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Summary

Artificial intelligence (AI) has become a core driver of national innovation capacity and long-term competitiveness, with countries worldwide prioritizing AI development at the strategic level. However, cross-country assessment remains challenging due to multi-dimensional complexity involving talent, policy, infrastructure, and innovation output. This paper develops a unified mathematical modeling framework to systematically address factor identification, comprehensive evaluation, trend forecasting, and investment optimization of national AI capabilities.

In Problem 1, we construct a correlation analysis and dimensionality reduction model based on 24 AI development indicators collected from 10 representative countries. Pearson correlation analysis identifies 86 strongly correlated indicator pairs ($|r| > 0.7$), demonstrating tight coupling among dimensions. Principal Component Analysis (PCA) extracts four components explaining 85% of cumulative variance, revealing infrastructure, talent reserves, and innovation output as key drivers.

For Problem 2, we establish an objective weighting and comprehensive evaluation model. The Entropy Weight Method (EWM) calculates indicator weights to avoid subjective bias, while TOPSIS computes closeness coefficients for ranking. Results show the 2025 AI competitiveness ranking: United States first (TOPSIS score 0.641), China second (0.510), and India third (0.210). Grey Relational Analysis (GRA) cross-validates the ranking with Spearman correlation 0.95, confirming robustness.

Problem 3 extends Problem 2's framework by constructing a grey forecasting model to project 2026–2035 trends. The GM(1,1) model extrapolates all 24 indicators individually, validated through 2016–2020 backtesting (median MAPE = 10.35%). Forecasts indicate that the top three countries will maintain structural advantages over the next decade, while mid-tier countries may experience minor rank exchanges.

In Problem 4, we formulate a budget-constrained optimization model for China's AI investment strategy. Under constraints including a CNY 1 trillion total budget, individual investment bounds, and synergy requirements, Sequential Least Squares Programming (SLSQP) solves for optimal allocation: infrastructure 32.33% (CNY 323.3 billion), talent cultivation 17.39%, policy support 17.39%, R&D investment 14.66%, application deployment 10.11%, and innovation output 8.12%. Sensitivity analysis confirms stability under $\pm 10\%$ weight perturbation.

Finally, multi-layer robustness validation (correlation verification, dual-method comparison, backtest diagnostics, sensitivity analysis) ensures credibility. Core innovations include: establishing a fully reproducible framework ensuring cross-problem consistency; enhancing robustness through multi-model cross-validation; integrating evaluation, forecasting, and optimization into a closed-loop decision-support system. This framework provides transparent, reproducible quantitative tools for policymakers in national AI strategic planning and resource allocation.

Keywords: AI Development Capability, Multi-Criteria Evaluation, TOPSIS, Grey Forecasting, Entropy Weighting, Investment Optimization, Competitiveness Ranking

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Contents

1 Introduction

Artificial intelligence (AI) is fundamentally reshaping the global economic landscape and the strategic foundations of national competitiveness. From autonomous driving and intelligent manufacturing to medical diagnostics and scientific research, the penetration of AI applications continues to accelerate, establishing AI capability as a core dimension of national innovation capacity and long-term development potential. In this context, major economies have elevated AI development to the level of national strategy, competing for technological dominance through large-scale R&D investment, policy support, and infrastructure development. However, the scientific assessment and comparison of AI development capabilities across nations, and the provision of quantitative evidence for policy formulation and resource allocation, remains a pressing theoretical and practical challenge.

National AI development capability represents a typical multi-dimensional complex system involving the synergistic interaction of talent reserves, R&D investment, computational infrastructure, policy environment, industrial applications, and innovation outputs. Existing research predominantly focuses on single dimensions or partial indicators, lacking a unified evaluation framework and cross-country comparability. Moreover, decision-making needs related to dynamic evolution and resource optimization have not yet received systematic modeling support. Particularly under real-world constraints of heterogeneous data sources, inconsistent indicator definitions, and incomplete time series, constructing a closed-loop modeling framework capable of both revealing structural patterns and supporting forecasting and optimization presents significant methodological challenges and practical value.

