```
CONTEXT
   System_Ctx
CONSTANTS
   TIME
   sigma
   plantV0
AXIOMS
   axm1   :   TIME = RRealPlus
   axm2   :   sigma ∈ RRealPlus ∧ sigma ↦Rzero ∈gt
   axm3   :   plantV0∈ RReal
END
```

CONTEXT

Theorems

AXIOMS

axm1 : ∀a,b,c,d·a↦b∈ leq ∧ c↦d ∈leq ⇒ plus(a↦c) ↦plus(b↦d) ∈ leq

axm2 : ∀a,b,c,d·Rzero↦a ∈ leq ∧ Rzero↦b ∈ leq ∧ Rzero↦c ∈ leq ∧ Rzero↦d ∈ leq ∧ a↦b∈ leq ∧ c↦d ∈leq ⇒ times (a↦c) ↦times(b↦d) ∈ leq

axm3 : ∀a,b,c· a↦b ∈ leq ∧ b↦c ∈ leq⇒ a ↦c ∈ leq

axm4 : ∀a,b· a∈ RReal ∧ b ∈ RReal
⇒
minus(times(a↦a) ↦times(b↦b))=times(plus(a↦b)↦minus(a↦b))

axm5 : ∀a· a∈ RReal ⇒ uminus(a)=minus(Rzero↦a)

axm6 : ∀a· a∈ RReal ⇒
a=plus(
times(divide(Rone ↦ Rtwo) ↦a)
↦
times(divide(Rone ↦ Rtwo) ↦a)
)

axm7 : ∀a,b· a∈ RReal ∧ b∈ RReal ∧ times(a↦b)∈ RRealStar
⇒
inverse(times(a↦b))=times(inverse(a)↦inverse(b))

END

```
MACHINE
    System_M
SEES
    System_Ctx
    Theorems
VARIABLES
    t
    plantV
INVARIANTS
    inv1   :   t ∈ TIME
    inv2   :   plantV ∈ Closed2Closed(Rzero, t) ⇸ RReal
EVENTS
    INITIALISATION   ≙
    STATUS
      ordinary
    BEGIN
      act1   :   t≔Rzero
      act2   :   plantV ≔{Rzero↦plantV0}
    END

    Progress   ≙
    STATUS
      ordinary
    BEGIN
      act1   :   t :| t' ∈ TIME ∧ (t ↦ t' ∈ lt ∧ minus(t'↦t) ↦ sigma ∈ geq)
    END

    Plant   ≙
    STATUS
      ordinary
    ANY
      e
      plant1
    WHERE
      grd1   :   e ∈ DE(RReal)
      grd2   :   Solvable(Closed2Closed(Rzero, t)\dom(plantV),e)
      grd3   :   plant1 ∈ Closed2Closed(Rzero, t)\dom(plantV) ⟶ RReal ∧
                 AppendSolutionBAP(e,
                 Closed2Closed(Rzero, t)\dom(plantV),
                 Closed2Closed(Rzero, t)\dom(plantV), plant1)
    THEN
      act1   :   plantV≔plantV⩤plant1
    END

END
```

