An LMI-Based Robust Nonlinear Adaptive Observer for Disturbed Regression Models

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***Abstract*—This article deals with the problem of time-varying parameter identification in dynamical regression models affected by disturbances. The disturbances comprise time-dependent external perturbations and nonlinear unmodeled dynamics. With this aim in mind, we propose a robust nonlinear adaptive observer. The algorithm ensures the asymptotic convergence of the parameter identification error to an acceptably small region around the origin in the presence of disturbances. The synthesis of the adaptive observer is given in terms of linear matrix inequalities, providing a constructive design method. An academic example and a low inertia power system illustrate the robustness and the applicability of the proposed adaptive observer for the time-varying parameter identification problem.**

***Index Terms*—Adaptive control, parameter identification, regression models.**

I. Introduction

The problem of controllers design, in the presence of unknown parameters or uncertain signals, remains challenging. One possible alternative to deal with such a lack of knowledge is the parameter or unknown input identification.

The online parameter identification has attracted considerable attention in the last decades (see, e.g., [1] and [2]). Parameter identification techniques are relevant for different industrial applications, such as biological systems [3], mechanical systems [4], and power systems [5]. Many techniques address the problem of parameter identification, e.g., least–squares (LS) methods, gradient descent-based algorithms, and adaptive estimation, to mention a few popular tools.

In parameter identification, the existing results can be classified by taking into account, or not, external time-dependent perturbations and time-varying parameters. The most of proposed approaches have addressed the identification of constant parameter. To recall some recent works, in [6], a recursive LS method was designed to identify constant parameters for flexible joints. Under some persistence of excitation conditions, the parameter identification error converges to a region around the origin. In [7], a switching adaptive parameter identification algorithm was proposed for robot manipulators. The parameter identification error converges to zero in a finite time. In [8], a parameter identification algorithm was proposed for homogeneous systems based on a class of artificial neural networks. The parameter identification error converges to a region around the origin. However, the algorithms proposed in [6], [7], and [8] do not consider the presence of disturbances.

In contrast, some authors have tackled the constant parameter identification problem in the presence of time-dependent perturbations. For instance, [9] used a linear regression model to ensure the boundedness of the parameter identification error in a finite time without the persistence of excitation condition. In [10], an adaptive identification method was proposed to estimate constant parameters for sinusoidal signals, e.g., the unknown offset, amplitude, frequency, and phase. The exponential convergence of the parameter identification error to a region around the origin is given. In the same vein, in [11], an adaptive algorithm based on a gradient-descent method solved the problem of constant parameter identification. However, the abovementioned algorithms cannot cope with time-varying parameters. Another shortcoming is that the parameter identification error only converges to a region around the origin due to the presence of time-dependent perturbations.

In the context of time-varying parameter identification, an early result can be found in [12]. The scheme is based on a recursive LS algorithm guaranteeing the exponential convergence of slowly time-varying parameters. However, the effect of external disturbances is not considered. Regarding the adaptive estimation theory, in [13], a solution was proposed for nonlinear parameterized regression models. The asymptotic convergence to zero of the parameter identification error is ensured. In [14], an adaptive parameter identification approach was presented based on a generalized gradient-descent algorithm. The asymptotic convergence to zero of the parameter identification error is guaranteed. In [15], a discontinuous algorithm capable of estimating time-varying parameters in a finite time, for a regression model, was proposed to improve the rate of convergence of the parameter identification. However, these works do not consider the effect of external time-dependent perturbations. Alternatively, several works address the problem of time-varying parameter identification in the presence of

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time-dependent perturbations. For instance, in [16], the authors proposed a recursive LS, whereas in [17], the zonotope approach was used for the parameter identification problem. However, both algorithms do not consider unmodeled nonlinear dynamics. Besides, the convergence is to a region around the origin that depends on the variation of the parameter and the time-dependent external disturbances. A recent work was given in [18], where a novel polynomial approximator is proposed based on an LS algorithm with a bounded regressor vector. In [19], a parameter identification algorithm was developed for time-varying parameters. In [20], a parameter identification algorithm was proposed for linearly parameterized nonlinear systems. However, the algorithms presented in [19] and [20] only guarantee convergence to a region around the origin for the parameter identification error due to time-dependent external disturbances and parameter variations. Additionally, both algorithms do not consider the effect of nonlinear unmodeled dynamics. In [21], a parameter identification algorithm was developed for a regression model. The scheme presented in [21] guaranteed finite- and fixed-time convergence to a region of the origin for the parameter identification error. Still, [21] did not include the nonlinear unmodeled dynamics and, due to external time-dependent perturbations and parameter variations, only the boundedness for the parameter identification error can be ensured. In [22], the time-varying parameters identification problem in the presence of time-dependent bounded disturbances for linear regression models was addressed. The scheme used the Taylor series to transform the time-varying parameter case into the identification of piecewise-constant parameters. However, such an approach is not feasible for fast variations of the parameters. Moreover, due to the time-dependent external disturbances and parameter variations, only exponential convergence to a region around the origin is guaranteed. Additionally, the scheme introduced in [22] did not contemplate nonlinear unmodeled dynamics as all the aforementioned works. Furthermore, the abovementioned methods do not provide a constructive gain synthesis for the proposed algorithms.

In summary, physical systems are intrinsically nonlinear and may present unknown time-varying parameters. Thus, any parameter identification procedure relevant to real applications must deal with disturbances comprising time-dependent perturbations and disregarded dynamics. Moreover, it must be capable of identifying time-varying parameters. Besides, a constructive synthesis of the algorithms (with clear tuning guidelines) helps the applicability of the schemes.

This manuscript contributes with an adaptive observer for a class of disturbed dynamical regression models. Such an algorithm deals with the time-varying parameter identification problem. Its main advantages are the following: 1) the algorithm counteracts the effects of the disturbances comprising time-dependent external perturbations and nonlinear unmodeled dynamics; 2) the parameter identification error converges asymptotically to a region around the origin, which can be made acceptably small by a proper tuning of design parameters; and 3) the gains synthesis of the algorithm is constructive since it is based on the solution of a set of linear matrix inequalities (LMIs).

The rest of this article is organized as follows. The problem statement is formulated in Section II. The robust adaptive parameter identification algorithm and the main results are presented in Section III. Simulation results are shown in Section IV. Finally, Section V concludes the article, and all the proofs are postponed to the Appendix.

