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Summary

Here is the summary of paper.

Please replace this text with your own summary. This summary should briefly describe the problem you are addressing, the methods you used, and the key results or conclusions you reached. Make sure to keep it concise and informative.

Keywords: KFC V50 Chicken

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1 Intorduction

1.1 Background

In the contemporary era, artificial intelligence (AI) has emerged as one of the core domains of global technological competition, exerting profound and systemic influences on economic development, social progress, and national security. With the acceleration of a new wave of technological revolution and industrial transformation, AI technologies are fundamentally reshaping traditional industrial structures, modes of production, and governance systems, and have gradually become a key indicator of a nation's scientific strength and overall competitiveness.

Against this backdrop, countries around the world have elevated artificial intelligence to a strategic priority at the national level, continuously increasing investments in algorithmic research, computing infrastructure, data resource development, and the expansion of application scenarios, with the aim of securing a leading position in the global AI competitive landscape.

1.2 Problem Restatement

This study aims to quantitatively evaluate national artificial intelligence (AI) development capabilities, compare global competitiveness, and analyze future development trends through a systematic mathematical modeling framework. The problem is decomposed into four sequential and interrelated tasks:

Task 1: Factor Identification and Correlation Analysis

Relevant data are collected and integrated to identify the key factors influencing national AI development. These factors are quantified, and their intrinsic correlations and interaction mechanisms are analyzed using statistical and visualization methods.

Task 2: Comprehensive Evaluation and Ranking

Based on the quantified factors and their correlations obtained in Task 1, a multi-criteria evaluation model is constructed to assess and rank the AI competitiveness of ten selected countries.

Task 3: Competitiveness Trend Prediction

Using historical data from 2016 to 2025, the future evolution of AI development factors during the period 2026–2035 is predicted. The evaluation model established in Task 2 is then applied to analyze the dynamic changes in national competitiveness rankings over time.

Task 4: Optimal Fund Allocation Strategy

Under a fixed budget constraint of a 1 trillion yuan special fund, a multi-objective optimization model is developed to determine the optimal allocation of resources across AI development factors, with the goal of maximizing China's comprehensive AI competitiveness by 2035.

By sequentially accomplishing these tasks, this study provides a coherent framework for factor identification, comparative evaluation, future trend analysis, and strategic decision support in the global AI competition landscape.

1.3 Our Work

这是我们的工作介绍

2 Basic Assumption

To ensure the feasibility, consistency, and interpretability of the proposed models, the following basic assumptions are made.

► **Hypothesis 1: Assume that national AI development capability is a latent attribute that can be approximated by a finite set of observable and quantifiable indicators.**

Legitimacy: At the national level, AI development is manifested through measurable outcomes and resource inputs recorded in public statistics. Although the true capability cannot be observed directly, its major characteristics can be reasonably inferred from aggregated, quantifiable indicators.

► **Hypothesis 2: Assume that all indicators within the same evaluation year are cross-sectionally consistent.**

Legitimacy: Although data may be collected from slightly different release years, AI development is a long-term process. Minor temporal discrepancies do not significantly affect national-level competitiveness comparisons and help simplify the modeling process.

► **Hypothesis 3: Assume that the indicators are independent of each other in the weighting and evaluation stages.**

Justification : While interactions among indicators exist, explicitly modeling such dependencies would increase complexity and reduce interpretability. Treating indicators as independent avoids double counting and ensures the applicability of entropy-based and multi-criteria evaluation methods.

► **Hypothesis 4: Assume that the fundamental mechanisms of AI development remain stable during the forecasting and optimization period.**

Justification : National AI strategies, infrastructure construction, and talent cultivation generally evolve gradually. This stability makes trend-based prediction and investment optimization reasonable and analytically tractable.

3 Symbols

4 Data Explanation

4.1 Data Description and Sources

The dataset covers ten representative countries and consists of multiple quantitative indicators describing national AI development capability. For organizational clarity, the indicators were grouped into six dimensions reflecting different aspects of AI development. All data corresponded to the same evaluation period and were obtained from publicly available and widely recognized sources, ensuring cross-country comparability.

4.2 Data Preprocessing

Basic preprocessing was conducted prior to analysis. Minor missing or abnormal values were handled through reasonable estimation and consistency checks. All indicators were defined as benefit-type variables and were normalized to eliminate dimensional differences before being used in subsequent models.

5 Task 1: Analysis of AI Development Factors

The objective of Task 1 is to uncover the internal structure of national AI development capability from a data-driven perspective. Instead of imposing predefined theoretical relationships, this task examines how multiple AI-related indicators co-evolve across countries and interact to jointly shape overall competitiveness. The analysis provides a structural foundation for the evaluation and forecasting tasks that follow.

5.1 Quantification of Key Indicators

National AI development capability is represented by a standardized indicator matrix

$$X = [x_{ij}] \in \mathbb{R}^{n \times p}, \quad (1)$$

where x_{ij} denotes the normalized value of indicator j for country i , with $n = 10$ countries and $p = 24$ indicators.

To ensure comparability among indicators with heterogeneous scales, min–max normalization is applied:

$$x_{ij} = \frac{x_{ij}^{\text{raw}} - \min(x_j)}{\max(x_j) - \min(x_j)} \in [0, 1]. \quad (2)$$

The resulting matrix X provides a unified quantitative description of AI development factors and serves as the common input for all analyses in Task 1.

5.2 Correlation Structure and Interaction Mechanism

Overall correlation patterns. Based on the standardized data, Pearson correlation analysis is first employed to examine linear associations among AI development indicators. The correlation coefficient between indicator j and k is defined as

$$r_{jk} = \frac{\sum_{i=1}^n (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{\sqrt{\sum_{i=1}^n (x_{ij} - \bar{x}_j)^2 \sum_{i=1}^n (x_{ik} - \bar{x}_k)^2}}, \quad (3)$$

forming the correlation matrix

$$R = [r_{jk}] \in \mathbb{R}^{p \times p}. \quad (4)$$

To focus on meaningful interactions, strong correlations are identified by

$$\mathcal{E} = \{(j, k) \mid |r_{jk}| > \tau, j < k\}, \quad (5)$$

where τ denotes a predefined threshold.

The correlation results reveal a densely connected structure among AI development factors. Many indicators exhibit strong positive correlations, indicating that progress in one aspect of AI capability is often accompanied by simultaneous improvement in others. In particular, talent-related indicators, R&D investment measures, market scale variables, and computing infrastructure show pronounced co-movement, reflecting coordinated national development strategies rather than isolated factor growth.

