c)

$$\sum_{k=-2}^{4} y[k]\delta[n-k]$$
$$= \delta[n+1] + 3\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4]$$

Answer 4

a)

$$x_1[n] = cos(\frac{5\pi}{2}n)$$
$$cos(\frac{5\pi}{2}n) = cos(\frac{5\pi}{2}n + \frac{5\pi}{2}N_0)$$
$$\frac{5\pi}{2}N_0 = 2\pi k$$

for k=5, $N_0 = 4$

b)

$$x_2[n] = sin(5n)$$
$$sin(5n) = sin(5n + 5N_0)$$
$$5N_0 = 2\pi k$$

There is no integer N_0 for any integer value k

c)

$$x_3(t) = 5sin(4t + \frac{\pi}{3})$$
$$5sin(4t + \frac{\pi}{3}) = 5sin(4t + 4T_0 + \frac{\pi}{3})$$
$$4T_0 + \frac{\pi}{3} = 2\pi$$
$$T_0 = \frac{5\pi}{12}$$

Answer 5

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$|a|\delta(at) = \delta(t)$$

$$\text{Step impulse function property: } \int_{-\infty}^{\infty} \delta(t)dt = \delta(t)$$

$$|a| \int_{-\infty}^{\infty} \delta(at)dt = \int_{-\infty}^{\infty} \delta(t)dt = \delta(t)$$

$$\text{On the left side, replace parameter t with } \frac{t}{a}$$

$$|a| \int_{-\infty}^{\infty} \delta(t)\frac{dt}{a} = \int_{-\infty}^{\infty} \delta(t)dt$$

$$\frac{a}{|a|} \int_{-\infty}^{\infty} \delta(t)dt = \int_{-\infty}^{\infty} \delta(t)dt$$

$1 = 1$ **(For any positive value a)**

Answer 6

a)

$$y_1[n] = S_1(x_1[n]) = 4x_1[n] + 2x_1[n-1]$$

$$y_2[n] = S_2(y_1[n]) = y_1[n-2]$$

$$y_2[n] = y_1[n-2] = 4x_1[n-2] + 2x_1[n-3]$$

Difference equation of the overall system in terms of x[n] and y[n] :

$$y[n] = S(x[n]) = 4x[n-2] + 2x[n-3]$$

b)

$$S_2(x_1[n]) = x_1[n-2]$$

$$S_1(x_1[n-2]) = 4x_1[n-2] + 2x_1[n-3]$$

Difference equation of this system in terms of x[n] and y[n] :

$$y[n] = 4x[n-2] + 2x[n-3]$$

The series connection of the sub systems is commutative.

c)

$$c_1y_1[n] = S(c_1x_1[n]) = 4c_1x_1[n-2] + 2c_1x_1[n-3]$$
$$c_2y_2[n] = S(c_2x_2[n]) = 4c_2x_2[n-2] + 2c_2x_2[n-3]$$
$$c_1y_1[n] + c_2y_2[n] = 4(c_1x_1[n-2] + c_2x_2[n-2]) + 2(c_1x_1[n-3] + c_2x_2[n-3])$$
$$= S(c_1x_1 + c_2x_2)$$

Conclusion:

$$S(c_1x_1) + S(c_2x_2) = c_1y_1[n] + c_2y_2[n] = S(c_1x_1 + c_2x_2)$$

Superposition property holds for the system S.

d)

Let $x_3[n] = x[n-n_0]$

$$y_3[n] = S(x_3[n]) = 4x_3[n-2] + 2x_3[n-3]$$
$$= 4x[n-n_0-2] + 2x_3[n-n_0-3]$$
$$= y[n-n_0]$$

$S(x[n-n_0]) = y[n-n_0]$ for any integer n_0

System S is Time invariant.