Tutorial: Normal location model

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Setting

This script applies the proposed ABC-SMC with Wasserstein distance to approximate the posterior distribution in a Normal location model.

The model specifies $Y \sim \mathcal{N}(\mu, \sigma^2)$, the parameters are (μ, σ), the prior is standard Normal on μ, and Gamma$(2, 1)$ on σ. The data are generated from a Gamma$(10, 5)$ distribution, with $n = 1000$ observations.

We begin by loading the package, registering multiple cores, setting the random number generator, etc.

```
# load package
library(winference)
# register parallel cores
registerDoMC(cores = detectCores())
# remove all
rm(list = ls())
# apply preferences for ggplotting
require(gridExtra)
theme_set(theme_bw())
# set RNG seed
set.seed(11)
```

Data and model

We generate some data.

```
# number of observations
nobservations <- 1000
# observations from a Gamma model, with mean 2
obs <- rgamma(nobservations, shape = 10, rate = 5)
```

We define a model.

```
# function to generate from prior distribution first argument is number of
# desired samples second argument contains hyper-parameters
rprior <- function(N, parameters) {
    particles <- matrix(nrow = N, ncol = 2)
    particles[, 1] <- rnorm(N, mean = parameters$mu_0, sd = 1/sqrt(parameters$nu))
    particles[, 2] <- rgamma(N, shape = parameters$alpha, rate = parameters$beta)
    return(particles)
}
# function to evaluate prior log-density first argument is a matrix of
# parameters (one per row) second argument contains hyper-parameters
dprior <- function(thetas, parameters) {
    logdensities <- dnorm(thetas[, 1], mean = parameters$mu_0, sd = 1/sqrt(parameters$nu),
        log = TRUE)
    logdensities <- logdensities + dgamma(thetas[, 2], shape = parameters$alpha,
        rate = parameters$beta, log = TRUE)
    return(logdensities)
```

```
}
# we specify a data-generating mechanism, given a parameter theta
simulate <- function(theta) {
    observations <- theta[1] + rnorm(nobservations) * theta[2]
    return(observations)
}
# we collect everything in a list; the 'parameters' list contains the
# hyper-parameters
target <- list(simulate = simulate, rprior = rprior, dprior = dprior, parameter_names = c("mu",
    "sigma"), parameters = list(mu_0 = 0, nu = 1, alpha = 2, beta = 1), thetadim = 2,
    ydim = 1)
```

Distance calculation and Monte Carlo algorithm

We define a way of calculating a distance between fake data and the observed data. Here we use the 1-Wasserstein distance, which is the distance between sorted samples.

```
y_obs_sorted <- sort(obs)
# function to compute 1-Wasserstein distance between observed data and fake
# data given as argument
compute_distance <- function(y_fake) {
    y_fake <- sort(y_fake)
    return(mean(abs(y_obs_sorted - y_fake)))
}
```

We now specify algorithmic parameters specified in a list.

```
# algorithmic parameters: number of particles, number of moves per
# rejuvenation step, proposal distribution, the number of steps to perform
# in total, the diversity parameter used in the threshold adaptation, the
# number of hits to use in the r-hit kernel, and the maximum number of
# trials to use in the r-hit kernel before rejecting.
param_algo <- list(nthetas = 1024, nmoves = 1, proposal = mixture_rmixmod(),
    minimum_diversity = 0.5, R = 2, maxtrials = 1e+05)
```

