\}$

$E = \{0, 1\}$

$B = \{1, 2, 3, 4\}$

$F = \{8, 4, 12\}$

$C = \{4, 8, 12\}$

$H = \{p, q\}$

$D = \{3, 1, 2, 4\}$

$G = \{1, 5, 7, 11\}$

$A \approx E \approx H$

$B \approx D \approx G$

$C \approx F$

} are equivalent sets.

Equal Set : Two set A & B are said to be equal if every member of A is a member of B & vice-versa.

e.g. $A = \{a, b\}$, $B = \{b, a\}$ $A = B$

Subset - Let A & B be two sets.

If every element of A is an element of B then A is called subset of B.

e.g.

Denoted By $A \subseteq B$

i.e. $A \subseteq B : \forall a \in A \Rightarrow a \in B$ $a \rightarrow$ element

e.g. $A = \{a, b\}$, $B = \{a, b, c\} \Rightarrow A \subseteq B$
 $A = \{d, b\}$, $B = \{a, b, c\} \Rightarrow A \not\subseteq B$

■ A is a subset of B $A \subseteq B$
 OR

B is a superset of A $B \supseteq A$

e.g.

$A \subset B \Rightarrow A \neq B$

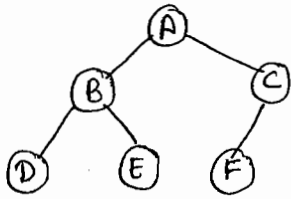
e.g.

Proper Subset : If $A \subseteq B$ & $A \neq B$ then A is called proper subset of B

i.e. $A \subset B$

BINARY SEARCH TREE :- Binary tree with search property

e.g.

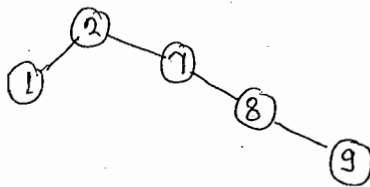


Above tree is BST then

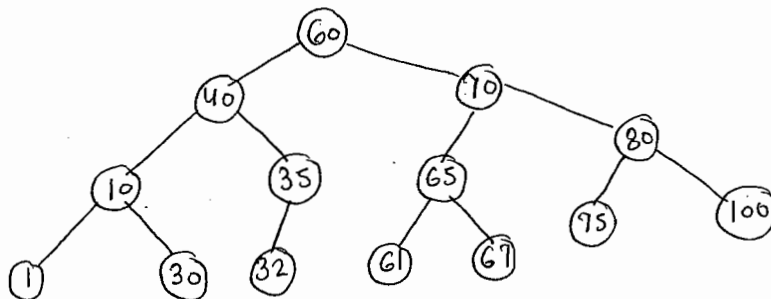
- (a) $D < B < A < E < C < F$
 (b) $A < B < D < E < C < F$
 ✓ (c) $D < B < E < A < F < C$
 (d) Not a

e.g. if 2, 7, 8, 1, 9 is inserted into a BST in that order.
 How many BST's are possible.

- ✓ (a) 1 (c) 5
 (b) 42 (d) Not a



e.g.

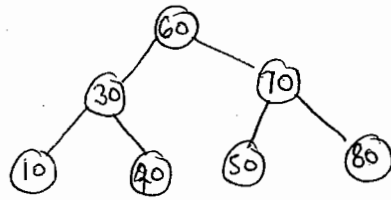


Traverse in Inorder in Binary search tree will always
 in Increasing Order.

Cost of Search in BST :- $O(\text{Height})$

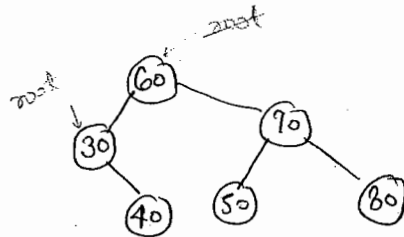
Search 61 cost = $O(\log n)$

Minimum of BST : left most leaf node but not always



Minimum = 10

But if ~



then Minimum will be that node whose left child is NULL in the left subtree.

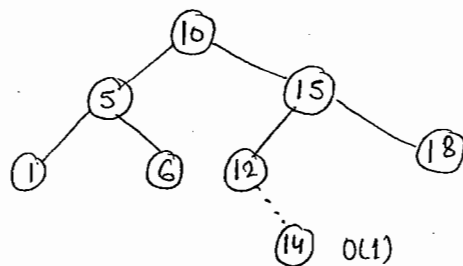
```
int smallest (tree * root)
{
    if (root != NULL)
    {
        while (root -> left != NULL)
            root = root -> left;
        return root -> data;
    }
}
```

Maximum of BST : Right child is NULL

```
int largest (Tree * root)
{
    if (root != NULL)
    {
        while (root -> right != NULL)
            root = root -> right;
        return root -> data;
    }
}
```

Cost of finding maximum & Minimum is BEST case = $O(1)$

Insert 14 :



Cost (insertion) = O (height of BST)

```

graph TD
    10((10)) --> 5((5))
    10 --> 15((15))
    5 --> 1((1))
    5 --> 6((6))
    15 --> 12((12))
    15 --> 20((20))
    12 --> 14((14))
  
```

Inorder predecessor of 12 = 10

Inorder Successor of 12 = 14

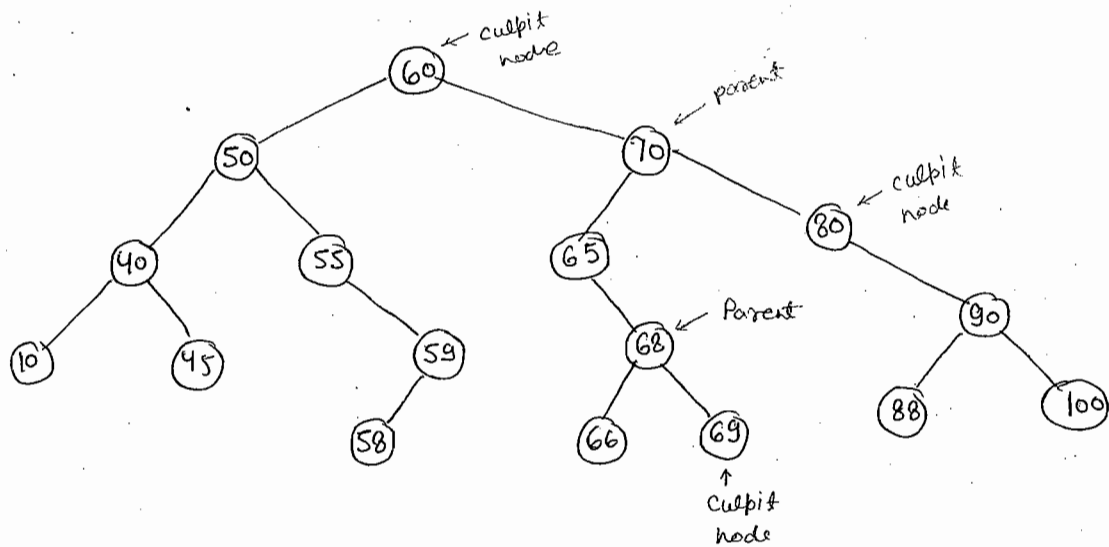
Inorder Predecessor : Is the largest element of the left subtree if it exists.

Inorder Successor : The smallest element of the right sub tree if it exists.

Case - I : When the culprit (Delete) node is a leaf node.

Case - II : When the culprit node has only one child.

Case - III : When the culprit node has both left & right child.



Case I : Delete 69

First find the position of 69

then find the parent of 69

then if it is right child

then remove the right link.

Case II : Delete 80

First traverse the tree & find location

Find parent of culprit node

if deleted node is right child

then link the right child of parent culprit node with
the right link of parent.

Case - III : Delete 60

It can be deleted by the Predecessor or Successor

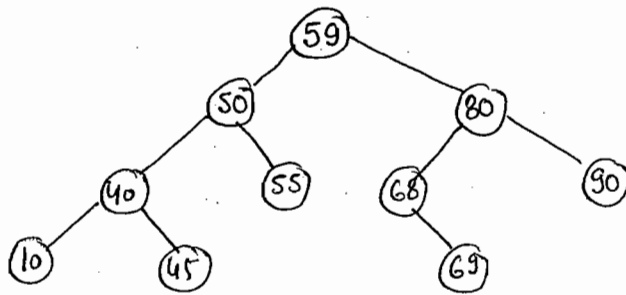
Replace culprit node with its successor

or then use case I

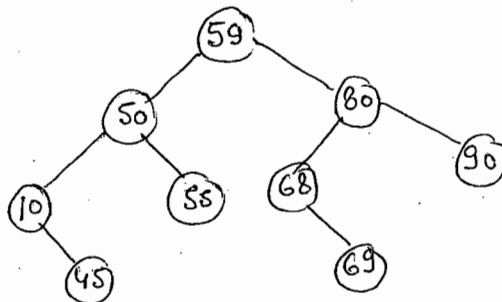
OR

Replace the culprit node with the predecessor & use case II.

eg.



Delete 40



Delete by predecessor.

