CENG 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 2

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April 2, 2024

Answer 1

$$x(t) = \begin{cases} 1 & -3 \leq t \leq 7 \\ 0 & otherwise \end{cases}$$

$$h(t) = \begin{cases} 1 & 1 \leq t \leq 15 \\ 0 & otherwise \end{cases}$$

Three ranges of t values, integrated seperately:

$$-3 \leq \tau \leq 7$$

$$t - 15 \leq \tau \leq t - 1$$

(I) partially overlap: $-2 \leq t \leq 8$

$$-3 \leq \tau \leq t - 1$$

$$\int_{-3}^{t-1} d\tau = t + 2$$

(II) fully overlap: $8 \leq t \leq 12$

$$-3 \leq \tau \leq 7$$

$$\int_{-3}^{7} d\tau = 10$$

(III) partially overlap: $12 \leq t \leq 22$

$$t - 15 \leq \tau \leq 7$$

$$\int_{t-15}^{7} d\tau = 22 - t$$

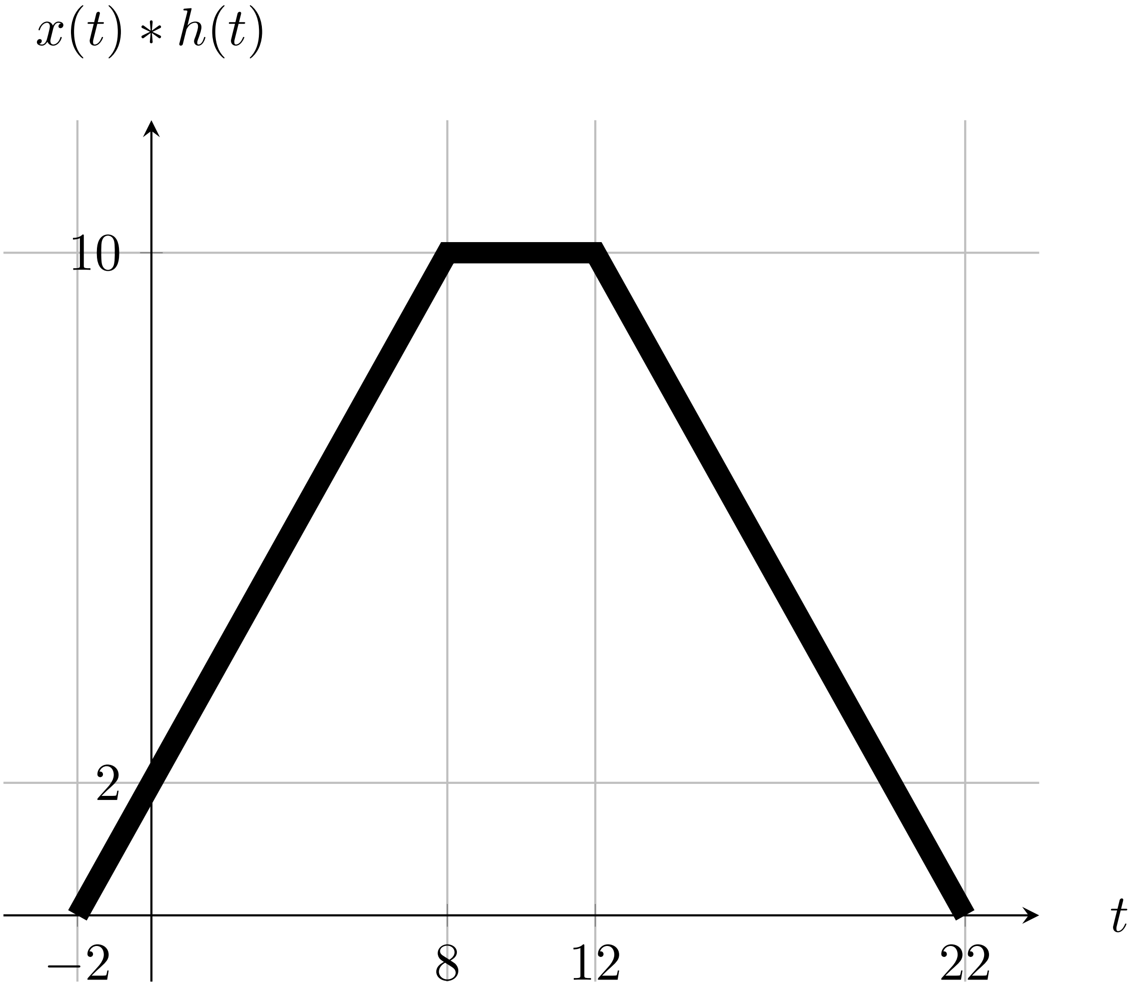

Figure 1: t vs. $x(\frac{1}{2}t - 2)$.

