TOYOTA
InfoTechnology
Center, U.S.A., Inc.
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Analysis of Probability of # of Transmissions and Persistent Collisions in Rel-14 PC5 LTE-V2X Mode 4

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Network Division

TOYOTA InfoTechnology Center, U.S.A., Inc.

Problem 1: Distribution of # of Transmissions

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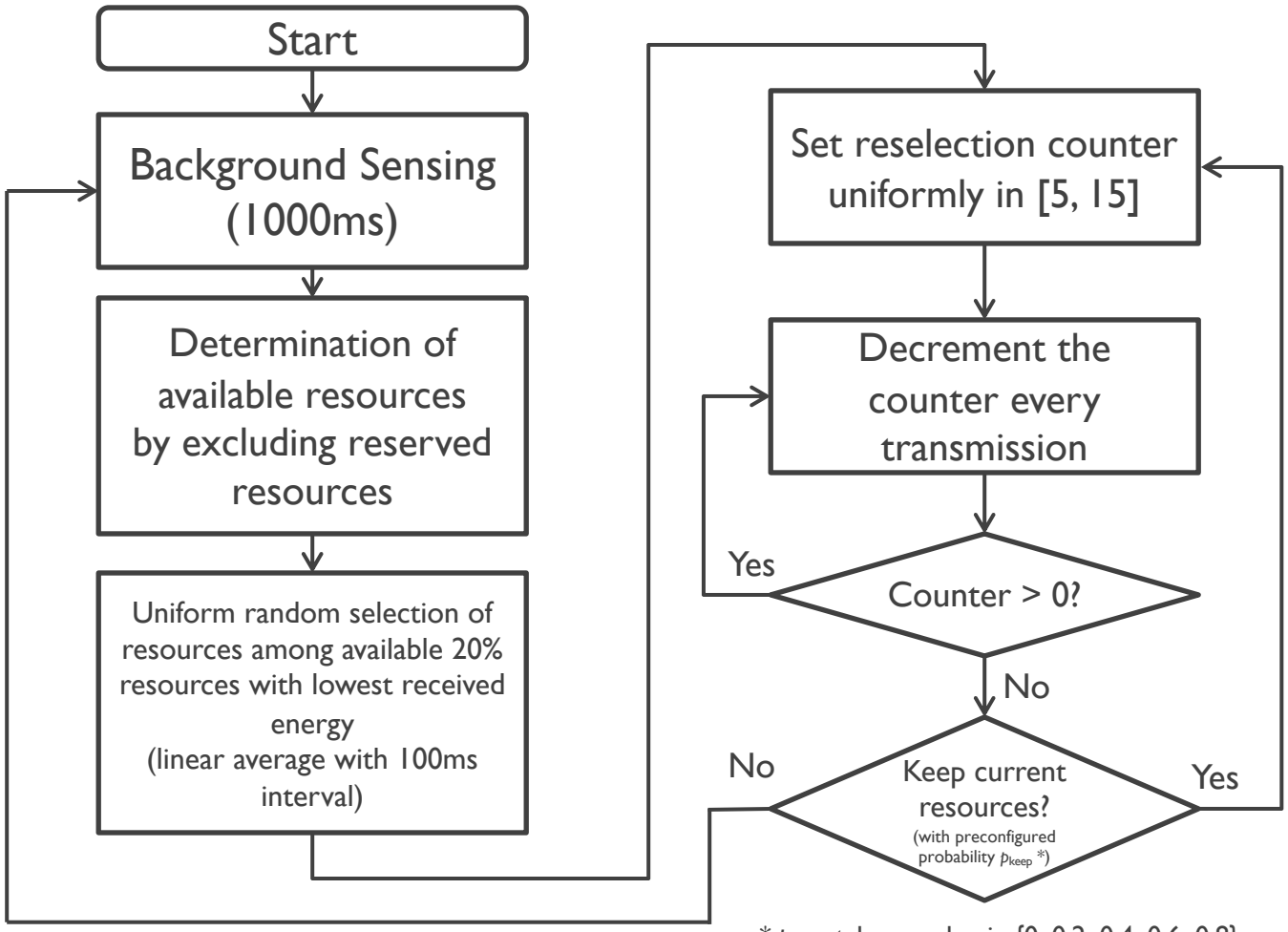
- **Problem 1**

  What's the probability distribution of # of transmissions between resource reselections?

