PLANT DIVERSITY

Taxonomy :

Taxis = arrangement, nomos = law → Taxonomy is the study of principles and procedures of classification.

This word was proposed by **A.P. de. Candolle** in his book "**Theories elementaire de la botanique**" (Theory of elementary botany)

Taxonomy includes study of following points

(1) **Identification :** A process by which an organism is recognised from the others by already known organism and is assigned to a particular taxonomic group is called identification.

(2) **Nomenclature :** Naming of organism according to international scientific rules is called nomenclature.

(3) **Classification :** A process by which any organism is grouped into convenient categories on the basis of some easily observable characters.

TYPES OF TAXONOMY

1. Systematics :-

The term "**Systematics**" was proposed by **Linnaeus**. The word systematics is derived from the latin word "systema" which means systematic arrangement of organisms.

Note : It includes description of external morphological characters of plants or living organisms.

eg. Morphological characters of Root, Stem, Leaves, Flowers

2. New systematics or Neo systematics :-

(1) Neo - systematics - A new branch - Name given by **Julian Huxley** (1940)

(2) It includes description of all external and internal structure, along with the structure of cell, development process and ecological informations of organisms.

(3) It is used to know the inter relationship among living organism.

Note : New systematics is mainly based on evolutionary as well as genetic relationship(experimental taxonomy) as compared to morphological characters.

3. Cytotaxonomy : The use of cytological characters of plants in classification or in solving taxonomic problems is called cytotaxonomy. Cytotaxonomy is based on cytological information like chromosome number, structure and behaviour.

4. Chemotaxonomy : The uses of chemical characters of plants in classification or in solving taxonomic problems is called chemotaxonomy or chemical taxonomy. It is based on the chemical constitution of plants. The fragrance and taste vary from species to species.

The basic chemical compounds used in chemotaxonomy are alkaloids, carotenoids, tannins, polysaccharide, nucleic acids, fatty acids, amino acids, aromatic compounds etc.

Some Informations :

- Practical significance of taxonomy is → Identification of unknown organism.
- Maximum diversity is found in tropical rain forests.
- Second maximum diversity is found in coral reefs
- The number of species that are known and described range between **1.7 – 1.8 million**. This refers to biodiversity or the number and types of organism present on earth.

NOMENCLATURE

Binomial system :

Given by Carolus Linnaeus

Carolus Linnaeus :- Linnaeus used this nomenclature system for the first time on large scale and proposed scientific name of all the plants and animals.

- Linnaeus is the founder of binomial system.
- Linnaeus proposed scientific name of plants in his book **"Species plantarum"**. It was published on 1 May 1753. So this was the initiation of binomial system for plants.So any name proposed (for plants) before this date is not accepted today.
- Linnaeus proposed scientific name of animals in his book "Systema naturae" (10[th] edition).
- This 10[th] edition of **Systema naturae** was published on 1 August 1758. So initiation of binomial system for animals is believed to be started on 1 Aug, 1758.

ICBN

"International Code of Botanical Nomenclature"

- Collection of rules regarding scientific - nomenclature of plants is known as ICBN.
- ICBN was first accepted in **1961**.

Main rules of ICBN :-

(1) According to binomial system name of any species consists of two components or words -

(i) Generic name - Name of genus

(ii) Specific epithet

e.g. ***Solanum tuberosum*** (Potato) ***Mangifera indica*** (Mango)

↓ ↓ ↓ ↓

Generic name Specific epithet Generic name Specific epithet

(2) In plant nomenclature (ICBN) tautonyms are not valid i.e. generic name and specific epithet should not be same in plants.

eg. *Mangifera mangifera*

But tautonyms are valid in animal nomenclature (ICZN-International Code of Zoological Nomenclature)

eg. *Naja naja (Indian cobra) , Rattus rattus (Rat)*

(3) First letter of generic name should be in capital letter and first letter of specific epithet should be in small letter.

eg. *Mangifera indica*

(4) When written with free hand or typed, then generic name and specific epithet should be separately underlined. But during printing name should be in italics to indicate their latin orgin.

(5) Name of scientist (who proposed nomenclature) should be written in short after the specific epithet

eg. *Mangifera indica* Linn.

(6) Name of scientist should be neither underlined nor in italics, but written in Roman letters (simple alphabets)

(7) Scientific names should be derived from **Latin** (usually) or **Greek** languages because they are dead languages.

(8) Type specimen (Herbarium Sheet) of newly discovered plant should be placed in herbarium (Dry garden).

Trinomial system :-

- According to this system name of any organism is composed of three words -

  (i) Generic name (ii) Specific epithet (iii) Subspecific epithet (Name of variety)

  eg. ***Brassica oleracea botrytis*** (Cauliflower)

  Brassica oleracea capitata (Cabbage)

  Brassica oleracea caulorapa (Knol-Khol)

  ↓ Generic name ↓ Specific epithet ↓ Variety

CLASSIFICATION

Biological classification :-

The art of identifying distinctions among organisms and placing them into groups that reflect their most significant features and relationship is called biological classification.

The purpose of biological classification is to organise the vast number of known organisms into categories that could be named, remembered and studied.

Type of Biological classification

(i) Practical classification :- In this type of classification, plants are classified on the basis of their economic importance or **human use**. This classification system is the **earliest system.**

e.g. Oil yielding plants → Coconut, Walnut, Soyabean

Fibre yielding plants → Jute, Cotton

Medicinal plants → *Rauwolfia, Cinchona, Eucalyptus*

Note : In this classification, any one plant can be a member of more than one group.

eg. Turmeric : Multi uses plant, it gives both medicines and spices.

(ii) Artificial classification :- In this type of classification plants are classified on the basis of one or two morphological characters. i.e. over all morphology is not considered.

for e.g. - Classification proposed by Linnaeus is Artificial

Linnaeus classified plant kingdom on the basis **stamen** into 24 classes.

Note : In the book "Genera Plantarum" Linnaeus classified the plant kingdom into 24 classes on the basis of stamen so, Linnaeus classification is also called sexual classification.

Note : Linnaeus divided flowering plants into 23 classes starting with class monandria with a single stamen (eg. *Canna*) and plants with twenty or more stamens attached with calyx were assigned to class **Icosandria**. He also included all non-flowering plants such as algae, fungi, mosses and ferns in a separate class called cryptogamia.

(iii) Natural classification :- In this type, plants are classified on the basis of their **complete morphological** characters of stem, root, leaves, flowers etc.

Importance –

Natural classification is believed to be the **best classification**, because it represents the natural similarities and dissimilarities of plants i.e. it represents the interrelationship among plants.

In this classification, the plants belonging to the same group shows many similarities, while in artificial classification, the plants belonging to the same group shows only, 1 or 2 similar characters. They have many dissimilarities.

Natural classification is of two types

(a) Natural formal (b) Natural phylogenetic

(a) Natural formal → In this classification, the phylogeny of the plant is not considered i.e. only the morphology of the plant is considered.

(b) Natural phylogenetic → In this classification, both morphology and phylogeny are considered. In phylogenetic classification, the plants are arranged on the basis of their evolution.

Lamarck	—	Proposed the term phylogeny.
Charles Darwin	—	Gave detailed explanation of phylogeny in his book "Origin of species" (1859). It was most popular book of it's time.

Thallophyta → Bryophyta → Pteridophyta → Gymnosperm → Angiosperm (most advanced plants)
Note : Phylogenetic classification also known as cladistic classification

(iv) Adansonian system or phenetic classification or Numerical classification :-

In it plants are classified on the basis of numbers of similarities and dissimilarities. This classification is easily carried out by using computers and it is based on all observable characteristics. In this classfiiciation number and codes are assigned to all the characters and the data are prepared and then processed. Those organism which have maximum similarities are placed in same group. In this way each character is given equal importance and at the same time hundreds of characters can be considered.

Note :

In this classification importance to any one character is not given , all characters have same importance. While in natural classification floral (reproductive) characters have more importance than vegetative (root, stem and leaves) characters.

TAXONOMIC CATEGORIES

Classification is not a single step process but involves heirarchy of steps in which each step represent a rank or category since the category is a part of overall taxonomic arrangement, it is called the taxonomic category and all categories together constitute the taxonomic hierarchy. Each category referred to as a unit of classification, infact, represents a rank and is commonly termed as taxon (Pl....taxa)

Species : Taxonomic studies consider a group of indiviual organism with fundamental similarities as a species. One should be able to distinguish one species from the other closely related species based on the distinct morphological differences.

Genus : Genus comperises a group of related spcies which has more characters in common in comparison to species of other genera.

For example : Potato and brinjal are two different species but both belong to the genus ***Solanum***.

Family : Family has a group of related genera with still less number of similarities as compared to genus and species. Families are characterised on the basis of both vegetative and reproductive features of plant species but reproductive or sexual or floral characters are used mainly.

For example : Three different genera **Solanum**, **Petunia** and **Datura** are included in family solanaceae.

Order : Order being a higher category is the assemblage of families which exhibit a few similar character.

For example : Plant families like **Convolvulaceae**, **Solanaceae** are included in the order **Polymoniales mainly based on the vegetative and floral or reproductive or sexual characters.**

Class : Class includes organism of related orders having less similarities than orders.

Division : Division includes all organisms belonging to different classes having a few common characters.

Biggest group → Kingdom
- Division /Phylum
- Class
- Order
- Family
- Genus
- Species

There are **7 main** taxonomic categories.They are **obligate or essential or broad categories** i.e. they are strictly used at the time of any plant classification.

There are some extra or sub categories, like sub division, sub order, sub family, etc. They are used only when they are needed.

