Indexing

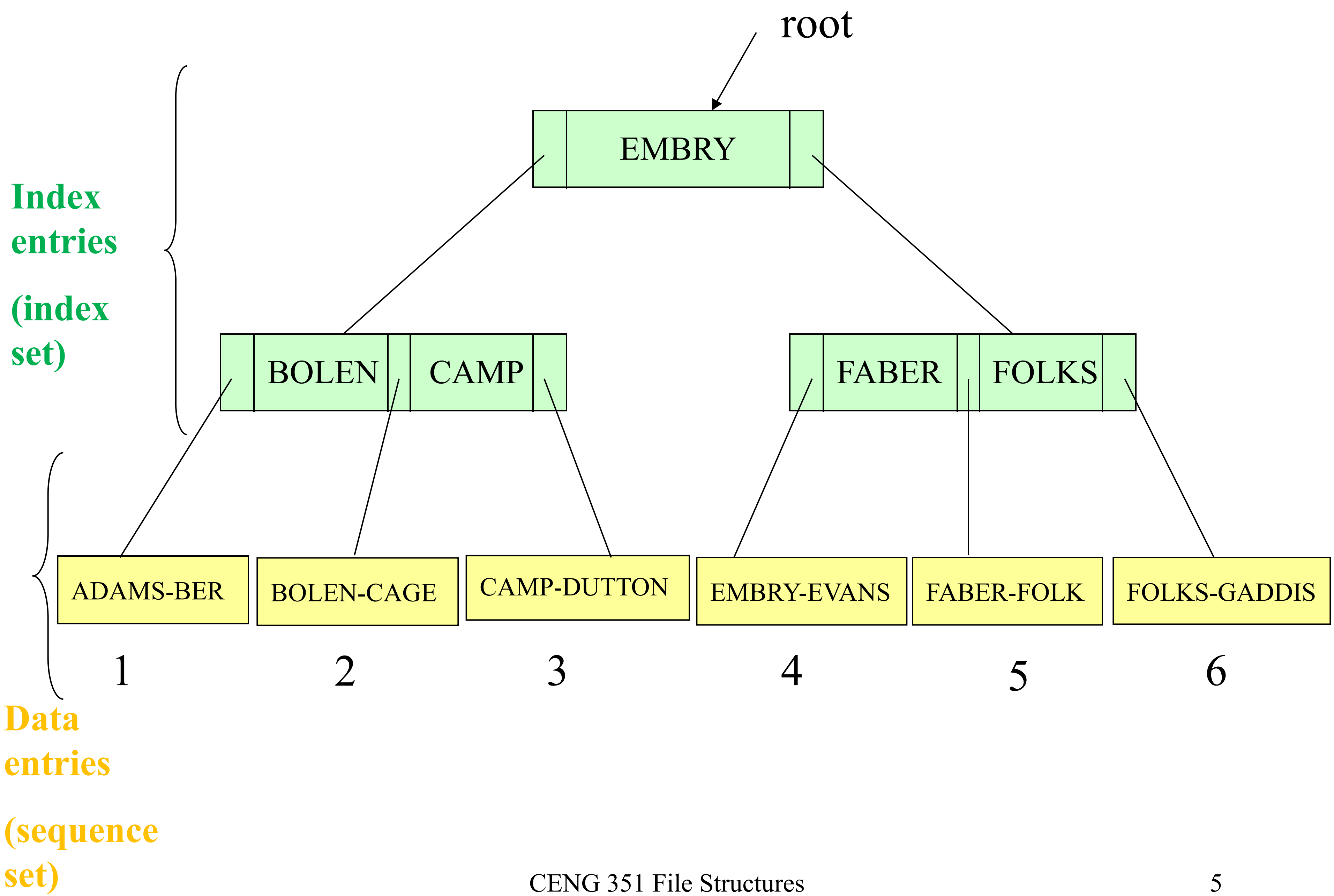
Part 2

Tree indexes

- If index doesn't fit in memory:
  - Divide the index structure into blocks,
  - Organize these blocks similarly building a tree structure.
- Tree indexes:
  - B Trees
  - B+ Trees
  - Simple prefix B+ Trees
  - …

B+ Trees

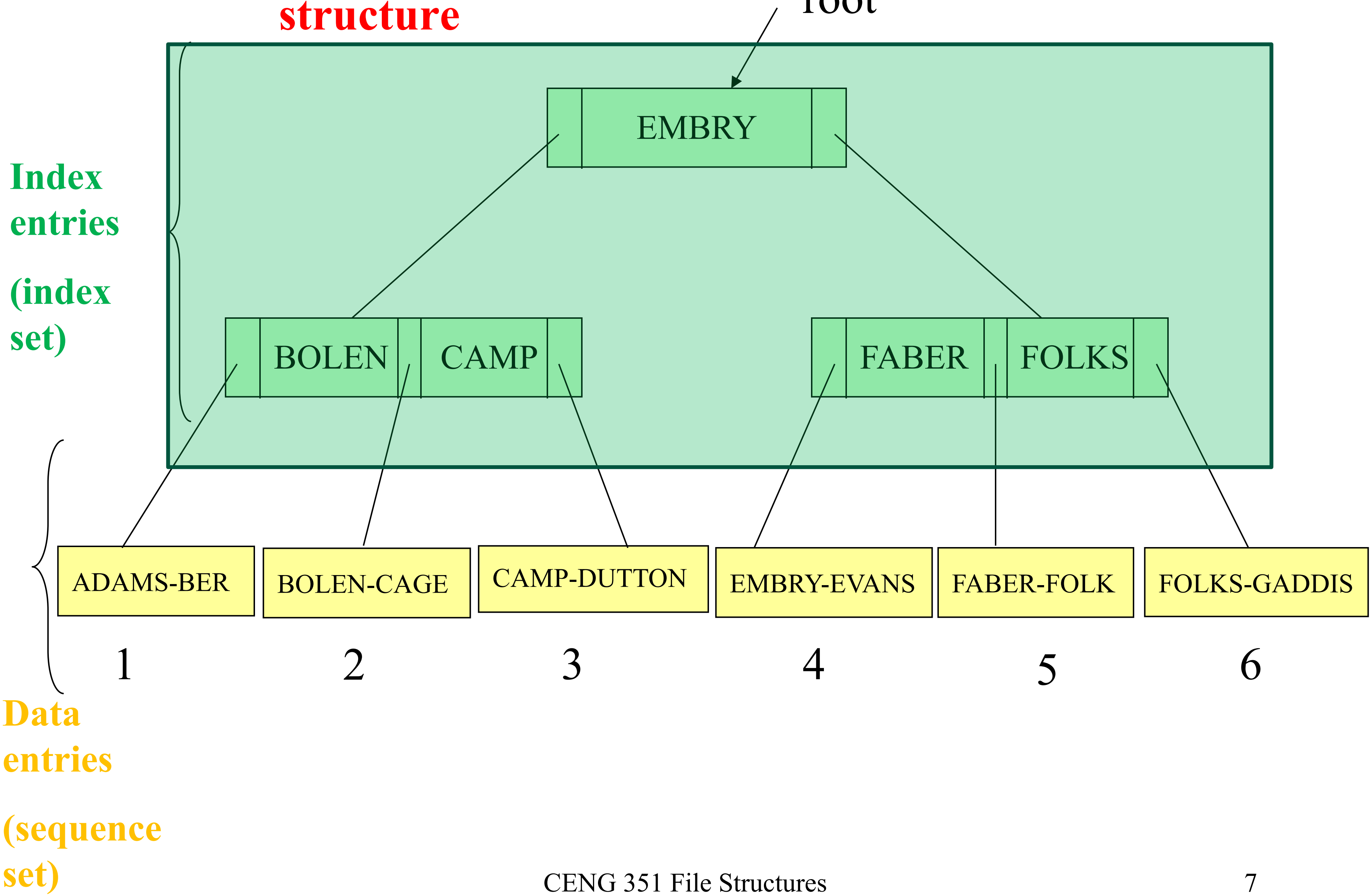
- B-tree is one of the most important data structures in computer science.
- What does B stand for? (Not binary!)
- B-tree is a **multiway search** tree.
- Several versions of B-trees have been proposed, but only B+ Trees have been used with large files.
- A B+tree is a B-tree in which data records are in leaf nodes, and faster sequential access is possible.


root
EMBRY
BOLEN
CAMP
FABER
FOLKS
ADAMS-BER
BOLEN-CAGE
CAMP-DUTTON
EMBRY-EVANS
FABER-FOLK
FOLKS-GADDIS
1
2
3
4
5
6
Index entries
(index set)
Data entries
(sequence set)

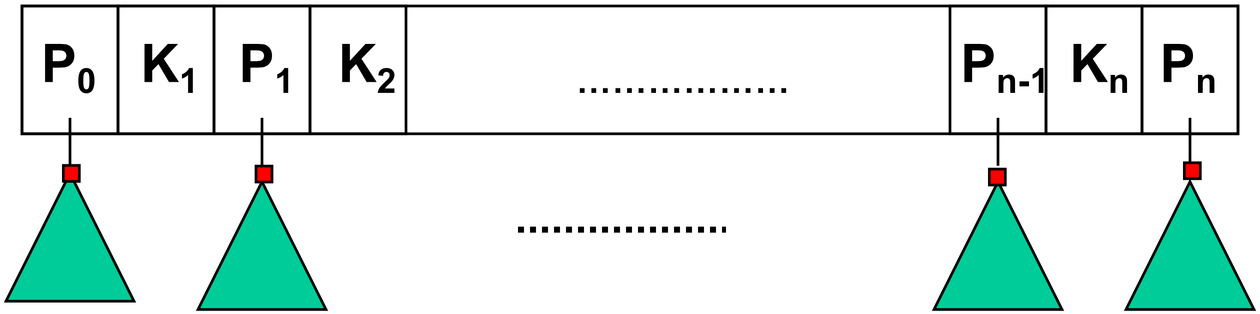
Formal definition of B+ Tree Properties

- Properties of a B+ Tree of order **d**:
  - All internal nodes (except root) have **at least d keys** and **at most 2d** keys.
  - Root can have at least **1 key** and **at most 2d** keys.
  - An internal node with **n** keys has **n+1** children
  - The root has at least 2 children unless it's a leaf.
  - All leaves are on the same level (balanced tree).

B+ tree: Internal/root node structure



B+ tree: Internal/root node structure

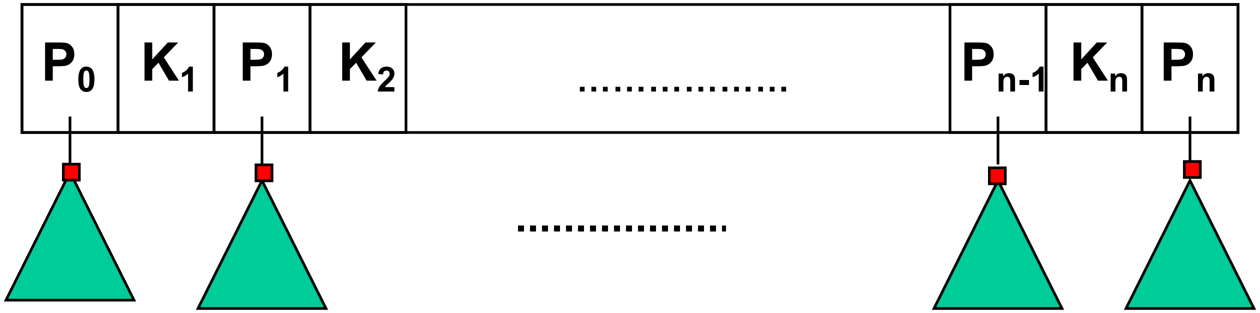


Each P_i is a pointer to a child node; each K_i is a search key value
# of search key values = n, # of pointers = n+1

In a B+ Tree of order d:

- All internal nodes (except root) have **at least d keys** and **at most 2d** keys ($d \le n \le 2d$).
- Root can have at least **1 key** and **at most 2d** keys. ($1 \le n \le 2d$).
- An internal node with **n** keys has **n+1** children.
- The root has at least 2 children unless it's a leaf.

B+ tree: Internal/root node structure



Each P_i is a pointer to a child node; each K_i is a search key value
\# of search key values = n, # of pointers = n+1

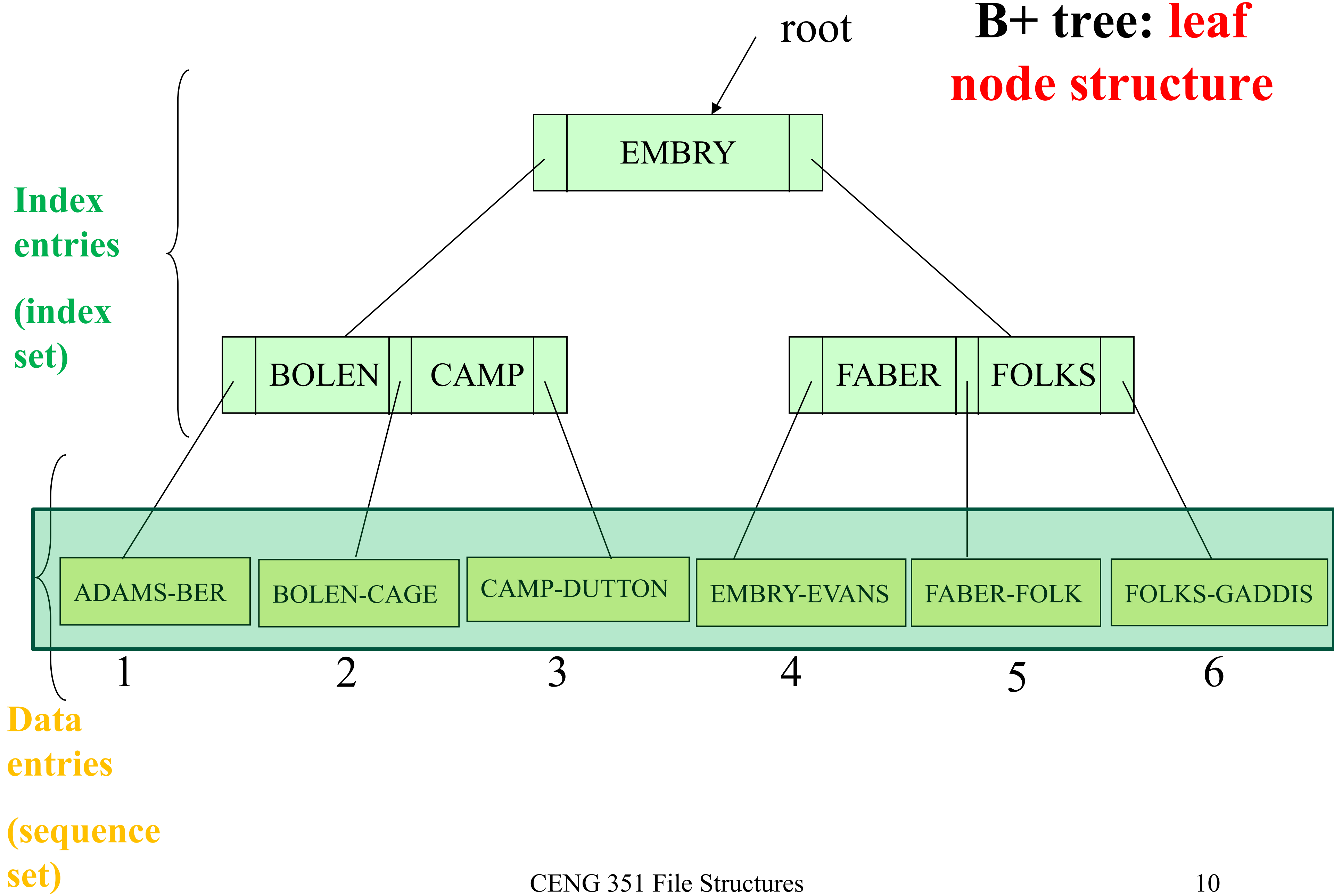
- Requirements:
- $K_1 < K_2 < \ldots < K_n$
- For any search key value K in the subtree pointed by Pi,