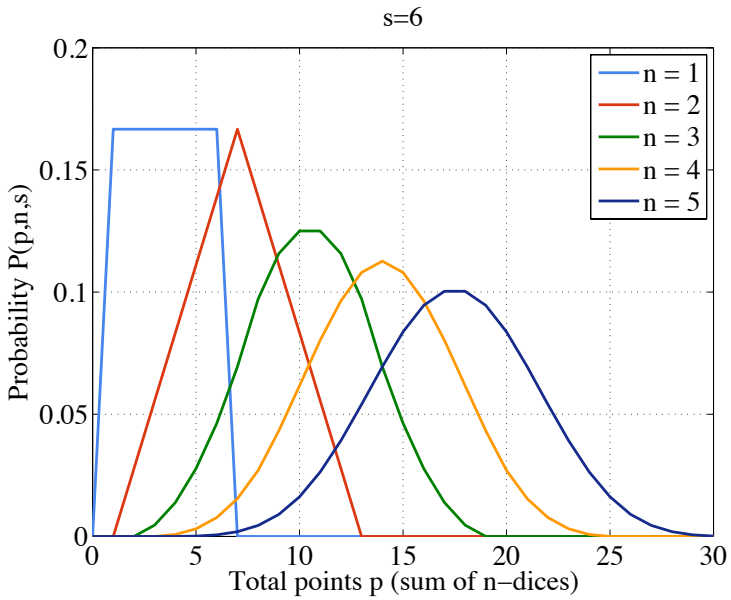


* p_{keep} takes a value in {0, 0.2, 0.4, 0.6, 0.8}.

Reference: Rolling n Dice

- For n dice with s sides each with numbers of 1, 2, …, s, the probability of obtaining p points as the sum of rolling n dice, $P_{\text{dice}}(p, n, s)$, is given by:

$$P_{\text{dice}}(p,n,s) = \frac{1}{s^n} \sum_{k=0}^{\lfloor (p-n)/s \rfloor} (-1)^k \binom{n}{k} \binom{p-sk-1}{n-1}$$



http://mathworld.wolfram.com/Dice.html

Solution for Problem 1

- For the given probability to keep the current resource p_{keep}, the probability distribution of # of transmissions x between resource reselection, $P(x, p_{\text{keep}})$, is given by:

$$P(x, p_{\text{keep}}) = \sum_{n=1}^{\infty} \underbrace{p_{\text{keep}}^{n-1}(1 - p_{\text{keep}})}_{\text{Probability of having } (n\text{-}1)\text{-th resource reselection}} \underbrace{P_{\text{dice}}(p = x - n(C_{\min} - 1), n, s = C_{\max} - C_{\min} + 1)}_{\text{Probability of having } x \text{ transmissions at } (n\text{-}1)\text{-th resource reselection}}$$