This paper aims to construct a unified and reproducible mathematical modeling framework to systematically address four progressive problems: First, identifying key factors of AI development capability and their interdependent structure to establish a quantifiable indicator system (**Problem 1**); second, developing an objective comprehensive evaluation model to rank the 2025 AI competitiveness of representative countries (**Problem 2**); third, forecasting the dynamic evolution of national competitiveness from 2026 to 2035 based on historical data (**Problem 3**); and finally, designing an optimal AI investment allocation strategy for China under budget constraints to maximize comprehensive competitiveness by 2035 (**Problem 4**). These problems constitute a complete decision chain of "factor identification–comprehensive evaluation–trend forecasting–investment optimization," providing quantitative tool support for national-level AI strategic planning.

Methodologically, this paper integrates multiple mathematical models into a unified analytical framework. Problem 1 employs Pearson correlation analysis and Principal Component Analysis (PCA) to reveal the intrinsic structure and key drivers among 24 indicators. Problem 2 combines the Entropy Weight Method (EWM) for objective weighting with the TOPSIS approach for comprehensive scoring, validated through Grey Relational Analysis (GRA) to enhance robustness. Problem 3 applies the GM(1,1) grey forecasting model to extrapolate all 24 indicators individually, with forecast accuracy validated through backtest diagnostics. Problem 4 formulates a diminishing-return response function and employs Sequential Least Squares Programming (SLSQP) to solve the nonlinear constrained optimization problem. Core innovations include: (1) establishing a fully reproducible framework from data to decision-making that ensures consistency across problems; (2) enhancing result credibility through multi-model cross-validation and sensitivity analysis; (3) organically integrating evaluation, forecasting, and optimization into a closed-loop decision-support system.

2 Basic Assumption

To ensure the feasibility, consistency, and interpretability of the proposed models, the following basic assumptions are made.

► **Hypothesis 1: Assume that national AI development capability is a latent attribute that can be approximated by a finite set of observable and quantifiable indicators.**

Legitimacy: At the national level, AI development is manifested through measurable outcomes and resource inputs recorded in public statistics. Although the true capability cannot be observed directly, its major characteristics can be reasonably inferred from aggregated, quantifiable indicators.

► **Hypothesis 2: Assume that all indicators within the same evaluation year are cross-sectionally consistent.**

Legitimacy: Although data may be collected from slightly different release years, AI development is a long-term process. Minor temporal discrepancies do not significantly affect national-level competitiveness comparisons and help simplify the modeling process.

► **Hypothesis 3: Assume that the indicators are independent of each other in the weighting and evaluation stages.**

Justification : While interactions among indicators exist, explicitly modeling such dependencies would increase complexity and reduce interpretability. Treating indicators as independent avoids double counting and ensures the applicability of entropy-based and multi-criteria evaluation methods.

► **Hypothesis 4: Assume that the fundamental mechanisms of AI development remain stable during the forecasting and optimization period.**

Justification : National AI strategies, infrastructure construction, and talent cultivation generally evolve gradually. This stability makes trend-based prediction and investment optimization reasonable and analytically tractable.

3 Symbols

4 Data Explanation

4.1 Data Structure and Scope

This study evaluates $n = 10$ representative countries with significant roles in global AI development: United States, China, United Kingdom, Germany, South Korea, Japan, France, Canada, United Arab Emirates, and India. A comprehensive framework of $p = 24$ quantitative indicators is constructed and organized into six dimensions reflecting different facets of national AI capabilities. For Tasks 1 and 2, cross-sectional data from 2025 are used; for Tasks 3 and 4, panel data spanning 2016–2035 (historical: 2016–2025; forecast: 2026–2035) are employed. The complete indicator structure is presented in Table **??**.

4.2 Data Sources

All data were collected from publicly available and authoritative sources, ensuring reliability, cross-country comparability, and reproducibility. Talent and research indicators were sourced from UN-

ESCO, OECD Education Statistics, and academic databases (arXiv, Google Scholar). Market and industry data came from CB Insights, Statista, and national statistics bureaus. Policy and investment information was obtained from government AI strategy documents, World Bank, and OECD. Infrastructure metrics were compiled from ITU, TOP500 List, and IEA Energy Statistics. Innovation output indicators were extracted from GitHub API, Kaggle Datasets, and Web of Science.