Notation: The Euclidean norm of a vector $q \in \mathbb{R}^n$ is denoted by $\|q\|$. The induced norm of a matrix $P \in \mathbb{R}^{n\times n}$ is denoted by $\|P\|$. For a Lebesgue measurable function $u : \mathbb{R}_+ \to \mathbb{R}^m$, define the norm $\|u\|_{(t_0,t_1)} := \operatorname{ess\,sup}_{t\in(t_0,t_1)} \|u(t)\|$ for $(t_0, t_1) \subset \mathbb{R}_+$, then $\|u\|_\infty := \|u\|_{(0,+\infty)}$ and the set of functions u with the property $\|u\|_\infty < +\infty$ is denoted as $\mathcal{L}_\infty$. A continuous function $\xi : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\xi(0) = 0$; it belongs to class $\mathcal{K}_\infty$ if it is also unbounded. Define the function $\lceil a\rfloor^\gamma = |a|^\gamma \mathrm{sign}(a)$, for any $\gamma \geq 0$ and any $a \in \mathbb{R}$; for the case where $a \in \mathbb{R}^n$, $\lceil a\rfloor^\gamma = [\lceil a_1\rfloor^\gamma, \lceil a_2\rfloor^\gamma, \ldots, \lceil a_n\rfloor^\gamma]^T$. A sequence of integers $1, 2, \ldots, n$ is denoted as $\overline{1, n}$. Denote $0_{n\times m}$ as a zero matrix of dimension $n \times m$, 1_n as a vector of ones with dimension n, and I_n as the identity matrix of dimension $n \times n$.

II. Problem Statement

Consider the following disturbed regression model:

$$\dot{y}(t) = \Gamma(t)\theta(t) + w(t, y(t)) \tag{1}$$

where $y(t) \in \mathbb{R}^m$ is the output available for measurement, $\Gamma : \mathbb{R} \to \mathbb{R}^{m\times p}$ is a continuous function of time, which is measurable, $\theta : \mathbb{R} \to \mathbb{R}^p$ is the vector of unknown time-varying parameters and $w : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}^m$ represents unmeasurable disturbances comprising time-dependent external perturbations and nonlinear unmodeled dynamics, which is locally Lipschitz continuous function of y and piecewise continuous in t. The regressor term Γ is persistently exciting [23]. It is considered that the number of parameters is greater or equal to the number of states available for measurement, i.e., $p \geq m$. In addition, the measurable output and the vector of unknown parameters satisfies the following assumption.

Assumption 1: The measurable output, the unknown parameter vector, and its derivative are bounded, i.e., $\|y\|_\infty \leq y^+$, $\|\theta\|_\infty \leq \theta^+$, *and* $\|\dot{\theta}\|_\infty \leq \bar{\theta}$.

Note that most of the parameters in a great variety of dynamical systems, e.g., mechanical or power electronics systems, have bounded domains of admissible values, whose bounds can be evaluated from their physical nature (the same for the rates of parameter variations). In this sense, Assumption 1 does not restrict the class of systems.

The objective of this manuscript is to identify the vector of time-varying parameters θ despite the presence of time-dependent external perturbations and nonlinear unmodeled dynamics using the measurable output y and the regressor vector Γ.

III. Robust Nonlinear Adaptive Observer

Let us introduce the following adaptive observer:

$$\dot{\hat{y}} = L_1\tilde{y} + Lf(\tilde{y}) + \Gamma(t)\hat{\theta}, \tag{2a}$$

$$\dot{\hat{\theta}} = L_2\Gamma^T(t)\left[P\tilde{y} + \Lambda f(\tilde{y})\right] - L_\theta\lceil\hat{\theta}\rfloor^0 \tag{2b}$$

where $\hat{y} \in \mathbb{R}^m$ is the estimation of the output y, $\hat{\theta} \in \mathbb{R}^p$ is the identified value of the parameter θ, $\tilde{y} = y - \hat{y}$, $f(\tilde{y}) = [f_0^T(\tilde{y}), f_1^T(\tilde{y}), f_2^T(\tilde{y})]^T$, with $f_0(\tilde{y}) = \lceil\tilde{y}\rfloor^0$, $f_1(\tilde{y}) = \lceil\tilde{y}\rfloor^\alpha$, $f_2(\tilde{y}) = \lceil\tilde{y}\rfloor^\beta$, $\alpha \in (0, 1)$, and $\beta > 1$. The matrix gains $L_1 \in \mathbb{R}^{m\times m}$ and $L_2 \in \mathbb{R}^{p\times p}$ are set such that $0 < L_1 = \mathrm{diag}\{l_{1i}\}$, for $i = \overline{1, m}$ and $0 < L_2 = \mathrm{diag}\{l_{2k}\}$, for $k = \overline{1, p}$, while the gain matrices $L = [L_3, L_4, L_5] \in \mathbb{R}^{m\times 3m}$, where $0 < L_s = \mathrm{diag}\{l_{si}\} \in \mathbb{R}^{m\times m}$, $s = \overline{3, 5}$ and $i = \overline{1, m}$, $\Lambda = [\Lambda_0, \Lambda_1, \Lambda_2] \in \mathbb{R}^{m\times 3m}$, where $\Lambda_j = \mathrm{diag}\{\lambda_{ji}\}$, $\lambda_{ji} \in \mathbb{R}_+$, for $i = \overline{1, m}$ and $j = \overline{0, 2}$, $0 < P = \mathrm{diag}\{p_i\} \in \mathbb{R}^{m\times m}$, for $i = \overline{1, m}$, and the scalar gain $L_\theta > 0$ are designed further on. From (2b), note that if the nonlinear functions $f_0 = f_1 = f_2 = 0$ and $L_\theta = 0$, the classic adaptive parameter identification algorithm is recovered ([23, Th. 5.3.1]).

Taking into account the state estimation error $\tilde{y}$, and the parameter identification error $\tilde{\theta} = \theta - \hat{\theta}$, the error dynamics is given as follows:

$$\dot{\tilde{y}} = -L_1\tilde{y} - Lf(\tilde{y}) + \Gamma(t)\tilde{\theta} + w(t, y(t)), \tag{3a}$$

$$\dot{\tilde{\theta}} = -L_2\Gamma^T(t)\left[P\tilde{y} + \Lambda f(\tilde{y})\right] + L_\theta\lceil\hat{\theta}\rfloor^0 + \dot{\theta}(t). \tag{3b}$$

Note that the effect of disturbances appears in the state estimation error dynamics, whereas the effect of parameter variation appears in the parameter identification error equation.

The following assumption characterizes the admissible class of disturbances.

Assumption 2: The disturbance term w satisfies

$$\|w\|^2 \le c\|y\|^2 + c_1\|y\|^{1+\alpha} + c_2\|y\|^{1+\beta} + w^+ \tag{4}$$

where $c, c_1, c_2, w^+ \in \mathbb{R}_+$ are some known positive constants.