CONTEXT
EventTriggered_Ctx
EXTENDS
System_Ctx
SETS
EXEC
PROP
CONSTANTS
prop_safe
prop_evt_trig
ctrl
plant
prg
f_evol
f_evol_plantV
prop_evade_values
AXIOMS
axm1 : prop_safe ∈ PROP⟶((RReal × RReal) ⟶ BOOL)
axm2 : prop_evt_trig ∈ PROP⟶((RReal×RReal)×RReal ⟶ BOOL)
axm3 : partition(EXEC, {ctrl},{plant},{prg})
axm4 : f_evol ∈ RReal ⟶ RReal
axm5 : f_evol_plantV ∈ (RReal ⟶ (TIME × RReal ⟶ RReal))
axm6 : ∀ ctrlV · ctrlV ∈ RReal ⇒ (f_evol_plantV(ctrlV) =
(λ t ↦ plantV · t ∈ TIME ∧ plantV ∈ RReal | f_evol(ctrlV)))
axm7 : prop_evade_values∈PROP⟶ℙ1(RReal)
END

```
MACHINE
    EventTriggered_M
REFINES
    System_M
SEES
    EventTriggered_Ctx
VARIABLES
    t
    plantV
    ctrlV
    exec
INVARIANTS
    inv1   :   ctrlV ∈ RReal
    inv2   :   exec ∈ EXEC
    inv3   :   exec≠plant ⇒ dom(plantV)=Closed2Closed(Rzero, t)
    inv4   :   exec=plant ⇒ t∉ dom(plantV)
EVENTS
    INITIALISATION   ≙
      extended
    STATUS
      ordinary
    BEGIN
      act1   ;   t≔Rzero
      act2   ;   plantV ≔{Rzero↦plantV0}
      act3   :   ctrlV :∈ RReal
      act4   :   exec ≔ ctrl
    END

    Progress   ≙
    STATUS
      ordinary
    REFINES
      Progress
    ANY
      t1
    WHERE
      grd1   :   exec=prg
      grd2   :   t1 ∈ TIME ∧ (t ↦ t1 ∈ lt ∧ minus(t1↦t) ↦ sigma ∈ geq)
      grd3   :   ∀x· x∈ PROP ⇒
                   (ctrlV∉ prop_evade_values(x)⇒
                        (prop_evt_trig(x))(plantV(t)↦minus(t1↦t)↦ctrlV) = TRUE)
    THEN
      act1   :   t≔t1
      act2   :   exec ≔ plant
    END

    Plant   ≙
    STATUS
      ordinary
    REFINES
      Plant
    ANY
      plant1
    WHERE
      grd1   :   exec=plant
      grd2   :   plant1 ∈ Closed2Closed(Rzero, t)\dom(plantV) ⟶ RReal
      grd3   :   ode(f_evol_plantV(ctrlV),plant1(t),t) ∈ DE(RReal)
      grd4   :   Solvable(Closed2Closed(Rzero, t)\dom(plantV),
                          ode(f_evol_plantV(ctrlV),plant1(t),t))
      grd5   :   AppendSolutionBAP(ode(f_evol_plantV(ctrlV),plant1(t),t),
                 Closed2Closed(Rzero, t)\dom(plantV),
                 Closed2Closed(Rzero, t)\dom(plantV), plant1)
    WITH
      e    :    e = ode(f_evol_plantV(ctrlV),plant1(t),t)
    THEN
      act1   :   plantV≔plantV⩤plant1
      act2   :   exec≔ctrl
    END
```

```
  Ctrl   ≙
  STATUS
    ordinary
  ANY
    value
  WHERE
    grd1   :   exec = ctrl
    grd2   :   value∈RReal
               ∀x· x∈ PROP ⇒
    grd3   :     (value∉ prop_evade_values(x)
               ⇒(prop_safe(x))(plantV(t)↦value) = TRUE)
  THEN
    act1   :   ctrlV ≔value
    act2   :   exec ≔ prg
  END

END
```

```
CONTEXT
    TimeTriggered_Ctx
EXTENDS
    EventTriggered_Ctx
CONSTANTS
    epsilon
    prop_safeEpsilon
AXIOMS
    axm1   :   epsilon ∈ TIME ∧ sigma↦epsilon ∈leq
    axm2   :   prop_safeEpsilon ∈ PROP⟶((RReal × RReal) ⟶ BOOL)
    axm3   :   Rzero↦epsilon ∈lt
END
```

MACHINE
TimeTriggered_M
REFINES
EventTriggered_M
SEES
TimeTriggered_Ctx
Theorems
VARIABLES
t
plantV
ctrlV
exec
EVENTS
INITIALISATION ≙
extended
STATUS
ordinary
BEGIN
act1 : t≔Rzero
act2 : plantV ≔{Rzero↦plantV0}
act3 : ctrlV :∈ RReal
act4 : exec ≔ ctrl
END

Progress ≙
STATUS
ordinary
REFINES
Progress
ANY
t1
WHERE
grd1 : exec=prg
grd2 : t1 ∈ TIME ∧ (t ↦ t1 ∈ lt ∧ minus(t1↦t) ↦ sigma ∈ geq)
grd3 : ∀x· x∈ PROP ⇒
(ctrlV∉ prop_evade_values(x)⇒
(prop_evt_trig(x))(plantV(t)↦minus(t1↦t)↦ctrlV) = TRUE)
grd4 : t1 ∈ TIME ∧ (t ↦ t1 ∈ lt) ∧ minus(t1↦t) ↦ sigma ∈ geq ∧ minus(t1↦t) ↦ epsilon ∈ leq
THEN
act1 : t≔t1
act2 : exec ≔ plant
END

Plant ≙
STATUS
ordinary
REFINES
Plant
ANY
plant1
WHERE
grd1 : exec=plant
grd2 : plant1 ∈ Closed2Closed(Rzero, t)\dom(plantV) ⟶ RReal
grd3 : ode(f_evol_plantV(ctrlV),plant1(t),t) ∈ DE(RReal)
grd4 : Solvable(Closed2Closed(Rzero, t)\dom(plantV),
ode(f_evol_plantV(ctrlV),plant1(t),t))
grd5 : AppendSolutionBAP(ode(f_evol_plantV(ctrlV),plant1(t),t),
Closed2Closed(Rzero, t)\dom(plantV),
Closed2Closed(Rzero, t)\dom(plantV), plant1)
THEN
act1 : plantV≔plantV⩤plant1
act2 : exec≔ctrl
END