Classification of Mango :-

Categories		Taxon(Pl.-Taxa)
Kingdom	—	Plantae
Division	—	Angiospermae
Class	—	Dicotyledonae
Order	—	Sapindales
Family	—	Anacardiaceae
Genus	—	Mangifera
Species	—	*Mangifera indica*

- The classification of any plant or animal is written in **descending** or ascending order.
- Hierarchy - **Descending** or ascending arrangement of taxonomic categories is known as hierarchy.
- **Species** :- Smallest taxonomic category → It is basic unit of classification

Note :As we go higher from species to kingdom, number of common characters decreases. Lower the taxa more are the characterstics that the members with in the taxon share. Higher the category, greater is the difficulty of determining the relationship to other taxa at the same level.

Suffix for taxa (Taxon)

Division	—	phyta
Sub div	—	phytina
Class	—	opsida, phyceae,ae
Order	—	ales
Family	—	aceae

Note : There is no suffix for Genus, Species and Kingdom

TAXONOMICAL AIDS

Biologists have established certain procedures and techniques to store and preserve the information as well as the specimens. some of these are explained to help you understand the usage of these aids.

(1) Herbarium :- Herbarium is a store house of collected plant specimens that are dried, pressed and preserved on sheet. Standard size of herbarium sheet is **11.5 ×16.5** inches. Herbarium is a store house of collected plant specimens that are dried, pressed and preserved on sheets. Further, these sheets are arranged according to a universally accepted system of classification. These specimens, along with their descriptions on herbarium sheets, become a store house or repository for future use. The herbarium sheets also carry a label providing information about date and place of collection, English, local and botanical names, family, collector's name, etc. Herbaria also serve as quick referral systems in taxonomical studies.

(2) Botanical Gardens :- These specialised gardens have collections of living plants for reference. Plant species in these gardens are grown for identification purposes and each plant is labelled indicating its botanical/ scientific name and its family. The famous botanical gardens are at **Kew (England), Indian Botanical Garden, Howrah (India) and at National Botanical Research Institute, Lucknow (India).**

(3) Museum :- Biological museums are generally set up in educational institutes such as schools and colleges. Museums have collections of preserved plant and animal specimens for study and reference. Specimens are preserved in the containers or jars in preservative solutions. Plant and animal specimens may also be preserved as dry specimens. Insects are preserved in insect boxes after collecting, killing and pinning. Larger animals like birds and mammals are usually stuffed and preserved. Museums often have collections of skeletons of animals too.

(4) Zoological Parks :- These are the places where wild animals are kept in protected environments under human care and which enable us to learn about their food habits and behaviour. All animals in a zoo are provided, as far as possible, the conditions similar to their natural habitats. Children love visiting these parks, commonly called Zoos.

(5) Key :- Key is used for identification of plants and animals based on the similarities and dissimilarities

- The keys are based on the contrasting characters generally in a pair called couplet. It represents the choice made between the two opposite options. This result in acceptance of only one and rejection of the other.
- Separate taxonomic keys are required for each taxonomic category such as family, genus and species for identification purpose.
- Each statement of couplet in the key is called a lead.
- Keys are generally analytical in nature.

Catalogue - It is a small booklet which gives account for books related to botanical titles, full name of authors and their publication.

Flora - It contains the actual account of habitat and distribution of plants of a given area.

Manuals - They are useful in providing information for identification of names of species found in an area.

Monographs - They contain information on any one taxon.

ICNB = International Code of Nomenclature for Bacteria

SPECIES CONCEPT

John Ray :- Proposed the **term** and **concept of species.** He described more than 18 thousands plants and animals in his book Historia generalis plantarum.

Biological concept of species :-

(1) **Ernst Mayr** (Darwin of 20th century) proposed the **biological concept of species**.

(2) **Mayr** defined the "species" in the form of biological concept.

(3) According to Mayr "All the members that can interbreed among themself and can produce fertile offsprings are the members of same species"

But this definition of Mayr was incomplete because this definition is applicable to sexually reproducing living beings because there are many organisms that have only asexual mode of reproduction.

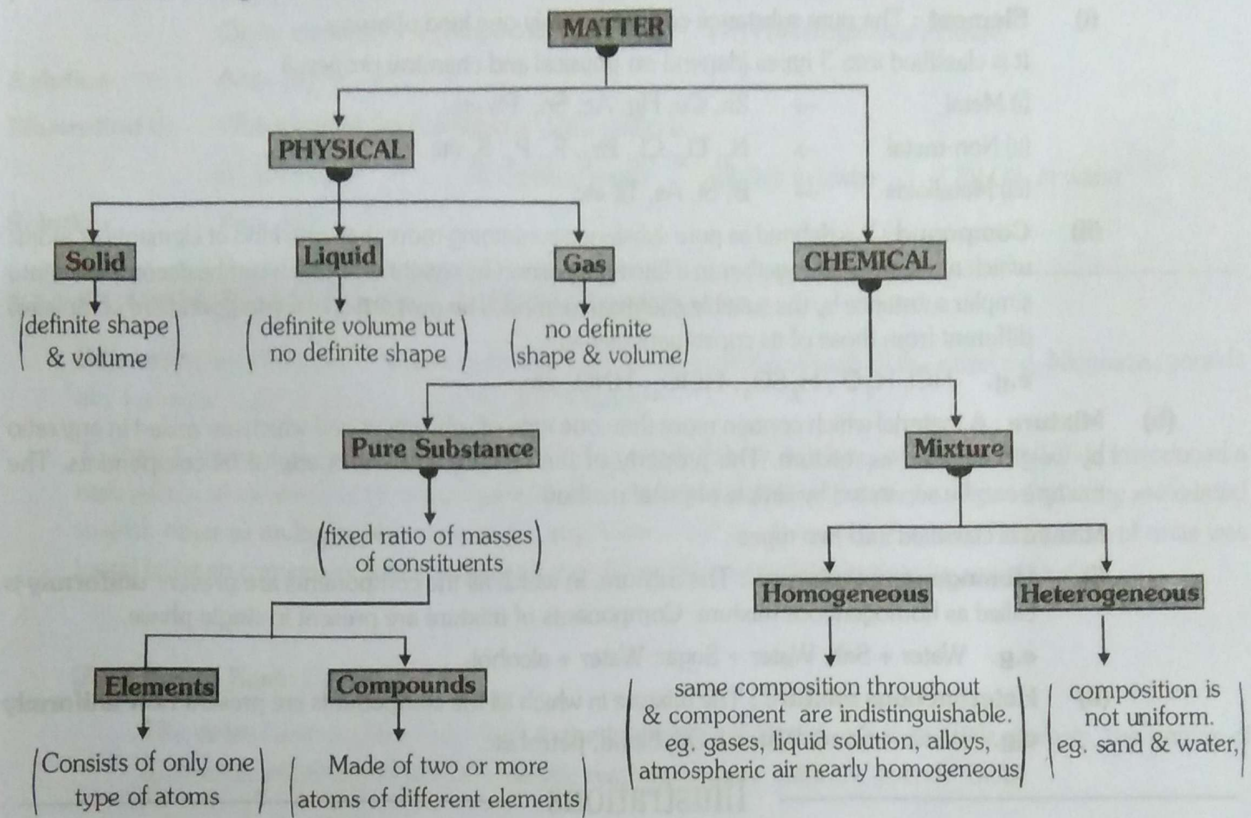
eg. Bacteria, Mycoplasma, BGA

(4) The main character in determination of any species is **interbreeding**. But this character is not used in taxonomy. **In taxonomy**, the determination of species is **mainly based on morphological characters.**

SOME BASIC CONCEPTS OF CHEMISTRY

1.0 INTRODUCTION

Chemistry deals with the composition, structure and properties of matter. These aspects can be best described and understood in terms of basic constituents of matter: **atoms** and **molecules**. That is why chemistry is called the science of atoms and molecules. Can we see, weight and perceive these entities? Is it possible to count the number of atoms and molecules in a given mass of matter and have a quantitative relationship between the mass and number of these particles (atoms and molecules)? We will like to answer some of these questions in this Unit. We would further describe how physical properties of matter can be quantitatively described using numerical values with suitable units.



Classification of universe

Universe is classified into two types, matter and energy.

(A) MATTER : The thing which occupy space and having mass which can be felt by our five senses is called matter.

Matter is further classified into two categories :

(I) Physical classification (II) Chemical classification

PHYSICAL CLASSIFICATION

It is based on physical state under ordinary conditions of temperature and pressure, so on the basis of two nature of forces matter can be classified into the following three ways :

(a) Solid (b) Liquid (c) Gas

(a) Solid : A substance is said to be solid if it possesses a definite volume and a definite shape.

e.g. sugar, iron, gold, wood etc.

(b) Liquid : A substance is said to be liquid if it possesses a definite volume but not definite shape. They take up the shape of the vessel in which they are put.

e.g. water, milk, oil, mercury, alcohol etc.

(c) Gas : A substance is said to be gas if it neither possesses a definite volume nor a definite shape. This is because they fill up the whole vessel in which they are put.

e.g. hydrogen(H_2), oxygen(O_2), carbon dioxide(CO_2) etc.

Chemical Classification

It may be classified into two types :

(a) Pure Substance (b) Mixture

(a) Pure Substance : A material containing only one type of substance. Pure Substance can not be separated into simpler substance by physical method.

e.g. : Element = Na, Mg, Ca .... etc.

Compound = HCl, H_2O, CO_2, HNO_3 .... etc.

Pure substance is classified into two types :

(a) Element (b) Compound

(i) Element : The pure substance containing only one kind of atoms.

It is classified into 3 types (depend on physical and chemical property)

(i) Metal → Zn, Cu, Hg, Ac, Sn, Pb etc.

(ii) Non-metal → N_2, O_2, Cl_2, Br_2, F_2, P_4, S_8 etc.

(iii) Metalloids → B, Si, As, Te etc.

(ii) Compound : It is defined as pure substance containing more than one kind of elements or atoms which are combined together in a fixed proportion by weight and which can be decomposed into simpler substance by the suitable chemical method. The properties of a compound are completely different from those of its constituent element.

e.g. HCl, H_2O , H_2SO_4, $HClO_4$, HNO_3 etc.

