Problem Chosen	2026HSB MCM/ICM Summary Sheet	Team Number
B		2601258

Assessment, Forecasting, and Optimization of National AI Competitiveness

Summary

Artificial intelligence has become a core driver of national competitiveness. This paper develops a unified framework to evaluate, forecast, and optimize national AI capabilities based on 24 indicators from 10 countries (2016–2025).

In Problem 1, Pearson correlation analysis identifies 86 strongly correlated indicator pairs ($|r| > 0.7$). Principal Component Analysis extracts four components explaining 85% cumulative variance, revealing infrastructure, talent, and innovation as key drivers.

For Problem 2, we employ Entropy Weight Method and TOPSIS to rank 2025 AI competitiveness: United States first (0.641), China second (0.510), India third (0.210). Grey Relational Analysis validates the ranking with Spearman correlation $\rho = 0.95$.

Problem 3 applies GM(1,1) grey forecasting to project 2026–2035 trends for all 24 indicators. Backtest diagnostics show median MAPE 10.35%. Results indicate top-three countries maintain structural advantages over the decade.

In Problem 4, Sequential Least Squares Programming optimizes China's CNY 1 trillion investment allocation: infrastructure 32.33%, talent cultivation 17.39%, policy support 17.39%. Sensitivity analysis confirms robustness under weight perturbation.

The framework integrates evaluation, forecasting, and optimization with multi-layer validation, providing reproducible decision support for national AI strategic planning.

Keywords: AI Development Capability, TOPSIS, Grey Forecasting, Entropy Weighting, Investment Optimization, Competitiveness Ranking

Contents

1 Intorduction

1.1 Background

In the contemporary era, artificial intelligence (AI) has emerged as one of the core domains of global technological competition, exerting profound and systemic influences on economic development, social progress, and national security. With the acceleration of a new wave of technological revolution and industrial transformation, AI technologies are fundamentally reshaping traditional industrial structures, modes of production, and governance systems, and have gradually become a key indicator of a nation's scientific strength and overall competitiveness.

Against this backdrop, countries around the world have elevated artificial intelligence to a strategic priority at the national level, continuously increasing investments in algorithmic research, computing infrastructure, data resource development, and the expansion of application scenarios, with the aim of securing a leading position in the global AI competitive landscape.

1.2 Problem Restatement

This study aims to quantitatively evaluate national artificial intelligence (AI) development capabilities, compare global competitiveness, and analyze future development trends through a systematic mathematical modeling framework. The problem is decomposed into four sequential and interrelated tasks:

Task 1: Factor Identification and Correlation Analysis

Relevant data are collected and integrated to identify the key factors influencing national AI development. These factors are quantified, and their intrinsic correlations and interaction mechanisms are analyzed using statistical and visualization methods.

Task 2: Comprehensive Evaluation and Ranking

Based on the quantified factors and their correlations obtained in Task 1, a multi-criteria evaluation model is constructed to assess and rank the AI competitiveness of ten selected countries.

Task 3: Competitiveness Trend Prediction

Using historical data from 2016 to 2025, the future evolution of AI development factors during the period 2026–2035 is predicted. The evaluation model established in Task 2 is then applied to analyze the dynamic changes in national competitiveness rankings over time.

Task 4: Optimal Fund Allocation Strategy

Under a fixed budget constraint of a 1 trillion yuan special fund, a multi-objective optimization model is developed to determine the optimal allocation of resources across AI development factors, with the goal of maximizing China's comprehensive AI competitiveness by 2035.

By sequentially accomplishing these tasks, this study provides a coherent framework for factor identification, comparative evaluation, future trend analysis, and strategic decision support in the global AI competition landscape.

1.3 Our Work

To systematically address these challenges, this study establishes a closed-loop framework of "data–modeling–validation–decision." Figure 1 illustrates the complete workflow from 24-

indicator data processing through four progressive problems, multi-layer validation, to final policy recommendations.

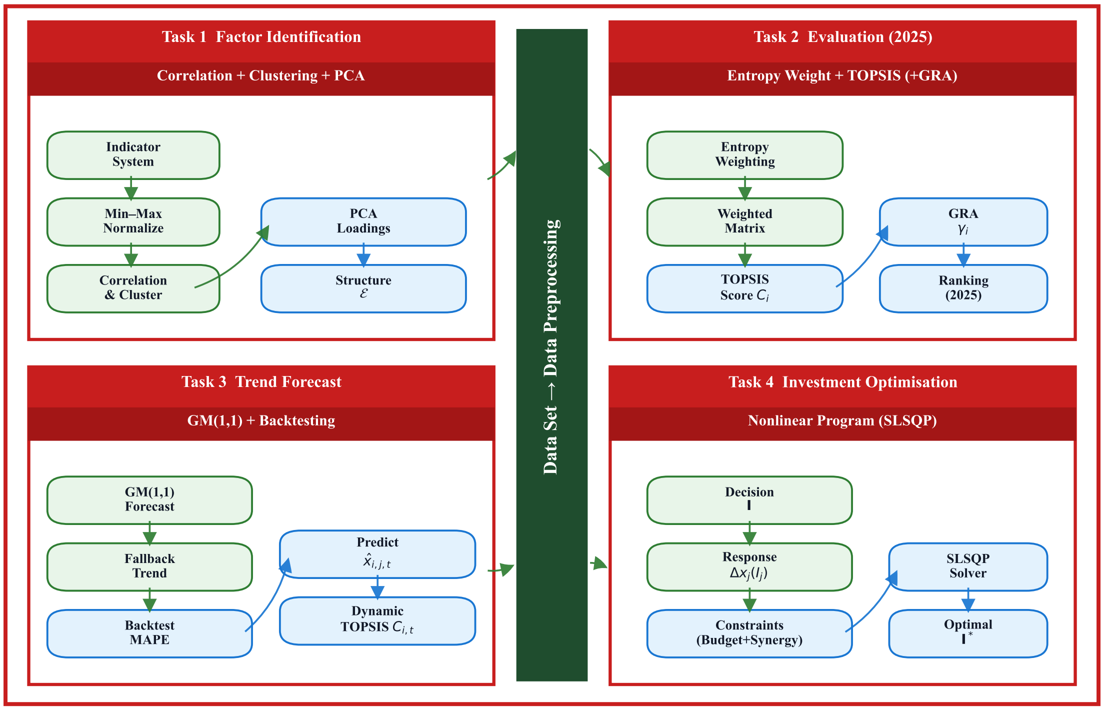


Figure 1: Overall modeling framework for assessing, forecasting, and optimizing national AI competitiveness.

2 Basic Assumption

To ensure the feasibility, consistency, and interpretability of the proposed models, the following basic assumptions are made.

► **Hypothesis 1: Assume that national AI development capability is a latent attribute that can be approximated by a finite set of observable and quantifiable indicators.**

Legitimacy: At the national level, AI development is manifested through measurable outcomes and resource inputs recorded in public statistics. Although the true capability cannot be observed directly, its major characteristics can be reasonably inferred from aggregated, quantifiable indicators.

► **Hypothesis 2: Assume that all indicators within the same evaluation year are cross-sectionally consistent.**

Legitimacy: Although data may be collected from slightly different release years, AI development is a long-term process. Minor temporal discrepancies do not significantly affect national-level competitiveness comparisons and help simplify the modeling process.

► **Hypothesis 3: Assume that the indicators are independent of each other in the weighting and evaluation stages.**

Justification : While interactions among indicators exist, explicitly modeling such dependencies would increase complexity and reduce interpretability. Treating indicators as independent avoids double counting and ensures the applicability of entropy-based and multi-criteria evaluation methods.

▶ **Hypothesis 4: Assume that the fundamental mechanisms of AI development remain stable during the forecasting and optimization period.**

Justification : National AI strategies, infrastructure construction, and talent cultivation generally evolve gradually. This stability makes trend-based prediction and investment optimization reasonable and analytically tractable.

3 Symbols

Table 1: Notation and Symbol Definitions

Symbol	Definition
n	Number of countries (samples)
p	Number of indicators
x_{ij}	Raw value of indicator j for country i
I_j	Comprehensive importance score of indicator j
w_j	Entropy weight of indicator j
C_i	TOPSIS closeness score for country i
$\hat{x}_{i,j,t}$	Predicted value of indicator j for country i in year t
$C_{i,t}$	AI competitiveness score of country i in year t
I_j	Investment allocation for indicator j
$\Delta x_j(\mathbf{I})$	Growth increment of indicator j under investment $\mathbf{I}$

4 Data Explanation

4.1 Data Structure and Scope

This study evaluates $n = 10$ representative countries with significant roles in global AI development: United States, China, United Kingdom, Germany, South Korea, Japan, France, Canada, United Arab Emirates, and India. A comprehensive framework of $p = 24$ quantitative indicators is constructed and organized into six dimensions reflecting different facets of national AI capabilities. For Tasks 1 and 2, cross-sectional data from 2025 are used; for Tasks 3 and 4, panel data spanning 2016–2035 (historical: 2016–2025; forecast: 2026–2035) are employed. The complete indicator structure is presented in Table 2.