Q. If the preorder of a BST is 5, 3, 1, 4, 7, 8, 6, 9 then what is the postorder of BST

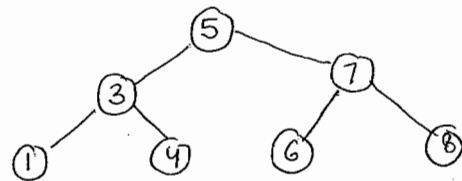
(a) 1, 3, 4, 5, 6, 7, 8

(c) 3, 4, 7, 6, 1, 8, 5

~~✓(6)~~ 1, 4, 3, 6, 7, 8, 5

(d) Nota

Inorder = 1, 3, 4, 5, 6, 7, 8



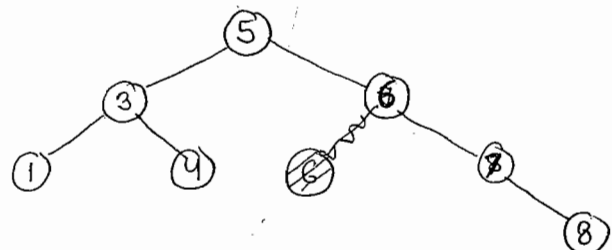
Q. If the BST contains 1, 3, 4, 5, 6, 7, 8 keys then which of the following can be a correct preorder traversal.

(a) $5, \underset{\uparrow}{3}, 1, 4, \underset{\uparrow}{7}, 8, 6$

(b) 5, 3, 4, 1, 7, 6, 8

✓(c) 5, (3, 1, 4), (6, 7, 8)

(d) Nota



Q. If set of data item ranging 10 — 100 is given in a BST & we want to search 55 then which is the current sequence of examining the nodes.

✓(a) 10, 20, 30, 40, 50, 55

(b) 40, 100, 90, 30, 45, 55

✓(c) 80, 70, 60, 45, 55

(d) Not a.

Q. What is the maximum distance b/w 2 nodes in a perfect Binary tree

(a) $\log n$

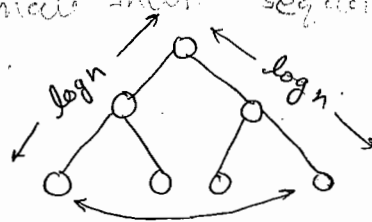
(b) $3 \log n$

✓(c) $2 \log n$

(d) Not a.

If we search any number then start with any where but check searching is greater than starting no. so all element is small with.

if starting no. is small then after this all element greater if any no. is small then sequence is wrong.



GRAPHICS :- "Graphics involves storing, displaying and manipulating the picture / image" [Pictorial Representation]

COMPUTER GRAPHICS :- Graphics involves storing, displaying and manipulating the picture / image and related data for the proper visualization through the computer.

Types of Graphics -

1. Passive Graphics :- User can't change or update the information or picture on the console.

e.g. Screen Saver

2. Interactive Graphics :- Acc. to the input of user the information or picture is changed on console.

e.g. video games.

These are widely used

3. Business Graphics :- Use of PPT, Bar chart, Pie chart

4. Scientific Graphics :- Use of curve fitting, contour curve, regression etc.

5. Cartoons & Animation :- Animation is continuous flow of static images.

Applications of Computer Graphics -

1. CAD / CADD :- Use to design the architecture of building.

2. Education :- CAL (Computer Aided Learning) . Online learning classes
3. Entertainment :- use graphics in movies
4. Medical :-
5. Science :-
6. Computer Vision :- It is a part of AI . It's aim is design some computer which can understand the real circumstance through images.
7. Image Processing :- Enhance the image (image correction, image reduction is the part of image processing.

Display devices are used to display the picture on the console.

Types of Display devices :-

- DVST (Direct view Storage tube)
 - Calligraphic (Random scan device display device)
 - Refresh raster display devices
 - Flat panel devices
 - LCD
 - LED
- } CRT Commonly used.

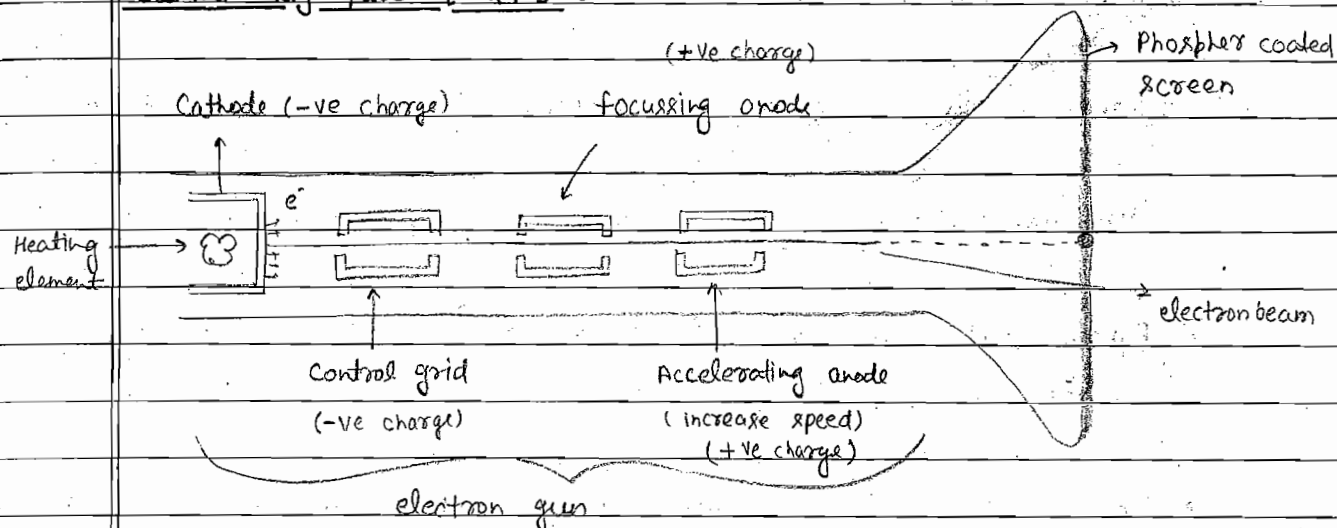
Generation of colors :- → Beam Penetration Method

→ Shadow mask approach or

Delta - Delta Approach

ring

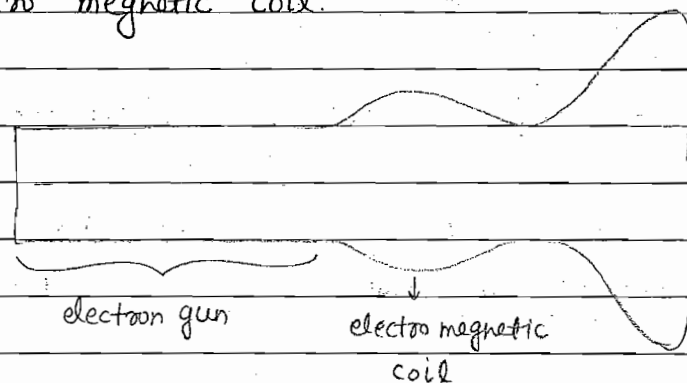
Cathod Ray Tube [CRT] :-



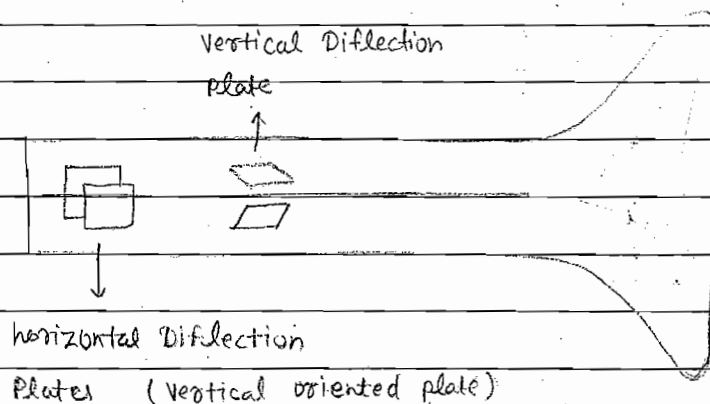
Brightness of screen is controlled by the control grid.
 Brightness depends upon the intensity on electron beam.
 Intensity depends upon the no. of electrons.

The electron beam is pointing to a single point on the screen but we want to reflect the electron beam on the whole screen, so we use Diffraction by using electro magnetic coil.

picture



monoly



Display Devices / System

1) Random scan / Calligraphic Display System :- It is also known as

"Vectored / Stroke / Line Drawing Display system."

(direction & magnitude)

- Fundamental component of a picture / image is a line (short line). A

- In it we can use any pattern for display.

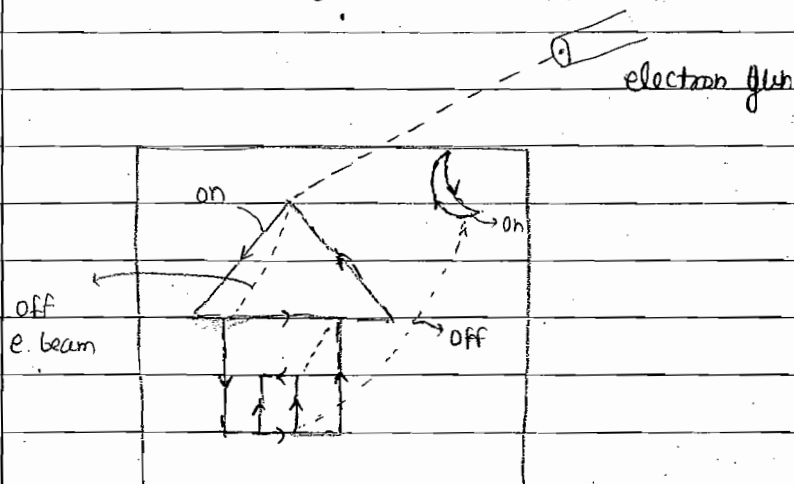
Short persistence a phosphor coated screen & delay + 100ms

In it we have to draw the picture again & again
so need to refresh

refresh rate = # of times the picture on the screen
is drawn in per unit time

$$= 30 \text{ Hz} = 30 \text{ fps} \quad (\text{frame per sec})$$

If the refresh rate is less than 30 Hz then the picture will be flicking & we can't see the picture clearly.



