

CONTEXT**System_Ctx****CONSTANTS**TIME
sigma
plantV0**AXIOMS**axm1 : TIME = RRealPlus
axm2 : $\sigma \in \text{RRealPlus} \wedge \sigma \neq 0 \rightarrow \sigma \in \text{Rgt}$
axm3 : plantV0 \in RReal**END**

MACHINE**System_M****SEES****System_Ctx****Theorems****VARIABLES****t****plantV****INVARIANTS****inv1** : $t \in \text{TIME}$ **inv2** : $\text{plantV} \in \text{Closed2Closed}(\text{Rzero}, t) \leftrightarrow \text{RReal}$ **EVENTS****INITIALISATION** \triangleq **STATUS****ordinary****BEGIN****act1** : $t = \text{Rzero}$ **act2** : $\text{plantV} = \{\text{Rzero} \mapsto \text{plantV0}\}$ **END****Progress** \triangleq **STATUS****ordinary****BEGIN****act1** : $t : | t' \in \text{TIME} \wedge (t \mapsto t' \in \text{lt} \wedge \text{minus}(t' \mapsto t) \mapsto \text{sigma} \in \text{geq})$ **END****Plant** \triangleq **STATUS****ordinary****ANY****e****plant1****WHERE****grd1** : $e \in \text{DE}(\text{RReal})$ **grd2** : $\text{Solvable}(\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}), e)$ $\text{plant1} \in \text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}) \rightarrow \text{RReal} \wedge$ **grd3** : $\text{AppendSolutionBAP}(e,$
 $\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}),$ $\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}), \text{plant1})$ **THEN****act1** : $\text{plantV} = \text{plantV} \uparrow \text{plant1}$ **END****END**

CONTEXT**EventTriggered_Ctx****EXTENDS****System_Ctx****SETS**

EXEC

PROP

CONSTANTS

prop_safe

prop_evt_trig

ctrl

plant

prg

f_evol

f_evol_plantV

prop_evade_values

AXIOMSaxm1 : prop_safe \in PROP \rightarrow ((RReal \times RReal) \rightarrow BOOL)axm2 : prop_evt_trig \in PROP \rightarrow ((RReal \times RReal) \times RReal \rightarrow BOOL)

axm3 : partition(EXEC, {ctrl},{plant},{prg})

axm4 : f_evol \in RReal \rightarrow RRealaxm5 : f_evol_plantV \in (RReal \rightarrow (TIME \times RReal \rightarrow RReal))axm6 : \forall ctrlV \cdot ctrlV \in RReal \Rightarrow (f_evol_plantV(ctrlV) =
(λ t \mapsto plantV \cdot t \in TIME \wedge plantV \in RReal | f_evol(ctrlV)))axm7 : prop_evade_values \in PROP \rightarrow $\mathbb{P}1$ (RReal)**END**

MACHINE

EventTriggered_M

REFINES

System_M

SEES

EventTriggered_Ctx

VARIABLES

t
 plantV
 ctrlV
 exec

INVARIANTS

inv1 : ctrlV ∈ RReal
 inv2 : exec ∈ EXEC
 inv3 : exec ≠ plant ⇒ dom(plantV) = Closed2Closed(Rzero, t)
 inv4 : exec = plant ⇒ t ∉ dom(plantV)

EVENTS**INITIALISATION** \triangleq

extended

STATUS

ordinary

BEGIN

act1 : t = Rzero
 act2 : plantV = {Rzero → plantV0}
 act3 : ctrlV ∈ RReal
 act4 : exec = ctrl

END**Progress** \triangleq **STATUS**

ordinary

REFINES

Progress

ANY

t1

WHERE

grd1 : exec = prg
 grd2 : t1 ∈ TIME ∧ (t → t1 ∈ lt ∧ minus(t1 → t) → sigma ∈ geq)
 $\forall x. x \in \text{PROP} \Rightarrow$
 grd3 : (ctrlV ∉ prop_evade_values(x) ⇒
 (prop_evt_trig(x))(plantV(t) → minus(t1 → t) → ctrlV) = TRUE)