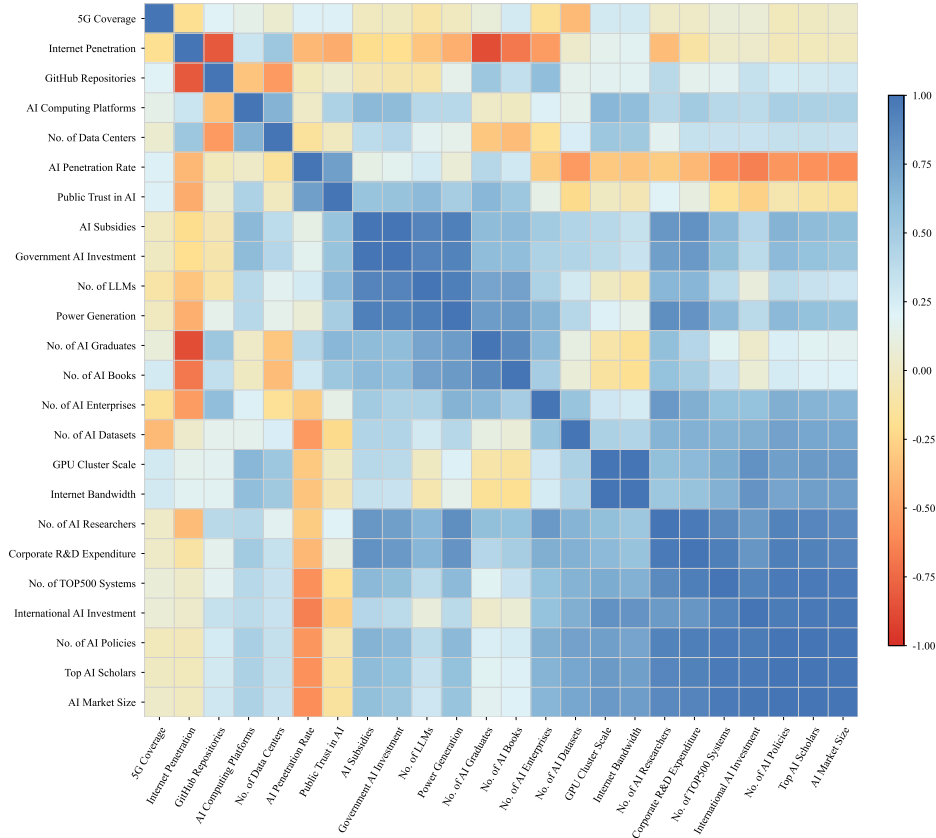


Figure 1: Correlation Heatmap of AI Development Factors

Structural grouping of indicators. While pairwise correlations describe local relationships, they do not directly reveal higher-level organization. To uncover such structure, hierarchical clustering is applied using correlation-based distances:

$$d_{jk} = 1 - |r_{jk}|. \quad (6)$$

The distance between clusters is computed using the average linkage criterion:

$$D(C_a, C_b) = \frac{1}{|C_a||C_b|} \sum_{j \in C_a} \sum_{k \in C_b} d_{jk}. \quad (7)$$

The resulting dendrogram reveals several coherent clusters that align with intuitive dimensions of AI development. Indicators related to investment intensity and market outcomes form a tightly connected group, computing infrastructure indicators cluster together and remain closely linked to investment-related factors, while talent and knowledge production indicators constitute another distinct cluster. These results confirm that AI development factors form a multi-dimensional yet internally coordinated system.

Principal component structure. The strong correlations and clustering patterns suggest substantial redundancy among indicators. To extract the dominant structural dimensions of AI development,

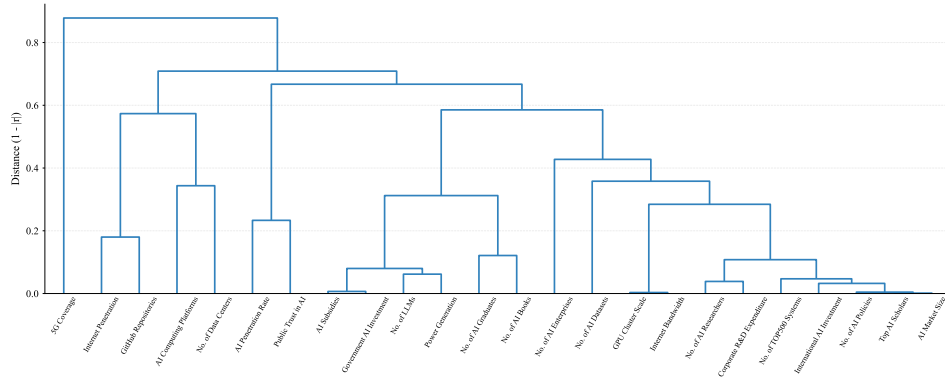


Figure 2: Hierarchical Clustering of AI Development Factors

principal component analysis (PCA) is conducted. Let

$$\tilde{X} = X - \mathbf{1}\bar{X}^T, \quad C = \frac{1}{n-1} \tilde{X}^T \tilde{X} \quad (8)$$

denote the centered data matrix and its covariance matrix. Eigen-decomposition yields

$$C = V\Lambda V^T, \quad (9)$$

where Λ is the diagonal matrix of eigenvalues.

The number of retained components m is determined by the cumulative variance criterion

$$\sum_{k=1}^m \frac{\lambda_k}{\sum_{j=1}^p \lambda_j} \geq \eta. \quad (10)$$

The PCA results indicate that variance is highly concentrated in the leading components. More than 90% of the total variance is explained by the first four principal components, suggesting that national AI development capability can be effectively characterized by a low-dimensional structure driven by a common underlying dimension.

Factor importance and system-level interactions. Building on the PCA results, the relative importance of each indicator is quantified by combining component loadings and variance contributions:

$$I_j = \sum_{k=1}^m v_{jk}^2 \cdot \frac{\lambda_k}{\sum_{l=1}^p \lambda_l}. \quad (11)$$

The importance ranking shows that explanatory power is concentrated in a limited number of indicators. Factors related to human capital, research output, investment scale, and advanced computing infrastructure consistently exhibit higher importance scores, highlighting their dominant role in shaping cross-country differences in AI capability.

From a system-level perspective, a strong-correlation network further illustrates the interaction mechanism among indicators. Investment- and infrastructure-related factors occupy central positions in the network, acting as hubs that connect multiple dimensions, while peripheral indicators exhibit weaker coupling. This structure indicates that AI development emerges from coordinated interactions among a core set of drivers rather than independent contributions.