4.2 Data Sources

All data were collected from publicly available and authoritative sources, ensuring reliability, cross-country comparability, and reproducibility. Talent and research indicators were sourced from

Table 2: AI Development Indicator Framework

Dimension	Indicators
Talent (T)	No. of AI Researchers, Top AI Scholars, No. of AI Graduates
Application (A)	No. of AI Enterprises, AI Market Size, AI Penetration Rate, No. of LLMs
Policy (P)	No. of AI Policies, AI Subsidies, Public Trust in AI
R&D (R)	Corporate R&D Expenditure, Government AI Investment, International AI Investment
Infrastructure (I)	5G Coverage, GPU Cluster Scale, Internet Bandwidth, Internet Penetration, Power Generation, AI Computing Platforms, No. of Data Centers, No. of TOP500 Systems
Output (O)	No. of AI Books, No. of AI Datasets, GitHub Repositories

UNESCO, OECD Education Statistics, and academic databases (arXiv, Google Scholar). Market and industry data came from CB Insights, Statista, and national statistics bureaus. Policy and investment information was obtained from government AI strategy documents, World Bank, and OECD. Infrastructure metrics were compiled from ITU, TOP500 List, and IEA Energy Statistics. Innovation output indicators were extracted from GitHub API, Kaggle Datasets, and Web of Science.

4.3 Data Preprocessing

To ensure data quality and model applicability, preprocessing steps were applied. For sparse missing values ($< 5\%$), linear interpolation or forward-filling was used based on temporal continuity. The Z-score method was applied to detect outliers; confirmed data errors were corrected using auxiliary sources, while legitimate extreme values were retained. All indicators were treated as benefit-type variables and normalized using Min-Max scaling to eliminate dimensional effects:

$$x'_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}$$

This normalization ensures comparability across indicators with different units and scales.

5 Task 1: Analysis of AI Development Factors

Task 1 aims to *reveal the internal structure* of national AI capability from the 24-indicator system. Rather than prespecifying causal links, we use cross-country co-movement to identify (i) tightly coupled factor groups, (ii) dominant low-dimensional directions, and (iii) a small set of high-leverage indicators. These structural outputs support Task 2 (objective evaluation) and inform Task 4 (synergy-aware constraints).

5.1 Quantification of Key Indicators

Let the min–max normalized indicator matrix be

$$X = [x_{ij}] \in \mathbb{R}^{n\times p}, \qquad n = 10,\ p = 24, \tag{1}$$

where x_{ij} is the normalized value of indicator j for country i. To remove scale effects across heterogeneous indicators, we apply

$$x_{ij} = \frac{x_{ij}^{\mathrm{raw}} - \min(x_j)}{\max(x_j) - \min(x_j)} \in [0, 1]. \tag{2}$$

Matrix X is the common input for correlation, clustering, and PCA in this task.

5.2 Correlation Structure and Interaction Mechanism

Correlation map and strong-link set. We quantify linear association by Pearson correlation:

$$r_{jk} = \frac{\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{\sqrt{\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)^2}\sqrt{\sum_{i=1}^{n}(x_{ik} - \bar{x}_k)^2}}, \qquad R = [r_{jk}] \in \mathbb{R}^{p\times p}. \tag{3}$$

To focus on interpretable dependencies, we define the strong-correlation edge set

$$\mathcal{E} = \{(j, k) \mid |r_{jk}| > \tau,\ j < k\}, \tag{4}$$

where τ is a fixed threshold (used consistently in Task 4 to impose synergy constraints). The heatmap in Fig. 2a shows a dense positive-correlation backbone, suggesting that talent, R&D investment, market scale, and compute infrastructure often advance jointly.

Hierarchical clustering (group-level structure). To move beyond pairwise links, we cluster indicators using correlation distance

$$d_{jk} = 1 - |r_{jk}|, \tag{5}$$

and average linkage between clusters C_a and C_b:

$$D(C_a, C_b) = \frac{1}{|C_a||C_b|}\sum_{j\in C_a}\sum_{k\in C_b} d_{jk}. \tag{6}$$

The dendrogram in Fig. 2b recovers coherent modules, typically separating (i) *investment–market–infrastructure* factors from (ii) *talent–knowledge production* factors. This validates that the indicator system is multidimensional but internally coordinated.

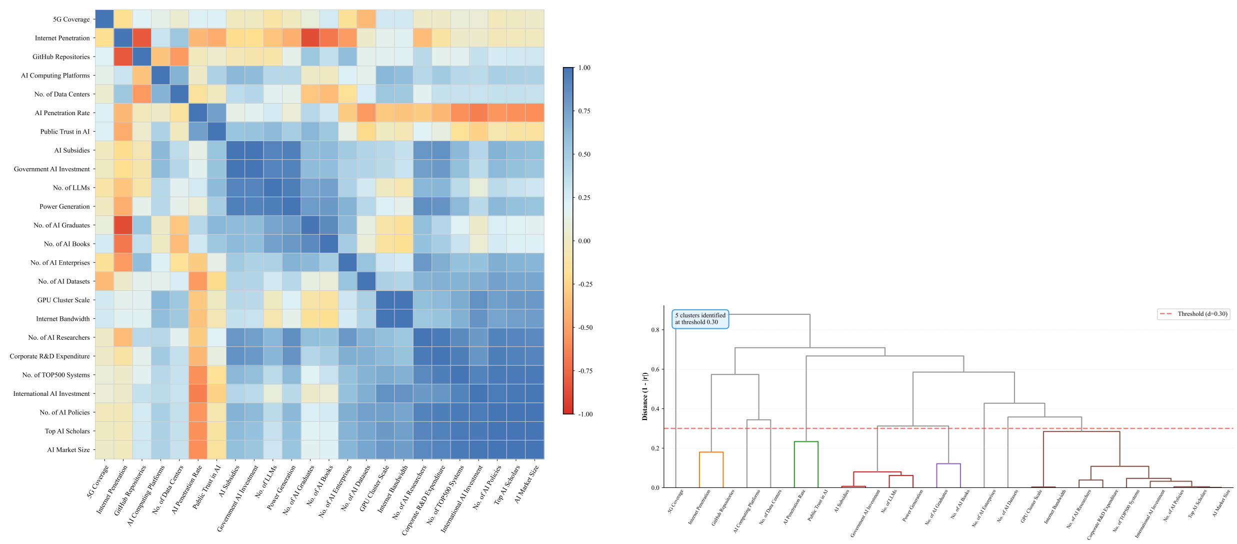
Principal component structure (dominant dimensions). High correlations imply redundancy, so we use PCA to extract dominant directions. Let

$$\tilde{X} = X - \mathbf{1}\bar{X}^T, \qquad C = \frac{1}{n-1}\tilde{X}^T\tilde{X}, \tag{7}$$

and eigen-decompose

$$C = V\Lambda V^T. \tag{8}$$

We retain the smallest m components that explain a high share of variance (in our results, the first four PCs explain $> 90\%$; see Fig. 3a). This indicates that cross-country AI capability differences can be summarized by a low-dimensional latent structure.



(a) Correlation heatmap of 24 indicators. (b) Hierarchical clustering dendrogram.

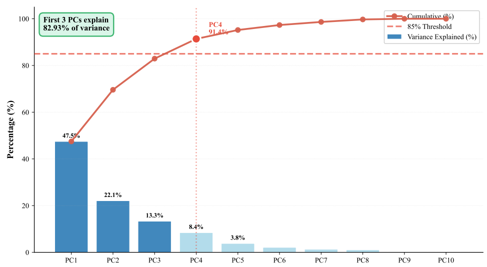
Figure 2: Pairwise association and group structure of AI development factors.