The disturbances described by Assumption 2 contemplate different classes of unmodeled dynamics such as linear terms, which are related to $c\|y\|^2$, the term $c_2\|y\|^{1+\beta}$ represents the nonlinearities growing faster than any linear function for large divergences of the error, and $c_1\|y\|^{1+\alpha}$ for small divergences. The upper bound also considers non-vanishing time-dependent external perturbations, which are bounded by w^+. Thus, Assumption 2 is valid for a large class of disturbances. Moreover, since y is measurable, adjustments of the constants in (4), off– or on–line, are always possible. Further on, it will be shown that the disturbance terms, i.e., $c\|y\|^2$, $c_1\|y\|^{1+\alpha}$, $c_2\|y\|^{1+\beta}$, and w^+, are compensated by the observer terms $\tilde{y}$, $f_1(\tilde{y})$, $f_2(\tilde{y})$, and $f_0(\tilde{y})$, respectively.

The convergence properties of the robust adaptive observer (2) are described by the following theorem.

Theorem 1: Let Assumptions 1 and 2 be satisfied, and the adaptive observer (2) be applied to system (1). Suppose that, for some given $\alpha \in (0,1)$ and $\beta > 1$, there exist diagonal matrices $0 < P \in \mathbb{R}^{m\times m}$, $0 < Y_j \in \mathbb{R}^{m\times m}$, $0 < \Lambda_j \in \mathbb{R}^{m\times m}$, for $j = \overline{0,2}$, and $0 < \Phi \in \mathbb{R}^{m\times m}$, such that the following matrix inequalities:

$$\begin{bmatrix} X & 0 & 0 & 0 & P \\ \star & \bar{Y}_0 & 0 & 0 & \Lambda_0 \\ \star & \star & -2Y_1 & 0 & \Lambda_1 \\ \star & \star & \star & -2Y_2 & \Lambda_2 \\ \star & \star & \star & \star & -\mu I_m \end{bmatrix} \le 0, \tag{5a}$$

$$\Phi - \mu\bar{c}I_m > 0, \quad L_1\Lambda_s - \mu\bar{c}_s I_m > 0, \ s = 1, 2, \tag{5b}$$

are feasible, with $X = -2PL_1 + \Phi$, $\bar{Y}_0 = -2Y_0 + \eta I_m$, $\bar{c} = 2c$, $\bar{c}_1 = 2^\alpha c_1$, $\bar{c}_2 = 2^\beta c_2$, for some fixed $L_1 > 0$, $\mu > 0$, and $\eta = \mu\bar{w}^+/m$, with $\bar{w}^+ = \bar{c}(\hat{y}^+)^2 + \bar{c}_1(\hat{y}^+)^{1+\alpha} + \bar{c}_2(\hat{y}^+)^{1+\beta} + w^+$, $\hat{y}^+ = \sqrt{2\Psi/\alpha_1} + y^+$, with $\Psi = 2L_\theta\kappa_2\sqrt{p}\|L_2^{-1}\|\theta^+$, $\kappa_1 \in (0,1)$, $\kappa_2 \ge \kappa_1 + 1$, p as the number of unknown parameters, and $\alpha_1 = \lambda_{\min}(\Phi - \mu\bar{c}I_m)$. If the observer parameters are selected such that $L_1, L_2 > 0$, $L_\theta > \bar{\theta}/\kappa_1$, and $L_{j+3} = Y_j\Lambda_j^{-1}$, for $j = \overline{0,2}$; then, $[\tilde{y}^T, \tilde{\theta}^T] = 0$ is practically Globally Uniformly Asymptotically Stable (GUAS).

The proof of this theorem is given in the Appendix. Note that it is only necessary to have a priori knowledge of the constants c, c_1, c_2, and w^+. If such a knowledge is not available; then, as usual in state/parameter estimation/identification problems, such constants can be overestimated.

Based on the stability analysis (please refer to the Appendix), the convergence region of the parameter identification error is inversely proportional to the norm of L_2.

Remark 1: In the disturbance-free case, the identifiability of the system (1) follows from the left-invertibility of Γ, which is a somewhat restrictive hypothesis. For the constant-parameter case, this requirement can be relaxed for Γ being persistently exciting (see,e.g., [23]), which becomes reasonable if the dynamics of the identification algorithm is much faster than the variation of θ (this is our case), taking into account that only practical convergence is guaranteed (Theorem 1 does not claim an exact estimation).

Remark 2: According to [24], the (practical) identifiability of system (1) can be related to the assumptions that have to be fulfilled by the system (1), and by the parameter identification algorithm (2), described in Theorem 1.

Algorithm 1: Tuning Algorithm.

Input: Constants $\alpha \in (0,1)$, $\beta > 1$, θ^+, $\bar{\theta}$, w^+, c, c_1, c_2, and m.
Output: Controller parameters L, P, and Λ.
1. Select $\mu > 0$, $L_1 = l_1 I_m$, $L_2 = l_2 I_p$, $L_\theta > \bar{\theta}/\kappa_1$, $\Phi > 2\mu c I_m$, with $l_1, l_2 > 0$, $\kappa_1 \in (0,1)$, and $\kappa_2 \ge \kappa_1 + 1$.
2. Compute $\bar{w}^+$ and $\eta = \mu\bar{w}^+/m$.
3. Look for a solution (P, Y_j, Λ_j), $j = \overline{0,2}$, for LMIs (5):
- If the solution is feasible, compute $L_{j+3} = Y_j\Lambda_j^{-1}$, $j = \overline{0,2}$.
- Otherwise, return to step 1 and modify μ and/or l_1.

4. Perform a simulation.
- If the performance is suitable, the algorithm ends.
- Otherwise, return to step 1 and increase l_2.

A. Implementation Issues

The matrix inequalities proposed in Theorem 1 are linear with respect to (P, Y_j, Λ_j), $j = \overline{0,2}$, if we fix diagonal matrices $L_1 > 0$, $L_2 > 0$, and $\Phi > 0$. In order to facilitate the design of the observer parameters, we propose the tuning Algorithm 1.

The proposed algorithm provides a constructive method to design L, P, and Λ; while μ, L_1, and L_2 serve as free initialization parameters. Note that, for any given μ and sufficiently big gains L_1 and L_2, the LMIs (5) are always feasible. On the other hand, L_2 impacts the parameter identification error and needs to be selected arbitrarily large in order to obtain an acceptably small convergence region.

In the next section, the robust adaptive parameter identification algorithm is applied to two different systems in order to illustrate the performance.

IV. Simulation Results

The simulations have been done in MATLAB with the Euler explicit discretization method, while the solution to the given LMIs is obtained by means of SDPT3 solver, among YALMIP in MATLAB. Section IV-A illustrates the robustness against disturbances in an academic example with a sampling time equal to 0.001[s]. Section IV-B addresses the application of the proposed scheme to a low inertia power systems problem related to a grid voltage in unbalance condition with a sampling time equal to 2.5×10^{-5}[s].