2) Refresh Raster Display system :- This screen is considered as a matrix of pixels

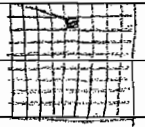
Pixel - 'Picture element' :- Smallest addressable unit / position on the screen.

pixel contains 3 component -

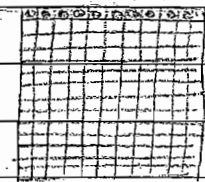
(x, y) coordinate

color

pixel
(x, y)
color



M Rows of pixels = m scan line



scan line

Each Row & Column is a scan line.

"A picture is a collection of points"

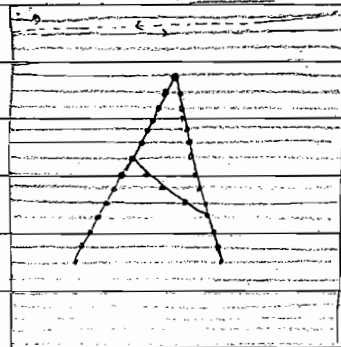
Display program

Refresh Buffer - A memory / Buffer the instruction / program to display the image on the screen is stored. It is read by the Display Processor & it interprete it & draw the corresponding picture on the screen

Change in picture, the content of the buffer will be change.

electron beam

E.G.

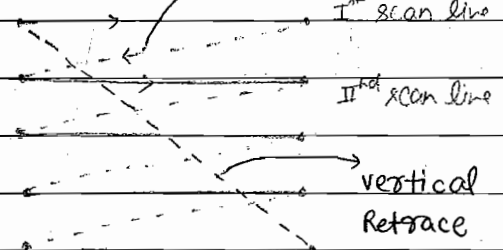


Horizontal Retrace

Ist scan line

IInd scan line

vertical Retrace



m Scan line $\Rightarrow (m-1)$ Horizontal Retrace

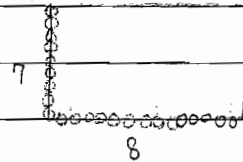
If Refresh rate is $= r$ fps

then # of vertical retrace in 1 second $= (r-1)$

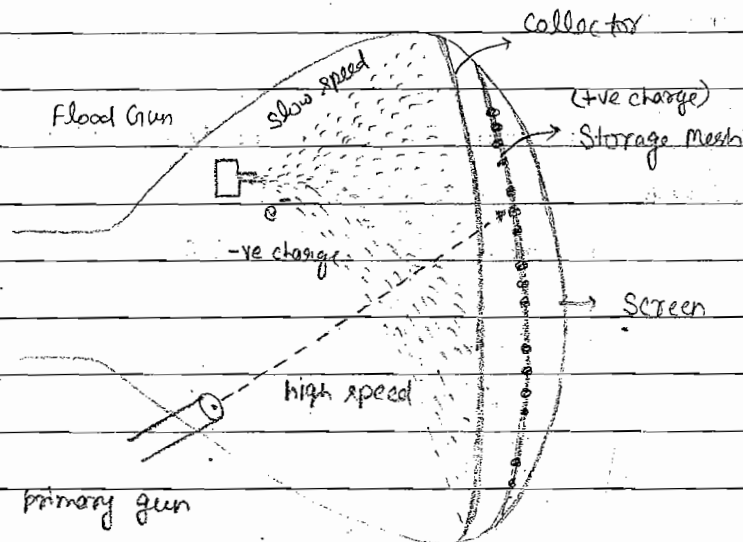
Resolution: # of pixels in unit horizontal direction *
of pixels in unit vertical direction

i.e # of pixels in per unit ^{square} Area

$$\begin{aligned}\text{resolution} &= 7 \times 8 \\ &= 56 \text{ pixels}\end{aligned}$$



- 3) DVST (Direct view storage tube) - It has long persistence phosphor coated screen.
No need to refresh so flicker free.



When computational Devices are moveable in nature.

Communicational Classification : It is three types.

1) 1-G First Generation Mobile - Analog voice is sent. It uses circuit switching.

Bandwidth 0 - 4 KHz

2) 2-G Second Generation Mobile - Digital voice and data, text
Packet switching. (virtual connection)

Bandwidth - 64 Kbps [Digital Radio]

3) 3-G Third Generation Mobile - Digital voice & Data (Text, audio, video)
Packet switching.

→ Telegram uses message switching

Mobile Station (MS) - Moving unit

Base Station (BS) - Stationary unit

mobile tower

Cell - The shape of cell is Hexagonal
radius of cell 1 to 12 meter



large area

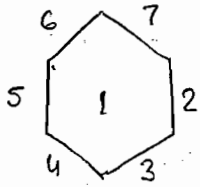
If the Mobile Station moves outside the cell the Roaming
will charge MS.

Handoff - If you are moving from one cell another cell.

1) Hard Handoff connection broken from BS₁ then connection establish
to second BS (call drop)

Soft Handoff - Connection of MS establish to second BS then connection will break from first BS. No call dropping

Frequency Re-Use Factor : (i) Re-Use factor 7



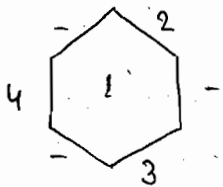
In this case frequency Re-use factor is 7. ($\frac{1}{7}$)

All cells has diff frequency if any will same then frequency interference can occur.

More frequency Required.

1 cell \leftarrow 1 BS

Re-Use Factor 4:



In this case adjacent left blank

frequency Re-use factor is 4 ($\frac{1}{4}$)

3 - In the following which is frequency re-use factor

(a) N^2

(b) N^3

✓ (c) $\frac{1}{N}$

(d) \sqrt{N}

Mobile Switching Center [MSC] :- Each BS is controlled by switching office called MSC.

[or Communication]

Transmitting - To place a call from MS, caller enters 7 or 10 digits to BS] digits & (Phone number) and presses the call button.

MS scans the band, seeking setup channel with strong signal & send to closest BS. BS sends that signal to MSC.

Mobile to Landline communication \rightarrow MSC sends the (data) signal to telephone office. If the

called party is available then connection is made & result is

played back MSC. AT this point MSC assigns an unused voice channel to call & connection is established

communication begins.
[forward Comm.]

Receiving - When MS is called, the Telephone central office sends [BS to MS] the number to MSC. MSC searches for the location of MS by sending query signal to each cell in a process called. Paging.

Once Mobile Station is found, the MSC transmits a ringing tone & MS answers, then assigns an unused voice channel to call. Then communication is established.

• First Generation (1G) Mobile : AMPS [Advanced Mobile Phone System] is one of analog cellular system developed by North America. AMPS uses FDMA to separate links.

AMPS - Transmitting frequency Band - ISM [Interium Standard]

ISM 800 - MHz

869 - 894 MHz

824 - 849

Receiving bandwidth

Bandwidth = 25

AMPS reuse factor 7

Hard Handoff

Each bandwidth is divided into 832 channels

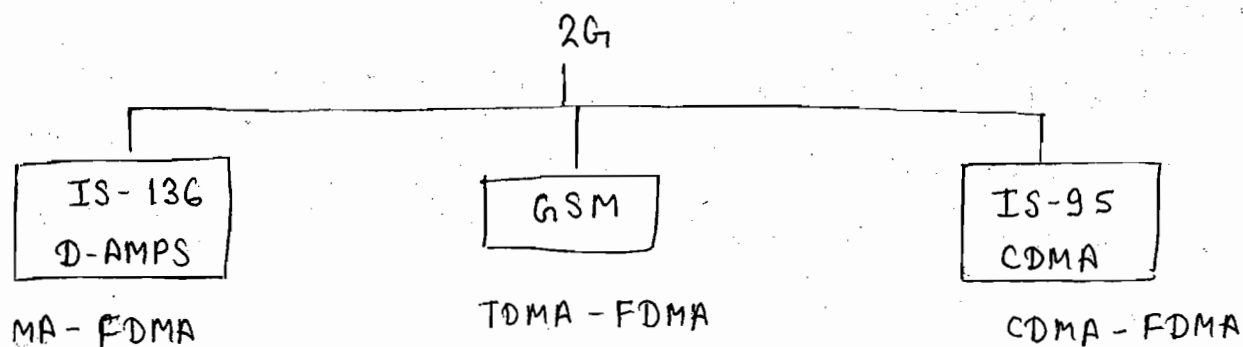
↓
2 provides each for 416

21 channels used for control

395 channels used for calling phase. $[416 - 21] = 395$

Disadvantage - Analog voice rate (bandwidth is low).

& circuit switching is used.