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Answer 2

a)

$$x[n] = \delta[n] + 2\delta[n-2] - 3\delta[n-4]$$
$$h[n] = 2\delta[n+2] + \delta[n-2]$$

By the distributive property of convolution

$$x[n] * h[n] = \delta[n] * h[n] + 2\delta[n-2] * h[n] - 3\delta[n-4] * h[n]$$

$$\delta[n] * h[n] = 2\delta[n+2] + \delta[n-2]$$
$$\delta[n-2] * h[n] = 2\delta[n] + \delta[n-4]$$
$$\delta[n-4] * h[n] = 2\delta[n-2] + \delta[n-6]$$

$$x[n] * h[n] = 2\delta[n+2] + 4\delta[n] - 5\delta[n-2] + 2\delta[n-4] - 3\delta[n-6]$$

$$y_1[n] = 2\delta[n+2] + 4\delta[n] - 5\delta[n-2] + 2\delta[n-4] - 3\delta[n-6]$$

b)

$$y_2[n] = x[n+2] * h[n]$$

By the time shifting property of LTI systems:

$$y[n] = x[n] * h[n] => x[n+k] * h[n] = y[n+k]$$

$$y_2[n] = x[n+2] * h[n] = y_1[n+2]$$
$$y_2[n] = 2\delta[n+4] + 4\delta[n+2] - 5\delta[n] + 2\delta[n-2] - 3\delta[n-4]$$

c)

$$x[n+2] = \delta[n+2] + 2\delta[n] - 3\delta[n-2]$$
$$h[n-2] = 2\delta[n] + \delta[n-4]$$

Same amount of shifts to opposite sides

for two functions defined on impulse functions:

$$y_3[n] = y_1[n] = 2\delta[n+2] + 4\delta[n] - 5\delta[n-2] + 2\delta[n-4] - 3\delta[n-6]$$

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Answer 3

$$y[n] = \frac{1}{5}x[n-1] + x[n]$$

a)

$$y[n] = \frac{1}{5}x[n-1] + x[n]$$

By feeding the system with unit impulse signal $x[n] = \delta[n]$

$$h[n] = \frac{1}{5}\delta[n-1] + \delta[n]$$

b)

Convoluting h[n] with $x[n] = \delta[n-2]$

$$\delta[n-2] * h[n] = \frac{1}{5}\delta[n-3] + \delta[n-2]$$

Resulting in a time shift of impulse response.

Output y[n] : $y[n] = \frac{1}{5}\delta[n-3] + \delta[n-2]$

c)

d)

$$h[n] \neq K\delta[n]$$

System has memory.

e)

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Answer 5

$$y[n] = \frac{1}{5}y[n-1] + 2x[n-2]$$

a)

Feeding the system y[n] with unit impulse signal:

$y[n] \to h[n]$ **for** $x[n] \to \delta[n]$

$$h[n] = \frac{1}{5}h[n-1] + 2\delta[n-2]$$

System is initially at rest: $h[0] = 0$

By using the recursive method, we can obtain the impulse response.

$$h[0] = 0$$

$$h[1] = \frac{1}{5}h[0] + 2\delta[-1] = 0$$

$$h[2] = \frac{1}{5}h[1] + 2\delta[0] = 2$$

$$h[3] = \frac{1}{5}h[2] + 2\delta[1] = (\frac{1}{5})(2)$$

$$h[4] = \frac{1}{5}h[3] + 2\delta[2] = (\frac{1}{5})^2(2)$$

$$h[5] = \frac{1}{5}h[4] + 2\delta[3] = (\frac{1}{5})^3(2)$$

...