We now run the algorithm.

```
# now run the algorithm
wsmcresults <- wsmc(compute_distance, target, param_algo, maxtime = 10)
```

```
## step 1 completed, in 0.611 seconds, 1024 distances calculated
## step 2... running until time >= 10 seconds
##   acceptance rates: 79.98047 %, threshold = 2.333193 , min. dist. = 0.09226229
##   total # distances calculated: 4267 (for this step: 3243)
##   total time spent: 1.733 seconds (for this step: 1.122 seconds)
## step 3... running until time >= 10 seconds
##   acceptance rates: 78.41797 %, threshold = 1.684126 , min. dist. = 0.06826041
##   total # distances calculated: 7553 (for this step: 3286)
##   total time spent: 2.49 seconds (for this step: 0.757 seconds)
## step 4... running until time >= 10 seconds
##   acceptance rates: 77.63672 %, threshold = 1.275666 , min. dist. = 0.06422625
##   total # distances calculated: 10854 (for this step: 3301)
##   total time spent: 3.311 seconds (for this step: 0.821 seconds)
## step 5... running until time >= 10 seconds
##   acceptance rates: 78.02734 %, threshold = 0.9583558 , min. dist. = 0.04986393
```

```
##   total # distances calculated: 14168 (for this step: 3314)
##   total time spent: 4.007 seconds (for this step: 0.696 seconds)
## step 6... running until time >= 10 seconds
##   acceptance rates: 76.5625 %, threshold = 0.6968365 , min. dist. = 0.04986393
##   total # distances calculated: 17491 (for this step: 3323)
##   total time spent: 4.848 seconds (for this step: 0.841 seconds)
## step 7... running until time >= 10 seconds
##   acceptance rates: 75.09766 %, threshold = 0.4796462 , min. dist. = 0.05039149
##   total # distances calculated: 20871 (for this step: 3380)
##   total time spent: 5.562 seconds (for this step: 0.714 seconds)
## step 8... running until time >= 10 seconds
##   acceptance rates: 75.09766 %, threshold = 0.3468625 , min. dist. = 0.05641825
##   total # distances calculated: 24304 (for this step: 3433)
##   total time spent: 6.608 seconds (for this step: 1.046 seconds)
## step 9... running until time >= 10 seconds
##   acceptance rates: 71.875 %, threshold = 0.2451915 , min. dist. = 0.05192055
##   total # distances calculated: 27770 (for this step: 3466)
##   total time spent: 7.298 seconds (for this step: 0.69 seconds)
## step 10... running until time >= 10 seconds
##   acceptance rates: 71.67969 %, threshold = 0.1817934 , min. dist. = 0.05093453
##   total # distances calculated: 31301 (for this step: 3531)
##   total time spent: 8.399 seconds (for this step: 1.101 seconds)
## step 11... running until time >= 10 seconds
##   acceptance rates: 69.62891 %, threshold = 0.1364398 , min. dist. = 0.04472092
##   total # distances calculated: 34955 (for this step: 3654)
##   total time spent: 9.165 seconds (for this step: 0.766 seconds)
## step 12... running until time >= 10 seconds
##   acceptance rates: 69.33594 %, threshold = 0.1055347 , min. dist. = 0.03240912
##   total # distances calculated: 38868 (for this step: 3913)
##   total time spent: 10.02 seconds (for this step: 0.855 seconds)
```

Now we can look at the ouput, with various plots.

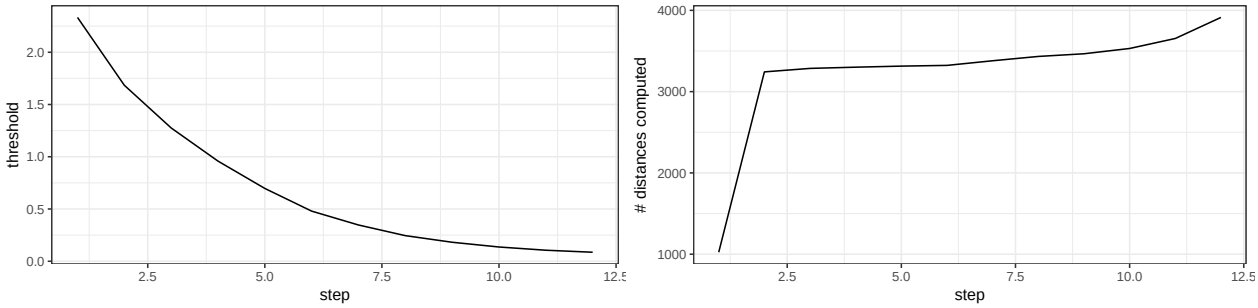
```
# we have access to all the generated particles, distances, and thresholds
names(wsmcresults)
```

```
##  [1] "thetas_history"    "distances_history" "threshold_history"
##  [4] "param_algo"        "latest_y"          "ncomputed"
##  [7] "compute_d"         "target"            "normcst"
## [10] "compute_times"
```

```
# latest_y contains the latest generated data and distances_history stores
# all the calculated distances, so
tail(wsmcresults$distances_history, n = 1)[[1]][1] == compute_distance(wsmcresults$latest_y[[1]])
```