  If $P_i = P_0$, we require $K < K_1$

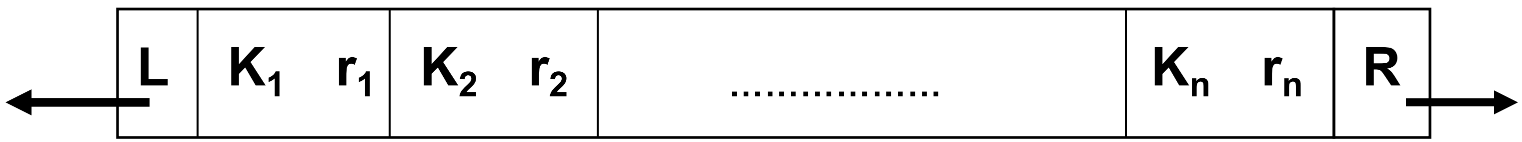
  If $P_i = P_n$, $K_n <= K$

  If $P_i = P_1, \ldots, P_{n-1}$, $K_i <= K < K_{i+1}$

B+ tree: leaf node structure



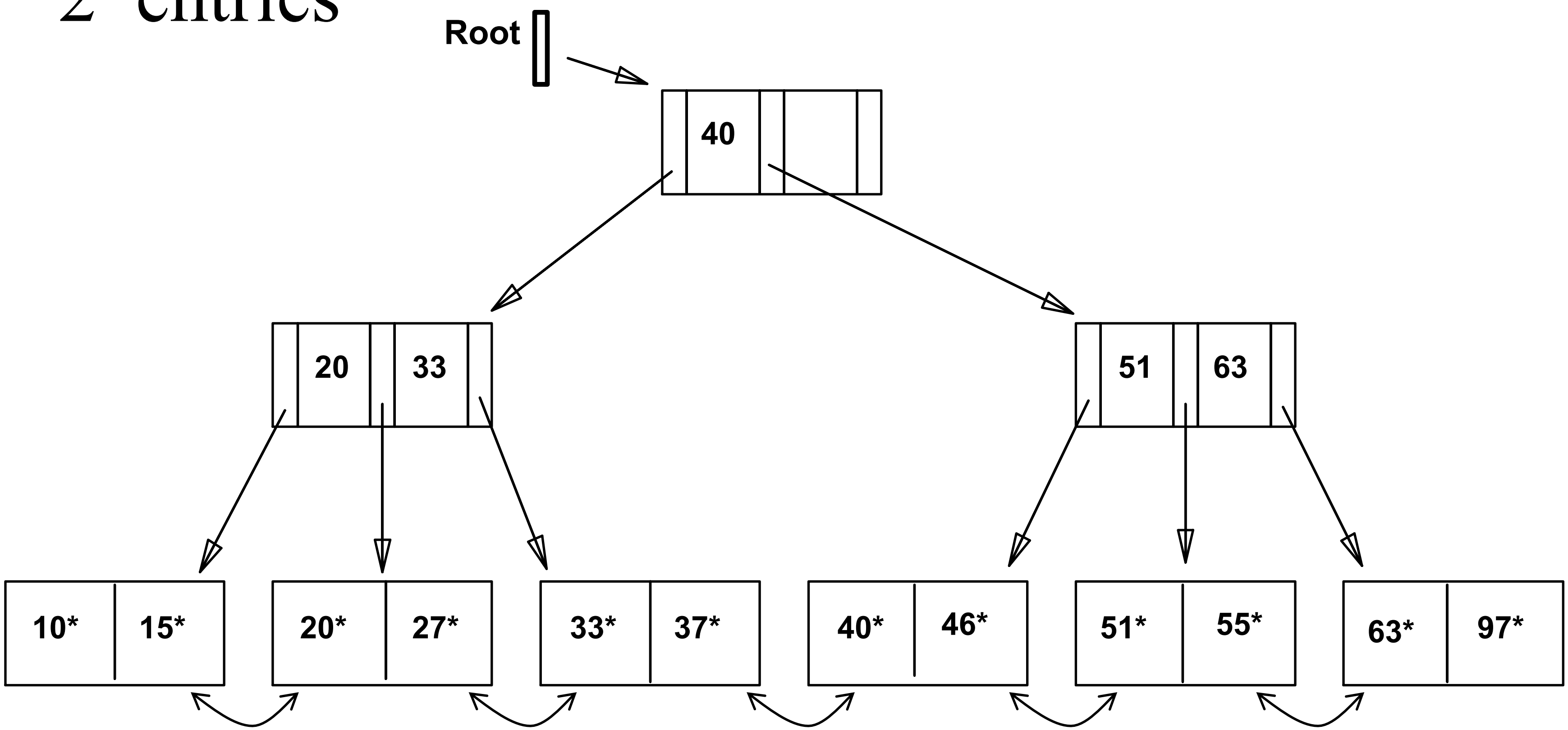
B+ tree: leaf node structure



- Pointer L points to the left neighbor; R points to the right neighbor (**doubly linked list**)
- $K_1 < K_2 < \ldots < K_n$
- $d \le n \le 2d$ (d is the order of this B+ tree)
- We will use K_i* for the pair $<K_i, r_i>$ and omit L and R for simplicity
- All leaves are on the same level (balanced tree).

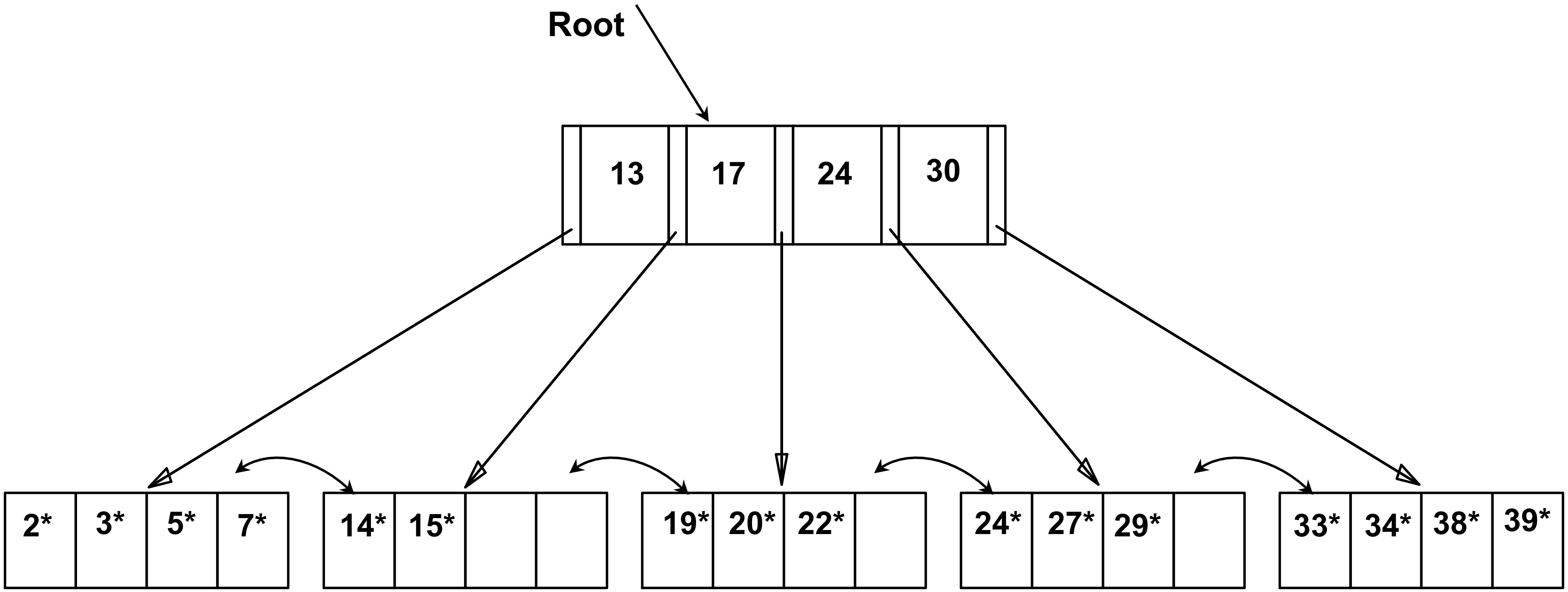
Example: B+ tree with order of 1

- Each node must hold at least 1 entry, and at most 2 entries



Example: Search in a B+ tree order 2

- Search: how to find the records with a given search key value?
  - Begin at root, and use key comparisons to go to leaf
- Examples: search for 5*, 16*, all data entries >= 24* ...
  - The last one is a range search, we need to do the sequential scan, starting from the first leaf containing a value >= 24.

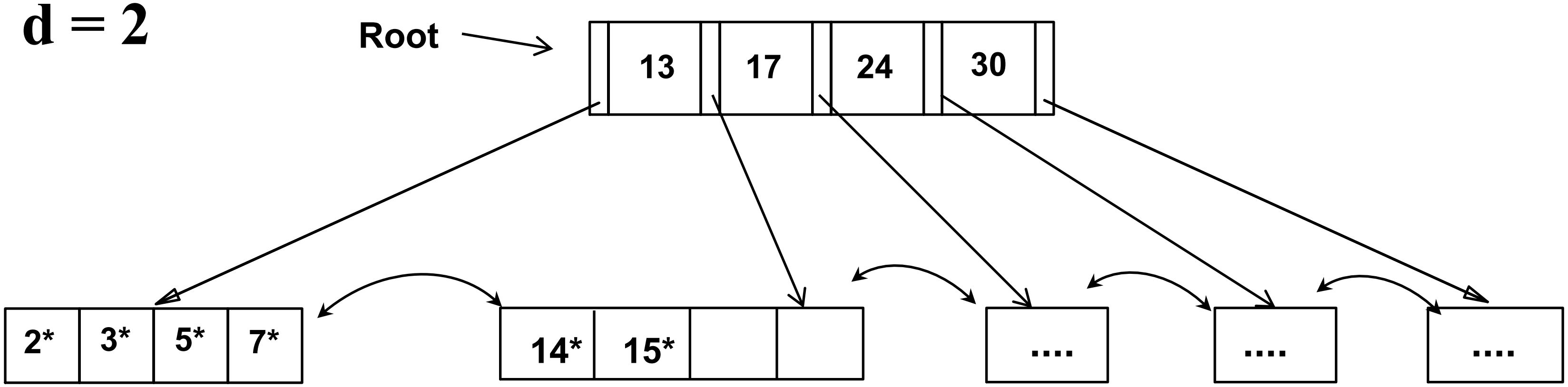


How to Insert a Data Entry into a B+ Tree?

- Let’s look at several examples first.

Inserting 16*, 8* into Example B+ tree

d = 2



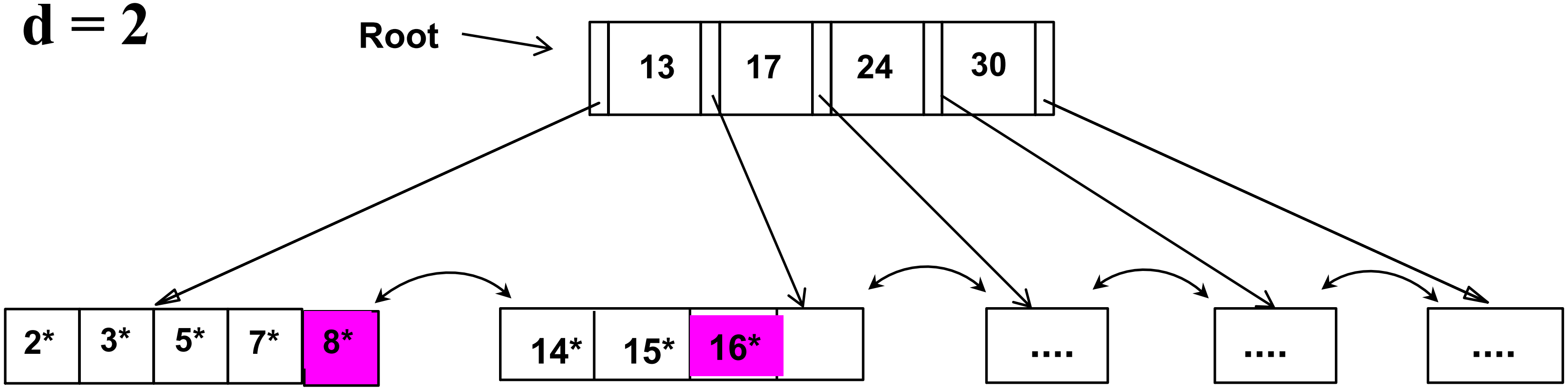
Leaf nodes:

$d \leq n \leq 2d$

$2 \leq n \leq 4$

Inserting 16*, 8* into Example B+ tree

d = 2



Leaf node overflows!!!

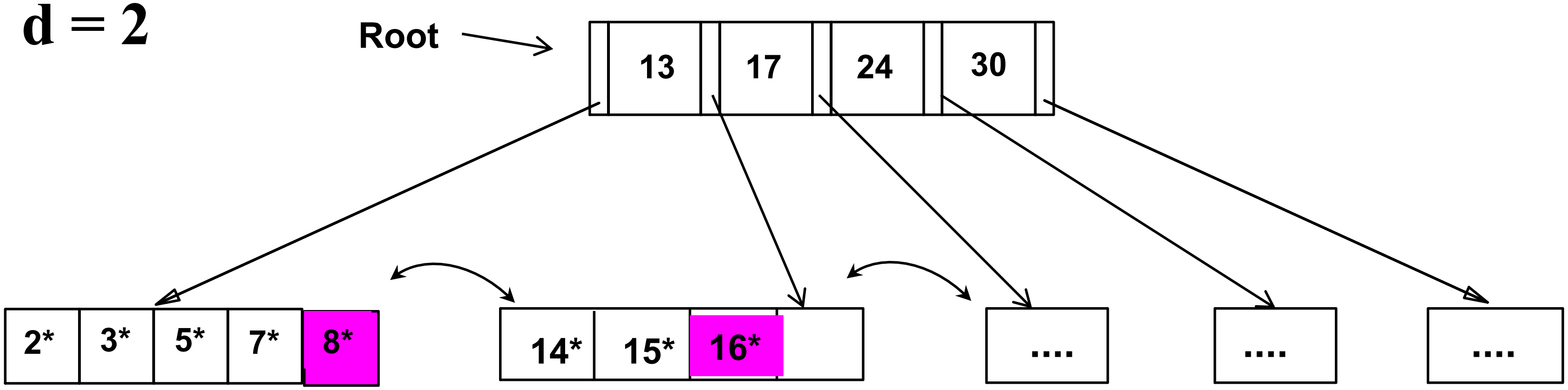
Leaf nodes:

$d \leq n \leq 2d$

$2 \leq n \leq 4$

Inserting 16*, 8* into Example B+ tree

d = 2

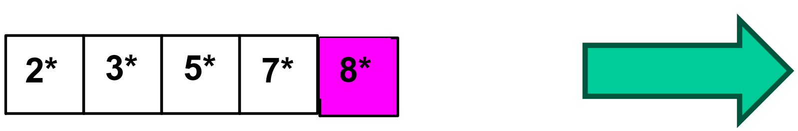


Leaf node overflows!!!

When a <u>leaf node</u> overflows:

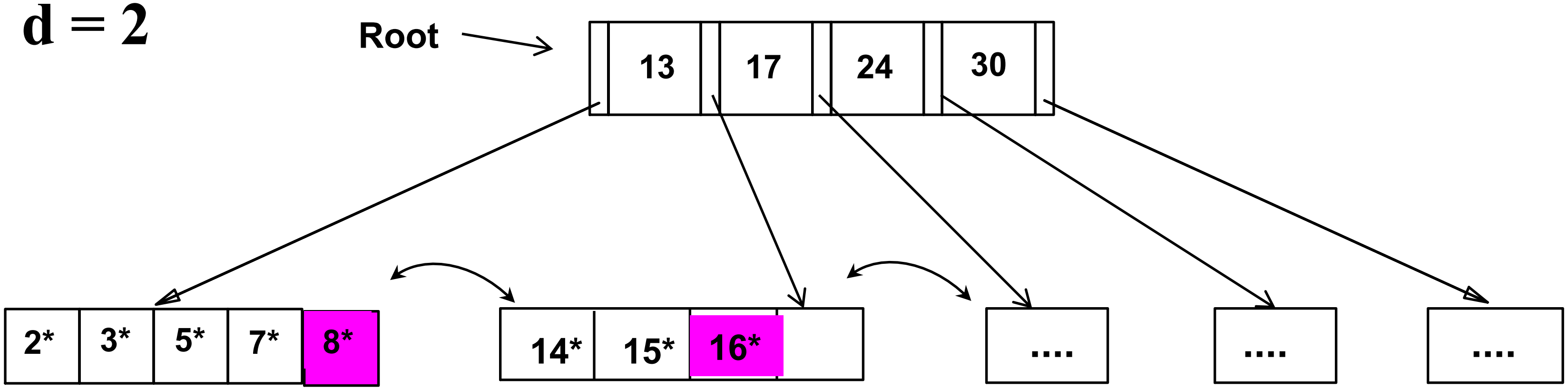
1) Split the node

First **d** entries stay in old node , move rest of entries to new node



Inserting 16*, 8* into Example B+ tree

d = 2

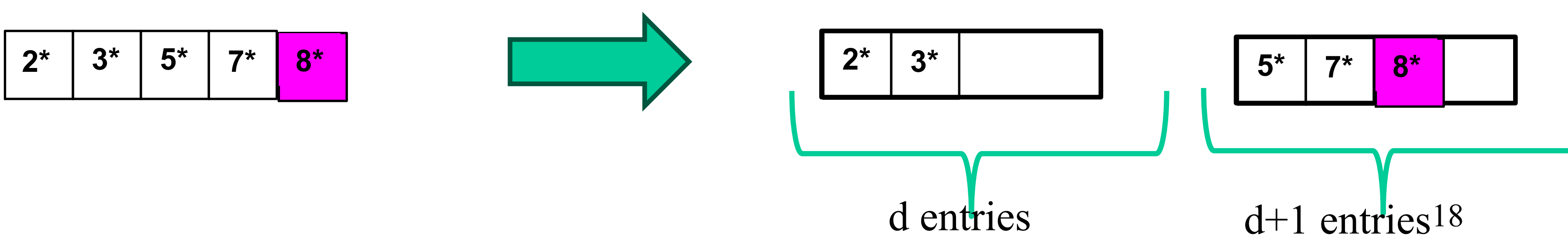


Leaf node overflows!!!