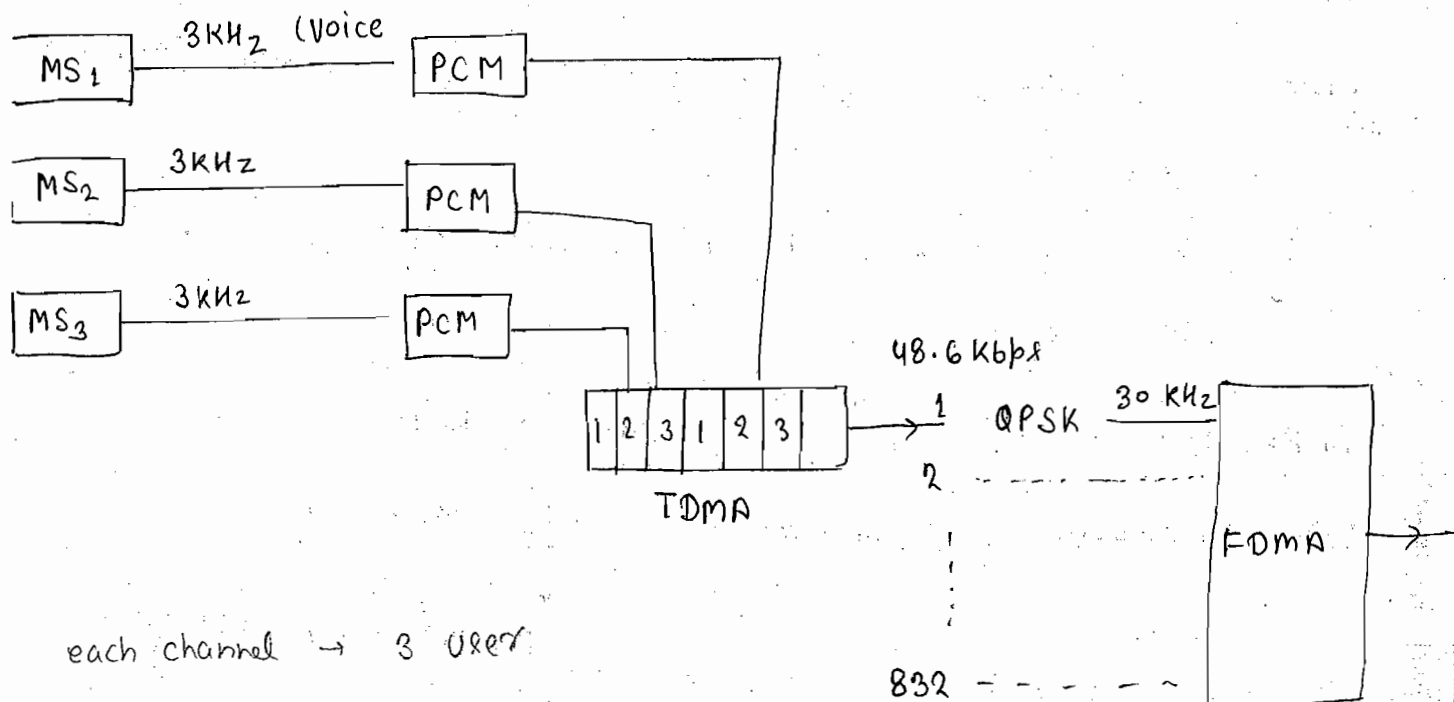


AMPS [Digital AMPS] : Initially D-AMPS denoted by IS-54 (Interim Standard -54) and later revised

- 136.

frequency band \rightarrow ~~FDMA~~ Same as AMPS

each voice channel digitized uses PCM [30 kHz \rightarrow 7.95 kbps]
[Pulse Code Modulation]



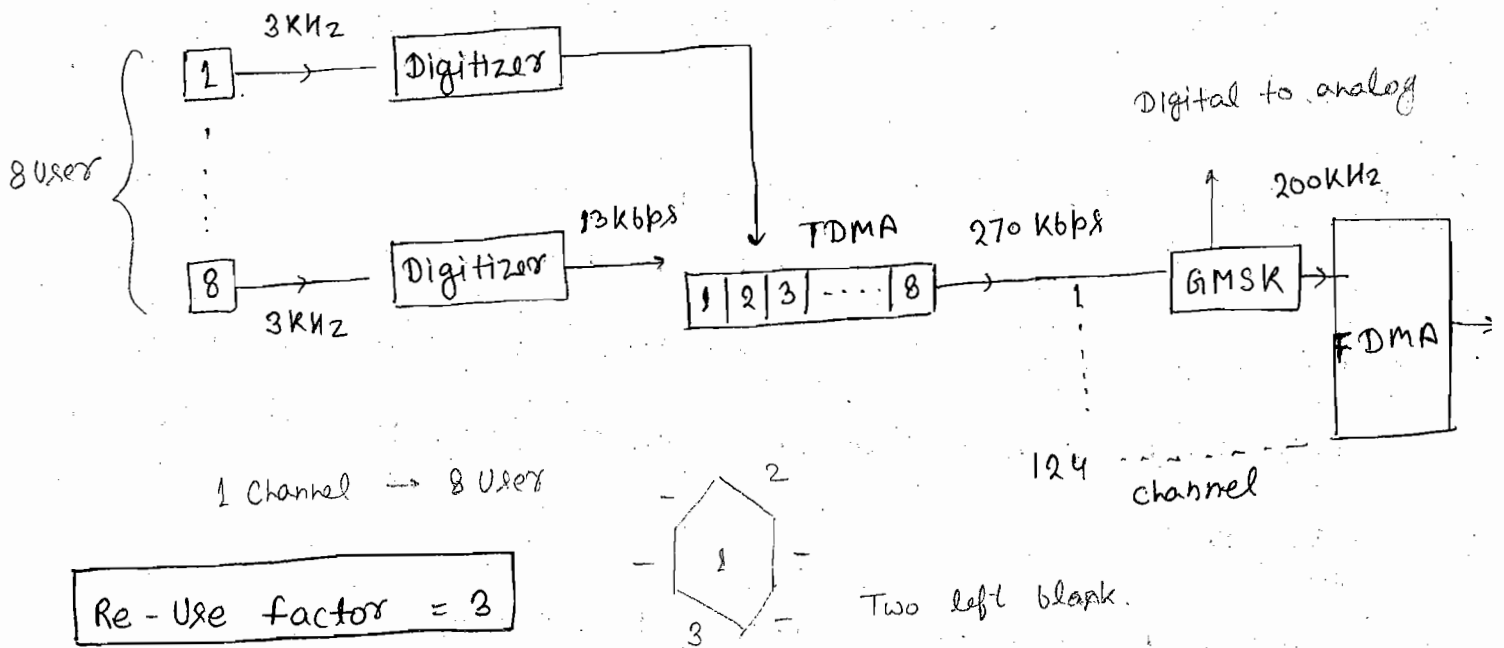
Reuse factor = 7

Standard.

Band - GSM uses two bands for duplex communication. Each band has 25 MHz bandwidth

Reverse Band - 124 channel : 890 - 915 MHz

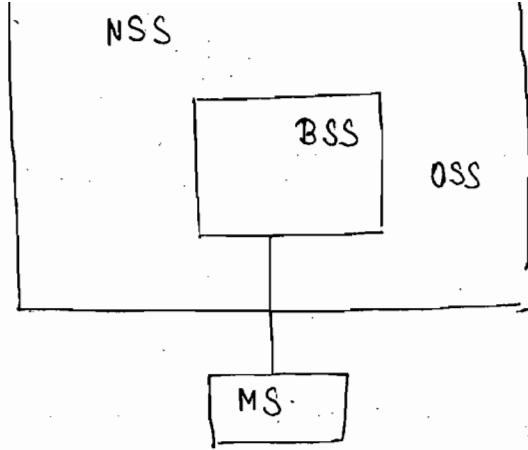
[Receiving] Forward Band - 124 channel : 935 - 960 MHz



GSM Architecture :

Terminology used in GSM :

1. MS (Mobile Station)
2. BSS (Base Station Subsystem)
3. NSS (Network Switching Subsystem)
4. OSS (Operation support Subsystem)



Optimally Bloom
Statement

operation research

The GSM network is divided into the following 3 major components
SS, BSS, OSS

IS-95 [Interim Standard] :- One of the dominant 2G standard
in North America.

Band - It works on traditional ISM 800 MHz or ISM 1900 MHz

Each band is divided into 20 channels of 1.25 MHz separated
by guard band.

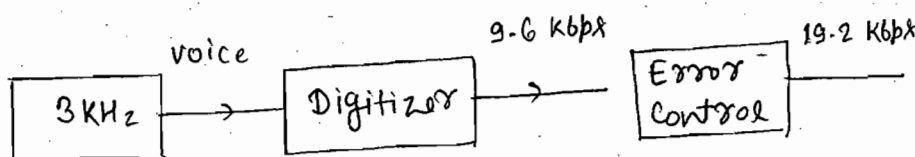
Each service provider is allotted 10 channels.

IS-95 is equivalent to 41 AMPS. $= 41 \times 30 \text{ KHz}$
 $= 1230 \text{ KHz}$
 $= 1.23 \text{ MHz}$

Synchronization - By using CDMA synchronization is achieved.