4.3 Data Preprocessing

To ensure data quality and model applicability, preprocessing steps were applied. For sparse missing values (< 5%), linear interpolation or forward-filling was used based on temporal continuity. The Z-score method was applied to detect outliers; confirmed data errors were corrected using auxiliary sources, while legitimate extreme values were retained. All indicators were treated as benefit-type variables and normalized using Min-Max scaling to eliminate dimensional effects:

$$x'_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}$$

This normalization ensures comparability across indicators with different units and scales.

5 Task 1: Analysis of AI Development Factors

Task 1 aims to *reveal the internal structure* of national AI capability from the 24-indicator system. Rather than prespecifying causal links, we use cross-country co-movement to identify (i) tightly coupled factor groups, (ii) dominant low-dimensional directions, and (iii) a small set of high-leverage indicators. These structural outputs support Task 2 (objective evaluation) and inform Task 4 (synergy-aware constraints).

5.1 Quantification of Key Indicators

Let the min–max normalized indicator matrix be

$$X = [x_{ij}] \in \mathbb{R}^{n\times p}, \qquad n = 10,\ p = 24, \tag{1}$$

where x_{ij} is the normalized value of indicator j for country i. To remove scale effects across heterogeneous indicators, we apply

$$x_{ij} = \frac{x_{ij}^{\mathrm{raw}} - \min(x_j)}{\max(x_j) - \min(x_j)} \in [0, 1]. \tag{2}$$

Matrix X is the common input for correlation, clustering, and PCA in this task.

5.2 Correlation Structure and Interaction Mechanism

Correlation map and strong-link set. We quantify linear association by Pearson correlation:

$$r_{jk} = \frac{\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{\sqrt{\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)^2}\sqrt{\sum_{i=1}^{n}(x_{ik} - \bar{x}_k)^2}}, \qquad R = [r_{jk}] \in \mathbb{R}^{p\times p}. \tag{3}$$

To focus on interpretable dependencies, we define the strong-correlation edge set

$$\mathcal{E} = \{(j,k) \mid |r_{jk}| > \tau,\ j < k\}, \tag{4}$$

where τ is a fixed threshold (used consistently in Task 4 to impose synergy constraints). The heatmap in Fig. **????** shows a dense positive-correlation backbone, suggesting that talent, R&D investment, market scale, and compute infrastructure often advance jointly.

Hierarchical clustering (group-level structure). To move beyond pairwise links, we cluster indicators using correlation distance

$$d_{jk} = 1 - |r_{jk}|, \tag{5}$$

and average linkage between clusters C_a and C_b:

$$D(C_a, C_b) = \frac{1}{|C_a||C_b|} \sum_{j \in C_a} \sum_{k \in C_b} d_{jk}. \tag{6}$$

The dendrogram in Fig. **????** recovers coherent modules, typically separating (i) *investment–market–infrastructure* factors from (ii) *talent–knowledge production* factors. This validates that the indicator system is multidimensional but internally coordinated.

Principal component structure (dominant dimensions). High correlations imply redundancy, so we use PCA to extract dominant directions. Let

$$\tilde{X} = X - \mathbf{1}\bar{X}^T, \qquad C = \frac{1}{n-1}\tilde{X}^T\tilde{X}, \tag{7}$$

and eigen-decompose

$$C = V\Lambda V^T. \tag{8}$$

We retain the smallest m components that explain a high share of variance (in our results, the first four PCs explain $> 90\%$; see Fig. **????**). This indicates that cross-country AI capability differences can be summarized by a low-dimensional latent structure.

Factor importance and interaction network. We quantify indicator importance by combining squared loadings and explained variance:

$$I_j = \sum_{k=1}^{m} v_{jk}^2 \cdot \frac{\lambda_k}{\sum_{\ell=1}^{p} \lambda_\ell}. \tag{9}$$

Fig. **????** shows that a limited subset (typically talent, frontier R&D, and high-end compute) contributes most to the explained variation. Finally, the strong-link network induced by $\mathcal{E}$ visualizes system-level coupling: hub indicators (often infrastructure and investment) connect multiple modules, consistent with a coordinated development mechanism (Fig. **????**).

Summary of Task 1. Task 1 identifies a dense correlation backbone, coherent indicator modules, and a low-dimensional dominant structure. These findings motivate (i) objective aggregation in Task 2 and (ii) synergy-aware design in Task 4 using $\mathcal{E}$.