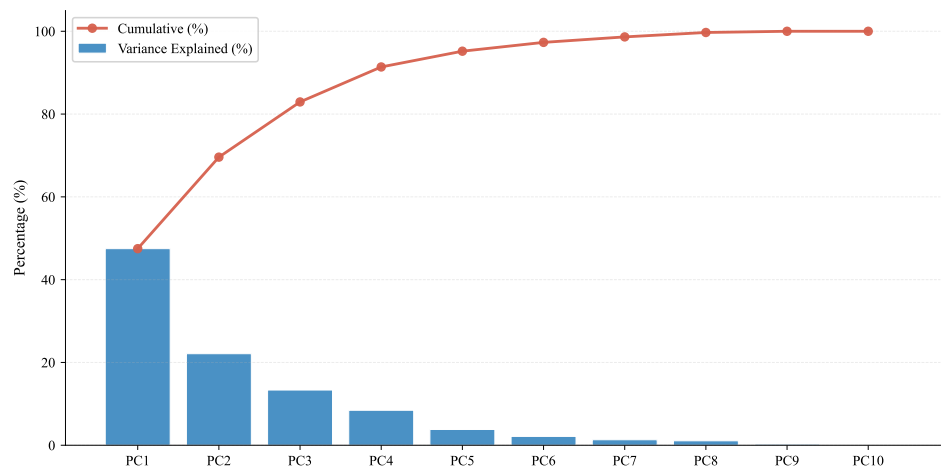


Figure 3: Variance Explained by Principal Components

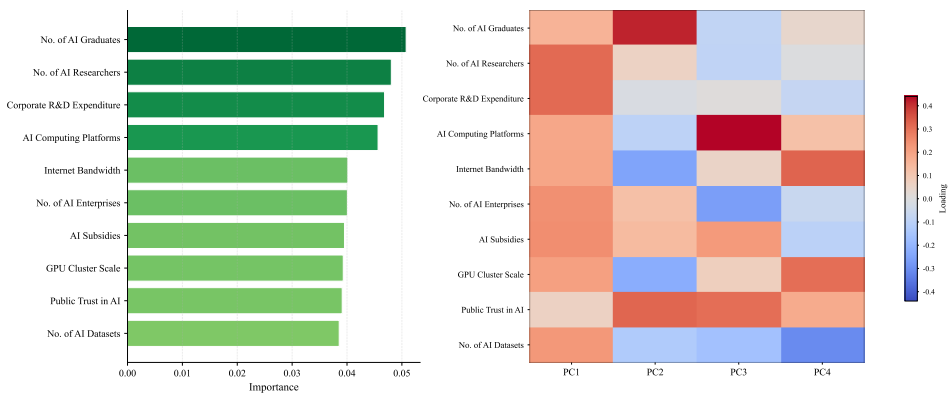


Figure 4: Factor Importance Ranking Bar Chart

Summary of Task 1. Through correlation analysis, hierarchical clustering, dimensionality reduction, and interaction exploration, Task 1 reveals that AI development factors form a highly interconnected and structured system. A small number of dominant dimensions and key indicators drive most cross-country variation, while remaining factors play supporting roles. These findings provide a concise and rigorous structural basis for the comprehensive evaluation in Task 2 and the forecasting analysis in Task 3.

6 Task 2: Evaluation of National AI Competitiveness in 2025

6.1 Comprehensive Evaluation Methodology

Based on the indicator system established in Task 1, this task constructs an objective framework to evaluate national AI competitiveness in 2025.

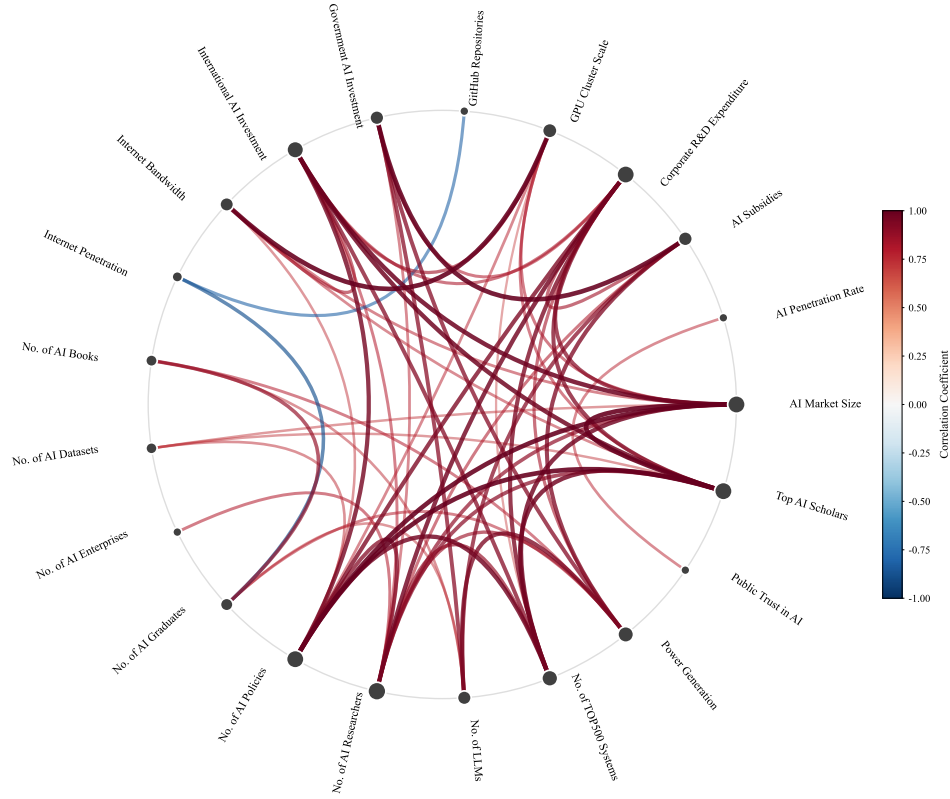


Figure 5: Interaction Network of AI Development Factors

Let the standardized indicator matrix be

$$X' = (x'_{ij})_{n \times p}, \quad n = 10, \quad p = 24, \quad (12)$$

where all indicators are benefit-type variables. The objective is to synthesize multidimensional information into a single, interpretable competitiveness score.

(1) Entropy-based indicator weighting. Indicator importance is determined using the entropy weight method. For indicator j ,

$$p_{ij} = \frac{x'_{ij}}{\sum_{i=1}^n x'_{ij}}, \quad 0 \ln 0 := 0, \quad (13)$$

$$e_j = -k \sum_{i=1}^n p_{ij} \ln p_{ij}, \quad k = \frac{1}{\ln n}, \quad (14)$$

and the entropy weight is

$$w_j = \frac{1 - e_j}{\sum_{j=1}^p (1 - e_j)}. \quad (15)$$

Indicators with greater cross-country dispersion receive higher weights.

The resulting weight distribution is shown in Figure 6, with representative indicators summarized in Table 1.

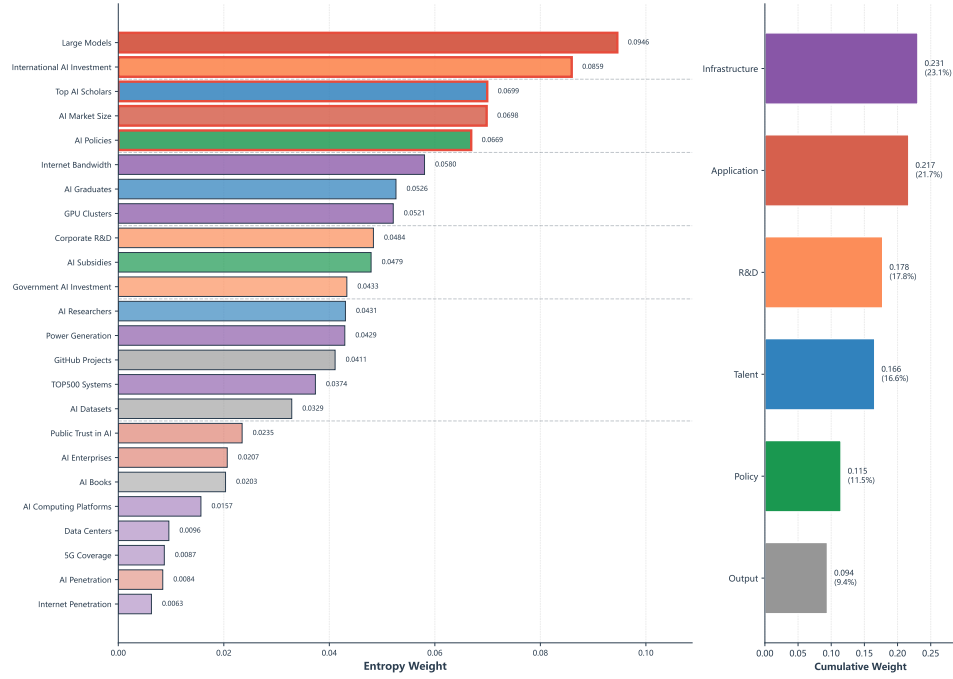


Figure 6: Indicator Weights Distribution (Entropy Weight Method)