(b) Mixture : A material which contain more than one type of substances and which are mixed in any ratio by weight is called as mixture. The property of the mixture is the property of its components. The mixture can be separated by simple physical method.

Mixture is classified into two types :

(i) Homogeneous mixture : The mixture, in which all the components are present **uniformly** is called as homogeneous mixture. Components of mixture are present in single phase.

e.g. Water + Salt, Water + Sugar, Water + alcohol,

(ii) Heterogenous mixture : The mixture in which all the components are present **non-uniformly**

e.g. Water + Sand, Water + Oil, blood, petrol etc.

Illustrations

Illustration 1. Which is an example of matter according to physical state at room temperature and pressure.

(1) solid (2) liquid (3) gas (4) all of these

Solution **Ans. (4)** According to the physical state at room temperature and pressure, the matter is present in 3 state solid, liquid & gas

Illustration 2. What are the types of the compound.

(1) Organic compound (2) Inorganic compound

(3) Both (1) and (2) (4) None of these

Solution **Ans. (3)** Compound is divided in 2 types. Inorganic compound & Organic compound

Illustration 3. Which of the following example of a Homogeneous mixture.

(1) Water + Alcohol (2) Water + Sand (3) Water + Oil (4) None of these

Solution **Ans. (1)** Water and alcohol are completely mixed and form uniform solution.

Illustration 4. Which mixture is called as solution.

(1) Heterogeneous mixture (2) Homogeneous mixture

(3) Both (1) and (2) (4) None of these

Solution **Ans. (2)** Homogeneous mixture is called as solution.

Illustration 5. Which of the following is a compound

(1) graphite (2) producer gas (3) cement (4) marble

Solution **Ans. (4)** Marble = $CaCO_3$ = compound.

Illustration 6. Which of the following statements is/are true :

(1) An element of a substance contains only one kind of atoms.

(2) A compound can be decomposed into its components.

(3) All homogeneous mixtures are called as solutions.

(4) All of these

Solution **Ans. (4)**

Illustration 7. A pure substance can only be :-

(1) A compound (2) An element

(3) An element or a compound (4) A heterogenous mixture

Solution **Ans. (3)**

Illustration 8. Which one of the following is not a mixture :

(1) Tap water (2) Distilled water (3) Salt in water (4) Oil in water

Solution **Ans. (2)**

1.1 S.I. UNITS (INTERNATIONAL SYSTEM OF UNITS)

Different types of units of measurements have been in use in different parts of the world e.g. kilograms, pounds etc. for mass ; miles, furlongs, yards etc. for distance.

To have a common system of units throughout the world. French Academy of Science, in 1791, introduced a new system of measurements called metric system in which the different units of a physical quantity are related to each other as multiples of powers of 10, e.g. 1 km = 10^3 m, 1cm = 10^{-2} m etc. This system of units was found to be so convenient that scientists all over the world adopted this system for scientific data.

(A) Seven Basic Units

The seven basic physical quantities in the International System of Units, their symbols, the names of their units (called the base units) and the symbols of these units are given in Table.

TABLE : SEVEN BASIC PHYSICAL QUANTITIES AND THEIR S.I. UNITS

Physical Quantity	Symbol	S.I. Unit	Symbol
Length	ℓ	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Electric current	I	ampere	A
Thermodynamic temperature	T	kelvin	K
Luminous intensity	I_u	candela	cd
Amount of the substance	n	mole	mol

(B) Prefixes Used With Units

The S.I. system recommends the multiples such as 10^3, 10^6, 10^9 etc. and fraction such as 10^{-3}, 10^{-6}, 10^{-9} etc., i.e. the powers are the multiples of 3. These are indicated by special prefixes. These along with some other fractions or multiples in common use, along with their prefixes are given below in Table and illustrated for length (m)

TABLE : SOME COMMONLY USED PREFIXES WITH THE BASE UNITS.

Prefix	Symbol	Multiplication Factor	Example
deci	d	10^{-1}	1 decimetre (dm) = 10^{-1} m
centi	c	10^{-2}	1 centimetre (cm) = 10^{-2} m
milli	m	10^{-3}	1 millimetre (mm) = 10^{-3} m
micro	μ	10^{-6}	1 micrometre (μm) = 10^{-6} m
nano	n	10^{-9}	1 nanometre (nm) = 10^{-9} m
pico	p	10^{-12}	1 picometre (pm) = 10^{-12} m
femto	f	10^{-15}	1 femtometre (fm) = 10^{-15} m
atto	a	10^{-18}	1 attometre (am) = 10^{-18} m
deka	da	10^{1}	1 dekametre(dam) = 10^{1} m
hecto	h	10^{2}	1 hectometre (hm) = 10^{2} m
kilo	k	10^{3}	1 kilometre (km) = 10^{3} m
mega	M	10^{6}	1 megamerte(Mm) = 10^{6} m
giga	G	10^{9}	1 gigametre (Gm) = 10^{9} m
tera	T	10^{12}	1 teramerte (Tm) = 10^{12} m
peta	P	10^{15}	1 petametre (Pm) = 10^{15} m
exa	E	10^{18}	1 exametre (Em) = 10^{18} m

As volume is very often expressed in litres, it is important to note that the equivalence in S.I. units for volume is as under: **1 litre (1L) = 1 dm³ = 1000 cm³**

and **1 millilitre (1mL) = 1 cm³ = 1cc**

(C) SOME IMPORTANT UNIT CONVERSIONS

1. Length :
1 mile = 1760 yards
1 yard = 3 feet
1 foot = 12 inches
1 inch = 2.54 cm
1Å = 10^{-10}m or 10^{-8} cm

2. Mass :
1 Ton = 1000 kg
1 Quintal = 100 kg
1 kg = 2.205 Pounds (lb)
1 kg = 1000 g
1 gram = 1000 milli gram
1 a.m.u. = 1.67×10^{-24} g

3. Volume :
1 L = 1 dm³ = 10^{-3} m³ = 10^{3} cm³ = 10^{3} mL = 10^{3} cc
1 mL = 1 cm³ = 10^{-6} m³
= 1 cc

4. Energy :
1 calorie = 4.184 joules $\simeq$ 4.2 joules
1 joule = 10^{7} ergs
1 litre atmosphere (L-atm) = 101.3 joule
1 electron volt (eV) = 1.602×10^{-19} joule

5. Pressure : 1 atmosphere (atm) = 760 torr
= 760 mm of Hg
= 76 cm of Hg
= 1.01325×10^{5} pascal (Pa)
= 1.01325×10^{5} N/m²

Some More Prefixes :

Semi	=	$\frac{1}{2}$	Mono	=	1
Sesqui	=	$\frac{3}{2} = 1.5$	Di or Bi	=	2
Tri	=	3	Tetra	=	4
Penta	=	5	Hexa	=	6
Hepta	=	7	Octa	=	8
Nona	=	9	Deca	=	10
Undeca	=	11	Do deca	=	12
Trideca	=	13	Tetra deca	=	14
Pentadeca	=	15	Hexa deca	=	16
Hepta deca	=	17	Octa deca	=	18
Nonadeca	=	19	Eicoso/Icoso	=	20

GOLDEN KEY POINTS

- The unit named after a scientist is started with a small letter and not with a capital letter e.g. unit of force is written as newton and not as Newton.
  Likewise unit of heat and work is written as joule and not as Joule.
- Symbols of the units do not have a plural ending like 's'. For example we have 10 cm and not 10 cms.
- Words and symbols should not be mixed e.g. we should write either joules per mole or J mol^{-1} and not joules mol^{-1}
- Prefixes are used with the basic units e.g. kilometer means 1000 m (because meter is the basic unit).
  Exception. Though kilogram is the basic unit of mass, yet prefixes are used with gram because in kilogram, kilo is already a prefix.
- A unit written with a prefix and a power is a power for the complete unit e.g. cm^3 means $(centimeter)^3$ and not centi $(meter)^3$.

Illustrations

Illustration 9. Which one of the following forms part of seven basic SI units :

(1) Joule (2) Candela (3) Newton (4) Pascal

Solution **Ans. (2)**

Illustration 10 Convert 2 litre atmosphere into erg.

Solution 2 litre atmosphere = 2×101.3 joule = $2\times101.3\times10^7$ erg. = 202.6×10^7 erg.

{1 litre atmosphere = 101.3J

Illustration 11 Convert 2 atm into cm of Hg.

Solution 2 atm = 2×76 cm of Hg = 152 cm of Hg

{1 atmosphere = 76 cm of Hg

Illustration 12 Convert 20 dm^3 into mL.

Solution 20 dm^3 = 20 litre = 20×1000 mL = 2×10^4 mL

1 dm^3 = 1 litre = 1000 mL

Illustration 13 Convert 59 F into °C.

Solution $°C = \frac{5}{9}(F - 32) = \frac{5}{9}(59 - 32) = \frac{5}{9} \times 27 = 15°C$

1.2 MOLE CONCEPT

In SI Units we represent mole by the symbol 'mol'. It is defined as follows :

(i) **A mole is the amount of a substance that contains as many entities (atoms, molecules or other particles) as there are atoms in exactly 12g of the carbon - 12 isotope.**

It may be emphasised that the mole of a substance always contains the same number of entities, no matter what the substance may be. In order to determine this number precisely, the mass of a carbon-12 atom was determined by a mass spectrometer and found to be equal to 1.992648×10^{-23}g Knowing that 1 mole of carbon weighs 12g, the number of atoms in it is equal to :

$$\frac{12\text{g/mol } C^{12}}{1.992648 \times 10^{-23}\text{g}/\ C^{12}\text{ atom}} = 6.0221367 \times 10^{23}\text{ atoms/mol}$$

(ii) **In a simple way, we can say that mole has 6.0221367×10^{23} entities (atom, molecules or ions etc.)**

The number of entities in 1 mol is so important that it is given a separate name and symbol, known as '**Avogadro constant**' denoted by N_A.