THEN

act1 : t = t1
 act2 : exec = plant

END**Plant** \triangleq **STATUS**

ordinary

REFINES

Plant

ANY

plant1

WHERE

grd1 : exec = plant
 grd2 : plant1 ∈ Closed2Closed(Rzero, t) \ dom(plantV) → RReal
 grd3 : ode(f_evol_plantV(ctrlV), plant1(t), t) ∈ DE(RReal)
 grd4 : Solvable(Closed2Closed(Rzero, t) \ dom(plantV),
 ode(f_evol_plantV(ctrlV), plant1(t), t))
 AppendSolutionBAP(ode(f_evol_plantV(ctrlV), plant1(t), t),
 Closed2Closed(Rzero, t) \ dom(plantV),
 Closed2Closed(Rzero, t) \ dom(plantV), plant1)

WITH

e : e = ode(f_evol_plantV(ctrlV), plant1(t), t)

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THEN
  act1 : plantV:=plantV◁plant1
  act2 : exec:=ctrl
END

Ctrl ≐
STATUS
  ordinary
ANY
  value
WHERE
  grd1 : exec = ctrl
  grd2 : value∈RReal
  grd3 :
    ∀x. x∈ PROP ⇒
      (value≠ prop_evade_values(x)
      ⇒(prop_safe(x))(plantV(t)⇒value) = TRUE)
THEN
  act1 : ctrlV :=value
  act2 : exec := prg
END

END

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CONTEXT**TimeTriggered_Ctx****EXTENDS****EventTriggered_Ctx****CONSTANTS**

epsilon

prop_safeEpsilon

AXIOMS*axm1* : epsilon \in TIME \wedge sigma \Rightarrow epsilon \in leq*axm2* : prop_safeEpsilon \in PROP \rightarrow ((RReal \times RReal) \rightarrow B00L)*axm3* : Rzero \Rightarrow epsilon \in lt**END**

MACHINE

TimeTriggered_M

REFINES

EventTriggered_M

SEES

TimeTriggered_Ctx

Theorems

VARIABLES

t

plantV

ctrlV

exec

EVENTS**INITIALISATION** \triangleq

extended

STATUS

ordinary

BEGIN*act1* : $t := Rzero$ *act2* : $plantV := \{Rzero \mapsto plantV0\}$ *act3* : $ctrlV : \in RReal$ *act4* : $exec := ctrl$ **END****Progress** \triangleq

extended

STATUS

ordinary

REFINES

Progress

ANY*t1***WHERE***grd1* : $exec = prg$ *grd2* : $t1 \in TIME \wedge (t \mapsto t1 \in lt \wedge minus(t1 \mapsto t) \mapsto sigma \in geq)$ $\forall x. x \in PROP \Rightarrow$ *grd3* : $(ctrlV \notin prop_evade_values(x) \Rightarrow$
 $(prop_evt_trig(x))(plantV(t) \mapsto minus(t1 \mapsto t) \mapsto ctrlV) = TRUE)$ *grd4* : $t1 \in TIME \wedge (t \mapsto t1 \in lt) \wedge minus(t1 \mapsto t) \mapsto sigma \in geq \wedge minus(t1 \mapsto t) \mapsto epsilon \in leq$ **THEN***act1* : $t := t1$ *act2* : $exec := plant$ **END****Plant** \triangleq

extended

STATUS

ordinary

REFINES

Plant

ANY*plant1***WHERE***grd1* : $exec = plant$ *grd2* : $plant1 \in Closed2Closed(Rzero, t) \setminus dom(plantV) \rightarrow RReal$ *grd3* : $ode(f_evol_plantV(ctrlV), plant1(t), t) \in DE(RReal)$ *grd4* : $Solvable(Closed2Closed(Rzero, t) \setminus dom(plantV),$
 $ode(f_evol_plantV(ctrlV), plant1(t), t))$ *grd5* : $AppendSolutionBAP(ode(f_evol_plantV(ctrlV), plant1(t), t),$
 $Closed2Closed(Rzero, t) \setminus dom(plantV),$
 $Closed2Closed(Rzero, t) \setminus dom(plantV), plant1)$ **THEN***act1* : $plantV := plantV \smallfrown plant1$ *act2* : $exec := ctrl$