Factor importance and interaction network. We quantify indicator importance by combining squared loadings and explained variance:

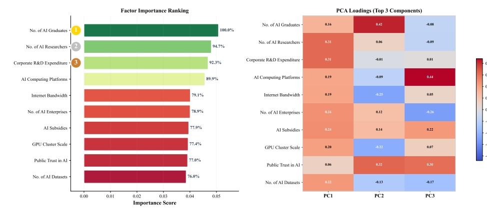
$$I_j = \sum_{k=1}^{m} v_{jk}^2 \cdot \frac{\lambda_k}{\sum_{\ell=1}^{p} \lambda_\ell}. \tag{9}$$

Fig. 3b shows that a limited subset (typically talent, frontier R&D, and high-end compute) contributes most to the explained variation. Finally, the strong-link network induced by $\mathcal{E}$ visualizes system-level coupling: hub indicators (often infrastructure and investment) connect multiple modules, consistent with a coordinated development mechanism (Fig. 3c).

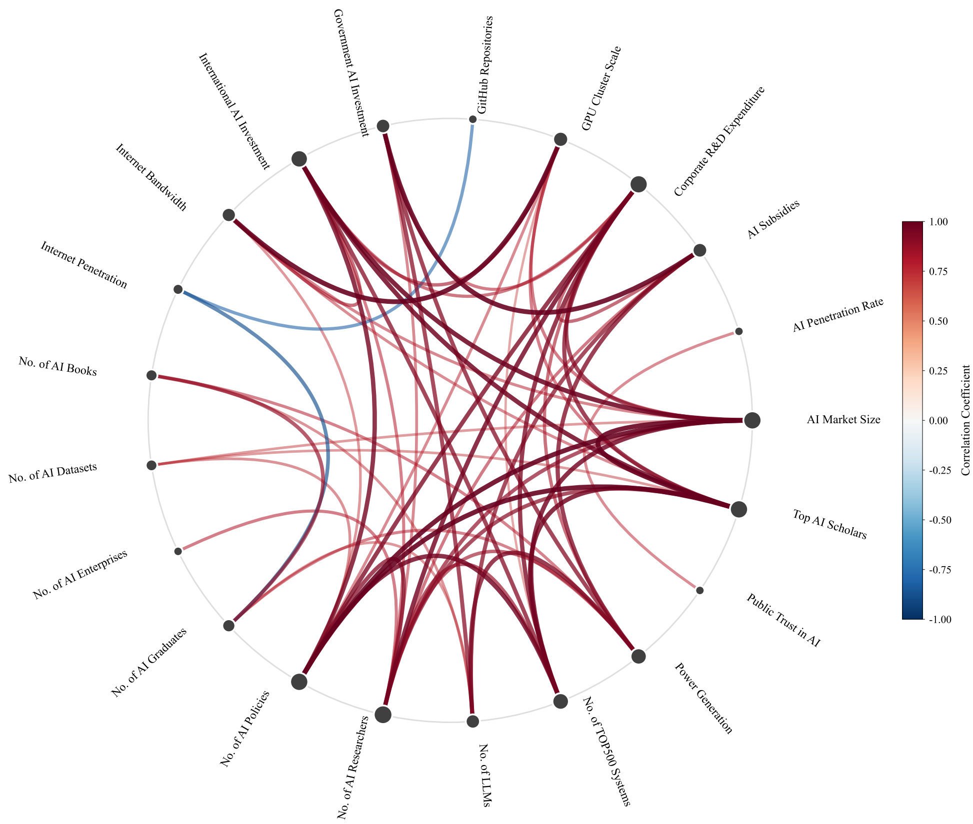
Summary of Task 1. Task 1 identifies a dense correlation backbone, coherent indicator modules, and a low-dimensional dominant structure. These findings motivate (i) objective aggregation in Task 2 and (ii) synergy-aware design in Task 4 using $\mathcal{E}$.



(a) Variance explained by PCs.



(b) Importance ranking I_j.



(c) Strong-correlation interaction network.

Figure 3: Low-dimensional structure and key drivers of AI development capability.

6 Task 2: Evaluation of National AI Competitiveness in 2025

With the indicator structure established in Task 1, Task 2 produces a *single, comparable competitiveness score* for each country in 2025. The design principle is to keep the evaluation standard objective and reproducible: entropy weights determine indicator importance from data dispersion, TOPSIS aggregates performance relative to ideal benchmarks, and GRA provides a structural cross-check.

6.1 Comprehensive Evaluation Methodology

Fig. 4 presents the overall evaluation framework adopted in Task 2. The procedure consists of two sequential phases. In Phase 1, objective indicator weights are determined using the Entropy Weight Method (EWM) after data standardization. In Phase 2, the weighted indicators are aggregated through TOPSIS, while Grey Relational Analysis (GRA) provides a structural validation and complementary perspective. The final competitiveness score is obtained by fusing distance-based and relational measures.

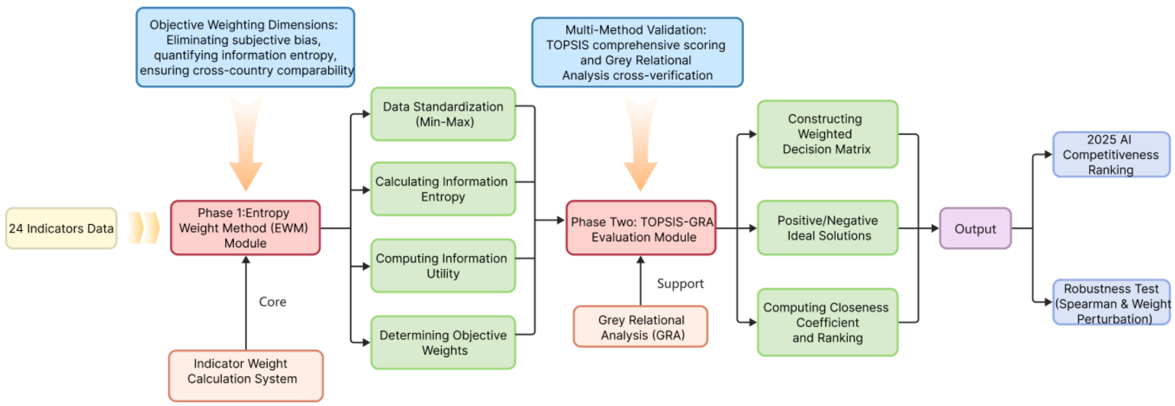


Figure 4: Overall evaluation framework of Task 2 based on EWM–TOPSIS–GRA integration.

Let the normalized indicator matrix for evaluation be

$$X' = (x'_{ij})_{n\times p}, \qquad n = 10,\ p = 24, \tag{10}$$

where all indicators are treated as benefit-type (larger is better).

(1) Entropy weight method (EWM). Define

$$p_{ij} = \frac{x'_{ij}}{\sum_{i=1}^{n} x'_{ij}}, \qquad 0 \ln 0 := 0, \tag{11}$$

$$e_j = -k \sum_{i=1}^{n} p_{ij} \ln p_{ij}, \qquad k = \frac{1}{\ln n}, \tag{12}$$

and the entropy weight

$$w_j = \frac{1 - e_j}{\sum_{j=1}^{p} (1 - e_j)}. \tag{13}$$

Indicators with higher cross-country dispersion obtain larger weights.

(2) TOPSIS aggregation. Construct the weighted matrix

$$v_{ij} = w_j x'_{ij}, \tag{14}$$

with ideal solutions

$$A_j^+ = \max_i v_{ij}, \qquad A_j^- = \min_i v_{ij}. \tag{15}$$

Distances to the ideals are

$$D_i^{\pm} = \sqrt{\sum_{j=1}^{p} (v_{ij} - A_j^{\pm})^2}, \tag{16}$$

and the closeness score is

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \in [0, 1]. \tag{17}$$

(3) Grey relational analysis (GRA) validation and fusion. Let the ideal profile be $v_{0j} = \max_i v_{ij}$ and define

$$\Delta_{ij} = |v_{0j} - v_{ij}|, \quad \Delta_{\min} = \min_{i,j} \Delta_{ij}, \quad \Delta_{\max} = \max_{i,j} \Delta_{ij}. \tag{18}$$