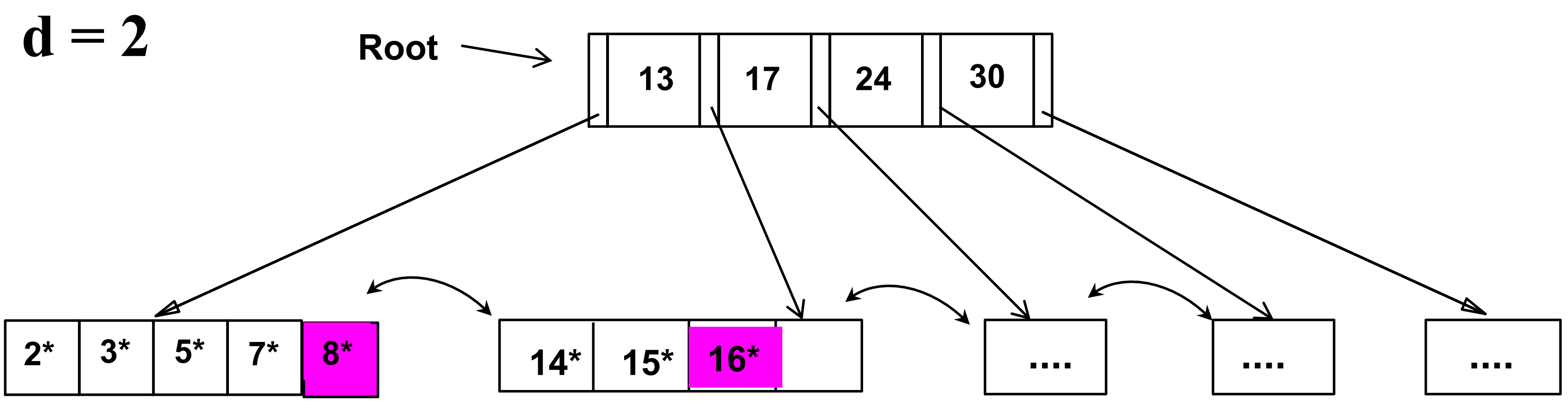
When a <u>leaf node</u> overflows:

1) Split the node

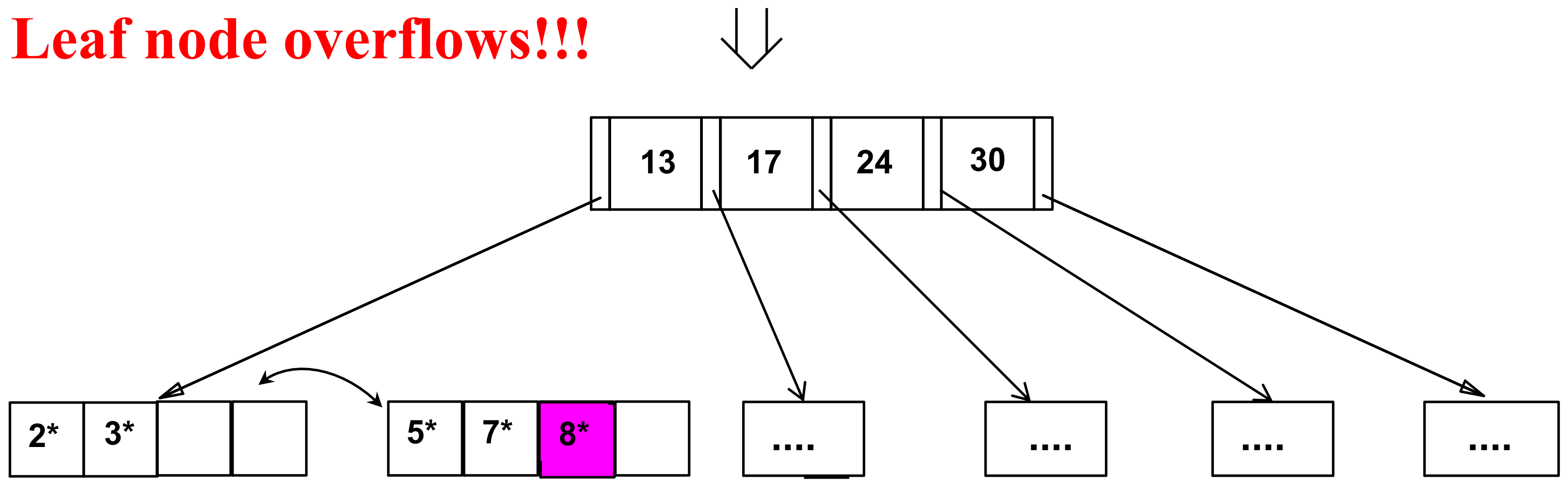
First **d** entries stay in old node , move rest of entries to new node



Inserting 16*, 8* into Example B+ tree



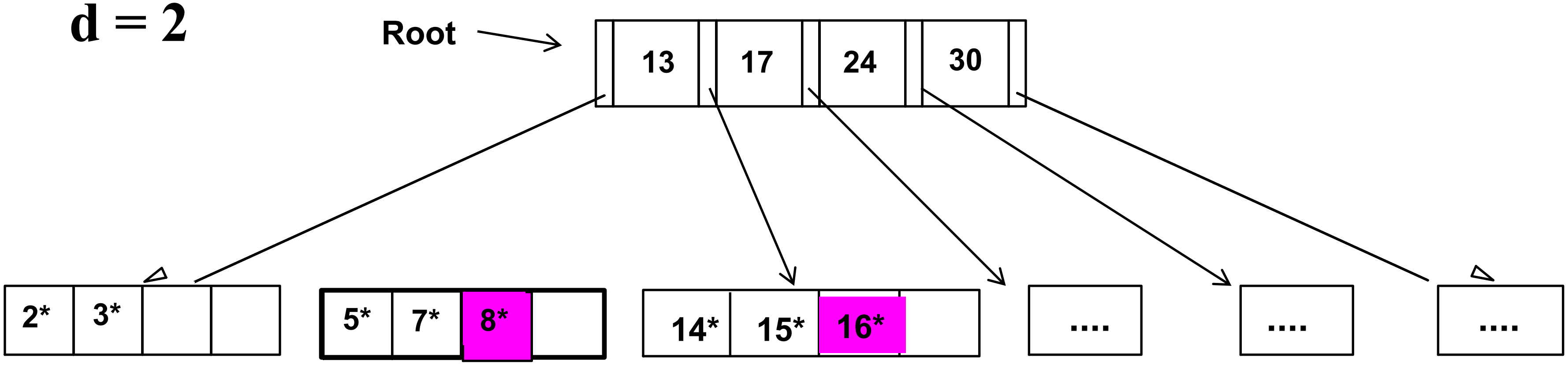
Leaf node overflows!!!



One new child (leaf node) generated; must add **one more pointer** to its parent, thus **one more key value** as well.

Inserting 16*, 8* into Example B+ tree

d = 2



Leaf node overflows!!!

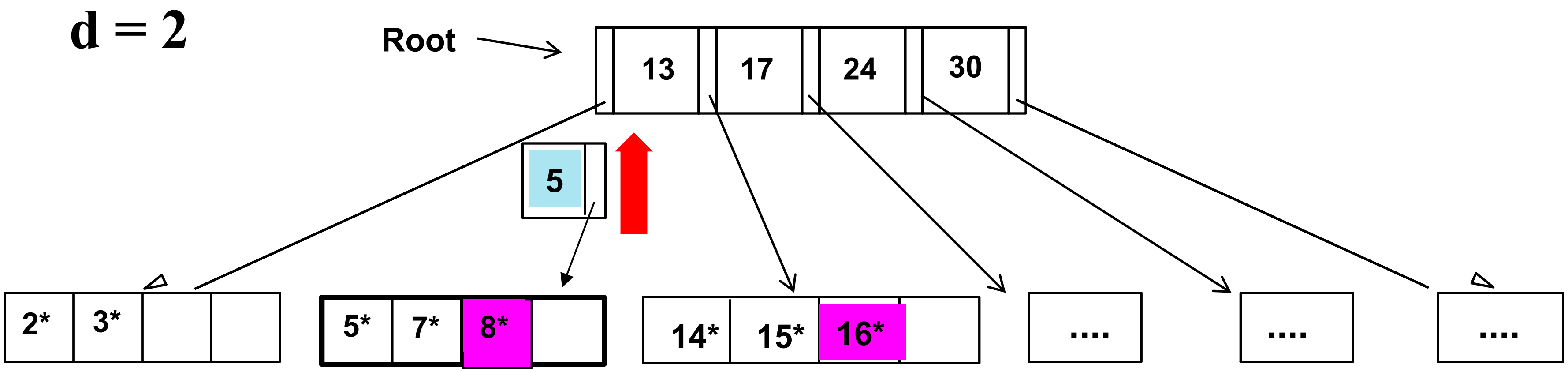
When a <u>leaf node</u> overflows:

1) Split the node

First **d** entries stay, move rest to new node

2) We need a pointer to the new block for the search: **COPY UP** the *middle key* as the search key. Also, add pointer to the new block

Inserting 16*, 8* into Example B+ tree



Leaf node overflows!!!

When a <u>leaf node</u> overflows:

1) Split the node

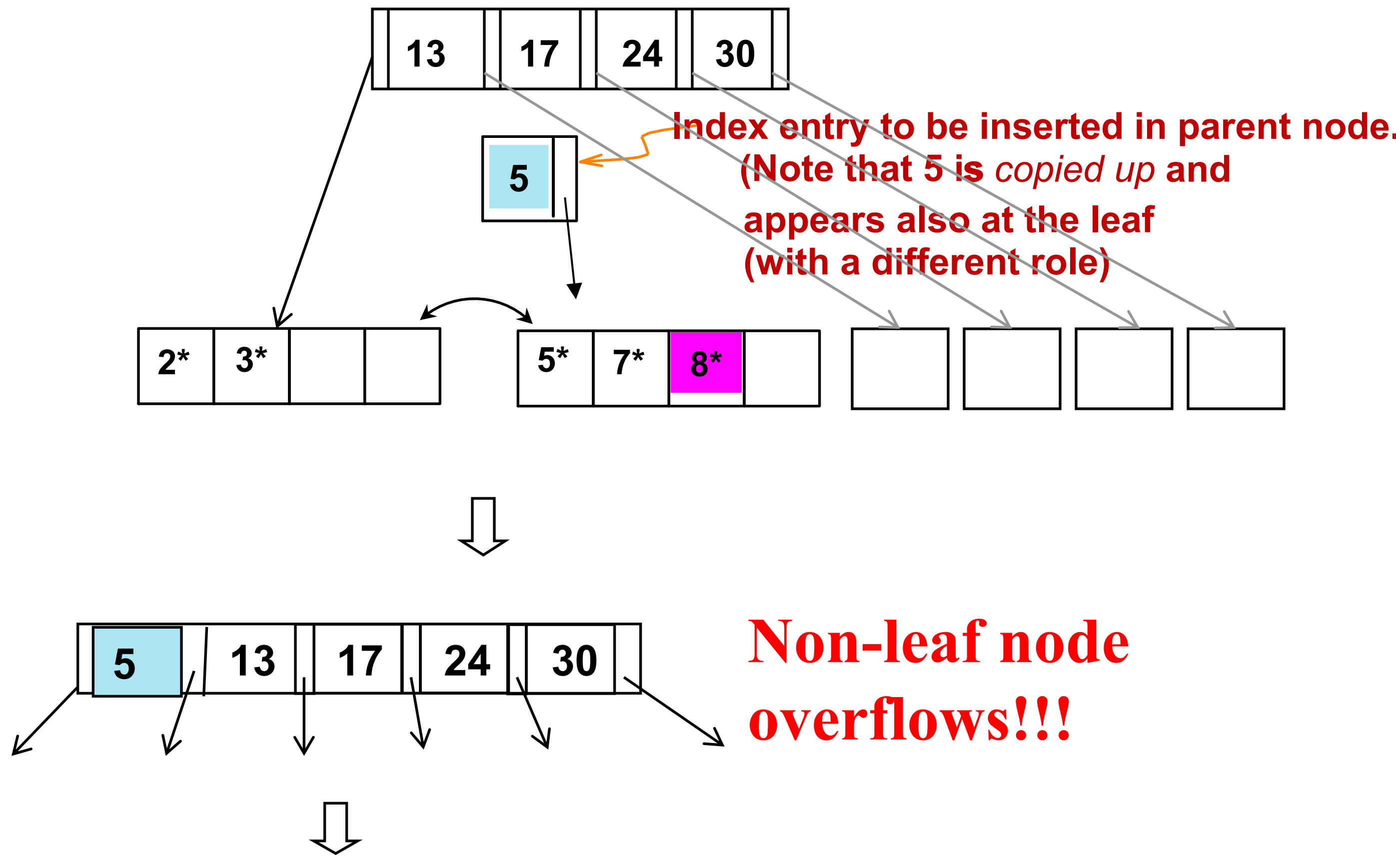
First **d** entries stay, move rest to new node

2) **COPY UP** the *middle key* as the search key. Also, add pointer to the new block

Inserting 8* (cont.)

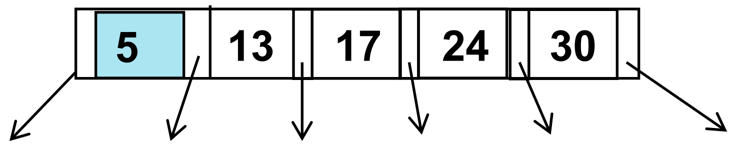
d = 2

- Copy up the middle value (leaf split)



Inserting 8* (cont.)

d = 2



Non-leaf node overflows!!!

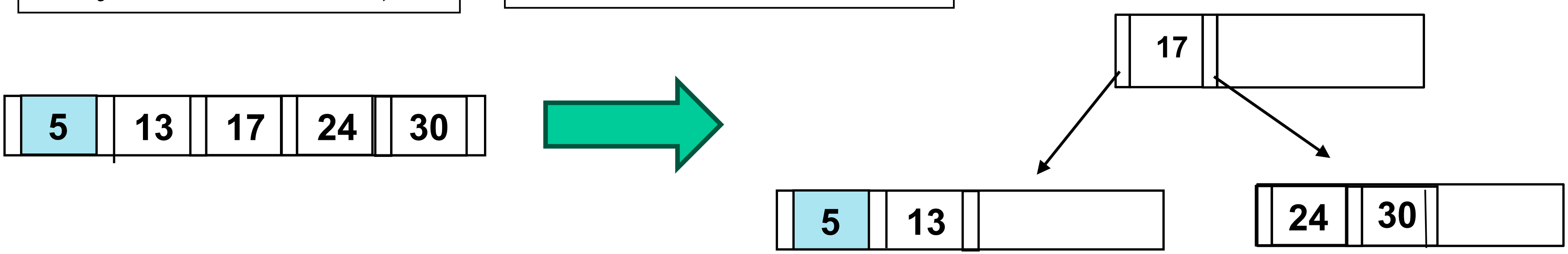
When a non-leaf node overflows:

1) Split the node

First **d** keys (and d+1 pointers) stay in old node)

Last **d** keys (and d+1 pointers) move to new node

2) **PUSH UP** middle key (17) and (pointers to the blocks)!

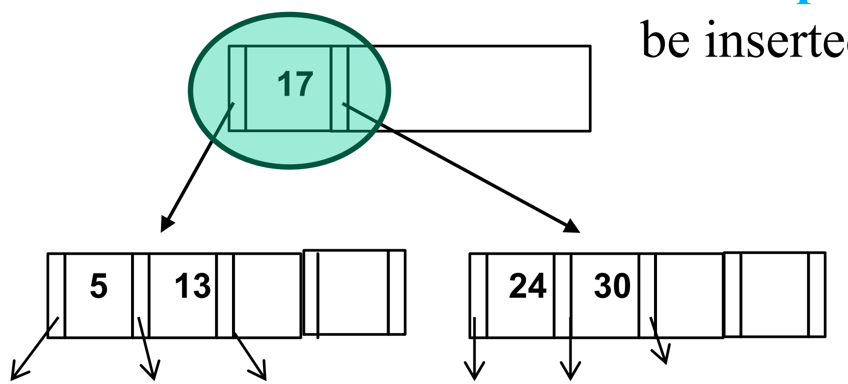


Insertion into B+ tree (cont.)

- Understand difference between copy-up and push-up
- Observe how min number of entries (d) is guaranteed in both leaf and index node splits.

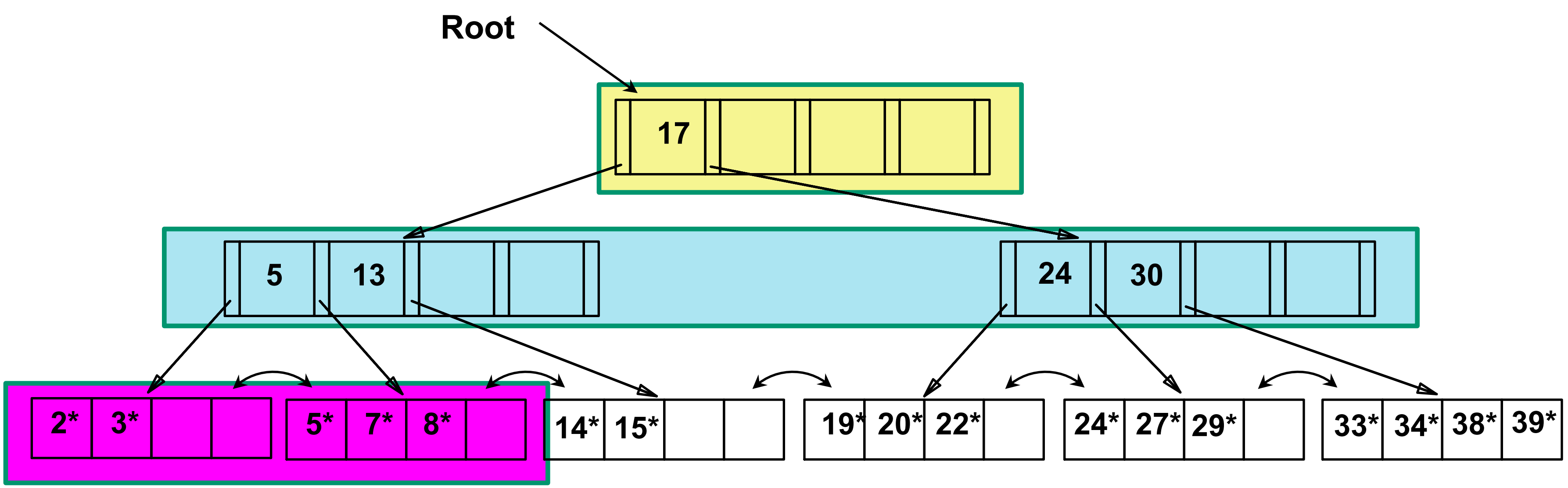
Recall: Root can have at least **1 key** and **at most 2d** keys.

Push up the index entry to be inserted in parent node



Note that 17 is pushed up and only **appears once** in the index. (Contrast this with a leaf split.)

Example B+ Tree After Inserting 8*



Notice that root was split, leading to increase in height.

B+ trees grow **bottom-up** dynamically!