Ctrl ≙
STATUS
ordinary
REFINES
Ctrl

```
    ANY
      value
    WHERE
      grd1   :   exec = ctrl
      grd2   :   value∈RReal
      grd3   :   ∀ x· x∈ PROP ⇒
                   (value∉ prop_evade_values(x)
                 ⇒(prop_safe(x))(plantV(t)↦value) = TRUE)
      grd4   :   ∀ x· x∈ PROP ⇒
                          (value∉ prop_evade_values(x)
                 ⇒(prop_safeEpsilon(x))(plantV(t)↦value) = TRUE)
    THEN
      act1   :   ctrlV ≔value
      act2   :   exec ≔ prg
    END

END
```

```
CONTEXT
   Desolve
EXTENDS
   TimeTriggered_Ctx
CONSTANTS
   B_desolve
   prop
AXIOMS
   axm1   :   B_desolve ∈  ℕ × RReal × (TIME ⇸ RReal ) ×TIME × (TIME × RReal) ⟶ (RReal ⇸ RReal)
   axm2   :   prop∈ RReal ⟶BOOL
   axm3   :   prop(plantV0)=TRUE
END
```

```
MACHINE
    TimeTriggered_desolve_M
REFINES
    TimeTriggered_M
SEES
    Desolve
    Theorems
VARIABLES
    t
    plantV
    ctrlV
    exec
INVARIANTS
    inv1   :   ∀x· x∈ dom(plantV)⇒prop(plantV(x))=TRUE
EVENTS
    INITIALISATION   ≙
      extended
    STATUS
      ordinary
    BEGIN
      act1   :   t≔Rzero
      act2   :   plantV ≔{Rzero↦plantV0}
      act3   :   ctrlV :∈ RReal
      act4   :   exec ≔ ctrl
    END

    Progress   ≙
    STATUS
      ordinary
    REFINES
      Progress
    ANY
      t1
    WHERE
      grd1   :   exec=prg
      grd2   :   t1 ∈ TIME ∧ (t ↦ t1 ∈ lt ∧ minus(t1↦t) ↦ sigma ∈ geq)
      grd3   :   ∀x· x∈ PROP ⇒
                   (ctrlV∉ prop_evade_values(x)⇒
                        (prop_evt_trig(x))(plantV(t)↦minus(t1↦t)↦ctrlV) = TRUE)
      grd4   :   t1 ∈ TIME ∧ (t ↦ t1 ∈ lt) ∧ minus(t1↦t) ↦ sigma ∈ geq ∧ minus(t1↦t) ↦ epsilon ∈ leq
    THEN
      act1   :   t≔t1
      act2   :   exec ≔ plant
    END

    Plant   ≙
    STATUS
      ordinary
    REFINES
      Plant
    ANY
      plant1
      lastTime
    WHERE
      grd1   :   exec=plant
      grd2   :   lastTime∈ TIME ∧ dom(plantV)=Closed2Closed(Rzero,lastTime)
      grd3   :   plant1 =B_desolve(1 ↦ ctrlV ↦ plantV ↦ t ↦ (lastTime↦plantV(lastTime)))
      grd4   :   plant1 ∈ Closed2Closed(Rzero, t)\dom(plantV) ⟶ RReal
      grd5   :    ode(f_evol_plantV(ctrlV),plant1(t),t) ∈ DE(RReal)
      grd6   :   Solvable(Closed2Closed(Rzero, t)\dom(plantV),
                          ode(f_evol_plantV(ctrlV),plant1(t),t))
      grd7   :     AppendSolutionBAP(ode(f_evol_plantV(ctrlV),plant1(t),t),
                 Closed2Closed(Rzero, t)\dom(plantV),Closed2Closed(Rzero, t)\dom(plantV), plant1)
      grd8   :    ∀xx· xx∈ dom(plant1)⇒ prop(plant1(xx))=TRUE
    THEN
      act1   :   plantV≔plantV⩤plant1
      act2   :   exec≔ctrl
    END
```

```
  Ctrl  ≙
  STATUS
    ordinary
  REFINES
    Ctrl
  ANY
    value
  WHERE
    grd1  :  exec = ctrl
    grd2  :  value∈RReal
             ∀x· x∈ PROP ⇒
    grd3  :    (value∉ prop_evade_values(x)
             ⇒(prop_safe(x))(plantV(t)↦value) = TRUE)
             ∀x· x∈ PROP ⇒
    grd4  :          (value∉ prop_evade_values(x)
             ⇒(prop_safeEpsilon(x))(plantV(t)↦value) = TRUE)
  THEN
    act1  :  ctrlV ≔value
    act2  :  exec ≔ prg
  END

END
```