Here entities may represent atoms, ions, molecules or other subatomic entities. Chemists count the number of atoms and molecules by weighing. In a reaction we require these particles (atoms, molecules and ions) in a definite ratio. We make use of this relationship between numbers and masses of the particles for determining the stoichiometry of reactions .

Formula to get moles are following :

(i) $\boxed{\text{Number of moles (n)} = \frac{\text{weight(g)}}{\text{molar mass}}}$

Where molar mass = gram atomic mass or gram molecular mass or gram ionic mass

(ii) $\boxed{\text{Number of moles (n)} = \frac{V_{(L)}}{22.4}}$ (Where V = Volume of gas in L at NTP or STP)

(iv) $\boxed{\text{Number of moles (n)} = \frac{N}{N_A}}$ (Where N = Number of particles)

$$\text{mole atoms} = \frac{\text{number of atoms}}{N_A} \quad \text{and} \quad \text{mole molecules} = \frac{\text{number of molecules}}{N_A}$$

SOME RELATED DEFINITIONS :

Atomic Mass (Relative Atomic Mass)

It is defined as the number which indicates how many times the mass of one atom of an element is heavier in comparison to $\frac{1}{12}$th part of the mass of one atom of C-12.

Atomic mass unit (a.m.u.) : The quantity $\frac{1}{12}$th mass of an atom of C^{12} is known as atomic mass unit.

Since mass of 1 atom of C - 12 = 1.9924×10^{-23} g

$$\therefore \frac{1}{12}\text{th part of the mass of 1 atom} = \frac{1.9924 \times 10^{-23}\text{g}}{12} = 1.67 \times 10^{-24}\text{ g} = 1\text{ a.m.u.} = \frac{1}{6.023 \times 10^{23}}$$

It may be noted that the atomic masses as obtained above are the relative atomic masses and not the actual masses of the atoms. These masses on the atomic mass scale are expressed in terms of atomic mass units (abbreviated as amu). Today, 'amu' has been replaced by 'u' which is known as unified mass.

One atomic mass unit (amu) is equal to $\frac{1}{12}$th of the mass of an atom of carbon - 12 isotope.

Thus the atomic mass of hydrogen is 1.008 amu while that of oxygen is 15.9994 amu (or taken as 16 amu).

Gram Atomic Mass (or Mass of 1 Gram Atom)

When numerical value of atomic mass of an element is expressed in grams then the value becomes gram atomic mass.

gram atomic mass = mass of 1 **gram atom** = mass of 1 **mole atom**

= mass of N_A atoms = mass of 6.023×10^{23} atoms.

Ex. gram atomic mass of oxygen = mass of 1 **g atom** of oxygen = mass of 1 **mol atom** of oxygen.

= mass of N_A atoms of oxygen. $= \left(\frac{16}{N_A} g\right) \times N_A = 16$ g

Molecular Mass (Relative Molecular Mass)

The number which indicates how many times the mass of one molecule of a substance is heavier in comparison to $\frac{1}{12}$ th part of the mass of an atom of C–12.

Gram Molecular Mass (Mass of 1 Gram Molecule)

When numerical value of molecular mass of the substance is expressed in grams then the value becomes gram molecular mass.

gram molecular mass = mass of 1 **gram molecule** = mass of 1 **mole molecule**

= mass of N_A molecules = mass of 6.023×10^{23} molecules

Ex. gram molecular mass of H_2SO_4 = mass of 1 **gram molecule** of H_2SO_4

= mass of 1 **mole molecule** of H_2SO_4

= mass of N_A molecules of H_2SO_4

$$= \left(\frac{98}{N_A} g\right) \times N_A = 98 \text{ g}$$

Actual Mass

The mass of one atom or one molecule of a substance is called as actual mass.

Ex. (i) Actual mass of O_2 = 32 amu = $32 \times 1.67 \times 10^{-24}$ g → Actual mass

(ii) Actual mass of H_2O = (2 + 16) amu = $18 \times 1.67 \times 10^{-24}$g = 2.99×10^{-23} g

Atomicity – Total number of atoms in **a molecule** of elementary substance is called as atomicity.

Ex.

Molecule	Atomicity
H_2	2
O_2	2
O_3	3
NH_3	4

Illustrations

Illustration 14. Find out the volume and mole in 56 g nitrogen at STP

Solution Molecular weight of N_2 is 28 g

(a) Calculation of volume : $\because$ 28 g of N_2 occupies = 22.4 litre at STP

$\therefore$ 56 g of N_2 occupies $= \frac{22.4}{28} \times 56$ litre = 44.8 litre at STP

(b) Calculation of mole : $\because$ 28 g of N_2 = 1 mol of N_2

$\therefore$ 56 g of $N_2 = \frac{1}{28} \times 56$ = 2 mol of N_2

Illustration 15. Calculate the volume and mass of 0.2 mol of O_3 at STP.

Solution

(a) Calculation of volume : $\because$ volume of 1 mole of O_3 at STP = 22.4 litre

$\therefore$ volume of 0.2 mole of O_3 at STP = 22.4 × 0.2 = 4.48 litre

(b) Calculation of mass : $\because$ mass of 1 mol of O_3 = 48 g

$\therefore$ mass of 0.2 mol of O_3 = 48 × 0.2 gm = 9.6 g

Illustration 16. Find out the moles & mass in 1.12 litre O_3 at STP.

Solution

(a) Calculation of mole: $\because$ at STP 22.4 litre of O_3 contain = 1 mol of O_3

$\therefore$ at STP 1.12 litre of O_3 contain $= \frac{1}{22.4} \times 1.12$

= 0.05 mol of O_3

(b) Calculation of mass : Molecular weight of O_3 = 48 g

$\because$ weight of 22.4 litre of O_3 at STP is = 48 g

$\therefore$ weight of 1.12 litre of O_3 at STP is $= \frac{48}{22.4} \times 1.12 = 2.4$ g

Illustration 17. Find out the mass of 10^{21} molecules of Cu.

Solution For Cu (i.e. mono atomic substance) number of atoms = number of molecules

$$\text{Number of moles of Cu} = \frac{N}{N_A} = \frac{10^{21}}{6.023 \times 10^{23}} = \frac{\text{weight}}{\text{Atomic weight}} = \frac{\text{weight}}{63.5}$$

$$\text{weight of Cu} = \frac{10^{21}}{6.023 \times 10^{23}} \times 63.5 = 0.106 \text{ g}$$

Illustration 18. Calculate the number of molecules and number of atoms present in 1 g of nitrogen ?

Solution Number of moles $(n) = \frac{\text{weight}}{M_w} = \frac{1}{28} \Rightarrow$ Number of molecules $(N) = \frac{N_A}{28}$

$\because$ 1 molecule of N_2 gas contain = 2 atoms

$\therefore$ $\frac{N_A}{28}$ molecules of N_2 gas contain $= 2 \times \frac{N_A}{28} = \frac{N_A}{14}$ atoms

Illustration 19. Calculate the number of moles in 11.2 litre at STP of oxygen.

Solution Number of moles of O_2 $(n) = \frac{V}{22.4} = \frac{11.2}{22.4} = 0.5$ mol

Illustration 20. $\frac{1}{2}$ **g molecule** of oxygen. Find (i) mass, (ii) number of molecules, (iii) volume at STP (iv) No. of oxygen atoms.

Solution

(i) $n = \frac{1}{2}\text{ mol} = \frac{\text{weight}}{M_w} = \frac{\text{weight}}{32}$ $\Rightarrow$ weight of oxygen = 16 g

(ii) $n = \frac{1}{2}\text{ mol} = \frac{N}{N_A}$ $\Rightarrow$ Number of molecules of oxygen (N) = $\frac{N_A}{2}$

(iii) $n = \frac{1}{2}\text{ mol} = \frac{V}{22.4}$ $\Rightarrow$ V = 11.2 litre

(iv) 1 molecule of O_2 contain = 2 oxygen atoms.

$\frac{N_A}{2}$ molecules of O_2 contain = $\frac{N_A}{2} \times 2 = N_A$ oxygen atoms.

BEGINNER'S BOX-1

1. The modern atomic weight scale is based on.
(1) C^{12} (2) O^{16} (3) H^1 (4) C^{13}

2. Gram atomic weight of oxygen is
(1) 16 amu (2) 16 g (3) 32 amu (4) 32 g

3. Molecular weight of SO_2 is :
(1) 64 g (2) 64 amu (3) 32 g (4) 32 amu

4. 1 amu is equal to :-
(1) $\frac{1}{12}$ of C–12 (2) $\frac{1}{14}$ of O–16 (3) 1 g of H_2 (4) 1.66×10^{-24} kg

5. The actual molecular mass of chlorine is :
(1) 58.93×10^{-24} g (2) 117.86×10^{-24} g (3) 58.93×10^{-24} kg (4) 117.86×10^{-24} kg

RELATION BETWEEN MOLECULAR WEIGHT AND VAPOUR DENSITY :

Vapour density (V.D) : Vapour density of a gas is the ratio of densities of gas & hydrogen at the same temperature & pressure.

$$\text{Vapour Density (V.D)} = \frac{\text{Density of gas}}{\text{Density of hydrogen}} = \frac{d_{gas}}{d_{H_2}} \qquad \left\{ d = \frac{m(\text{mass})(g)}{V(\text{Volume})(mL)} \right.$$

$$\text{V.D} = \frac{(m_{gas})\text{for certain V litre volume}}{(m_{H_2})\text{for certain V litre volume}}$$

If N molecules are present in the given volume of a gas and hydrogen under similar condition of temperature and pressure.

$$\text{V.D.} = \frac{(m_{gas})\text{ of N molecules}}{(m_{H_2})\text{of N molecules}} = \frac{(m_{gas})\text{ of 1 molecule}}{(m_{H_2})\text{of 1 molecule}} = \frac{\text{Molecular mass of gas}}{2}$$

$$\therefore \quad \boxed{\textbf{Molecular mass of gas } (M_w) = 2 \times \text{V.D}}$$

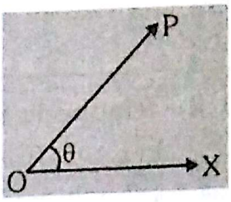
BASIC MATHEMATICS USED IN PHYSICS

Mathematics is the supporting tool of Physics. Elementary knowledge of basic mathematics is useful in problem solving in Physics. In this chapter we study *Elementary Algebra, Trigonometry, Coordinate Geometry* and *Calculus (differentiation and integration).*

1. TRIGONOMETRY

1.1 Angle

Consider a revolving line OP.