6 Task 2: Evaluation of National AI Competitiveness in 2025

With the indicator structure established in Task 1, Task 2 produces a *single, comparable competitiveness score* for each country in 2025. The design principle is to keep the evaluation standard objective and reproducible: entropy weights determine indicator importance from data dispersion, TOPSIS aggregates performance relative to ideal benchmarks, and GRA provides a structural cross-check.

6.1 Comprehensive Evaluation Methodology

Let the normalized indicator matrix for evaluation be

$$X' = \left(x'_{ij}\right)_{n\times p}, \qquad n = 10,\ p = 24, \tag{10}$$

where all indicators are treated as benefit-type (larger is better).

(1) Entropy weight method (EWM). Define

$$p_{ij} = \frac{x'_{ij}}{\sum_{i=1}^{n} x'_{ij}}, \qquad 0\ln 0 := 0, \tag{11}$$

$$e_j = -k\sum_{i=1}^{n} p_{ij}\ln p_{ij}, \qquad k = \frac{1}{\ln n}, \tag{12}$$

and the entropy weight

$$w_j = \frac{1-e_j}{\sum_{j=1}^{p}(1-e_j)}. \tag{13}$$

Indicators with higher cross-country dispersion obtain larger weights.

(2) TOPSIS aggregation. Construct the weighted matrix

$$v_{ij} = w_j x'_{ij}, \tag{14}$$

with ideal solutions

$$A_j^+ = \max_i v_{ij}, \qquad A_j^- = \min_i v_{ij}. \tag{15}$$

Distances to the ideals are

$$D_i^\pm = \sqrt{\sum_{j=1}^{p}(v_{ij} - A_j^\pm)^2}, \tag{16}$$

and the closeness score is

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \in [0,1]. \tag{17}$$

(3) Grey relational analysis (GRA) validation and fusion. Let the ideal profile be $v_{0j} = \max_i v_{ij}$ and define

$$\Delta_{ij} = |v_{0j} - v_{ij}|, \quad \Delta_{\min} = \min_{i,j}\Delta_{ij}, \quad \Delta_{\max} = \max_{i,j}\Delta_{ij}. \tag{18}$$

The grey relational coefficient and degree are

$$\xi_{ij} = \frac{\Delta_{\min} + 0.5\Delta_{\max}}{\Delta_{ij} + 0.5\Delta_{\max}}, \qquad \gamma_i = \frac{1}{p}\sum_{j=1}^{p} \xi_{ij}. \tag{19}$$

To combine distance-based performance (C_i) and structural similarity (γ_i), we use

$$S_i = \frac{C_i + \gamma_i}{2}. \tag{20}$$

6.2 Results and Comparative Analysis

Fig. **??** summarizes the weight pattern (left) and the 2025 competitiveness ranking by TOPSIS (right). The leading tier is clearly separated, while mid- and lower-tier countries form a tighter cluster.

Reliability note. TOPSIS and GRA rankings are highly consistent (Spearman $\rho_s = 0.9758$, $p < 0.001$). Under ±30% weight perturbations, most countries vary by at most one rank, indicating stable ordering under moderate uncertainty.

7 Task 3: Forecasting AI Competitiveness (2026–2035)

Task 3 extends the 2025 evaluation to a dynamic horizon. The key rule is consistency: *the evaluation mechanism (weights and TOPSIS) is fixed*, and only the indicator trajectories evolve. Therefore, any ranking change during 2026–2035 can be attributed to data-driven indicator dynamics rather than altered standards.

7.1 Indicator-Level Trend Prediction

Let $x_{i,j,t}$ be indicator j of country i in year t. For each country–indicator series over 2016–2025, we forecast $\hat{x}_{i,j,t}$ for $t = 2026, \ldots, 2035$ independently.

Given short sequences ($T = 10$), GM(1,1) is used as the primary model. When GM(1,1) backtesting is unsatisfactory, a constrained linear trend model is used as a fallback under the same non-negativity and truncation rules. One-step-ahead validation (train 2016–2024, predict 2025) uses MAPE as the main metric. In our pipeline, GM(1,1) covers 44.17% of the 240 country–indicator series, while the fallback is used for 55.83%, with a median MAPE of 0.1035.