Inserting a Data Entry into a B+ Tree: Summary

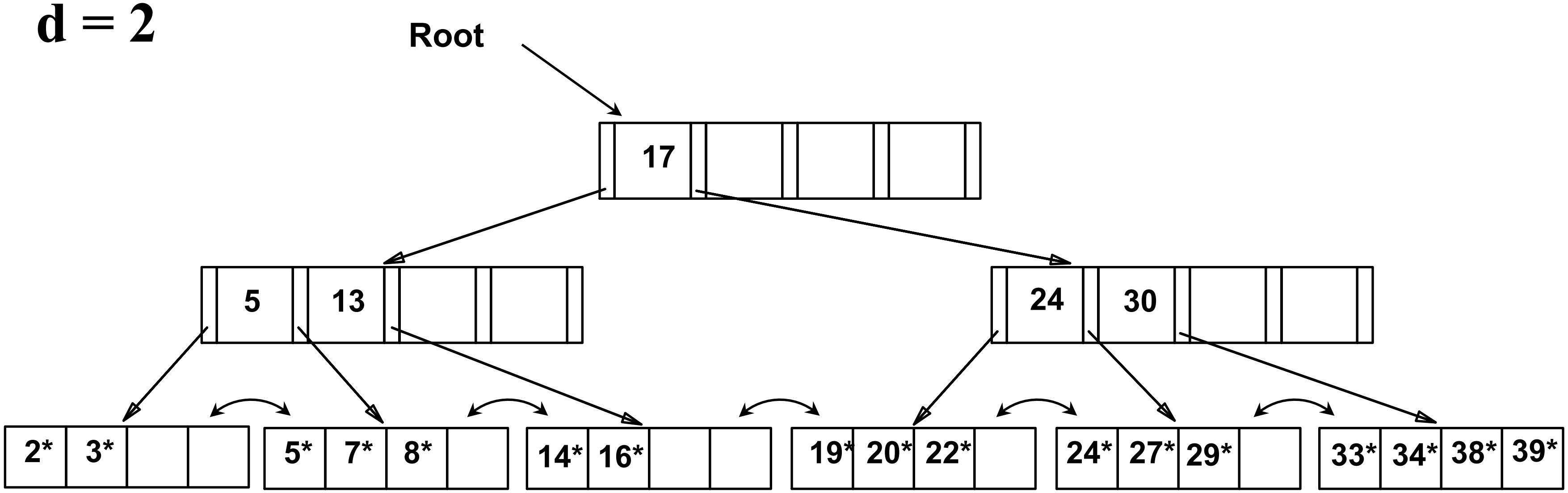
- Find correct leaf *L.*
- Put data entry onto *L*.
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, put middle key in L2
    - **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.
- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- Splits "grow" tree; root split increases height.
  - Tree growth: gets *wider* or *one level taller at top.*

Deleting a Data Entry from a B+ Tree

- Examine examples first …

Delete 19* and 20*

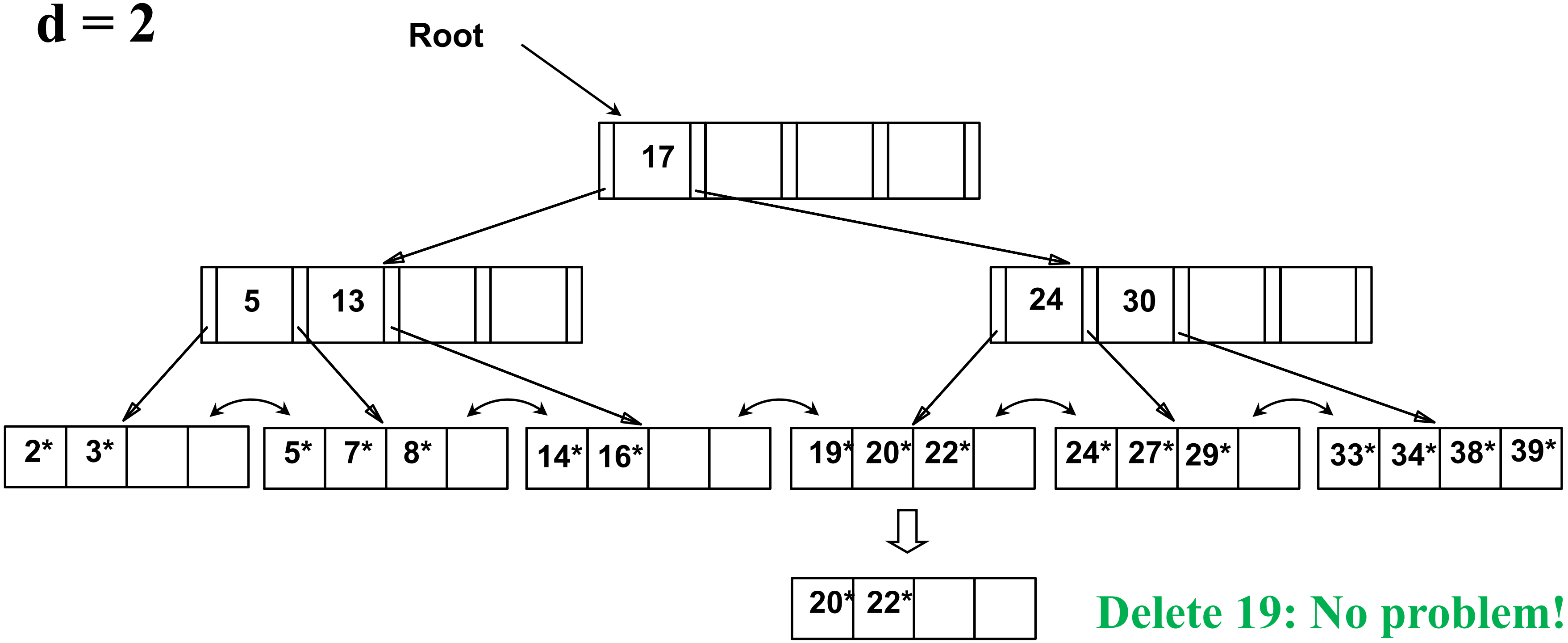
d = 2



Leaf nodes:
d ≤ n ≤ 2d
2 ≤ n ≤ 4

Delete 19* and 20*

d = 2



Delete 19: No problem!

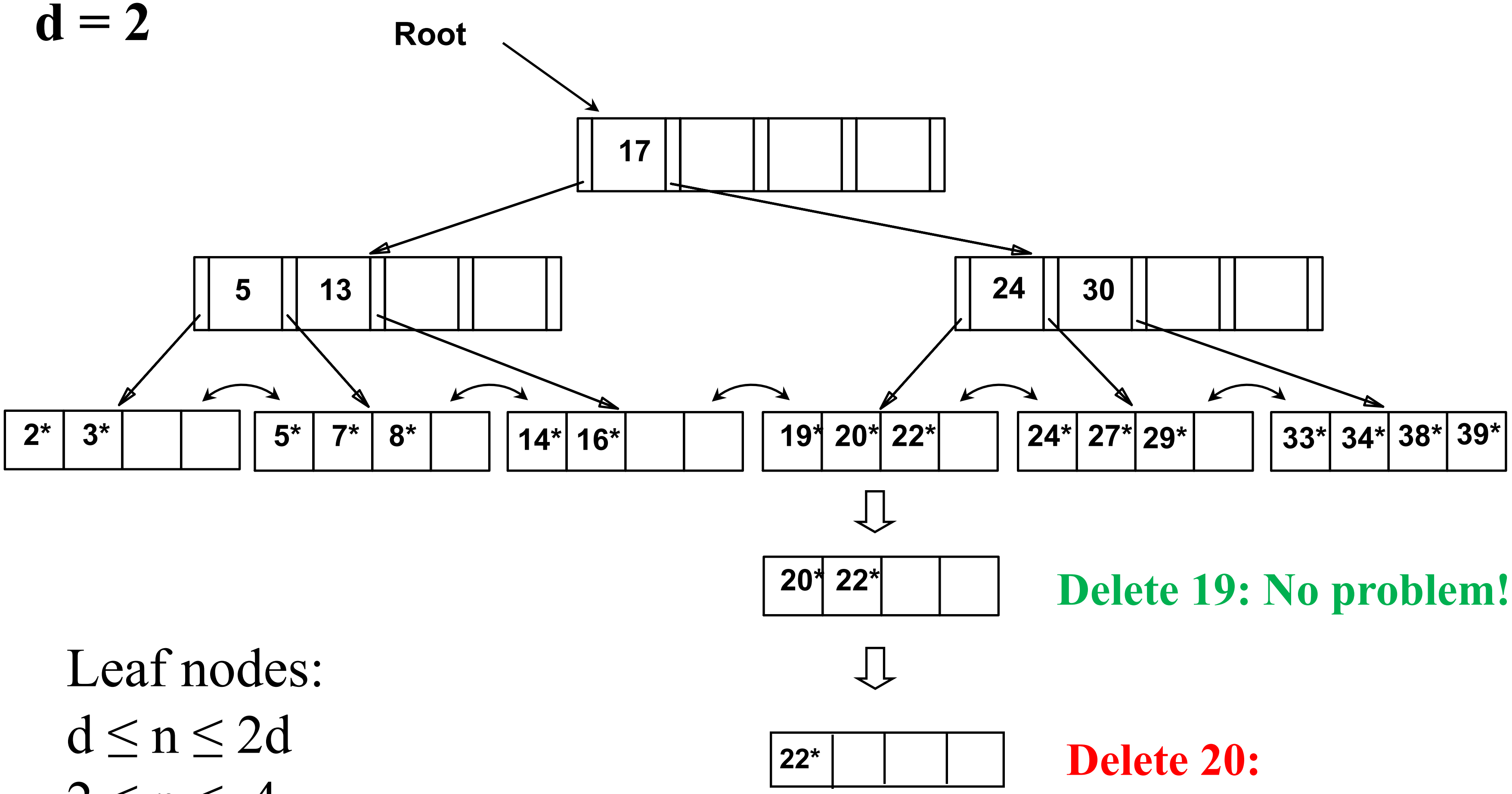
Leaf nodes:

$d \leq n \leq 2d$

$2 \leq n \leq 4$

Delete 19* and 20*

d = 2



Delete 19: No problem!

Delete 20:
Leaf node underflows!

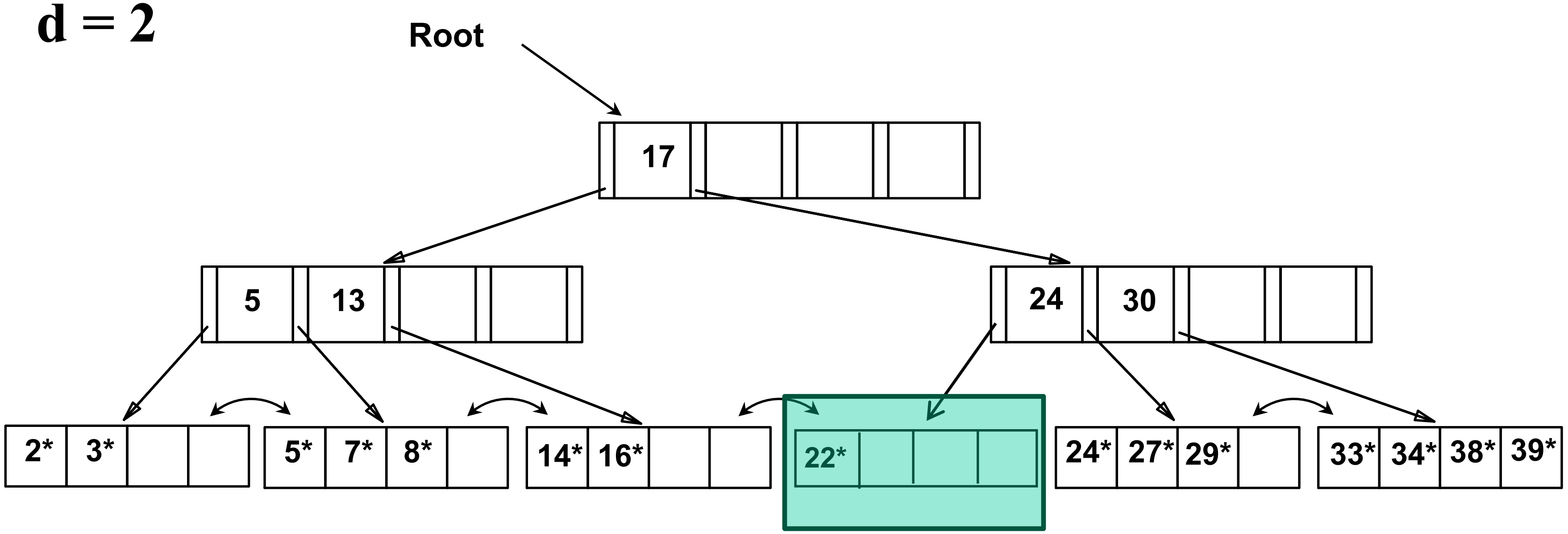
Leaf nodes:

$d \le n \le 2d$

$2 \le n \le 4$

Delete 19* and 20*

d = 2



Leaf node underflows!

When a <u>leaf node</u> underflows:

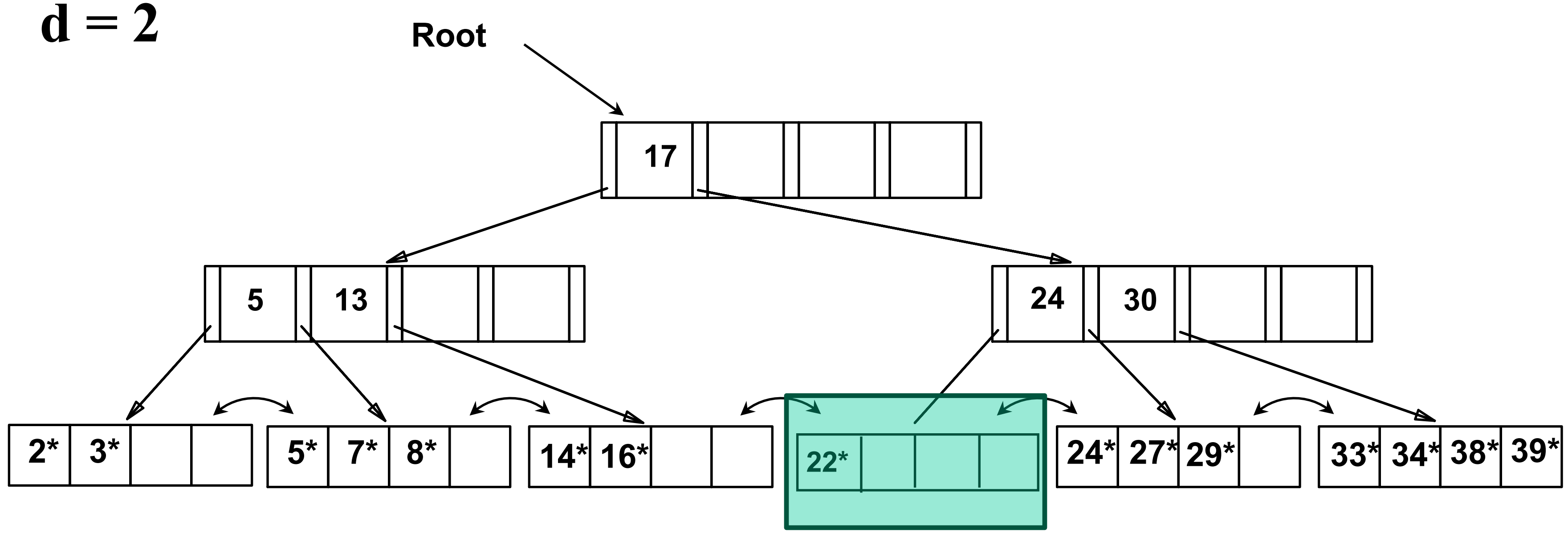
Two options (try in order):

1- Redistribute among sibling nodes evenly, and if this is not possible,

2- Merge nodes

Delete 19* and 20*

d = 2

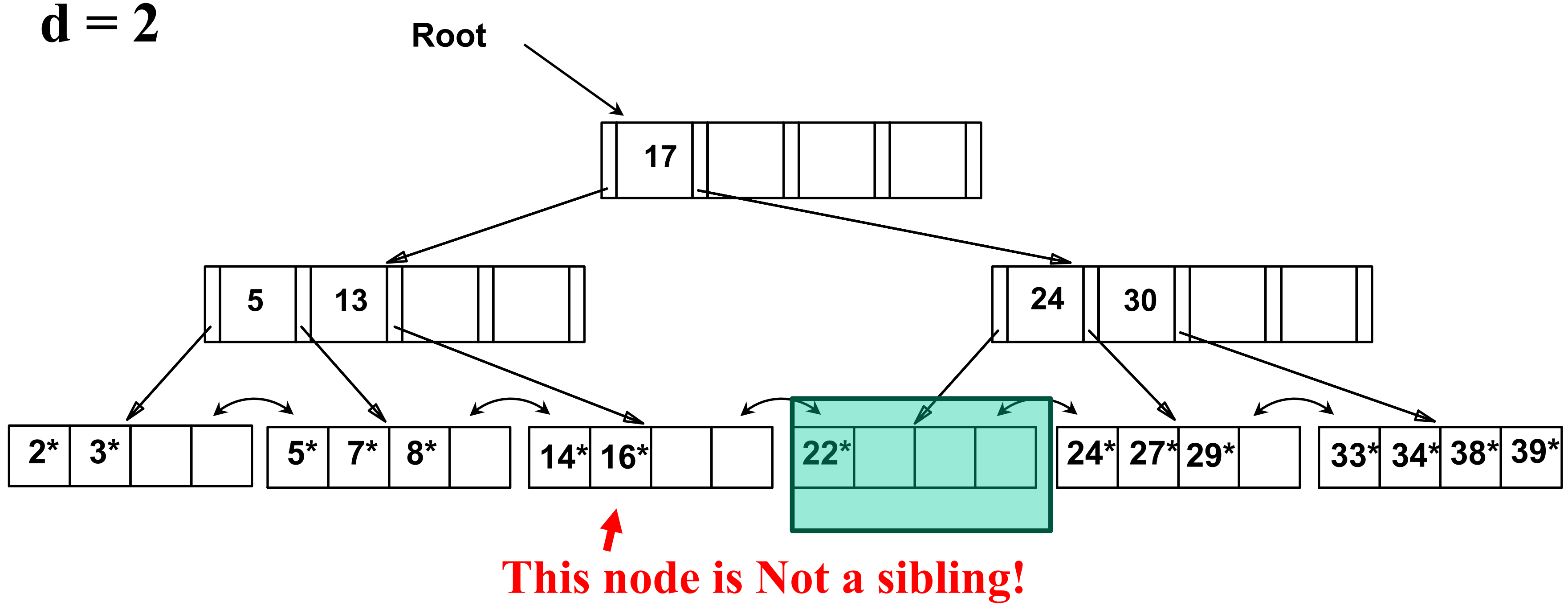


Redistribute for leaf nodes:

A **sibling** of a node is the node that is <u>adjacent</u> (immediate to left or right) to it, and has the <u>same parent</u>

Delete 19* and 20*

d = 2

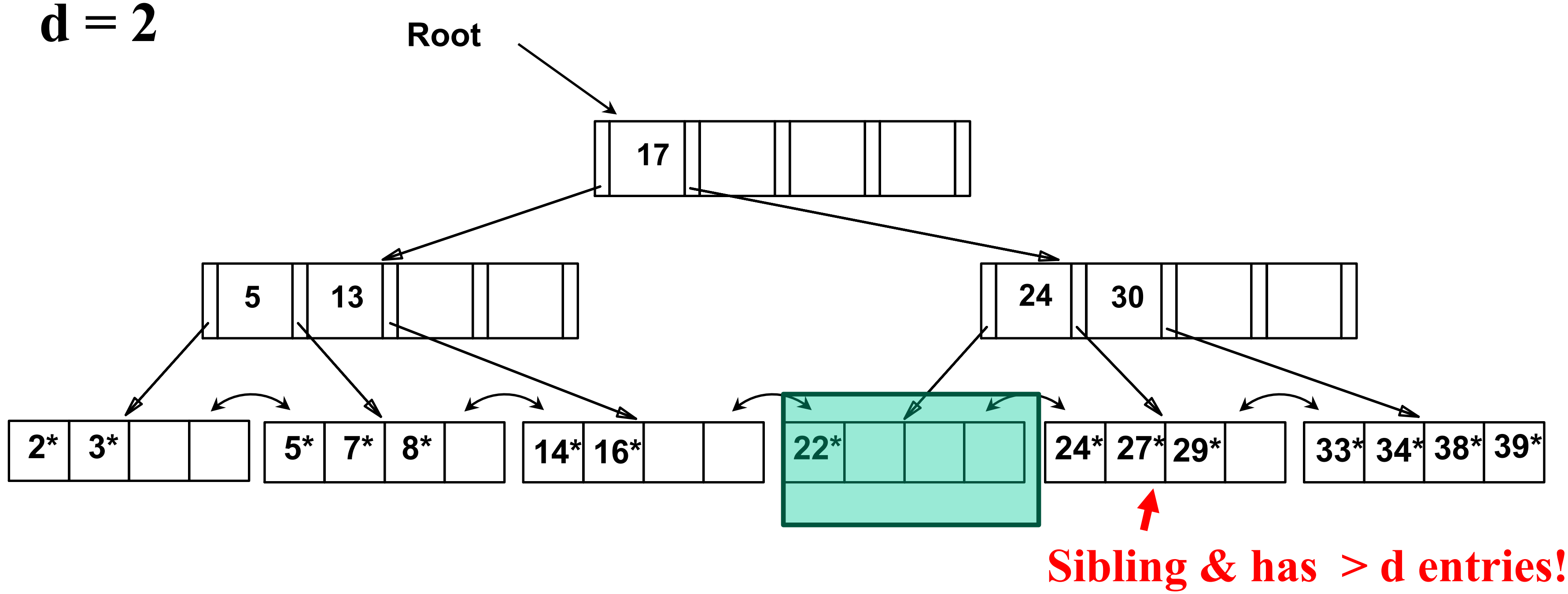


Redistribute for leaf nodes:

A **sibling** of a node is the node that is adjacent (immediate to left or right) to it, and has the same parent

Delete 19* and 20*

d = 2

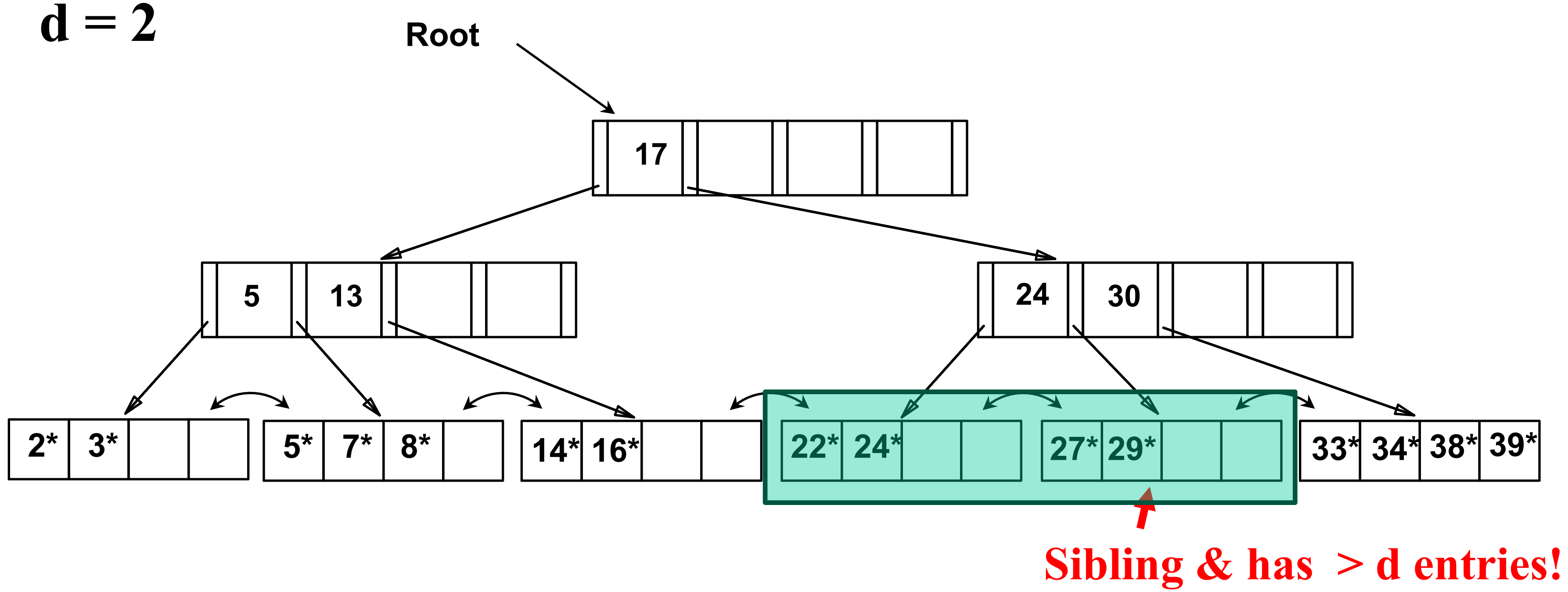


Redistribute for leaf nodes:

A **sibling** of a node is the node that is adjacent (immediate to left or right) to it, and has the same parent

Delete 19* and 20*

d = 2



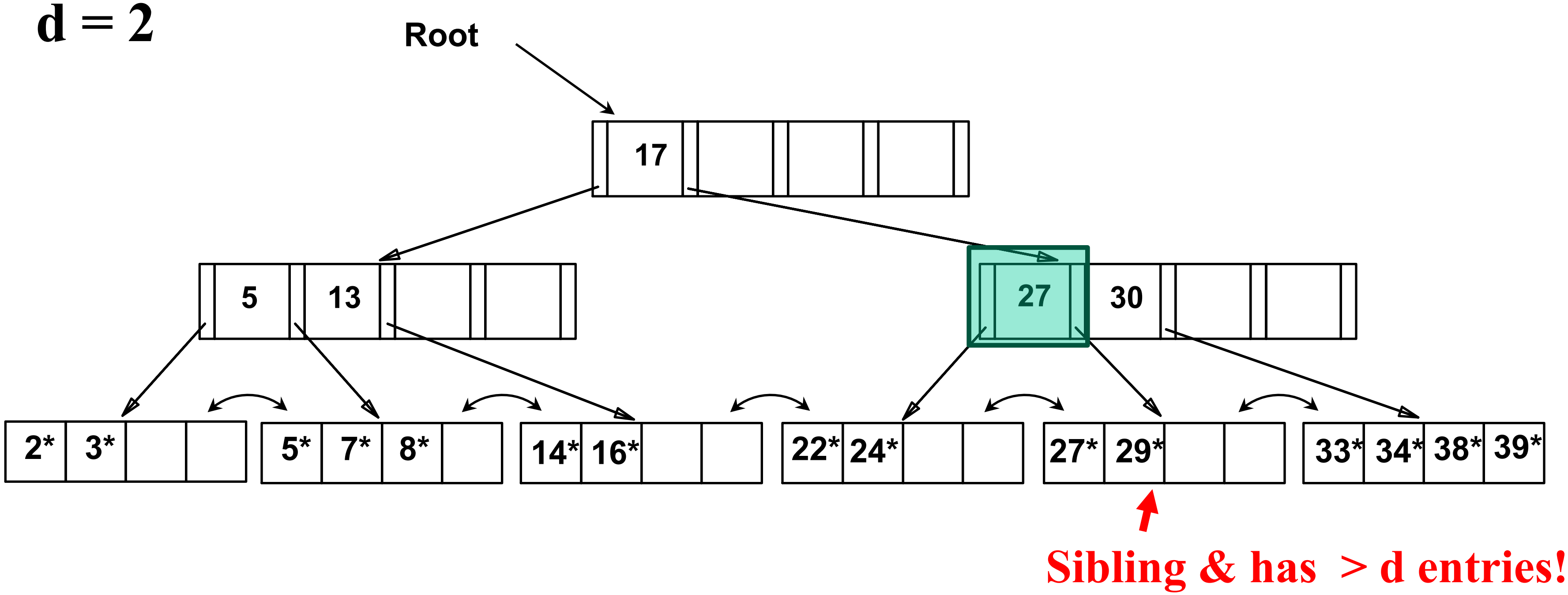
Redistribute for leaf nodes:

A **sibling** of a node is the node that is adjacent (immediate to left or right) to it, and has the same parent

1) Redistribute among siblings

Delete 19* and 20*

d = 2

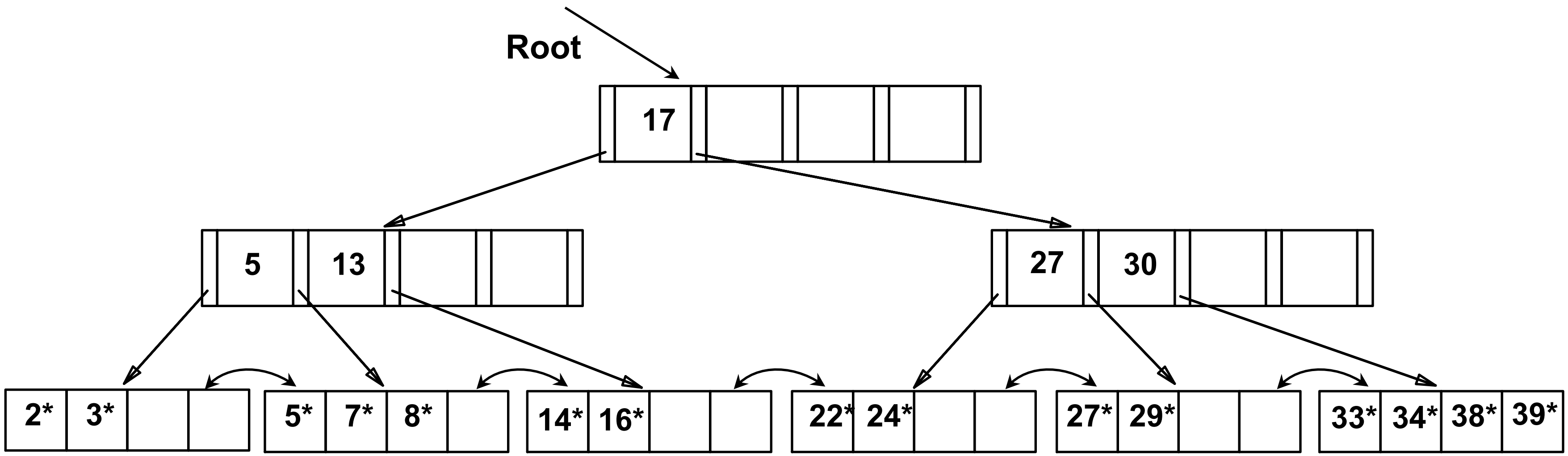


Redistribute for leaf nodes:

A **sibling** of a node is the node that is <u>adjacent</u> (immediate to left or right) to it, and has the <u>same parent</u>

1) Redistribute among siblings
2) COPY-UP (Update) the middle key as the search key

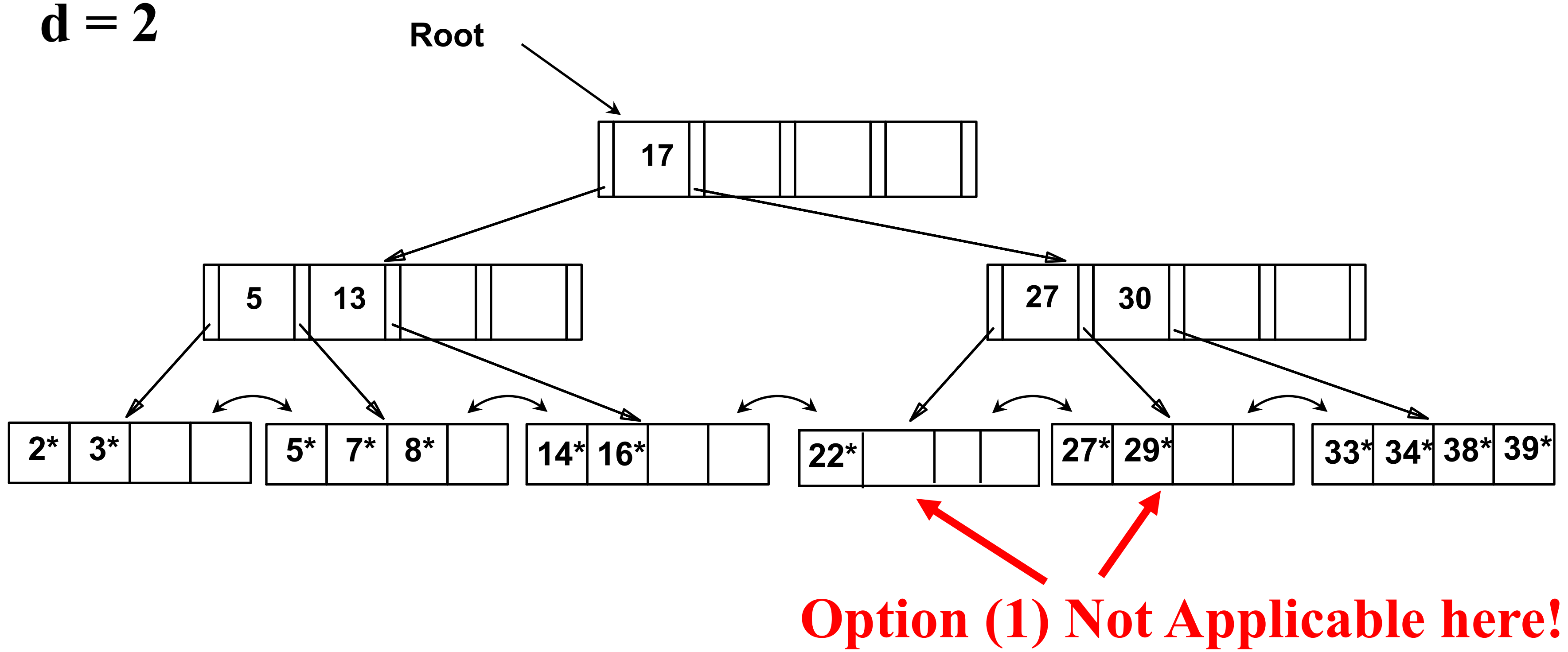
Deleting 19* and 20* (cont.)



- Notice how 27 is *copied up*.
- But can we move it up?
- Now we want to delete 24
- Underflow again!