Suppose that it revolves in anticlockwise direction starting from its initial position OX.

The angle is defined as the amount of revolution that the revolving line makes with its initial position.

From fig. the angle covered by the revolving line OP is $\theta = \angle POX$

The angle

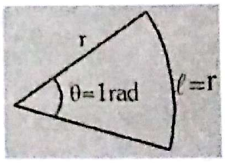
is taken **positive** if it is traced by the revolving line in anticlockwise direction and

is taken **negative** if it is covered in clockwise direction.

$$1^\circ = 60' \text{ (minute)}$$
$$1' = 60'' \text{ (second)}$$

1 right angle = 90° (degrees) also 1 right angle = $\frac{\pi}{2}$ rad (radian)

One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle. 1 rad = $\frac{180^\circ}{\pi} \approx 57.3^\circ$



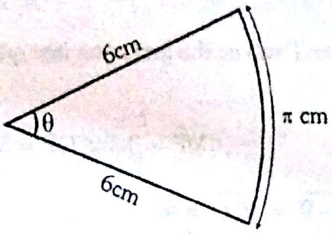
To convert an angle from degree to radian multiply it by $\frac{\pi}{180^\circ}$

To convert an angle from radian to degree multiply it by $\frac{180^\circ}{\pi}$

Illustrations

Illustration 1.

A circular arc is of length π cm. Find angle subtended by it at the centre in radian and degree.



Solution

$$\theta = \frac{s}{r} = \frac{\pi \text{ cm}}{6 \text{ cm}} = \frac{\pi}{6}\text{rad} = 30^\circ \quad \text{As } 1 \text{ rad} = \frac{180^\circ}{\pi} \quad \text{So } \theta = \frac{\pi}{6} \times \frac{180^\circ}{\pi} = 30^\circ$$

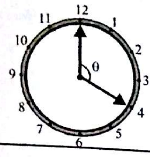
Illustration 2.

When a clock shows 4 o'clock, how much angle do its minute and hour needles make?

(1) 120° (2) $\frac{\pi}{3}$ rad (3) $\frac{2\pi}{3}$ rad (4) 160°

Solution

Ans. (1,3)



From diagram angle $\theta = 4 \times 30° = 120° = \frac{2\pi}{3}$ rad

1.2 Trigonometrical ratios (or T ratios)

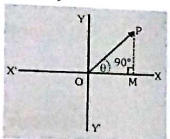
Let two fixed lines XOX' and YOY' intersect at right angles to each other at point O Then,

(i) Point O is called origin.

(ii) XOX' is known as X-axis and YOY' as Y-axis.

(iii) Portions XOY, YOX', X'OY' and Y'OX are called I, II, III and IV quadrant respectively

Consider that the revolving line OP has traced out angle θ (in I quadrant) in anticlockwise direction From P, draw perpendicular PM on OX. Then, side OP (in front of right angle) is called hypotenuse,side MP (in front of angle θ) is called **opposite side or perpendicular** and side OM (making angle θ with hypotenuse) is called **adjacent side or base.**



The three sides of a right angled triangle are connected to each other through six different ratios, called trigonometric ratios or simply T-ratios :

$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP}$ $\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP}$

$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM}$ $\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM}$ $\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$

It can be easily proved that :

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

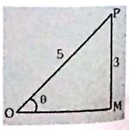
$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \text{cosec}^2\theta$

Illustrations

Illustration 3.

Given sin θ = 3/5. Find all the other T-ratios, if θ lies in the first quadrant

Solution



In Δ OMP, $\sin\theta = \frac{3}{5}$ so MP = 3 and OP = 5

$\because$ OM = $\sqrt{(5)^2 - (3)^2} = \sqrt{25-9} = \sqrt{16} = 4$

Now, $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$ $\tan\theta = \frac{MP}{OM} = \frac{3}{4}$

$\cot\theta = \frac{OM}{MP} = \frac{4}{3}$ $\sec\theta = \frac{OP}{OM} = \frac{5}{4}$ $\text{cosec}\,\theta = \frac{OP}{MP} = \frac{5}{3}$

Table : The T-ratios of a few standard angles ranging from 0° to 180°

Angle (θ)	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	−1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	−1	$-\frac{1}{\sqrt{3}}$	0

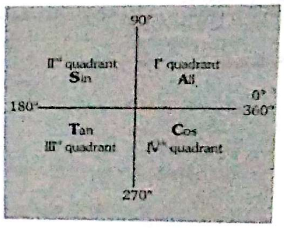
1.3 Four Quadrants and ASTC Rule*

In first quadrant, all trigonometric ratios are positive

In second quadrant, only sinθ and cosecθ are positive

In third quadrant, only tanθ and cotθ are positive.

In fourth quadrant, only cosθ and secθ are positive



* Remember as Add Sugar To Coffee or After School To College.

1.4 Trigonometrical Ratios of General Angles (Reduction Formulae)

(i) Trigonometric function of an angle (2nπ + θ) where n=0, 1, 2, 3, ... will be remain same.

$\sin(2n\pi + \theta) = \sin\theta$ $\cos(2n\pi + \theta) = \cos\theta$ $\tan(2n\pi + \theta) = \tan\theta$

(ii) Trigonometric function of an angle $\left(\frac{n\pi}{2} + \theta\right)$ will remain same if n is even and sign of trigonometric function will be according to value of that function in quadrant

$\sin(\pi - \theta) = +\sin\theta$ $\cos(\pi - \theta) = -\cos\theta$ $\tan(\pi - \theta) = -\tan\theta$

$\sin(\pi + \theta) = -\sin\theta$ $\cos(\pi + \theta) = -\cos\theta$ $\tan(\pi + \theta) = +\tan\theta$

$\sin(2\pi - \theta) = -\sin\theta$ $\cos(2\pi - \theta) = +\cos\theta$ $\tan(2\pi - \theta) = -\tan\theta$

(iii) Trigonometric function of an angle $\left(\frac{n\pi}{2} + \theta\right)$ will be changed into co-function if n is odd and sign of trigonometric function will be according to value of that function in quadrant.

$\sin\left(\frac{\pi}{2} + \theta\right) = +\cos\theta$ $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$ $\tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta$

$\sin\left(\frac{\pi}{2} - \theta\right) = +\cos\theta$ $\cos\left(\frac{\pi}{2} - \theta\right) = +\sin\theta$ $\tan\left(\frac{\pi}{2} - \theta\right) = +\cot\theta$

(iv) Trigonometric function of an angle −θ (negative angles)

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = +\cos\theta$ $\tan(-\theta) = -\tan\theta$

$\sin(90°+\theta) = \cos\theta$ $\cos(90°+\theta) = -\sin\theta$ $\tan(90°+\theta) = -\cot\theta$	$\sin(180°-\theta) = \sin\theta$ $\cos(180°-\theta) = -\cos\theta$ $\tan(180°-\theta) = -\tan\theta$	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$	$\sin(90°-\theta) = \cos\theta$ $\cos(90°-\theta) = \sin\theta$ $\tan(90°-\theta) = \cot\theta$
$\sin(180°+\theta) = -\sin\theta$ $\cos(180°+\theta) = -\cos\theta$ $\tan(180°+\theta) = \tan\theta$	$\sin(270°-\theta) = -\cos\theta$ $\cos(270°-\theta) = -\sin\theta$ $\tan(270°-\theta) = \cot\theta$	$\sin(270°+\theta) = -\cos\theta$ $\cos(270°+\theta) = \sin\theta$ $\tan(270°+\theta) = -\cot\theta$	$\sin(360°-\theta) = -\sin\theta$ $\cos(360°-\theta) = \cos\theta$ $\tan(360°-\theta) = -\tan\theta$

Illustrations

Illustration 4.

Find the value of
(i) $\cos(-60°)$ (ii) $\tan 210°$ (iii) $\sin 300°$ (iv) $\cos 120°$

Solution

(i) $\cos(-60°) = \cos 60° = \frac{1}{2}$

(ii) $\tan 210° = \tan(180° + 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

(iii) $\sin 300° = \sin(270° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(iv) $\cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$

BEGINNER'S BOX-1

1. Find the values of :
(i) $\tan(-30°)$ (ii) $\sin 120°$ (iii) $\sin 135°$ (iv) $\cos 150°$ (v) $\sin 270°$ (vi) $\cos 270°$

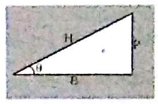
2. If $\sec\theta = \frac{5}{3}$ and $0 < \theta < \frac{\pi}{2}$. Find all the other T-ratios.

1.5 A few important trigonometric formulae

$\sin(A+B) = \sin A\cos B + \cos A\sin B$	$\cos(A+B) = \cos A\cos B - \sin A\sin B$
$\sin(A-B) = \sin A\cos B - \cos A\sin B$	$\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$	$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$
$\sin 2A = 2\sin A\cos A$	$\cos 2A = \cos^2 A - \sin^2 A$
$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$	$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$
	$1 + \cos A = 2\cos^2\frac{A}{2}$, $1 - \cos A = 2\sin^2\frac{A}{2}$

1.6 Range of trigonometric functions

As $\sin\theta = \frac{P}{H}$ and $P \le H$ so $-1 \le \sin\theta \le 1$



As $\cos\theta = \frac{B}{H}$ and $B \le H$ so $-1 \le \cos\theta \le 1$

As $\tan\theta = \frac{P}{B}$ so $-\infty < \tan\theta < \infty$

Remember : $-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

1.7 Small Angle Approximation

If θ is small (say $< 5°$) then $\sin\theta \approx \theta$, $\cos\theta \approx 1$ & $\tan\theta \approx \theta$. Here θ must be in radians.