Table 1: Entropy Weights of Representative Indicators (Top and Bottom)

Indicator	Entropy e_j	Redundancy $(1 - e_j)$	Weight w_j
Large Models	0.2144	0.7856	0.0946
International AI Investment	0.2862	0.7138	0.0859
Top AI Scholars	0.4192	0.5808	0.0699
AI Market Size	0.4201	0.5799	0.0698
AI Policies	0.4444	0.5556	0.0669
AI Penetration	0.9300	0.0700	0.0084
5G Coverage	0.9275	0.0725	0.0087
Internet Penetration	0.9478	0.0522	0.0063

(2) TOPSIS-based comprehensive evaluation. Using the entropy weights, the weighted decision matrix is

$$v_{ij} = w_j x'_{ij}. \quad (16)$$

The positive and negative ideal solutions are

$$A_j^+ = \max_i v_{ij}, \quad A_j^- = \min_i v_{ij}, \quad (17)$$

and the distances to these benchmarks are

$$D_i^\pm = \sqrt{\sum_{j=1}^p (v_{ij} - A_j^\pm)^2}. \quad (18)$$

The TOPSIS score is defined as

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}. \quad (19)$$

(3) Structural validation by grey relational analysis. Let the ideal profile be

$$v_{0j} = \max_i v_{ij}. \quad (20)$$

Define

$$\Delta_{ij} = |v_{0j} - v_{ij}|, \quad \Delta_{\min} = \min_{i,j} \Delta_{ij}, \quad \Delta_{\max} = \max_{i,j} \Delta_{ij}. \quad (21)$$

The grey relational coefficient and degree are

$$\xi_{ij} = \frac{\Delta_{\min} + 0.5 \Delta_{\max}}{\Delta_{ij} + 0.5 \Delta_{\max}}, \quad \gamma_i = \frac{1}{p} \sum_{j=1}^p \xi_{ij}. \quad (22)$$

6.2 Results and Comparative Analysis

(1) Indicator weight structure. Entropy weights concentrate on frontier and output-oriented indicators, indicating that cross-country differences are primarily driven by advanced innovation capacity and global resource integration, while basic digital penetration indicators contribute marginally.

(2) National competitiveness ranking in 2025. The TOPSIS results are reported in Table 2 and Figure 7. The United States and China form a clear leading tier, followed by India and the UAE, while the remaining countries cluster closely.

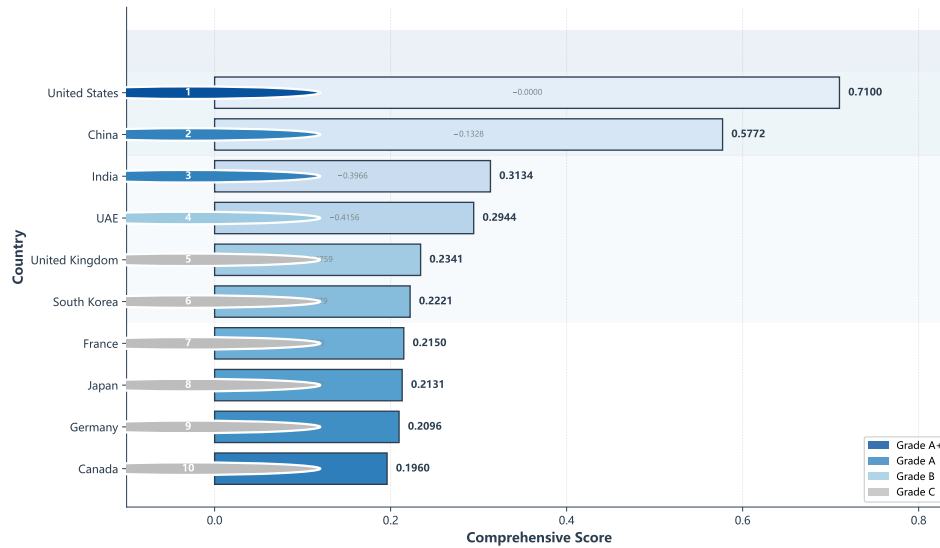


Figure 7: AI Competitiveness Ranking in 2025 (TOPSIS Score)

(3) Cross-method validation and fusion ranking. The grey relational results are summarized in Table 3. To integrate distance-based performance and structural similarity, the fusion score is defined as

$$S_i = \frac{C_i + \gamma_i}{2}. \quad (23)$$

Table 2: TOPSIS Comprehensive Evaluation Results (2025)

Country	D_i^+	D_i^-	TOPSIS Score C_i	Rank
United States	0.0978	0.1744	0.6407	1
China	0.1320	0.1377	0.5104	2
India	0.2006	0.0533	0.2098	3
UAE	0.1998	0.0449	0.1836	4
United Kingdom	0.2040	0.0225	0.0995	5
South Korea	0.2063	0.0165	0.0740	6
France	0.2065	0.0153	0.0688	7
Japan	0.2064	0.0141	0.0640	8
Germany	0.2063	0.0138	0.0625	9
Canada	0.2091	0.0090	0.0414	10

Table 3: Grey Relational Degrees and Rankings

Country	Grey Relational Degree γ_i	Rank
United States	0.7793	1
China	0.6440	2
India	0.4170	3
UAE	0.4052	4
South Korea	0.3703	5
United Kingdom	0.3686	6
Japan	0.3623	7
France	0.3612	8
Germany	0.3567	9
Canada	0.3506	10

Table 4: Final AI Competitiveness Ranking (2025) by Fusion Score

Country	TOPSIS Rank	GRA Rank	TOPSIS C_i	GRA γ_i	Fusion Score S_i	Grade
United States	1	1	0.6407	0.7793	0.7100	A+
China	2	2	0.5104	0.6440	0.5772	A
India	3	3	0.2098	0.4170	0.3134	A
UAE	4	4	0.1836	0.4052	0.2944	B
United Kingdom	5	6	0.0995	0.3686	0.2341	C
South Korea	6	5	0.0740	0.3703	0.2221	C
France	7	8	0.0688	0.3612	0.2150	C
Japan	8	7	0.0640	0.3623	0.2131	C
Germany	9	9	0.0625	0.3567	0.2096	C
Canada	10	10	0.0414	0.3506	0.1960	C

The final ranking is reported in Table 4.

(4) Reliability and robustness assessment. Consistency between TOPSIS and GRA is measured using Spearman's rank correlation coefficient, yielding $\rho_s = 0.9758$ ($p < 0.001$). Sensitivity analysis under $\pm 30\%$ weight perturbations confirms that most countries exhibit rank variations of at most one position. The validation results are illustrated in Figure 8 and summarized in Table 5.

