CENG 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 3

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1.

$$a_k = \begin{cases} -1 & \textbf{k is even} \\ 1 & \textbf{k is odd} \end{cases}$$

$$a_k = -cos(\pi k)$$

$$-cos(\pi k) = -\frac{1}{2}e^{j\pi k} - \frac{1}{2}e^{-j\pi k}$$

$$x(t) = (-\frac{1}{2})\sum_{k=-\infty}^{\infty} e^{jk(\omega_0+\pi)t} + e^{-jk(\omega_0-\pi)t}$$

$$\omega_0 = 2\pi/T = \pi/2$$

$$x(t) = (-\frac{1}{2})\sum_{k=-\infty}^{\infty} e^{jk(3\pi/2)t} + e^{-jk(\pi/2)t}$$

2. (a)

$$a_k = \frac{1}{4}\int_0^4 x(t)e^{-jk\omega_o t}dt$$

$$a_k = \frac{1}{4}\int_0^2 2te^{-jk\omega_o t}dt + \frac{1}{4}\int_2^4 (4-t)e^{-jk\omega_o t}dt$$

By evaluating the integrals, we get:

$$a_k = -\frac{(jkwt+1)\,\mathrm{e}^{-jkwt}}{2j^2k^2w^2} + \frac{(jkw\cdot(t-4)+1)\,\mathrm{e}^{-jkwt}}{4j^2k^2w^2}$$

(b)

$$x(t) \longleftrightarrow_{FS} a_k$$

$$\frac{dx(t)}{dt} \longleftrightarrow_{FS} jk\omega_o a_k$$

$$a`_k = jk\omega_o a_k$$

$$a`_k = -\frac{(jkwt+1)\,\mathrm{e}^{-jkwt}}{2jkw} + \frac{(jkw\cdot(t-4)+1)\,\mathrm{e}^{-jkwt}}{4jkw}$$

3. (a)

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}$$

a-1) Spectral coefficients of $x_1[n] = cos(\frac{\pi}{2}n)$

$$\frac{\pi}{2}N_0 = 2\pi m \rightarrow N_0 = 4$$

$$a_k = \frac{1}{4}\sum_{n=0}^{3} cos(\frac{\pi}{2}n)e^{-jk(\pi/2)n}$$

$$a_k = \frac{1}{4}(1 + 0 + (-1)e^{-jk\pi} + 0)$$

$$a_k = \frac{1-(-1)^k}{4}$$

$$a_k = \frac{1-(-1)^k}{4}$$

$$a_k = \begin{cases} 0 & \textbf{k is even} \\ 0.5 & \textbf{k is odd} \end{cases}$$

a-2) Spectral coefficients of $x_2[n] = sin(\frac{\pi}{2}n)$

$$\frac{\pi}{2}N_0 = 2\pi m \rightarrow N_0 = 4$$

$$a_k = \frac{1}{4}\sum_{n=0}^{3} sin(\frac{\pi}{2}n)e^{-jk(\pi/2)n}$$

$$a_k = \frac{1}{4}(0 + e^{-jk(\pi/2)} + 0 - e^{-jk(3\pi/2)})$$

$$a_k = \frac{e^{-jk(\pi/2)} - e^{-jk(3\pi/2)}}{4}$$

a-3) Spectral coefficients of $x_3[n] = cos(\frac{\pi}{2}n)sin(\frac{\pi}{2}n)$

$x_3[n] = 0$ for all integer n.

(b)

$$x_1(t) \longleftrightarrow_{FS} a_k$$

$$x_2(t) \longleftrightarrow_{FS} b_k$$

$$x(t) \longleftrightarrow_{FS} a_k * b_k$$

$$a_k * b_k = \sum_{l=0}^{3} = a_l b_{k-l}$$

$$= a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0$$

$$= 0(b_3) + (a_1)0 + 0(b_1) + (a_3)0$$

$$= 0$$

The result is the same due to the multiplication property.
The signal that is the multiplication of the signals in time domain has the coefficients in frequency domain which are the convolution of the coefficients of these two signals.