where

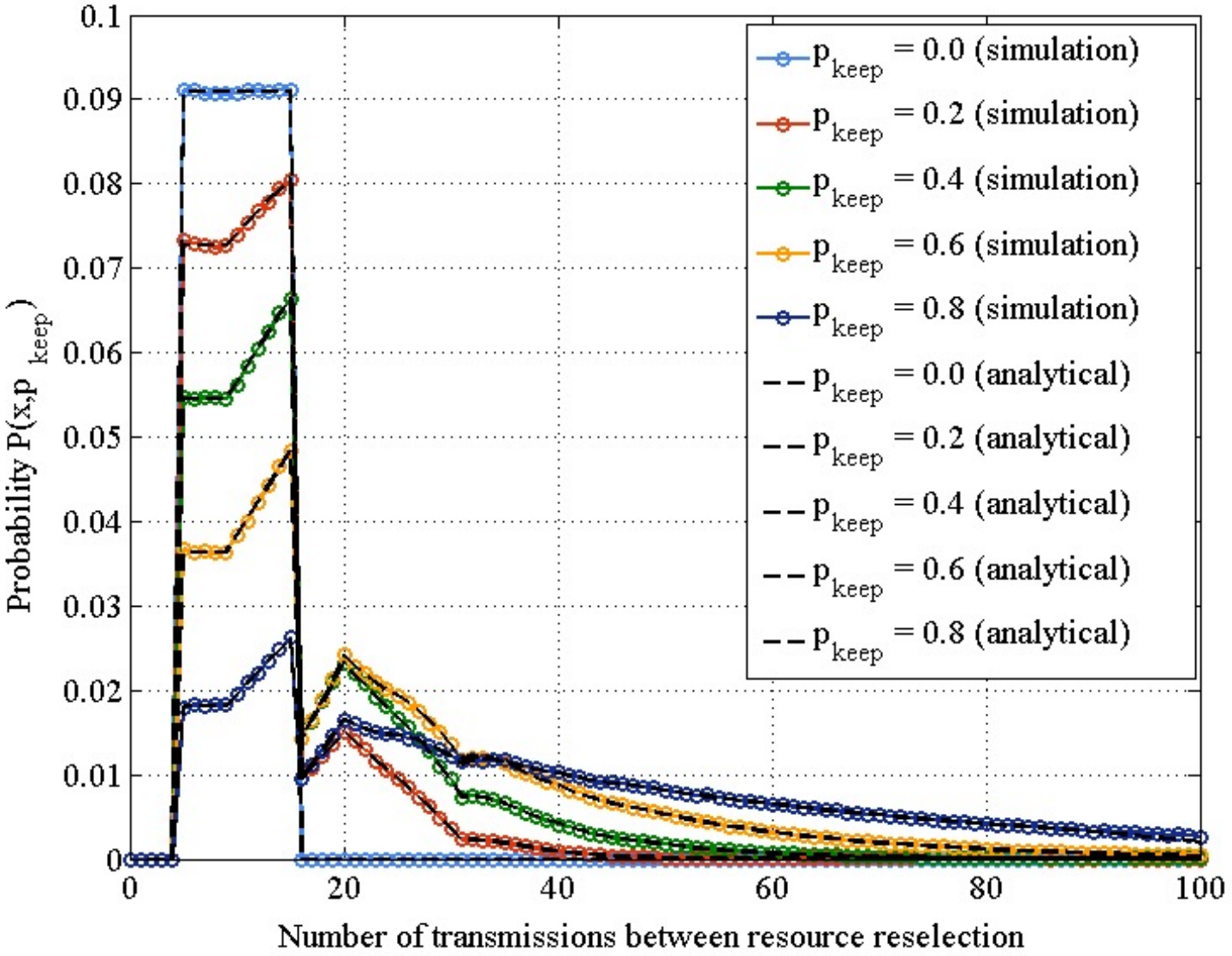
$$P_{\text{dice}}(p, n, s) = \frac{1}{s^n} \sum_{k=0}^{\lfloor (p-n)/s \rfloor} (-1)^k \binom{n}{k} \binom{p - sk - 1}{n - 1}$$

$C_{\min} = 5$ (Minimum value of reselection counter)

$C_{\max} = 15$ (Maximum value of reselection counter)