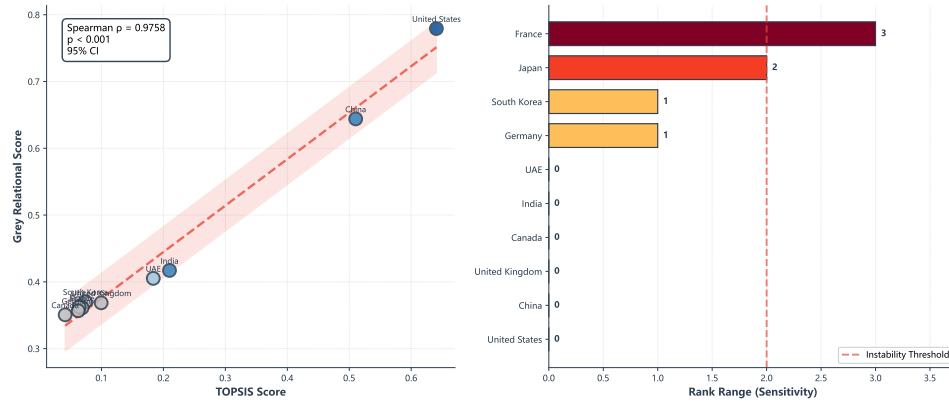


Figure 8: Validation and Robustness Analysis (Method Consistency and Sensitivity)

Table 5: Ranking Sensitivity under Weight Perturbation (Rank Range)

Country	Rank Range
United States	0
China	0
India	0
UAE	0
United Kingdom	0
Canada	0
Germany	1
South Korea	1
Japan	2
France	3

7 Task 3: Forecasting AI Competitiveness (2026–2035)

Building on the indicator system identified in Task 1 and the objective evaluation framework established in Task 2, Task 3 extends the analysis from a static assessment to a dynamic forecasting perspective. The central purpose of this task is to examine how national AI competitiveness may evolve over the period 2026–2035 if historical development trajectories continue, without introducing any additional policy shocks or subjective assumptions.

The forecasting strategy follows a unified logic: future competitiveness is inferred indirectly through the predicted evolution of underlying indicators, while the evaluation mechanism itself remains

unchanged. In this way, any variation in future rankings can be attributed solely to data-driven indicator dynamics.

7.1 Indicator-Level Trend Prediction

Let $x_{i,j,t}$ denote the observed value of indicator j for country i in year t . The dataset forms a balanced panel with $n = 10$ countries and $p = 24$ indicators over the historical period 2016–2025. For each country–indicator pair, future values $\hat{x}_{i,j,t}$ for $t = 2026, \dots, 2035$ are predicted independently.

Given the limited length of historical sequences, the Grey Forecasting Model GM(1,1) is adopted as the primary prediction tool. For a fixed sequence

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(T)), \quad T = 10,$$

the model applies first-order accumulated generation (AGO), estimates the corresponding differential equation, and recovers the predicted original series through inverse transformation. GM(1,1) is particularly suitable in this context due to its robustness under small-sample conditions and its ability to capture long-term monotonic trends.

To ensure numerical stability and realistic forecasts across heterogeneous indicators, several unified engineering treatments are applied. Near-zero or sparse series are translated to maintain non-negativity, forecast values are truncated according to indicator-specific bounds, and logarithmic scaling is used when necessary for indicators with large magnitude dispersion. Importantly, all treatments are rule-based, reversible, and applied consistently across countries.

Model reliability is examined through one-step-ahead backtesting. Using data from 2016–2024, the year 2025 is predicted and evaluated using the mean absolute percentage error (MAPE). For sequences where GM(1,1) performs poorly, a linear trend model under identical constraints is adopted as a fallback. Across all 240 country–indicator sequences, GM(1,1) is used for 44.17% of cases, while the fallback model is activated for 55.83%. The overall error distribution remains well controlled, with a median MAPE of 0.1035. A compact diagnostic summary of forecasting accuracy and model usage is presented in Fig. 9.

7.2 Annual Evaluation and Score Evolution

Once indicator-level forecasts are obtained, national AI competitiveness is evaluated on a yearly basis using the same methodology as in Task 2. Let $W = (w_1, \dots, w_p)$ denote the entropy-based indicator weights derived previously. These weights are held fixed throughout the forecasting horizon to ensure temporal comparability.

For each forecast year t , the predicted indicator matrix

$$\hat{X}_t = (\hat{x}_{i,j,t})_{n \times p}$$

is normalized and evaluated using the TOPSIS method, yielding the relative closeness score $C_{i,t} \in [0, 1]$ for each country.

Table 6 reports TOPSIS scores for three representative years (2026, 2030, and 2035). To illustrate the dynamic evolution of competitiveness more intuitively, Fig. 10 visualizes the score trajectories and inter-country gaps over the entire forecast horizon.

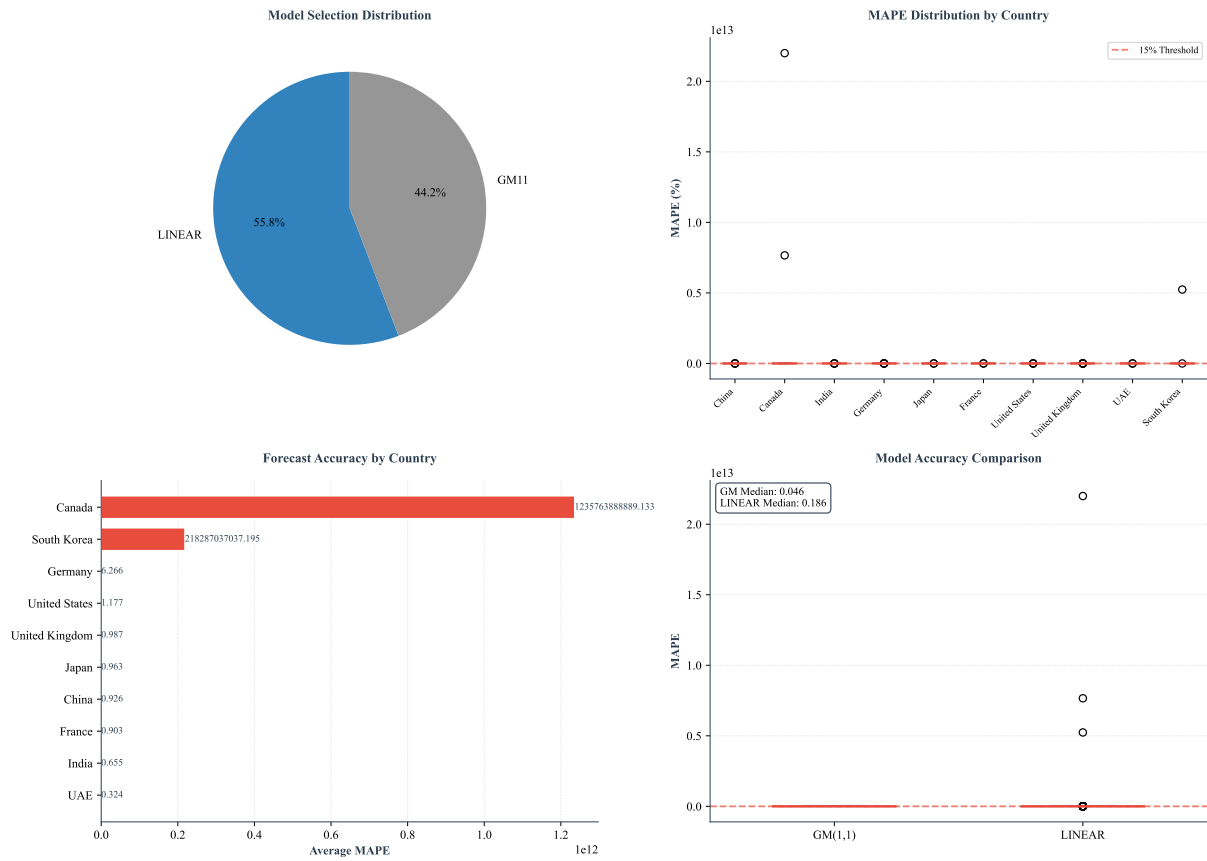


Figure 9: Forecast diagnostics panel for the indicator-level prediction stage.