To provide synchronization GPS is also used

IS-95 Forward Transmission : 64 Channel



Operation Research

Linear Programming Problem (LPP)

- | | |
|---------------------------|---------------|
| 1) Graphical method | → Numerical |
| 2) Simplex method | } Theoretical |
| (i) two phase method | |
| (ii) Big-M method | |
| 3) Duality | |
| 4) Transportation Problem | } Numerical |
| 5) Assignment problem | |

Linear Programming Problem (LPP) :

General form of a LPP -

$x_1, x_2, x_3, \dots, x_n$ decision variables

Objective function

$$Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \quad \text{--- (1)}$$

$(c_1, c_2, \dots, c_n) \leftarrow$ cost vector

Subject to condition,

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n (\leq, =, \geq) b_1$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n (\leq, =, \geq) b_2$$

\vdots

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n (\leq, =, \geq) b_m$$

(2)

Non-negative constraints,

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0 \quad \text{--- (3)}$$

all the decision variable should be Non-negative.

Solution of LPP - Any n -tuple (x_1, x_2, \dots, x_n) which satisfies (2) is called a solution to the LPP.

Feasible solution - Any solution of a LPP which also satisfies (3) is called a feasible solution.

All values are non negative

Optimal Solution: Any feasible solution which optimizes (max/min) the value of Z is called an optimal solution.

Unbounded Solution: If we can increase / decrease the value of Z infinitely, then we say the problem has unbounded optimal solution.

Graphical Solution: when two variable problem

$$\text{Max } Z = 3x_1 + 5x_2$$

subject to condition

$$x_1 + 2x_2 = 2000 \quad x_1 + 2x_2 \leq 2000$$

$$x_1 + x_2 = 1500 \quad x_1 + x_2 \leq 1500$$

$$x_2 = 600 \quad x_2 \leq 600$$

$$\& \quad x_1, x_2 \geq 0$$

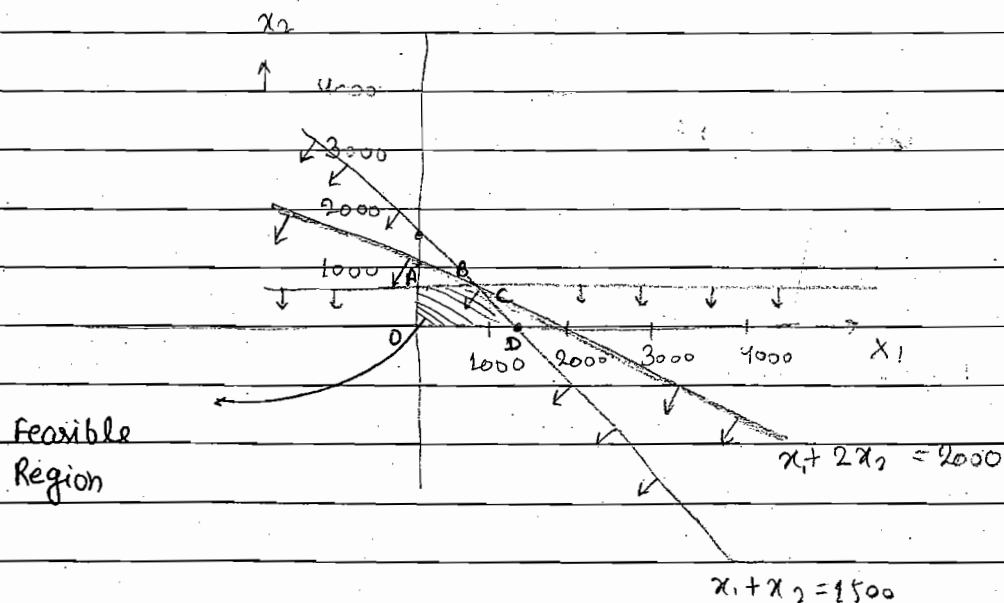
$$x_1 + 2x_2 = 2000$$

$$\text{Let } x_1 = 0$$

$$x_2 = 1000 \quad (0, 1000)$$

$$\text{Let } x_2 = 0$$

$$x_1 = 2000 \quad (2000, 0)$$



$$x_1 + x_2 = 15000$$

$$x_2 = 15000$$

$$(0, 15000)$$

$$\text{Let } x_1 = 0$$

$$x_1 = 1500$$

$$(1500, 0)$$

$$\text{Let } x_2 = 0$$

$$O (0, 0)$$

$$Z_0 = 0$$

$$A (0, 600)$$

$$Z_A = 3000$$

$$x_1 + 2x_2 = 2000$$

$$B (800, 600)$$

$$Z_B = 5400$$

$$x_1 + 2 \times 600 = 2000 \quad x_1 = 800$$

$$C (1000, 500)$$

$$Z_C = 5500$$

$$x_1 + 2x_2 = 2000$$

$$D (1500, 0)$$

$$Z_D = 4500$$

$$x_1 + x_2 = 1500$$

$$x_2 = 500$$

$$x_1 = 1000$$

Optimal solⁿ

$$C (1000, 500)$$

$$x_1 = 1000$$

$$\& \quad x_2 = 500$$

Optimal value

$$Z = 5500$$

eg

$$\text{Max } Z = x_1 + x_2$$

Subject to condition

$$x_1 + 2x_2 \leq 2000$$

$$x_1 + x_2 \leq 1500$$

$$x_2 \leq 600$$

$$\& \quad x_1, x_2 \geq 0$$

$$O (0, 0)$$

$$Z_0 = 0$$

$$A (0, 600)$$

$$Z_A = 600$$

$$B (800, 600)$$

$$Z_B = 1400$$

$$C (1000, 500)$$

$$Z_C = 1500$$

$$D (1500, 0)$$

$$Z_D = 1500$$

C & D gives optimal solⁿ

Above CD all are optimal solⁿ

Optimal solⁿ \rightarrow infinitely many

Optimal value $Z \rightarrow 1500$

- If you will get infinitely many optimal solⁿ, then objective fuⁿ line must be parallel to be one of the constraint

eg.

$$\text{Min } Z = 1.5x_1 + 2.5x_2$$

$$\text{sub. to constr. } x_1 + 3x_2 \geq 3$$

$$x_1 + x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

$$x_1 + 3x_2 = 3$$

$$\text{let } x_1 = 0$$

$$x_2 = 1$$

$$\text{let } x_2 = 0$$

$$x_1 = 3$$

$$(0, 1)$$

$$(3, 0)$$

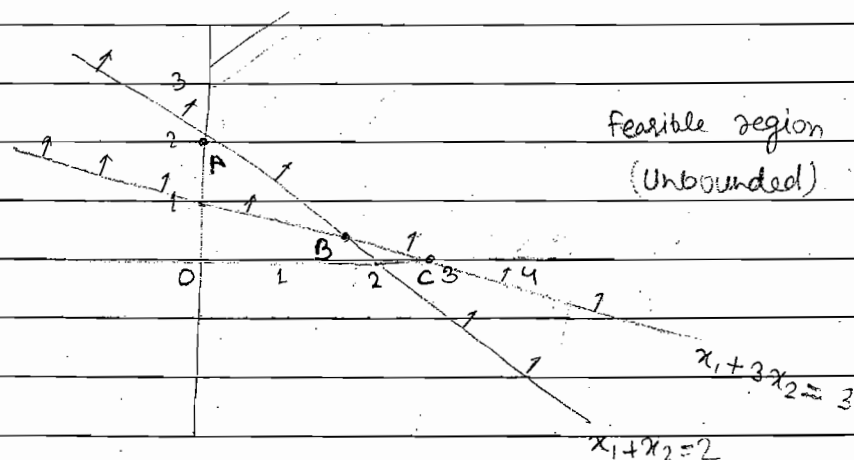
$$x_1 + x_2 = 2$$

$$x_1 = 2$$

$$x_2 = 2$$

$$(0, 2)$$

$$(2, 0)$$



$$A (0, 2)$$

$$Z_A = 5$$

$$x_1 + 3x_2 = 3$$

$$B (1.5, 0.5)$$

$$Z_B = 3.5$$

$$x_1 + x_2 = 2$$

$$C (3, 0)$$

$$Z_C = 4.5$$

$$2x_2 = 5$$

$$x_2 = 2.5$$

Optimal solⁿ

$$x_1 = 1.5, x_2 = 0.5$$

Optimal val

$$Z = 3.5 \quad \text{finite value}$$

eg

$$\max Z = 1.5x_1 + 2.5x_2$$

$$x_1 + 3x_2 \geq 3$$

$$x_1 + x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

Unbounded optimal solⁿ. because region is unbounded.

eg

$$\min Z = -1.5x_1 - 2.5x_2$$

$$x_1 + 3x_2 \geq 3$$

$$x_1 + x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

as increase the value of x_1 & x_2 , the value of Z will decrease

so it is also unbounded optimal solⁿ.

Q.

$$\max Z = 3x_1 + 2x_2$$

sub. to

$$x_1 - x_2 \leq 1$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

$$x_1 - x_2 = 1$$

$$x_1 = 1$$

$$x_2 = 0$$

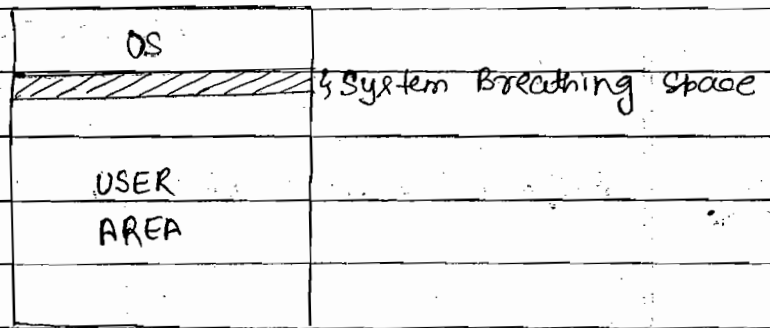
$$(1, 0)$$

Process State

Non - Preemptive OR Non Preemption

When the process releases CPU voluntarily, it is known as Non - Preemption otherwise Preemption.