Delete 24*

d = 2



When a <u>leaf node</u> underflows:

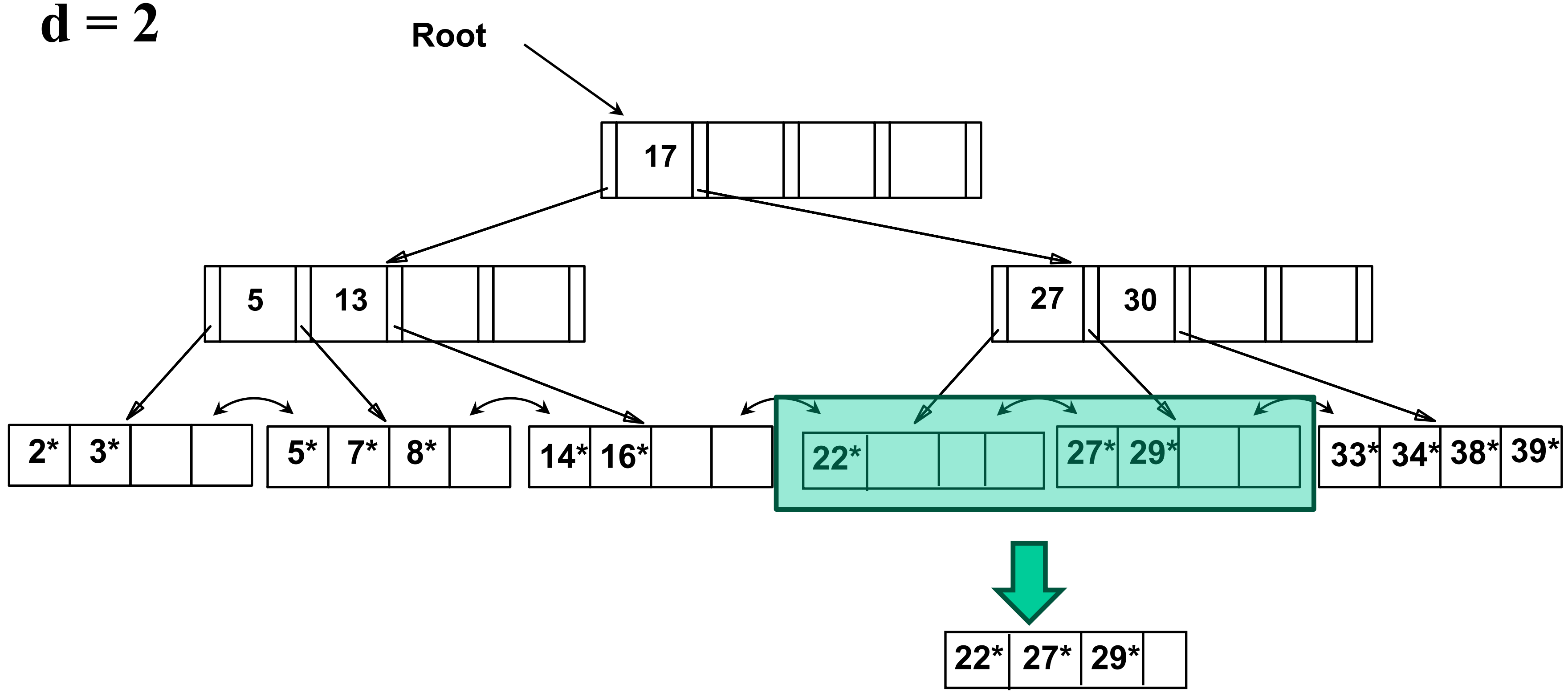
Two options (try in order):

1- Redistribute among sibling nodes evenly, and if this is not possible,

2- Merge nodes

Delete 24*

d = 2

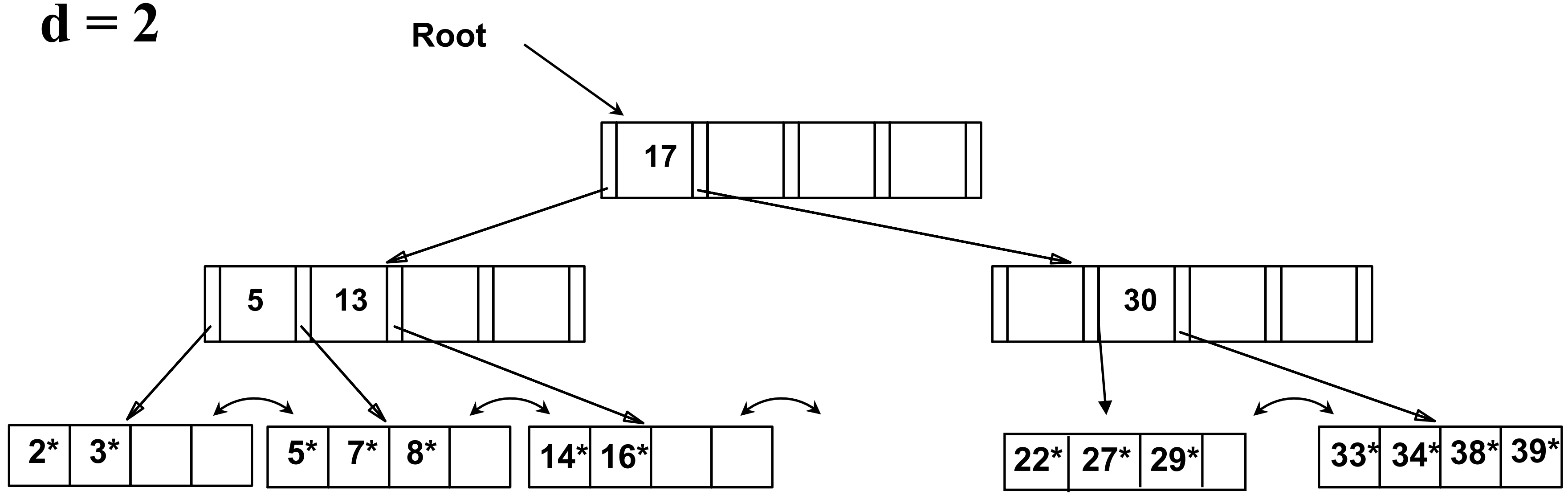


Option (2): Merge leaf nodes

Step 1: Merge leaf nodes

Delete 24*

d = 2

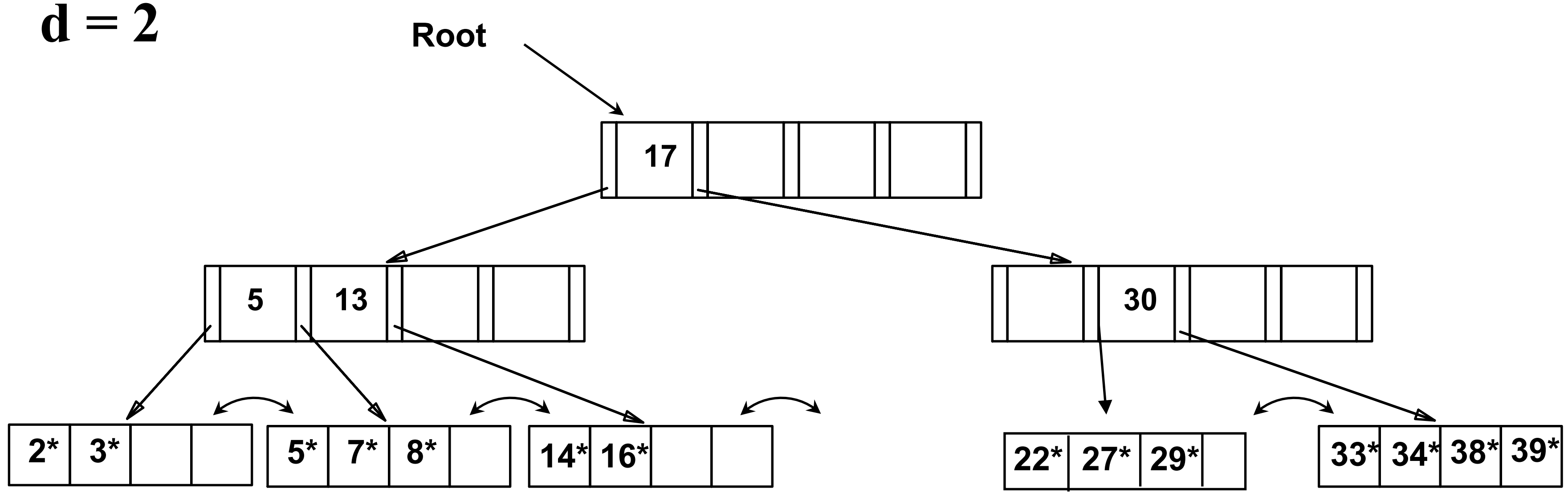


Option (2): Merge leaf nodes

Step 1: Merge leaf nodes
Step 2: Remove the search key entry and pointer to the discarded node

Delete 24*

d = 2



Is it good like this?

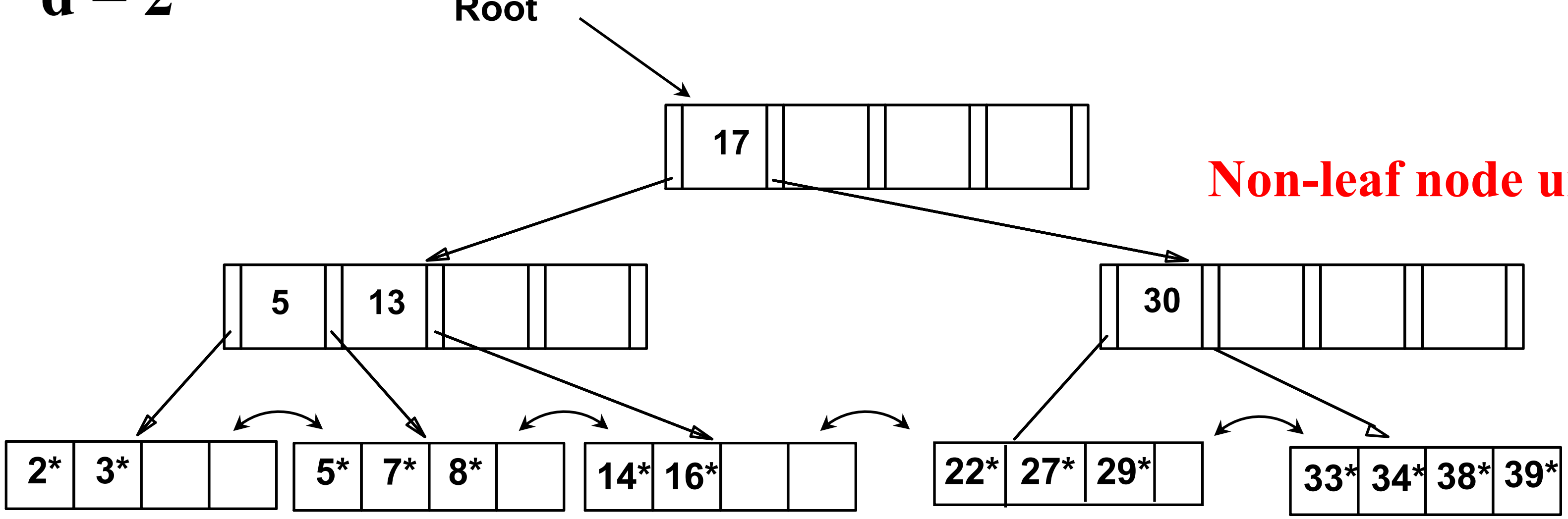
Option (2): Merge leaf nodes

Step 1: Merge leaf nodes

Step 2: Remove the search key entry and pointer to the discarded node

Delete 24*

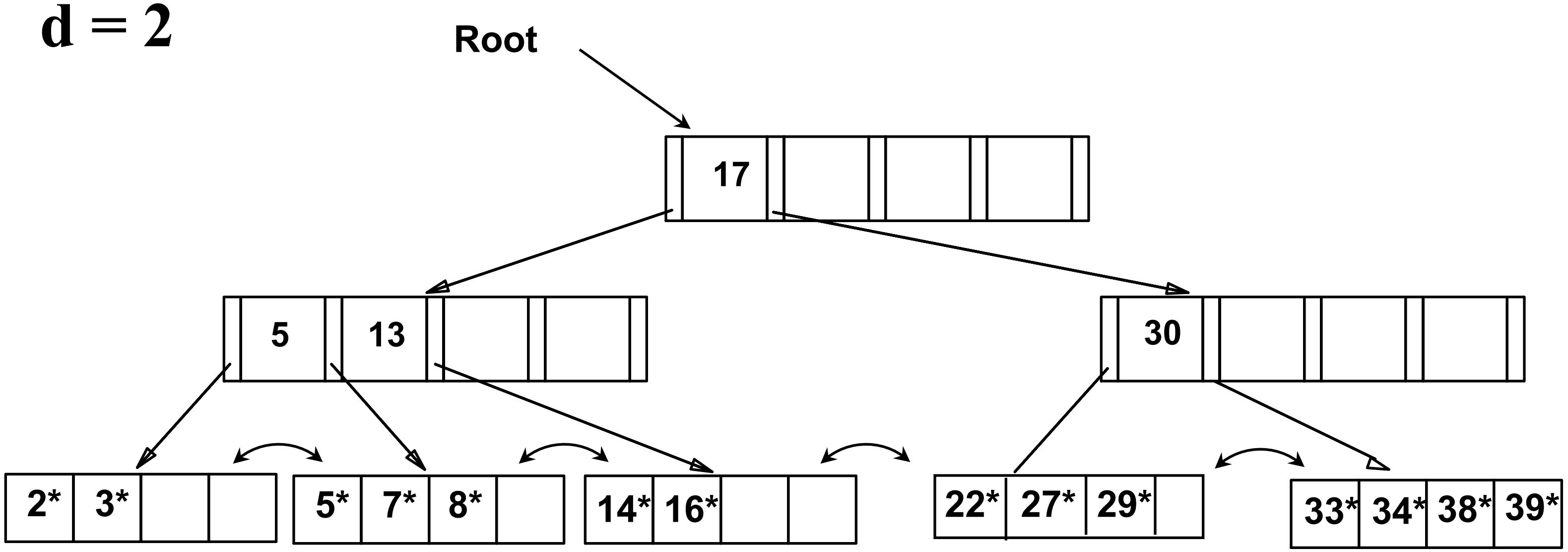
d = 2



Non-leaf node underflows!

Delete 24*

d = 2



When a <u>non-leaf node</u> underflows:

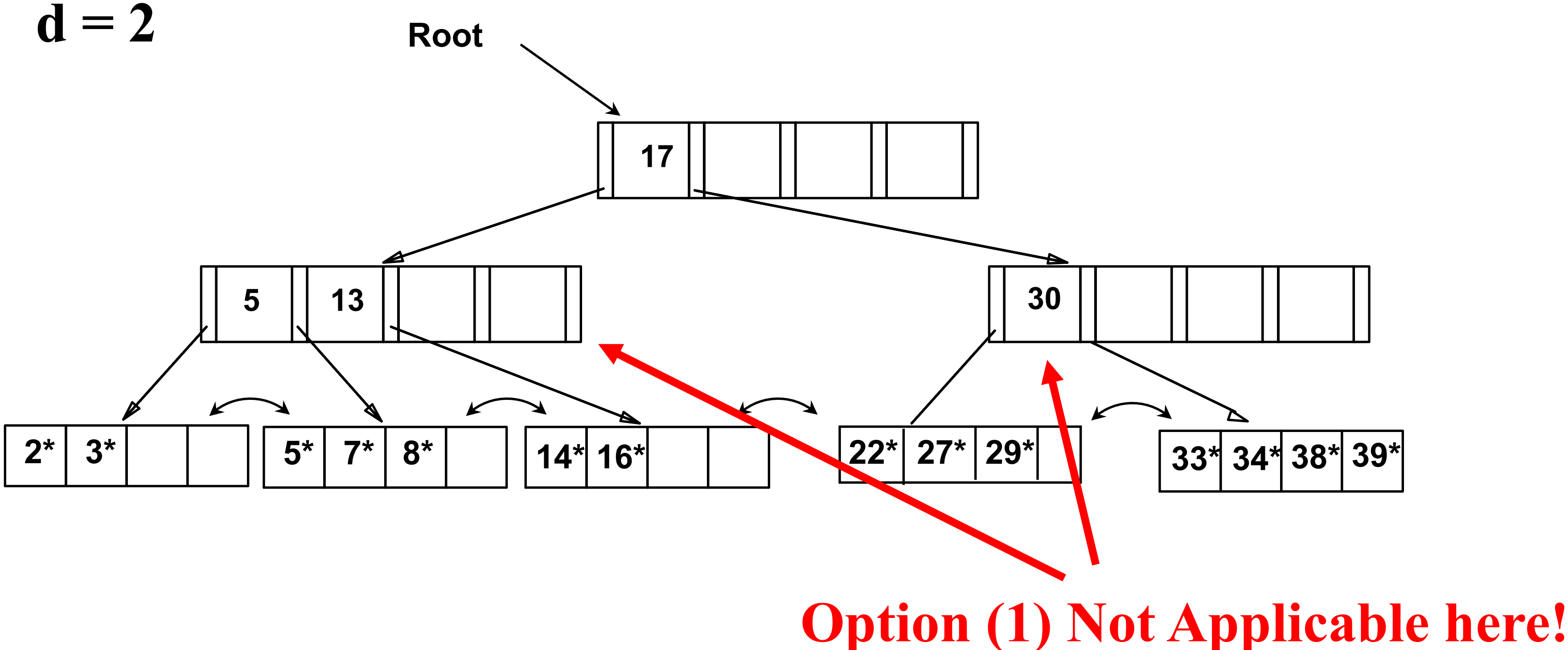
Two options (try in order):

1- Redistribute among sibling nodes evenly evenly, and if this is not possible,

2- Merge nodes

Delete 24*

d = 2



When a <u>non-leaf node</u> underflows:

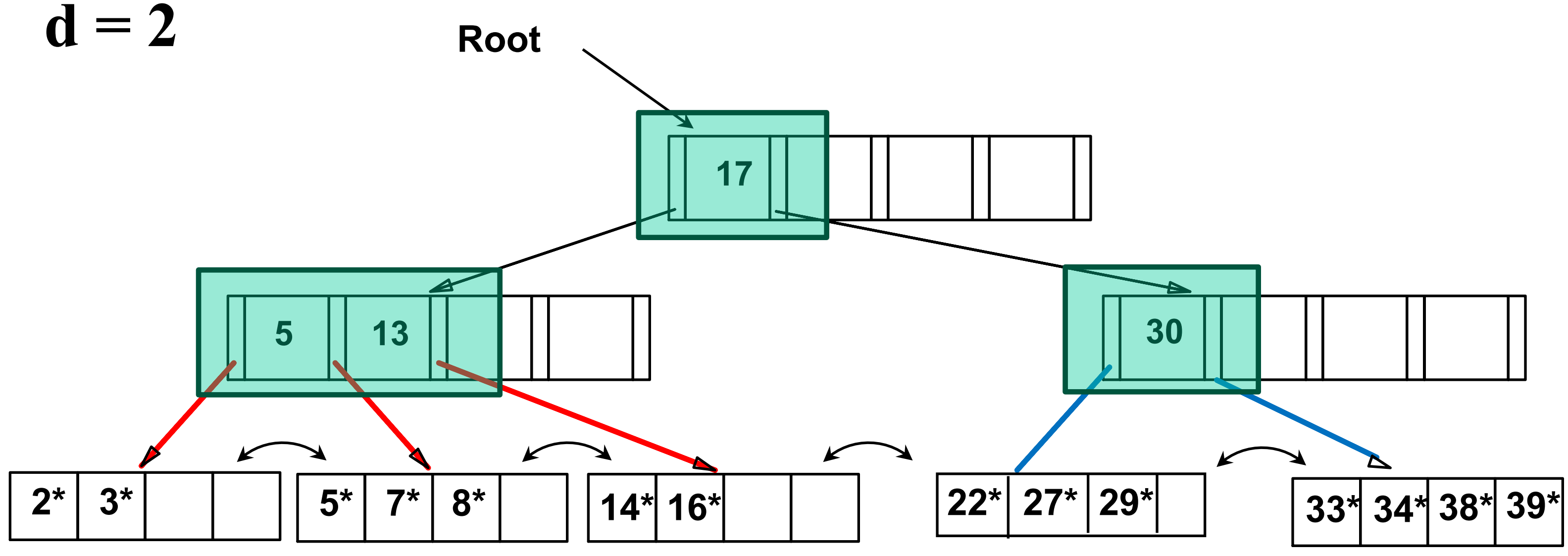
Two options (try in order):