Table 6: Selected TOPSIS scores $C_{i,t}$ for 2026, 2030, and 2035.

Country	2026	2030	2035
United States	0.653	0.644	0.633
China	0.515	0.505	0.507
India	0.213	0.221	0.244
United Arab Emirates	0.160	0.162	0.178
France	0.069	0.080	0.168
Germany	0.108	0.121	0.143
United Kingdom	0.069	0.073	0.102
Canada	0.042	0.057	0.101
South Korea	0.055	0.066	0.097
Japan	0.054	0.060	0.093

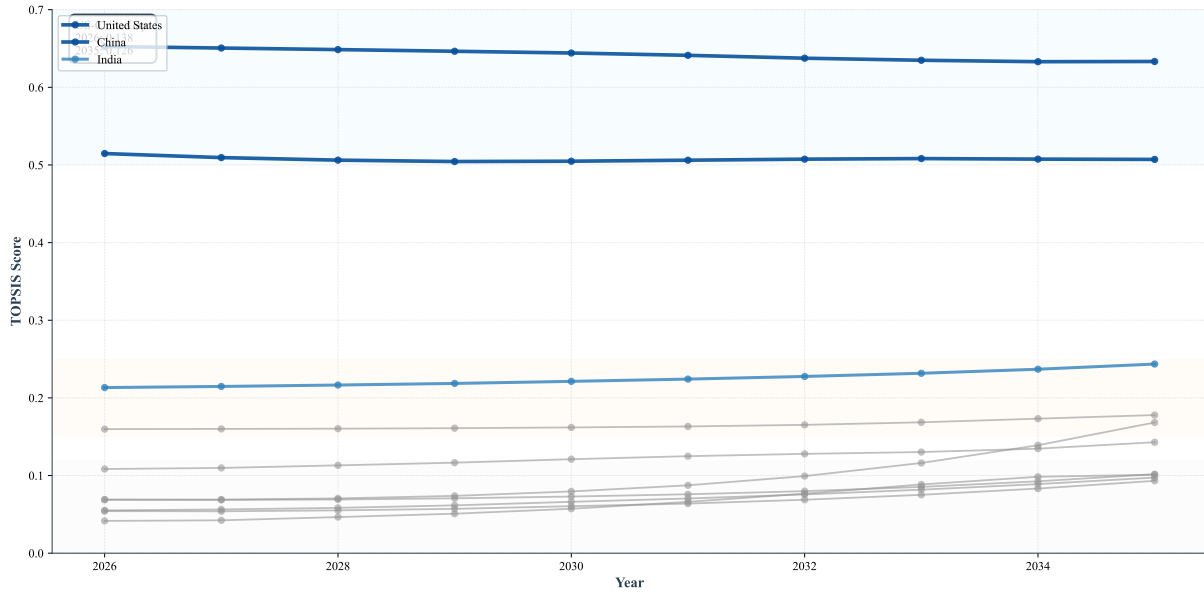


Figure 10: Score gap dynamics of TOPSIS closeness scores (2026–2035).

Two score-level patterns emerge clearly. First, the leading position remains stable, although the score gap between the top two countries narrows slightly over time. Second, the overall dispersion of scores decreases, indicating gradual convergence in comprehensive AI competitiveness.

7.3 Ranking Evolution and Stability

For each year, countries are ranked in descending order of $C_{i,t}$. The resulting ranking trajectories are shown in Fig. 11, while overall rank stability is summarized in Fig. 12.

The forecast indicates a highly stable top tier, with the United States, China, India, and the United Arab Emirates consistently occupying the first four positions throughout the period. Rank changes occur primarily among mid- and lower-tier countries, where score differences are comparatively small.

Quantitative rank stability statistics are reported in Table 7. These results confirm that observed rank fluctuations are generally limited to one-position changes and do not reflect structural reversals.

7.4 Interpretation and Robustness

Because both the indicator weights and the evaluation procedure are fixed, all ranking changes arise solely from predicted indicator trajectories. For mid-tier countries, ranking swaps reflect the accumulation of small advantages across several high-weight indicators rather than abrupt changes in any single metric. This structural interpretation is consistent with the observed score convergence shown in Fig. 10.

Overall robustness is supported by the forecasting diagnostics (Fig. 9) and by the clear separation between top-tier and mid-tier score levels. A baseline-to-forecast comparison is provided in Fig. 13, illustrating that most rank adjustments occur locally among closely competing countries.

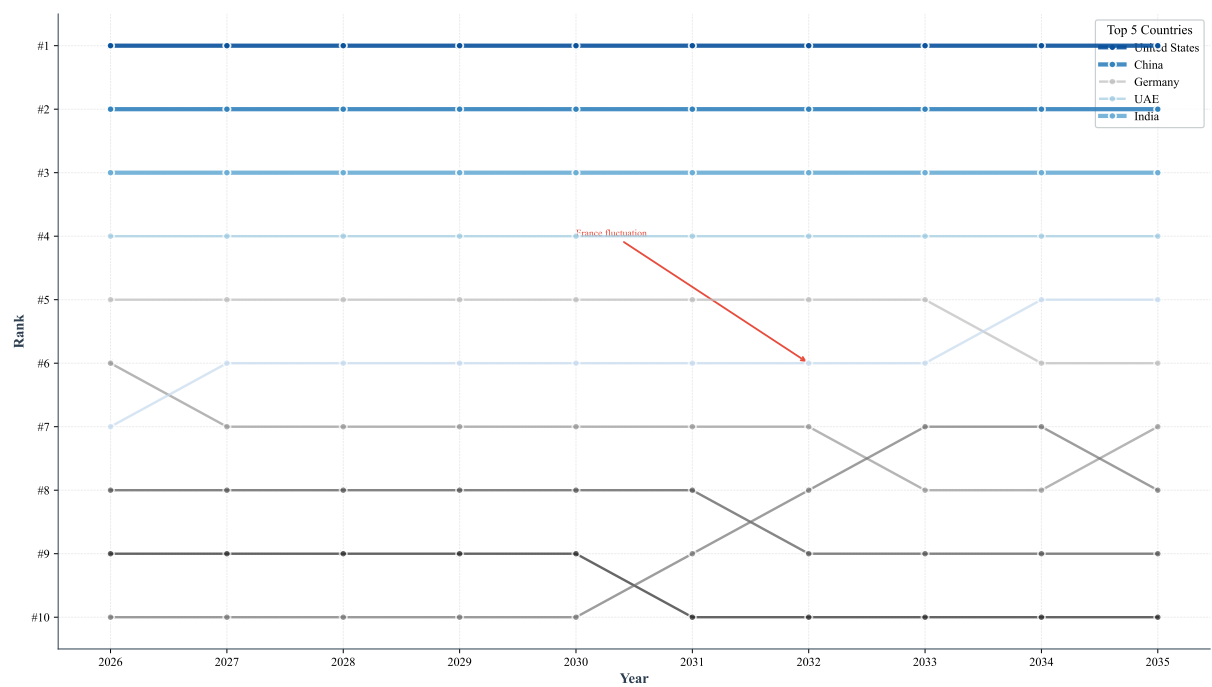


Figure 11: Ranking evolution (bump chart) from 2026 to 2035.