Main Memory



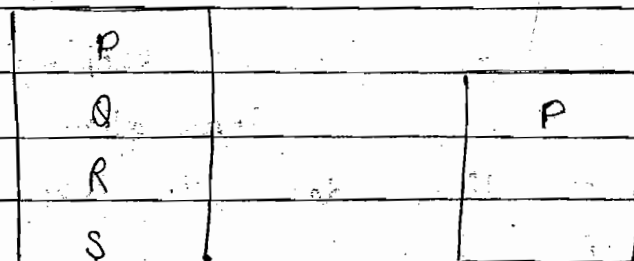
Kernel - That part of OS which resides permanently in main memory is known as Kernel.

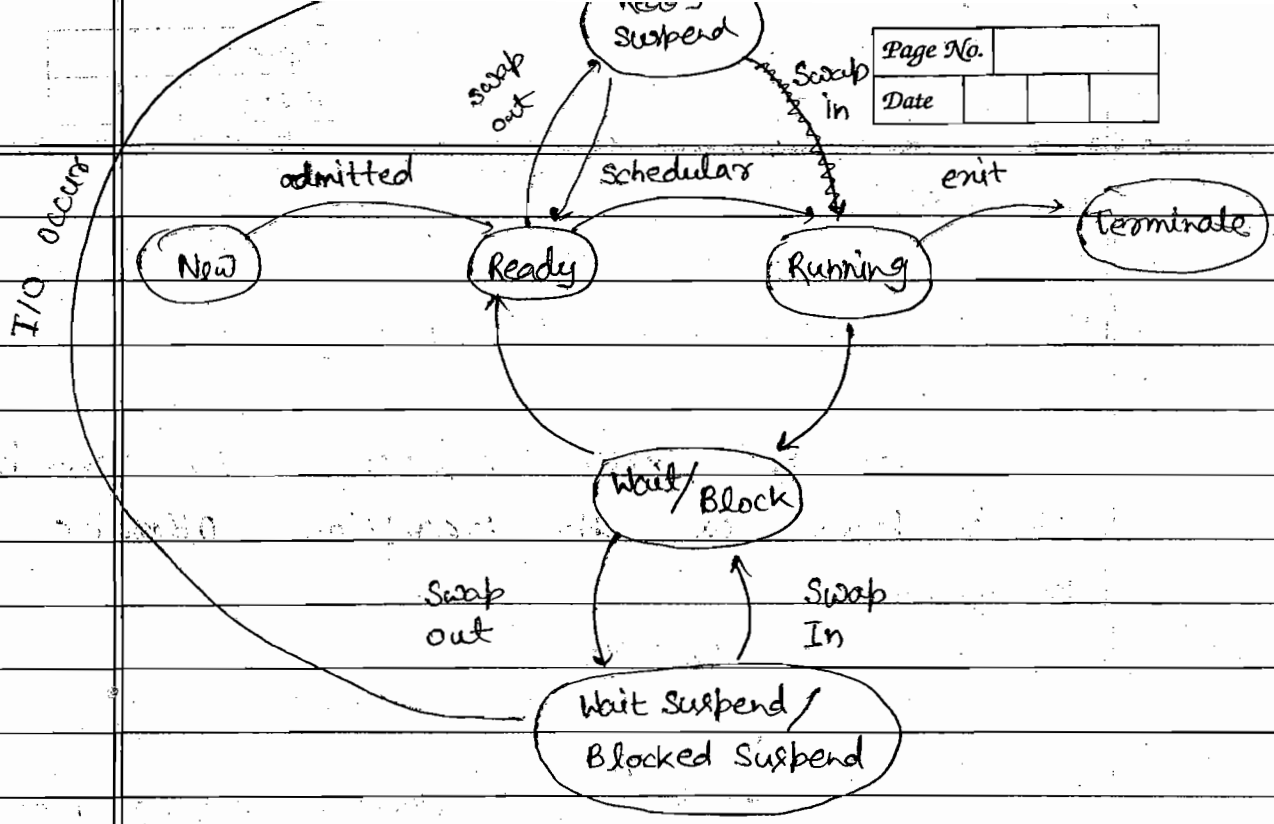
Kernel performs necessary operations like Process management, Process scheduling, memory management & I/O management.

Main Memory

Secondary Memory

B →





Suspend State will be in secondary memory.

Q. Consider a system with n CPU then what can be the maximum and minimum no. of processes that may present in ready, running & blocked state

	Min	Max
ready	0	no limit
running	0	n
block	0	no limit

Process Control Block (PCB) — OS maintains a
OR data structure for

Task Control Block (TCB) — keeping all the information related with a process that data structure is known as PCB.

PCB contains the following information —

ninate

Process State
Process Id
Program Counter
Registers
Memory limit
List of open file
⋮
⋮
⋮

- Process State
- Program Counter (A Register tells about the next execute instruction).
- Process id (A unique no. assigned to process)
- CPU Register
- CPU Scheduling information
- Memory management information.
- Accounting information of process
- I/O Status information

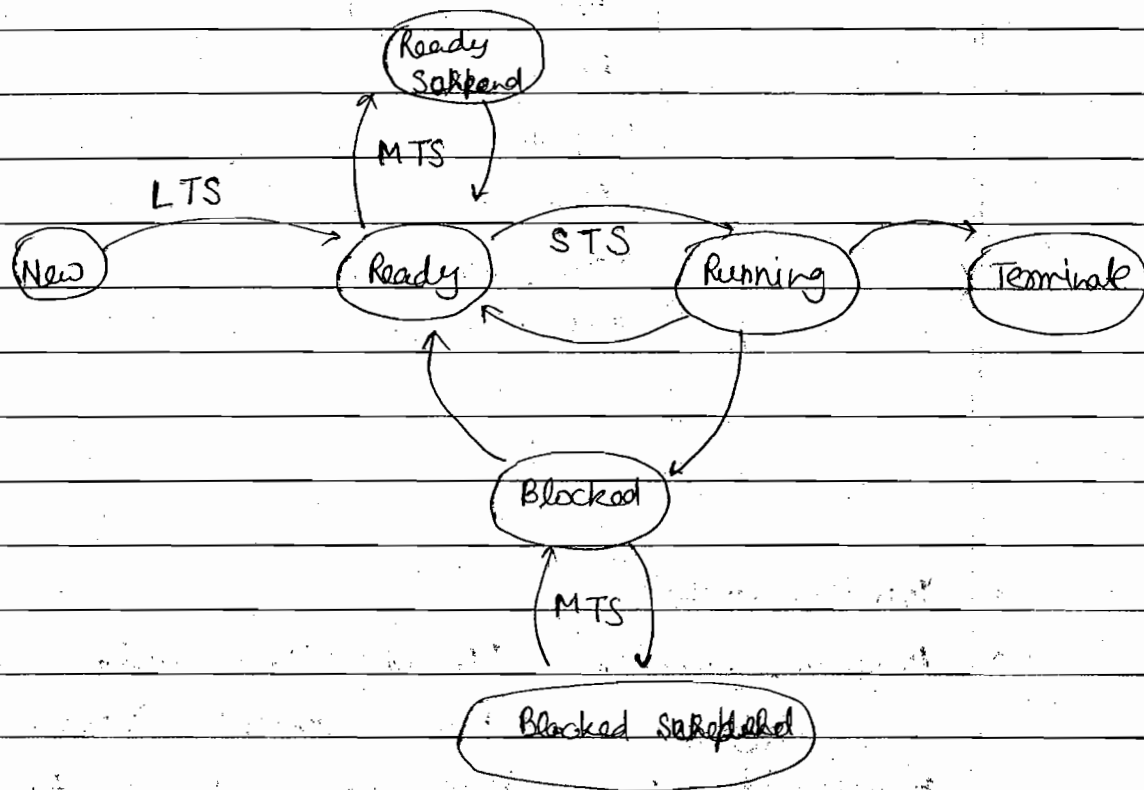
Scheduling of Process

Scheduler — It is a program that schedules the process from source to destination.