The grey relational coefficient and degree are

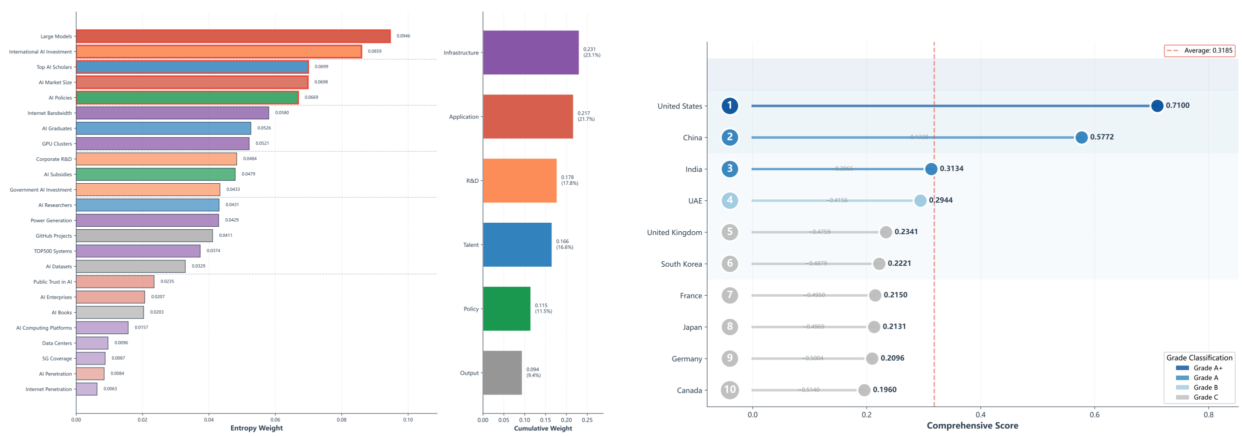
$$\xi_{ij} = \frac{\Delta_{\min} + 0.5\Delta_{\max}}{\Delta_{ij} + 0.5\Delta_{\max}}, \qquad \gamma_i = \frac{1}{p} \sum_{j=1}^{p} \xi_{ij}. \tag{19}$$

To combine distance-based performance (C_i) and structural similarity (γ_i), we use

$$S_i = \frac{C_i + \gamma_i}{2}. \tag{20}$$

6.2 Results and Comparative Analysis

Fig. 5 summarizes the weight pattern (left) and the 2025 competitiveness ranking by TOPSIS (right). The leading tier is clearly separated, while mid- and lower-tier countries form a tighter cluster.



(a) Entropy weight distribution. (b) TOPSIS ranking (2025).

Figure 5: Key outputs of Task 2: indicator weights and the 2025 ranking.

Reliability note. TOPSIS and GRA rankings are highly consistent (Spearman $\rho_s = 0.9758$, $p < 0.001$). Under $\pm 30\%$ weight perturbations, most countries vary by at most one rank, indicating stable ordering under moderate uncertainty.

Table 3: TOPSIS Comprehensive Evaluation Results (2025)

Country	D_i^+	D_i^-	TOPSIS Score C_i	Rank
United States	0.0978	0.1744	0.6407	1
China	0.1320	0.1377	0.5104	2
India	0.2006	0.0533	0.2098	3
UAE	0.1998	0.0449	0.1836	4
United Kingdom	0.2040	0.0225	0.0995	5
South Korea	0.2063	0.0165	0.0740	6
France	0.2065	0.0153	0.0688	7
Japan	0.2064	0.0141	0.0640	8
Germany	0.2063	0.0138	0.0625	9
Canada	0.2091	0.0090	0.0414	10

Table 4: Final AI Competitiveness Ranking (2025) by Fusion Score

Country	TOPSIS Rank	GRA Rank	TOPSIS C_i	GRA γ_i	Fusion S_i	Grade
United States	1	1	0.6407	0.7793	0.7100	A+
China	2	2	0.5104	0.6440	0.5772	A
India	3	3	0.2098	0.4170	0.3134	A
UAE	4	4	0.1836	0.4052	0.2944	B
United Kingdom	5	6	0.0995	0.3686	0.2341	C
South Korea	6	5	0.0740	0.3703	0.2221	C
France	7	8	0.0688	0.3612	0.2150	C
Japan	8	7	0.0640	0.3623	0.2131	C
Germany	9	9	0.0625	0.3567	0.2096	C
Canada	10	10	0.0414	0.3506	0.1960	C

7 Task 3: Forecasting AI Competitiveness (2026–2035)

Fig. 6 illustrates the integrated three-phase pipeline adopted in Task 3. Phase 1 prepares historical indicator sequences and estimates GM(1,1) parameters. Phase 2 projects country–indicator trajectories for 2026–2035. Phase 3 evaluates annual AI competitiveness by inheriting the fixed weights and TOPSIS framework from Task 2, yielding ranking evolution and diagnostic outputs.

Task 3 extends the 2025 evaluation to a dynamic horizon. The key rule is consistency: *the evaluation mechanism (weights and TOPSIS) is fixed*, and only the indicator trajectories evolve. Therefore, any ranking change during 2026–2035 can be attributed to data-driven indicator dynamics rather than altered standards.

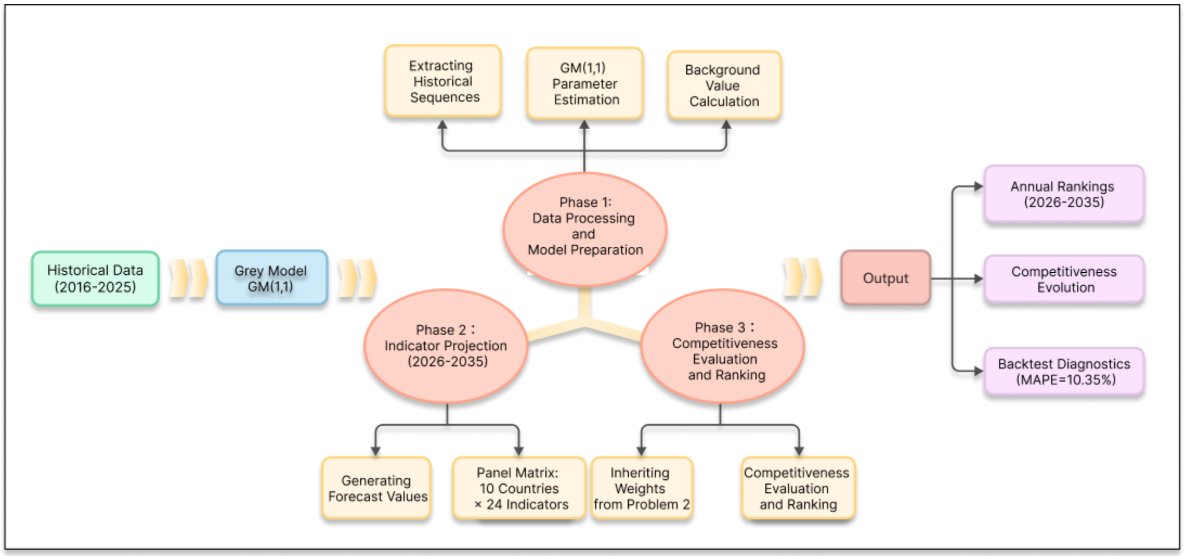
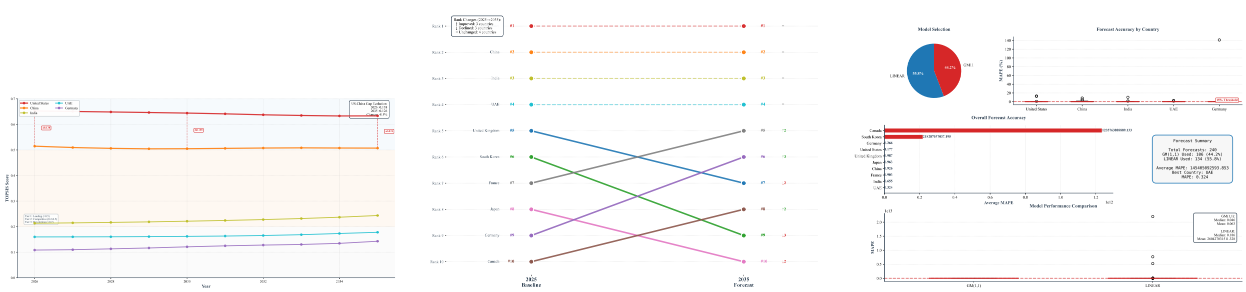


Figure 6: Overall forecasting and evaluation pipeline of Task 3.

7.1 Indicator-Level Trend Prediction

Let $x_{i,j,t}$ be indicator j of country i in year t. For each country–indicator series over 2016–2025, we forecast $\hat{x}_{i,j,t}$ for $t = 2026, \ldots, 2035$ independently.

Given short sequences ($T = 10$), GM(1,1) is used as the primary model. When GM(1,1) backtesting is unsatisfactory, a constrained linear trend model is used as a fallback under the same non-negativity and truncation rules. One-step-ahead validation (train 2016–2024, predict 2025) uses MAPE as the main metric. In our pipeline, GM(1,1) covers 44.17% of the 240 country–indicator series, while the fallback is used for 55.83%, with a median MAPE of 0.1035.



(a) Score gaps (2026–2035). (b) 2025 vs. 2035 slope chart. (c) Forecast diagnostics panel.