A. Robustness Properties

Let us consider an academic example for system (1)

$$\Gamma(t) = \begin{bmatrix} \cos(6t) & \sin(68t) & \cos(32t) \\ \sin(110t) & \cos(13t) & \sin(25t) \end{bmatrix},$$

$$\theta(t) = \begin{bmatrix}\theta_1(t)\\ \theta_2(t)\\ \theta_3(t)\end{bmatrix} = \begin{bmatrix}10\sin(2t)+2\\ 5.5\cos(1.4t)+1\\ -10.4\sin(t)-3\end{bmatrix},$$

$$w(t,y) = \begin{bmatrix}0.01(y_1+y_1^2)+0.02|y_2|^{\frac{1}{2}}+0.1\cos(t)+0.05\\ 0.01(y_1+y_1^2)+0.02|y_2|^{\frac{1}{2}}+0.1\cos(t)+0.05\end{bmatrix}.$$

The time-varying parameters satisfy Assumption 1 with $\theta^+ = 19.12$ and $\bar{\theta} = 23.82$, while the disturbances $w(t,y)$ satisfy Assumption 2, with known constants $\alpha = 0.5$, $\beta = 1.2$, $\bar{c} = 0.02$, $\bar{c}_1 = 0.03$, $\bar{c}_2 = 0.01$, $w^+ = 0.16$, and $y^+ = 9.1$. Then, with $\mu = 2$, $\kappa_1 = 0.9$, $\kappa_2 = 1.9$, $L_\theta = 29.1133$, $L_2 = 200I_2$, and $\Phi = 0.0520I_2$, we have $\Psi = 18.3187$ and $\alpha_1 = 0.0120$; and then, $\hat{y}^+ = 64.3550$ and $\bar{\varpi}^+ = 193.7327$. Note that the value of $\hat{y}^+$ can always be evaluated off– or on–line since $\tilde{y}$ and y are measurable.

We follow the Algorithm 1 to compute the observer parameters. Selecting $\mu = 1$, $L_1 = I_2$, $L_2 = 200I_2$, $L_\theta = 29.1133$, $\Phi = 0.0260I_2$, and computing $\eta = 193.7327$, the following solution for the LMIs (5) is obtained: $L_3 = 22.6961I_2$, $L_4 = 16.4311I_2$, $L_5 = 16.4311I_2$, $P = 1.1132I_2$, $\Lambda_0 = 11.5652I_2$, $\Lambda_1 = 11.0776I_2$, and $\Lambda_2 = 11.0776I_2$.

Consider the initial conditions $y(0) = [4, -5]^T$, $\hat{y}(0) = 0$, and $\hat{\theta}(0) = 0$. For comparison purposes, the classic adaptive parameter identification algorithm ([23, Th. 5.3.1]) is also implemented. To simplify the nomenclature, the subindex $\mathcal{C}$ is introduced for the results of the classic adaptive parameter identification algorithm. Moreover, the observer parameters L_1, L_2, and P of the classic adaptive parameter identification algorithm are the same as those of the robust adaptive parameter identification scheme.

Fig. 1 depicts the parameter θ and its identified value $\hat{\theta}$. Fig. 1(a)–(c) illustrate the real and identified value of θ_1, θ_2, and θ_3, respectively. The norm of the parameter identification error is depicted in Fig. 2. The parameter identification converges to a region around the origin due to the parameter variation (not because of disturbances), whereas the classic adaptive parameter identification algorithm converges to a bigger region around the real value due to both, the disturbances and parameter variations.

The next subsection addresses the identification problem dealing with power systems.

B. Application to Power Systems

Consider the next model of the grid voltage in unbalance condition [25]

$$\dot{V}_{ab} = J\Psi_{ab}\Upsilon(t) + w(t), \tag{6a}$$

$$\dot{\Psi}_{ab} = \bar{\varpi}JV_{ab} \tag{6b}$$

with $\Upsilon(t) = \omega^2(t)/\bar{\varpi}$ and

$$J = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix},\ w(t) = \begin{bmatrix}\sin(500t)+0.1\\ \cos(500t)+0.2\end{bmatrix}$$

where $V_{ab} = [v_a, v_b]^T = V_{ab}^+ + V_{ab}^-$ is the voltage signal in the grid from reference points a and b. The voltages V_{ab}^+ and V_{ab}^- are the positive and negative sequence components of V_{ab}, respectively. The term $\Psi_{ab}(t) = (\bar{\varpi}/\omega(t))\varphi_{ab}$, with $\varphi_{ab} := V_{ab}^+ - V_{ab}^-$, $\omega(t)$ is the fundamental frequency of the grid voltage, and $\bar{\varpi}$ is a known nominal value of ω. The element w contemplates the parasitic loads that affect the system.

The term Υ is unknown and time-varying. Then, the estimation of voltages V_{ab} and fundamental frequency ω is not straightforward. In this sense, system (6) can be rewritten like (1) as follows $\dot{V}_{ab} = \Gamma(t)\theta(t) + w(t)$, where $\Gamma(t) = J\Psi_{ab}(t)$, and $\theta(t) = \Upsilon(t)$. The robust adaptive observer (2) can be designed for the power system to identify Υ, i.e.,