Illustrations

Illustration 5.

Find the approximate values of (i) $\sin 1°$ (ii) $\tan 2°$ (iii) $\cos 1°$

Solution

(i) $\sin 1° = \sin\left(1° \times \frac{\pi}{180°}\right) = \sin\left(\frac{\pi}{180}\right) \approx \frac{\pi}{180}$ (ii) $\tan 2° = \tan\left(2° \times \frac{\pi}{180°}\right) = \tan\left(\frac{\pi}{90}\right) \approx \frac{\pi}{90}$ (iii) $\cos 1° \approx 1$

2. COORDINATE GEOMETRY

To specify the position of a point in space, we use right handed rectangular axes coordinate system. This system consists of (i) origin (ii) axis or axes. If a point is known to be on a given line or in a particular direction, only one coordinate is necessary to specify its position, if it is in a plane, two coordinates are required, if it is in space three coordinates are needed.

- **Origin**

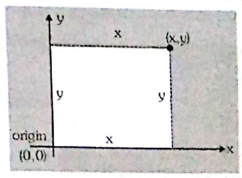
This is any fixed point which is convenient to you. All measurements are taken w.r.t. this fixed point.

- **Axis or Axes**

Any fixed direction passing through origin and convenient to you can be taken as an axis. If the position of a point or position of all the points under consideration always happen to be in a particular direction, then only one axis is required. This is generally called the x-axis. If the positions of all the points under consideration are always in a plane, two perpendicular axes are required. These are generally called x and y-axis. If the points are distributed in a space, three perpendicular axes are taken which are called x, y and z-axis.

2.1 Position of a point in xy plane

The position of a point is specified by its distances from origin along (or parallel to) x and y-axis as shown in figure. Here x-coordinate and y-coordinate is called abscissa and ordinate respectively.



2.2 Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) is given by $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Note : In space $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

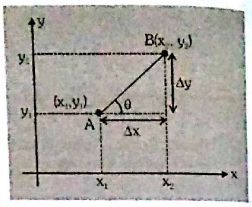
2.3 Slope of a Line

The slope of a line joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is denoted by m and is given by

$m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}=\tan\theta$ [If both axes have identical scales]

Here θ is the angle made by line with positive x-axis.

Slope of a line is a quantitative measure of inclination.



Illustrations

Illustration 6.

For point (2, 14) find abscissa and ordinate. Also find distance from y and x-axis.

Solution

Abscissa = x-coordinate = 2 = distance from y-axis.
Ordinate = y-coordinate = 14 = distance from x-axis.

Illustration 7.

Find value of a if distance between the points (–9 cm, a cm) and (3 cm, 3cm) is 13 cm.

Solution

By using distance formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \Rightarrow 13=\sqrt{[3-(-9)]^2+[3-a]^2}$

$\Rightarrow 13^2 = 12^2 + (3-a)^2 \Rightarrow (3-a)^2 = 13^2 - 12^2 = 5^2 \Rightarrow (3-a) = \pm 5 \Rightarrow a = -2$ cm or 8 cm

Illustration 8.

A dog wants to catch a cat. The dog follows the path whose equation is $y-x = 0$ while the cat follows the path whose equation is $x^2 + y^2 = 8$. The coordinates of possible points of catching the cat are :

(1) (2, –2) (2) (2, 2) (3) (–2, 2) (4) (–2, –2)

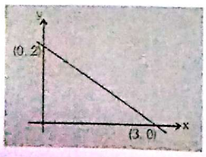
Solution **Ans. (2,4)**

Let catching point be (x_1, y_1) then, $y_1-x_1=0$ and $x_1^2+y_1^2=8$

Therefore, $2x_1^2 = 8 \Rightarrow x_1^2 = 4 \Rightarrow x_1 = \pm 2$; So possible points are (2, 2) and (–2, –2).

BEGINNER'S BOX-2

1. Distance between two points (8, – 4) and (0, a) is 10. All the values are in the same unit of length. Find the positive value of a.
2. Calculate the distance between two points (0, –1, 1) and (3, 3, 13).
3. Calculate slope of shown line



3. DIFFERENTIATION

3.1 Function

Constant: A quantity, whose value remains unchanged during mathematical operations, is called a constant quantity. The integers, fractions like π ,e etc are all constants.

Variable: A quantity, which can take different values, is called a variable quantity. A variable is usually represented as x, y, z, etc.

Function: A quantity y is called a function of a variable x, if corresponding to any given value of x, there exists a single definite value of y. The phrase 'y is function of x' is represented as $y = f(x)$

For example, consider that y is a function of the variable x which is given by $y = 3x^2 + 7x + 2$

If x = 1, then $y = 3(1)^2 + 7(1) + 2 = 12$ and when x = 2, $y = 3(2)^2 + 7(2) + 2 = 28$

Therefore, when the value of variable x is changed, the value of the function y also changes but corresponding to each value of x, we get a single definite value of y. Hence, $y = 3x^2 + 7x + 2$ represents a function of x.

3.2 Physical meaning of $\frac{dy}{dx}$

(i) The ratio of small change in the function y and the variable x is called the average rate of change of y w.r.t. x.

For example, if a body covers a small distance Δs in small time Δt, then

average velocity of the body, $v_{av} = \frac{\Delta s}{\Delta t}$

Also, if the velocity of a body changes by a small amount Δv in small time Δt, then average acceleration of the body, $a_{av} = \frac{\Delta v}{\Delta t}$

(ii) When $\Delta x \to 0$ The limiting value of $\frac{\Delta y}{\Delta x}$ is $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{dy}{dx}$

It is called the instantaneous rate of change of y w.r.t. x.

The *differentiation of a function w.r.t. a variable implies the instantaneous rate of change of the function w.r.t. that variable.*

Like wise, instantaneous velocity of the body $v=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\frac{ds}{dt}$

and instantaneous acceleration of the body $a=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{dv}{dt}$

3.3 Theorems of differentiation

1. If c = constant, $\frac{d}{dx}(c)=0$
2. y = c u, where c is a constant and u is a function of x, $\frac{dy}{dx}=\frac{d}{dx}(cu)=c\frac{du}{dx}$
3. $y = u \pm v \pm w$, where, u, v and w are functions of x, $\frac{dy}{dx}=\frac{d}{dx}(u\pm v\pm w)=\frac{du}{dx}\pm\frac{dv}{dx}\pm\frac{dw}{dx}$
4. y = u v where u and v are functions of x, $\frac{dy}{dx}=\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
5. $y=\frac{u}{v}$, where u and v are functions of x, $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
6. $y = x^n$, n real number, $\frac{dy}{dx}=\frac{d}{dx}(x^n)=nx^{n-1}$

Illustrations

Illustration 9.

Find $\frac{dy}{dx}$, when (i) $y = \sqrt{x}$ (ii) $y = x^5 + x^4 + 7$ (iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Solution

(i) $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

(ii) $y = x^5 + x^4 + 7 \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^5 + x^4 + 7) = \frac{d}{dx}(x^5) + \frac{d}{dx}(x^4) + \frac{d}{dx}(7)$

$= 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3$

(iii) $y = x^2 + 4x^{-1/2} - 3x^{-2} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2}) = \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$

$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2}) = 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)x^{-3}$

$= 2x - 2x^{-3/2} + 6x^{-3}$

BEGINNER'S BOX-3

1. Find $\frac{dy}{dx}$ for the following :

   (i) $y = x^{7/2}$ (ii) $y = x^{-3}$ (iii) $y = x$ (iv) $y = x^5 + x^3 + 4x^{1/2} + 7$

   (v) $y = 5x^4 + 6x^{3/2} + 9x$ (vi) $y = ax^2 + bx + c$ (vii) $y = 3x^5 - 3x - \frac{1}{x}$

2. Given $s = t^2 + 5t + 3$, find $\frac{ds}{dt}$.

3. If $s = ut + \frac{1}{2}at^2$, where u and a are constants. Obtain the value of $\frac{ds}{dt}$

4. The area of a blot of ink is growing such that after t seconds, its area is given by $A = (3t^2+7)$ cm². Calculate th rate of increase of area at t=5 second.

5. The area of a circle is given by $A = \pi r^2$, where r is the radius. Calculate the rate of increase of area w r t radius.

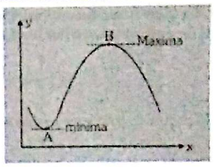
6. Obtain the differential coefficient of the following :

   (i) $(x - 1)(2x + 5)$ (ii) $\frac{1}{2x+1}$ (iii) $\frac{3x+4}{4x+5}$ (iv) $\frac{x^2}{x^3+1}$

3.4 Formulae for differential coefficients of trigonometric, logarithmic and exponentia functions

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$
- $\frac{d}{dx}(\log_e x) = \frac{1}{x}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(e^{ax}) = ae^{ax}$

3.5 Maximum and Minimum value of a Function



Higher order derivatives are used to find the maximum and minimum values of a function. At the points of maxima and minima, first derivative (i.e. $\frac{dy}{dx}$) becomes zero.

At point 'A' (minima) :

As we see in figure, in the neighbourhood of A, slope increases so $\frac{d^2y}{dx^2} > 0$

Condition for minima : $\boxed{\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} > 0}$

At point 'B' (maxima) : As we see in figure, in the neighbourhood of B, slope decreases so $\frac{d^2y}{dx^2} < 0$

Condition for maxima : $\boxed{\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} < 0}$

Illustrations

Illustration 10.