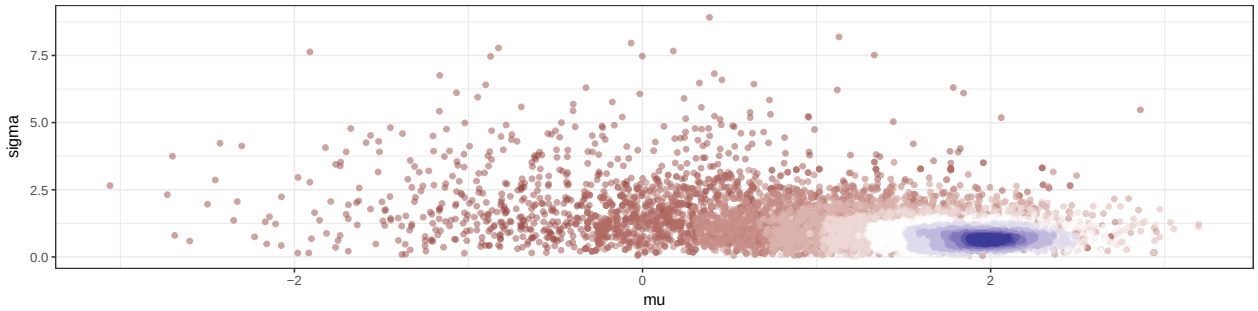
```
## [1] TRUE
```

```
# let's plot some of the output, for instance the evolution of the distance
# thresholds and the number of simulations per step
grid.arrange(plot_threshold(wsmcresults), plot_ncomputed(wsmcresults), ncol = 2)
```

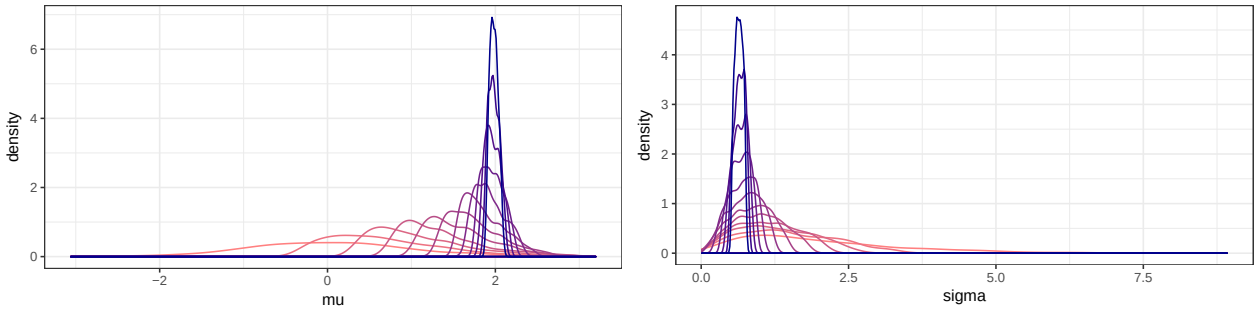


We can look at the distributions of parameters.

```
# and let's look at the parameters themselves
plot_bivariate(wsmcresults, i1 = 1, i2 = 2)
```




```
grid.arrange(plot_marginal(wsmcresults, i = 1), plot_marginal(wsmcresults, i = 2),
    ncol = 2)
```