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END

Ctrl ≐
  extended
STATUS
  ordinary
REFINES
  Ctrl
ANY
  value
WHERE
  grd1 : exec = ctrl
  grd2 : value ∈ ℝReal
        ∀x. x ∈ PROP ⇒
  grd3 : (value ∉ prop_evade_values(x)
        ⇒ (prop_safe(x))(plantV(t) ↦ value) = TRUE)
        ∀x. x ∈ PROP ⇒
  grd4 : (value ∉ prop_evade_values(x)
        ⇒ (prop_safeEpsilon(x))(plantV(t) ↦ value) = TRUE)
THEN
  act1 : ctrlV := value
  act2 : exec := prg
END

END

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CONTEXT**Theorems****AXIOMS**

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axm1 :  $\forall a, b, c, d. a \mapsto b \in \text{leq} \wedge c \mapsto d \in \text{leq} \Rightarrow \text{plus}(a \mapsto c) \mapsto \text{plus}(b \mapsto d) \in \text{leq}$ 
axm2 :  $\forall a, b, c, d. \text{Rzero} \mapsto a \in \text{leq} \wedge \text{Rzero} \mapsto b \in \text{leq} \wedge \text{Rzero} \mapsto c \in \text{leq} \wedge \text{Rzero} \mapsto d \in \text{leq} \wedge a \mapsto b \in \text{leq} \wedge c \mapsto d \in \text{leq} \Rightarrow \text{times}(a \mapsto c) \mapsto \text{times}(b \mapsto d) \in \text{leq}$ 
axm3 :  $\forall a, b, c. a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{leq} \Rightarrow a \mapsto c \in \text{leq}$ 
axm4 :  $\forall a, b. a \in \text{RReal} \wedge b \in \text{RReal} \Rightarrow$ 
       $\text{minus}(\text{times}(a \mapsto a) \mapsto \text{times}(b \mapsto b)) = \text{times}(\text{plus}(a \mapsto b) \mapsto \text{minus}(a \mapsto b))$ 
axm5 :  $\forall a. a \in \text{RReal} \Rightarrow \text{uminus}(a) = \text{minus}(\text{Rzero} \mapsto a)$ 
       $\forall a. a \in \text{RReal} \Rightarrow$ 
       $a = \text{plus}(\text{times}(\text{divide}(\text{Rone} \mapsto \text{Rtwo}) \mapsto a)$ 
axm6 :  $\mapsto \text{times}(\text{divide}(\text{Rone} \mapsto \text{Rtwo}) \mapsto a)$ 
       $)$ 
       $\forall a, b. a \in \text{RReal} \wedge b \in \text{RReal} \wedge \text{times}(a \mapsto b) \in \text{RRealStar}$ 
axm7 :  $\Rightarrow \text{inverse}(\text{times}(a \mapsto b)) = \text{times}(\text{inverse}(a) \mapsto \text{inverse}(b))$ 

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END

CONTEXT**Desolve****EXTENDS****TimeTriggered_Ctx****CONSTANTS****B_desolve****prop****AXIOMS****axm1** : $B_desolve \in N \times RReal \times (TIME \leftrightarrow RReal) \times TIME \times (TIME \times RReal) \rightarrow (RReal \leftrightarrow RReal)$ **axm2** : $prop \in RReal \rightarrow BOOL$ **axm3** : $prop(plantV0)=TRUE$ **END**