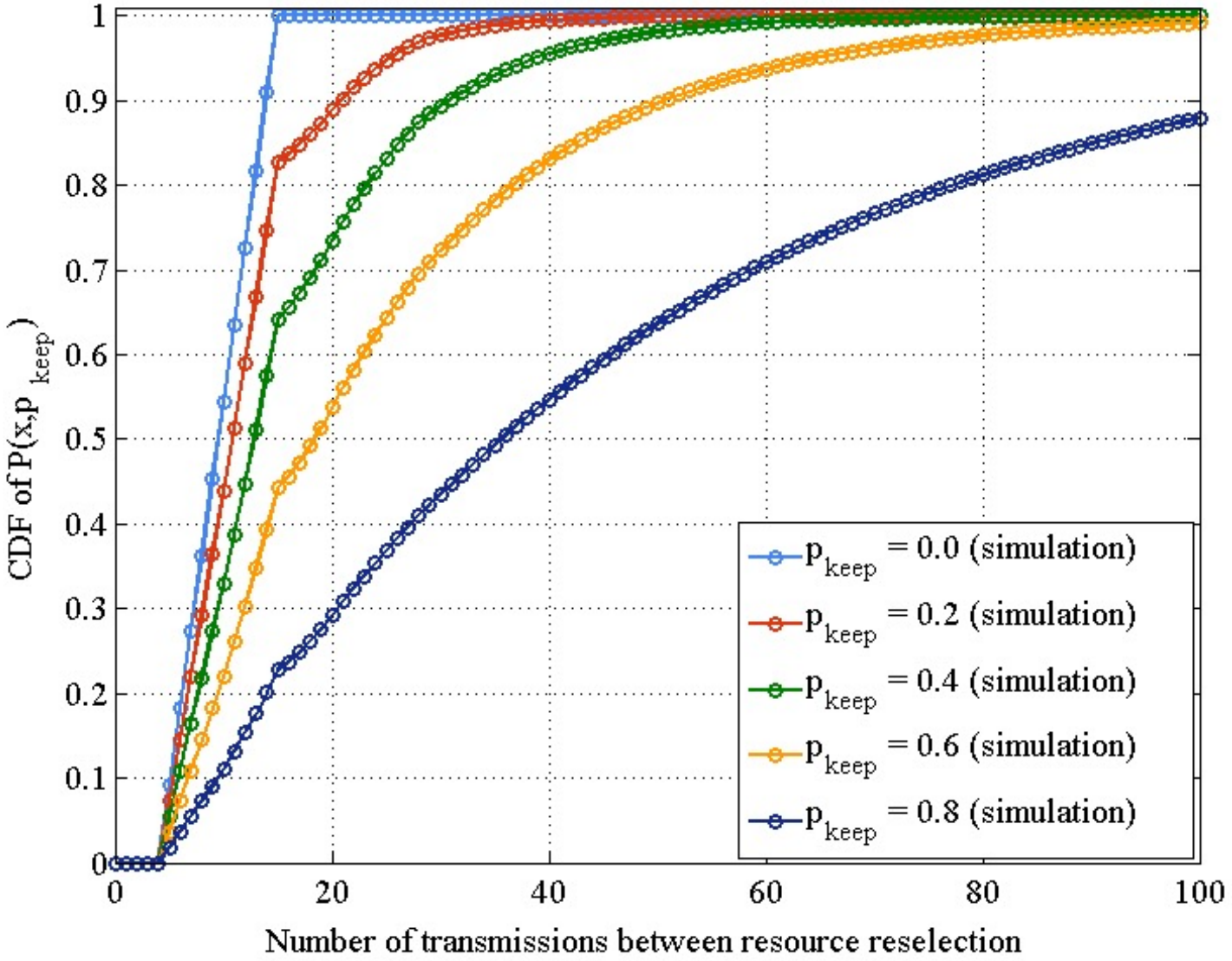
Comparison of Theoretical Analysis and Simulation Results for Problem 1

Probability Distribution Function



Comparison of Theoretical Analysis and Simulation Results for Problem I

Cumulative Distribution Function



- **Problem 2**
  What's the probability distribution of # of persistent collisions if two nodes select the same resource at the same time?

Solution for Problem 2

- Let X_1 and X_2 denote the random variables of # of transmission before resource reselection for Node 1 and Node 2, respectively, i.e., $\Pr(X_1) = \Pr(X_2) = P(x, p_{\text{keep}})$
- Also let Y denote the random variable of # of persistent collisions between Node 1 and Node 2, i.e., $Y = \min(X_1, X_2)$
- The probability $\Pr(Y)$ is given by:

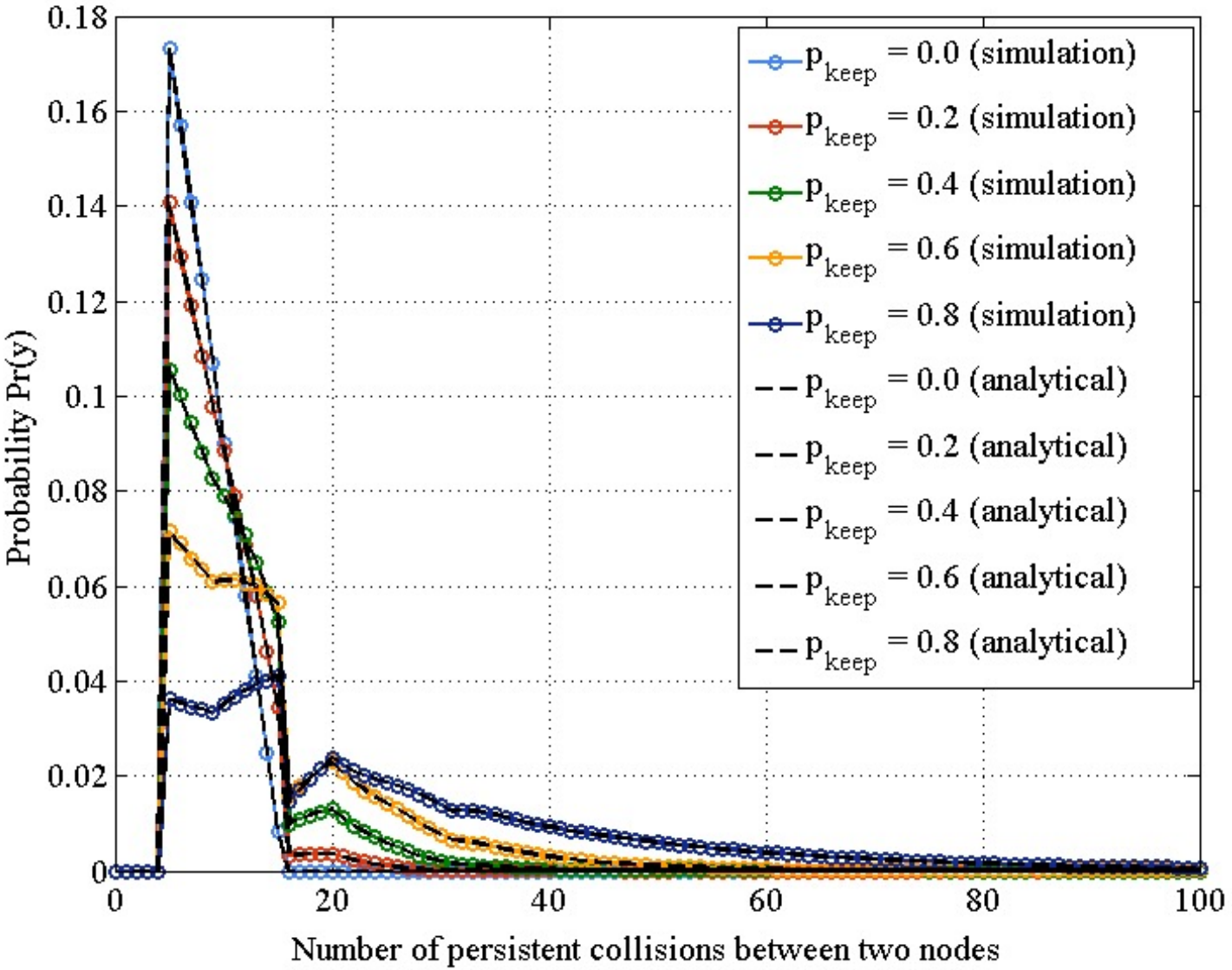
$$\begin{aligned}\Pr(Y=y) &= \Pr(X_1=y)\Pr(X_2=y) + \Pr(X_1=y)\Pr(X_2>y) + \Pr(X_1>y)\Pr(X_2=y) \\ &= P^2(x=y, p_{\text{keep}}) + 2P(x=y, p_{\text{keep}})\left(1 - \sum_{i=1}^{y} P(x=i, p_{\text{keep}})\right)\end{aligned}$$

where

$$P(x, p_{\text{keep}}) = \sum_{n=1}^{\infty} p_{\text{keep}}^{n-1}(1-p_{\text{keep}})P_{\text{dice}}(p = x - n(C_{\min}-1), n, s = C_{\max} - C_{\min} + 1)$$