Figure 7: Forecasting and evaluation diagnostics for Task 3.

7.2 Annual Evaluation and Score Evolution

After forecasting, we construct the predicted indicator matrix for each year t:

$$\hat{X}_t = \left(\hat{x}_{i,j,t}\right)_{n \times p}.$$

We keep the entropy weights from Task 2 fixed as $W = (w_1, \ldots, w_p)$, and apply the same TOPSIS procedure to obtain the annual closeness scores $C_{i,t} \in [0, 1]$.

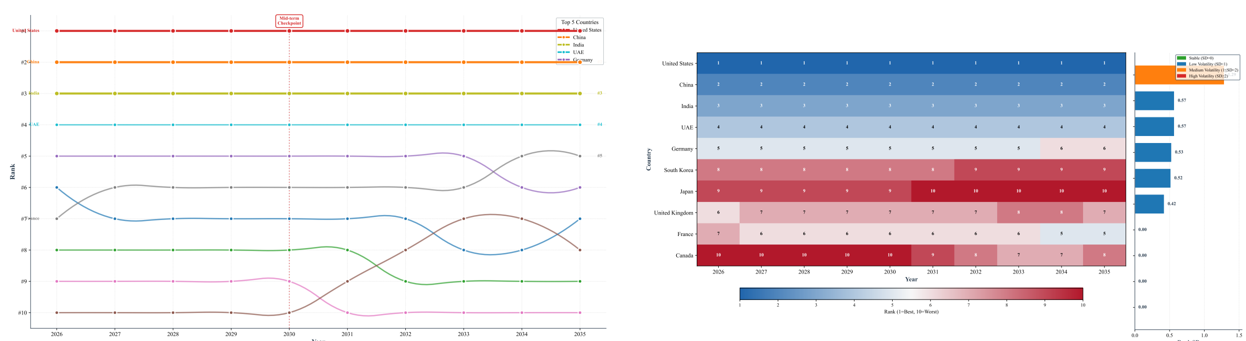
Table 5 reports the TOPSIS scores for representative years (2026, 2030, 2035), while Fig. 7a visualizes the score convergence and the evolution of cross-country gaps.

Table 5: Selected TOPSIS scores $C_{i,t}$ for 2026, 2030, and 2035.

Country	2026	2030	2035
United States	0.653	0.644	0.633
China	0.515	0.505	0.507
India	0.213	0.221	0.244
United Arab Emirates	0.160	0.162	0.178
France	0.069	0.080	0.168
Germany	0.108	0.121	0.143
United Kingdom	0.069	0.073	0.102
Canada	0.042	0.057	0.101
South Korea	0.055	0.066	0.097
Japan	0.054	0.060	0.093

7.3 Ranking Evolution and Stability

Countries are ranked annually by $C_{i,t}$. Fig. 8 shows both the ranking trajectories (bump chart) and the stability heatmap. The top tier remains stable, while rank swaps occur mainly among closely competing mid-/lower-tier countries.



(a) Bump chart (2026–2035).

(b) Rank stability heatmap.

Figure 8: Ranking evolution and stability over the forecast horizon.

7.4 Interpretation and Robustness

Because weights and evaluation rules are fixed, ranking changes come solely from predicted indicator trajectories. Observed swaps are local (small score gaps) rather than structural reversals, consistent with the convergence pattern in Fig. 7a. Forecast reliability is supported by the diagnostics in Fig. 7c.

Table 6: Rank stability summary (2026–2035).

Country	AvgRank	StdRank	BestRank	WorstRank
United States	1.00	0.00	1	1
China	2.00	0.00	2	2
India	3.00	0.00	3	3
United Arab Emirates	4.00	0.00	4	4
Germany	5.20	0.40	5	6
France	5.90	0.54	5	7
United Kingdom	7.10	0.54	6	8
South Korea	8.40	0.49	8	9
Canada	8.90	1.22	7	10
Japan	9.50	0.50	9	10

7.5 Summary

Task 3 couples indicator-level forecasting with the fixed Task 2 evaluation to project 2026–2035 competitiveness. The results suggest stable global leadership, gradual score convergence, and limited, interpretable mid-tier rank changes, providing the scenario baseline required by Task 4.

8 Task 4: Optimization of AI Development Investment Strategy

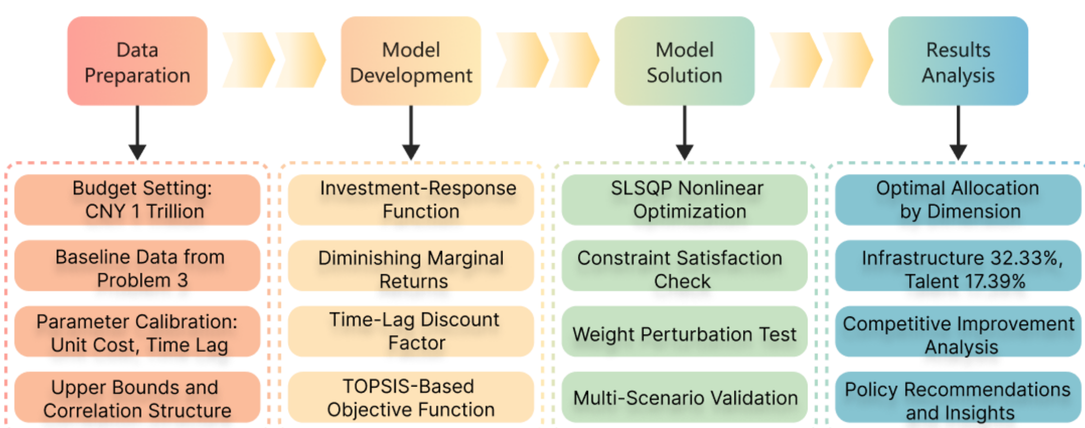


Figure 9: Overall decision and optimization framework of Task 4.

Fig. 9 summarizes the end-to-end optimization pipeline of Task 4. Starting from the baseline trajectories obtained in Task 3 and a fixed total budget, an investment–response model with diminishing returns and time-lag effects is constructed. The resulting nonlinear constrained problem is solved using SLSQP, and the optimized allocation is evaluated under the same TOPSIS framework as in Task 2, yielding allocation patterns, competitiveness gains, and policy insights.

Tasks 1–3 establish a consistent pipeline: indicators → objective weights → TOPSIS scores → multi-year scenario. Task 4 turns this pipeline into a decision problem. Starting in 2026, China allocates an additional *1 trillion RMB* special fund, and the goal is to maximize China's *2035* comprehensive AI competitiveness under the *same* evaluation standard and comparison set.

8.1 Model Formulation and Constraints

Decision variables and budget. Let $\mathbf{I} = (I_1, \ldots, I_p)^\top$ be the allocation across $p = 24$ indicators. To match the policy budget and the numerical outputs, we measure investment in *hundred-million RMB* (100 million yuan). Thus, the total budget is

$$\sum_{j=1}^{p} I_j = B, \qquad B = 10000 \text{ (100 million yuan)}. \tag{21}$$

Objective (fixed evaluation standard). Let $S_{\text{CN}}(\cdot)$ denote China's TOPSIS closeness coefficient under the fixed weight vector $\mathbf{w}$ obtained in Task 2. The optimization objective is

$$\mathbf{I}^* = \arg\max_{\mathbf{I}} \ S_{\text{CN}}\Big(X^{2035}(\mathbf{I}); \mathbf{w}\Big), \tag{22}$$

where $X^{2035}(\mathbf{I})$ is the 2035 evaluation matrix: all non-China rows are fixed at the Task 3 scenario values, and only China's row is updated by the investment response.

Inputs from previous tasks.

$$\mathbf{w} \leftarrow \text{Task 2 (EWM weights)}, \tag{23}$$
$$\mathbf{x}_{\text{CN}}^{\text{base}} \leftarrow \text{Task 3 (China baseline trajectory)}, \tag{24}$$
$$X_{2035}^{\text{scen}} \leftarrow \text{Task 3 (2035 scenario for all countries)}, \tag{25}$$
$$\mathcal{E} \leftarrow \text{Task 1 (strong correlation structure)}. \tag{26}$$

Investment–indicator response (diminishing returns & time lag). For indicator j, introduce: unit cost C_j, time-lag discount γ_j, and saturation upper bound L_j. The investment-induced increment is modeled by

$$\Delta x_j(\mathbf{I}) = \frac{I_j}{C_j}\left(1 - \frac{x_j^{\text{base}}}{L_j}\right)\gamma_j, \qquad j = 1, \ldots, p, \tag{27}$$

and the post-investment level is truncated by feasibility:

$$x_{\text{CN},j}^{2035}(\mathbf{I}) = \min\left\{x_j^{\text{base}} + \Delta x_j(\mathbf{I}),\ L_j\right\}. \tag{28}$$

Upper bounds follow a relative-competitiveness rule:

$$L_j = \begin{cases} 1.5\, x_{j,2025}^{\text{CN}}, & x_{j,2025}^{\text{CN}} \ge x_{j,2025}^{\text{US}}, \\ 3.0\, x_{j,2025}^{\text{US}}, & x_{j,2025}^{\text{CN}} < x_{j,2025}^{\text{US}}, \end{cases} \qquad L_j \le 100 \text{ (ratio-type indicators)}. \tag{29}$$

Time-lag discounts are grouped as $\gamma_j \in \{1.0, 0.8, 0.6\}$ for short-/medium-/long-horizon effects.