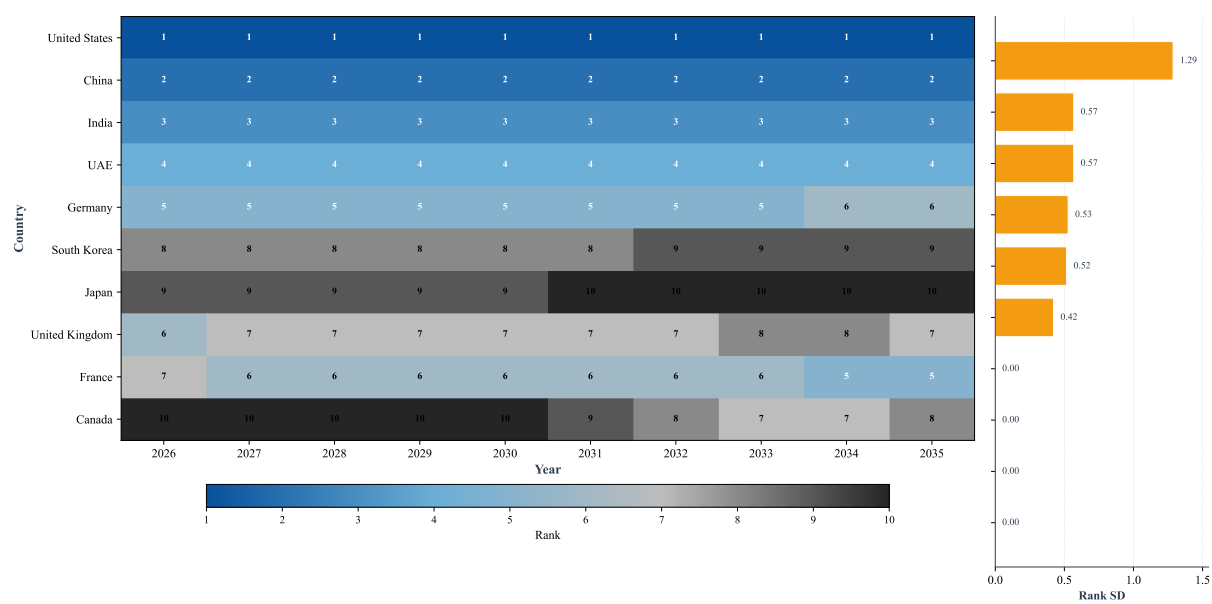


Figure 12: Rank stability heatmap over the forecast horizon.

Table 7: Rank stability statistics over 2026–2035.

Country	AvgRank	StdRank	BestRank	WorstRank	MaxRise	MaxFall
United States	1.00	0.00	1	1	0	0
China	2.00	0.00	2	2	0	0
India	3.00	0.00	3	3	0	0
United Arab Emirates	4.00	0.00	4	4	0	0
Germany	5.20	0.40	5	6	0	1
France	5.90	0.54	5	7	1	0
United Kingdom	7.10	0.54	6	8	1	1
South Korea	8.40	0.49	8	9	0	1
Canada	8.90	1.22	7	10	1	1
Japan	9.50	0.50	9	10	0	1

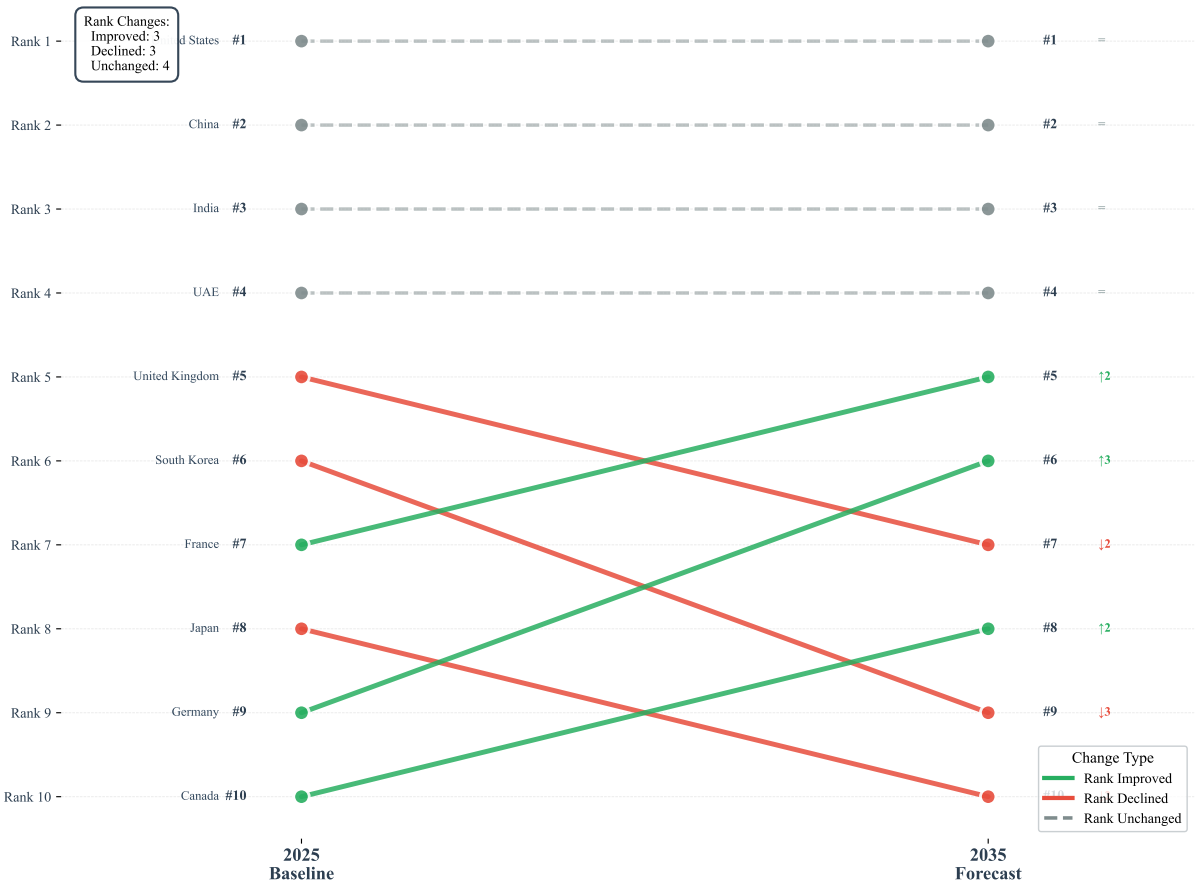


Figure 13: Comparison between 2025 baseline and 2035 forecasted rankings.