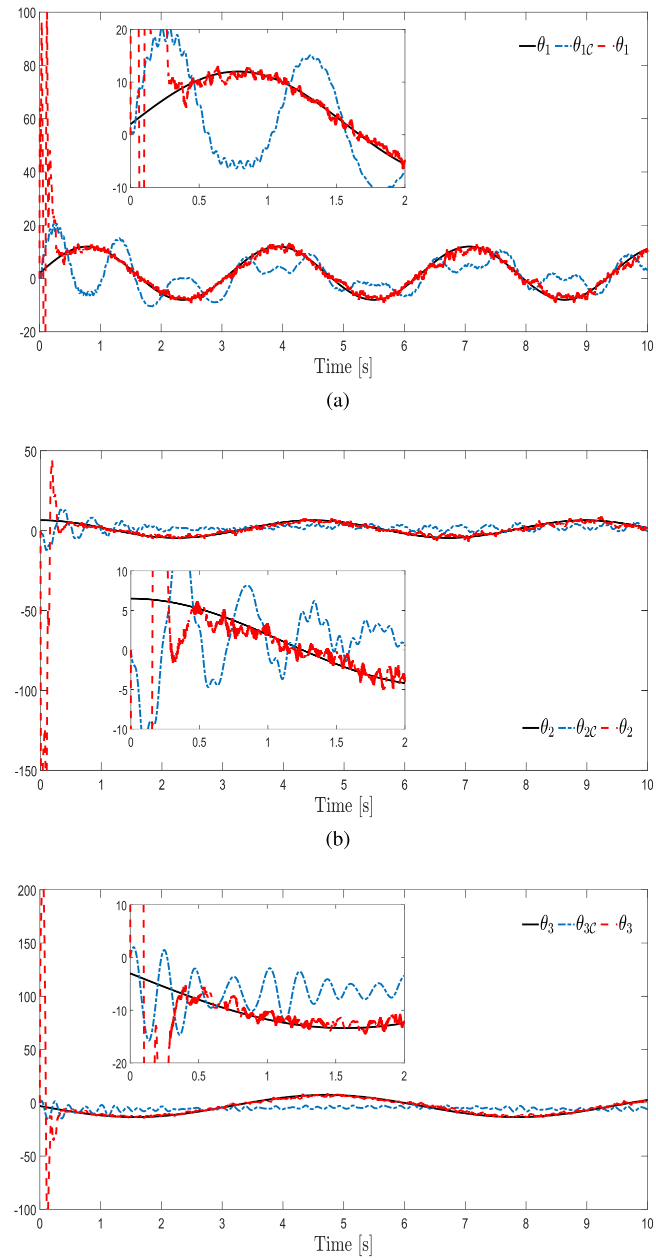

Fig. 1. Parameter identification. (a) Identification of the parameter θ_1. (b) Identification of the parameter θ_2. (c) Identification of the parameter θ_3.

$$\dot{\hat{V}}_{ab} = L_1\tilde{V}_{ab} + Lf(\tilde{V}_{ab}) + \Gamma(t)\hat{\theta},$$

$$\dot{\hat{\theta}} = L_2\Gamma^T(t)[P\tilde{V}_{ab} + \Lambda f(\tilde{V}_{ab})] - L_\theta\lceil\hat{\theta}\rfloor^0$$

where $\tilde{V}_{ab} = V_{ab} - \hat{V}_{ab}$. The fundamental frequency of the three-phase voltage is time-varying, and the voltage amplitude is $|V_{ab}| = 100$ [V]. To consider the effect of an Inertia and Rate of Change of Frequency, which is present in power systems, the fundamental frequency changes smoothly as follows $\omega(t) = \omega_b(t) + 48$ for $0 \le t < 5$, $\omega(t) =$

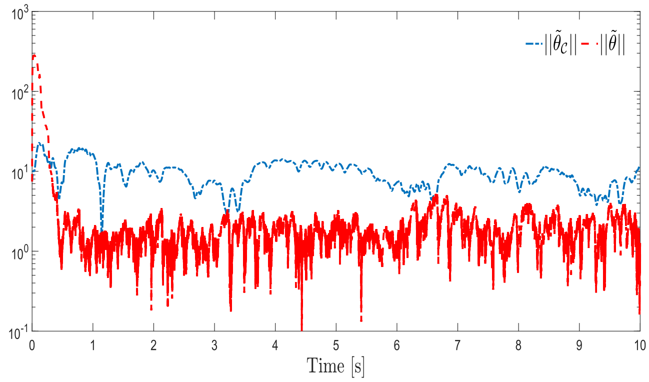

Fig. 2. Norm of the parameter identification error.

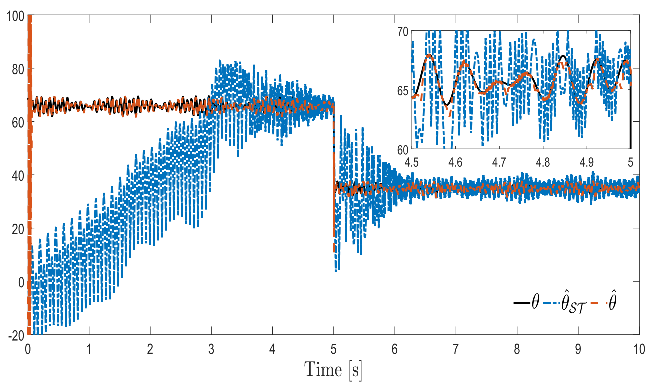

Fig. 3. Parameter identification.

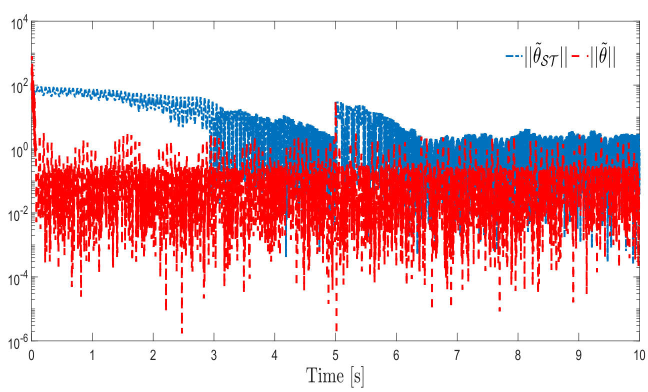

Fig. 4. Norm of the parameter identification error.

$\omega_b(t) + 35$, for $t \geq 5$, where $\omega_b(t) = 0.3\sin(60t) + 0.4\sin(92t) + 0.1\cos(10t) + 0.7\cos(87t)$. The nominal value of ω is considered as $\bar{\omega} = 35$. Then, the time-varying parameters satisfy Assumption 1 with $\theta^+ = 50$ and $\bar{\theta} = 118.7$, while the disturbances satisfy Assumption 2, with $\bar{c} = \bar{c}_1 = \bar{c}_2 = 0$, $\bar{w}^+ = w^+ = 3.2$, and $y^+ = 83$. Note that in this case we do not need to compute $\hat{y}^+$.

We follow the Algorithm 1 to compute the observer parameters. Selecting $\mu = 0.05$, $L_1 = I_2$, $L_2 = 200$, $L_\theta = 145.0778$, $\Phi = 0.0150I_2$, and computing $\eta = 0.0800$, the following solution for the LMIs (5) is obtained: $L_3 = 40.0594I_2$, $L_4 = 39.9826I_2$, $L_5 = 39.9826I_2$, $P = 0.0268I_2$, $\Lambda_0 = 0.7467I_2$, $\Lambda_1 = 0.7472I_2$, and $\Lambda_2 = 0.7472I_2$.

The initial conditions are $V_{ab}(0) = [0, 0]^T$ and the initial conditions for the observer are $\hat{V}_{ab}(0) = [0, 0]^T$ and $\hat{\theta}_{ab}(0) = 0$. For comparison purposes, the Super Twisting-based adaptive parameter identification algorithm (ST), proposed in [26], i.e.,

$$\dot{\hat{V}}_{ab} = -k_1 \frac{\hat{V}_{ab} - V_{ab}}{\|\hat{V}_{ab} - V_{ab}\|^{\frac{1}{2}}} + \Gamma(t)\hat{\theta}_{ST},$$

$$\dot{\hat{\theta}}_{ST} = -k_2 \Gamma^T(t) \frac{\hat{V}_{ab} - V_{ab}}{\|\hat{V}_{ab} - V_{ab}\|}$$

is also implemented. The subindex ST is introduced for the results of the algorithm presented in [26]. The gain selection is based on the method proposed in [26, Th. 1]. For $k_1 = 7.51$, the variables $A = 1$ and $c = 4.51$ are fixed. For k_2, an additional condition under the class of admissible disturbances is introduced, i.e., the disturbances satisfies $||\dot{w}||_\infty \leq \Delta$. For this case, $\Delta = 500$, and thus, $k_2 = 98.24$. It is important to highlight that the class of disturbances, studied here, is larger than the class of disturbances considered in [26], which are only time-dependent perturbations and it is required to be Lipschitz. Opposite to [26], in this research, the disturbances comprise time-dependent perturbations and nonlinear unmodeled dynamics. Moreover, no Lipschitz condition is needed.