The minimum value of $y = 5x^2 - 2x + 1$ is

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) $\frac{4}{5}$ (4) $\frac{3}{5}$

Solution **Ans. (3)**

For maximum/minimum value $\frac{dy}{dx} = 0 \Rightarrow 5(2x) - 2(1) + 0 = 0 \Rightarrow x = \frac{1}{5}$. Now at $x = \frac{1}{5}$, $\frac{d^2y}{dx^2} = 10$ which is positive

so y has minimum value at $x = \frac{1}{5}$. Therefore $y_{min} = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right) + 1 = \frac{4}{5}$

4. INTEGRATION

In integral calculus, the differential coefficient of a function is given. We are required to find the function. Integration is basically used for summation . Σ is used for summation of discrete values, while $\int$ sign is used for continous function.

If I is integration of f(x) with respect to x then $I = \int f(x)\,dx$ [we can check $\frac{dI}{dx} = f(x)$] $\because \int f'(x)\,dx = f(x) + c$

where c = an arbitrary constant

Let us proceed to obtain intergral of x^n w.r.t. x. $\frac{d}{dx}(x^{n+1}) = (n+1)x^n$

Since the process of integration is the reverse process of differentiation,

$$\int (n+1)x^n dx = x^{n+1} \quad \text{or} \quad (n+1)\int x^n dx = x^{n+1} \Rightarrow \int x^n dx = \frac{x^{n+1}}{n+1}$$

The above formula holds for all values of n, except n = –1.

It is because, for n = –1, $\int x^n dx = \int x^{-1} dx = \int \frac{1}{x} dx$

$$\because \frac{d}{dx}(\log_e x) = \frac{1}{x} \qquad \therefore \int \frac{1}{x} dx = \log_e x$$

Similarly, the formulae for integration of some other functions can be obtained if we know the differential coefficients of various functions.

4.1 Few basic formulae of integration

Following are a few basic formulae of integration :

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, Provided $n \neq -1$

2. $\int \sin x\, dx = -\cos x + c$ $\quad (\because \frac{d}{dx}(\cos x) = -\sin x)$

3. $\int \cos x\, dx = \sin x + c$ $\quad (\because \frac{d}{dx}(\sin x) = \cos x)$

4. $\int \frac{1}{x} dx = \log_e x + c$ $\quad (\because \frac{d}{dx}(\log_e x) = \frac{1}{x})$

5. $\int e^x\, dx = e^x + c$ $\quad (\because \frac{d}{dx}(e^x) = e^x)$

Illustrations

Illustration 11.

Integrate w.r.t. x. : (i) $x^{11/2}$ (ii) x^{-7} (iii) $x^{p/q}\ (p/q \neq -1)$

Solution

(i) $\int x^{11/2} dx = \frac{x^{11/2+1}}{\frac{11}{2}+1} + c = \frac{2}{13} x^{13/2} + c$ (ii) $\int x^{-7} dx = \frac{x^{-7+1}}{-7+1} + c = -\frac{1}{6} x^{-6} + c$ (iii) $\int x^{\frac{p}{q}} dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1} + c = \frac{q}{p+q} x^{(p+q)/q} +$

Illustration 12.

Evaluate $\int \left(x^2 - \cos x + \frac{1}{x}\right) dx$

Solution

$$I = \int x^2 dx - \int \cos x dx + \int \frac{1}{x} dx = \frac{x^{2+1}}{2+1} - \sin x + \log_e x + c = \frac{x^3}{3} - \sin x + \log_e x + c$$

BEGINNER'S BOX-4

1. Evaluate the following integrals :

(i) $\int x^{15} dx$ (ii) $\int x^{-1/2} dx$ (iii) $\int (3x^{-7} + x^{-1}) dx$ (iv) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$

(v) $\int \left(x + \frac{1}{x}\right) dx$ (vi) $\int \left(\frac{a}{x^2} + \frac{b}{x}\right) dx$ (a and b are constant)

4.2 Definite Integrals

When a function is integrated between a lower limit and an upper limit, it is called a definite integral.

If $\frac{d}{dx}(f(x)) = f'(x)$, then

$\int f'(x) dx$ is called indefinite integral and $\int_a^b f'(x) dx$ is called definite integral

Here, a and b are called lower and upper limits of the variable x.

After carrying out integration, the result is evaluated between upper and lower limits as explained below :

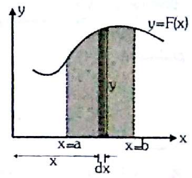
$$\int_a^b f'(x) dx = |f(x)|_a^b = f(b) - f(a)$$

4.3 Area under a curve and definite integration

Area of small shown darkly shaded element = ydx = f(x) dx

If we sum up all areas between x=a and x= b then

$\int_a^b f(x) dx$ = shaded area between curve and x-axis.



Illustrations

Illustration 13.

The integral $\int_1^5 x^2 dx$ is equal to

(1) $\frac{125}{3}$ (2) $\frac{124}{3}$ (3) $\frac{1}{3}$ (4) 45

Solution **Ans. (2)**

$$\int_1^5 x^2 dx = \left[\frac{x^3}{3}\right]_1^5 = \left[\frac{5^3}{3} - \frac{1^3}{3}\right] = \frac{125}{3} - \frac{1}{3} = \frac{124}{3}$$

BEGINNER'S BOX-5

1. Evaluate the following integrals

(i) $\int_R^\infty \frac{GMm}{x^2} dx$ (ii) $\int_{r_1}^{r_2} -k\frac{q_1 q_2}{x^2} dx$ (iii) $\int_u^v Mv\, dv$ (iv) $\int_0^\infty x^{-1/2} dx$

(v) $\int_0^{\pi/2} \sin x\, dx$ (vi) $\int_0^{\pi/2} \cos x\, dx$ (vii) $\int_{-\pi/2}^{\pi/2} \cos x\, dx$

4.4 Average value of a continuous function in an interval

Average value of a function y = f(x), over an interval $a \le x \le b$ is given by $y_{av} = \frac{\int_a^b y dx}{\int_a^b dx} = \frac{\int_a^b y dx}{b-a}$

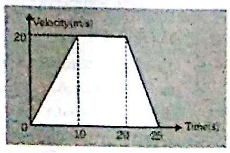
Illustrations

Illustration 14.

The velocity-time graph of a car moving along a straight road is shown in figure. The average velocity of the car in first 25 seconds is

(1) 20 m/s (2) 14 m/s

(3) 10 m/s (4) 17.5 m/s



Solution : **Ans. (2)**

$$\text{Average velocity} = \frac{\int_0^{25} v dt}{25-0} = \frac{\text{Area of v-t graph between t=0 to t = 25 s}}{25} = \frac{1}{25}\left[\left(\frac{25+10}{2}\right)(20)\right] = 14 \text{ m/s}$$

Illustration 15.

Determine the average value of $y = 2x + 3$ in the interval $0 \le x \le 1$.

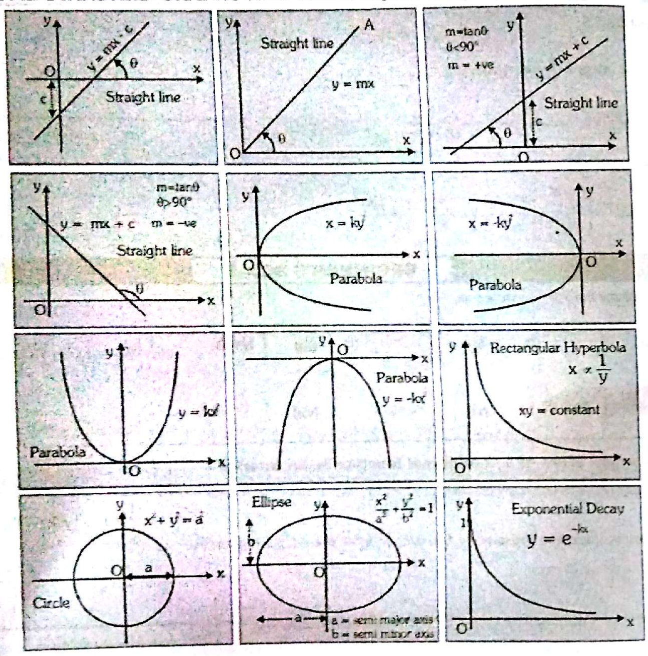
(1) 1 (2) 5 (3) 3 (4) 4

Solution

$$y_{av} = \frac{\int_0^1 y\,dx}{1-0} = \int_0^1 (2x+3)\,dx = \left[2\left(\frac{x^2}{2}\right) + 3x\right]_0^1 = 1^2 + 3(1) - 0^2 - 3(0) = 1 + 3 = 4$$

Ans.

5. SOME STANDARD GRAPHS AND THEIR EQUATIONS



6. ALGEBRA

6.1 Quadratic equation and its solution :

An algebraic equation of second order (highest power of the variable is equal to 2) is called a quadratic equ

Equation $ax^2 + bx + c = 0$ is the general quadratic equation.

The general solution of the above quadratic equation or value of variable x is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots $= x_1 + x_2 = -\frac{b}{a}$ and product of roots $= x_1 x_2 = \frac{c}{a}$

For real roots $b^2 - 4ac \ge 0$ and for imaginary roots $b^2 - 4ac < 0$

Illustrations

Illustration 16.

Solve the equation $2x^2 + 5x - 12 = 0$

Solution

By comparision with the standard quadratic equation $a = 2$, $b = 5$ and $c = -12$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4} = \frac{+6}{4}, \frac{-16}{4} \text{ or } x = \frac{3}{2}, -4$$

Illustration 17.

The speed (v) of a particle moving along a straight line is given by $v = t^2 + 3t - 4$ where v is in m/s and t in seconds. Find time t at which the particle will momentarily come to rest.

Solution

When particle comes to rest, $v = 0$.

So $t^2 + 3t - 4 = 0 \Rightarrow t = \frac{-3 \pm \sqrt{9 - 4(1)(-4)}}{2(1)} \Rightarrow t = 1 \text{ or } -4$

Neglect negative value of t, Hence $t = 1$ s

Illustration 18.