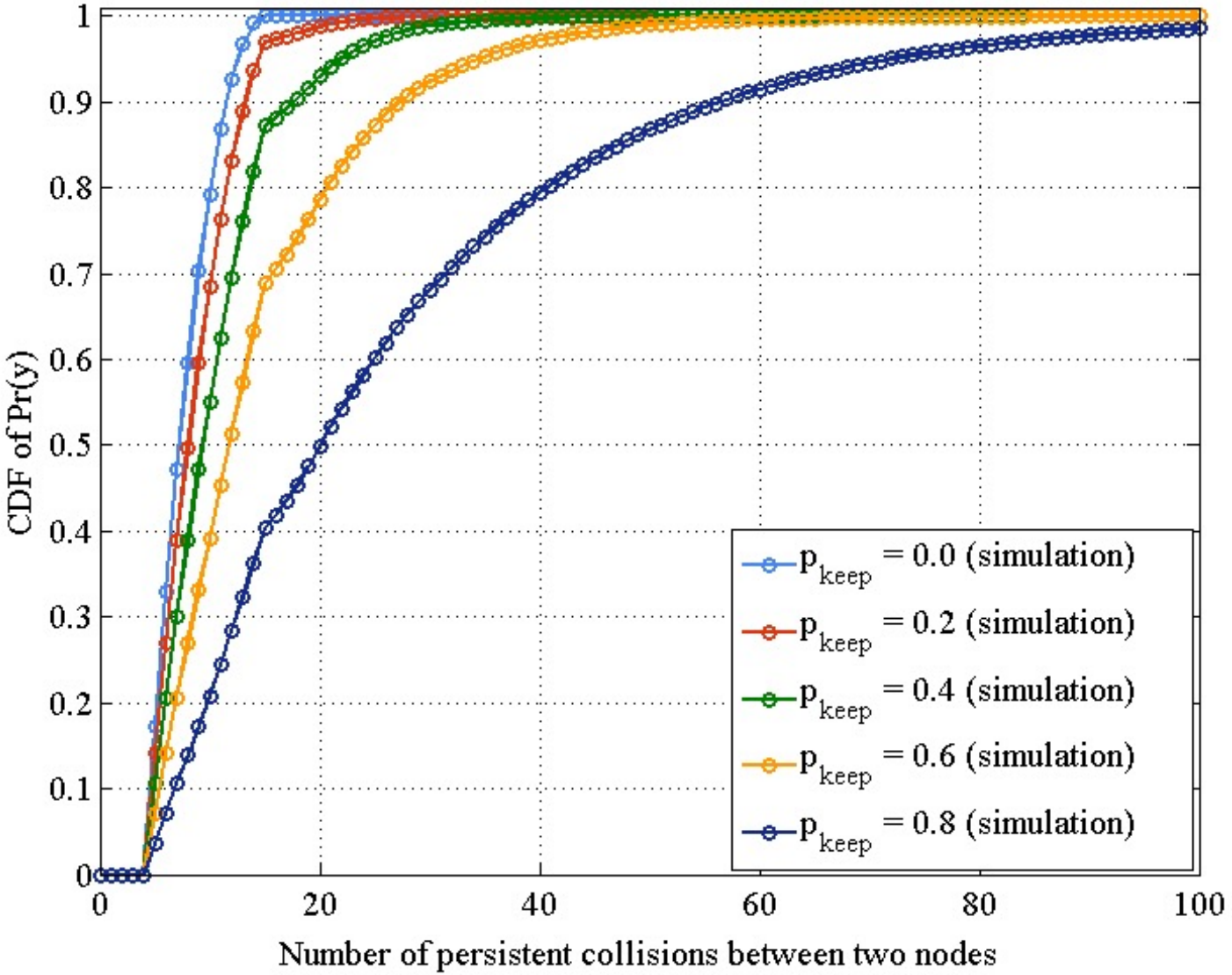
Comparison of Theoretical Analysis and Simulation Results for Problem 2

Probability Distribution Function



Comparison of Theoretical Analysis and Simulation Results for Problem 2

Cumulative Distribution Function



Average # of Persistent Collisions Based on Simulation Results

p_{keep}	Average # of Persistent Collision
0.0	8.1821
0.2	9.1718
0.4	11.0323
0.6	15.0242
0.8	27.3359