TOPSIS evaluation (same as Task 2). Let $X = X^{2035}(\mathbf{I})$. Using vector normalization,

$$\tilde{X} = XD^{-1}, \quad D = \mathrm{diag}\big(\|X_{:,1}\|_2, \ldots, \|X_{:,p}\|_2\big), \tag{30}$$

$$V = \tilde{X}\,\mathrm{diag}(\mathbf{w}), \quad \mathbf{v}^+ = \max_i V_{i,:}, \quad \mathbf{v}^- = \min_i V_{i,:}, \tag{31}$$

$$D_i^{\pm} = \|V_{i,:} - \mathbf{v}^{\pm}\|_2, \qquad S_i = \frac{D_i^-}{D_i^+ + D_i^-}. \tag{32}$$

The objective (22) maximizes S_{CN}.

Constraints. (1) Budget and bounds:

$$\sum_{j=1}^{p} I_j = B, \qquad I_{\min} \le I_j \le I_{\max}. \tag{33}$$

(2) Synergy constraints (from Task 1 strong links). To avoid structurally imbalanced growth, we impose ratio-type coupling constraints:

$$\begin{aligned} x_{\text{Large Models}} &\le 200\, x_{\text{GPU}}, \\ x_{\text{Top AI Scholars}} &\le 5.0\, x_{\text{Researchers}}, \\ x_{\text{AI Publications}} &\le 0.24\, x_{\text{Researchers}}, \\ x_{\text{AI Enterprises}} &\le 78\, x_{\text{AI Market}}, \\ x_{\text{AI Datasets}} &\le 0.75\, x_{\text{Enterprise R\&D}}. \end{aligned} \tag{34}$$

Solution method. The nonlinear constrained program is solved by SLSQP with equal-allocation initialization $I_j = B/p$, maximum 500 iterations, and tolerance 10^{-6}.

8.2 Optimal Allocation Results and Insights

All allocations below are in *100 million yuan (RMB)*.

Overall allocation pattern. The optimized plan prioritizes *infrastructure–policy–market*, with secondary emphasis on enterprise R&D and high-end talent. Table 7 and Fig. 10 summarize the TAPRIO dimension distribution.

Table 7: Dimension-level distribution of the 1 trillion RMB special fund.

Dimension	Investment (100 million yuan)	Share (%)
Infrastructure (I)	3232.64	32.33
Talent (T)	1739.28	17.39
Policy (P)	1739.11	17.39
Application (A)	1512.40	15.12
R&D (R)	1264.80	12.65
Output (O)	511.84	5.12

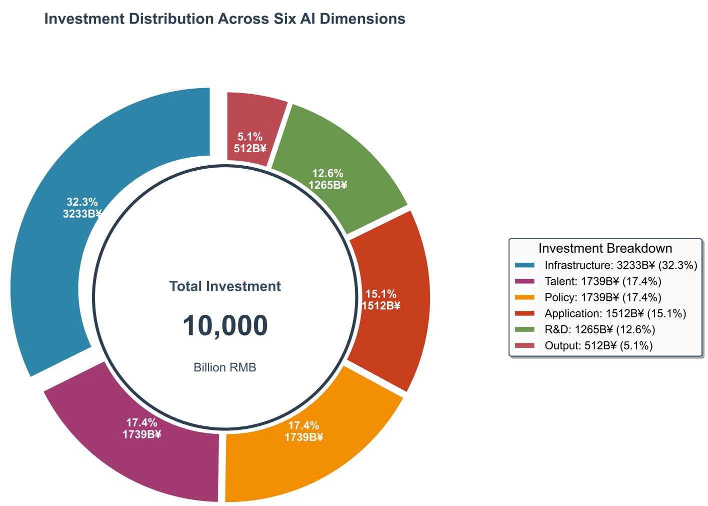


Figure 10: Dimension-level investment distribution (donut chart).

Indicator-level priorities. Top-10 funded indicators account for the majority of the budget (Table 8), and Fig. 11 visualizes the allocation rank.

Table 8: Top-10 funded indicators under the optimized allocation.

Rank	Indicator	Investment (100 million yuan)	Share (%)
1	GPU cluster scale	1500.00	15.00
2	Number of AI policies	1474.23	14.74
3	AI market size	1121.23	11.21
4	Enterprise R&D expenditure	927.66	9.28
5	Top AI scholars	820.92	8.21
6	AI researchers	816.90	8.17
7	TOP500 supercomputer count	731.54	7.32
8	GitHub AI-related projects	308.86	3.09
9	Number of data centers	281.74	2.82
10	Internet bandwidth	232.86	2.33

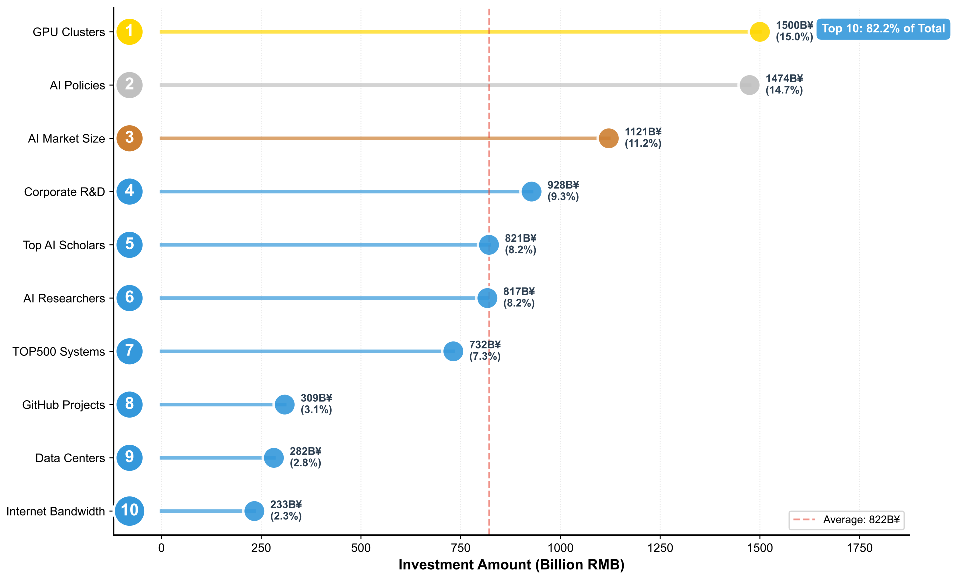


Figure 11: Top-10 indicator investments (lollipop chart).

Full 24-indicator allocation (reproducibility).

Table 9: Full investment allocation across all 24 indicators.

Rank	Indicator	Investment (100 million yuan)	Share (%)
1	GPU cluster scale	1500.000	15.000
2	Number of AI policies	1474.225	14.742
3	AI market size	1121.225	11.212
4	Enterprise R&D expenditure	927.661	9.277
5	Top AI scholars	820.922	8.209
6	AI researchers	816.904	8.169
7	TOP500 supercomputer count	731.544	7.315
8	GitHub AI-related projects	308.863	3.089
9	Number of data centers	281.743	2.817
10	Internet bandwidth	232.859	2.329
11	Number of AI enterprises	218.799	2.188
12	International AI investment	189.042	1.890
13	AI subsidy amount	149.758	1.498
14	Government AI investment	148.102	1.481
15	AI computing platforms	145.773	1.458
16	Electricity production	138.211	1.382
17	AI application penetration	122.376	1.224
18	AI social trust	115.123	1.151
19	Number of AI datasets	101.499	1.015
20	Number of AI books	101.482	1.015
21	Number of AI graduates	101.459	1.015
22	5G coverage rate	101.438	1.014
23	Internet penetration rate	101.075	1.011
24	Number of large models	50.000	0.500

Indicator improvements under the response function. Using Eqs. (27)–(28), we compute China's post-investment indicator levels in 2035. Table 10 reports selected indicators (baseline vs. post-investment), and Fig. 12 visualizes growth rates (log-scale).