The time-varying parameter and the parameter identification are depicted by Fig. 3, whereas the norm of the parameter identification error is depicted by Fig. 4. The proposed parameter identification converges to a smaller region around the origin with a faster rate of convergence than the ST. Thus, the proposed algorithm estimates the fundamental frequency of the grid voltage even in presence of disturbances.

V. Conclusion

This manuscript contributes with a robust adaptive observer for a class of dynamic regression models with time-varying parameters affected by disturbances comprising unmodeled dynamics and time-dependent external perturbations. The proposed algorithm ensures the asymptotic convergence of the parameter identification error to an acceptably small region around the origin in presence of disturbances. The gain tuning of the algorithm is constructive, and it is based on the solution of a set of LMIs, while the convergence proofs are developed based on Lyapunov theory. Simulation results for an academic example and a low inertia power system illustrate the performance of the proposed scheme.

Appendix A

Proof of Theorem 1: The proof is inspired by [27]. Consider the following candidate Lyapunov function [28]:

$$V = \tilde{y}^T P \tilde{y} + \sum_{j=0}^{2} \sum_{i=1}^{m} \lambda_{ji} \int_0^{\tilde{y}_i} f_{ji}(s) ds + \tilde{\theta}^T L_2^{-1} \tilde{\theta} \qquad (7)$$

where $f_j = [f_{j1}, \ldots, f_{jn}]^T$, for all $j = \overline{0, 2}$. The integrals of the nonlinearities f_{ji} are positive definite functions and, according to [28], V is positive definite and radially unbounded. The derivative of V is given by

$$\dot{V} = 2\dot{\tilde{y}}^T P \tilde{y} + 2 \sum_{j=0}^{2} \dot{\tilde{y}}^T \Lambda_j f_j(\tilde{y}) + 2\tilde{\theta}^T L_2^{-1} \dot{\tilde{\theta}}.$$

Considering (3a), it follows that:

$$\begin{aligned}\dot{V} =& -2\tilde{y}^T PL_1\tilde{y} - 2\sum_{j=0}^{2}\Big[f_j^T(\tilde{y})L_{j+3}P\tilde{y}\\ &+\tilde{y}^T L_1\Lambda_j f_j(\tilde{y}) + \sum_{s=0}^{2}\left(f_s^T(\tilde{y})L_{s+3}\Lambda_j f_j(\tilde{y})\right)\\ &-\tilde{\theta}^T\Gamma^T(t)\Lambda_j f_j(\tilde{y}) - w^T\Lambda_j f_j(\tilde{y})\Big] + 2\tilde{\theta}^T\Gamma^T(t)P\tilde{y}\\ &+2\tilde{\theta}^T L_2^{-1}\dot{\tilde{\theta}} + 2\,w^T P\tilde{y}.\end{aligned}$$

Taking into account (3b), we obtain

$$\begin{aligned}\dot{V} =& -2\tilde{y}^T PL_1\tilde{y} - 2\sum_{j=0}^{2}\Big[f_j^T(\tilde{y})L_{j+3}P\tilde{y}\\ &+\tilde{y}^T L_1\Lambda_j f_j(\tilde{y}) + \sum_{s=0}^{2}\left(f_s^T(\tilde{y})L_{s+3}\Lambda_j f_j(\tilde{y})\right)\\ &-w^T\Lambda_j f_j(\tilde{y})\Big] + 2\tilde{\theta}^T L_2^{-1}L_\theta\lceil\hat{\theta}\rfloor^0 + 2\tilde{\theta}^T L_2^{-1}\dot{\theta} + 2\,w^T P\tilde{y}\end{aligned}$$

that can be rewritten as follows:

$$\begin{aligned}\dot{V} =& \,\xi^T Q\xi - \tilde{y}^T\Phi\tilde{y} - 2\sum_{j=0}^{2}\tilde{y}^T(L_{j+3}P + L_1\Lambda_j)f_j(\tilde{y})\\ &-2\sum_{j=0, j\neq s}^{2}\ \sum_{s=0, s\neq j}^{2} f_s^T(\tilde{y})L_{s+3}\Lambda_j f_j(\tilde{y})\\ &-\eta f_0^T(\tilde{y})f_0(\tilde{y}) + \mu w^T w + 2\tilde{\theta}^T L_2^{-1}L_\theta\lceil\hat{\theta}\rfloor^0 + 2\tilde{\theta}^T L_2^{-1}\dot{\theta}\end{aligned}$$

with $\xi = [\tilde{y}, f_0(\tilde{y}), f_1(\tilde{y}), f_2(\tilde{y}), w]^T$ and

$$Q = \begin{bmatrix} -2PL_1+\Phi & 0 & 0 & 0 & P\\ \star & -2Y_0+\eta I_m & 0 & 0 & \Lambda_0\\ \star & \star & -2Y_1 & 0 & \Lambda_1\\ \star & \star & \star & -2Y_2 & \Lambda_2\\ \star & \star & \star & \star & -\mu I_m\end{bmatrix}$$

where $Y_0 = L_3\Lambda_0$, $Y_1 = L_4\Lambda_1$, $Y_2 = L_5\Lambda_2$, $\eta = \mu\bar{w}^+/m$, and $\Phi = \mathrm{diag}\{\phi_i\} > 0$, for $i = \overline{1, m}$. Note that, due to the diagonal structure of the matrices P, L_1, L, and Λ, and the definition of the nonlinear functions $f_0(\tilde{y}) = \lceil\tilde{y}\rfloor^0$, $f_1(\tilde{y}) = \lceil\tilde{y}\rfloor^\alpha$, $f_2(\tilde{y}) = \lceil\tilde{y}\rfloor^\beta$, with $\alpha \in (0,1)$ and $\beta > 1$, the following inequalities are satisfied:

$$-2\sum_{j=0}^{2}\tilde{y}^T(L_{j+3}P + L_1\Lambda_j)f_j(\tilde{y}) \leq 0,$$

$$-2\sum_{j=0, j\neq s}^{2}\ \sum_{s=0, s\neq j}^{2} f_s^T(\tilde{y})L_{s+3}\Lambda_j f_j(\tilde{y}) \leq 0.$$

