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Answer 1

$$z = \frac{\sqrt{2}+\sqrt{2}i}{2+2\sqrt{3}i}$$
$$= \frac{1}{\sqrt{2}} * \frac{1+i}{1+\sqrt{3}i}$$

Exponential Representation of (1+i):

$$r = \sqrt{1^2+1^2} = \sqrt{2}$$
$$\theta = arctan(\frac{1}{1}) = \frac{\pi}{4}$$
$$1+i = \sqrt{2}e^{i\frac{\pi}{4}}$$

Similarly, Exponential Representation of $1+\sqrt{3}i$:

$$r = \sqrt{1^2+\sqrt{3}^2} = 2$$
$$\theta = arctan(\frac{\sqrt{3}}{1}) = \frac{\pi}{3}$$
$$1+\sqrt{3}i = 2e^{i\frac{\pi}{3}}$$

Using these representations:

$$z = \frac{1}{\sqrt{2}} * \frac{\sqrt{2}e^{i\frac{\pi}{4}}}{2e^{i\frac{\pi}{3}}}$$
$$z = \frac{1}{2}e^{-i\frac{\pi}{12}} \text{ (*)}$$
$$= \frac{1}{2}(cos(-\frac{\pi}{12}) + i * sin(-\frac{\pi}{12}))$$
$$z = 0.483 - 0.1294i \text{ (**)}$$

a)

By equation (**):

$$Rez = 0.483$$
$$Imz = -0.1294$$

b)

By equation (*):

$$Magnitude : r = 0.5$$

$$Phase : \theta = -\pi/12$$

Answer 2

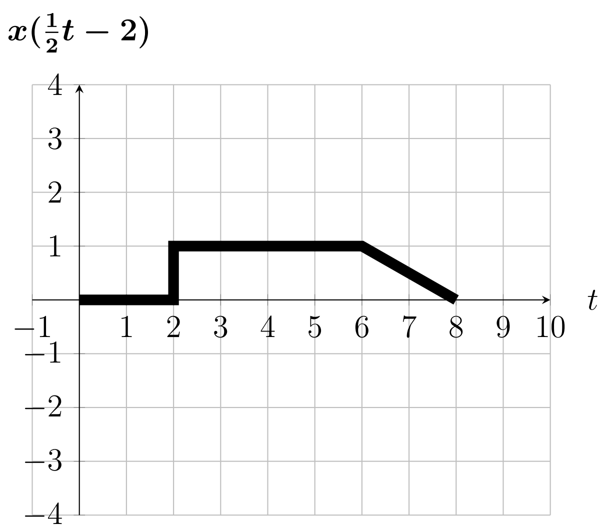

Figure 1: t vs. $x(\frac{1}{2}t - 2)$.

Answer 3

a)

$$\sum_{k=-3}^{3} x[k]\delta[n-k]$$
$$= \delta[n+3] - \delta[n+2] - \delta[n+1] - \delta[n] + \delta[n-1] + 2\delta[n-2] + \delta[n-3]$$

b)

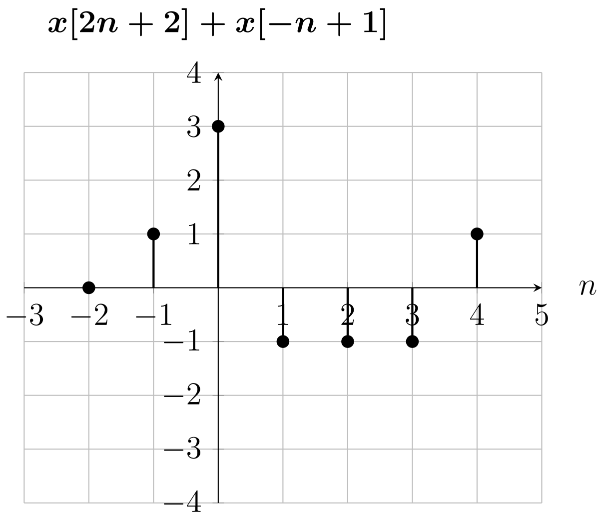

Figure 2: n vs. $x[2n+2]+x[-n+1]$.

c)

$$\sum_{k=-2}^{4} y[k]\delta[n-k]$$
$$= \delta[n+1] + 3\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4]$$