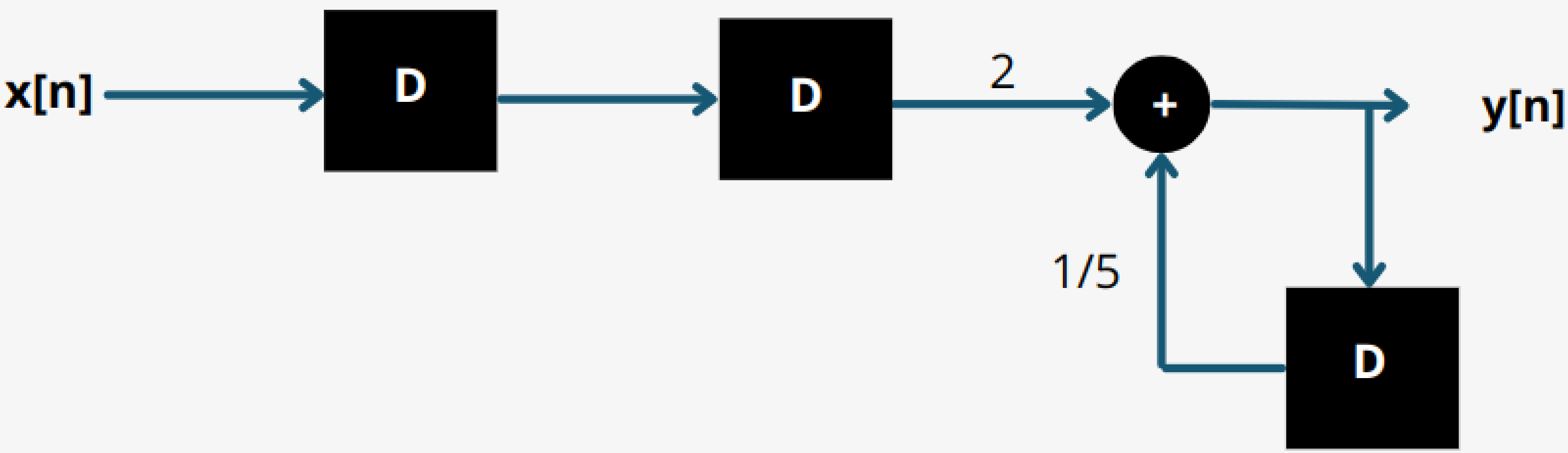
$$h[n] = (\frac{1}{5})^{n-2}(2)$$

for $n > 1$

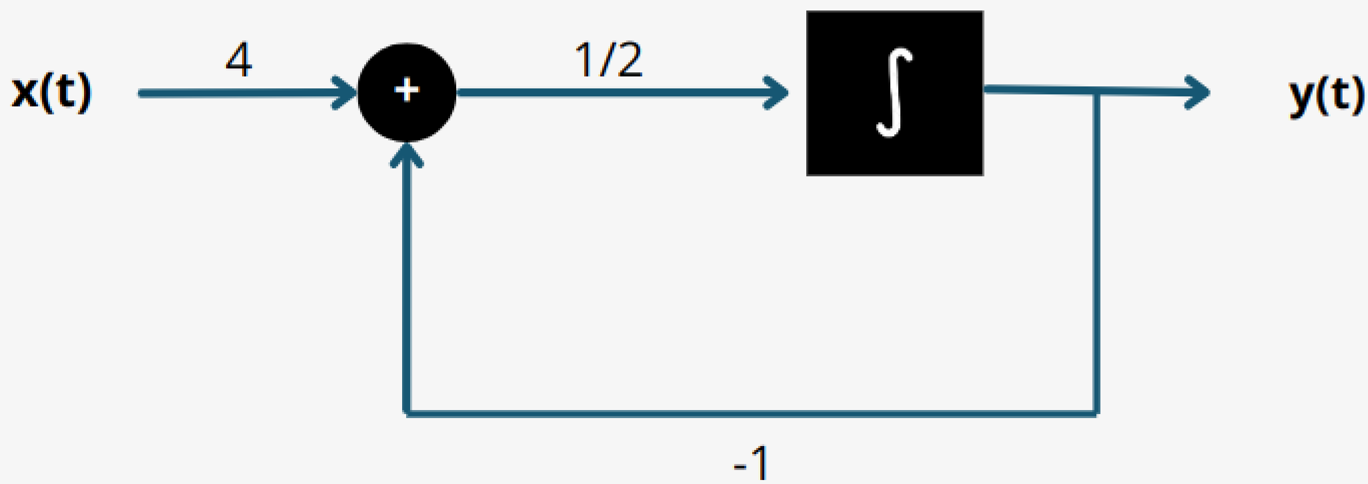
$$h[n] = (\frac{1}{5})^{n-2}(2)\mu[n-2]$$

b)

5_c)



6_a)



6_b)