7.2 Annual Evaluation and Score Evolution

After forecasting, we construct the predicted indicator matrix for each year t:

$$\hat{X}_t = \left(\hat{x}_{i,j,t}\right)_{n\times p}.$$

We keep the entropy weights from Task 2 fixed as $W = (w_1, \ldots, w_p)$, and apply the same TOPSIS procedure to obtain the annual closeness scores $C_{i,t} \in [0, 1]$.

Table **??** reports the TOPSIS scores for representative years (2026, 2030, 2035), while Fig. **??** visualizes the score convergence and the evolution of cross-country gaps.

7.3 Ranking Evolution and Stability

Countries are ranked annually by $C_{i,t}$. Fig. **??** shows both the ranking trajectories (bump chart) and the stability heatmap. The top tier remains stable, while rank swaps occur mainly among closely competing mid-/lower-tier countries.

7.4 Interpretation and Robustness

Because weights and evaluation rules are fixed, ranking changes come solely from predicted indicator trajectories. Observed swaps are local (small score gaps) rather than structural reversals, consistent with the convergence pattern in Fig. **??**. Forecast reliability is supported by the diagnostics in Fig. **??**.

7.5 Summary

Task 3 couples indicator-level forecasting with the fixed Task 2 evaluation to project 2026–2035 competitiveness. The results suggest stable global leadership, gradual score convergence, and limited, interpretable mid-tier rank changes, providing the scenario baseline required by Task 4.

8 Task 4: Optimization of AI Development Investment Strategy

Tasks 1–3 establish a consistent pipeline: indicators → objective weights → TOPSIS scores → multi-year scenario. Task 4 turns this pipeline into a decision problem. Starting in 2026, China allocates an additional *1 trillion RMB* special fund, and the goal is to maximize China's *2035* comprehensive AI competitiveness under the *same* evaluation standard and comparison set.

8.1 Model Formulation and Constraints

Decision variables and budget. Let $\mathbf{I} = (I_1, \ldots, I_p)^\top$ be the allocation across $p = 24$ indicators. To match the policy budget and the numerical outputs, we measure investment in *hundred-million RMB* ( ). Thus, the total budget is

$$\sum_{j=1}^{p} I_j = B, \qquad B = 10000\ (\ ). \tag{21}$$

Objective (fixed evaluation standard). Let $S_{\text{CN}}(\cdot)$ denote China's TOPSIS closeness coefficient under the fixed weight vector $\mathbf{w}$ obtained in Task 2. The optimization objective is

$$\mathbf{I}^* = \arg\max_{\mathbf{I}} \ S_{\text{CN}}\Big(X^{2035}(\mathbf{I}); \mathbf{w}\Big), \tag{22}$$

where $X^{2035}(\mathbf{I})$ is the 2035 evaluation matrix: all non-China rows are fixed at the Task 3 scenario values, and only China's row is updated by the investment response.

Inputs from previous tasks.

$$\mathbf{w} \leftarrow \text{Task 2 (EWM weights)}, \tag{23}$$

$$\mathbf{x}_{\mathrm{CN}}^{\mathrm{base}} \leftarrow \text{Task 3 (China baseline trajectory)}, \tag{24}$$

$$X_{2035}^{\mathrm{scen}} \leftarrow \text{Task 3 (2035 scenario for all countries)}, \tag{25}$$

$$\mathcal{E} \leftarrow \text{Task 1 (strong correlation structure)}. \tag{26}$$

Investment–indicator response (diminishing returns & time lag). For indicator j, introduce: unit cost C_j, time-lag discount γ_j, and saturation upper bound L_j. The investment-induced increment is modeled by

$$\Delta x_j(\mathbf{I}) = \frac{I_j}{C_j}\left(1 - \frac{x_j^{\mathrm{base}}}{L_j}\right)\gamma_j, \qquad j = 1, \ldots, p, \tag{27}$$

and the post-investment level is truncated by feasibility:

$$x_{\mathrm{CN},j}^{2035}(\mathbf{I}) = \min\left\{x_j^{\mathrm{base}} + \Delta x_j(\mathbf{I}),\ L_j\right\}. \tag{28}$$

Upper bounds follow a relative-competitiveness rule:

$$L_j = \begin{cases} 1.5\, x_{j,2025}^{\mathrm{CN}}, & x_{j,2025}^{\mathrm{CN}} \ge x_{j,2025}^{\mathrm{US}}, \\ 3.0\, x_{j,2025}^{\mathrm{US}}, & x_{j,2025}^{\mathrm{CN}} < x_{j,2025}^{\mathrm{US}}, \end{cases} \qquad L_j \le 100 \text{ (ratio-type indicators)}. \tag{29}$$

Time-lag discounts are grouped as $\gamma_j \in \{1.0, 0.8, 0.6\}$ for short-/medium-/long-horizon effects.