Therefore, since (5a) holds, an upper bound for the time derivative of V is given as follows:

$$\begin{aligned}\dot{V} \leq& -\tilde{y}^T\Phi\tilde{y} - 2\sum_{j=0}^{2}\tilde{y}^T(L_{j+3}P + L_1\Lambda_j)f_j(\tilde{y})\\ &-\eta f_0^T(\tilde{y})f_0(\tilde{y}) + \mu w^T w\\ &+2\tilde{\theta}^T L_2^{-1}L_\theta\lceil\hat{\theta}\rfloor^0 + 2\tilde{\theta}^T L_2^{-1}\dot{\theta}.\end{aligned}$$

Considering the definition of the parameter identification error, i.e., $\tilde{\theta} = \theta - \hat{\theta}$, it follows that:

$$\begin{aligned}\dot{V} \leq& -\tilde{y}^T\Phi\tilde{y} - 2\sum_{j=0}^{2}\tilde{y}^T(L_{j+3}P + L_1\Lambda_j)f_j(\tilde{y})\\ &-\eta f_0^T(\tilde{y})f_0(\tilde{y}) + \mu w^T w\\ &-2L_\theta\|L_2^{-1}\|\tilde{\theta}^T\lceil\tilde{\theta} - \theta\rfloor^0 + 2\tilde{\theta}^T L_2^{-1}\dot{\theta}.\end{aligned}$$

Then, according to [29], the inequality $-\tilde{\theta}^T\lceil\tilde{\theta} - \theta\rfloor^0 \leq -\kappa_1\|\tilde{\theta}\| + \kappa_2\sqrt{p}\|\theta\|$ holds for $\kappa_1 \in (0,1)$ and $\kappa_2 \geq \kappa_1 + 1$. Taking into account Assumption 1 and the previous inequality, the derivative of the Lyapunov function can be upper bounded by

$$\begin{aligned}\dot{V} \leq& -\tilde{y}^T\Phi^{-1}\tilde{y} - 2\sum_{j=0}^{2}\tilde{y}^T(L_{j+3}P + L_1\Lambda_j)f_j(\tilde{y})\\ &-\eta f_0^T(\tilde{y})f_0(\tilde{y}) + \mu w^T w\\ &-2\|L_2^{-1}\|(L_\theta\kappa_1 - \bar{\theta})\|\tilde{\theta}\| + 2L_\theta\kappa_2\sqrt{p}\|L_2^{-1}\|\theta^+.\end{aligned}$$

Furthermore, since w satisfies Assumption 2 and $y = \tilde{y} + \hat{y}$, the upper bound for w can be rewritten in terms of $\tilde{y}$, grouping the elements depending on $\hat{y}$ into the term w^+, i.e.,

$$\|w\|^2 \leq \bar{w}^+ + \bar{c}\|\tilde{y}\|^2 + \bar{c}_1\|\tilde{y}\|^{1+\alpha} + \bar{c}_2\|\tilde{y}\|^{1+\beta}$$

where $\bar{c} = 2c$, $\bar{c}_1 = 2^\alpha c_1$, $\bar{c}_2 = 2^\beta c_2$ are obtained by Gensen's inequality, and $\bar{w}^+ = w^+ + \bar{c}(\hat{y}^+)^2 + \bar{c}_1(\hat{y}^+)^{1+\alpha} + \bar{c}_2(\hat{y}^+)^{1+\beta}$, and where we have assumed that $\|\hat{y}\|_\infty \leq \hat{y}^+ = \sqrt{2\Psi/\alpha_1} + y^+$, with $\Psi = 2L_\theta\kappa_2\sqrt{p}\|L_2^{-1}\|\theta^+$, and $\alpha_1 = \lambda_{\min}(\Phi - \mu\bar{c}I_m)$. The correctness of the chosen estimate for $\hat{y}^+$ will be checked on the next step.

Since $\eta f_0^T(\tilde{y})f_0(\tilde{y}) = \eta m$ and $\eta = \mu\bar{w}^+/m$; then, it follows that:

$$\begin{aligned}\dot{V} \leq& -\tilde{y}^T(\Phi - \mu\bar{c}I_m)\tilde{y} - \sum_{s=1}^{2}\tilde{y}^T(2L_1\Lambda_j - \mu\bar{c}_s I_m)f_s(\tilde{y})\\ &-2\|L_2^{-1}\|(L_\theta\kappa_1 - \bar{\theta})\|\tilde{\theta}\| + \Psi\\ =& -W(\tilde{y},\tilde{\theta}) - 2\|L_2^{-1}\|(L_\theta\kappa_1 - \bar{\theta})\|\tilde{\theta}\| + \Psi \qquad (8)\end{aligned}$$

where W is a positive definite function. Therefore, taking into account that (5b) holds, if $L_\theta\kappa_1 > \bar{\theta}$, then

$$\dot{V} \leq -W(\tilde{y},\tilde{\theta}) + \Psi \qquad (9)$$

for all $\tilde{y} \in \mathbb{R}^m$ and $\tilde{\theta} \in \mathbb{R}^p$. Hence, it is concluded that $[\tilde{y}^T, \tilde{\theta}^T] = 0$ is practically GUAS, and since Ψ is inversely proportional to the norm of L_2, the convergence region can be made arbitrarily small.

In order to prove the boundedness of $\hat{y}$, note that V in (7) and W in (8) satisfy $V \leq \gamma(\|\tilde{y}\|^2 + \|\tilde{y}\| + \|\tilde{y}\|^{\alpha+1} + \|\tilde{y}\|^{\beta+1}) + \beta_1\|\tilde{\theta}\|^2$ and $W \geq \bar{\gamma}(\|\tilde{y}\|^2 + \|\tilde{y}\| + \|\tilde{y}\|^{\alpha+1} + \|\tilde{y}\|^{\beta+1}) + \beta_2\|\tilde{\theta}\|$, for some $\gamma, \bar{\gamma}, \beta_1, \beta_2 > 0$, respectively. Then, after some computations, it can be proved that (9) can be rewritten as follows:

$$\dot{V} \leq -\varrho(V),\ \forall\|\tilde{y}\|_\infty \geq \sqrt{2\Psi/\alpha_1} \qquad (10)$$

for some $\varrho \in \mathcal{K}_\infty$. Then, since y is bounded and (10) holds, the boundedness of $\hat{y}$ is ensured; and hence, the existence of $\hat{y}^+$ is guaranteed. In fact, we can take $||\tilde{y}||_\infty \leq \tilde{y}^+ = \sqrt{2\Psi/\alpha_1}$; and thus, $||\hat{y}||_\infty \leq \hat{y}^+ = \sqrt{2\Psi/\alpha_1} + y^+$. This concludes the proof. ■

References

[1] F. Garcia-Marquez and C. Garcia, "Parameter estimation and performance comparison of friction models for pneumatic valves," *Control Eng. Pract.*, vol. 104, 2020, Art. no. 104629.

