Student Information

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Answer 1

a)

$$\sum_{n=1}^{5} \frac{N}{x} = 1$$

$$N * (\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}) = 1$$

$$N * (\frac{60}{137}) = 1$$

$$N = 0.438$$

b)

$$\sum_{n=1}^{5} xP(x) = \sum_{n=1}^{5} N = 5N = 2.19$$

c)

$$Var(X) = E(X^2) - \mu^2$$

$$= (\sum_{n=1}^{5} x^2 P(x)) - \mu^2$$

$$= (\sum_{n=1}^{5} xN) - \mu^2$$

$$= 15N - \mu^2$$

$$= 6.57 - (2.19)^2$$

$$= 1.774$$

d)

$$E(Y) = \sum_{n=1}^{5} yP(y) = \sum_{n=1}^{5} \frac{y^2}{15} = \frac{1}{15} * (1^2 + 2^2 + 3^2 + 4^2 + 5^2)$$

$$E(Y) = 3.667$$

$$E(XY) = \sum_{n=1}^{5} xyP(x,y) = \sum_{n=1}^{5} xyP(x)P(y)$$

$$\sum_{n=1}^{5} N\frac{y^2}{15} = 8.03$$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$= 8.03 - (2.19)(3.667) = 0$$

Since P(x,y) = P(x)P(y) ; X and Y are independent random variables.

As a result, E(XY) = E(X)E(Y) and Cov(X,Y) = E(XY) - E(X)E(Y) = 0.

Conclusion: Covariance of two independent random variables is zero.

Answer 2

a)

X: Number of success in 1000 trials.

$$P(X \geq 1) = 1 - P(X < 1) = 1 - F(0) = 0.95$$

$$F(0) = 0.05$$

For F(0) = 0.05, $\lambda = 3$, using Poisson approximation to Binomial

$$\lambda = np = 1000p = 3$$

$$p = 0.003$$

b)

(i)

Y: Number of games to play in order to win two times

$$P(Y > 500)$$

X: Number of wins against IM in 500 games.

(Negative Binomial to Binomial)

$$P(Y > 500) = P(X < 2) = F(1)$$

Using Poisson approximation to Binomial with

$$\lambda = np = 500 * 3 * 10^{-1} = 1.5$$

$$F(1) = 0.558$$

(ii)

Y: Number of games to play in order to win two times

$$P(Y > 10000)$$

X: Number of wins against GM in 10000 games.

(Negative Binomial to Binomial)

$$P(Y > 10000) = P(X < 2) = F(1)$$

Using Poisson approximation to Binomial with

$$\lambda = np = 10000 * 10^{-4} = 1$$

$$F(1) = 0.736$$

c)

Y: Number of days that one feels sick

$$P(Y \leq 6) = F(6) \text{ with } \lambda = np = (366)(0.02) = 7.32$$

$$F(6) \text{ for } \lambda = 7.32 \cong \frac{(0.450 + 0.378)}{2} = 0.414$$

Answer 3

a)

```
#Total number of days
n = 366;

#probability of not feeling sick on any given day
p = 0.98;


#sum pmf from x = 360 to x = 366

totalProb = 0;

for k = 360:366
    prob_k = nchoosek(n, k) * p^k * (1-p)^(n-k);
    totalProb = totalProb + prob_k;
end

#Print result
fprintf('total = %.3f\n', totalProb);
```

total = 0.401

The found value is lower than the calculated one due to approximation.

The poisson approximated value is slightly higher than preciously calculated Binomial value

b)

```
p = 0.98;
x = 6;

#Range variable for n
N = 50:400;

#Array: Binomial cdf values for ranging n
cdfBinom = zeros(length(N), 1);

#Array: Poisson cdf values for ranging n
cdfPoisson = zeros(length(N), 1);

#Fill arrays for ranging n
```

```
for i = 1:length(N)
    n = N(i);

    cdfBinom(i) = binocdf(x, n, 0.02);

    cdfPoisson(i) = poisscdf(x, (n * 0.02));
end


figure;

plot(N, cdfBinom, 'b-', 'LineWidth', 2);
hold on;
plot(N, cdfPoisson, 'r--', 'LineWidth', 2);

xlabel('n');
ylabel('P');

legend('Binomial', 'Poisson');

grid on;
```