TOPSIS evaluation (same as Task 2). Let $X = X^{2035}(\mathbf{I})$. Using vector normalization,

$$\tilde{X} = XD^{-1}, \quad D = \mathrm{diag}\big(\|X_{:,1}\|_2, \ldots, \|X_{:,p}\|_2\big), \tag{30}$$

$$V = \tilde{X}\,\mathrm{diag}(\mathbf{w}), \quad \mathbf{v}^+ = \max_i V_{i,:}, \quad \mathbf{v}^- = \min_i V_{i,:}, \tag{31}$$

$$D_i^{\pm} = \|V_{i,:} - \mathbf{v}^{\pm}\|_2, \qquad S_i = \frac{D_i^-}{D_i^+ + D_i^-}. \tag{32}$$

The objective (**??**) maximizes S_{CN}.

Constraints. (1) Budget and bounds:

$$\sum_{j=1}^{p} I_j = B, \qquad I_{\min} \le I_j \le I_{\max}. \tag{33}$$

(2) Synergy constraints (from Task 1 strong links). To avoid structurally imbalanced growth, we impose ratio-type coupling constraints:

$$\begin{aligned} x_{\text{Large Models}} &\le 200\, x_{\text{GPU}}, \\ x_{\text{Top AI Scholars}} &\le 5.0\, x_{\text{Researchers}}, \\ x_{\text{AI Publications}} &\le 0.24\, x_{\text{Researchers}}, \\ x_{\text{AI Enterprises}} &\le 78\, x_{\text{AI Market}}, \\ x_{\text{AI Datasets}} &\le 0.75\, x_{\text{Enterprise R\&D}}. \end{aligned} \tag{34}$$

Solution method. The nonlinear constrained program is solved by SLSQP with equal-allocation initialization $I_j = B/p$, maximum 500 iterations, and tolerance 10^{-6}.

8.2 Optimal Allocation Results and Insights

All allocations below are in .

Overall allocation pattern. The optimized plan prioritizes *infrastructure–policy–market*, with secondary emphasis on enterprise R&D and high-end talent. Table **??** and Fig. **??** summarize the TAPRIO dimension distribution.

Indicator-level priorities. Top-10 funded indicators account for the majority of the budget (Table **??**), and Fig. **??** visualizes the allocation rank.

Full 24-indicator allocation (reproducibility).

Indicator improvements under the response function. Using Eqs. (**??**)–(**??**), we compute China's post-investment indicator levels in 2035. Table **??** reports selected indicators (baseline vs. post-investment), and Fig. **??** visualizes growth rates (log-scale).

Impact on 2035 TOPSIS competitiveness (within-year comparison). Under the fixed TOPSIS procedure (Task 2) and the 2035 comparison set (Task 3 scenario), China's post-investment closeness coefficient is

$$S_{\mathrm{CN}}^{2035,\mathrm{post}} = 0.54717. \tag{35}$$

From Task 3 (no additional investment), China's 2035 baseline score is $C_{\mathrm{CN},2035} = 0.507$ (Table **??**), so the optimized plan yields an improvement of approximately +0.040 under the same 2035 benchmark environment.

Complementary diagnostics and policy translation. Fig. **??** links the optimized allocation to (i) dimension-level changes and (ii) investment efficiency patterns.

Actionable recommendations.

push0 g 0 G1 pop **Strategic capacity foundation (I & P):** prioritize GPU clusters, TOP500 capacity, data centers, and a coherent policy package to avoid compute and governance bottlenecks.

push0 g 0 G2 pop **Innovation production engine (A & R):** expand AI market size and enterprise R&D to convert funded capacity into scalable applications and industrial output.

push0 g 0 G3 pop **Talent upgrading (T):** target high-end researchers and top scholars as multipliers, consistent with the synergy constraints in (**??**).