Types of Scheduler —

1. Long term Scheduler (very low frequency)
2. Medium term Scheduler

3. Short term scheduler (more frequency) →

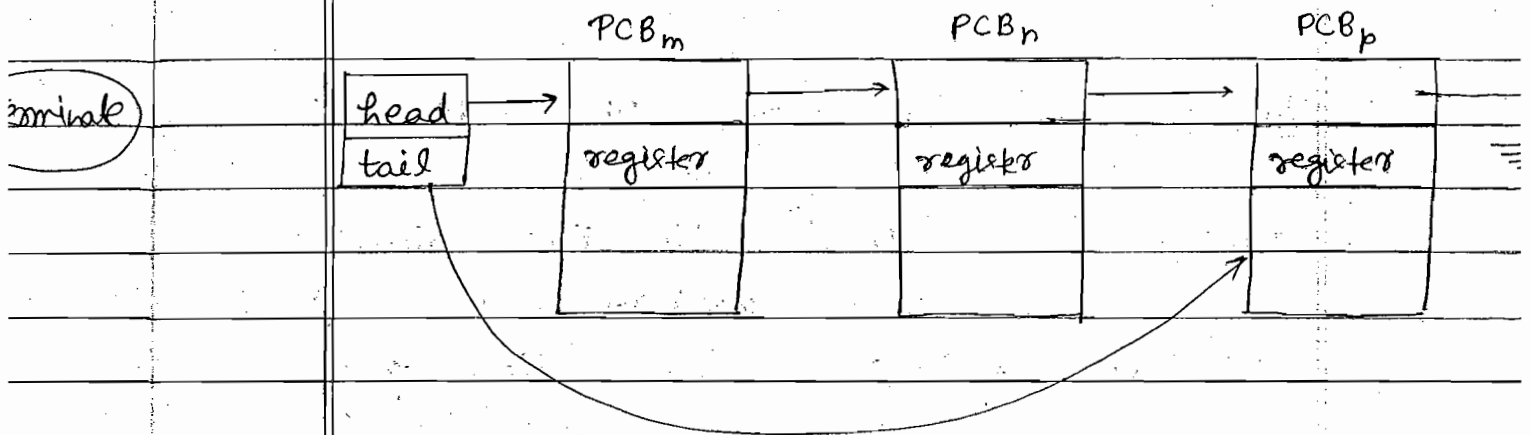


- Long Term Scheduler — The decision to add to the pool of processes to be executed.
- Medium Term Scheduler — The decision to add the no. of process that are partially or full in main memory.
- Short Term Scheduler — The decision as to which available processes will be executed by the processor.

⇒ STS is also known as 'CPU Scheduler'

⇒ The frequency of SITS is very high.

Scheduling Queues —



- The processes that are residing in main memory & are ready to & waiting to execute are kept in ready queue.
- This ready queue is generally stored as link list as shown in above.

✓ Metrics Related with Process —

1. Arrival Time (AT) — Arrival time is the time when the process arrives into ready state.
2. Burst Time — Burst time is time required for the process for its execution.
3. Completion Time — It is time when process completes its execution.

4. Turn Around Time - It is the time difference
b/w completion time &
Arrival time

$$TAT = CT - AT$$

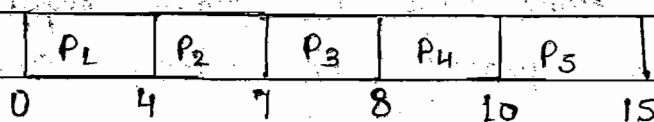
5. Waiting Time = Turn around time - Burst time.

6. Response Time - It is the time difference
b/w submission of a job
& first response from the system.

CPU Scheduling Policies - / Uniprocessor Scheduling

(1) First Come First Served (FCFS) - ~~closed~~ e.g.
Criteria - Arrival Time
Mode - Non - Preemptive

Process No.	AT	BT	Find
1	0	4	(i) Avg WT
2	1	3	(ii) Avg TAT
3	2	1	
4	3	2	
5	4	5	



Gantt Chart

1. Feasibility — Any solⁿ will follow some constraints & Bounds.

→ Technical Feasibility : Man Power

→ Economical Feasibility : Negotiation b/w. client & Service provider.

→ Environmental Feasibility :

2. S/W Requirements — R

→ S/W Requirement Gathering



Refinement



Specification

→ Functional Requirement

→ Non-functional Requirement

→ Goals of Implementations

3. S/W Design — DS + Algo

4. S/W Coding / (Implementation) / Development

5. S/W Testing — S/W Debugger is a part of compiler calls system

Cycles	Avg Cost (Duration)
S/w Analysis	7%
Design	6%
Coding	5%
Testing	15%
Maintenance	67%

Q. In the following which phase required maximum duration.

- (a) Analysis (✓) (b) Design
(c) coding (d) None of these

- S/w always goes under Maintenance.

Maintenance	1. Corrective	20% of 67 = 13.4
	2. Adaptive	19% of 67 = 12.73
	3. Perfective	60% of 67 = 40.2
	4. Preventive	1% of 67 = 0.67
	[Enhancement]	

Q. In the following which phase required maximum duration.

- (a) Analysis (c) Design
(b) Testing (✓) (d) Adaptive Maintenance

Q. Which phase required minimum duration.

- (a) Analysis (c) coding
(b) Preventive (✓) (d) None of these

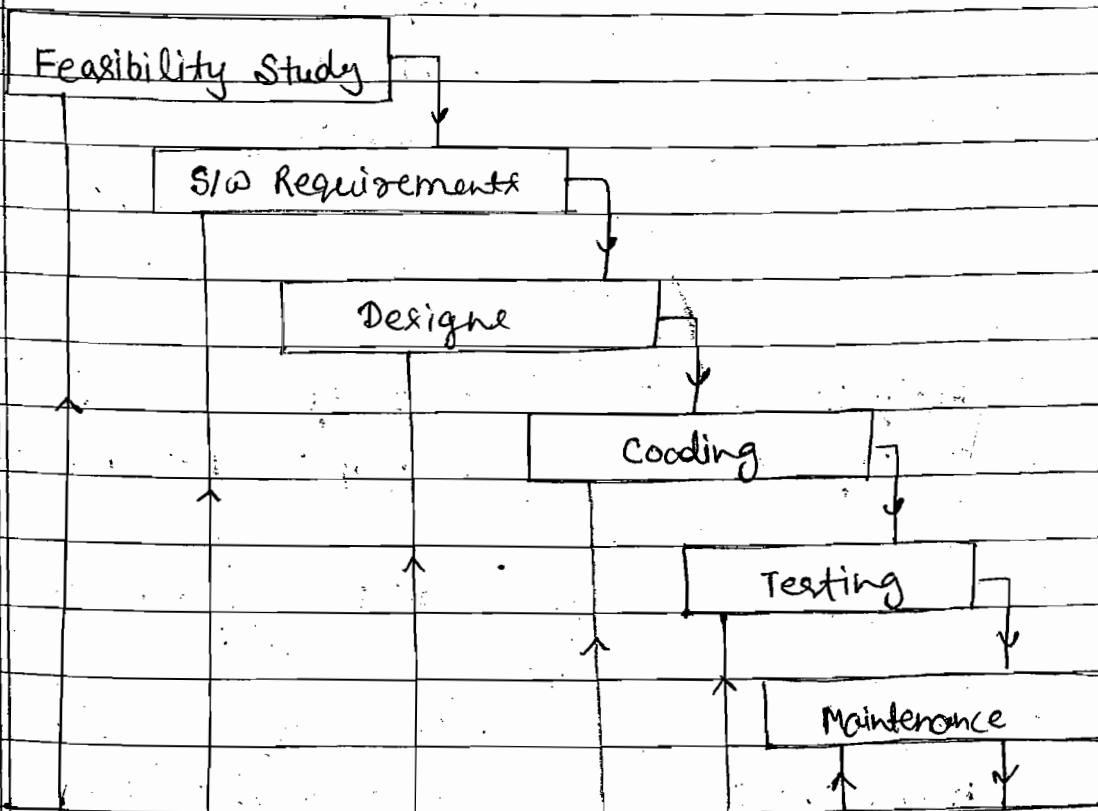
Advantages of Waterfall Model -

1. Simple & Sequential
2. First Model

Short coming / Pitfalls of Waterfall Model -

1. Initially all requirements should be known.
(Requirements are frozen)
2. Heavy document Driven.
3. Error Removal's very costly.
4. Customer Unsatisfaction.
5. Due to many errors S/W fails.
6. Big - Bang Model [early failure]

(2) Iterative Waterfall Model :



- Verification is done in each phase of SDLC
 ↳ "We are developing product right"
- Validation "we are developing right product"
 ↳ done in last (before delivery of S/W).
 ↳ Remove (fix) residual errors.
- Corrective Maintenance - There are some errors in S/W so corrective maintenance uses
 (Intermediate Code)
- Adaptive Maintenance → For platform independent we use adaptive maintenance
- Perfective Maintenance [Enhancement] - Initially all requirement are not known. So adding new features we use perfective.
- Preventive Maintenance - For correcting any error or how you correct error either in all places. [Prevent the change]

When the probability of error is high then you perform 'Preventive Maintenance'

[Residual Error Removal → Patch : Service Pack]
 Remaining

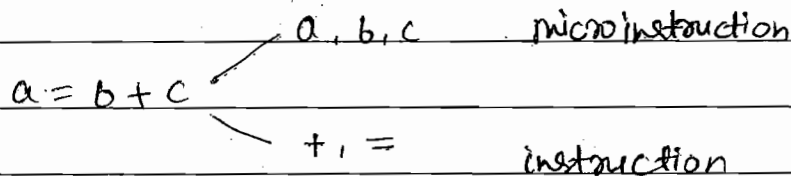
for removing thousands of errors.

SOFTWARE -

- ① Basic unit of ^{Software} System is Microinstruction

Microinstruction interacts with Microprocessors

- ② Instruction - one or more than one micro-instruction make Instruction
Used in Architecture



- ③ Program - Set of Instruction.

- Firmware has a unit instruction.