4.

$$x_1(t) = cos(k\frac{\pi}{3})$$

$$x_1(t) \longleftrightarrow_{FS} b_k$$

$$cos(k\frac{\pi}{3}) = \frac{1}{2}e^{jk(\pi/3)} + \frac{1}{2}e^{-jk(\pi/3)}$$

$$cos(k\frac{\pi}{3}) \rightarrow N = 6$$

$$b_k = \frac{1}{6}\sum_{n=0}^{5} x_1[n]e^{-jkn(\pi/3)} = \frac{1}{2}e^{jk(\pi/3)} + \frac{1}{2}e^{-jk(\pi/3)}$$

$$x_1[-1] = x_1[1] = 3$$

$$x_2(t) = cos(k\frac{\pi}{4})$$

$$x_2(t) \longleftrightarrow_{FS} c_k$$

$$cos(k\frac{\pi}{4}) = \frac{1}{2}e^{jk(\pi/4)} + \frac{1}{2}e^{-jk(\pi/4)}$$

$$cos(k\frac{\pi}{4}) \rightarrow N = 8$$

$$c_k = \frac{1}{8}\sum_{n=0}^{7} x_2[n]e^{-jkn(\pi/4)} = \frac{1}{2}e^{jk(\pi/4)} + \frac{1}{2}e^{-jk(\pi/4)}$$

$$x_2[-1] = x_2[1] = 4$$

By linear property:

$$x[n] = 4\delta[n+1] + 4\delta[n-1] + 3\delta[n+3] + 3\delta[n-3]$$

5. (a)

$$x[n] = sin(\frac{6\pi}{13}n + \frac{\pi}{2}) = cos(\frac{6\pi}{13}n)$$

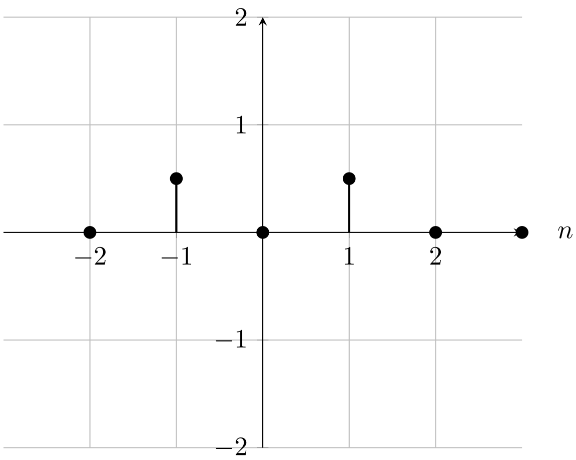
$$\frac{6\pi}{13}N_0 = 2\pi m \rightarrow N_0 = 13$$

(b)

$$cos(\frac{6\pi}{13}n) = \frac{1}{2}e^{j(6\pi/13)n} + \frac{1}{2}e^{-j(6\pi/13)n}$$

$$a_{-1} = a_1 = \frac{1}{2}$$

$a_k = 0$ for any other integer k value.



6. (a)

$$H(j\omega) = (\frac{1}{4})\frac{1}{\frac{3}{4} + j\omega}$$

Using inverse transform (by table 4.2) :

$$\frac{1}{(a + j\omega)} \longleftrightarrow_{FS^{-1}} e^{-at}\mu(t)$$

$$h(t) = (\frac{1}{4})e^{(-3/4)t}\mu(t)$$

$$h(t) = \frac{e^{(-3/4)t}\mu(t)}{4}$$

(b)

$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau) e^{((-3/4)t-\tau)} \mu(t-\tau) d\tau$$
$$x(t) = e^{at}\mu(t)$$
$$= \int_{-\infty}^{\infty} e^{a\tau}\mu(\tau) e^{((-3/4)t-\tau)} \mu(t-\tau) d\tau$$
$$= \int_{0}^{t} e^{a\tau} e^{((-3/4)t-\tau)} d\tau$$
$$e^{-5t} - e^{-10t} = e^{(-3/4)t} \int_{0}^{t} e^{(a-1)\tau} d\tau$$

7.