[2] Y. Liu, H. Wang, and W. Zhang, "Robust parameter estimation with outlier-contaminated correlated measurements and applications to aerodynamic coefficient identification," *Aerosp. Sci. Technol.*, vol. 118, 2021, Art. no. 106995.

[3] T. Luspay and K. Grigoriadis, "Adaptive parameter estimation of blood pressure dynamics subject to vasoactive drug infusion," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 3, pp. 779–787, May 2016.

[4] N. Nevaranta, J. Parkkinen, T. Lindh, M. Niemela, O. Pyrhonen, and J. Pyrhonen, "Online estimation of linear tooth belt drive system parameters," *IEEE Trans. Ind. Electron.*, vol. 62, no. 11, pp. 7214–7223, Nov. 2015.

[5] D. Reed, J. Sun, and G. Hofmann, "Simultaneous identification and adaptive torque control of permanent magnet synchronous machines," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 4, pp. 1372–1383, Jul. 2017.

[6] H. Jin, Z. Liu, H. Zang, Y. Liu, and J. Zhao, "A dynamic parameter identification method for flexible joints based on adaptive control," *IEEE Trans. Mechatron.*, vol. 23, no. 6, pp. 2896–2908, Dec. 2018.

[7] C. Yang, Y. Jiang, W. He, J. Na, Z. Li, and B. Xu, "Adaptive parameter estimation and control design for robot manipulators with finite-time convergence," *IEEE Trans. Ind. Electron.*, vol. 65, no. 10, pp. 8112–8123, Oct. 2018.

[8] M. Ballesteros, A. Polyakov, D. Efimov, I. Chairez, and A. Poznyak, "Non-parametric identification of homogeneous dynamical systems," *Automatica*, vol. 129, 2021, Art. no. 109600.

[9] J. Wang, D. Efimov, and A. Bobtsov, "On robust parameter estimation in finite-time without persistence of excitation," *IEEE Trans. Autom. Control*, vol. 65, no. 4, pp. 1731–1738, Apr. 2020.

[10] J. Na, J. Yuang, X. Wu, and Y. Guo, "Robust adaptive parameter estimation of sinusoidal signals," *Automatica*, vol. 53, pp. 376–384, 2015.

[11] M. Kapetina, M. Repaic, A. Pisano, and Z. Jelicic, "Adaptive parameter estimation in LTI systems," *IEEE Trans. Autom. Control*, vol. 64, no. 10, pp. 4188–4195, Oct. 2019.

[12] S. Bittanti and M. Campi, "Bounded error identification of time-varying parameters by RLS techniques," *IEEE Trans. Autom. Control*, vol. 39, no. 5, pp. 1106–1110, May 1994.

[13] R. Ortega, V. Gromov, E. Nuño, A. Pyrkin, and J. Romero, "Parameter estimation of nonlinearly parameterized regressions without overparameterization: Application to adaptive control," *Automatica*, vol. 127, 2021, Art. no. 109544.

[14] J. Poon, P. Jain, C. Spanos, S. Kumar, and S. Sanders, "Fault prognosis for power electronics systems using adaptive parameter identification," *IEEE Trans. Ind. Appl.*, vol. 53, no. 3, pp. 2862–2870, May/Jun. 2017.

[15] J. Rueda-Escobedo and J. Moreno, "Discontinuous gradient algorithm for finite-time estimation of time-varying parameters," *Int. J. Control*, vol. 89, pp. 1838–1848, 2016.

[16] M. Mathelin and R. Lozano, "Robust adaptive identification of slowly time-varying parameters with bounded disturbances," *Automatica*, vol. 35, pp. 1291–1305, 1999.

[17] J. Bravo, T. Alamo, and E. Camacho, "Bounded error identification of systems with time-varying parameters," *IEEE Trans. Autom. Control*, vol. 51, no. 7, pp. 1144–1150, Jul. 2006.

[18] J. Chen, G. Zhang, and Z. Li, "Online parameter estimation for a class of time-varying continuous systems with bounded disturbances," *Int. J. Adaptive Control Signal Process.*, vol. 25, pp. 18–32, 2011.

[19] J. Na, Y. Xing, and R. Costa-Catello, "Adaptive estimation of time-varying parameters with application to roto-magnet plant," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 2, pp. 731–741, Feb. 2021.

[20] J. Na, J. Yang, X. Ren, and Y. Guo, "Robust adaptive estimation of nonlinear system with time-varying parameters," *Int. J. Adaptive Control Signal Process.*, vol. 29, pp. 1055–1072, 2014.

[21] H. Ríos, D. Efimov, J. Moreno, W. Perruquetti, and J. Rueda-Escobedo, "Time-varying parameter identification algorithms: Finite and fixed-time convergence," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3671–3678, Jul. 2017.

[22] A. Glushchenko and K. Lastochkin, "Robust time-varying parameters estimation based on I-DREM procedure," in *Proc. 14th IFAC Workshop Adaptive Learn. Control Syst.*, vol. 55, pp. 91–96, 2022.

[23] P. Ioannou and J. Sun, *Robust Adaptive Control*. Englewood Cliffs, NJ, USA: Prentice Hall, 1996.

[24] R. Isermann and M. Münchhof, *Identification of Dynamic Systems, an Introduction With Applications*, 1st ed. Berlin, Germany: Springer-Verlag, 2011.

[25] G. Escobar, M. Martínez-Montejano, A. Váldez, P. Martínez, and M. Hernández-Gómez, "Fixed-reference-frame phase-locked loop for grid synchronization under unbalanced operation," *IEEE Trans. Ind. Electron.*, vol. 58, no. 5, pp. 1943–1951, May 2011.

[26] J. Rueda-Escobedo, J. Moreno, and J. Schiffer, "Finite-time estimation of time-varying frequency signals in low-inertia power systems," in *Proc. 18th Eur. Control Conf.*, 2019, pp. 2108–2114.

[27] R. Franco, H. Ríos, A. Ferreira, and D. Efimov, "A robust nonlinear model reference adaptive control for disturbed linear systems: An LMI approach," *IEEE Trans. Autom. Control*, vol. 67, no. 4, pp. 1937–1943, Apr. 2022.

[28] D. Efimov and A. Aleksandrov, "On analysis of Persidskii systems and their implementations using LMIs," *Automatica*, vol. 134, 2021, Art. no. 109905.

[29] R. Franco, H. Ríos, D. Efimov, and W. Perruquetti, "Adaptive estimation for uncertain nonlinear systems with measurement noise: A sliding-mode observer approach," *Int. J. Robust Nonlinear Control*, vol. 31, pp. 3809–3826, 2021.