Answer 4

a)

$$x_1[n] = cos(\frac{5\pi}{2}n)$$
$$cos(\frac{5\pi}{2}n) = cos(\frac{5\pi}{2}n + \frac{5\pi}{2}N_0)$$
$$\frac{5\pi}{2}N_0 = 2\pi k$$

for k=5, $N_0 = 4$

b)

$$x_2[n] = sin(5n)$$
$$sin(5n) = sin(5n + 5N_0)$$
$$5N_0 = 2\pi k$$

There is no integer N_0 for any integer value k

c)

$$x_3(t) = 5sin(4t + \frac{\pi}{3})$$
$$5sin(4t + \frac{\pi}{3}) = 5sin(4t + 4T_0 + \frac{\pi}{3})$$
$$4T_0 + \frac{\pi}{3} = 2\pi$$
$$T_0 = \frac{5\pi}{12}$$

Answer 5

$$\delta(at) = \frac{1}{|a|}\delta(t)$$
$$|a|\delta(at) = \delta(t)$$
$$\text{Step impulse function property: } \int_{-\infty}^{\infty} \delta(t)dt = \delta(t)$$
$$|a| \int_{-\infty}^{\infty} \delta(at)dt = \int_{-\infty}^{\infty} \delta(t)dt = \delta(t)$$
$$\text{On the left side, replace parameter t with } \frac{t}{a}$$
$$|a| \int_{-\infty}^{\infty} \delta(t)\frac{dt}{a} = \int_{-\infty}^{\infty} \delta(t)dt$$
$$\frac{a}{|a|} \int_{-\infty}^{\infty} \delta(t)dt = \int_{-\infty}^{\infty} \delta(t)dt$$
$$1 = 1 \textbf{ (For any positive value a)}$$

Answer 6

a)

$$y_1[n] = S_1(x_1[n]) = 4x_1[n] + 2x_1[n-1]$$
$$y_2[n] = S_2(y_1[n]) = y_1[n-2]$$
$$y_2[n] = y_1[n-2] = 4x_1[n-2] + 2x_1[n-3]$$

Difference equation of the overall system in terms of x[n] and y[n] :

$$y[n] = S(x[n]) = 4x[n-2] + 2x[n-3]$$

b)

$$S_2(x_1[n]) = x_1[n-2]$$

$$S_1(x_1[n-2]) = 4x_1[n-2] + 2x_1[n-3]$$

Difference equation of this system in terms of x[n] and y[n] :

$$y[n] = 4x[n-2] + 2x[n-3]$$

The series connection of the sub systems is commutative.

c)

$$c_1y_1[n] = S(c_1x_1[n]) = 4c_1x_1[n-2] + 2c_1x_1[n-3]$$

$$c_2y_2[n] = S(c_2x_2[n]) = 4c_2x_2[n-2] + 2c_2x_2[n-3]$$

$$c_1y_1[n] + c_2y_2[n] = 4(c_1x_1[n-2] + c_2x_2[n-2]) + 2(c_1x_1[n-3] + c_2x_2[n-3])$$

$$= S(c_1x_1 + c_2x_2)$$

Conclusion:

$$S(c_1x_1) + S(c_2x_2) = c_1y_1[n] + c_2y_2[n] = S(c_1x_1 + c_2x_2)$$

Superposition property holds for the system S.

d)

Let $x_3[n] = x[n-n_0]$

$$y_3[n] = S(x_3[n]) = 4x_3[n-2] + 2x_3[n-3]$$

$$= 4x[n-n_0-2] + 2x_3[n-n_0-3]$$

$$= y[n-n_0]$$

$S(x[n-n_0]) = y[n-n_0]$ for any integer n_0

System S is Time invariant.

Answer 7

```
from sympy import symbols, Function

# Define symbolic variables
n = symbols('n', integer=True)
x = Function('x')(n)  # Input signal x[n]

# Constants (coefficients) for the linear combination and seperate outputs
c1, c2 = symbols('c1 c2')

# Define two different input signals
x1 = Function('x1')(n)
x2 = Function('x2')(n)


def isLinear(system):
    if(  system(c1*x1 + c2*x2)  == ( c1*system(x1) + c2*system(x2) ) ) :
        print("The given system is a Linear system. \n")
    else:
        print("The given system is a Non-Linear system. \n")


# Test

# y[n] = n*x[n]
def system_a(x):
    return n * x

isLinear( system_a )


# y[n] = x[n]**2
def system_b(x):
    return x**2

isLinear(system_b )
```

a) The given system is a Non-Linear system

b) The given system is a Non-Linear system