7.5 Summary

Task 3 integrates indicator-level time series forecasting with a fixed and validated evaluation framework to project the future evolution of national AI competitiveness. The results suggest a stable global leadership structure, gradual convergence among developing competitors, and limited, interpretable ranking changes concentrated in the middle tier. This dynamic assessment provides a coherent quantitative basis for the investment optimization analysis conducted in Task 4.

8 Task 4: Optimization of China's AI Development Investment (2026–2035)

In Tasks 1–3, we have established a unified analytical framework for evaluating national AI development capability: (i) Task 1 identified a 24-indicator system and revealed key structural correlations among indicators; (ii) Task 2 determined objective indicator weights and fixed the TOPSIS evaluation scheme; (iii) Task 3 provided baseline and forecasted indicator trajectories for all countries up to 2035.

Under the assumption that China allocates an additional 1 trillion RMB in special funds starting from 2026, Task 4 aims to determine an optimal investment allocation strategy that maximizes China's comprehensive AI competitiveness in 2035, while maintaining full consistency with the established evaluation framework.

8.1 Problem Formulation

Let $\mathbf{I} = (I_1, I_2, \dots, I_p)^\top$ denote the investment allocation vector over the $p = 24$ indicators, where I_j represents the investment (in billion RMB) allocated to indicator j . The optimization objective is defined as

$$\mathbf{I}^* = \arg \max_{\mathbf{I}} S_{\text{CN}}(X^{2035}(\mathbf{I}); \mathbf{w}), \quad (24)$$

subject to the total budget constraint

$$\sum_{j=1}^p I_j = B, \quad B = 10000. \quad (25)$$

Here, $S_{\text{CN}}(\cdot)$ denotes China's TOPSIS closeness coefficient under the fixed weight vector \mathbf{w} obtained in Task 2, and $X^{2035}(\mathbf{I})$ represents the 2035 indicator matrix in which only China's indicator values are affected by the investment decision.

8.2 Symbols and Data Interfaces

To ensure comparability with previous tasks, all structural, weighting, and forecasting information is treated as exogenous input:

$$\mathbf{w} \leftarrow \text{Task 2 (Entropy Weight Method)}, \quad (26)$$

$$\mathbf{x}_{\text{CN}}^{\text{base}} \leftarrow \text{Task 3 (China baseline, 2026)}, \quad (27)$$

$$X_{2035}^{\text{scen}} \leftarrow \text{Task 3 (2035 forecast scenario)}, \quad (28)$$

$$\mathcal{E} \leftarrow \text{Task 1 (strong correlation structure)}. \quad (29)$$

For each indicator j , the following parameters are introduced:

- C_j : unit investment cost required to increase indicator j by one unit;
- γ_j : time-lag discount factor reflecting realization speed;
- L_j : upper bound representing saturation or feasible growth limits;
- I_{\min}, I_{\max} : lower and upper bounds on single-indicator investment.

8.3 Investment–Indicator Response Function

Considering diminishing marginal returns and heterogeneous realization horizons across indicator types, the incremental change of indicator j induced by investment I_j is modeled as

$$\Delta x_j(\mathbf{I}) = \frac{I_j}{C_j} \left(1 - \frac{x_j^{\text{base}}}{L_j} \right) \gamma_j, \quad j = 1, \dots, p. \quad (30)$$

Accordingly, China's indicator value in 2035 after investment is given by

$$x_{\text{CN},j}^{2035}(\mathbf{I}) = \min \left\{ x_j^{\text{base}} + \Delta x_j(\mathbf{I}), L_j \right\}. \quad (31)$$

In vector form,

$$\Delta \mathbf{x}(\mathbf{I}) = (\boldsymbol{\gamma} \oslash \mathbf{C}) \odot \left(\mathbf{1} - \mathbf{x}^{\text{base}} \oslash \mathbf{L} \right) \odot \mathbf{I}, \quad (32)$$

where \odot and \oslash denote element-wise multiplication and division, respectively.

8.4 Upper Bounds and Time-Lag Settings

Indicator upper bounds are determined according to a relative competitiveness rule:

$$L_j = \begin{cases} 1.5 x_{j,2025}^{\text{CN}}, & x_{j,2025}^{\text{CN}} \geq x_{j,2025}^{\text{US}}, \\ 3.0 x_{j,2025}^{\text{US}}, & x_{j,2025}^{\text{CN}} < x_{j,2025}^{\text{US}}, \end{cases} \quad L_j \leq 100 \text{ (ratio-type indicators)}. \quad (33)$$

Time-lag discount factors are grouped by indicator characteristics:

$$\gamma_j \in \{1.0, 0.8, 0.6\}, \quad (34)$$

corresponding respectively to short-term (e.g., infrastructure), medium-term (e.g., R&D and applications), and long-term (e.g., talent cultivation) effects.

8.5 Evaluation Matrix Construction

The 2035 evaluation matrix $X^{2035}(\mathbf{I}) \in \mathbb{R}^{n \times p}$ is constructed as follows: all non-China rows are fixed at their forecasted values from Task 3, while China's row is replaced by $\mathbf{x}_{\text{CN}}^{2035}(\mathbf{I})$. This design guarantees that the comparison set and evaluation standard remain unchanged.

8.6 TOPSIS-Based Objective Function

Let $X = X^{2035}(\mathbf{I})$. The TOPSIS procedure follows exactly the formulation in Task 2:

$$\tilde{X} = XD^{-1}, \quad D = \text{diag}(\|X_{:,1}\|_2, \dots, \|X_{:,p}\|_2), \quad (35)$$

$$V = \tilde{X} \text{diag}(\mathbf{w}), \quad \mathbf{v}^+ = \max_i V_{i,:}, \quad \mathbf{v}^- = \min_i V_{i,:}, \quad (36)$$

$$D_i^\pm = \|V_{i,:} - \mathbf{v}^\pm\|_2, \quad S_i = \frac{D_i^-}{D_i^+ + D_i^-}. \quad (37)$$

The optimization objective is to maximize S_{CN} .

8.7 Constraints

Budget and Bound Constraints

$$\sum_{j=1}^p I_j = B, \quad I_{\min} \leq I_j \leq I_{\max}. \quad (38)$$

Synergy Constraints To prevent structural imbalance caused by isolated investment surges, synergy constraints are imposed based on strong correlations identified in Task 1:

$$x_{\text{Large Models}} \leq 200 x_{\text{GPU}}, \quad (39)$$

$$x_{\text{Top AI Scholars}} \leq 5.0 x_{\text{Researchers}}, \quad (40)$$

$$x_{\text{AI Publications}} \leq 0.24 x_{\text{Researchers}}, \quad (41)$$

$$x_{\text{AI Enterprises}} \leq 78 x_{\text{AI Market}}, \quad (42)$$

$$x_{\text{AI Datasets}} \leq 0.75 x_{\text{Enterprise R\&D}}. \quad (43)$$

8.8 Solution Method

The resulting optimization problem is a nonlinear constrained programming problem. It is solved using the Sequential Least Squares Programming (SLSQP) algorithm, with an equal-allocation initialization $I_j = B/p$, a maximum of 500 iterations, and a convergence tolerance of 10^{-6} .

This formulation yields a reproducible investment allocation strategy fully consistent with the structural insights, evaluation methodology, and forecast scenarios established in Tasks 1–3.