See Figure 1

c)

```
p = 0.78;
x = 6;

#Range variable for n
N = 50:400;

#Array: Binomial cdf values for ranging n
cdfBinom = zeros(length(N), 1);

#Array: Poisson cdf values for ranging n
cdfPoisson = zeros(length(N), 1);

#Fill arrays for ranging n
for i = 1:length(N)
    n = N(i);

    cdfBinom(i) = binocdf(x, n, 0.22);
```

```
    cdfPoisson(i) = poisscdf(x, (n * 0.22));
end


figure;

plot(N, cdfBinom, 'b-', 'LineWidth', 2);
hold on;
plot(N, cdfPoisson, 'r--', 'LineWidth', 2);

xlabel('n');
ylabel('P');

legend('Binomial', 'Poisson');

grid on;
```

See Figure 2

The total probability drastically decreases with p.

The difference between Binomial and Poisson CDF is noticable.

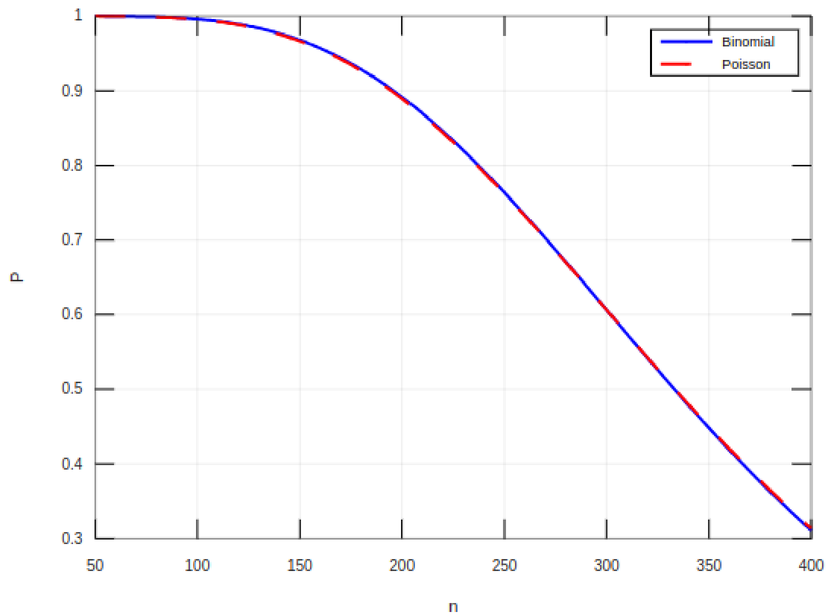

Figure 1: p = 0.98

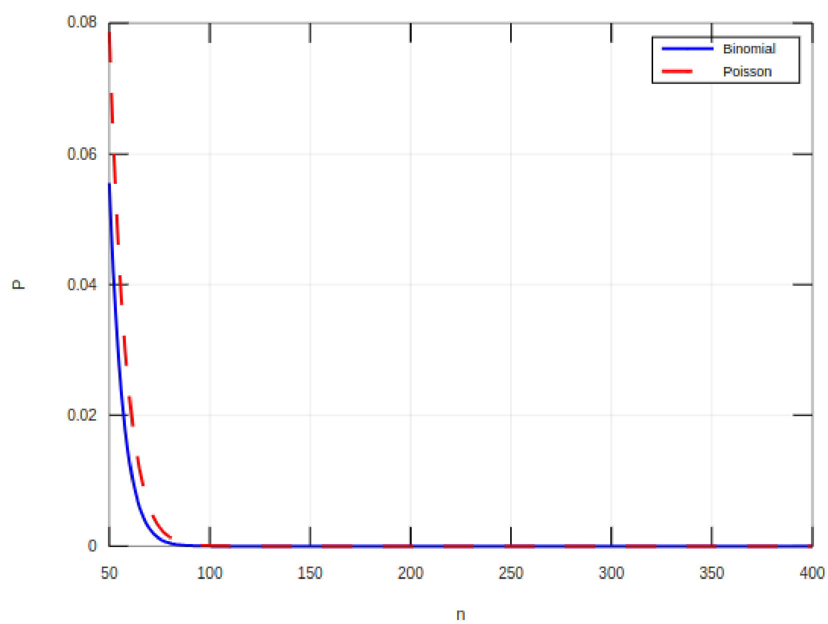

Figure 2: p = 0.78