- Instruction will represent basic computation of microprocessor

- Instruction represented by binary

- Microprocessor does following for on instruction

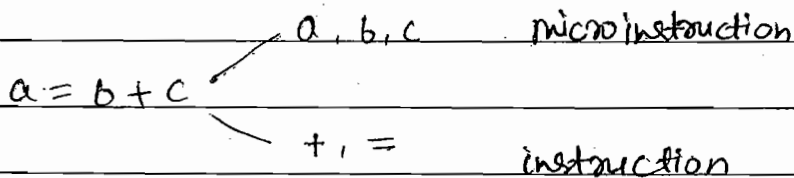
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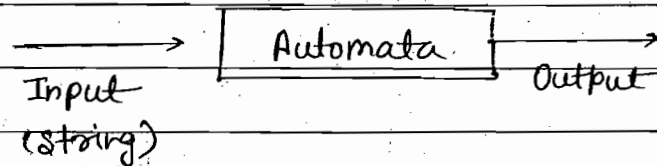
TDC

Motivation & Application :- Construction of Compiler
Text Editor (Regular exp),
Pattern Recognition (NFA, Regular Expression).

TDC → Computational Model -

- F.A [Finite Automata]
- P.D.A [Pushdown Automata]
- L.B.A [Linear Bounded Automata]
- T.m. [Turing Machine]

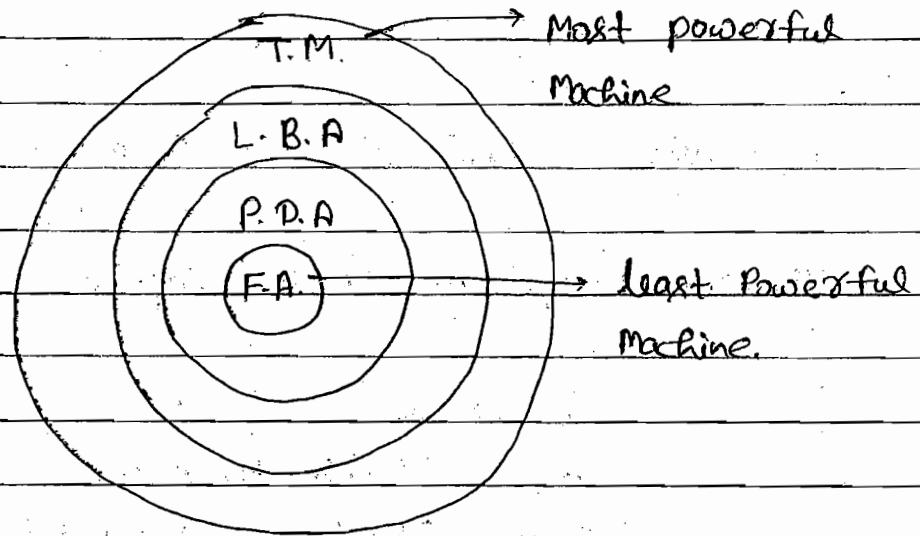
Turing Church Thesis :- "Every work done by
digital computer can also
be done by T.M."



'Automata' is a fundamental computation machine
Which take an input & process without
interference gives an output.

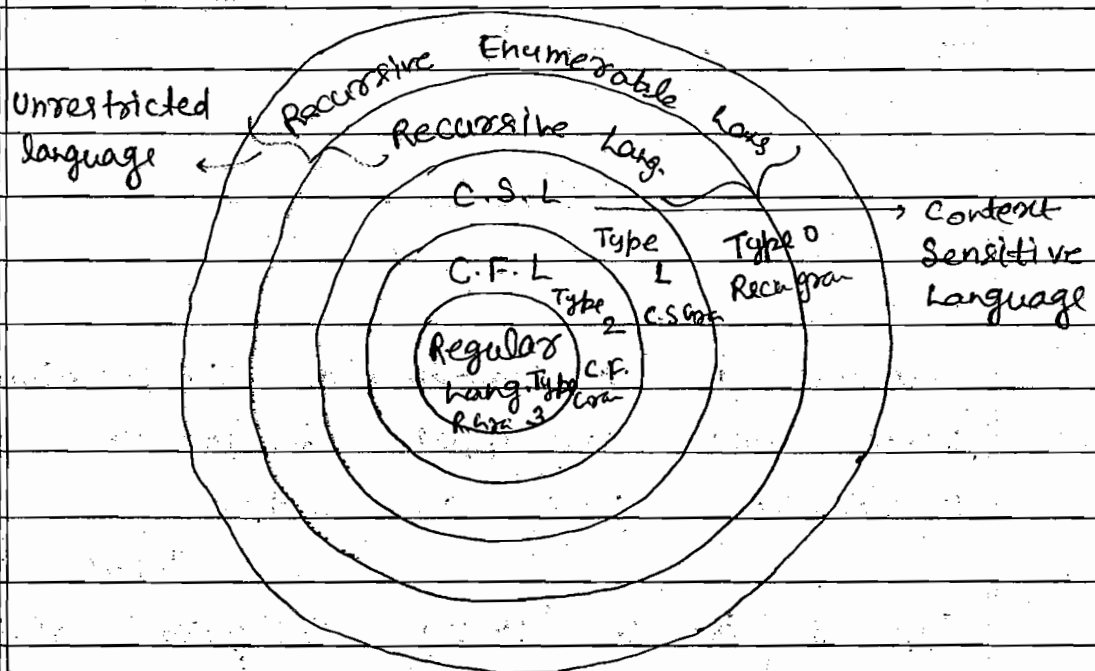
Input will
typicall be a 'string'. Collection of string
is called 'language' or 'formal Language.'

Language (string) may be meaningful or
may not be.



Hierarchy w.r. to Power of Machine

$$FA < P.D.A < L.B.A < T.M.$$



Neo chomsky Hierarchy

- All Regular Languages are C.F.L. (Context Free Language).

- Any language outside the boundary will not be Recursive enumerable language.

- Regular languages are accepted by F.A.

- C.F.L are accepted by P.D.A.

- C.S.L are accepted by L.B.A

- T.M use both Recursive lang. & Recursive Enumerable lang.

- Finite Automata is limited within Regular language

Terminal Symbols — are denoted by small letters, digits / Special symbols.

like $a, b, c, x, y, \dots, 0, 1, 2, \dots, \#, @, \dots$

Non-Terminal Symbols — denoted by capital letters.

Alphabet (Σ) — A finite collection of terminal symbols

e.g. $\Sigma_1 = \{a, b\}$ & $\Sigma_2 = \{0, 1\}$ Binary Alphabet

Word / String — A Sequence of symbols taken from Σ

e.g. $\Sigma = \{a, b\}$
 $w = ab \dots w = aaaa$

Operations on the strings / words

1. Finding the length - $\Sigma = \{a, b\}$

$$w = abba$$

length of string $|w| = 4$

Null String / Empty String (λ / ϵ) - A string whose length is '0'

$$|\lambda| = 0$$

2. Concatenation - $\Sigma = \{a, b\}$

$$w_1 = abb$$

$$w_2 = ab$$

$$w_1 w_2 = abbab$$

$$w_2 w_1 = ababb$$

$$w_1 w_2 \neq w_2 w_1$$

When $w_1 = w_2$ then

$$w_1 w_2 = w_2 w_1$$

$$|w_1 w_2| = |w_2 w_1| = |w_1| + |w_2|$$

$$\lambda w = w \lambda = w$$

$$w^0 = \lambda$$

$$w^1 = w$$

$$w^2 = w \cdot w$$

$$w^3 = w \cdot w^2 = w \cdot w \cdot w$$

$$w^n = w \cdot w^{n-1} = w \cdot w \cdot w \cdots (n \text{ times})$$

♦ $|W^n| = n * |W|$

$$[W.W.W \dots W] = |W| + |W| + \dots + |W|$$

$$|W^n| = n * |W|$$

3. Reversal - $W = abbbab$

show

$W^R = \text{Reverse of } W$

$W^R = babbbab$

$$a^R = a$$

$$a^R = a$$

Palindrome

$$|W| = |W^R|$$

$$(xy)^R = y^R x^R$$

♦ Palindrome - $W = aba$

$W^R = aba$

$$W = W^R$$

→ Even Palindrome - (i) $W = W^R$

(ii) $|W|$ is Even

→ Odd Palindrome - (i) $W = W^R$

(ii) $|W|$ is odd

a is an Even Palindrome

a is an Odd Palindrome

K^{th} symbol from beginning = K^{th} symbol from last

e.g. # of Palindromes of length 8 over $\Sigma = \{0, 1\}$

$$\underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{2} \quad | \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$2^4 = 16$$

e.g. # of Palindromes of length 9 over $\Sigma = \{0, 1\}$

$$\underline{2} \quad \underline{2} \quad \underline{2} \quad \underline{2} \quad | \quad \underline{2} \quad | \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$

$$2^5 = 32$$

e.g. # of Palindromes of length n over Σ with

$$|\Sigma| = k$$

$$\boxed{k^{\lceil n/2 \rceil}}$$

Kleen Stars / Kleen Closure = If $\Sigma = \{a, b\}$

Σ^* = The set of all string which can be constructed by using the symbols from Σ including ϵ .

e.g. $\Sigma = \{a, b\}$

$\Sigma^* = \{ \epsilon, \underline{a}, \underline{b}, \underline{aa}, \underline{bb}, \underline{ab}, \underline{ba}, \underline{aaa}, \dots \}$

$\epsilon, bbb, aab, aba, baa, bba, bab, abb, \dots$

\hookrightarrow Universal Language