The speed (v) and time (t) for an object moving along straight line are related as $t^2 + 100 = 2vt$ where v is in meter/second and t is in second. Find the possible positive values of v.

Solution

For possible values of v, time t must be real so from $b^2 - 4ac \ge 0$

we have $(-2v)^2 - 4(1)(100) \ge 0$

$\Rightarrow 4v^2 - 400 \ge 0 \Rightarrow v^2 - 100 \ge 0$

$\Rightarrow (v - 10)(v + 10) \ge 0 \Rightarrow v \ge 10 \text{ and } v \le -10$

Hence possible positive values of v are $v \ge 10$ m/s

BEGINNER'S BOX-6

1. Solve for x : (i) $10x^2 - 27x + 5 = 0$ (ii) $pqx^2 - (p^2 + q^2)x + pq = 0$

2. In quadratic equation $ax^2 + bx + c = 0$, if discriminant is $D = b^2 - 4ac$, then roots of the quadratic equation are : (choose the correct alternative)

(1) Real and distinct, if $D > 0$ (2) Real and equal (i.e., repeated roots), if $D = 0$

(3) Non-real (i.e. imaginary), if $D < 0$ (4) All of the above are correct

6.2 Binomial Expression :

An algebraic expression containing two terms is called a binomial expression.

For example $(a+b)$, $(a+b)^3$, $(2x-3y)^{-1}$, $\left(x + \frac{1}{y}\right)$ etc. are binomial expressions.

Binomial Theorem

$$(a+b)^n = a^n + na^{n-1}b^1 + \frac{n(n-1)}{2 \times 1}a^{n-2}b^2 + \ldots\ldots, \qquad (1+x)^n = 1 + nx + \frac{n(n-1)}{2 \times 1}x^2 + \ldots\ldots$$

Binomial Approximation

If x is very small, compared to 1, then terms containing higher powers of x can be neglected so $(1+x)^n \approx 1 + nx$

Illustrations

Illustration 19.

Calculate $\sqrt{0.99}$

Solution

$$\sqrt{0.99} = (1-0.01)^{1/2} \approx 1 - \frac{1}{2}(0.01) \approx 1 - 0.005 \approx 0.995$$

Illustration 20.

The mass m of a body moving with a velocity v is given by $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where m_0 = rest mass of body = 10 kg and c = speed of light = 3×10^8 m/s. Find the value of m at $v = 3 \times 10^7$ m/s.

Solution

$$m = m_0\left(1-\frac{v^2}{c^2}\right)^{-1/2} = 10\left[1-\left(\frac{3\times10^7}{3\times10^8}\right)^2\right]^{-1/2} = 10\left[1-\frac{1}{100}\right]^{-1/2} \approx 10\left[1-\left(-\frac{1}{2}\right)\left(\frac{1}{100}\right)\right] = 10 + \frac{10}{200} \approx 10.05$$

6.3 Logarithm

Common formulae :

- $\log mn = \log m + \log n$
- $\log \frac{m}{n} = \log m - \log n$
- $\log m^n = n \log m$
- $\log_e m = 2.303 \log_{10} m$

6.4 Componendo and Dividendo Rule : If $\frac{p}{q} = \frac{a}{b}$ then $\frac{p+q}{p-q} = \frac{a+b}{a-b}$

6.5 Arithmetic progression (AP)

General form : a, a + d, a + 2d, ..., a + (n–1)d. Here a = first term, d = common difference

Sum of n terms $S_n = \frac{n}{2}[a+a+(n-1)d] = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[1^{st} \text{ term} + n^{th} \text{ term}]$

Illustrations

Illustration 21.

Find the sum of given Arithmetic Progression 4 + 8 + 12 +.........+ 64

(1) 464 (2) 540 (3) 544 (4) 646

Ans. (

Solution

Here a = 4, d = 4, n = 16 So, sum $= \frac{n}{2}$[First term + last term] $= \frac{16}{2}[4 + 64] = 8(68) = 544$

Note :

(i) Sum of first n natural numbers. $S_n = 1 + 2 + 3 + ... + n = \frac{n}{2}[1+n] = \frac{n(n+1)}{2}$

(ii) Sum of first n squared natural numbers $S_{n^2} = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+}{6}$

BEGINNER'S BOX-7

1. Find sum of first 50 natural numbers.

2. Find $1^2 + 2^2 + ... + 10^2$.

6.6 Geometric Progression (GP)

General form : a, ar, ar^2,..., ar^{n-1} Here a = first term, r = common ratio

Sum of n terms $S_n = \frac{a(1-r^n)}{1-r}$ Sum of ∞ term $S_\infty = \frac{a}{1-r}$ ($\because r < 1$ ∴ $r^\infty \to 0$)

Illustrations

Illustration 22.

Find the sum of given series 1 + 2 + 4 + 8 +.............+ 256

(1) 510 (2) 511 (3) 512 (4) 513

Solution : **Ans.[2]**

Here a = 1, r = 2, n = 9 ($\because$ 256 = 2^8) So $S_9 = \frac{(1)(1-2^9)}{(1-2)} = 2^9 - 1 = 512 - 1 = 511$

Illustration 23.

Find $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$upto ∞.

(1) ∞ (2) 1 (3) 2 (4) 1.925

Solution : **Ans.[3]**

Here, a = 1, r = $\frac{1}{2}$ So, $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$

BEGINNER'S BOX-8

1. Find $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + ... \infty$.

2. Find $F_{net} = GMm\left[\frac{1}{r^2} + \frac{1}{2r^2} + \frac{1}{4r^2} + ...\text{upto } \infty\right]$.

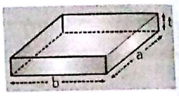
7. GEOMETRY

7.1 Formulae for determination of area :

1. Area of a square = $(\text{side})^2$
2. Area of rectangle = length × breadth
3. Area of a triangle = $\frac{1}{2}$ (base × height)
4. Area of trapezoid = $\frac{1}{2}$ (distance between parallel sides) × (sum of parallel sides)
5. Area enclosed by a circle = πr^2 (r = radius)
6. Surface area of a sphere = $4\pi r^2$ (r = radius)
7. Area of a parallelogram = base × height
8. Area of curved surface of cylinder = $2\pi r\ell$ (r = radius and ℓ = length)
9. Area of ellipse = π ab (a and b are semi major and semi minor axes respectively)
10. Surface area of a cube = $6(\text{side})^2$
11. Total surface area of cone = $\pi r^2 + \pi r\ell$ where $\pi r\ell = \pi r\sqrt{r^2+h^2}$ = lateral area

7.2 Formulae for determination of volume :

1. Volume of a rectangular slab = length × breadth × height = abt
2. Volume of a cube = $(\text{side})^3$



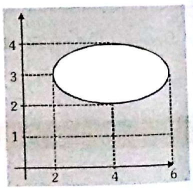
3. Volume of a sphere = $\frac{4}{3}\pi r^3$ (r = radius)
4. Volume of a cylinder = $\pi r^2\ell$ (r = radius and ℓ is length)
5. Volume of a cone = $\frac{1}{3}\pi r^2 h$ (r = radius and h is height)

Note : $\pi = \frac{22}{7} = 3.14$, $\pi^2 = 9.8776 \approx 10$ and $\frac{1}{\pi} = 0.3182 \approx 0.3$.

Illustrations

Illustration 24.

Calculate the unshaded area.



Solution

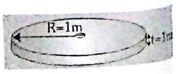
Shaded area = Area of ellipse = πab

Here $a = 6 - 4 = 2$ and $b = 4 - 3 = 1$

$\Rightarrow$ Area = $\pi \times 2 \times 1 = 2\pi$ units

Illustration 25.

Calculate the volume of given disk.



Solution

Volume = $\pi R^2 t = (3.14)(1)^2(10^{-3}) = 3.14 \times 10^{-3}\ m^3$

VECTORS

Scalar Quantities

A physical quantity which can be described completely by its magnitude only and does not require a direction is known as a scalar quantity.

It obeys the ordinary rules of algebra.

Ex : Distance, mass, time, speed, density, volume, temperature, current etc.

Vector Quantities

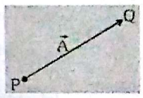
A physical quantity which requires magnitude and a particular direction, when it is expressed.

Ex. : Displacement, velocity, acceleration, force etc.

$\boxed{\text{Vector quantities must obey the rules of vector algebra}}$

A vector is represented by a line headed with an arrow.

Its length is proportional to its magnitude.



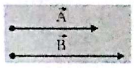
$\vec{A}$ is a vector.

$\vec{A} = \overrightarrow{PQ}$

Magnitude of $\vec{A} = |\vec{A}|$ or A

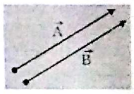
1.1 Types of vector

- **Parallel Vectors :-**



Those vectors which have same direction are called parallel vectors.
Angle between two parallel vectors is always 0°

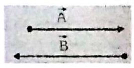
- **Equal Vectors**



Vectors which have equal magnitude and same direction are called equal vectors.

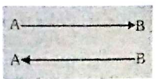
$\vec{A} = \vec{B}$

- **Anti-parallel Vectors :**



Those vectors which have opposite direction are called anti-parallel vector.
Angle between two anti-parallel vectors is always 180°

- **Negative (or Opposite) Vectors**

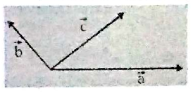


Vectors which have equal magnitude but opposite direction are called negative vectors of each other.

$\overline{AB}$ and $\overline{BA}$ are negative vectors

$\overline{AB} = -\overline{BA}$

- **Co-initial vector**



Co-initial vectors are those vectors which have the same initial point.
In figure $\vec{a}, \vec{b}$ and $\vec{c}$ are co-initial vectors.

- **Collinear Vectors :**

The vectors lying in the same line are known as collinear vectors.
Angle between collinear vectors is either 0° or 180°.