1- Redistribute among sibling nodes evenly evenly, and if this is not possible,

2- Merge nodes

Delete 24*

d = 2



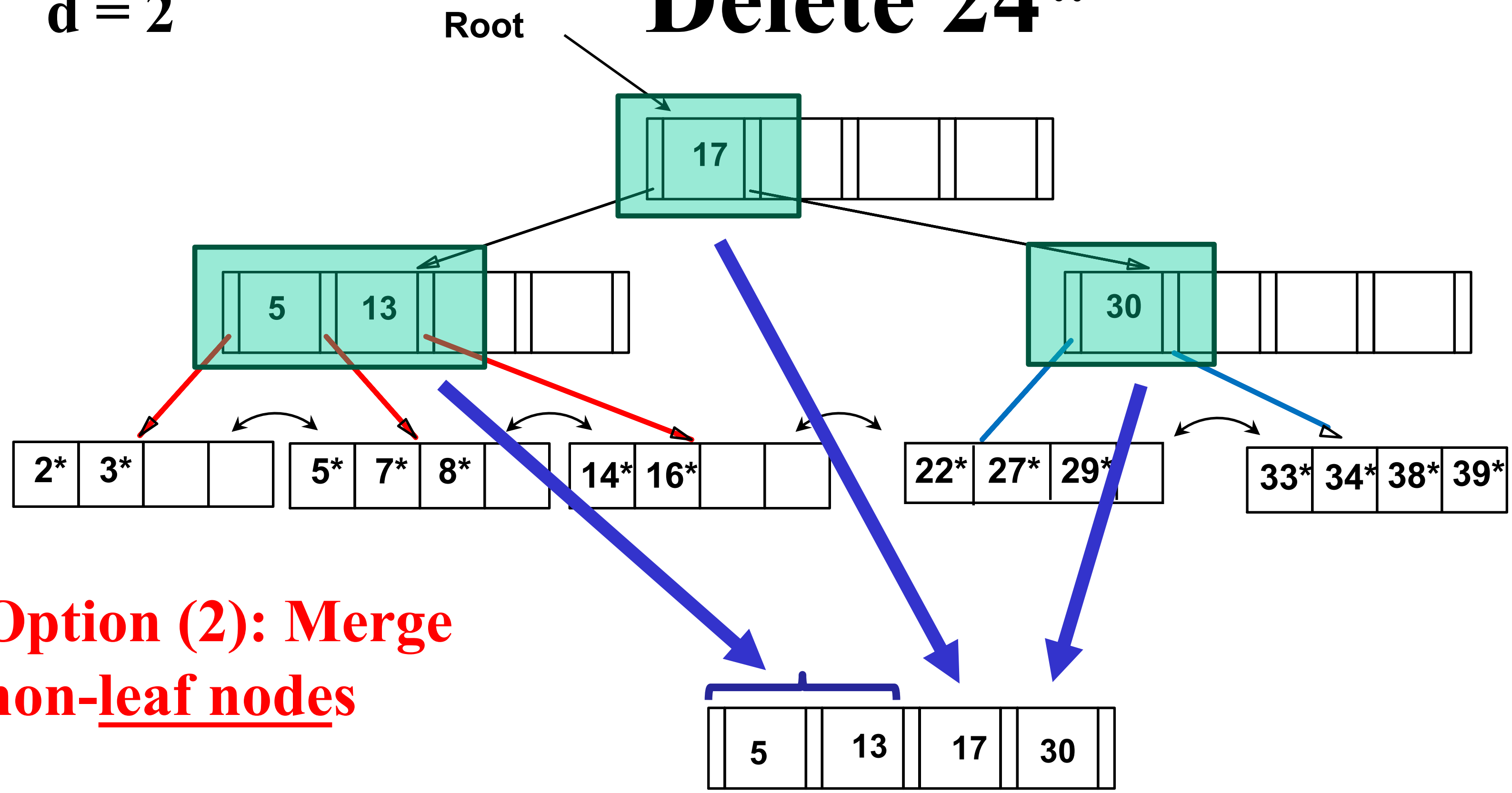
Option (2): Merge non-leaf nodes

Merge:

- Entries in first non-leaf node (together with pointers),
- **PULL DOWN** the splitting search key,
- followed by the entries in the second non-leaf node (together with pointers)

Delete 24*

d = 2

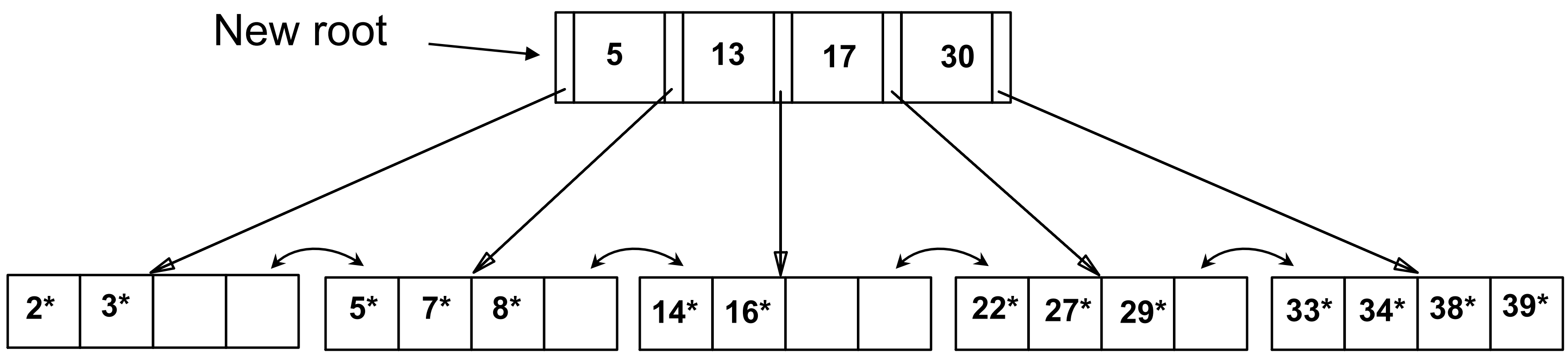


Option (2): Merge non-leaf nodes

Merge:

- Entries in first non-leaf node (together with pointers),
- **PULL DOWN** the splitting search key,
- followed by the entries in the second non-leaf node (together with pointers)

Delete 24*



Deleting a Data Entry from a B+ Tree: Summary

- Start at root, find leaf *L* where entry belongs.
- Remove the entry.
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from *sibling* *(adjacent node with same parent as L)*.
    - If re-distribution fails, *merge* *L* and sibling.
- If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.
- Merge could propagate to root, decreasing height.

Non-leaf Node Redistribution

When a <u>non-leaf node</u> underflows:

Two options (try in order):

1- Redistribute among nodes evenly, and if this is not possible,

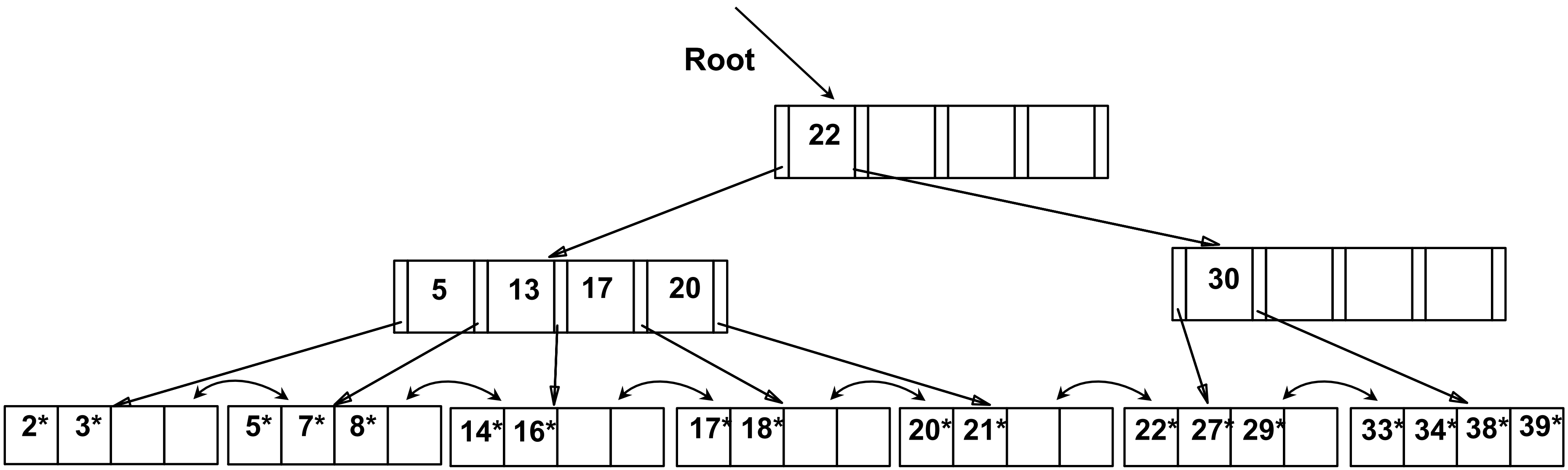
2- Merge nodes

We have already seen an example for the second case.

How about the first case!

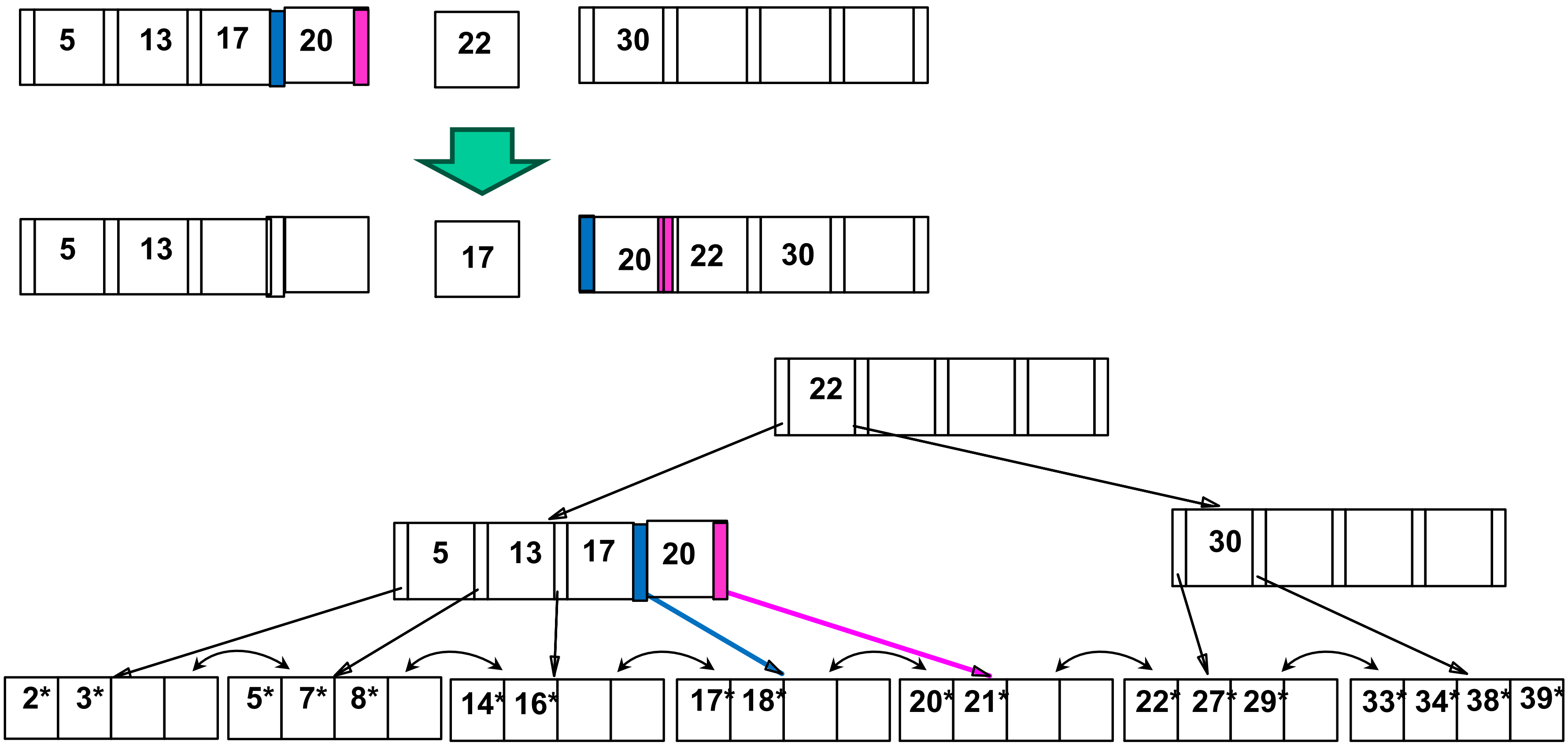
Example of Non-leaf Re-distribution

- Tree is shown below *during deletion* of 24*. (What could be a possible initial tree?)
- In contrast to previous example, can re-distribute entry from left child of root to right child.

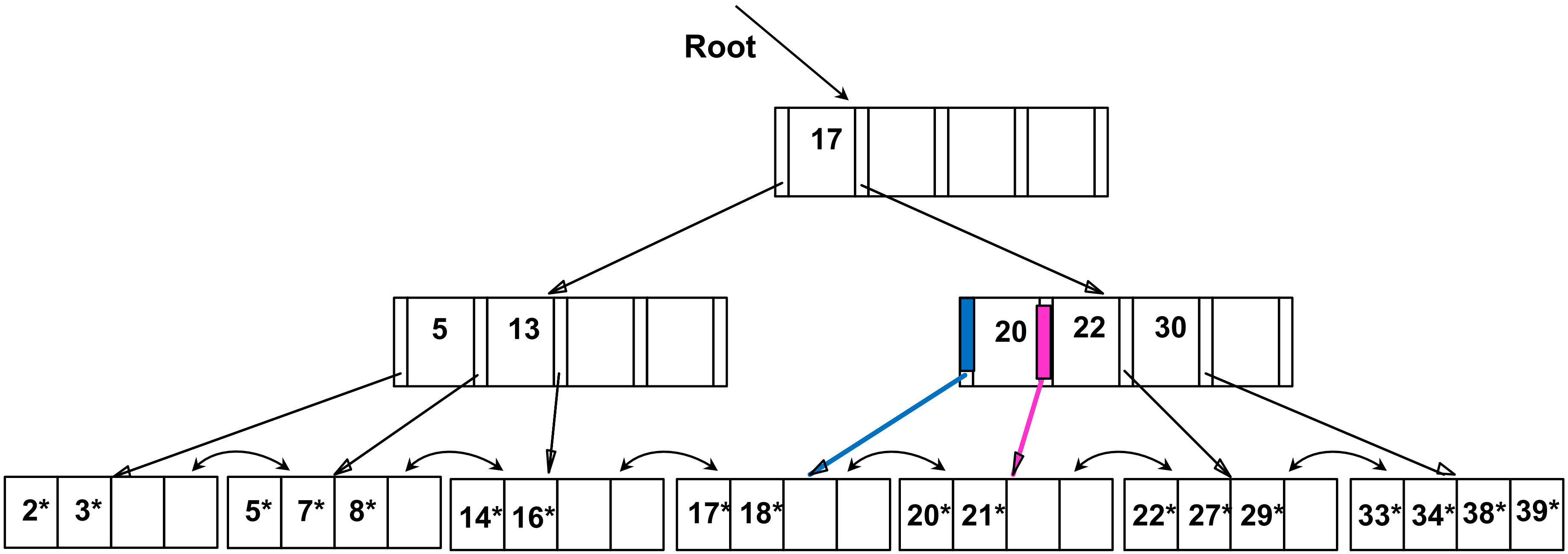


Re-distribution

- Intuitively, entries are re-distributed by `*pushing through*' the splitting entry in the parent node.
- Consider all search keys together