MACHINE

TimeTriggered_desolve_M

REFINES

TimeTriggered_M

SEESDesolve
Theorems**VARIABLES**t
plantV
ctrlV
exec**INVARIANTS**inv1 : $\forall x. x \in \text{dom}(\text{plantV}) \Rightarrow \text{prop}(\text{plantV}(x)) = \text{TRUE}$ **EVENTS****INITIALISATION** \triangleq

extended

STATUS

ordinary

BEGINact1 : $t := \text{Rzero}$
act2 : $\text{plantV} := \{\text{Rzero} \mapsto \text{plantV0}\}$
act3 : $\text{ctrlV} : \in \text{RReal}$
act4 : $\text{exec} := \text{ctrl}$ **END****Progress** \triangleq

extended

STATUS

ordinary

REFINES

Progress

ANY

t1

WHEREgrd1 : $\text{exec} = \text{prg}$
grd2 : $t1 \in \text{TIME} \wedge (t \mapsto t1 \in \text{lt} \wedge \text{minus}(t1 \mapsto t) \mapsto \text{sigma} \in \text{geq})$
 $\forall x. x \in \text{PROP} \Rightarrow$
grd3 : $(\text{ctrlV} \notin \text{prop_evade_values}(x) \Rightarrow$
 $(\text{prop_evt_trig}(x))(\text{plantV}(t) \mapsto \text{minus}(t1 \mapsto t) \mapsto \text{ctrlV}) = \text{TRUE})$
grd4 : $t1 \in \text{TIME} \wedge (t \mapsto t1 \in \text{lt}) \wedge \text{minus}(t1 \mapsto t) \mapsto \text{sigma} \in \text{geq} \wedge \text{minus}(t1 \mapsto t) \mapsto \text{epsilon} \in \text{leq}$ **THEN**act1 : $t := t1$
act2 : $\text{exec} := \text{plant}$ **END****Plant** \triangleq **STATUS**

ordinary

REFINES

Plant

ANYplant1
lastTime**WHERE**grd1 : $\text{exec} = \text{plant}$
grd2 : $\text{lastTime} \in \text{TIME} \wedge \text{dom}(\text{plantV}) = \text{Closed2Closed}(\text{Rzero}, \text{lastTime})$
grd3 : $\text{plant1} = \text{B_desolve}(1 \mapsto \text{ctrlV} \mapsto \text{plantV} \mapsto t \mapsto (\text{lastTime} \mapsto \text{plantV}(\text{lastTime})))$
grd4 : $\text{plant1} \in \text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}) \rightarrow \text{RReal}$
grd5 : $\text{ode}(\text{f_evol_plantV}(\text{ctrlV}), \text{plant1}(t), t) \in \text{DE}(\text{RReal})$
grd6 : $\text{Solvable}(\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}),$
 $\text{ode}(\text{f_evol_plantV}(\text{ctrlV}), \text{plant1}(t), t))$
grd7 : $\text{AppendSolutionBAP}(\text{ode}(\text{f_evol_plantV}(\text{ctrlV}), \text{plant1}(t), t),$
 $\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}),$

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Closed2Closed(Rzero, t)\dom(plantV), plant1)
  grd8 :  $\forall x \cdot x \in \text{dom}(\text{plant1}) \Rightarrow \text{prop}(\text{plant1}(x)) = \text{TRUE}$ 
THEN
  act1 :  $\text{plantV} := \text{plantV} \ast \text{plant1}$ 
  act2 :  $\text{exec} := \text{ctrl}$ 
END

Ctrl  $\triangleq$ 
  extended
STATUS
  ordinary
REFINES
  Ctrl
ANY
  value
WHERE
  grd1 :  $\text{exec} = \text{ctrl}$ 
  grd2 :  $\text{value} \in \text{RReal}$ 
           $\forall x \cdot x \in \text{PROP} \Rightarrow$ 
  grd3 :  $(\text{value} \notin \text{prop\_evade\_values}(x))$ 
           $\Rightarrow (\text{prop\_safe}(x))(\text{plantV}(t) \ast \text{value}) = \text{TRUE}$ 
           $\forall x \cdot x \in \text{PROP} \Rightarrow$ 
  grd4 :  $(\text{value} \notin \text{prop\_evade\_values}(x))$ 
           $\Rightarrow (\text{prop\_safeEpsilon}(x))(\text{plantV}(t) \ast \text{value}) = \text{TRUE}$ 
THEN
  act1 :  $\text{ctrlV} := \text{value}$ 
  act2 :  $\text{exec} := \text{prg}$ 
END
END

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