Table 10: Selected indicator changes from baseline (2026) to post-investment level (2035).

Indicator	Baseline 2026	Post-invest 2035	Increment	Growth (%)
GitHub AI-related projects	5094.574	99000.000	93905.426	1843.244
AI graduates	70.000	97.500	27.500	39.286
GPU cluster scale	3.967	32.968	29.001	731.056
Internet bandwidth	1.607	6.201	4.594	285.874
Number of AI policies	72.933	270.580	197.647	270.998
AI researchers	279.342	900.000	620.658	222.186
TOP500 supercomputer cnt	83.200	205.780	122.580	147.332
AI market size	138.000	295.159	157.159	113.883
AI enterprises	5901.133	11830.887	5929.754	100.485
Enterprise R&D exp.	665.333	1172.337	507.004	76.203

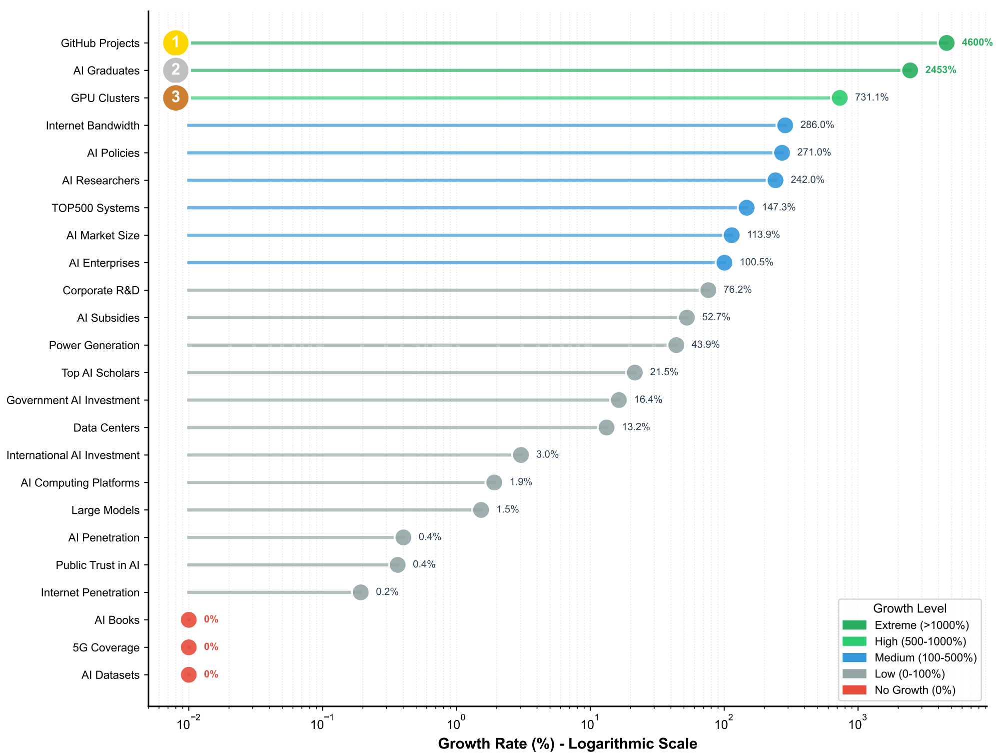


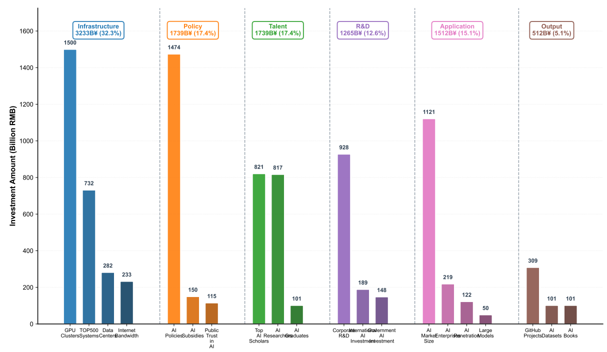
Figure 12: Indicator growth rates under the optimized investment (log-scale lollipop).

Impact on 2035 TOPSIS competitiveness (within-year comparison). Under the fixed TOPSIS procedure (Task 2) and the 2035 comparison set (Task 3 scenario), China's post-investment closeness coefficient is

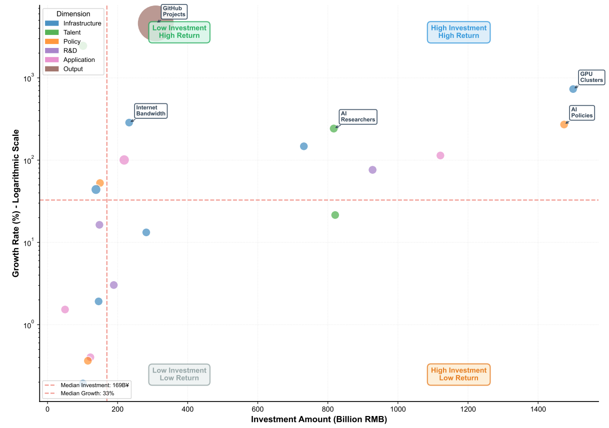
$$S_{\mathrm{CN}}^{2035,\mathrm{post}} = 0.54717. \tag{35}$$

From Task 3 (no additional investment), China's 2035 baseline score is $C_{\mathrm{CN},2035} = 0.507$ (Table 5), so the optimized plan yields an improvement of approximately +0.040 under the same 2035 benchmark environment.

Complementary diagnostics and policy translation. Fig. 13 links the optimized allocation to (i) dimension-level changes and (ii) investment efficiency patterns.



(a) TAPRIO dimension comparison (2035).



(b) Efficiency quadrant (bubble chart).

Figure 13: Interpretation tools for the optimized investment plan.

Actionable recommendations.

1. **Strategic capacity foundation (I & P):** prioritize GPU clusters, TOP500 capacity, data centers, and a coherent policy package to avoid compute and governance bottlenecks.
2. **Innovation production engine (A & R):** expand AI market size and enterprise R&D to convert funded capacity into scalable applications and industrial output.
3. **Talent upgrading (T):** target high-end researchers and top scholars as multipliers, consistent with the synergy constraints in (34).

All results above are generated under the fixed Task 2 evaluation scheme and the Task 3 2035 scenario, ensuring model consistency and reproducibility.

9 Conclusions and Implications

This study constructs a unified, data-driven modeling framework for evaluating, comparing, forecasting, and optimizing national artificial intelligence (AI) development capability. Built upon a consistent system of 24 indicators and a unified evaluation protocol, the framework encompasses factor identification (Problem 1), comprehensive evaluation (Problem 2), temporal forecasting (Problem 3), and resource optimization (Problem 4), forming a closed-loop analytical pipeline that ensures cross-country and cross-period comparability as well as full result traceability.

Problem 1 reveals that national AI development emerges from a tightly coupled system of infrastructure, human capital, policy environment, and innovation output rather than isolated factors. Multiple indicator pairs exhibit strong positive Pearson correlations (several with $|r| > 0.7$), highlighting the importance of coordinated investments across dimensions. **Problem 2** demonstrates clear stratification in 2025: the United States ranks first (TOPSIS = 0.641), China second

(0.510), and India third (0.210). High consistency between TOPSIS and Grey Relational Analysis rankings reinforces robustness, suggesting that stratification reflects structural rather than transient drivers. **Problem 3** extends the analysis to 2026–2035: under structural-stability assumptions, the global AI competitiveness landscape exhibits strong inertia, with leading countries (U.S., China, India) maintaining advantages in the medium term and only limited rank exchanges among mid-tier countries. Backtest diagnostics report a median MAPE = 0.1035 (10.35%), indicating acceptable forecast accuracy. **Problem 4**, under the scenario of an additional CNY 1 trillion for China from 2026, recommends prioritizing infrastructure investment (32.33%, $\approx$ CNY 323.3 billion), followed by talent cultivation and policy support (each 17.39%). This allocation pattern indicates high marginal returns to computational capacity and foundational capabilities and underscores the need for coordinated institutional and human-capital measures.

Methodologically, this study integrates entropy-based weighting (Entropy Weight Method, EWM), multi-model cross-validation (TOPSIS and Grey Relational Analysis, GRA), time-series forecasting (Grey model GM(1,1) with backtest diagnostics), and nonlinear constrained optimization (Sequential Least Squares Programming, SLSQP). Robustness checks and error diagnostics support the credibility of the findings, suggesting that observed ranking patterns likely reflect structural characteristics rather than methodological artifacts.