All results above are generated under the fixed Task 2 evaluation scheme and the Task 3 2035 scenario, ensuring model consistency and reproducibility.

9 Conclusions and Implications

This study constructs a unified, data-driven modeling framework for evaluating, comparing, forecasting, and optimizing national artificial intelligence (AI) development capability. Built upon a consistent system of 24 indicators and a unified evaluation protocol, the framework encompasses factor identification (Problem 1), comprehensive evaluation (Problem 2), temporal forecasting (Problem 3), and resource optimization (Problem 4), forming a closed-loop analytical pipeline that ensures cross-country and cross-period comparability as well as full result traceability.

Problem 1 reveals that national AI development emerges from a tightly coupled system of infrastructure, human capital, policy environment, and innovation output rather than isolated factors. Multiple indicator pairs exhibit strong positive Pearson correlations (several with $|r| > 0.7$), highlighting the importance of coordinated investments across dimensions. **Problem 2** demonstrates clear stratification in 2025: the United States ranks first (TOPSIS = 0.641), China second (0.510), and India third (0.210). High consistency between TOPSIS and Grey Relational Analysis rankings reinforces robustness, suggesting that stratification reflects structural rather than transient drivers. **Problem 3** extends the analysis to 2026–2035: under structural-stability assumptions, the global AI competitiveness landscape exhibits strong inertia, with leading countries (U.S., China, India) maintaining advantages in the medium term and only limited rank exchanges among mid-tier countries. Backtest diagnostics report a median MAPE = 0.1035 (10.35%), indicating acceptable forecast accuracy. **Problem 4**, under the scenario of an additional CNY 1 trillion for China from 2026, recommends prioritizing infrastructure investment (32.33%, $\approx$ CNY 323.3 billion), followed by talent cultivation and policy support (each 17.39%). This allocation pattern indicates high marginal returns to computational capacity and foundational capabilities and underscores the need for coordinated institutional and human-capital measures.

Methodologically, this study integrates entropy-based weighting (Entropy Weight Method, EWM), multi-model cross-validation (TOPSIS and Grey Relational Analysis, GRA), time-series forecasting (Grey model GM(1,1) with backtest diagnostics), and nonlinear constrained optimization (Sequential Least Squares Programming, SLSQP). Robustness checks and error diagnostics support the credibility of the findings, suggesting that observed ranking patterns likely reflect structural characteristics rather than methodological artifacts.

Several limitations merit acknowledgment. First, the analysis relies on publicly available data that may be lagged and thus not capture the most recent technological breakthroughs or abrupt policy changes. Second, forecasts assume structural stability and do not explicitly model disruptive events (e.g., major policy shifts or breakthrough technologies). Third, the investment-response formulation adopts simplifying assumptions (such as diminishing marginal returns); real-world input-output dynamics may be affected by complex institutional and market feedback. We have incorporated sensitivity analyses where feasible; future work should consider higher-frequency proxy indicators and explicit shock-scenario modeling.

Based on these findings, we offer the following recommendations to policymakers:

push0 g 0 G1 pop Adopt long-term, systemic AI development strategies rather than fragmented short-term measures;

push0 g 0 G2 pop Prioritize computational capacity and infrastructure investment—allocating materially above-average shares in the short to medium term to remove binding bottlenecks;

push0 g 0 G3 pop Coordinate investments in talent, policy, and R&D so financial, human, and institutional capacities reinforce one another;

push0 g 0 G4 pop For latecomer countries, design cross-cycle catch-up strategies combining large-scale investment, institutional innovation, and international cooperation to mitigate path-dependence disadvantages.

Overall, this study provides a transparent, reproducible, and extensible framework for assessing and optimizing national AI capability, offering quantitative, evidence-based guidance for policy formulation. Despite data and model limitations, the framework delivers valuable insights into structural drivers and long-term evolution of national AI capacity. Future extensions should increase data frequency, incorporate shock scenarios, and strengthen dynamic investment-to-output modeling.

Overall, this study provides a transparent, reproducible, and extensible framework for national AI capability assessment and optimization, offering quantitative and evidence-based reference for policy formulation. Despite limitations in data and model assumptions, the framework yields valuable insights into the structural drivers and long-term evolutionary pathways of national AI capacity building. Future work may advance the field by improving data frequency, introducing shock scenarios, and strengthening dynamic investment-output modeling.