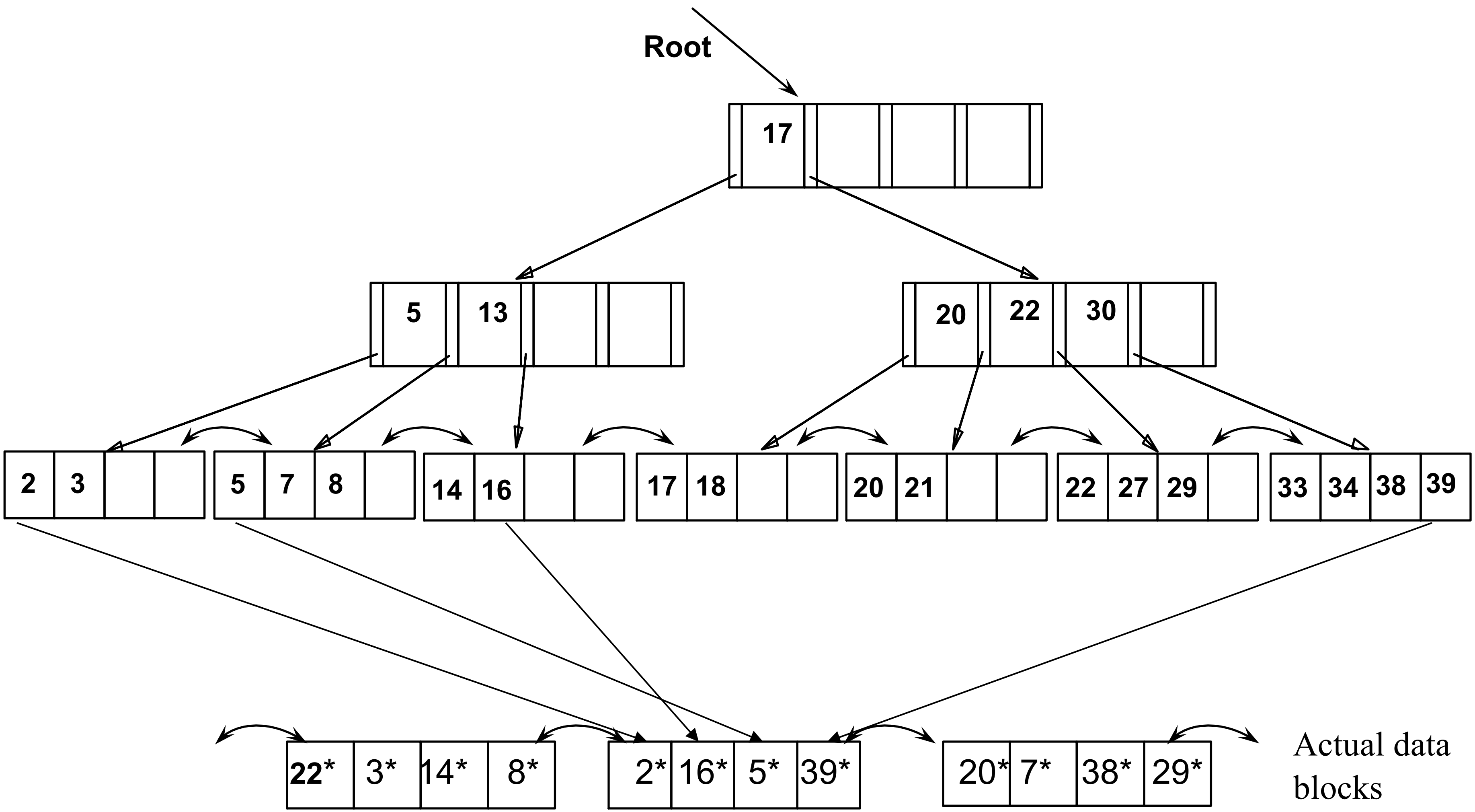
After Re-distribution



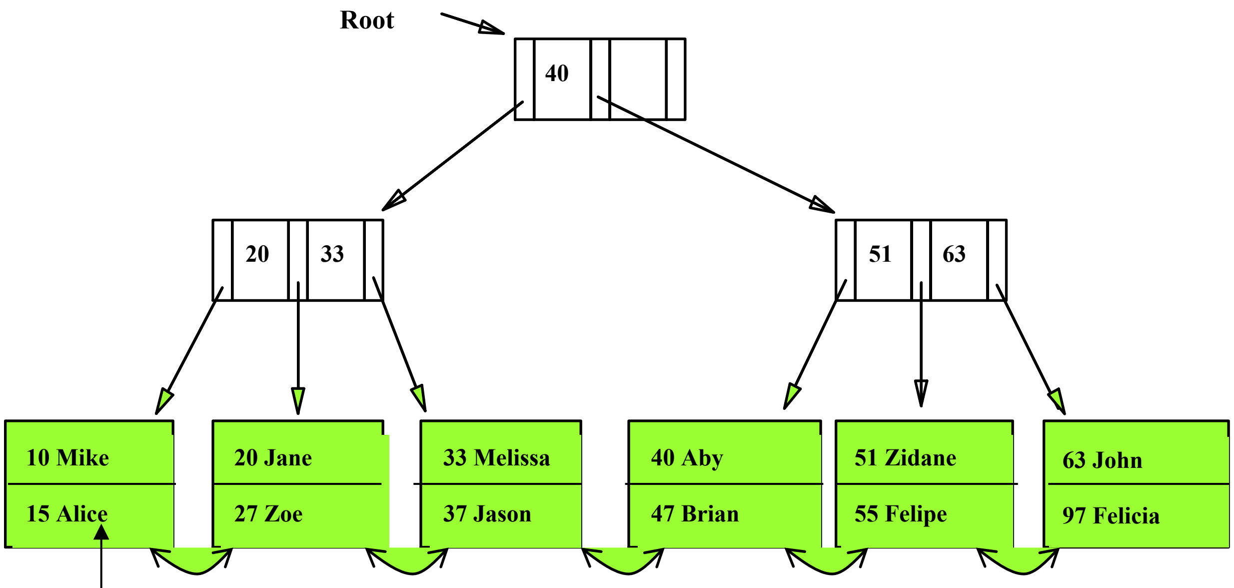
Primary vs Secondary Index

- Note: We were assuming the data items were in sorted order
  - This is called *primary/clustered B+tree* index
- *Secondary B+tree* index:
  - Built on an attribute that the file is not sorted on.
- Can have many different indexes on the same file.

A Secondary B+-Tree index



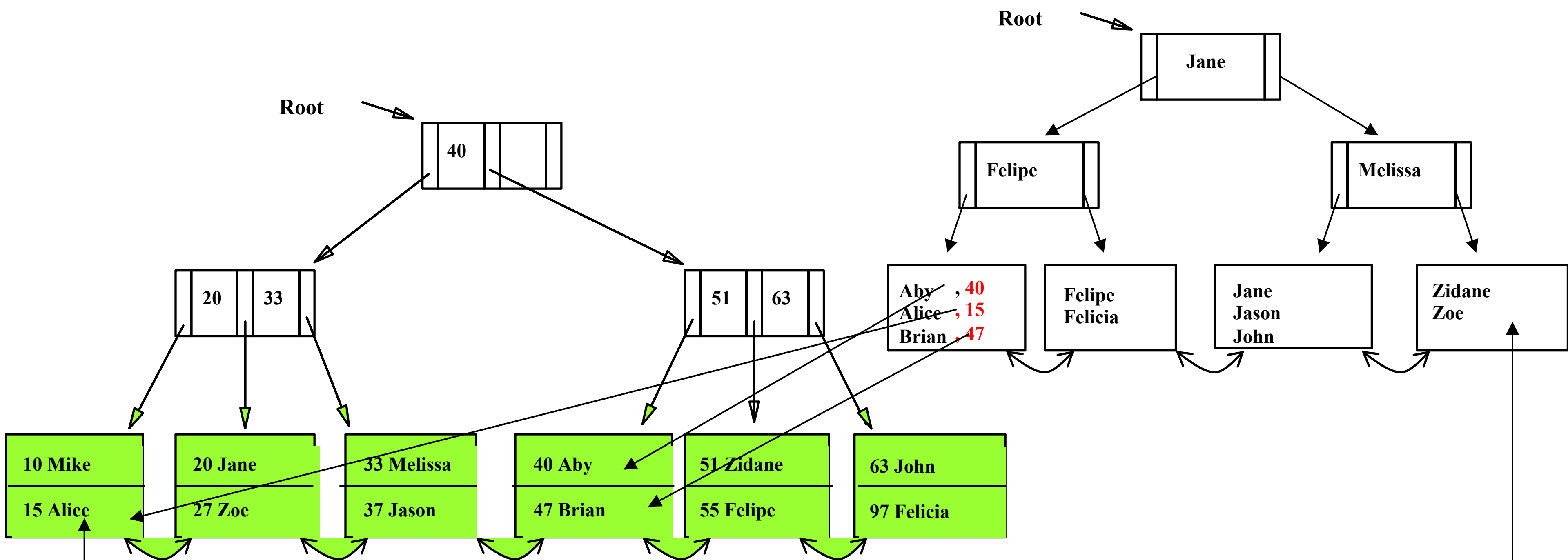
A file organized as (or, has) a **Primary B+-Tree** index on *ssn*



- As **15***, we store the **actual data record** with key value 15 (**Alternative-1**)
- In this case, the leaf nodes can be larger, say a few blocks (pages)

A file organized as (or, has) a **Primary B+-Tree** index on *ssn*

The same file also has a **Secondary B+-Tree** index on *name*

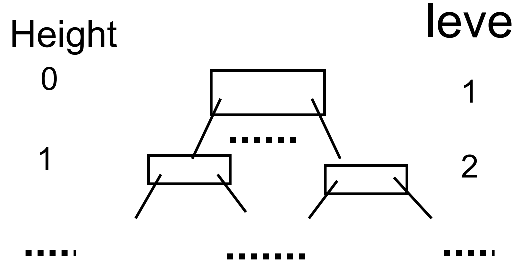


- As **15***, we store the **actual data record** with key value 15 (**Alternative-1**)
- In this case, the leaf nodes can be larger, say a few blocks (pages)

- We have k* as <key, rid> (**Alternative 2**)
- We can have rid's as pointers, or use PK as rid. We show both above

Cost for searching a value in B+ tree

- Assumptions:
  - Each interior node is a disk block
  - Each leaf node is also a disk block and data entries (K*) are of the form <key, ptr>. There are D data entries.
  - Let F be the average number of pointers in a node (for internal nodes, it is called *fanout,* **i.e.,** avg. number of children)
- Observe: Let H be the height of the B+ tree: we need to read H+1 nodes (blocks) to reach a data entry in a leaf node
- How do we find H?
  - Level 1 = 1 page = F^0 page
  - Level 2 = F pages = F^1 pages
  - Level 3 = F * F pages = F^2 pages
  - Level H+1 = …….. = F^H pages (i.e., leaf nodes)
  - F pointers → F-1 keys, so there must be D/(F-1) leaf nodes
  - $D/(F-1) = F^H$. That is, $H = \log_F(\frac{D}{F-1})$



B+ Trees in Practice

- Typical order: 100. Typical fill-factor: 66%.
  - average fanout = 133 (i.e, # of pointers in internal node)
- Can often hold top levels in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes
- Suppose there are 1,000,000,000 data entries.
  - $H = \log_{133}(1000000000/132) < 4$
  - The cost is reading H+1 = 5 pages

Cost Computation: Another Example

Leaves would store the **actual records**

- A primary B+ tree index on key field giftID.
- 2.500.000 gift records, each record: 400 bytes.
- giftID: 12 bytes, address pointer: 4 bytes
- A bucket can hold 500 records
  - So we have larger leaf nodes (called **buckets**), as we store actual records
  - No claim for interior nodes, assume each is a block!
- B+ tree will have a fill factor of 50% [min occupancy]
- B (block size): 1600
- s: 10 ms, r: 5 ms, btt: 1 ms.

a) No of index nodes and their total size

We need to find i) fanout of the nodes, and ii) no of leaves.

i) fanout: Assume , **n keys (n+1) ptrs** can fit to an index node:

n x 12 + (n+1) x 4 = 1600 bytes → 16n = 1596 / 16 → n= 99

So at most 99 keys in a node (2d= 99, d (tree order) is floor(99/2))

Tree fill factor 50%; max 99 keys x 50% = 49 keys

fanout: 49 + 1 = 50 ptrs per node

ii) no of leaves:

500 rec/leaf * fill factor (50%) = 250 recs/leaf

2.5M records / 250 = 10000 leaf nodes (i.e., buckets)

a) No of index nodes and their total size

- Tree height = $\log_{50} 10000 = 3$
- So, there are H+1 = 4 levels

Level 4: 10000 leaf nodes (data buckets)

Level 3: ceil (10000 / 50 ptrs) = 200 nodes

Level 2: ceil (200/50) = 4 nodes

Level 1: ceil (4/50) = 1 node (root)

Index nodes: 1 + 4 + 200 = 205

Total Size: 205 x 1600 bytes

b) Time cost of reading an arbitrary record

- Three has H=3, so 4 levels
- At the first 3 levels, we fetch index nodes:

3 x (s + r+ btt) = 3x (10 +5+1) = 48 ms

- At the fourth level we fetch the leaf node (data bucket)
  - But how many blocks is a data bucket?
  - (500 recs x 400 bytes/rec) / 1600 = 125 blocks
  - So, cost s + r+ 125 x btt = 10+ 5+ 125 x 1 = 140 ms
- Total cost: 48 + 140 = 188 ms

c) Cost of reading all records in sorted manner

- Reach to leftmost leaf node, as before:
- at the first 3 levels, we fetch index nodes:

3 x (s + r+ btt) = 3x (10 +5+1) = 48 ms

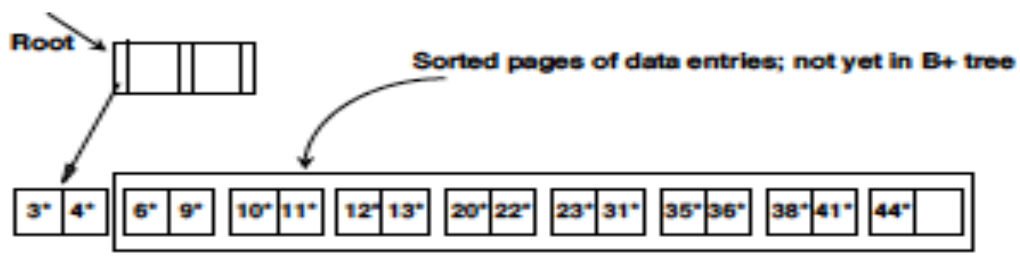
- Read all the leaf nodes (using doubly linked list pointers)
  - 10000 (s + r + 125 x btt)
- Think: What if this is a secondary B+ tree and we store <key, ptr> pairs at leaf nodes (data buckets)?

Terminology

- **Blocking Factor:** the number of records which can fit in a leaf node.
- **Fan-out :** the average number of children of an internal node.
- A B+tree index can be used either as a primary index or a secondary index.
  - **Primary index:** determines the way the records are actually stored
  - **Secondary index:** the records in the file are not grouped in blocks according to keys of secondary indexes

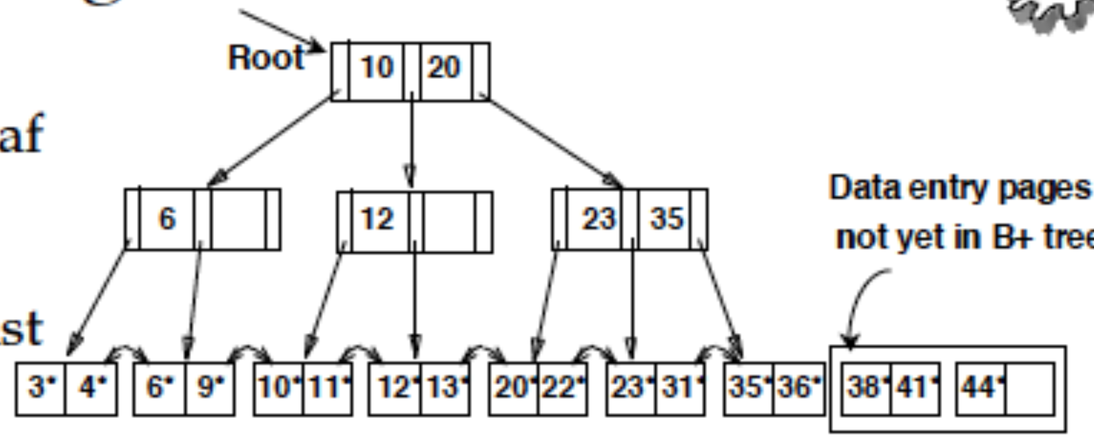
Bulk Loading of a B+ Tree

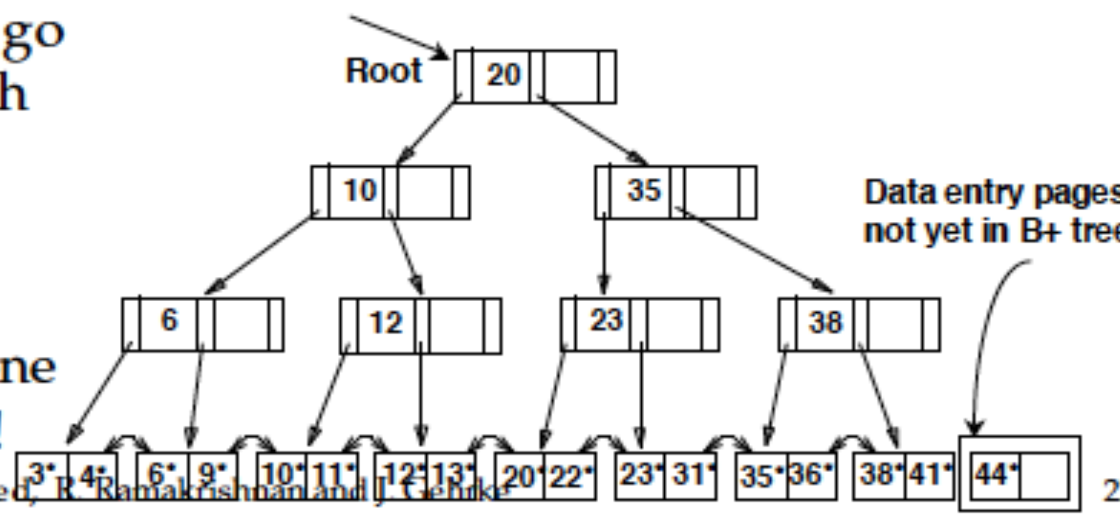
- If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.
- Bulk Loading can be done much more efficiently.
  - Initialization: Sort all data entries, insert pointer to first (leaf) page in a new (root) page



Bulk Loading (Contd.)

- Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)
- Much faster than repeated inserts, especially when one considers locking!





Summary

- Tree-structured indexes are ideal for range-searches, also good for equality searches.
- B+ tree is a dynamic structure.
  - Inserts/deletes leave tree height-balanced; High fanout (**F**) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.
  - Typically, 67% occupancy on average.
  - If data entries are data records, splits can change rids!
- Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.

More…

- Hash-based Indexes
  - Static Hashing
  - Extendible Hashing
  - Linear Hashing
- Grid-files
- R-Trees
- etc…
- A nice animation site for B+ trees: https://www.cs.usfca.edu/~galles/visualization/BPlusTree.html