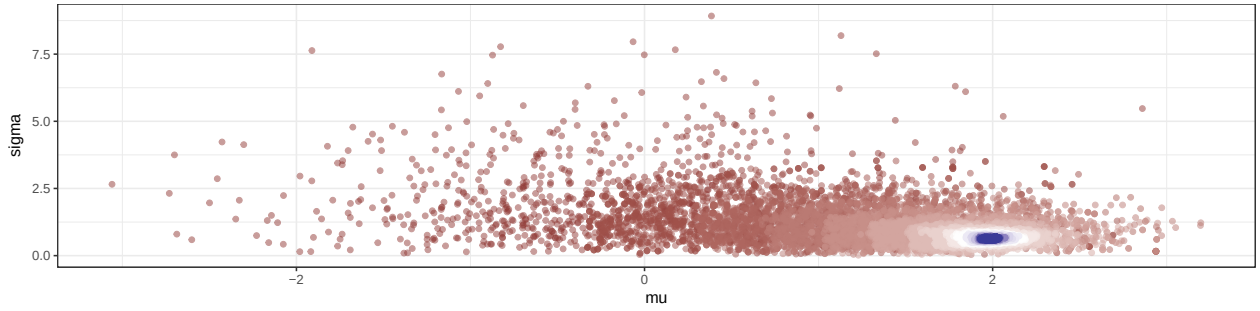
Finally, we can proceed to more steps of the algorithm, and plot the resulting output.

```
# let's do 10 more steps
wsmcresults_continued <- wsmc_continue(wsmcresults, maxtime = 10)
```

```
## result file contains the result of 12 steps
## step 13... running until time >= 10 seconds (since continue)
##   acceptance rates: 61.81641 %, threshold = 0.08615183 , min. dist. = 0.03240912
##   total # distances calculated: 43321 (for this step: 4453)
##   total time spent: 10.936 seconds (since continue: 0.916 s; this step: 0.916 s)
## step 14... running until time >= 10 seconds (since continue)
##   acceptance rates: 57.8125 %, threshold = 0.07437552 , min. dist. = 0.03923658
##   total # distances calculated: 48758 (for this step: 5437)
##   total time spent: 11.845 seconds (since continue: 1.825 s; this step: 0.909 s)
## step 15... running until time >= 10 seconds (since continue)
##   acceptance rates: 52.53906 %, threshold = 0.06679423 , min. dist. = 0.04160277
##   total # distances calculated: 55996 (for this step: 7238)
##   total time spent: 12.735 seconds (since continue: 2.715 s; this step: 0.89 s)
```

```
## step 16... running until time >= 10 seconds (since continue)
##   acceptance rates: 48.53516 %, threshold = 0.06164923 , min. dist. = 0.03594095
##   total # distances calculated: 66756 (for this step: 10760)
##   total time spent: 13.767 seconds (since continue: 3.747 s; this step: 1.032 s)
## step 17... running until time >= 10 seconds (since continue)
##   acceptance rates: 45.01953 %, threshold = 0.05796125 , min. dist. = 0.03430539
##   total # distances calculated: 82884 (for this step: 16128)
##   total time spent: 15.042 seconds (since continue: 5.022 s; this step: 1.275 s)
## step 18... running until time >= 10 seconds (since continue)
##   acceptance rates: 45.70312 %, threshold = 0.055216 , min. dist. = 0.03407409
##   total # distances calculated: 105945 (for this step: 23061)
##   total time spent: 16.958 seconds (since continue: 6.938 s; this step: 1.916 s)
## step 19... running until time >= 10 seconds (since continue)
##   acceptance rates: 44.72656 %, threshold = 0.05288579 , min. dist. = 0.03594095
##   total # distances calculated: 141016 (for this step: 35071)
##   total time spent: 18.88 seconds (since continue: 8.86 s; this step: 1.922 s)
## step 20... running until time >= 10 seconds (since continue)
##   acceptance rates: 40.72266 %, threshold = 0.05079637 , min. dist. = 0.03484967
##   total # distances calculated: 198207 (for this step: 57191)
##   total time spent: 21.678 seconds (since continue: 11.658 s; this step: 2.798 s)
```

```
#
plot_bivariate(wsmcresults_continued, i1 = 1, i2 = 2)
```




```
grid.arrange(plot_marginal(wsmcresults_continued, i = 1), plot_marginal(wsmcresults_continued,
    i = 2), ncol = 2)
```