Several limitations merit acknowledgment. First, the analysis relies on publicly available data that may be lagged and thus not capture the most recent technological breakthroughs or abrupt policy changes. Second, forecasts assume structural stability and do not explicitly model disruptive events (e.g., major policy shifts or breakthrough technologies). Third, the investment-response formulation adopts simplifying assumptions (such as diminishing marginal returns); real-world input-output dynamics may be affected by complex institutional and market feedback. We have incorporated sensitivity analyses where feasible; future work should consider higher-frequency proxy indicators and explicit shock-scenario modeling.

Based on these findings, we offer the following recommendations to policymakers:

1. Adopt long-term, systemic AI development strategies rather than fragmented short-term measures;

2. Prioritize computational capacity and infrastructure investment—allocating materially above-average shares in the short to medium term to remove binding bottlenecks;

3. Coordinate investments in talent, policy, and R&D so financial, human, and institutional capacities reinforce one another;

4. For latecomer countries, design cross-cycle catch-up strategies combining large-scale investment, institutional innovation, and international cooperation to mitigate path-dependence disadvantages.

Overall, this study provides a transparent, reproducible, and extensible framework for assessing and optimizing national AI capability, offering quantitative, evidence-based guidance for policy formulation. Despite data and model limitations, the framework delivers valuable insights into structural drivers and long-term evolution of national AI capacity. Future extensions should increase data frequency, incorporate shock scenarios, and strengthen dynamic investment-to-output modeling.

Overall, this study provides a transparent, reproducible, and extensible framework for national AI capability assessment and optimization, offering quantitative and evidence-based reference for policy formulation. Despite limitations in data and model assumptions, the framework yields valuable insights into the structural drivers and long-term evolutionary pathways of national AI capacity building. Future work may advance the field by improving data frequency, introducing shock scenarios, and strengthening dynamic investment-output modeling.

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Appendix

Appendix A. Indicator System (24 Indicators) and Dimension Mapping

This paper uses a consistent set of $p = 24$ benefit-type indicators across Tasks 1–4 (Table 2 in the main text). The six-dimension TAPRIO structure is: Talent (T), Application (A), Policy (P), R&D (R), Infrastructure (I), and Output (O).

Table 11: A-1. Indicator list and dimension mapping (consistent across Tasks 1–4).

Dimension	Indicator (as used in the main text)
T	No. of AI Researchers
T	Top AI Scholars
T	No. of AI Graduates
A	No. of AI Enterprises
A	AI Market Size
A	AI Penetration Rate
A	No. of LLMs
P	Public Trust in AI
P	No. of AI Policies
P	AI Subsidies
R	Corporate R&D Expenditure
R	Government AI Investment
R	International AI Investment
I	5G Coverage
I	GPU Cluster Scale
I	Internet Bandwidth
I	Internet Penetration
I	Power Generation
I	AI Computing Platforms
I	No. of Data Centers
I	No. of TOP500 Systems
O	No. of AI Books
O	No. of AI Datasets
O	GitHub Repositories

Data scope and usage. The evaluation set includes $n = 10$ countries (Section 4.1). Tasks 1–2 use the 2025 cross-sectional data, while Tasks 3–4 use the 2016–2025 historical panel and the 2026–2035 forecasts (Section 4.1). All indicators are treated as benefit-type variables and normalized using min–max scaling (Section 4.3 and Eq. (2)).

Appendix B. Evaluation Pipeline Reference (EWM + TOPSIS + Optional GRA)

For brevity, the full derivations are not duplicated here. The paper follows a fixed evaluation standard across years and tasks:

- **Entropy Weight Method (EWM)**: Eqs. (11)–(13) in Section 6.1 define the proportion matrix, entropy, and weights w_j.
- **TOPSIS aggregation**: Eqs. (14)–(17) in Section 6.1 define weighted performance, ideal solutions, distances $D_i^{\pm}$, and the closeness score C_i.
- **Grey Relational Analysis (GRA) cross-check and fusion (optional)**: Eqs. (18)–(20) in Section 6.1 provide the GRA degree γ_i and fusion score $S_i = (C_i + \gamma_i)/2$. The main ranking discussion uses the TOPSIS score and reports GRA consistency as robustness evidence (Section 6.2).

Consistency rule used in Task 3/4. Task 3 keeps the weight vector $\mathbf{w}$ fixed from Task 2 and applies the same TOPSIS procedure to annual predicted matrices $\hat{X}_t$ (Section 7.2). Task 4 also evaluates the post-investment 2035 matrix $X_{2035}(\mathbf{I})$ using the same fixed weights and TOPSIS standard (Eqs. (30)–(32)).

Appendix C. Forecasting Implementation Notes (GM(1,1) + Fallback)

Task 3 forecasts each country–indicator series independently over 2016–2025 and projects 2026–2035 (Section 7.1). Given the short time length ($T = 10$), GM(1,1) is used as the primary model, with a constrained linear trend fallback when GM(1,1) backtesting is unsatisfactory. One-step-ahead validation (train 2016–2024, predict 2025) uses MAPE as the main accuracy metric (Section 7.1 and Fig. 4c). In the reported pipeline, GM(1,1) covers 44.17% of series and the fallback covers 55.83%, with a median MAPE of 0.1035.

Appendix D. Task 4 Optimization Settings (Condensed)

Decision variables and budget. The allocation vector is $\mathbf{I} = (I_1, \ldots, I_p)^\top$ over $p = 24$ indicators. Investment is measured in 100 million RMB; the total budget is $B = 10000$ (Eq. (21)).

Objective. The objective maximizes China's 2035 TOPSIS closeness under the fixed weight vector $\mathbf{w}$ from Task 2:

$$\mathbf{I}^* = \arg\max_{\mathbf{I}} S_{\mathrm{CN}}(X_{2035}(\mathbf{I}); \mathbf{w}),$$

as defined in Eq. (22).

Investment–indicator response. Indicator increments follow diminishing returns with saturation and time-lag discount (Eq. (27)), and the post-investment level is truncated by an upper bound (Eq. (28)). Upper bounds follow the relative-competitiveness rule in Eq. (29), including a cap for ratio-type indicators.

Constraints. Budget and per-indicator bounds are given in Eq. (33). Synergy constraints are ratio-type coupling constraints derived from the strong-correlation structure in Task 1 (Eq. (34)), used to avoid structurally imbalanced growth.

Solver. The nonlinear constrained program is solved by SLSQP with equal-allocation initialization $I_j = B/p$, maximum 500 iterations, and tolerance 10^{-6} (end of Section 8.1).

Appendix E. Reproducibility Checklist (Minimal)

All reported figures and tables are generated from:

- **Task 1 outputs:** correlation heatmap, clustering dendrogram, PCA variance and importance, and interaction network (Figs. 1–2).
- **Task 2 outputs:** entropy weight distribution, TOPSIS/GRA ranking, and robustness statements (Fig. 3; Tables 3–4).
- **Task 3 outputs:** forecasting diagnostics, representative-year TOPSIS scores, and rank stability summaries (Figs. 4–5; Tables 5–6).
- **Task 4 outputs:** optimized allocation and indicator changes under the response function (Figs. 6–9; Tables 7–10).

The evaluation standard (weights and TOPSIS rules) is fixed once established in Task 2 and is reused without modification in Tasks 3–4, ensuring cross-year and cross-scenario comparability (Sections 7.2 and 8.1).

AI Use Report

In the preparation of this paper, artificial intelligence (AI) tools were used solely as *auxiliary support tools* to assist with language organization, structural refinement, and consistency checking. The use of AI tools did **not** replace any part of the mathematical modeling, data analysis, computational implementation, or result interpretation conducted by the team.

Specifically, AI tools were used for the following purposes:

- Assisting with English language polishing and improving clarity and coherence of written expressions;
- Providing suggestions on LaTeX formatting and structural organization of the paper;
- Helping to summarize and restate model descriptions and results based strictly on outputs generated by the team;
- Supporting logical consistency checks of explanations without introducing new assumptions or results.

All mathematical models, assumptions, parameter selections, data preprocessing procedures, algorithm implementations, numerical experiments, figures, tables, and conclusions were **independently designed, implemented, and verified by the team**. No AI tool was used to generate raw data, perform numerical computation, determine model structure, or make substantive analytical decisions.

The team takes full responsibility for the originality, correctness, and integrity of the models and results presented in this paper. The use of AI tools strictly complies with the competition requirements and academic integrity standards.